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OF

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By JOHN C. TRAUTWINE, C.E.,

AUTHOR OF "THE MEASUREMENT AND COST OF EARTHWORK," "THE FIELD PRACTICE OF LAYING OUT CIRCULAR CURVES FOR RAILROADS," ETC.

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Twelfth Edition, Twenty-eighth Thousand.

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THE LATE

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JOHN WILEY & SONS,
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PREFACE TO FIRST EDITION.

SHOULD experts in engineering complain that they do not find anything of interest in this volume, the writer would merely remind them that it was not his intention that they should. The book has been prepared for *young* members of the profession; and one of the leading objects has been to elucidate in plain English, a few important elementary principles which the savants have enveloped in such a haze of mystery as to render pursuit hopeless to any but a confirmed mathematician.

Comparatively few engineers are good mathematicians; and in the writer's opinion, it is fortunate that such is the case; for nature rarely combines high mathematical talent, with that practical tact, and observation of outward things, so essential to a successful engineer.

There have been, it is true, brilliant exceptions; but they are very rare. But few even of those who have been tolerable mathematicians when young, can, as they advance in years, and become engaged in business, spare the time necessary for retaining such accomplishments.

Nearly all the scientific principles which constitute the foundation of civil engineering are susceptible of complete and satisfactory explanation to any person who *really* possesses only so much elementary knowledge of arithmetic and natural philosophy as is *supposed* to be taught to boys of twelve or fourteen in our public schools.*

* Let two little boys weigh each other on a platform scale. Then when they balance each other on their board see-saw, let them see (and measure for themselves) that the lighter one is farther from the fence-rail on which their board is placed, in the same proportion as the heavier boy outweighs the lighter one. They will then have learned the grand principle of the *lever*. Then let them measure and see that the light one see-saws farther than the heavy one, in the same proportion; and they will have acquired the principle of *virtual velocities*. Explain to them that *equality of moments* means nothing more than that when they seat themselves at their mea-

The little that is beyond this, might safely be intrusted to the savants. Let them work out the results, and give them to the engineer *in intelligible language.* We could afford to take their words for it, because such things are their specialty; and because we know that they are the best qualified to investigate them. On the same principle we intrust our lives to our physician, or to the captain of the vessel at sea. Medicine and seamanship are their respective specialties.

If there is any point in which the writer may hope to meet the approbation of proficients, it is in the accuracy of the tables. The pains taken in this respect have been very great. Most of the tables have been entirely recalculated expressly for this book; and one of the results has been the detection of a great many errors in those in common use. He trusts that none will be found exceeding one, or sometimes two, in the last figure of any table in which great accuracy is required. There are many errors to that amount, especially where

---

ured distances on their see-saw, *they balance each other.* Let them see that the weight of the heavy boy, when multiplied by his distance in feet from the fence-rail amounts to just as much as the weight of the light one when multiplied by his distance. Explain to them that each of the amounts is in *foot-pounds.* Tell them that the lightest one, because he see-saws so much faster than the other, will bump against the ground just as hard as the heavy one; and that this means that *their momentums are equal.* The boys may then go in to dinner, and probably puzzle their big lout of a brother who has just passed through college with high honors. They will not forget what they have learned, for they learned it *as play*, without any ear-pulling, spanking, or keeping in. Let their bats and balls, their marbles, their swings, &c, once become their philosophical apparatus, and children may be taught (*really* taught) many of the most important principles of engineering before they can read or write.

It is the ignorance of these principles, so easily taught even to children, that constitutes what is popularly called "THE PRACTICAL ENGINEER;" which, in the great majority of cases, means simply an ignoramus, who blunders along without knowing any other reason for what he does, than that he has seen it done so before. And it is this same ignorance that causes employers to prefer this practical man to one who is conversant with principles. They, themselves, were spanked, kept in, &c, when boys, because they could not master leverage, equality of moments, and virtual velocities, enveloped in x's, p's, Greek letters, square-roots, cube-roots, &c, and they naturally set down any man as a fool who could. They turn up their noses at science, not dreaming that the word means simply, *knowing why.* And it must be confessed that they are not altogether without reason; for the savants appear to prepare their books with the express object of preventing purchasers, (they have but few *readers,*) from learning why.

the recalculation was very tedious, and where, consequently, interpolation was resorted to. They are too small to be of practical importance. He knows, however, the almost impossibility of avoiding larger errors *entirely;* and will be glad to be informed of any that may be detected, except the final ones alluded to, that they may be corrected in case another edition should be called for. Tables which are absolutely reliable, possess an intrinsic value that is not to be measured by money alone. With this consideration the volume has been made a trifle larger than would otherwise have been necessary, in order to admit the stereotyped sines and tangents from his book on railroad curves. These have been so thoroughly compared with standards prepared independently of each other, that the writer believes them to be absolutely correct.

In order to reduce the volume to pocket-size, smaller type has been used than would otherwise have been desirable.

Many abbreviations of common words in frequent use have been introduced, such as abut, cen, diag, hor, vert, pres, &c, instead of abutment, center, diagonal, horizontal, vertical, pressure, &c. They can in no case lead to doubt; while they appreciably reduce the thickness of the volume.

Where prices have been added, they are placed in footnotes. They are intended merely to give an approximate or comparative idea of value; for constant fluctuations prevent anything farther.

The addresses of a few manufacturing establishments have also been inserted in notes, in the belief that they might at times be found convenient. They have been given without the knowledge of the proprietors.

The writer is frequently asked to name good elementary books on civil engineering; but regrets to say that there are very few such in our language. "Civil Engineering," by Prof. Mahan of West Point; "Roads and Railroads," by the late Prof. Gillespie; and the "Manual for Railroad Engineers," by George L. Vose, Civ. Eng, and Professor of Civil Engineering in Bowdoin College, Brunswick, Maine, are

the best. The last, published by Lee & Shepard, Boston, 1873, is the most complete work of its class with which the writer is acquainted.

Many of Weale's series are excellent. Some few of them are behind the times; but it is to be hoped that this may be rectified in future editions. Among pocket-books, Haswell, Hamilton's Useful Information, Henck, Molesworth, Nystrom, Weale, &c, abound in valuable matter.

The writer does not include Rankine, Moseley, and Weisbach, because, although their books are the productions of master-minds, and exhibit a profundity of knowledge beyond the reach of ordinary men, yet their language also is so profound that very few engineers can read them. The writer himself, having long since forgotten the little higher mathematics he once knew, cannot. To him they are but little more than striking instances of how completely the most simple facts may be buried out of sight under heaps of mathematical rubbish.

Where the word "ton" is used in this volume, it always means 2240 lbs, because that is its meaning in U. S. law.

JOHN C. TRAUTWINE.

PHILADELPHIA, September, 1876.

PREFACE TO THE TWENTY-SECOND THOUSAND.

SINCE the appearance of its last edition (the twentieth thousand) in 1883, the "Pocket-Book" has been thoroughly revised, and many important additions and other alterations have been made. These necessitated considerable change in the places of the former matter, and it was deemed best to turn this necessity to advantage, and to make a thorough re-arrangement, putting all of the articles, as far as possible, in a rational order, so that the reader might, in many cases, be able to readily find a desired subject without the use of the index, which, notwithstanding, has been greatly enlarged. All of the articles formerly in the appendix, and such of those in the glossary as seemed out of place there, have been transferred to their proper places in the body of the work.

In making the re-arrangement, the rule has been to proceed from the abstract to the concrete, from the theoretical to the practical, from the general to the particular. All this will appear from a glance at the new and very full table of contents on pages xxiii to xxxii. Thus, beginning with Arithmetic, we proceed to Mensuration of lines, angles, surfaces, solids, the simple, in each case, preceding the complex. Next come the various branches of Surveying, with surveying and engineering instruments. Then Sound, Heat, Air and Water, Hydrostatics, Hydraulics, Water Supply, and Mechanics. Next, under Materials and their Properties, we have Specific Gravity, Weight, Weights and Measures, Weights, Dimensions, Prices, etc, of many manufactured and other articles, and Strength of Materials. This last leads naturally to strengths of beams and trusses, after which follow Suspension Bridges. Proceeding next to subjects more closely connected with actual construction, and in which theory plays a less important part, we come to Well Boring, Dredging, Foundations, and Stonework, the latter including Quarrying and Masonry, Bricks, Mortar, Cements and Concretes, Retaining Walls, and Arches of brick and stone. Then follows the article on Railroads, under which head have been grouped, in order, Railroad Construction (including Grades, Curves, Earthwork, Tunnels, Trestles, Roadway, and Turnouts), Railroad Equipment (including Turntables, Water Stations, &c), Railroad Rolling-Stock and Operation, and Railroad Statistics. By the transfer to the body of the book of the various articles in the appendix, the glossary comes to its proper place at the end of the volume, just before the index.

It is believed that this thorough re-arrangement will, in many cases, enable the reader to find a subject more readily by the aid of the table

of contents than by that of the index; or, indeed, as already suggested, without the aid of either, and by the natural place of the subject in the order of arrangement.

The following are some of the principal changes of subject matter in this edition. (The former pages of revised articles are given in parentheses.) On page 34, a table of Fractions reduced to exact decimals has been added. On pages 128 to 140, are two new Tables of Circumferences and Areas of Circles, to not less than four places of decimals. These tables have been calculated independently by two persons, expressly for this work, and the two results compared after the casting of the plates. They are therefore believed to be correct.* The rules for finding chords, radii, &c, of Circular Arcs, p 141 (16, 17) have been greatly extended, re-modeled, re-illustrated, and systematized. Under Thermometers, pp 213, &c (309, &c), the rules for conversion of Fahrenheit to Centigrade readings, and *vice versa*, have been put into more convenient shape, and others added for the conversion of both of these scales to that of Réaumur and *vice versa*. In consonance with this change, and to make this subject complete, three new Conversion Tables have been substituted for the one formerly given. The article on Flotation, &c, p 235 (635) has been corrected and amplified. The opening articles in Hydraulics, pp 237, &c (535, &c), in which are explained the divisions of the total head and their several offices, have been re-written, and it is believed they will be found to give a fuller and more satisfactory explanation than before, of the various phenomena connected with this subject. On p 254 will be found a new formula for the discharge through a pipe of varying diameter. The remarks on City Water-Pipes, Valves and Fire-Hydrants, pp 293 to 305 (572 to 577) have been re-written, enlarged, and modernized. New forms of Pipe-Joint, Stop-Valve, and Fire-Hydrant, and several modern appliances, are described and illustrated; and recent data are given for the Cost of Pipe and Laying.

* And as nearly accurate as their number of decimal places permits, except that if 1 be added to the final decimal of the circumferences and areas corresponding to the following diameters, their error in excess will be a little less than their present error in deficiency. (*Here followed, in the 22d thousand, a list of these diameters.*) The greatest error in these is now *less than* 1 *in the final decimal.* It will then be *less than* ½, as in all the others. (*These trifling inaccuracies have been corrected for the 25th thousand.*)

On p 364 (587) the Velocities, &c, of Falling Bodies are tabulated in convenient form. The remarks on Centrifugal Force, p 366 (494), have been revised, and the explanation made clearer. On p 396 will be found a concise but full account of the arrangement of the new Standard Railway Time. The various articles on pp 398 to 433 (357 to 382) in which are given the Dimensions, Weights, Prices, &c, of various manufactured articles in common use, have been thoroughly overhauled, and corrected up to the present time. Among these may be noted especially Iron Pipes, p 405 (364) to which Boiler Tubes have been added; Bolts, Nuts, and Washers, pp 406 to 408 (374 to 376); a new list of Wire Gauges, giving the new British Standard Gauge, p 410 (368); Wire Ropes, p 413 (380); Tin Plates, p 419 (379); and Window-Glass, p 432 (514).

On p 435 (174) is a new table of crushing loads of American woods, deduced from the last census. The former table was based upon Hodgkinson's experiments, made chiefly with foreign woods. The coefficients for transverse strengths of timber, p 493 (185), have also been changed, where necessary, to correspond with results of the census experiments.

On p 449 (234) is a revised and complete list of Phœnix Segment Columns, illustrated. A number of changes have been made in Rivets and Riveting, pp 468, &c (653, &c).

Under Transverse Strength, pp 478, &c (183, &c), have been grouped the several articles on Moments of Rupture, of Inertia, and of Resistance, Open and Closed Beams, Shearing of Beams, &c, which were formerly scattered through widely separated parts of the book (183, &c, 194, &c, 217, &c, 642 to 650). These have been carefully revised and amended, and arranged into a systematic whole, from which the student can now obtain a clear and correct notion of this somewhat troublesome subject. On pp. 521 to 523 (211 to 213) are new rules and tables for I and channel beams; and, on pp 525 to 527 (373) similar rules and tables for angle and T iron. These have been carefully compiled from the latest tables of the Pencoyd Works, which were selected on account of the large range of sizes made by those works, and the completeness of the data obtainable in regard to them. On p 524 (304) are illustrations of recent bridges of I beams in use on the Penna R R. The article on Shearing of (or vertical strains in) Beams, p 532 (642), is entirely rewritten and re-illustrated. A new article on Riveted Girders is given, pp 537 to 546 (214 to 217), in which much fuller rules, used in the best practice of to-day, are substituted for those of Fairbairn; and illustrations from the present standards of the Penna R R are given in place of those from the Charing Cross Railway of a quarter of a century ago.

In Trusses, the remarks on Counter-bracing, pp 564 to 570 (275 to 281), the treatment of the Fink Roof Truss, pp 574, &c (264, &c), and that of the Braced Arch, p 592 (274), have been corrected and simplified. Increased dimensions have been given in the tables of Wooden Howe

and Pratt Trusses, pp 595 and 596 (284 and 285), to provide for the modern weights of such loads as would probably be placed upon such structures.

Considerable changes are made in Suspension Bridges, pp 615 to 625 (588 to 597). The formulæ for the strains in the main chains have been re-arranged, and those for the strains in the back-stays and on the piers have been much extended and illustrated, and made to cover the ground more fully. The description of the Pierce Well-borer, p 626 (636), has been corrected, other boring tools described and illustrated, and a new article on Artesian Well-Drilling added. The Nasmyth Steam-Hammer Pile-Driver is described on p 642. The articles on Stonework, pp 651 to 668 (310 to 313) have been greatly enlarged by the addition of new articles on Machine Rock-Drills, Air Compressors, and Modern Explosives. In the first named, sectional views and descriptions of several of the more prominent modern drills are given.

Under Railroads, pp 722 to 818 (409 to 428, &c), will be found collected most, or all, of the former matter of the book, relating exclusively to that subject, and much that is new. A Table of Curves, with Radii, &c, in *metres*, has been added, on p 728, for the convenience of those who have occasion to work by the metric system. Under Earthwork, pp 747, &c (418 to 428, 435 to 441), original rules and tables have been added for estimating the cost of moving earth by means of the modern wheeled and drag scrapers, and that of moving earth and rock by cars and locomotives. In this connection, also, the operation of the modern steam excavators, or land dredges, is described, and data given concerning their capacity and their cost of operation.

The article on Trestles, p 755, &c (307, &c), is greatly enlarged by descriptions of the new Portage, Kinzua, and other recent *iron* trestles. The former articles (390 to 408) on Rail-joints and Turnouts have been replaced by new, enlarged, and re-illustrated articles, pp 763 to 789, based upon present practice. Under Rail-joints the principal place has been given to the fish and angle plates now so generally used, after which follow the modern Fisher and Gibbon joints. The older forms, including most of those illustrated in former editions, are briefly referred to for purposes of comparison. Under Turnouts, the lengthy details of the stub-switch stand, &c, are omitted, and the point and Wharton switches are given greater prominence. Several important improvements having been made in the Wharton since our last edition (in which a new description of that switch was given), a further revision became necessary, and the new article is accompanied by a new cut, showing the switch in its latest form. Under Frogs, prominence is given to the Steel-Rail Frog, which has now supplanted the older forms on first-class lines; the Mansfield Elastic Frog is described and illustrated; and an explanation of the working of the Spring-Rail Frog is added, with original cuts. The directions for laying out turnouts from curves have been simplified.

Under Turntables, the description of the Sellers table has been re-written and brought into line with recent improvements; and several modern forms of plate and cast-iron turntables are described and illustrated. The remarks on Track-Tanks, p 802 (434), have been extended, and a cut added, showing a cross-section of the Penna R R standard tank.

The scattered data (411 to 413) on Locomotives and Cars, and the Railroad Statistics formerly given (409, 410) have made room for tabulated modern data, pp 805 to 818, on these subjects, much fuller than those which they have replaced, and covering both standard and narrow-gauge roads.

The following are among the more important of the minor changes.

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55	Exterior Angles	62
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149 &c	Ellipse	25, 26, 630
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220	Records of Rain-falls	518
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384	Foot-Note.	
425	Creosote, &c	359 to 362
433	Paper	151, 152
436	Crushing Loads of Masonry and of Ice (foot-notes)	175
438, 439	Iron Pillars	222, 223, 234
504 to 506	Deflections, &c, of Beams	196 to 198
572	First paragraph	262
573	Foot-Note	263
710	Brick Arches, Penna R R, Phila.	
803	Track Scales	409

A number of minor additions have been made to the glossary.

Most of the new matter is in nonpareil, the *larger* of the two types heretofore used. **Boldfaced** type has been freely used; but only for the purpose of guiding the reader rapidly to a desired division of a subject. For *emphasis*, *italics* have been employed.

Illustrations which were lacking in clearness or neatness have been re-touched and re-lettered, or replaced with new and better cuts. The new matter is very freely illustrated.

New rules have been put in the shape of formulæ, and many of the

old rules have been re-cast into the same form. In doing this, the terms of the formulæ have, except where this would be exceedingly cumbersome, been written out in full, as in former editions, so that the reader is not compelled to look back over a number of pages to find the meaning of arbitrary symbols, and to tax his mind with remembering them when found. It is believed that the formulæ will be found at least as easy to use as the rules. Take, for example, the second formula in Art 10, p 262 (556). In its former shape it read:

"To find the time reqd to fill the reservoir, m, from any level, c, above the top of the opening, to any upper level, d." (This should have read "to the level a of the upper reservoir.")

"RULE. First find the area in sq ft of a hor section of the reservoir m, which is supposed to be of uniform section throughout its depth. Mult together this area, the constant number 2, and the sq rt of the vert height, $a\,c$, in ft. Call the prod p. Mult together the area of the opening o, in sq ft; the coeff of contraction (usually about .62, whether the disch be into the air or under water); and the constant 8.02. Call the prod y. Div p by y. The quot will be the reqd time in secs."

It is safe to say that any one, capable of using the above rule, can as readily use the following formula, in which it now appears

$$\text{Seconds required to } \textit{raise level} \text{ in } m \text{ from } c \text{ to } a = \frac{\sqrt{\text{height } a\,c \text{ in ft}} \times \text{hor area of } m \text{ in sq ft} \times 2}{\text{area of opening } o \text{ in sq ft} \times .62 \times 8.03}$$

and, moreover, the formula has the great advantages of showing the whole operation at a glance, of making its *principle* more apparent and of being much more convenient for reference. The formula might have been slightly condensed by giving the *product* of $.62 \times 8.03$, instead of the *factors;* but it was preferred to give the latter because they show the principle of the formula. The coefficient .62 had already been explained. The article referred to has also been otherwise improved.

The addition of new matter, and a number of blank spaces necessarily left in making the re-arrangement, have increased the number of pages about one-fifth.

The new index is in stricter alphabetical order than that of former editions, and contains more than twice as many entries, although much repetition has been avoided by the free use of cross-references, without which this part of the work might have been indefinitely extended.

The selection of articles of manufacture or merchandise for illustration, has been guided by no other consideration than their fitness for the purpose, and the courtesy of the parties representing them, in supplying information.

The writer gratefully acknowledges the kindness of those who have assisted in furnishing and arranging data.

J. C. T., JR.

PHILADELPHIA, January, 1885.

PREFACE TO THE TWENTY-FIFTH THOUSAND.

For this edition, the articles on Flow of Water in Channels, pp 268 etc., Friction, pp 370 etc., and Timber Preservation, pp 425 etc., have been re-written. In the first named, Kutter's formula is given, with tables to facilitate its use, and instructions for preparing a diagram from which its results may be taken by inspection. Under Friction are given the results of recent researches, including the now famous experiments of Capt. Douglas Galton on brake friction. The new article on Timber Preservation embodies, besides other matter, results recently published by a Committee of the American Society of Civil Engineers.

Prices and descriptions of manufactured articles etc., have been revised to date.

A number of changes have been made in the articles on Force in Rigid Bodies, and Trusses, in order to further simplify the treatment of those subjects. These changes include the enlargement, and the re-arrangement in more convenient form, of the articles on Gravity and Falling Bodies, Descent on Inclined Planes, Angular Velocity, Moment of Inertia, Radius of Gyration and Centrifugal Force.

On p 230 *a*, the subject of Distribution of Pressure in Plane Surfaces is explained more fully than in earlier editions. The empirical formula, p 243, for flow of water in pipes, has been so modified as to give more closely approximate results. An article on the Fatigue of Materials has been added, p 435. The rules for Strengths of Pillars, pp 439, etc., and those for Limited Deflections of Beams, pp 510 etc., have been simplified. On p 678 is given a summary of the results of Mr. Eliot C. Clarke's recent experiments on the Strength etc. of Cements. Many other minor improvements have been made.

Advantage has been taken of these changes to still further extend the substitution of the larger for the smaller type.

J. C. T. Jr.

Philadelphia, April, 1886.

PREFACE TO THE TWENTY-SEVENTH THOUSAND.

THE present edition contains revised formulæ for thicknesses of cylinders under internal pressure, p. 232, a revised table of thicknesses for cast-iron pipe, p. 233, additional tables of coefficient "c" by Kutter's formula, pp. 275 etc., and a new and fuller table of values of foreign coins, p. 386. Mr. Pegram's suggested uniform loading for railroad bridges, to be substituted for the usual wheel loads in specifications, is given on p. 546. The formulæ for approximate weights of truss bridges, p. 605, have been revised, and the table of dimensions and weights of large locomotives, p. 807, has been made to include some still heavier engines now in use. For minor changes see pp. 154, 155, 156, 159, 192, 358, 368, 397, 432 etc. About thirty pages more of the smaller type have been replaced by the larger.

J. C. T., JR.

PHILADELPHIA, March, 1887.

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In many cases, a subject may be found more quickly by means of this table of contents, than by the index, page 833. See also Glossary, page 819.

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THE

CIVIL ENGINEER'S POCKET-BOOK.

ARITHMETIC.

On this subject we shall merely give a few examples for refreshing the memory of those who for want of constant practice cannot always recall the processes at the moment.

Subtraction of Vulgar Fractions.

$\frac{1}{2} - \frac{1}{2} = 0.$ $\frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}.$ $\frac{6}{8} - \frac{2}{8} = \frac{4}{8} = \frac{1}{2}.$ $\frac{3}{4} - \frac{5}{9} = \frac{27}{36} - \frac{20}{36} = \frac{7}{36}.$

$3\frac{4}{7} - 1\frac{2}{3} = \frac{25}{7} - \frac{5}{3} = \frac{75}{21} - \frac{35}{21} = \frac{40}{21} = 1\frac{19}{21}.$

Addition of Vulgar Fractions.

$\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1.$ $\frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1.$ $\frac{6}{8} + \frac{2}{8} = \frac{8}{8} = 1.$ $\frac{3}{4} + \frac{5}{9} = \frac{27}{36} + \frac{20}{36} = \frac{47}{36} = 1\frac{11}{36}.$

$3\frac{4}{7} + 1\frac{2}{3} = \frac{25}{7} + \frac{5}{3} = \frac{75}{21} + \frac{35}{21} = \frac{110}{21} = 5\frac{5}{21}.$

$\frac{2}{3} + \frac{3}{5} + \frac{8}{4} = \frac{40}{60} + \frac{36}{60} + \frac{120}{60} = \frac{196}{60} = 3\frac{16}{60} = 3\frac{4}{15}.$

Multiplication of Vulgar Fractions.

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}.$ $\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}.$ $\frac{6}{8} \times \frac{2}{8} = \frac{12}{64} = \frac{3}{16}.$ $\frac{3}{4} \times \frac{5}{9} = \frac{15}{36} = \frac{5}{12}.$

$3\frac{4}{7} \times 1\frac{2}{3} = \frac{25}{7} \times \frac{5}{3} = \frac{125}{21}.$ $\frac{2}{3} \times \frac{3}{5} \times \frac{8}{4} = \frac{48}{60} = \frac{4}{5}.$

$\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{5}{3}$ of $\frac{7}{5} = \frac{3}{4} \times \frac{1}{2} \times \frac{5}{3} \times \frac{7}{5} = \frac{105}{120} = \frac{21}{24} = \frac{7}{8}.$

Division of Vulgar Fractions.

$\frac{1}{2} \div \frac{1}{2} = \frac{2}{2} = 1.$ $\frac{3}{4} \div \frac{1}{4} = \frac{12}{4} = \frac{3}{1} = 3.$ $\frac{6}{8} \div \frac{2}{8} = \frac{48}{16} = \frac{24}{8} = \frac{12}{4} = \frac{3}{1} = 3.$ $\frac{3}{4} \div \frac{5}{9} = \frac{27}{20} = 1\frac{7}{20}.$

$3\frac{4}{7} \div 1\frac{2}{3} = \frac{25}{7} \div \frac{5}{3} = \frac{75}{35} = \frac{15}{7} = 2\frac{1}{7}.$ $5 \div \frac{7}{8} = \frac{5}{1} \div \frac{7}{8} = \frac{40}{7} = 5\frac{5}{7}.$

To find the greatest common divisor of a Vulgar Fraction.

Ex. 1. Of $\frac{70}{175}$.

70)175(2
140

35)70(2
70
Ans 35.

Ex. 2. Of $\frac{84}{20}$.

20)84(4
80

4)20(5
20
Ans 4.

To reduce a Vulgar Fraction to its lowest terms.

First find the greatest common divisor; then divide both the numerator and denominator by it. Thus, in the preceding example $\frac{70}{175} = \frac{2}{5}$ Ans. And $\frac{84}{20} = \frac{21}{5}$ Ans.

To reduce a Vulgar Fraction to a decimal form.

Divide the numerator by the denominator. Thus,

$\frac{1}{2} = 2)1.0(0.5$ Ans.
10

$1\frac{3}{4} = 4)13(3.25$ Ans.
12
10
8
20
20

$\frac{3}{4}\frac{2}{0} = 40)32.0(0.8$ Ans.
320

Reduce 3 inches to the decimal of a foot. There are 12 ins in a foot; therefore, the question is to reduce $\frac{3}{12}$ to a decimal. Therefore, 12)3.0(0.25 of a foot. Ans.
24
60
60

Reduce 2 ft 3 ins to the decimal of a yard. There are 36 ins in a yard; and 27 ins in 2 ft 3 ins; therefore, $\frac{2}{3}\frac{7}{6}$ of a yard = 36)27.0(0.75 of a yd. Ans.
252
180
180

How many feet and ins are there in .75 of a yard? Here

.75
3 ft in a yd.
Ft 2).25
12 ins in a ft.
Ins 3.00 Ans 2 ft 3 ins.

How many feet and ins are there in .0625 of a yard?

.0625
3 feet in a yd.
No feet, .1875
12 ins in a ft.
Ins 2.2500

	ft.	ins.
Ans.	0	2.25.

How many cubic feet are there in .314 of a cub yard? And cub ins in .46 of a cub ft?

.314
27 cub ft in a yd.
2198
628
8.478 cub ft. Ans.

.46
1728 cub ins a cub ft.
368
92
322
46
794.88 cub ins. Ans.

Fractions reduced to exact decimals.

$\frac{1}{64}$	.015625	$\frac{17}{64}$	.265625	$\frac{33}{64}$	.515625	$\frac{49}{64}$	.765625
$\frac{1}{32}$	.03125	$\frac{9}{32}$	.28125	$\frac{17}{32}$	.53125	$\frac{25}{32}$	.78125
$\frac{3}{64}$	.046875	$\frac{19}{64}$	.296875	$\frac{35}{64}$	.546875	$\frac{51}{64}$	.796875
$\frac{1}{16}$	.0625	$\frac{5}{16}$	.3125	$\frac{9}{16}$	.5625	$\frac{13}{16}$	.8125
$\frac{5}{64}$	.078125	$\frac{21}{64}$	.328125	$\frac{37}{64}$	.578125	$\frac{53}{64}$	.828125
$\frac{3}{32}$	.09375	$\frac{11}{32}$	.34375	$\frac{19}{32}$	.59375	$\frac{27}{32}$	.84375
$\frac{7}{64}$	.109375	$\frac{23}{64}$	.359375	$\frac{39}{64}$	.609375	$\frac{55}{64}$	.859375
$\frac{1}{8}$	.125	$\frac{3}{8}$	.375	$\frac{5}{8}$	.625	$\frac{7}{8}$	.875
$\frac{9}{64}$	.140625	$\frac{25}{64}$	.390625	$\frac{41}{64}$	.640625	$\frac{57}{64}$	.890625
$\frac{5}{32}$	.15625	$\frac{13}{32}$	.40625	$\frac{21}{32}$	.65625	$\frac{29}{32}$	.90625
$\frac{11}{64}$	.171875	$\frac{27}{64}$	.421875	$\frac{43}{64}$	.671875	$\frac{59}{64}$	.921875
$\frac{3}{16}$	.1875	$\frac{7}{16}$	.4375	$\frac{11}{16}$	.6875	$\frac{15}{16}$	.9375
$\frac{13}{64}$	.203125	$\frac{29}{64}$	.453125	$\frac{45}{64}$	.703125	$\frac{61}{64}$	.953125
$\frac{7}{32}$	.21875	$\frac{15}{32}$	.46875	$\frac{23}{32}$	.71875	$\frac{31}{32}$	.96875
$\frac{15}{64}$	.234375	$\frac{31}{64}$	.484375	$\frac{47}{64}$	.734375	$\frac{63}{64}$	.984375
$\frac{1}{4}$	.25	$\frac{1}{2}$	5	$\frac{3}{4}$	.75	1	1.

Decimals.

ADDITION. Add together .25 and .75; also .006, 1.3472, and 43.

```
 .25                                  .006
 .75                                 1.3472
 ---                                43.
1.00  Ans.                          -------
                                    44.3532  Ans.
```

SUBTRACTION. Subtract .25 from .75; also .0001 from 1; also 6.30 from 9.01.

```
 .75          1.          9.01
 .25          .0001       6.30
 ---          -----       ----
 .50 Ans.     .9999 Ans.  2.71  Ans.
```

MULTIPLICATION. Mult 3 × .3; also .3 × .3; also .3 × .03; also 4.326 × .003.

```
 3          .3          .3          4.326
 .3         .3          .03         .003
 --         --          ---         -----
 .9 Ans.    .09 Ans.    .009 Ans.   .012978 Ans.
```

DIVISION. Divide 3. by .3; also .3 by .3; also .3 by .03; also 4.326 by .0003.

```
.3)3.0(10. Ans.    .3).3(1. Ans.    .03).30(10. Ans.    .0003)4.3260(14420. Ans.
   3                  3                 3                   3
   --                                   --                  ---
   0                                    0                   13
                                                            12
                                                            ---
                                                             12
                                                             12
                                                             ---
                                                              6
                                                              6
                                                              ---
                                                              0
```

Divide 62 by 87.042.

```
87.042)62.0000(0.712, &c. Ans.
       60.9294
       -------
        107060
         87042
        ------
        200180
```

Divide .006 by 20.

```
20.000).0060000(0.0003 Ans.
        60000
        -----
```

Duodecimals.

Duodecimals refer to square *feet* of 144 sq ins; to twelfths of a square or duodecimal foot; each such twelfth being called an *inch;* and being equal to 12 square inches; and to *twelfths*, each equal to the 12th of a duodecimal inch, or to one square inch. The dimensions of the thing to be measd are supposed to be taken in common feet, ins, and 12ths of an inch; but as ordinary measuring rules are divided into 8ths of an inch, it is usually guess-work to some extent. Duodecimals are very properly going out of use, in favor of decimals; we shall therefore give no rule for them. By means of our table of "Inches reduced to Decimals of a Foot," p. 388, all dimensions in feet, ins, and 8ths, &c, can be at once taken out in ft and decimals of a foot.

Single Rule of Three; or, Simple Proportion.

If 3 men lay 10000 bricks in a certain time, how many could 6 men lay in the same time? They will evidently lay more; therefore, the second term of the proportion must be greater than the first.

```
3 : 6 : : 10000 : 20000 Ans.
              6
         3)60000
         -----
          20000 Ans.
```

If 3 men require 10 hours to lay a certain number of bricks, how many hours would 6 men require? They will evidently require less time; therefore, the second term of the proportion must be less than the first.

```
6 : 3 : : 10 : 5 Ans.
            3
           --
         6)30
           --
           5 Ans
```

Double Rule of Three; or, Compound Proportion.

If three men can lay 4000 bricks in 2 days, how many men can lay 12000 in 3 days? Here we see that 4000 bricks require $3 \times 2 = 6$ *days' work;* therefore 12000 will require,

4000 : 12000 : : 6 : 18 days' work.

But there are only 3 days to do the 18 days work in; therefore the number of men must be $\frac{18}{3} = 6$ men. Ans.

A moment's reflection will suffice to reduce any case of double rule of three to this simple form.

Ratio.

Ratio. Simple ratio is a number denoting how often one quantity is contained in another. Thus, the ratio of 5 to 10 is $\frac{5}{10}$, or $\frac{1}{2}$; and the ratio of 10 to 5 is $\frac{10}{5}$, or 2. When, of four numbers, two have to each other the same ratio that the other two have, the numbers are said to be *in proportion* to each other. Thus, 6 has the same ratio (2) to 3, as 100 has to 50; therefore, 6, 3, 100, and 50, are said to be in proportion; or, as 6 : 3 : : 100 : 50. In other words, an *equality of ratios* is called proportion. Ratio and proportion are often confounded with one another; but the error is one of no importance. *Duplicate ratio* is that of the *squares* of numbers.

Arithmetical Progression,

In a series of numbers, is a progressive increase or decrease in each successive number, by the addition or subtraction of the same amount at each step; as in 1, 2, 3, 4, 5, &c., in which 1 is added at each step; or 10, 8, 6, 4, &c., in which 2 is subtracted at each step; or ¼, ½, ¾, 1, 1¼, &c. In any such series the numbers are called its *terms;* and the equal increase or decrease at each step its *common difference.*

To find the com diff, knowing the first and last terms; and the number of terms. Find the diff between the first and last terms. From the number of terms subtract 1. Div the diff just found, by the rem.

To find the last term, knowing the first term; the com diff; and the number of terms. From the number of terms take 1. Mult the rem by the com diff. To the prod add the first term.

To find the number of terms, having the first and last ones; and the com diff. Take the diff between the first and last terms. Div this diff by the com diff. To the quot add 1.

To find the sum of all the terms, having the first and last ones; and the number of terms. Add together the first and last terms. Div their sum by 2. Mult the quot by the number of terms.

Geometrical Progression,

In a series of numbers, is a progressive increase or decrease in each successive number, by the same multiplier or divisor at each step; as 3, 9, 27, 81, &c, where each succeeding term is increased by mult the preceding one by 3. Or 48, 24, 12, 6, &c, or 27, 13½, 6¾, 3⅜, &c, where each succeeding term is found by dividing the preceding one by 2. The multiplier or divisor is called the *common ratio* of the series, or progression.

To find the last term, knowing the first one; the ratio; and the number of terms. Raise the ratio to a power 1 less than the number of terms. Mult this power by the first term.

Ex. First term 10; ratio 3; number of terms 8; what is the last term? Here the number of terms being 8, the ratio 3 must be raised to the 7th power; thus:

$$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 2187, = 7\text{th power.}$$ And $2187 \times 10 = 21870$ last term. Ans.

A man agreed to buy 8 fine horses; paying $10 for the first; $30 for the second; $90 for the third, &c; how much will the last one cost him? Ans, $21870, as before.

To find the sum of all the terms, knowing the first one; the ratio; and the number of terms. Raise the ratio to a power equal to the whole number of terms. From this power subtract 1. Div the rem by 1 less than the ratio. Mult the quot by the first term.

Ex. As before. What is the sum of all the terms? Here the ratio must be raised to the 8th power; thus, $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 6561 = 8\text{th pow.}$ And 6560 div by 1 less than the ratio $3, = \frac{6560}{2} = 3280.$ And 3280×10 (or number of terms) $= 32800 =$ sum. Ans.

In the foregoing case, the 8 horses would cost $32800.

Permutation

Shows in how many positions any number of things can be arranged in a row. To do this, multiply together all the numbers used in counting the things. Thus, in how many positions in a row can 9 things be placed? Here,

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 = 362880 \text{ positions.}$$ Ans.

Combination

Shows how many combinations of a few things can be made out of a greater number of things. To do this, first set down that number which indicates the greater number of things; and after it a series of numbers, diminishing by 1, until there are *in all* as many as the number of the few things that are to form each combination. Then beginning under the last one, set down said number of few things; and going backward, set down another series, also diminishing by 1, until arriving under the first of the upper numbers. Mult together all the upper numbers to form one prod; and all the lower ones to form another. Div the upper prod by the lower one.

Ex. How many combinations of 4 figures each, can be made from the 9 figs 1, 2, 3, 4, 5, 6, 7, 8, 9, or from 9 any things?

$$\frac{9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4} = \frac{3024}{24} = 126 \text{ combinations.}$$ Ans.

Alligation

Shows the value of a mixture of different ingredients, when the quantity and value of each of these last is known.

Ex. What is the value of a pound of a mixture of 20 lbs of sugar worth 15 cts per lb; with 30 lbs worth 25 cts per lb?

lbs.	cts.	cts.
20	× 15 =	300
30	× 25 =	750
50 lbs.		1050 cts.

Therefore, $\frac{1050}{50} = 21$ cts. Ans.

Equation of Payments.

A owes B $1200; of which $400 are to be paid in 3 months; $500 in 4 months; and $300 in 6 months; all bearing interest until paid; but it has been agreed to pay all at once. Now, at what time must this payment be made so that neither party shall lose any interest?

$		months.		
400	×	3	=	1200
500	×	4	=	2000
300	×	6	=	1800
1200				5000

Therefore, $\frac{5000}{1200} = 4.16$, &c, months. Ans.

A owes B $1000 to be paid in 12 days; and $500 to be paid in 3 months. What would be the time for paying all at once?

$		days.		
1000	×	12	=	12000
500	×	90	=	45000
1500				57000

Therefore, $\frac{57000}{1500} = 38$ days. Ans.

Simple Interest.

What is the simple interest on $865.32 for one year, at 6 per ct per annum?

Principal.		Interest.		Principal.		Interest.
$100	:	$6	::	$865.32	:	$51.9192

6

100)5191.92(51.9192 Ans. = 51.91 92/100 $cts.

What is the interest on $865.32 for 1 year, 3 months, and 10 days, at 7 per cent per annum?

First calculate the interest for 1 year only; thus:

Prin.		Int.		Prin.		Int.
$100	:	$7	::	$865.32	:	$60.5724

7

100)6057.24(60.5724

Then say, If 1 year or 365 days give $60.5724 int, what will 465 days give? or

Days.		Int.		Days.		Int.
365	:	$60.5724	::	465	:	$77.16, &c. Ans.

At 5 per ct simple interest, money doubles itself in 20 years; at 6 per ct, in 16⅔ years; and at 7 per ct, in $14\frac{2}{7}$ years. Simple Interest is Single Rule of Three.

Compound Interest.

When money is borrowed for more than a year at compound interest, find the simple interest at the end of the first year, and add it to the principal, for a second principal. Find the simple interest on this second enlarged principal for the next year, and add it to the enlarged principal for a third principal; and so on for each successive year.

At 5 per ct compound interest, money doubles itself in about $14\frac{1}{5}$ years; at 6 per ct, in about 11.9 years; and at 7 per ct, in about 10¼ years.

Discount

Is a deduction of a part of the interest, when money at interest is paid before it is due. Or it is a deduction of the whole of the interest in advance, at the time the money is lent. In the first case, if I borrow $100 for 1 year at 8 per ct, I must at the end of the year pay back $108; but if I pay at the end of 3 months, I must add only $2, or the interest for those 3 months, paying back $102; and the diff of $6 is the discount. Therefore, to find the discount in such cases, first find the interest for the full time; then that for the short time; and take the diff.

In the second case, if I borrow $100 from a bank for one year, at 6 per ct, I receive but 100 − 6 = $94; but at the end of the year I must pay back $100. By discounting in this manner, the bank actually gains more than 6 per ct; for it gains $6 for the use of $94 for 1 year. In the United States, the banks deduct discount for 3 days more than the time stipulated in the note; these are called "days of grace." The borrower is not obliged to pay before the last of these 3 days.

Commission, or Brokerage,

Is a percentage (or so much per each $100) paid to commission merchants for selling our goods; or to brokers, or other kinds of agents, for transacting business for us. It is Single Rule of Three.

Ex. If a broker makes purchases for me to the amount of $9362, at 2 per ct, what is his brokerage? Say, as

Purchase.		Brokerage.		Purchase.		Brokerage.
$100	:	$2	::	$9362	:	$187.24

Insurance

Is a percentage (called a *premium*) paid to a company for insuring our property against fire, &c. The company, or insurers, (called also underwriters,) deliver to the person insured, a paper bearing their seal, &c, and called the *Policy of Insurance*; which contains the conditions of the transaction. Insurance is calculated like Commissions, &c.; being merely Single Rule of Three.

Fellowship.

A puts $6000 into a business in partnership with B, who puts in $9000. At the end of a year they have made $2400; how much is each one's share? Here, $6000 + $9000 = $15000 joint capital; then say,

Joint cap.		Total gain.		A's cap.		A's share.
$15000	:	$2400	::	$6000	:	$960

				B's cap.		B's share.
And	$15000	:	$2400	::	$9000	: $1440

Logarithms of Numbers, from 0 to 1000.*

No.	0	1	2	3	4	5	6	7	8	9	Prop.
0	0	00000	30103	47712	60206	69897	77815	84510	90309	95424	
10	00000	00432	00860	01283	01703	02118	02530	02938	03342	03742	415
11	04139	04532	04921	05307	05690	06069	06445	06818	07188	07554	379
12	07918	08278	08636	08990	09342	09691	10037	10380	10721	11059	349
13	11394	11727	12057	12385	12710	13033	13353	13672	13987	14301	323
14	14613	14921	15228	15533	15836	16136	16435	16731	17026	17318	300
15	17609	17897	18184	18469	18752	19033	19312	19590	19865	20139	281
16	20412	20682	20951	21218	21484	21748	22010	22271	22530	22788	264
17	23045	23299	23552	23804	24054	24303	24551	24797	25042	25285	249
18	25527	25767	26007	26245	26481	26717	26951	27184	27415	27646	236
19	27875	28103	28330	28555	28780	29003	29225	29446	29666	29885	223
20	30103	30319	30535	30749	30963	31175	31386	31597	31806	32014	212
21	32222	32428	32633	32838	33041	33243	33445	33646	33845	34044	202
22	34242	34439	34635	34830	35024	35218	35410	35602	35793	35983	194
23	36173	36361	36548	36735	36921	37106	37291	37474	37657	37839	185
24	38021	38201	38381	38560	38739	38916	39093	39269	39445	39619	177
25	39794	39967	40140	40312	40483	40654	40824	40993	41162	41330	171
26	41497	41664	41830	41995	42160	42324	42488	42651	42813	42975	164
27	43136	43296	43456	43616	43775	43933	44090	44248	44404	44560	158
28	44716	44870	45024	45178	45331	45484	45636	45788	45939	46089	153
29	46240	46389	46538	46686	46834	46982	47129	47275	47421	47567	148
30	47712	47856	48000	48144	48287	48430	48572	48713	48855	48995	143
31	49136	49276	49415	49554	49693	49831	49968	50105	50242	50379	138
32	50515	50650	50785	50920	51054	51188	51321	51454	51587	51719	134
33	51851	51982	52113	52244	52374	52504	52633	52763	52891	53020	130
34	53148	53275	53402	53529	53655	53781	53907	54033	54157	54282	126
35	54407	54530	54654	54777	54900	55022	55145	55266	55388	55509	122
36	55630	55750	55870	55990	56110	56229	56348	56466	56584	56702	119
37	56820	56937	57054	57170	57287	57403	57518	57634	57749	57863	116
38	57978	58092	58206	58319	58433	58546	58658	58771	58883	58995	113
39	59106	59217	59328	59439	59549	59659	59769	59879	59988	60097	110
40	60206	60314	60422	60530	60638	60745	60852	60959	61066	61172	107
41	61278	61384	61489	61595	61700	61804	61909	62013	62117	62221	104
42	62325	62428	62531	62634	62736	62838	62941	63042	63144	63245	102
43	63347	63447	63548	63648	63749	63848	63948	64048	64147	64246	99
44	64345	64443	64542	64640	64738	64836	64933	65030	65127	65224	98
45	65321	65417	65513	65609	65705	65801	65896	65991	66086	66181	96
46	66276	66370	66464	66558	66651	66745	66838	66931	67024	67117	94
47	67210	67302	67394	67486	67577	67669	67760	67851	67942	68033	92
48	68124	68214	68304	68394	68484	68574	68663	68752	68842	68930	90
49	69020	69108	69196	69284	69372	69460	69548	69635	69722	69810	88
50	69897	69983	70070	70156	70243	70329	70415	70500	70586	70671	86
51	70757	70842	70927	71011	71096	71180	71265	71349	71433	71516	84
52	71600	71683	71767	71850	71933	72015	72098	72181	72263	72345	82
53	72428	72509	72591	72672	72754	72835	72916	72997	73078	73158	81
54	73239	73319	73399	73480	73559	73639	73719	73798	73878	73957	80
55	74036	74115	74193	74272	74351	74429	74507	74585	74663	74741	78
56	74818	74896	74973	75050	75127	75204	75281	75358	75434	75511	77
57	75587	75663	75739	75815	75891	75966	76042	76117	76192	76267	75
58	76342	76417	76492	76566	76641	76715	76789	76863	76937	77011	74
59	77085	77158	77232	77305	77378	77451	77524	77597	77670	77742	73
60	77815	77887	77959	78031	78103	78175	78247	78318	78390	78461	72
61	78533	78604	78675	78746	78816	78887	78958	79028	79098	79169	71
62	79239	79309	79379	79448	79518	79588	79657	79726	79796	79865	70
63	79934	80002	80071	80140	80208	80277	80345	80413	80482	80550	69
64	80618	80685	80753	80821	80888	80956	81023	81090	81157	81224	68
65	81291	81358	81424	81491	81557	81624	81690	81756	81822	81888	67

*Each log is supposed to have the decimal sign . before it.

Logarithms of Numbers, from 0 to 1000*—(Continued.)

No.	0	1	2	3	4	5	6	7	8	9	Prop.
66	81954	82020	82085	82151	82216	82282	82347	82412	82477	82542	66
67	82607	82672	82736	82801	82866	82930	82994	83058	83123	83187	65
68	83250	83314	83378	83442	83505	83569	83632	83695	83758	83821	64
69	83884	83947	84010	84073	84136	84198	84260	84323	84385	84447	63
70	84509	84571	84633	84695	84757	84818	84880	84941	85003	85064	62
71	85125	85187	85248	85309	85369	85430	85491	85551	85612	85672	61
72	85733	85793	85853	85913	85973	86033	86093	86153	86213	86272	60
73	86332	86391	86451	86510	86569	86628	86687	86746	86805	86864	59
74	86923	86981	87040	87098	87157	87215	87273	87332	87390	87448	58
75	87506	87564	87621	87679	87737	87794	87852	87909	87966	88024	57
76	88081	88138	88195	88252	88309	88366	88422	88479	88536	88592	56
77	88649	88705	88761	88818	88874	88930	88986	89042	89098	89153	56
78	89209	89265	89320	89376	89431	89487	89542	89597	89652	89707	55
79	89762	89817	89872	89927	89982	90036	90091	90145	90200	90254	54
80	90309	90363	90417	90471	90525	90579	90633	90687	90741	90794	54
81	90848	90902	90955	91009	91062	91115	91169	91222	91275	91328	53
82	91381	91434	91487	91540	91592	91645	91698	91750	91803	91855	53
83	91907	91960	92012	92064	92116	92168	92220	92272	92324	92376	52
84	92427	92479	92531	92582	92634	92685	92737	92788	92839	92890	51
85	92941	92993	93044	93095	93146	93196	93247	93298	93348	93399	51
86	93449	93500	93550	93601	93651	93701	93751	93802	93852	93902	50
87	93951	94001	94051	94101	94151	94200	94250	94300	94349	94398	49
88	94448	94497	94546	94596	94645	94694	94743	94792	94841	94890	49
89	94939	94987	95036	95085	95133	95182	95230	95279	95327	95376	48
90	95424	95472	95520	95568	95616	95664	95712	95760	95808	95856	48
91	95904	95951	95999	96047	96094	96142	96189	96236	96284	96331	48
92	96378	96426	96473	96520	96567	96614	96661	96708	96754	96801	47
93	96848	96895	96941	96988	97034	97081	97127	97174	97220	97266	47
94	97312	97359	97405	97451	97497	97543	97589	97635	97680	97726	46
95	97772	97818	97863	97909	97954	98000	98045	98091	98136	98181	46
96	98227	98272	98317	98362	98407	98452	98497	98542	98587	98632	45
97	98677	98721	98766	98811	98855	98900	98945	98989	99033	99078	45
98	99122	99166	99211	99255	99299	99343	99387	99431	99475	99519	44
99	99563	99607	99651	99694	99738	99782	99825	99869	99913	99956	44

*Each log is supposed to have the decimal sign . before it.

The log of 2870 is 3.45788	The log of .287 is — 1.45788
" " " 287 is 2.45788	" " " .028 is — 2.44716
" " " 28.7 is 1.45788	" " " .002 is — 3.30103
" " " 2.87 is 0.45788	" " " .0002 is — 4.30103

What is the log of 2873?

Here, log of 2870 = 3.45788
And prop 153 × 3 = 459

3.458339

To find roots divide the log (with its index) of the given number, by that number which expresses the kind of root. The quotient will be the log of the required root.

Example. What is the cube root of 2870?

Here, the log of 2870, with its index, is 3.45788. And $\frac{3.45788}{3} = 1.15263$. Hence the cube root is 14.2

The Hyperbolic, or Napierian logarithm is the common log of the table multiplied by 2.3025851.

Square Roots and Cube Roots of Numbers from .1 to 28.

No errors

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Sq. Rt.	C. Rt.	No.	Sq. Rt.	C. Rt.
.1	.01	.001	.316	.464	.7	2.387	1.786	.4	3.661	2.375
.15	.0225	.0034	.387	.531	.8	2.408	1.797	.6	3.688	2.387
.2	.04	.008	.447	.585	.9	2.429	1.807	.8	3.715	2.399
.25	.0625	.0156	.500	.630	6.	2.449	1.817	14.	3.742	2.410
.3	.09	.027	.548	.669	.1	2.470	1.827	.2	3.768	2.422
.35	.1225	.0429	.592	.705	.2	2.490	1.837	.4	3.795	2.433
.4	.16	.064	.633	.737	.3	2.510	1.847	.6	3.821	2.444
.45	.2025	.0911	.671	.766	.4	2.530	1.857	.8	3.847	2.455
.5	.25	.125	.707	.794	.5	2.550	1.866	15.	3.873	2.466
.55	.3025	.1664	.742	.819	.6	2.569	1.876	.2	3.899	2.477
.6	.36	.216	.775	.843	.7	2.588	1.885	.4	3.924	2.488
.65	.4225	.2746	.806	.866	.8	2.608	1.895	.6	3.950	2.499
.7	.49	.343	837	.888	.9	2.627	1.904	.8	3.975	2.509
.75	.5625	.4219	.866	.909	7.	2.646	1.913	16.	4.	2.520
.8	.64	.512	.894	.928	.1	2.665	1.922	.2	4.025	2.530
.85	.7225	.6141	.922	.947	.2	2.683	1.931	.4	4.050	2.541
.9	.81	.729	.949	.965	.3	2.702	1.940	.6	4.074	2.551
.95	.9025	.8574	.975	.983	.4	2.720	1.949	.8	4.099	2.561
1.	1.000	1.000	1.000	1.000	.5	2 739	1.957	17.	4.123	2.571
.05	1.103	1.158	1.025	1.016	.6	2.757	1.966	.2	4.147	2.581
1.1	1.210	1.331	1.049	1.032	.7	2.775	1 975	.4	4.171	2.591
.15	1.323	1.521	1.072	1.048	.8	2.793	1.983	.6	4.195	2.601
1.2	1.440	1.728	1.095	1.063	.9	2.811	1.992	.8	4.219	2.611
.25	1.563	1.953	1.118	1.077	8.	2.828	2.000	18.	4.243	2.621
1.3	1.690	2.197	1.140	1.091	.1	2.846	2.008	.2	4.266	2.630
.35	1.823	2.460	1.162	1.105	.2	2.864	2.017	.4	4.290	2 640
1.4	1.960	2.744	1.183	1.119	.3	2.881	2.025	.6	4.313	2.650
.45	2.103	3.049	1.204	1.132	.4	2.898	2.033	.8	4.336	2.659
1.5	2.250	3.375	1.225	1.145	.5	2.915	2.041	19.	4.359	2.668
.55	2.403	3.724	1.245	1.157	.6	2.933	2.049	.2	4.382	2.678
1.6	2.560	4.096	1.265	1.170	.7	2.950	2.057	.4	4.405	2.687
.65	2.723	4.492	1.285	1.182	.8	2.966	2.065	.6	4.427	2.696
1.7	2.890	4.913	1.304	1.193	.9	2.983	2.072	.8	4.450	2.705
.75	3.063	5.359	1.323	1.205	9.	3.	2.080	20.	4.472	2.714
1.8	3.240	5.832	1.342	1.216	.1	3.017	2.088	.2	4.494	2.723
.85	3.423	6.332	1.360	1.228	.2	3.033	2.095	.4	4.517	2.732
1.9	3.610	6.859	1.378	1.239	.3	3.050	2.103	.6	4.539	2.741
.95	3.803	7.415	1.396	1.249	.4	3.066	2.110	.8	4.561	2.750
2.	4.000	8.000	1.414	1.260	.5	3.082	2.118	21.	4.583	2.759
.1	4.410	9.261	1.449	1.281	.6	3.098	2.125	.2	4.604	2.768
.2	4.840	10.65	1.483	1.301	.7	3.114	2.133	.4	4.626	2.776
.3	5.290	12.17	1.517	1.320	.8	3.130	2.140	.6	4.648	2.785
.4	5.760	13.82	1.549	1.339	.9	3.146	2.147	.8	4.669	2.794
.5	6.250	15.63	1.581	1.357	10.	3.162	2.154	22.	4.690	2.802
.6	6.760	17.58	1.612	1.375	.1	3.178	2.162	.2	4.712	2.810
.7	7.290	19.68	1.643	1.392	.2	3.194	2.169	.4	4.733	2.819
.8	7.840	21.95	1.673	1.409	.3	3.209	2.176	.6	4.754	2.827
.9	8.410	24.39	1.703	1.426	.4	3.225	2.183	.8	4.775	2.836
3.	9.	27.	1.732	1.442	.5	3.240	2.190	23.	4.796	2.844
.1	9.61	29.79	1.761	1.458	.6	3.256	2.197	.2	4.817	2.852
.2	10.24	32.77	1.789	1.474	.7	3.271	2.204	.4	4.837	2.860
.3	10.89	35.94	1.817	1.489	.8	3.286	2.210	.6	4.858	2.868
.4	11.56	39.30	1.844	1.504	.9	3.302	2.217	.8	4.879	2.876
.5	12.25	42.88	1.871	1.518	11.	3.317	2.224	24.	4.899	2.884
.6	12.96	46.66	1.897	1.533	.1	3.332	2.231	.2	4.919	2.892
.7	13.69	50.65	1.924	1.547	.2	3.347	2.237	.4	4.940	2.900
.8	14 44	54.87	1.949	1.560	.3	3.362	2.244	.6	4.960	2.908
.9	15.21	59.32	1.975	1.574	.4	3 376	2.251	.8	4.980	2.916
4.	16.	64.	2.	1.587	.5	3.391	2.257	25.	5.	2.924
.1	16.81	68.92	2.025	1.601	.6	3.406	2.264	.2	5.020	2.932
.2	17.64	74.09	2.049	1 613	.7	3.421	2.270	.4	5.040	2.940
.3	18.49	79.51	2.074	1.626	.8	3.435	2.277	.6	5.060	2.947
.4	19.36	85.18	2.098	1.639	.9	3.450	2.283	.8	5.079	2.955
.5	20.25	91.13	2.121	1.651	12.	3.464	2 289	26.	5.099	2.962
.6	21.16	97.34	2.145	1.663	.1	3.479	2.296	.2	5.119	2.970
.7	22.09	103.8	2.168	1.675	.2	3.493	2.302	.4	5.138	2.978
.8	23.04	110.6	2.191	1.687	.3	3.507	2.308	.6	5.158	2.985
.9	24.01	117.6	2.214	1.698	.4	3.521	2.315	.8	5.177	2.993
5.	25.	125.	2.236	1.710	.5	3.536	2.321	27.	5.196	3 000
.1	26.01	132.7	2.258	1.721	.6	3.550	2.327	.2	5.215	3 007
.2	27.04	140.6	2.280	1.732	.7	3.564	2.333	.4	5.235	3.015
.3	28.09	148.9	2.302	1.744	.8	3.578	2 339	.6	5.254	3.022
.4	29.16	157.5	2.324	1.754	.9	3.592	2.345	.8	5.273	3.029
.5	30.25	166.4	2.345	1.765	13.	3.606	2.351	28.	5.292	3.037
.6	31.36	175.6	2.366	1.776	.2	3.633	2.363	.2	5.310	3.044

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000.

REMARK ON THE FOLLOWING TABLE. Wherever the effect of a fifth decimal in the roots would be to add 1 to the fourth and final decimal in the table, the addition has been made. No errors.

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
1	1	1	1.0000	1.0000	61	3721	226981	7.8102	3.9365
2	4	8	1.4142	1.2599	62	3844	238328	7.8740	3.9579
3	9	27	1.7321	1.4422	63	3969	250047	7.9373	3.9791
4	16	64	2.0000	1.5874	64	4096	262144	8.0000	4.
5	25	125	2.2361	1.7100	65	4225	274625	8.0623	4.0207
6	36	216	2.4495	1.8171	66	4356	287496	8.1240	4.0412
7	49	343	2.6458	1.9129	67	4489	300763	8.1854	4.0615
8	64	512	2.8284	2.0000	68	4624	314432	8.2462	4.0817
9	81	729	3.0000	2.0801	69	4761	328509	8.3066	4.1016
10	100	1000	3.1623	2.1544	70	4900	343000	8.3666	4.1213
11	121	1331	3.3166	2.2240	71	5041	357911	8.4261	4.1408
12	144	1728	3.4641	2.2894	72	5184	373248	8.4853	4.1602
13	169	2197	3.6056	2.3513	73	5329	389017	8.5440	4.1793
14	196	2744	3.7417	2.4101	74	5476	405224	8.6023	4.1983
15	225	3375	3.8730	2.4662	75	5625	421875	8.6603	4.2172
16	256	4096	4.0000	2.5198	76	5776	438976	8.7178	4.2358
17	289	4913	4.1231	2.5713	77	5929	456533	8.7750	4.2543
18	324	5832	4.2426	2.6207	78	6084	474552	8.8318	4.2727
19	361	6859	4.3589	2.6684	79	6241	493039	8.8882	4.2908
20	400	8000	4.4721	2.7144	80	6400	512000	8.9443	4.3089
21	441	9261	4.5826	2.7589	81	6561	531441	9.	4.3267
22	484	10648	4.6904	2.8020	82	6724	551368	9.0554	4.3445
23	529	12167	4.7958	2.8439	83	6889	571787	9.1104	4.3621
24	576	13824	4.8990	2.8845	84	7056	592704	9.1652	4.3795
25	625	15625	5.0000	2.9240	85	7225	614125	9.2195	4.3968
26	676	17576	5.0990	2.9625	86	7396	636056	9.2736	4.4140
27	729	19683	5.1962	3.0000	87	7569	658503	9.3274	4.4310
28	784	21952	5.2915	3.0366	88	7744	681472	9.3808	4.4480
29	841	24389	5.3852	3.0723	89	7921	704969	9.4340	4.4647
30	900	27000	5.4772	3.1072	90	8100	729000	9.4868	4.4814
31	961	29791	5.5678	3.1414	91	8281	753571	9.5394	4.4979
32	1024	32768	5.6569	3.1748	92	8464	778688	9.5917	4.5144
33	1089	35937	5.7446	3.2075	93	8649	804357	9.6437	4.5307
34	1156	39304	5.8310	3.2396	94	8836	830584	9.6954	4.5468
35	1225	42875	5.9161	3.2711	95	9025	857375	9.7468	4.5629
36	1296	46656	6.0000	3.3019	96	9216	884736	9.7980	4.5789
37	1369	50653	6.0828	3.3322	97	9409	912673	9.8489	4.5947
38	1444	54872	6.1644	3.3620	98	9604	941192	9.8995	4.6104
39	1521	59319	6.2450	3.3912	99	9801	970299	9.9499	4.6261
40	1600	64000	6.3246	3.4200	100	10000	1000000	10.	4.6416
41	1681	68921	6.4031	3.4482	101	10201	1030301	10.0499	4.6570
42	1764	74088	6.4807	3.4760	102	10404	1061208	10.0995	4.6723
43	1849	79507	6.5574	3.5034	103	10609	1092727	10.1489	4.6875
44	1936	85184	6.6332	3.5303	104	10816	1124864	10.1980	4.7027
45	2025	91125	6.7082	3.5569	105	11025	1157625	10.2470	4.7177
46	2116	97336	6.7823	3.5830	106	11236	1191016	10.2956	4.7326
47	2209	103823	6.8557	3.6088	107	11449	1225043	10.3441	4.7475
48	2304	110592	6.9282	3.6342	108	11664	1259712	10.3923	4.7622
49	2401	117649	7.0000	3.6593	109	11881	1295029	10.4403	4.7769
50	2500	125000	7.0711	3.6840	110	12100	1331000	10.4881	4.7914
51	2601	132651	7.1414	3.7084	111	12321	1367631	10.5357	4.8059
52	2704	140608	7.2111	3.7325	112	12544	1404928	10.5830	4.8203
53	2809	148877	7.2801	3.7563	113	12769	1442897	10.6301	4.8346
54	2916	157464	7.3485	3.7798	114	12996	1481544	10.6771	4.8488
55	3025	166375	7.4162	3.8030	115	13225	1520875	10.7238	4.8629
56	3136	175616	7.4833	3.8259	116	13456	1560896	10.7703	4.8770
57	3249	185193	7.5498	3.8485	117	13689	1601613	10.8167	4.8910
58	3364	195112	7.6158	3.8709	118	13924	1643032	10.8628	4.9049
59	3481	205379	7.6811	3.8930	119	14161	1685159	10.9087	4.9187
60	3600	216000	7.7460	3.9149	120	14400	1728000	10.9545	4.9324

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000—(CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
121	14641	1771561	11.	4.9461	186	34596	6434856	13.6382	5.7083
122	14884	1815848	11.0454	4.9597	187	34969	6539203	13.6748	5.7185
123	15129	1860867	11.0905	4.9732	188	35344	6644672	13.7113	5.7287
124	15376	1906624	11.1355	4.9866	189	35721	6751269	13.7477	5.7388
125	15625	1953125	11.1803	5.	190	36100	6859000	13.7840	5.7489
126	15876	2000376	11.2250	5.0133	191	36481	6967871	13.8203	5.7590
127	16129	2048383	11.2694	5.0265	192	36864	7077888	13.8564	5.7690
128	16384	2097152	11.3137	5.0397	193	37249	7189057	13.8924	5.7790.
129	16641	2146689	11.3578	5.0528	194	37636	7301384	13.9284	5.7896
130	16900	2197000	11.4018	5.0658	195	38025	7414875	13.9642	5.7989
131	17161	2248091	11.4455	5.0788	196	38416	7529536	14.	5.8088
132	17424	2299968	11.4891	5.0916	197	38809	7645373	14.0357	5.8186
133	17689	2352637	11.5326	5.1045	198	39204	7762392	14.0712	5.8285
134	17956	2406104	11.5758	5.1172	199	39601	7880599	14.1067	5.8383
135	18225	2460375	11.6190	5.1299	200	40000	8000000	14.1421	5.8480
136	18496	2515456	11.6619	5.1426	201	40401	8120601	14.1774	5.8578
137	18769	2571353	11.7047	5.1551	202	40804	8242408	14.2127	5.8675
138	19044	2628072	11.7473	5.1676	203	41209	8365427	14.2478	5.8771
139	19321	2685619	11.7898	5.1801	204	41616	8489664	14.2829	5.8868
140	19600	2744000	11.8322	5.1925	205	42025	8615125	14.3178	5.8964
141	19881	2803221	11.8743	5.2048	206	42436	8741816	14.3527	5.9059
142	20164	2863288	11.9164	5.2171	207	42849	8869743	14.3875	5.9155
143	20449	2924207	11.9583	5.2293	208	43264	8998912	14.4222	5.9250
144	20736	2985984	12.	5.2415	209	43681	9129329	14.4568	5.9345
145	21025	3048625	12.0416	5.2536	210	44100	9261000	14.4914	5.9439
146	21316	3112136	12.0830	5.2656	211	44521	9393931	14.5258	5.9533
147	21609	3176523	12.1244	5.2776	212	44944	9528128	14.5602	5.9627
148	21904	3241792	12.1655	5.2896	213	45369	9663597	14.5945	5.9721
149	22201	3307949	12.2066	5.3015	214	45796	9800344	14.6287	5.9814
150	22500	3375000	12.2474	5.3133	215	46225	9938375	14.6629	5.9907
151	22801	3442951	12.2882	5 3251	216	46656	10077696	14.6969	6.
152	23104	3511808	12.3288	5.3368	217	47089	10218313	14.7309	6.0092
153	23409	3581577	12.3693	5.3485	218	47524	10360232	14.7648	6.0185
154	23716	3652264	12.4097	5.3601	219	47961	10503459	14.7986	6.0277
155	24025	3723875	12.4499	5.3717	220	48400	10648000	14.8324	6.0368
156	24336	3796416	12.4900	5.3832	221	48841	10793861	14.8661	6.0459
157	24649	3869893	12.5300	5.3947	222	49284	10941048	14.8997	6.0550
158	24964	3944312	12.5698	5.4061	223	49729	11089567	14.9332	6.0641
159	25281	4019679	12.6095	5.4175	224	50176	11239424	14.9666	6.0732
160	25600	4096000	12.6491	5.4288	225	50625	11390625	15.	6.0822
161	25921	4173281	12.6886	5.4401	226	51076	11543176	15.0333	6.0912
162	26244	4251528	12.7279	5.4514	227	51529	11697083	15.0665	6.1002
163	26569	4330747	12.7671	5.4626	228	51984	11852352	15.0997	6.1091
164	26896	4410944	12.8062	5.4737	229	52441	12008989	15.1327	6.1180
165	27225	4492125	12.8452	5.4848	230	52900	12167000	15.1658	6.1269
166	27556	4574296	12.8841	5.4959	231	53361	12326391	15.1987	6.1358
167	27889	4657463	12.9228	5.5069	232	53824	12487168	15.2315	6.1446
168	28224	4741632	12.9615	5.5178	233	54289	12649337	15.2643	6.1534
169	28561	4826809	13.	5.5288	234	54756	12812904	15.2971	6.1622
170	28900	4913000	13.0384	5.5397	235	55225	12977875	15.3297	6.1710
171	29241	5000211	13.0767	5.5505	236	55696	13144256	15.3623	6.1797
172	29584	5088448	13.1149	5.5613	237	56169	13312053	15.3948	6.1885
173	29929	5177717	13.1529	5.5721	238	56644	13481272	15.4272	6.1972
174	30276	5268024	13.1909	5.5828	239	57121	13651919	15.4596	6.2058
175	30625	5359375	13.2288	5.5934	240	57600	13824000	15.4919	6.2145
176	30976	5451776	13.2665	5.6041	241	58081	13997521	15.5242	6.2231
177	31329	5545233	13.3041	5.6147	242	58564	14172488	15.5563	6.2317
178	31684	5639752	13.3417	5.6252	243	59049	14348907	15.5885	6.2403
179	32041	5735339	13.3791	5.6357	244	59536	14526784	15.6205	6.2488
180	32400	5832000	13.4164	5.6462	245	60025	14706125	15.6525	6.2573
181	32761	5929741	13.4536	5.6567	246	60516	14886936	15.6844	6.2658
182	33124	6028568	13.4907	5.6671	247	61009	15069223	15.7162	6.2743
183	33489	6128487	13.5277	5.6774	248	61504	15252992	15.7480	6.2828
184	33856	6229504	13.5647	5.6877	249	62001	15438249	15.7797	6.2912
185	34225	6331625	13.6015	5.6980	250	62500	15625000	15.8114	6.2996

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000 — (CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt	No.	Square.	Cube.	Sq. Rt.	C. Rt.
251	63001	15813251	15.8430	6.3080	316	99856	31554496	17.7764	6.8113
252	63504	16003008	15.8745	6.3164	317	100489	31855013	17.8045	6.8185
253	64009	16194277	15.9060	6.3247	318	101124	32157432	17.8326	6.8256
254	64516	16387064	15.9374	6.3330	319	101761	32461759	17.8606	6.8328
255	65025	16581375	15.9687	6.3413	320	102400	32768000	17.8885	6.8399
256	65536	16777216	16.	6.3496	321	103041	33076161	17.9165	6.8470
257	66049	16974593	16.0312	6.3579	322	103684	33386248	17.9444	6.8541
258	66564	17173512	16.0624	6.3661	323	104329	33698267	17.9722	6.8612
259	67081	17373979	16.0935	6.3743	324	104976	34012224	18.	6.8683
260	67600	17576000	16.1245	6.3825	325	105625	34328125	18.0278	6.8753
261	68121	17779581	16.1555	6.3907	326	106276	34645976	18.0555	6.8824
262	68644	17984728	16.1864	6.3988	327	106929	34965783	18.0831	6.8894
263	69169	18191447	16.2173	6.4070	328	107584	35287552	18.1108	6.8964
264	69696	18399744	16.2481	6.4151	329	108241	35611289	18.1384	6.9034
265	70225	18609625	16.2788	6.4232	330	108900	35937000	18.1659	6.9104
266	70756	18821096	16.3095	6.4312	331	109561	36264691	18.1934	6.9174
267	71289	19034163	16.3401	6.4393	332	110224	36594368	18.2209	6.9244
268	71824	19248832	16.3707	6.4473	333	110889	36926037	18.2483	6.9313
269	72361	19465109	16.4012	6.4553	334	111556	37259704	18.2757	6.9382
270	72900	19683000	16.4317	6.4633	335	112225	37595375	18.3030	6.9451
271	73441	19902511	16.4621	6.4713	336	112896	37933056	18.3303	6.9521
272	73984	20123648	16.4924	6.4792	337	113569	38272753	18.3576	6.9589
273	74529	20346417	16.5227	6.4872	338	114244	38614472	18.3848	6.9658
274	75076	20570824	16.5529	6.4951	339	114921	38958219	18.4120	6.9727
275	75625	20796875	16.5831	6.5030	340	115600	39304000	18.4391	6.9795
276	76176	21024576	16.6132	6.5108	341	116281	39651821	18.4662	6.9864
277	76729	21253933	16.6433	6.5187	342	116964	40001688	18.4932	6.9932
278	77284	21484952	16.6733	6.5265	343	117649	40353607	18.5203	7.
279	77841	21717639	16.7033	6.5343	344	118336	40707584	18.5472	7.0068
280	78400	21952000	16.7332	6.5421	345	119025	41063625	18.5742	7.0136
281	78961	22188041	16.7631	6.5499	346	119716	41421736	18.6011	7.0203
282	79524	22425768	16.7929	6.5577	347	120409	41781923	18.6279	7.0271
283	80089	22665187	16.8226	6.5654	348	121104	42144192	18.6548	7.0338
284	80656	22906304	16.8523	6.5731	349	121801	42508549	18.6815	7.0406
285	81225	23149125	16.8819	6.5808	350	122500	42875000	18.7083	7.0473
286	81796	23393656	16.9115	6.5885	351	123201	43243551	18.7350	7.0540
287	82369	23639903	16.9411	6.5962	352	123904	43614208	18.7617	7.0607
288	82944	23887872	16.9706	6.6039	353	124609	43986977	18.7883	7.0674
289	83521	24137569	17.	6.6115	354	125316	44361864	18.8149	7.0740
290	84100	24389000	17.0294	6.6191	355	126025	44738875	18.8414	7.0807
291	84681	24642171	17.0587	6.6267	356	126736	45118016	18.8680	7.0873
292	85264	24897088	17.0880	6.6343	357	127449	45499293	18.8944	7.0940
293	85849	25153757	17.1172	6.6419	358	128164	45882712	18.9209	7.1006
294	86436	25412184	17.1464	6.6494	359	128881	46268279	18.9473	7.1072
295	87025	25672375	17.1756	6.6569	360	129600	46656000	18.9737	7.1138
296	87616	25934336	17.2047	6.6644	361	130321	47045881	19.	7.1204
297	88209	26198073	17.2337	6.6719	362	131044	47437928	19.0263	7.1269
298	88804	26463592	17.2627	6.6794	363	131769	47832147	19.0526	7.1335
299	89401	26730899	17.2916	6.6869	364	132496	48228544	19.0788	7.1400
300	90000	27000000	17.3205	6.6943	365	133225	48627125	19.1050	7.1466
301	90601	27270901	17.3494	6.7018	366	133956	49027896	19.1311	7.1531
302	91204	27543608	17.3781	6.7092	367	134689	49430863	19.1572	7.1596
303	91809	27818127	17.4069	6.7166	368	135424	49836032	19.1833	7.1661
304	92416	28094464	17.4356	6.7240	369	136161	50243409	19.2094	7.1726
305	93025	28372625	17.4642	6.7313	370	136900	50653000	19.2354	7.1791
306	93636	28652616	17.4929	6.7387	371	137641	51064811	19.2614	7.1855
307	94249	28934443	17.5214	6.7460	372	138384	51478848	19.2873	7.1920
308	94864	29218112	17.5499	6.7533	373	139129	51895117	19.3132	7.1984
309	95481	29503629	17.5784	6.7606	374	139876	52313624	19.3391	7.2048
310	96100	29791000	17.6068	6.7679	375	140625	52734375	19.3649	7.2112
311	96721	30080231	17.6352	6.7752	376	141376	53157376	19.3907	7.2177
312	97344	30371328	17.6635	6.7824	377	142129	53582633	19.4165	7.2240
313	97969	30664297	17.6918	6.7897	378	142884	54010152	19.4422	7.2304
314	98596	30959144	17.7200	6.7969	379	143641	54439939	19.4679	7.2368
315	99225	31255875	17.7482	6.8041	380	144400	54872000	19.4936	7.2432

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000—(CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
381	145161	55306341	19.5192	7.2495	446	198916	88716536	21.1187	7.6403
382	145924	55742968	19.5448	7.2558	447	199809	89314623	21.1424	7.6460
383	146689	56181887	19.5704	7.2622	448	200704	89915392	21.1660	7.6517
384	147456	56623104	19.5959	7.2685	449	201601	90518849	21.1896	7.6574
385	148225	57066625	19.6214	7.2748	450	202500	91125000	21.2132	7.6631
386	148996	57512456	19.6469	7.2811	451	203401	91733851	21.2368	7.6688
387	149769	57960603	19.6723	7.2874	452	204304	92345408	21.2603	7.6744
388	150544	58411072	19.6977	7.2936	453	205209	92959677	21.2838	7.6801
389	151321	58863869	19.7231	7.2999	454	206116	93576664	21.3073	7.6857
390	152100	59319000	19.7484	7.3061	455	207025	94196375	21.3307	7.6914
391	152881	59776471	19.7737	7.3124	456	207936	94818816	21.3542	7.6970
392	153664	60236288	19.7990	7.3186	457	208849	95443993	21.3776	7.7026
393	154449	60698457	19.8242	7.3248	458	209764	96071912	21.4009	7.7082
394	155236	61162984	19.8494	7.3310	459	210681	96702579	21.4243	7.7138
395	156025	61629875	19.8746	7.3372	460	211600	97336000	21.4476	7.7194
396	156816	62099136	19.8997	7.3434	461	212521	97972181	21.4709	7.7250
397	157609	62570773	19.9249	7.3496	462	213444	98611128	21.4942	7.7306
398	158404	63044792	19.9499	7.3558	463	214369	99252847	21.5174	7.7362
399	159201	63521199	19.9750	7.3619	464	215296	99897344	21.5407	7.7418
400	160000	64000000	20.	7.3681	465	216225	100544625	21.5639	7.7473
401	160801	64481201	20.0250	7.3742	466	217156	101194696	21.5870	7.7529
402	161604	64964808	20.0499	7.3803	467	218089	101847563	21.6102	7.7584
403	162409	65450827	20.0749	7.3864	468	219024	102503232	21.6333	7.7639
404	163216	65939264	20.0998	7.3925	469	219961	103161709	21.6564	7.7695
405	164025	66430125	20.1246	7.3986	470	220900	103823000	21.6795	7.7750
406	164836	66923416	20.1494	7.4047	471	221841	104487111	21.7025	7.7805
407	165649	67419143	20.1742	7.4108	472	222784	105154048	21.7256	7.7860
408	166464	67917312	20.1990	7.4169	473	223729	105823817	21.7486	7.7915
409	167281	68417929	20.2237	7.4229	474	224676	106496424	21.7715	7.7970
410	168100	68921000	20.2485	7.4290	475	225625	107171875	21.7945	7.8025
411	168921	69426531	20.2731	7.4350	476	226576	107850176	21.8174	7.8079
412	169744	69934528	20.2978	7.4410	477	227529	108531333	21.8403	7.8134
413	170569	70444997	20.3224	7.4470	478	228484	109215352	21.8632	7.8188
414	171396	70957944	20.3470	7.4530	479	229441	109902239	21.8861	7.8243
415	172225	71473375	20.3715	7.4590	480	230400	110592000	21.9089	7.8297
416	173056	71991296	20.3961	7.4650	481	231361	111284641	21.9317	7.8352
417	173889	72511713	20.4206	7.4710	482	232324	111980168	21.9545	7.8406
418	174724	73034632	20.4450	7.4770	483	233289	112678587	21.9773	7.8460
419	175561	73560059	20.4695	7.4829	484	234256	113379904	22.	7.8514
420	176400	74088000	20.4939	7.4889	485	235225	114084125	22.0227	7.8568
421	177241	74618461	20.5183	7.4948	486	236196	114791256	22.0454	7.8622
422	178084	75151448	20.5426	7.5007	487	237169	115501303	22.0681	7.8676
423	178929	75686967	20.5670	7.5067	488	238144	116214272	22.0907	7.8730
424	179776	76225024	20.5913	7.5126	489	239121	116930169	22.1133	7.8784
425	180625	76765625	20.6155	7.5185	490	240100	117649000	22.1359	7.8837
426	181476	77308776	20.6398	7.5244	491	241081	118370771	22.1585	7.8891
427	182329	77854483	20.6640	7.5302	492	242064	119095488	22.1811	7.8944
428	183184	78402752	20.6882	7.5361	493	243049	119823157	22.2036	7.8998
429	184041	78953589	20.7123	7.5420	494	244036	120553784	22.2261	7.9051
430	184900	79507000	20.7364	7.5478	495	245025	121287375	22.2486	7.9105
431	185761	80062991	20.7605	7.5537	496	246016	122023936	22.2711	7.9158
432	186624	80621568	20.7846	7.5595	497	247009	122763473	22.2935	7.9211
433	187489	81182737	20.8087	7.5654	498	248004	123505992	22.3159	7.9264
434	188356	81746504	20.8327	7.5712	499	249001	124251499	22.3383	7.9317
435	189225	82312875	20.8567	7.5770	500	250000	125000000	22.3607	7.9370
436	190096	82881856	20.8806	7.5828	501	251001	125751501	22.3830	7.9423
437	190969	83453453	20.9045	7.5886	502	252004	126506008	22.4054	7.9476
438	191844	84027672	20.9284	7.5944	503	253009	127263527	22.4277	7.9528
439	192721	84604519	20.9523	7.6001	504	254016	128024064	22.4499	7.9581
440	193600	85184000	20.9762	7.6059	505	255025	128787625	22.4722	7.9634
441	194481	85766121	21.	7.6117	506	256036	129554216	22.4944	7.9686
442	195364	86350888	21.0238	7.6174	507	257049	130323843	22.5167	7.9739
443	196249	86938307	21.0476	7.6232	508	258064	131096512	22.5389	7.9791
444	197136	87528384	21.0713	7.6289	509	259081	131872229	22.5610	7.9843
445	198025	88121125	21.0950	7.6346	510	260100	132651000	22.5832	7.9896

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000 — (CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
511	261121	133432831	22.6053	7.9948	576	331776	191102976	24.	8.3203
512	262144	134217728	22.6274	8.	577	332929	192100033	24.0208	8.3251
513	263169	135005697	22.6495	8.0052	578	334084	193100552	24.0416	8.3300
514	264196	135796744	22.6716	8.0104	579	335241	194104539	24.0624	8.3348
515	265225	136590875	22.6936	8.0156	580	336400	195112000	24.0832	8.3396
516	266256	137388096	22.7156	8.0208	581	337561	196122941	24.1039	8.3443
517	267289	138188413	22.7376	8.0260	582	338724	197137368	24.1247	8.3491
518	268324	138991832	22.7596	8.0311	583	339889	198155287	24.1454	8.3539
519	269361	139798359	22.7816	8.0363	584	341056	199176704	24.1661	8.3587
520	270400	140608000	22.8035	8.0415	585	342225	200201625	24.1868	8.3634
521	271441	141420761	22.8254	8.0466	586	343396	201230056	24.2074	8.3682
522	272484	142236648	22.8473	8.0517	587	344569	202262003	24.2281	8.3730
523	273529	143055667	22.8692	8.0569	588	345744	203297472	24.2487	8.3777
524	274576	143877824	22.8910	8.0620	589	346921	204336469	24.2693	8.3825
525	275625	144703125	22.9129	8.0671	590	348100	205379000	24.2899	8.3872
526	276676	145531576	22.9347	8.0723	591	349281	206425071	24.3105	8.3919
527	277729	146363183	22.9565	8.0774	592	350464	207474688	24.3311	8.3967
528	278784	147197952	22.9783	8.0825	593	351649	208527857	24.3516	8.4014
529	279841	148035889	23.	8.0876	594	352836	209584584	24.3721	8.4061
530	280900	148877000	23.0217	8.0927	595	354025	210644875	24.3926	8.4108
531	281961	149721291	23.0434	8.0978	596	355216	211708736	24.4131	8.4155
532	283024	150568768	23.0651	8.1028	597	356409	212776173	24.4336	8.4202
533	284089	151419437	23.0868	8.1079	598	357604	213847192	24.4540	8.4249
534	285156	152273304	23.1084	8.1130	599	358801	214921799	24.4745	8.4296
535	286225	153130375	23.1301	8.1180	600	360000	216000000	24.4949	8.4343
536	287296	153990656	23.1517	8.1231	601	361201	217081801	24.5153	8.4390
537	288369	154854153	23.1733	8.1281	602	362404	218167208	24.5357	8.4437
538	289444	155720872	23.1948	8.1332	603	363609	219256227	24.5561	8.4484
539	290521	156590819	23.2164	8.1382	604	364816	220348864	24.5764	8.4530
540	291600	157464000	23.2379	8.1433	605	366025	221445125	24.5967	8.4577
541	292681	158340421	23.2594	8.1483	606	367236	222545016	24.6171	8.4623
542	293764	159220088	23.2809	8.1533	607	368449	223648543	24.6374	8.4670
543	294849	160103007	23.3024	8.1583	608	369664	224755712	24.6577	8.4716
544	295936	160989184	23.3238	8.1633	609	370881	225866529	24.6779	8.4763
545	297025	161878625	23.3452	8.1683	610	372100	226981000	24.6982	8.4809
546	298116	162771336	23.3666	8.1733	611	373321	228099131	24.7184	8.4856
547	299209	163667323	23.3880	8.1783	612	374544	229220928	24.7386	8.4902
548	300304	164566592	23.4094	8.1833	613	375769	230346397	24.7588	8.4948
549	301401	165469149	23.4307	8.1882	614	376996	231475544	24.7790	8.4994
550	302500	166375000	23.4521	8.1932	615	378225	232608375	24.7992	8.5040
551	303601	167284151	23.4734	8.1982	616	379456	233744896	24.8193	8.5086
552	304704	168196608	23.4947	8.2031	617	380689	234885113	24.8395	8.5132
553	305809	169112377	23.5160	8.2081	618	381924	236029032	24.8596	8.5178
554	306916	170031464	23.5372	8.2130	619	383161	237176659	24.8797	8.5224
555	308025	170953875	23.5584	8.2180	620	384400	238328000	24.8998	8.5270
556	309136	171879616	23.5797	8.2229	621	385641	239483061	24.9199	8.5316
557	310249	172808693	23.6008	8.2278	622	386884	240641848	24.9399	8.5362
558	311364	173741112	23.6220	8.2327	623	388129	241804367	24.9600	8.5408
559	312481	174676879	23.6432	8.2377	624	389376	242970624	24.9800	8.5453
560	313600	175616000	23.6643	8.2426	625	390625	244140625	25.	8.5499
561	314721	176558481	23.6854	8.2475	626	391876	245314376	25.0200	8.5544
562	315844	177504328	23.7065	8.2524	627	393129	246491883	25.0400	8.5590
563	316969	178453547	23.7276	8.2573	628	394384	247673152	25.0599	8.5635
564	318096	179406144	23.7487	8.2621	629	395641	248858189	25.0799	8.5681
565	319225	180362125	23.7697	8.2670	630	396900	250047000	25.0998	8.5726
566	320356	181321496	23.7908	8.2719	631	398161	251239591	25.1197	8.5772
567	321489	182284263	23.8118	8.2768	632	399424	252435968	25.1396	8.5817
568	322624	183250432	23.8328	8.2816	633	400689	253636137	25.1595	8.5862
569	323761	184220009	23.8537	8.2865	634	401956	254840104	25.1794	8.5907
570	324900	185193000	23.8747	8.2913	635	403225	256047875	25.1992	8.5952
571	326041	186169411	23.8956	8.2962	636	404496	257259456	25.2190	8.5997
572	327184	187149248	23.9165	8.3010	637	405769	258474853	25.2389	8.6043
573	328329	188132517	23.9374	8.3059	638	407044	259694072	25.2587	8.6088
574	329476	189119224	23.9583	8.3107	639	408321	260917119	25.2784	8.6132
575	330625	190109375	23.9792	8.3155	640	409600	262144000	25.2982	8.6177

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000—(CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
641	410881	263374721	25.3180	8.6222	706	498436	351895816	26.5707	8.9043
642	412164	264609288	25.3377	8.6267	707	499849	353393243	26.5895	8.9085
643	413449	265847707	25.3574	8.6312	708	501264	354894912	26.6083	8.9127
644	414736	267089984	25.3772	8.6357	709	502681	3564008?9	26.6271	8.9169
645	416025	268336125	25.3969	8.6401	710	504100	357911000	26.6458	8.9211
646	417316	269586136	25.4165	8.6446	711	505521	359425431	26.6646	8.9253
647	418609	270840023	25.4362	8.6490	712	506944	360944128	26.6833	8.9295
648	419904	272097792	25.4558	8.6535	713	508369	362467097	26.7021	8.9337
649	421201	273359449	25.4755	8.6579	714	509796	363994344	26.7208	8.9378
650	422500	274625000	25.4951	8.6624	715	511225	365525875	26.7395	8.9420
651	423801	275894451	25.5147	8.6668	716	512656	367061696	26.7582	8.9462
652	425104	277167808	25.5343	8.6713	717	514089	368601813	26.7769	8.9503
653	426409	278445077	25.5539	8.6757	718	515524	370146232	26.7955	8.9545
654	427716	279726264	25.5734	8.6801	719	516961	371694959	26.8142	8.9587
655	429025	281011375	25.5930	8.6845	720	518400	373248000	26.8328	8.9628
656	430336	282300416	25.6125	8.6890	721	519841	374805361	26.8514	8.9670
657	431649	283593393	25.6320	8.6934	722	521284	376367048	26.8701	8.9711
658	432964	284890312	25.6515	8.6978	723	522729	377933067	26.8887	8.9752
659	434281	286191179	25.6710	8.7022	724	524176	379503424	26.9072	8.9794
660	435600	287496000	25.6905	8.7066	725	525625	381078125	26.9258	8.9835
661	436921	288804781	25.7099	8.7110	726	527076	382657176	26.9444	8.9876
662	438244	290117528	25.7294	8.7154	727	528529	384240583	26.9629	8.9918
663	439569	291434247	25.7488	8.7198	728	529984	385828352	26.9815	8.9959
664	440896	292754944	25.7682	8.7241	729	531441	387420489	27.	9.
665	442225	294079625	25.7876	8.7285	730	532900	389017000	27.0185	9.0041
666	443556	295408296	25.8070	8.7329	731	534361	390617891	27.0370	9.0082
667	444889	296740963	25.8263	8.7373	732	535824	392223168	27.0555	9.0123
668	446224	298077632	25.8457	8.7416	733	537289	393832837	27.0740	9.0164
669	447561	299418309	25.8650	8.7460	734	538756	395446904	27.0924	9.0205
670	448900	300763000	25.8844	8.7503	735	540225	397065375	27.1109	9.0246
671	450241	302111711	25.9037	8.7547	736	541696	398688256	27.1293	9.0287
672	451584	303464448	25.9230	8.7590	737	543169	400315553	27.1477	9.0328
673	452929	304821217	25.9422	8.7634	738	544644	401947272	27.1662	9.0369
674	454276	306182024	25.9615	8.7677	739	546121	403583419	27.1846	9.0410
675	455625	307546875	25.9808	8.7721	740	547600	405224000	27.2029	9.0450
676	456976	308915776	26.	8.7764	741	549081	406869021	27.2213	9.0491
677	458329	310288733	26.0192	8.7807	742	550564	408518488	27.2397	9.0532
678	459684	311665752	26.0384	8.7850	743	552049	410172407	27.2580	9.0572
679	461011	313046839	26.0576	8.7893	744	553536	411830784	27.2764	9.0613
680	462400	314432000	26.0768	8.7937	745	555025	413493625	27.2947	9.0654
681	463761	315821241	26.0960	8.7980	746	556516	415160936	27.3130	9.0694
682	465124	317214568	26.1151	8.8023	747	558009	416832723	27.3313	9.0735
683	466489	318611987	26.1343	8.8066	748	559504	418508992	27.3496	9.0775
684	467856	320013504	26.1534	8.8109	749	561001	420189749	27.3679	9.0816
685	469225	321419125	26.1725	8.8152	750	562500	421875000	27.3861	9.0856
686	470596	322828856	26.1916	8.8194	751	564001	423564751	27.4044	9.0896
687	471969	324242703	26.2107	8.8237	752	565504	425259008	27.4226	9.0937
688	473344	325660672	26.2298	8.8280	753	567009	426957777	27.4408	9.0977
689	474721	327082769	26.2488	8.8323	754	568516	428661064	27.4591	9.1017
690	476100	328509000	26.2679	8.8366	755	570025	430368875	27.4773	9.1057
691	477481	329939371	26.2869	8.8408	756	571536	432081216	27.4955	9.1098
692	478864	331373888	26.3059	8.8451	757	573049	433798093	27.5136	9.1138
693	480249	332812557	26.3249	8.8493	758	574564	435519512	27.5318	9.1178
694	481636	334255384	26.3439	8.8536	759	576081	437245479	27.5500	9.1218
695	483025	335702375	26.3629	8.8578	760	577600	438976000	27.5681	9.1258
696	484416	337153536	26.3818	8.8621	761	579121	440711081	27.5862	9.1298
697	485809	338608873	26.4008	8.8663	762	580644	442450728	27.6043	9.1338
698	487204	340068392	26.4197	8.8706	763	582169	444194947	27.6225	9.1378
699	488601	341532099	26.4386	8.8748	764	583696	445943744	27.6405	9.1418
700	490000	343000000	26.4575	8.8790	765	585225	447697125	27.6586	9.1458
701	491401	344472101	26.4764	8.8833	766	586756	449455096	27.6767	9.1498
702	492804	345948408	26.4953	8.8875	767	588289	451217663	27.6948	9.1537
703	494209	347428927	26.5141	8 8917	768	589824	452984832	27.7128	9.1577
704	495616	348913664	26.5330	8.8959	769	591361	454756609	27.7308	9.1617
705	497025	350402625	26.5518	8.9001	770	592900	456533000	27.7489	9.1657

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000 — (CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
771	594441	458314011	27.7669	9.1696	836	698896	584277056	28.9137	9.4204
772	595984	460099648	27.7849	9.1736	837	700569	586376253	28.9310	9.4241
773	597529	461889917	27.8029	9.1775	838	702244	588480472	28.9482	9.4279
774	599076	463684824	27.8209	9.1815	839	703921	590589719	28.9655	9.4316
775	600625	465484375	27.8388	9.1855	840	705600	592704000	28.9828	9.4354
776	602176	467288576	27.8568	9.1894	841	707281	594823321	29.	9.4391
777	603729	469097433	27.8747	9.1933	842	708964	596947688	29.0172	9.4429
778	605284	470910952	27.8927	9.1973	843	710649	599077107	29.0345	9.4466
779	606841	472729139	27.9106	9.2012	844	712336	601211584	29.0517	9.4503
780	608400	474552000	27.9285	9.2052	845	714025	603351125	29.0689	9.4541
781	609961	476379541	27.9464	9.2091	846	715716	605495736	29.0861	9.4578
782	611524	478211768	27.9643	9.2130	847	717409	607645423	29.1033	9.4615
783	613089	480048687	27.9821	9.2170	848	719104	609800192	29.1204	9.4652
784	614656	481890304	28.	9.2209	849	720801	611960049	29.1376	9.4690
785	616225	483736625	28.0179	9.2248	850	722500	614125000	29.1548	9.4727
786	617796	485587656	28.0357	9.2287	851	724201	616295051	29.1719	9.4764
787	619369	487443403	28.0535	9.2326	852	725904	618470208	29 1890	9.4801
788	620944	489303872	28.0713	9.2365	853	727609	620650477	29.2062	9 4838
789	622521	491169069	28.0891	9.2404	854	729316	622835864	29.2233	9.4875
790	624100	493039000	28.1069	9.2443	855	731025	625026375	29.2404	9.4912
791	625681	494913671	28.1247	9.2482	856	732736	627222016	29.2575	9.4949
792	627264	496793088	28.1425	9.2521	857	734449	629422793	29.2746	9.4986
793	628849	498677257	28.1603	9.2560	858	736164	631628712	29.2916	9.5023
794	630436	500566184	28.1780	9.2599	859	737881	633839779	29.3087	9.5060
795	632025	502459875	28.1957	9.2638	860	739600	636056000	29.3258	9.5097
796	633616	504358336	28.2135	9.2677	861	741321	638277381	29.3428	9.5134
797	635209	506261573	28.2312	9.2716	862	743044	640503928	29.3598	9.5171
798	636804	508169592	28.2489	9.2754	863	744769	642735647	29.3769	9.5207
799	638401	510082399	28.2666	9.2793	864	746496	644972544	29.3939	9.5244
800	640000	512000000	28.2843	9.2832	865	748225	647214625	29.4109	9.5281
801	641601	513922401	28.3019	9.2870	866	749956	649461896	29.4279	9.5317
802	643204	515849608	28.3196	9.2909	867	751689	651714363	29.4449	9.5354
803	644809	517781627	28.3373	9.2948	868	753424	653972032	29.4618	9.5391
804	646416	519718464	28.3549	9.2986	869	755161	656234909	29.4788	9.5427
805	648025	521660125	28.3725	9.3025	870	756900	658503000	29.4958	9.5464
806	649636	523606616	28.3901	9.3063	871	758641	660776311	29.5127	9.5501
807	651249	525557943	28.4077	9.3102	872	760384	663054848	29.5296	9.5537
808	652864	527514112	28.4253	9.3140	873	762129	665338617	29.5466	9.5574
809	654481	529475129	28.4429	9.3179	874	763876	667627624	29.5635	9.5610
810	656100	531441000	28.4605	9.3217	875	765625	669921875	29.5804	9.5647
811	657721	533411731	28.4781	9.3255	876	767376	672221376	29.5973	9.5683
812	659344	535387328	28.4956	9.3294	877	769129	674526133	29.6142	9.5719
813	660969	537367797	28.5132	9.3332	878	770884	676836152	29.6311	9.5756
814	662596	539353144	28.5307	9.3370	879	772641	679151439	29.6479	9.5792
815	664225	541343375	28.5482	9.3408	880	774400	681472000	29.6648	9.5828
816	665856	543338496	28.5657	9.3447	881	776161	683797841	29.6816	9.5865
817	667489	545338513	28.5832	9.3485	882	777924	686128968	29.6985	9.5901
818	669124	547343432	28.6007	9.3523	883	779689	688465387	29.7153	9.5937
819	670761	549353259	28.6182	9.3561	884	781456	690807104	29.7321	9.5973
820	672400	551368000	28.6356	9.3599	885	783225	693154125	29.7489	9.6010
821	674041	553387661	28.6531	9.3637	886	784996	695506456	29.7658	9.6046
822	675684	555412248	28.6705	9.3675	887	786769	697864103	29.7825	9.6082
823	677329	557441767	28.6880	9.3713	888	788544	700227072	29.7993	9.6118
824	678976	559476224	28.7054	9.3751	889	790321	702595369	29.8161	9.6154
825	680625	561515625	28.7228	9.3789	890	792100	704969000	29.8329	9.6190
826	682276	563559976	28.7402	9.3827	891	793881	707347971	29.8496	9.6226
827	683929	565609283	28.7576	9.3865	892	795664	709732288	29.8664	9.6262
828	685584	567663552	28.7750	9.3902	893	797449	712121957	29.8831	9.6298
829	687241	569722789	28.7924	9.3940	894	799236	714516984	29.8998	9.6334
830	688900	571787000	28.8097	9.3978	895	801025	716917375	29.9166	9.6370
831	690561	573856191	28.8271	9.4016	896	802816	719323136	29.9333	9.6406
832	692224	575930368	28.8444	9.4053	897	804609	721734273	29.9500	9.6442
833	693889	578009537	28.8617	9.4091	898	806404	724150792	29.9666	9.6477
834	695556	580093704	28.8791	9.4129	899	808201	726572699	29.9833	9 6513
835	697225	582182875	28.8964	9.4166	900	810000	729000000	30.	9.6549

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000—(CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
901	811801	731432701	30.0167	9.6585	951	904401	860085351	30.8383	9.8339
902	813604	733870808	30.0333	9.6620	952	906304	862301408	30.8545	9.8374
903	815409	736314327	30.0500	9.6656	953	908209	865523177	30.8707	9.8408
904	817216	738763264	30.0666	9.6692	954	910116	868250664	30.8869	9.8443
905	819025	741217625	30.0832	9.6727	955	912025	870983875	30.9031	9.8477
906	820836	743677416	30.0998	9.6763	956	913936	873722816	30.9192	9.8511
907	822649	746142643	30.1164	9.6799	957	915849	876467493	30.9354	9.8546
908	824464	748613312	30.1330	9.6834	958	917764	879217912	30.9516	9.8580
909	826281	751089429	30.1496	9.6870	959	919681	881974079	30.9677	9.8614
910	828100	753571000	30.1662	9.6905	960	921600	884736000	30.9839	9.8648
911	829921	756058031	30.1828	9.6941	961	923521	887503681	31.	9.8683
912	831744	758550528	30.1993	9.6976	962	925444	890277128	31.0161	9.8717
913	833569	761048497	30.2159	9.7012	963	927369	893056347	31.0322	9.8751
914	835396	763551944	30.2324	9.7047	964	929296	895841344	31.0483	9.8785
915	837225	766060875	30.2490	9.7082	965	931225	898632125	31.0644	9.8819
916	839056	768575296	30.2655	9.7118	966	933156	901428696	31.0805	9.8854
917	840889	771095213	30.2820	9.7153	967	935089	904231063	31.0966	9.8888
918	842724	773620632	30.2985	9.7188	968	937024	907039232	31.1127	9.8922
919	844561	776151559	30.3150	9.7224	969	938961	909853209	31.1288	9.8956
920	846400	778688000	30.3315	9.7259	970	940900	912673000	31.1448	9.8990
921	848241	781229961	30.3480	9.7294	971	942841	915498611	31.1609	9.9024
922	850084	783777448	30.3645	9.7329	972	944784	918330048	31.1769	9.9058
923	851929	786330467	30.3809	9.7364	973	946729	921167317	31.1929	9.9092
924	853776	788889024	30.3974	9.7400	974	948676	924010424	31.2090	9.9126
925	855625	791453125	30.4138	9.7435	975	950625	926859375	31.2250	9.9160
926	857476	794022776	30.4302	9.7470	976	952576	929714176	31.2410	9.9194
927	859329	796597983	30.4467	9.7505	977	954529	932574833	31.2570	9.9227
928	861184	799178752	30.4631	9.7540	978	956484	935441352	31.2730	9.9261
929	863041	801765089	30.4795	9.7575	979	958441	938313739	31.2890	9.9295
930	864900	804357000	30.4959	9.7610	980	960400	941192000	31.3050	9.9329
931	866761	806954491	30.5123	9.7645	981	962361	944076141	31.3209	9.9363
932	868624	809557568	30.5287	9.7680	982	964324	946966168	31.3369	9.9396
933	870489	812166237	30.5450	9.7715	983	966289	949862087	31.3528	9.9430
934	872356	814780504	30.5614	9.7750	984	968256	952763904	31.3688	9.9464
935	874225	817400375	30.5778	9.7785	985	970225	955671625	31.3847	9.9497
936	876096	820025856	30.5941	9.7819	986	972196	958585256	31.4006	9.9531
937	877969	822656953	30.6105	9.7854	987	974169	961504803	31.4166	9.9565
938	879844	825293672	30.6268	9.7889	988	976144	964430272	31.4325	9.9598
939	881721	827936019	30.6431	9.7924	989	978121	967361669	31.4484	9.9632
940	883600	830584000	30.6594	9.7959	990	980100	970299000	31.4643	9.9666
941	885481	833237621	30.6757	9.7993	991	982081	973242271	31.4802	9.9699
942	887364	835896888	30.6920	9.8028	992	984064	976191488	31.4960	9.9733
943	889249	838561807	30.7083	9.8063	993	986049	979146657	31.5119	9.9766
944	891136	841232384	30.7246	9.8097	994	988036	982107784	31.5278	9.9800
945	893025	843908625	30.7409	9.8132	995	990025	985074875	31.5436	9.9833
946	894916	846590536	30.7571	9.8167	996	992016	988047936	31.5595	9.9866
947	896809	849278123	30.7734	9.8201	997	994009	991026973	31.5753	9.9900
948	898704	851971392	30.7896	9.8236	998	996004	994011992	31.5911	9.9933
949	900601	854670349	30.8058	9.8270	999	998001	997002999	31.6070	9.9967
950	902500	857375000	30.8221	9.8305	1000	1000000	1000000000	31.6228	10.

To find the square or cube of any whole number ending with ciphers. First, omit all the final ciphers. Take from the table the square or cube (as the case may be) of the rest of the number. To this *square* add *twice* as many ciphers as there were final ciphers in the original number. To the *cube* add *three* times as many as in the original number. Thus, for 905000^2; $9052 = 819025$. Add twice 2 ciphers, obtaining 81902500000. For 905000^3, $9053 = 741217625$. Add 3 times 2 ciphers, obtaining 741217625000000.

For tables of **fifth roots, fifth powers,** and **square roots of fifth powers,** see pp 251 to 253.

Square Roots and Cube Roots of Numbers from 1000 to 10000.

No errors.

Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.
1005	31.70	10.02	1405	37.48	11.20	1805	42.49	12.18	2205	46.96	13.02
1010	31.78	10.03	1410	37.55	11.21	1810	42.54	12.19	2210	47.01	13.03
1015	31.86	10.05	1415	37.62	11.23	1815	42.60	12.20	2215	47.06	13.04
1020	31.94	10.07	1420	37.68	11.24	1820	42.66	12.21	2220	47.12	13.05
1025	32.02	10.08	1425	37.75	11.25	1825	42.72	12.22	2225	47.17	13.05
1030	32.09	10.10	1430	37.82	11.27	1830	42.78	12.23	2230	47.22	13.06
1035	32.17	10.12	1435	37.88	11.28	1835	42.84	12.24	2235	47.28	13.07
1040	32.25	10.13	1440	37.95	11.29	1840	42.90	12.25	2240	47.33	13.08
1045	32.33	10.15	1445	38.01	11.31	1845	42.95	12.26	2245	47.38	13.09
1050	32.40	10.16	1450	38.08	11.32	1850	43.01	12.28	2250	47.43	13.10
1055	32.48	10.18	1455	38.14	11.33	1855	43.07	12.29	2255	47.49	13.11
1060	32.56	10.20	1460	38.21	11.34	1860	43.13	12.30	2260	47.54	13.12
1065	32.63	10.21	1465	38.28	11.36	1865	43.19	12.31	2265	47.59	13.13
1070	32.71	10.23	1470	38.34	11.37	1870	43.24	12.32	2270	47.64	13.14
1075	32.79	10.24	1475	38.41	11.38	1875	43.30	12.33	2275	47.70	13.15
1080	32.86	10.26	1480	38.47	11.40	1880	43.36	12.34	2280	47.75	13.16
1085	32.94	10.28	1485	38.54	11.41	1885	43.42	12.35	2285	47.80	13.17
1090	33.02	10.29	1490	38.60	11.42	1890	43.47	12.36	2290	47.85	13.18
1095	33.09	10.31	1495	38.67	11.43	1895	43.53	12.37	2295	47.91	13.19
1100	33.17	10.32	1500	38.73	11.45	1900	43.59	12.39	2300	47.96	13.20
1105	33.24	10.34	1505	38.79	11.46	1905	43.65	12.40	2305	48.01	13.21
1110	33.32	10.35	1510	38.86	11.47	1910	43.70	12.41	2310	48.06	13.22
1115	33.39	10.37	1515	38.92	11.49	1915	43.76	12.42	2315	48.11	13.23
1120	33.47	10.38	1520	38.99	11.50	1920	43.82	12.43	2320	48.17	13.24
1125	33.54	10.40	1525	39.05	11.51	1925	43.87	12.44	2325	48.22	13.25
1130	33.62	10.42	1530	39.12	11.52	1930	43.93	12.45	2330	48.27	13.26
1135	33.69	10.43	1535	39.18	11.54	1935	43.99	12.46	2335	48.32	13.27
1140	33.76	10.45	1540	39.24	11.55	1940	44.05	12.47	2340	48.37	13.28
1145	33.84	10.46	1545	39.31	11.56	1945	44.10	12.48	2345	48.43	13.29
1150	33.91	10.48	1550	39.37	11.57	1950	44.16	12.49	2350	48.48	13.30
1155	33.99	10.49	1555	39.43	11.59	1955	44.22	12.50	2355	48.53	13.30
1160	34.06	10.51	1560	39.50	11.60	1960	44.27	12.51	2360	48.58	13.31
1165	34.13	10.52	1565	39.56	11.61	1965	44.33	12.53	2365	48.63	13.32
1170	34.21	10.54	1570	39.62	11.62	1970	44.38	12.54	2370	48.68	13.33
1175	34.28	10.55	1575	39.69	11.63	1975	44.44	12.55	2375	48.73	13.34
1180	34.35	10.57	1580	39.75	11.65	1980	44.50	12.56	2380	48.79	13.35
1185	34.42	10.58	1585	39.81	11.66	1985	44.55	12.57	2385	48.84	13.36
1190	34.50	10.60	1590	39.87	11.67	1990	44.61	12.58	2390	48.89	13.37
1195	34.57	10.61	1595	39.94	11.68	1995	44.67	12.59	2395	48.94	13.38
1200	34.64	10.63	1600	40.00	11.70	2000	44.72	12.60	2400	48.99	13.39
1205	34.71	10.64	1605	40.06	11.71	2005	44.78	12.61	2405	49.04	13.40
1210	34.79	10.66	1610	40.12	11.72	2010	44.83	12.62	2410	49.09	13.41
1215	34.86	10.67	1615	40.19	11.73	2015	44.89	12.63	2415	49.14	13.42
1220	34.93	10.69	1620	40.25	11.74	2020	44.94	12.64	2420	49.19	13.43
1225	35.00	10.70	1625	40.31	11.76	2025	45.00	12.65	2425	49.24	13.43
1230	35.07	10.71	1630	40.37	11.77	2030	45.06	12.66	2430	49.30	13.44
1235	35.14	10.73	1635	40.44	11.78	2035	45.11	12.67	2435	49.35	13.45
1240	35.21	10.74	1640	40.50	11.79	2040	45.17	12.68	2440	49.40	13.46
1245	35.28	10.76	1645	40.56	11.80	2045	45.22	12.69	2445	49.45	13.47
1250	35.36	10.77	1650	40.62	11.82	2050	45.28	12.70	2450	49.50	13.48
1255	35.43	10.79	1655	40.68	11.83	2055	45.33	12.71	2460	49.60	13.50
1260	35.50	10.80	1660	40.74	11.84	2060	45.39	12.72	2470	49.70	13.52
1265	35.57	10.82	1665	40.80	11.85	2065	45.44	12.73	2480	49.80	13.54
1270	35.64	10.83	1670	40.87	11.86	2070	45.50	12.74	2490	49.90	13.55
1275	35.71	10.84	1675	40.93	11.88	2075	45.55	12.75	2500	50.00	13.57
1280	35.78	10.86	1680	40.99	11.89	2080	45 61	12.77	2510	50.10	13.59
1285	35.85	10.87	1685	41.05	11.90	2085	45.66	12.78	2520	50.20	13.61
1290	35.92	10.89	1690	41.11	11.91	2090	45.72	12.79	2530	50.30	13.63
1295	35.99	10.90	1695	41.17	11.92	2095	45.77	12.80	2540	50.40	13.64
1300	36.06	10.91	1700	41.23	11.93	2100	45.83	12.81	2550	50 50	13.66
1305	36.12	10.93	1705	41.29	11.95	2105	45.88	12.82	2560	50.60	13 68
1310	36.19	10.94	1710	41.35	11.96	2110	45.93	12.83	2570	50.70	13.70
1315	36.26	10.96	1715	41.41	11.97	2115	45.99	12.84	2580	50.79	13.72
1320	86.33	10.97	1720	41.47	11.98	2120	46.04	12.85	2590	50.89	13.73
1325	36.40	10.98	1725	41.53	11.99	2125	46.10	12.86	2600	50.99	13.75
1330	36.47	11.00	1730	41.59	12.00	2130	46.15	12.87	2610	51.09	13.77
1335	36.54	11.01	1735	41.65	12.02	2135	46.21	12.88	2620	51.19	13.79
1340	36.61	11.02	1740	41.71	12.03	2140	46.26	12.89	2630	51.28	13.80
1345	36.67	11.04	1745	41.77	12.04	2145	46.31	12.90	2640	51.38	13.82
1350	36.74	11.05	1750	41.83	12.05	2150	46.37	12.91	2650	51.48	13.84
1355	36.81	11.07	1755	41.89	12.06	2155	46.42	12.92	2660	51.58	13.86
1360	36.88	11.08	1760	41.95	12.07	2160	46.48	12.93	2670	51.67	13.87
1365	36.95	11.09	1765	42.01	12.09	2165	46.53	12.94	2680	51.77	13.89
1370	37.01	11.11	1770	42.07	12.10	2170	46.58	12.95	2690	51.87	13.91
1375	37.08	11.12	1775	42.13	12.11	2175	46.64	12.96	2700	51.96	13.92
1380	37.15	11.13	1780	42.19	12.12	2180	46.69	12.97	2710	52.06	13.94
1385	37.22	11.15	1785	42.25	12.13	2185	46.74	12.98	2720	52.15	13.96
1390	37.28	11.16	1790	42.31	12.14	2190	46.80	12.99	2730	52.25	13.98
1395	37.35	11.17	1795	42.37	12.15	2195	46.85	13.00	2740	52.35	13.99
1400	37.42	11.19	1800	42.43	12.16	2200	46.90	13.01	2750	52.44	14.01

Square Roots and Cube Roots of Numbers from 1000 to 10000
—(CONTINUED.)

Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.
2760	52.54	14.03	3550	59.58	15.25	4340	65.88	16.31	5130	71.62	17.25
2770	52.63	14.04	3560	59.67	15.27	4350	65.95	16.32	5140	71.69	17.26
2780	52.73	14.06	3570	59.75	15.28	4360	66.03	16.34	5150	71.76	17.27
2790	52.82	14.08	3580	59.83	15.30	4370	66.11	16.35	5160	71.83	17.28
2800	52.92	14.09	3590	59.92	15.31	4380	66.18	16.36	5170	71.90	17.29
2810	53.01	14.11	3600	60.00	15.33	4390	66.26	16.37	5180	71.97	17.30
2820	53.10	14.13	3610	60.08	15.34	4400	66.33	16.39	5190	72.04	17.31
2830	53.20	14.14	3620	60.17	15.35	4410	66.41	16.40	5200	72.11	17.32
2840	53.29	14.16	3630	60.25	15.37	4420	66.48	16.41	5210	72.18	17.34
2850	53.39	14.18	3640	60.33	15.38	4430	66.56	16.42	5220	72.25	17.35
2860	53.48	14.19	3650	60.42	15.40	4440	66.63	16.44	5230	72.32	17.36
2870	53.57	14.21	3660	60.50	15.41	4450	66.71	16.45	5240	72.39	17.37
2880	53.67	14.23	3670	60.58	15.42	4460	66.78	16.46	5250	72.46	17.38
2890	53.76	14.24	3680	60.66	15.44	4470	66.86	16.47	5260	72.53	17.39
2900	53.85	14.26	3690	60.75	15.45	4480	66.93	16.49	5270	72.59	17.40
2910	53.94	14.28	3700	60.83	15.47	4490	67.01	16.50	5280	72.66	17.41
2920	54.04	14.29	3710	60.91	15.48	4500	67.08	16.51	5290	72.73	17.42
2930	54.13	14.31	3720	60.99	15.49	4510	67.16	16.52	5300	72.80	17.44
2940	54.22	14.33	3730	61.07	15.51	4520	67.23	16.53	5310	72.87	17.45
2950	54.31	14.34	3740	61.16	15.52	4530	67.31	16.55	5320	72.94	17.46
2960	54.41	14.36	3750	61.24	15.54	4540	67.38	16.56	5330	73.01	17.47
2970	54.50	14.37	3760	61.32	15.55	4550	67.45	16.57	5340	73.08	17.48
2980	54.59	14.39	3770	61.40	15.56	4560	67.53	16.58	5350	73.14	17.49
2990	54.68	14.41	3780	61.48	15.58	4570	67.60	16.59	5360	73.21	17.50
3000	54.77	14.42	3790	61.56	15.59	4580	67.68	16.61	5370	73.28	17.51
3010	54.86	14.44	3800	61 64	15.60	4590	67.75	16.62	5380	73.35	17.52
3020	54.95	14.45	3810	61.73	15.62	4600	67.82	16.63	5390	73.42	17.53
3030	55.05	14.47	3820	61.81	15.63	4610	67.90	16.64	5400	73.48	17.54
3040	55.14	14.49	3830	61.89	15.65	4620	67.97	16.66	5410	73.55	17.55
3050	55.23	14.50	3840	61.97	15.66	4630	68.04	16.67	5420	73.62	17.57
3060	55.32	14.52	3850	62.05	15.67	4640	68.12	16.68	5430	73.69	17.58
3070	55.41	14.53	3860	62.13	15.69	4650	68.19	16.69	5440	73.76	17.59
3080	55.50	14.55	3870	62.21	15.70	4660	68.26	16.70	5450	73.82	17.60
3090	55.59	14.57	3880	62.29	15.71	4670	68.34	16.71	5460	73.89	17.61
3100	55.68	14.58	3890	62.37	15.73	4680	68.41	16.73	5470	73.96	17.62
3110	55.77	14.60	3900	62.45	15.74	4690	68.48	16.74	5480	74.03	17.63
3120	55.86	14.61	3910	62.53	15.75	4700	68 56	16.75	5490	74.09	17.64
3130	55.95	14.63	3920	62.61	15.77	4710	68.63	16.76	5500	74.16	17.65
3140	56.04	14.64	3930	62.69	15.78	4720	68.70	16.77	5510	74.23	17.66
3150	56.12	14.66	3940	62.77	15.79	4730	68.77	16.79	5520	74.30	17.67
3160	56.21	14.67	3950	62.85	15.81	4740	68.85	16.80	5530	74.36	17.68
3170	56.30	14.69	3960	62.93	15.82	4750	68.92	16.81	5540	74.43	17.69
3180	56.39	14.71	3970	63.01	15.83	4760	68 99	16.82	5550	74.50	17.71
3190	56.48	14.72	3980	63.09	15.85	4770	69.07	16.83	5560	74.57	17.72
3200	56.57	14.74	3990	63.17	15.86	4780	69.14	16.85	5570	74.63	17.73
3210	56.66	14.75	4000	63.25	15.87	4790	69 21	16.86	5580	74.70	17.74
3220	56.75	14.77	4010	63.32	15.89	4800	69.28	16.87	5590	74.77	17.75
3230	56.83	14.78	4020	63.40	15.90	4810	69 35	16.88	5600	74.83	17.76
3240	56.92	14.80	4030	63.48	15.91	4820	69.43	16 89	5610	74.90	17.77
3250	57.01	14 81	4040	63.56	15.93	4830	69.50	16.90	5620	74.97	17.78
3260	57.10	14.83	4050	63.64	15.94	4840	69.57	16.92	5630	75.03	17.79
3270	57.18	14.84	4060	63.72	15.95	4850	69.64	16.93	5640	75.10	17.80
3280	57.27	14.86	4070	63 80	15.97	4860	69.71	16.94	5650	75.17	17.81
3290	57.36	14.87	4080	63.87	15.98	4870	69.79	16.95	5660	75.23	17.82
3300	57.45	14.89	4090	63.95	15.99	4880	69.86	16.96	5670	75.30	17.83
3310	57.53	14.90	4100	64.03	16.01	4890	69.93	16.97	5680	75.37	17.84
3320	57.62	14.92	4110	64.11	16.02	4900	70.00	16.98	5690	75.43	17.85
3330	57.71	14.93	4120	64.19	16.03	4910	70.07	17.00	5700	75.50	17.86
3340	57.79	14.95	4130	64.27	16.04	4920	70.14	17.01	5710	75.56	17.87
3350	57.88	14.96	4140	64.34	16.06	4930	70.21	17.02	5720	75.63	17.88
3360	57.97	14.98	4150	64.42	16.07	4940	70.29	17.03	5730	75.70	17.89
3370	58.05	14.99	4160	64.50	16.08	4950	70.36	17.04	5740	75.76	17.90
3380	58.14	15.01	4170	64.58	16.10	4960	70.43	17.05	5750	75.83	17.92
3390	58.22	15.02	4180	64.65	16.11	4970	70.50	17.07	5760	75.89	17.93
3400	58.31	15.04	4190	64.73	16.12	4980	70.57	17.08	5770	75.96	17.94
3410	58.40	15.05	4200	64.81	16.13	4990	70.64	17.09	5780	76.03	17.95
3420	58.48	15.07	4210	64.88	16.15	5000	70.71	17.10	5790	76.09	17.96
3430	58.57	15.08	4220	64.96	16.16	5010	70.78	17.11	5800	76.16	17.97
3440	58.65	15.10	4230	65.04	16.17	5020	70.85	17.12	5810	76.22	17.98
3450	58.74	15.11	4240	65.12	16.19	5030	70.92	17.13	5820	76.29	17.99
3460	58.82	15.12	4250	65.19	16.20	5040	70.99	17.15	5830	76.35	18.00
3470	58.91	15.14	4260	65.27	16.21	5050	71.06	17.16	5840	76.42	18.01
3480	58.99	15.15	4270	65.35	16.22	5060	71.13	17.17	5850	76.49	18.02
3490	59.08	15.17	4280	65.42	16.24	5070	71.20	17.18	5860	76.55	18.03
3500	59.16	15.18	4290	65.50	16.25	5080	71.27	17.19	5870	76.62	18.04
3510	59.25	15.20	4300	65 57	16.26	5090	71.34	17.20	5880	76.68	18.05
3520	59.33	15.21	4310	65.65	16.27	5100	71.41	17.21	5890	76.75	18.06
3530	59.41	15.23	4320	65.73	16.29	5110	71.48	17.22	5900	76.81	18.07
3540	59.50	15.24	4330	65.80	16.30	5120	71.55	17.24	5910	76.88	18.08

Square Roots and Cube Roots of Numbers from 1000 to 10000

—(CONTINUED.)

Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt
5920	76 94	18.09	6710	81.91	18.86	7500	86.60	19.57	8290	91.05	20.24
5930	77.01	18.10	6720	81.98	18.87	7510	86.66	19.58	8300	91.10	20.25
5940	77.07	18.11	6730	82.04	18.88	7520	86.72	19.59	8310	91.16	20.26
5950	77.14	18.12	6740	82.10	18.89	7530	86.78	19.60	8320	91.21	20.26
5960	77.20	18.13	6750	82.16	18.90	7540	86.83	19.61	8330	91.27	20.27
5970	77.27	18.14	6760	82.22	18.91	7550	86.89	19.62	8340	91.32	20.28
5980	77.33	18.15	6770	82.28	18.92	7560	86.95	19.63	8350	91.38	20.29
5990	77.40	18.16	6780	82.34	18.93	7570	87.01	19.64	8360	91.43	20.30
6000	77.46	18.17	6790	82.40	18.94	7580	87.06	19.64	8370	91.49	20.30
6010	77.52	18.18	6800	82.46	18.95	7590	87.12	19.65	8380	91.54	20.31
6020	77.59	18.19	6810	82.52	18.95	7600	87.18	19.66	8390	91.60	20.32
6030	77.65	18.20	6820	82.58	18.96	7610	87.24	19.67	8400	91.65	20.33
6040	77.72	18.21	6830	82.64	18.97	7620	87.29	19.68	8410	91.71	20.34
6050	77.78	18.22	6840	82.70	18.98	7630	87.35	19.69	8420	91.76	20.34
6060	77.85	18.23	6850	82.76	18.99	7640	87.41	19.70	8430	91.82	20.35
6070	77.91	18.24	6860	82.83	19.00	7650	87.46	19.70	8440	91.87	20.36
6080	77.97	18.25	6870	82.89	19.01	7660	87.52	19.71	8450	91.92	20.37
6090	78.04	18.26	6880	82.95	19.02	7670	87.58	19.72	8460	91.98	20.38
6100	78.10	18.27	6890	83.01	19.03	7680	87.64	19.73	8470	92.03	20.38
6110	78.17	18.28	6900	83.07	19.04	7690	87.69	19.74	8480	92.09	20.39
6120	78.23	18.29	6910	83.13	19.05	7700	87.75	19.75	8490	92.14	20.40
6130	78.29	18.30	6920	83.19	19.06	7710	87.81	19.76	8500	92.20	20.41
6140	78.36	18.31	6930	83.25	19.07	7720	87.86	19.76	8510	92.25	20.42
6150	78.42	18.32	6940	83.31	19.07	7730	87.92	19.77	8520	92.30	20.42
6160	78.49	18.33	6950	83.37	19.08	7740	87.98	19.78	8530	92.36	20.43
6170	78.55	18.34	6960	83.43	19.09	7750	88.03	19.79	8540	92.41	20.44
6180	78.61	18.35	6970	83.49	19.10	7760	88.09	19.80	8550	92.47	20.45
6190	78.68	18.36	6980	83.55	19.11	7770	88.15	19.81	8560	92.52	20.46
6200	78.74	18.37	6990	83.61	19.12	7780	88.20	19.81	8570	92.57	20.46
6210	78.80	18.38	7000	83.67	19.13	7790	88.26	19.82	8580	92.63	20.47
6220	78.87	18.39	7010	83.73	19.14	7800	88.32	19.83	8590	92.68	20.48
6230	78.93	18.40	7020	83.79	19.15	7810	88.37	19.84	8600	92.74	20.49
6240	78.99	18.41	7030	83.85	19.16	7820	88.43	19.85	8610	92.79	20.50
6250	79.06	18.42	7040	83.90	19.17	7830	88.49	19.86	8620	92.84	20.50
6260	79.12	18.43	7050	83.96	19.17	7840	88.54	19.87	8630	92.90	20.51
6270	79.18	18.44	7060	84.02	19.18	7850	88.60	19.87	8640	92.95	20.52
6280	79.25	18.45	7070	84.08	19.19	7860	88.66	19.88	8650	93.01	20.53
6290	79 31	18.46	7080	84.14	19.20	7870	88.71	19.89	8660	93.06	20.54
6300	79.37	18.47	7090	84.20	19.21	7880	88.77	19.90	8670	93.11	20.54
6310	79.44	18.48	7100	84.26	19.22	7890	88.83	19.91	8680	93.17	20.55
6320	79.50	18.49	7110	84.32	19.23	7900	88.88	19.92	8690	93.22	20.56
6330	79.56	18.50	7120	84.38	19.24	7910	88.94	19.92	8700	93.27	20.57
6340	79.62	18.51	7130	84.44	19.25	7920	88.99	19.93	8710	93.33	20.57
6350	79.69	18.52	7140	84.50	19.26	7930	89.05	19.94	8720	93.38	20.58
6360	79.75	18.53	7150	84.56	19.26	7940	89.11	19.95	8730	93.43	20.59
6370	79.81	18.54	7160	84.62	19.27	7950	89.16	19.96	8740	93.49	20.60
6380	79.87	18.55	7170	84.68	19.28	7960	89.22	19.97	8750	93.54	20.61
6390	79.94	18.56	7180	84.73	19.29	7970	89.27	19.97	8760	93.59	20.61
6400	80.00	18.57	7190	84.79	19.30	7980	89.33	19.98	8770	93.65	20.62
6410	80.06	18.58	7200	84.85	19.31	7990	89.39	19.99	8780	93.70	20.63
6420	80.12	18.59	7210	84.91	19.32	8000	89.44	20.00	8790	93.75	20.64
6430	80.19	18.60	7220	84.97	19.33	8010	89.50	20.01	8800	93.81	20.65
6440	80.25	18.60	7230	85.03	19.34	8020	89.55	20.02	8810	93.86	20.65
6450	80.31	18.61	7240	85.09	19.35	8030	89.61	20.02	8820	93.91	20.66
6460	80.37	18.62	7250	85.15	19.35	8040	89.67	20.03	8830	93.97	20.67
6470	80.44	18.63	7260	85.21	19.36	8050	89.72	20.04	8840	94.02	20.68
6480	80.50	18.64	7270	85.26	19.37	8060	89.78	20.05	8850	94.07	20.68
6490	80.56	18.65	7280	85.32	19.38	8070	89.83	20.06	8860	94.13	20.69
6500	80.62	18.66	7290	85.38	19.39	8080	89.89	20.07	8870	94.18	20.70
6510	80.68	18.67	7300	85.44	19.40	8090	89.94	20.07	8880	94.23	20.71
6520	80.75	18.68	7310	85.50	19.41	8100	90.00	20.08	8890	94.29	20.72
6530	80.81	18.69	7320	85.56	19.42	8110	90.06	20.09	8900	94.34	20.72
6540	80.87	18.70	7330	85.62	19.43	8120	90.11	20.10	8910	94.39	20.73
6550	80.93	18.71	7340	85.67	19.43	8130	90.17	20.11	8920	94.45	20.74
6560	80.99	18.72	7350	85.73	19.44	8140	90.22	20.12	8930	94.50	20.75
6570	81.06	18.73	7360	85.79	19.45	8150	90.28	20.12	8940	94.55	20.75
6580	81.12	18.74	7370	85.85	19.46	8160	90.33	20.13	8950	94.60	20.76
6590	81.18	18.75	7380	85.91	19.47	8170	90.39	20.14	8960	94.66	20.77
6600	81.24	18.76	7390	85.97	19.48	8180	90.44	20.15	8970	94.71	20.78
6610	81.30	18.77	7400	86.02	19.49	8190	90.50	20.16	8980	94.76	20.79
6620	81.36	18.78	7410	86.08	19.50	8200	90.55	20.17	8990	94.82	20.79
6630	81.42	18.79	7420	86.14	19.50	8210	90.61	20.17	9000	94.87	20.80
6640	81.49	18.80	7430	86.20	19.51	8220	90.66	20.18	9010	94.92	20.81
6650	81.55	18.81	7440	86.26	19.52	8230	90.72	20.19	9020	94.97	20.82
6660	81.61	18.81	7450	86.31	19.53	8240	90.77	20.20	9030	95.03	20.82
6670	81.67	18.82	7460	86.37	19.54	8250	90.83	20.21	9040	95.08	20.83
6680	81.73	18.83	7470	86.43	19.55	8260	90.88	20.21	9050	95.13	20.84
6690	81.79	18.84	7480	86.49	19.56	8270	90.94	20.22	9060	95.18	20.85
6700	81.85	18.85	7490	86.54	19.57	8280	90.99	20.23	9070	95.24	20.86

Square Roots and Cube Roots of Numbers from 1000 to 10000

—(CONTINUED.)

Num.	Sq. Rt.	Cu. Rt	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.
9080	95.29	20.86	9320	96.54	21.04	9550	97.72	21.22	9780	98.89	21.39
9090	95.34	20.87	9330	96.59	21.05	9560	97.78	21.22	9790	98.94	21.39
9100	95.39	20.88	9340	96.64	21.06	9570	97.83	21.23	9800	98.99	21.40
9110	95.45	20.89	9350	96.70	21.07	9580	97 88	21.24	9810	99.05	21.41
9120	95.50	20.89	9360	96.75	21.07	9590	97.93	21.25	9820	99.10	21.41
9130	95.55	20.90	9370	96.80	21.08	9600	97.98	21.25	9830	99.15	21.42
9140	95.60	20.91	9380	96.85	21.09	9610	98.03	21.26	9840	99.20	21.43
9150	95.66	20.92	9390	96.90	21.10	9620	98.08	21.27	9850	99.25	21 44
9160	95.71	20.92	9400	96.95	21.10	9630	98.13	21.28	9860	99.30	21.44
9170	95.76	20.93	9410	97.01	21.11	9640	98.18	21.28	9870	99.35	21.45
9180	95.81	20.94	9420	97.06	21.12	9650	98.23	21.29	9880	99.40	21.46
9190	95.86	20.95	9430	97.11	21.13	9660	98.29	21.30	9890	99.45	21.47
9200	95.92	20.95	9440	97.16	21.13	9670	98.34	21.30	9900	99.50	21.47
9210	95.97	20.96	9450	97.21	21.14	9680	98.39	21.31	9910	99.55	21 48
9220	96.02	20.97	9460	97.26	21.15	9690	98.44	21.32	9920	99.60	21.49
9230	96.07	20.98	9470	97.31	21.16	9700	98.49	21.33	9930	99.65	21.49
9240	96.12	20.98	9480	97.37	21.16	9710	98.54	21.33	9940	99.70	21.50
9250	96 18	20.99	9490	97.42	21.17	9720	98.59	21.34	9950	99.75	21.51
9260	96.23	21.00	9500	97.47	21.18	9730	98.64	21.35	9960	99.80	21.52
9270	96.28	21.01	9510	97.52	21.19	9740	98.69	21.36	9970	99.85	21.52
9280	96.33	21.01	9520	97.57	21.19	9750	98.74	21.36	9980	99.90	21.53
9290	96.38	21.02	9530	97.62	21.20	9760	98.79	21.37	9990	99.95	21.54
9300	96.44	21.03	9540	97.67	21.21	9770	98.84	21.38	10000	100.00	21.54
9310	96.49	21.04									

To find Square or Cube Roots of large numbers not contained in the column of numbers of the table.

Such roots may sometimes be taken at once from the table, by merely regarding the columns of powers as being columns of numbers; and those of numbers as being those of roots. Thus, if the *sq rt* of 25281 is reqd, first find that number in the column of *squares;* and opposite to it, in the column of numbers, is its sq rt 159. For the *cube rt* of 857375, find that number in the column of *cubes;* and opposite to it, in the col of numbers, is its cube rt 95. When the exact number is not contained in the column of squares, or cubes, as the case may be, we may use instead the number nearest to it, if no great accuracy is reqd. But when a considerable degree of accuracy is necessary, the following very correct methods may be used.

For the square root.

This rule applies both to whole numbers, and to those which are *partly* (not wholly) decimal. First in the foregoing manner, take out the tabular number, which is nearest to the given one; and also it tabular sq rt. Mult this tabular number by 3; to the prod add the given number. Call the sum A. Then mult the given number by 3; to the prod add the tabular number. Call the sum B. Then

A : B : : Tabular root : Reqd root.

Ex. Let the given number be 946.53. Here we find the nearest tabular number to be 947; and its tabular sq rt 30.7734. Hence,

947 = tab num
3
2841
946.53 = given num.
3787.53 = A.

and

946.53 = given num.
3
2839.59
947 = tab num.
3786.59 = B.

	A.		B.		Tab root.		Reqd root.
Then	3787.53	:	3786.59	: :	30.7734	:	30.7657 +.

The root as found by actual mathematical process is also 30.7657 +.

For the cube root.

This rule applies both to whole numbers, and to those which are *partly* decimal. First take out the tabular number which is nearest to the given one; and also its tabular cube rt. Mult this tabular number by 2; and to the prod add the given number. Call the sum A. Then mult the given number by 2; and to the prod add the tabular number. Call the sum B. Then

A : B : : Tabular root : Reqd root.

Ex. Let the given number be 7368. Here we find the nearest tabular number (in the column of *cubes*) to be 6859; and its tabular cube rt 19. Hence,

6859 = tab num.
2
13718
7368 = given num.
21086 = A.

and

7368 = given num.
2
14736
6859 = tab num.
21595 = B.

	A.		B.		Tab Root.		Reqd Rt.
Then, as	21086	:	21595	: :	19	:	19.4585

The root as found by correct mathematical process is 19.4588. The engineer rarely requires ev

this degree of accuracy; for his purposes, therefore, this process is greatly preferable to the ordinary laborious one.

To find the square root of a number which is wholly decimal.

Very simple, and correct to the third numeral figure inclusive. If the number does not contain at least five figures, *counting from the first numeral, and including it*, add one or more ciphers to make five. If, after that, the *whole number* is not separable into twos, add another cipher to make it so. Then beginning at the first numeral figure, and including it, assume the number to be a whole one. In the table find the number nearest to this assumed one; take out its tabular sq rt; move the decimal point of this tabular root to the left, *half* as many places as the finally modified *decimal* number has figures.

Ex. What is the sq rt of the decimal .002? Here, in order to have at least five decimal figures, counting from the first numeral (2), and including it, add ciphers thus, .00.20.00.0. But, as it is not now separable into twos, add another cipher, thus, .00,20,00,00. Then beginning at the first numeral (2), assume this decimal to be the whole number 200000. The nearest to this in the table is 199809; and the sq rt of this is 447. Now, the decimal number as finally modified, namely, .00.20,00.00, has eight figures; one-half of which is 4; therefore, move the decimal point of the root 447, four places to the left; making it .0447. This is the reqd sq rt of .002, correct to the third numeral 7 included.

To find the cube root of a number which is wholly decimal.

Very simple, and correct to the third numeral inclusive.

If the number does not contain at least five figures, counting from the first numeral, and including it, add one or more ciphers to make five. If, after that, the number is not separable into threes, add one or more ciphers to make it so. Then beginning at the first numeral, and including it, assume the number to be a whole one. In the table find the number nearest to this assumed one, and take out its tabular cub rt. Move the decimal point of this rt to the left, one-third as many places as the finally modified *decimal* number has figures.

Ex. What is the cube rt of the decimal .002? Here, in order to have at least five figures, counting from the first numeral (2), and including it, add ciphers thus, .002 000.0. But as it is not now separable into threes, add two more ciphers to make it so; thus, .002,000,000. Then beginning with the first numeral (2), assume the decimal to be the whole number 2000000. The nearest cube to this in the table in the column of cubes, is 2000376; and its tabular cube rt as found in the col of numbers, is 126. Now, the decimal number as finally modified, namely, .002 000 000, has nine figures; one-third of which is 3; therefore, move the decimal point of the root 126, three places to the left, making it .126. This is the reqd cube rt of the decimal .002, correct to the third numeral 6 included.

To find roots by logarithms, see p 39.

For tables of **fifth roots, fifth powers,** and **square roots of fifth powers,** see pp 251 to 253.

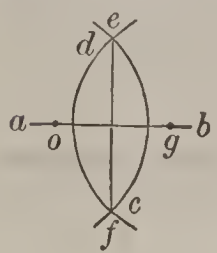
GEOMETRY.

Lines. Figures. Solids. defined. Strictly speaking a geometrical **line** is simply length, or distance. The lines we draw on paper have not only length, but breadth and thickness; still they are the most convenient symbol we can employ for denoting a geometrical line. **Straight** lines are also called **right** lines. A **vertical** line is one that points toward the center of the earth; and a **horizontal** one is at right angles to a vert one. **A plane figure** is merely any flat surface or area entirely enclosed by lines either straight or curved; which are called its outline, boundary, circumf, or periphery. We often confound the outline with the fig itself as when we speak of drawing circles, squares, &c; for we actually draw only their outlines. Geometrically speaking, a fig has length and breadth only; no thickness. **A solid** is any body; it has length, breadth, and thickness.

Geometrically similar figs or solids, are not necessarily of the same **size;** but only of precisely the same **shape.** Thus, any two squares are, scientifically speaking, similar to each other; so also any two circles, cubes, &c, no matter how different they may be in size. When they are not only of the same shape, but of the same size, they are said to be **similar,** and **equal.**

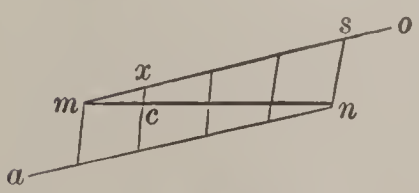
The **quantities of lines** are to each other simply as their **lengths;** but the quantities, or areas, or surfaces of similar **figures,** are as, or in proportion to, the **squares** of any one of the corresponding lines or sides which enclose the figures, or which may be drawn upon them; and the quantities, or solidities of similar **solids,** are as the **cubes** of any of the corresponding lines which form their edges, or the figures by which they are enclosed.

Rem.—Simple as the following operations appear, it is only by care, and good instruments, that they are made to give accurate results. Several of them can be much better performed by means of a metallic triangle having one perfectly accurate right angle. In the field, the tape-line, chain, or a measuring-rod will take the place of the dividers and ruler used indoors.



To divide a given line, *a b*, into two equal parts.

From its ends *a* and *b* as centers, and with any rad greater than one-half of *a b* describe the arcs c and *d*, and join *e f*. If the line *a b* is very long, first lay off equal dists *a o* and *b g*, each way from the ends, so as to approach conveniently near to each other; and then proceed as if *o g* were the line to be divided. Or measure *a b* by a scale, and thus ascertain its center.



To divide a given line, *m n*, into any given number of equal parts.

From *m* and *n* draw any two parallel lines *m* o and *n a* to an indefinite dist; and on them, from *m* and *n* step off the reqd number of equal parts of any convenient length: finally, join the corresponding points thus stepped off. Or only one line, as *m* o, may be drawn and stepped off, as to *s*, then join *s n*; and draw the other short lines parallel to it

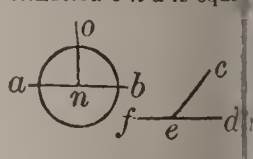
To divide a given line, *m n*, into two parts which shall have a given proportion to each other.

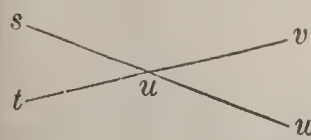
This is done on the same principle as the last; thus, let the proportion be as 1 to 3. First draw any line *m o*; and with any convenient opening of the dividers make *m x* equal to one step; and *x* equal to three steps. Join *s n*; and parallel to it draw *x c*. Then *m* c is to c *n* as 1 is to 3.

ANGLES.

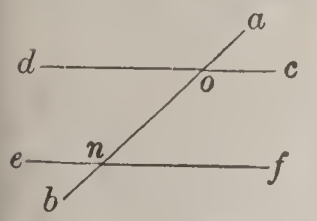
Angles. When two straight, or right lines meet each other at any inclination, the inclination is called an **angle;** and is measured by the degrees contained in the arc of a circle described from the point of meeting as a center. Since all circles, whether large or small, are supposed to be divided into 360 degrees, it follows that any number of degrees of small circle will measure the same degree of inclination as will the same number of a large one.

When two straight lines, as o *n* and *a b*, meet in such a manner that the inclination o *n a* is equal to the inclination *o n b*, then the two lines are said to be **perpendicular** to each other; and the angles *o n a* and *o n b*, are called **right angles;** and are each measd by, or are equal to, 90°, or one-fourth part of the circumf of a circle. Any angle, as *c e d*, smaller than a right angle, is called **acute** or sharp; and one *c e f*, larger than a right angle, is called **obtuse,** or blunt. When one line meets another, as in the first Fig on opposite page, the two angles on the same side of either line are called **contiguous,** or **adjacent.** Thus, *v u s* and *v u w* are adjacent; also *t u s* and *t u w*; *s u t* and *s u v*; *w u t* and *w u v*. The sum of two adjacent angles is always equal to two right angles; or to 180°. Therefore, if we know the number of degrees contained in one of them, and subtract it from 180°, we obtain the other.



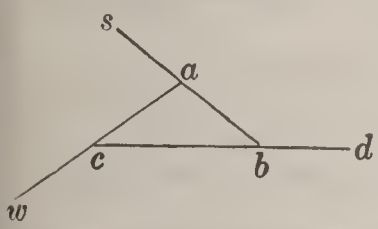


When two straight lines *cross* each other, forming four angles, either pair of those angles which point in exactly opposite directions are called **opposite,** or **vertical angles;** thus, the pair *s u t*, and *v u w* are opposite angles; also the pair *s u v* and *t u w*. The opposite angles of any pair are always equal to each other.



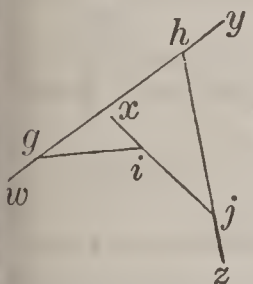
When a straight line *a b* crosses two parallel lines *c d*, *e f*, the **alternate** angles which form a kind of Z are equal to each other. Thus, the angles *d o n*, and *o n f*, are equal: as are also *c o n*, and *o n e*. Also the sum of the two internal angles on the same side of *a b*, is equal to two right angles, or 180°; thus, *c o n* + *o n f* = 180°; also *d o n* + *o n e* = 180°.

An interior angle,



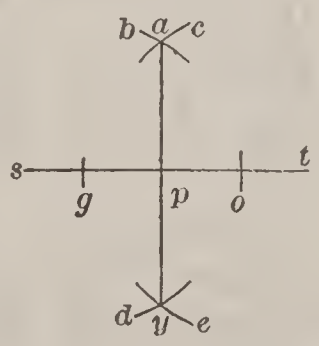
In any fig, is any angle formed *inside* of that fig, by the meeting of two of its sides, as the angles *c a b*, *a b c*, *b c a*, of this triangle. All the interior angles of any straight-lined figure of any number of sides whatever, are together equal to twice as many right angles minus four, as the figure has sides. Thus, a triangle has 3 sides; twice that number is 6; and 6 right angles, or 6 × 90° = 540°; from which take 4 right angles, or 360°; and there remain 180°, which is the number of degrees in every plane, or straight-lined triangle. This principle furnishes an easy means of testing our measurements of the angles of any fig; for if the sum of all our measurements does not agree with the sum given by the rule, it is a proof that we have committed some error.

An exterior angle



Of any straight-lined figure, is any angle, as *a b d*, formed OUTSIDE of that fig, by the meeting of any side, as *a b*, with the prolongation of an adjacent side, as *c b*; so likewise the angles *c a s*, and *b c w*.* All the exterior angles of any straight-lined fig, no matter how many sides it may have, amount to 360°; but if any of the angles are **re-entering,** *i e*, pointing *inwards*, as *g i j*, the supplements, as *g i x*, corresponding to such, must be taken as *negative* or *minus*. Thus *a b d* + *b c w* + *c a s* = 360°; and *y h j* + *z j i* − *g i x* + *i g w* = 360°. Angles, as *a*, *b*, c, *g*, *h*, and *j*, which point *outward*, are called **salient.**

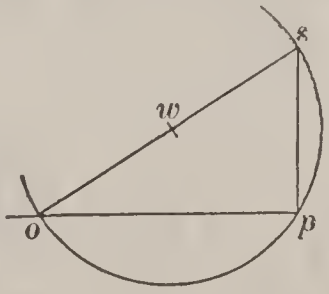
From any given point, *p*, on a line *s t*, to draw a perp, *p a*.



From *p*, with any convenient opening of the dividers, step off the equals *p o*, *p g*. From o and *g* as centers, with any opening greater than half o *g*, describe the two short arcs *b* and c; and join *a p*. Or still better, describe four arcs, and join *a y*.

Or from *p* with any convenient scale describe two short arcs *g* and c either one of them with a radius 3, and the other with a rad 4. Then from *g* with rad 5 describe the arc *b*. Join *p a*.

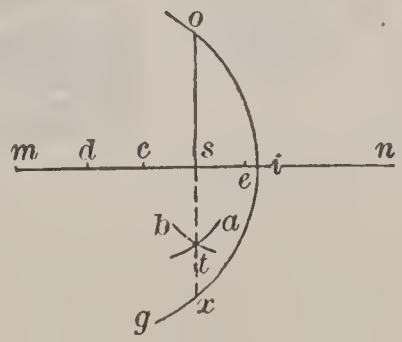
If the point *p* is at one end of the line, or very near it,



Extend the line, if possible, and proceed as above. But if this cannot be done, then from any convenient point, *w*, open the dividers to *p*, and describe the semicircle, *s p o*; through o *w* draw *o w s*; join *p s*.

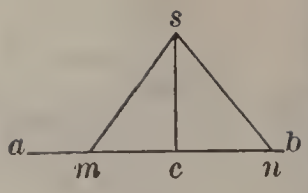
Or use the last foregoing process with rads 3, 4, and 5.

From a given point, *o*, to let fall a perp *o s*, to a given line, *m n*.



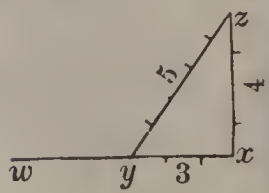
From o, measure to the line *m n*, any two equal dists, o c, o *e*; and from c and *e* as centers, with any opening greater than half of c *e*, describe the two arcs *a* and *b*; join o *t*. Or from any point, as *d* on the line, open the dividers to o, and describe the arc o *g*; make *i x* equal to *i o*; and join *o x*.

* An exterior angle *a b d*, or *y h j*, is the *supplement* (p 56) of the interior angle *a b* c, or *g h j*, at the same point *b*, or *h*.



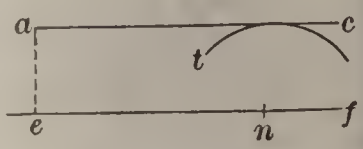
If the line, *a b*, is on the ground,

And a perp is reqd to be drawn from c, first measure off any two equal dists, c *m*, c *n*. At *m* and *n*, hold the ends of a piece of string, tape line, or chain, *m s n*; then tighten out the string, &c, as shown by *m s n*; *s* being its center. Then will *s* c be the reqd perp. Or if the perp *x z* is to be drawn from the *end* of the line *w x*, first measure *x y* upon the line, and equal to three feet; then holding the end of a tape-line at *x*, and its nine feet mark at *y*, hold the four feet mark at *z*, keeping *z x* and *z y* equally stretched. Then *z x* will be the reqd perp, because 3, 4, and 5, make the sides of a right-angled triangle. Instead of 3, 4, and 5, any multiples of those numbers may be used, such as 6, 8, and 10; or 9, 12, 15, &c: also instead of feet, we may use yards, chains, &c.



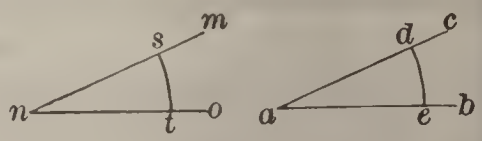
Through a given point, *a*, to draw a line, *a c*, parallel to another line, *e f*.

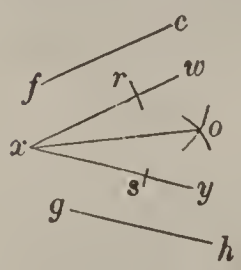
With the perp dist, *a e*, from any point, *n*, in *e f*, describe an arc, *t*; draw *a* c just touching the arc.



At any point, *a*, in a line *a b*, to make an angle *c a b*, equal to a given angle, *m n o*.

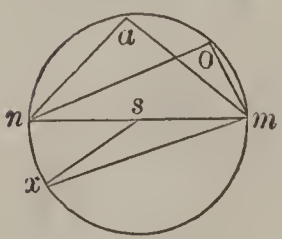
From *n* and *a*, with any convenient rad, describe the arcs *s t*. *d e*; measure *s t*, and make *e d* equal to it; through *a d* draw *a* c.



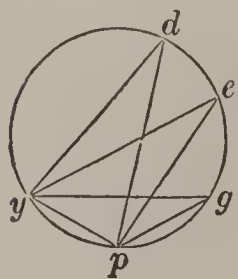


To bisect, or divide any angle, *w x y*, into two equal parts.

From *x* set off any two equal dists, *x r*, *x s*. From *r* and *s* with any rad describe two arcs intersecting, as at *o*; and join *o x*. If the two sides of the angle do not meet, as c *f* and *g h*, either first extend them until they do meet; or else draw lines *x w*, and *x y*, parallel to them, and at equal dists from them, so as to meet; then proceed as before.



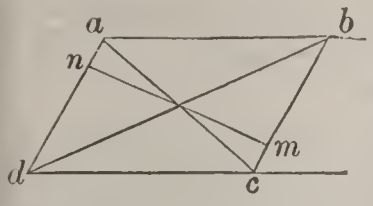
All angles, as *n a m*, *n o m*, at the circumf of a semicircle, and standing on its diam *n m*, are right angles; or, as it is usually expressed, **all angles in a semicircle are right angles.** An angle *n s x* at the centre of a circle, is twice as great as an angle *n m x* at the circumf, when both stand upon the same arc *n x*.



All angles, as *y d p*, *y e p*, *y g p*, at the circumf of a circle, and standing upon the same arc, as *y p*, are equal to each other; or, as usually expressed, **all angles in the same segment of a circle are equal.**

The **complement** of an angle is what it lacks of 90°. Thus, the complement of 80° is 90° − 80° = 10°; and that of 210° is 90° − 210° = − 120°. The **supplement** of an angle is what it lacks of 180°. Thus, the supplement of 80° is 180° − 80° = 100°; and that of 210° is 180° − 210° = − 30°. But ordinarily we may neglect the signs + and −, before complements and supplements, and call the complement of an angle its *diff* from 90°; and the supplement its *diff* from 180°.

Angles in a Parallelogram.



A parallelogram is any four-sided straight-lined figure whose opposite sides are equal, as *a b c d*; or a square, &c. Any line drawn across a parallelogram between 2 opposite angles, is called a *diagonal*, as *a c*, or *b d*. A diag divides a parallelogram into two equal parts; as does also any line *m n* drawn through the center of either diag; and moreover, the line *m n* itself is div into two equal parts by the diag. Two diags bisect each other; they also divide the parallelogram into four triangles of equal areas. The sum of the two angles at the ends of any one side is $= 180°$; thus, $d\ a\ b + a\ b\ c = a\ b\ c + b\ c\ d = 180°$; and the sum of the four angles, $d\ a\ b$, $a\ b\ c$, $b\ c\ d$, $c\ d\ a = 360°$.

The sum of the squares of the four sides, is equal to the sum of the squares of the two diags.

To reduce Minutes and Seconds to Degrees and decimals of a Degree, etc.

In any given angle—

Number of degrees = Number of minutes ÷ 60.
= Number of seconds ÷ 3600.

Number of minutes = Number of degrees × 60.
= Number of seconds ÷ 60.

Number of seconds = Number of degrees × 3600.
= Number of minutes × 60.

Table of Minutes and Seconds in decimals of a Degree.

Min.	Deg.	Min.	Deg.	Min.	Deg.	Sec.	Deg.	Sec.	Deg.	Sec.	Deg.
1	.016666	21	.350000	41	.683333	1	.000278‡	21	.005833	41	.011389
2	.033333	22	.366666	42	.700000	2	.000556	22	.006111	42	.011667
3	.050000	23	.383333	43	.716666	3	.000833	23	.006389	43	.011944
4	.066666	24	.400000	44	.733333	4	.001111	24	.006667	44	.012222
5	.083333	25	.416666	45	.750000	5	.001389	25	.006944	45	.012500
6	.100000	26	.433333	46	.766666	6	.001667	26	.007222	46	.012778
7	.116666	27	.450000	47	.783333	7	.001944	27	.007500	47	.013056
8	.133333	28	.466666	48	.800000	8	.002222	28	.007778	48	.013333
9	.150000	29	.483333	49	.816666	9	.002500	29	.008056	49	.013611
10	.166666	30	.500000	50	.833333	10	.002778	30	.008333	50	.013889
11	.183333	31	.516666	51	.850000	11	.003056	31	.008611	51	.014167
12	.200000	32	.533333	52	.866666	12	.003333	32	.008889	52	.014444
13	.216666	33	.550000	53	.883333	13	.003611	33	.009167	53	.014722
14	.233333	34	.566666	54	.900000	14	.003889	34	.009444	54	.015000
15	.250000	35	.583333	55	.916666	15	.004167	35	.009722	55	.015278
16	.266666	36	.600000	56	.933333	16	.004444	36	.010000	56	.015556
17	.283333	37	.616666	57	.950000	17	.004722	37	.010278	57	.015833
18	.300000	38	.633333	58	.966666	18	.005000	38	.010556	58	.016111
19	.316666	39	.650000	59	.983333	19	.005278	39	.010833	59	.016389
20	.333333	40	.666666	60	1.000000	20	.005556	40	.011111	60	.016667

‡ .0002777778.

To Measure Angles by a 2 Ft Rule, etc.

The four fingers of the hand, held at right angles to the arm, and at arms-length from the eye, cover about 7 degrees. And 7° corresponds to about 12.2 ft in 100 ft; or to 36.6 ft in 100 yds; or to 645 ft in a mile; or in the same proportion as the distance.

The following Table

may sometimes be found useful for the rough measurement *of angles*, either on a drawing, or between distant objects in the field. If the inner edges of a common two-foot rule be opened to the extent shown in the column of inches, its edges will be inclined at the angles shown in the columns of angles. Since an opening of ⅛ of an inch up to 19 inches or about 105°, corresponds to from about ½ to 1°, no great accuracy is to be expected; and beyond 105° still less; the liability to error increasing very rapidly as the opening becomes greater. Thus, the last ⅛ inch corresponds to about 12°.

As to the table itself, angles for openings intermediate of those therein given, may be calculated to the nearest minute or two, by simple proportion, up to 23 inches of opening, or about 147°.

Table of Angles corresponding to openings of a 2-foot rule.

(Original.)

D, degrees; M, minutes. Correct.

Ins.	D.	M.	Ins.	D.	M.	Ins.	D.	M.	Ins.	D.	M.	Ins.	D.	M.	Ins.	D.	M.
¼	1	12	4¼	20	24	8¼	40	13	12¼	61	23	16¼	85	14	20¼	115	5
	1	48		21			40	51		62	5		86	3		116	12
½	2	24	½	21	37	½	41	29	½	62	47	½	86	52	½	117	20
	3	00		22	13		42	7		63	28		87	41		118	30
¾	3	36	¾	22	50	¾	42	46	¾	64	11	¾	88	31	¾	119	40
	4	11		23	27		43	24		64	53		89	21		120	52
1	4	47	5	24	3	9	44	3	13	65	35	17	90	12	21	122	6
	5	23		24	39		44	42		66	18		91	3		123	20
¼	5	58	¼	25	16	¼	45	21	¼	67	1	¼	91	54	¼	124	36
	6	34		25	53		45	59		67	44		92	46		125	54
½	7	10	½	26	30	½	46	38	½	68	28	½	93	38	½	127	14
	7	46		27	7		47	17		69	12		94	31		128	35
¾	8	22	¾	27	44	¾	47	56	¾	69	55	¾	95	24	¾	129	59
	8	58		28	21		48	35		70	38		96	17		131	25
2	9	34	6	28	58	10	49	15	14	71	22	18	97	11	22	132	53
	10	10		29	35		49	54		72	6		98	5		134	24
¼	10	46	¼	30	11	¼	50	34	¼	72	51	¼	99	00	¼	135	58
	11	22		30	49		51	13		73	36		99	55		137	35
½	11	58	½	31	26	½	51	53	½	74	21	½	100	51	½	139	16
	12	34		32	3		52	33		75	6		101	48		141	1
¾	13	10	¾	32	40	¾	53	13	¾	75	51	¾	102	45	¾	142	51
	13	46		33	17		53	53		76	36		103	43		144	46
3	14	22	7	33	54	11	54	34	15	77	22	19	104	41	23	146	48
	14	58		34	33		55	14		78	8		105	40		148	58
¼	15	34	¼	35	10	¼	55	55	¼	78	54	¼	106	39	¼	151	17
	16	10		35	47		56	35		79	40		107	40		153	48
½	16	46	½	36	25	½	57	16	½	80	27	½	108	41	½	156	34
	17	22		37	3		57	57		81	14		109	43		159	43
¾	17	59	¾	37	41	¾	58	38	¾	82	2	¾	110	46	¾	163	27
	18	35		38	19		59	19		82	49		111	49		168	18
4	19	12	8	38	57	12	60	00	16	83	37	20	112	53	24	180	00
	19	48		39	35		60	41		84	26		113	58			

Or this table may be used thus. From any point measure 12 ft towards each object, and place marks. Measure the dist in ft between these marks. Suppose the first cols in the table to be ft instead of ins; then opposite the dist in ft will be the angle. **One-eighth of a ft is 1.5 ins.**

The following is a good way to measure an angle. Measure 100 or any other number of ft towards each object, and place marks. Measure the dist between the marks. Then

As dist measured toward one object . 1 . . **Half** the dist between marks · · · nat sine of **Half** the angle.

Find this nat sine in the table of nat sines, take out the corresponding angle, and multiply it by 2. See near foot of p 114.

Sines, Tangents, &c.

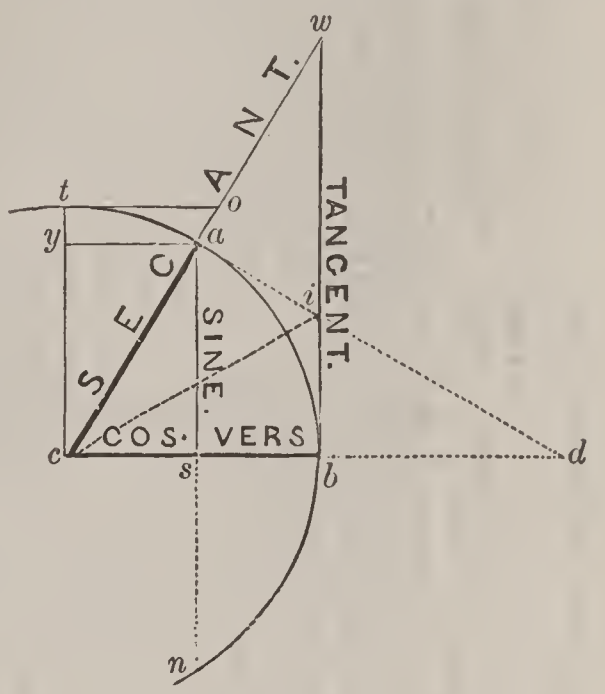
Sine, *a s*, of any angle, *a c b*, or which is the same thing, the sine of any circular arc, *a b*, which subtends or measures the angle, is a straight line drawn from one end, as *a*, of the arc, at right angles to, and terminating at, the rad *c b*, drawn to the other end *b* of the arc. It is, therefore, equal to half the chord *a n*, of the arc *a b n*, which is equal to twice the arc *a b*; or, the sine of an angle is always equal to half the chord of twice that angle; and vice versa, the chord of an angle is always equal to twice the sine of half the angle.

The sine *t c* of an angle *t c b*, or of an arc *t a b*, of 90°, is equal to the rad of the arc or of the circle; and this sine of 90° is greater than that of any other angle.

Cosine *c s* of an angle *a c b*, is that part of the rad which lies between the sine and the center of the circle. It is always equal to the sine *y a* of the complement *t c a* of *a c b*; or of what *a c b* wants of being 90°. The prefix co before sines, &c, means complement; thus, cosine means sine of the complement.

Versed sine *s b* of any angle *a c b*, is that part of the diam which lies between the sine, and the outer end *b*. It is very common, but erroneous, when speaking of bridges, &c, to call the rise or height *s b* of a circular arch *a b n*, its versed sine; while it is actually the versed sine of only half the arch. This absurdity should cease; for the word rise or height is not only more expressive, but is correct.

Tangent *b w* or *a d*, of any angle *a c b*, is a line drawn from, and at right angles to, the end *b* or *a* of either rad *c b*, or *c a*, which forms one of the legs of the angle; and terminating as at *w*, or *d*, in the prolongation of the rad which forms the other leg. This last rad thus prolonged, that is, *c w*, or *c d*, as the case may be, is the **secant** of the angle *a c b*. The angle *t c b* being supposed to be equal to 90°, the angle *t c a* becomes the complement of the angle *a c b*, or what *a c b* wants of being 90°; and the sine *y a* of this complement; its versed sine *t y*; its tangent *t o*; and its secant *c o*, are respectively the co-sine, co-versed sine; co-tangent; and co-secant, of the angle *a c b*. Or, vice versa, the sine, &c, of *a c b*, are the cosine, &c, of *t c a*; because the angle *a c b* is the complement of the angle *t c a*. When the rad *c b*, *c a*, or *c t*, is assumed to be equal to unity, or 1, the corresponding sines, tangents, &c, are called *natural* ones; and their several lengths for diff angles, for said rad of unity, have been calculated: constituting the well-known tables of nat sines, &c. In any circle whose rad is either larger or smaller than 1, the sines, &c, of the angles will be in the same proportion larger or smaller than those in the tables, and are consequently found by mult the sine, &c, of the table, by said larger or smaller rad.



The following table of natural sines, &c, does not contain nat versed sines, co-versed sines, secants, nor cosecants, but these may be found thus; for any angle not exceeding 90 degrees.

Versed Sine. From 1 take the nat cosine.
Co-versed Sine. From 1 take the nat sine.
Secant. Divide 1 by the nat cosine.
Cosecant. Divide 1 by the nat sine.

For angles exceeding 90°; to find the sine, cosine, tangent, cotang, secant, or cosec, (but not the versed sine or co-versed sine), take the angle from 180°: if between 180° and 270° take 180° from the angle; if bet 270° and 360°, take the angle from 360°. Then in each case take from the table the sine, cosine, tang, or cotang of the remainder. Find its secant or cosec as directed above. *For the versed sine;* if between 90° and 270°, add cosine to 1; if bet 270° and 360°, take cosine from 1. (The engineer seldom needs sines, &c, exceeding 180°.

To find the nat sine, cosine, tang, secant, versed sine, &c, of an angle containing seconds. First find that due to the given deg and min; then the next greater one. Take their diff. Then as 60 sec are to this diff, so are the sec only of the given angle to a dec quantity to be **added** to the one first taken out if it is a sine, tang, secant, &c; or to be **subtracted** from it if it is a **cosine, cotang, cosecant,** &c.

The tangents in the table are strict **trigonometrical** ones; that is, tangents to given *angles;* and which must extend to meet the secants of the angles to which they belong. **Ordinary,** or **geometrical tangents,** as those on p 124, may extend as far as we please. **In the field practice of railroad curves,** two trigonometrical tangents terminate where they meet each other. Each of these tangs is the tang of *half* the curve. It is usually, but improperly, called "the tang of the *curve.*" "Apex dist of the curve," as suggested by Mr Shunk, would be better.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
	0 Deg.						0 Deg.						0 Deg.				
0	·0000000	·000000	Infinite.	1·000000	60	21	·0061086	·006108	163·7001	·9999813	39	41	·0119261	·011927	83·84350	·9999289	19
1	·0002909	·000291	3437·746	1·000000	59	22	·0063995	·006399	156·2590	·9999795	38	42	·0122170	·012217	81·84704	·9999254	18
2	·0005818	·000582	1718·873	·9999998	58	23	·0066904	·006690	149·4650	·9999776	37	43	·0125079	·012508	79·94343	·9999218	17
3	·0008727	·000872	1145·915	·9999996	57	24	·0069813	·006981	143·2371	·9999756	36	44	·0127987	·012799	78·12634	·9999181	16
4	·0011636	·001163	859·4363	·9999993	56	25	·0072721	·007272	137·5075	·9999736	35	45	·0130896	·013090	76·39000	·9999143	15
5	·0014544	·001454	687·5488	·9999989	55	26	·0075630	·007563	132·2185	·9999714	34	46	·0133805	·013381	74·72916	·9999105	14
6	·0017453	·001745	572·9572	·9999985	54	27	·0078539	·007854	127·3213	·9999692	33	47	·0136713	·013672	73·13899	·9999065	13
7	·0020362	·002036	491·1060	·9999979	53	28	·0081448	·008145	122·7739	·9999668	32	48	·0139622	·013963	71·61507	·9999025	12
8	·0023271	·002327	429·7175	·9999973	52	29	·0084357	·008436	118·5401	·9999644	31	49	·0142530	·014254	70·15334	·9998984	11
9	·0026180	·002618	381·9709	·9999966	51	30	·0087265	·008726	114·5886	·9999619	30	50	·0145439	·014545	68·75008	·9998942	10
10	·0029089	·002908	343·7737	·9999958	50	31	·0090174	·009017	110·8920	·9999593	29	51	·0148348	·014836	67·40185	·9998900	9
11	·0031998	·003199	312·5213	·9999949	49	32	·0093083	·009308	107·4264	·9999567	28	52	·0151256	·015127	66·10547	·9998856	8
12	·0034907	·003490	286.4777	·9999939	48	33	·0095992	·009599	104·1709	·9999539	27	53	·0154165	·015418	64·85800	·9998812	7
13	·0037815	·003781	264·4408	·9999928	47	34	·0098900	·009890	101·1069	·9999511	26	54	·0157073	·015709	63·65674	·9998766	6
14	·0040724	·004072	245.5519	·9999917	46	35	·0101809	·010181	98·21794	·9999482	25	55	·0159982	·016000	62·49915	·9998720	5
15	·0043633	·004363	229·1816	·9999905	45	36	·0104718	·010472	95·48947	·9999452	24	56	·0162890	·016291	61·38290	·9998673	4
16	·0046542	·004654	214·8576	·9999892	44	37	·0107627	·010763	92·90848	·9999421	23	57	·0165799	·016582	60·30582	·9998625	3
17	·0049451	·004945	202·2187	·9999878	43	38	·0110535	·011054	90·46333	·9999389	22	58	·0168707	·016873	59·26587	·9998577	2
18	·0052360	·005236	190·9841	·9999863	42	39	·0113444	·011345	88·14357	·9999357	21	59	·0171616	·017164	58·26117	·9998527	1
19	·0055268	·005526	180·9322	·9999847	41	40	·0116353	·011636	85·93979	·9999323	20	60	·0174524	·017455	57·28996	·9998477	0
20	·0058177	·005817	171·8854	·9999831	40												
′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′
	Deg. 89						Deg. 89.						Deg. 89				

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
	1 Deg.						1 Deg.						1 Deg.				
0	·0174524	·017455	57·28996	·9998477	60	21	·0235598	·023566	42·43346	·9997224	39	41	·0293755	·029388	34·02730	·9995684	19
1	·0177432	·017746	56·35059	·9998426	59	22	·0238506	·023857	41·91579	·9997156	38	42	·0296662	·029679	33·69350	·9995599	18
2	·0180341	·018037	55·44151	·9998374	58	23	·0241414	·024148	41·41058	·9997086	37	43	·0299570	·029970	33·36619	·9995512	17
3	·0183249	·018328	54·56130	·9998321	57	24	·0244322	·024439	40·91741	·9997015	36	44	·0302478	·030261	33·04517	·9995424	16
4	·0186158	·018619	53·70858	·9998267	56	25	·0247230	·024730	40·43583	·9996943	35	45	·0305385	·030552	32·73026	·9995336	15
5	·0189066	·018910	52·88211	·9998213	55	26	·0250138	·025021	39·96546	·9996871	34	46	·0308293	·030843	32·42129	·9995247	14
6	·0191974	·019201	52·08067	·9998157	54	27	·0253046	·025312	39·50589	·9996798	33	47	·0311200	·031135	32·11809	·9995157	13
7	·0194883	·019492	51·30315	·9998101	53	28	·0255954	·025603	39·05677	·9996724	32	48	·0314108	·031426	31·82051	·9995066	12
8	·0197791	·019783	50·54850	·9998044	52	29	·0258862	·025894	38·61773	·9996649	31	49	·0317015	·031717	31·52839	·9994974	11
9	·0200699	·020074	49·81572	·9997986	51	30	·0261769	·026185	38·18845	·9996573	30	50	·0319922	·032008	31·24157	·9994881	10
10	·0203608	·020365	49·10388	·9997927	50	31	·0264677	·026477	37·76861	·9996497	29	51	·0322830	·032299	30·95992	·9994788	9
11	·0206516	·020656	48·41208	·9997867	49	32	·0267585	·026768	37·35789	·9996419	28	52	·0325737	·032591	30·68330	·9994693	8
12	·0209424	·020947	47·73950	·9997807	48	33	·0270493	·027059	36·95600	·9996341	27	53	·0328644	·032882	30·41158	·9994598	7
13	·0212332	·021238	47·08534	·9997745	47	34	·0273401	·027350	36·56265	·9996262	26	54	·0331552	·033173	30·14461	·9994502	6
14	·0215241	·021529	46·44886	·9997683	46	35	·0276309	·027641	36·17759	·9996182	25	55	·0334459	·033464	29·88229	·9994405	5
15	·0218149	·021820	45·82935	·9997620	45	36	·0279216	·027932	35·80055	·9996101	24	56	·0337366	·033755	29·62449	·9994308	4
16	·0221057	·022111	45·22614	·9997556	44	37	·0282124	·028223	35·43128	·9996020	23	57	·0340274	·034047	29·37110	·9994209	3
17	·0223965	·022402	44·63859	·9997492	43	38	·0285032	·028514	35·06954	·9995937	22	58	·0343181	·034338	29·12200	·9994110	2
18	·0226873	·022693	44·06611	·9997426	42	39	·0287940	·028805	34·71511	·9995854	21	59	·0346088	·034629	28·87708	·9994009	1
19	·0229781	·022984	43·50812	·9997360	41	40	·0290847	·029097	34·36777	·9995770	20	60	·0348995	·034920	28·63625	·9993908	0
20	·0232690	·023275	42·96407	·9997292	40												
′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′
	Deg 88.						Deg. 88.						Deg. 88				

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
	2 Deg.						2 Deg.						2 Deg.				
0	·0348995	·034920	28·63625	·9993908	60	21	·0410037	·041038	24·36750	·9991590	39	41	·0468159	·046867	21.33685	·9989035	19
1	·0351902	·035212	28·39939	·9993806	59	22	·0412944	·041329	24·19571	·9991470	38	42	·0471065	·047158	21·20494	·9988899	18
2	·0354809	·035503	28·16642	·9993704	58	23	·0415850	·041621	24·02632	·9991350	37	43	·0473970	·047450	21·07466	·9988761	17
3	·0357716	·035794	27·93723	·9993600	57	24	·0418757	·041912	23·85927	·9991228	36	44	·0476876	·047741	20·94596	·9988623	16
4	·0360623	·036085	27·71174	·9993495	56	25	·0421663	·042203	23·69453	·9991106	35	45	·0479781	·048033	20·81882	·9988484	15
5	·0363530	·036377	27·48985	·9993390	55	26	·0424569	·042495	23·53205	·9990983	34	46	·0482687	·048325	20·69322	·9988344	14
6	·0366437	·036668	27·27148	·9993284	54	27	·0427475	·042786	23·37177	·9990859	33	47	·0485592	·048616	20·56911	·9988203	13
7	·0369344	·036959	27·05655	·9993177	53	28	·0430382	·043078	23·21366	·9990734	32	48	·0488498	·048908	20·44648	·9988061	12
8	·0372251	·037250	26·84498	·9993069	52	29	·0433288	·043369	23·05767	·9990609	31	49	·0491403	·049199	20·32530	·9987919	11
9	·0375158	·037542	26·63669	·9992960	51	30	·0436194	·043660	22·90376	·9990482	30	50	·0494308	·049491	20·20555	·9987775	10
10	·0378065	·037833	26·43160	·9992851	50	31	·0439100	·043952	22·75189	·9990355	29	51	·0497214	·049782	20·08719	·9987631	9
11	·0380971	·038124	26·22963	·9992740	49	32	·0442006	·044243	22·60201	·9990227	28	52	·0500119	·050074	19·97021	·9987486	8
12	·0383878	·038416	26·03073	·9992629	48	33	·0444912	·044535	22·45409	·9990098	27	53	·0503024	·050366	19·85459	·9987340	7
13	·0386785	·038707	25·83482	·9992517	47	34	·0447818	·044826	22·30809	·9989968	26	54	·0505929	·050657	19·74029	·9987194	6
14	·0389692	·038998	25·64183	·9992404	46	35	·0450724	·045118	22·16398	·9989837	25	55	·0508835	·050949	19·62729	·9987046	5
15	·0392598	·039290	25·45170	·9992290	45	36	·0453630	·045409	22·02171	·9989706	24	56	·0511740	·051241	19·51558	·9986898	4
16	·0395505	·039581	25·26436	·9992176	44	37	·0456536	·045701	21·88125	·9989573	23	57	·0514645	·051532	19·40513	·9986748	3
17	·0398111	·039872	25·07975	·9992060	43	38	·0459442	·045992	21·74256	·9989440	22	58	·0517550	·051824	19·29592	·9986598	2
18	·0401318	·040164	24·89782	·9991944	42	39	·0462347	·046284	21·60563	·9989306	21	59	·0520455	·052116	19·18793	·9986447	1
19	·0404224	·040455	24·71851	·9991827	41	40	·0465253	·046575	21·47040	·9989171	20	60	·0523360	·052407	19·08113	·9986295	0
20	·0407131	·040746	24·54175	·9991709	40												
′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′
	Deg. 87.						Deg. 87.						Deg. 87				

3 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0523360	·052407	19·08113	·9986295	60
1	·0526264	·052699	18·97552	·9986143	59
2	·0529169	·052991	18·87106	·9985989	58
3	·0532074	·053282	18·76775	·9985835	57
4	·0534979	.053574	18·66556	·9985680	56
5	·0537883	·053866	18·56447	·9985524	55
6	·0540788	·054158	18·46447	·9985367	54
7	·0543693	·054449	18·36553	·9985209	53
8	·0546597	·054741	18·26765	·9985050	52
9	·0549502	·055033	18·17080	·9984891	51
10	·0552406	·055325	18·07497	·9984731	50
11	·0555311	·055616	17·98015	·9984570	49
12	·0558215	·055908	17.88631	·9984408	48
13	·0561119	·056200	17·79344	·9984245	47
14	·0564024	·056492	17·70152	·9984081	46
15	·0566928	·056784	17·61055	·9983917	45
16	·0569832	·057075	17·52051	·9983751	44
17	·0572736	·057367	17·43138	·9983585	43
18	·0575640	·057659	17·34315	·9983418	42
19	·0578544	·057951	17·25580	·9983250	41
20	·0581448	·058243	17·16933	·9983082	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 86.

3 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·0584352	·058535	17·08372	·9982912	39
22	·0587256	·058827	16·99895	·9982742	38
23	·0590160	·059119	16·91502	·9982570	37
24	·0593064	·059410	16·83191	·9982398	36
25	·0595967	·059702	16·74961	·9982225	35
26	·0598871	·059994	16·66811	·9982052	34
27	·0601775	·060286	16·58739	·9981877	33
28	·0604678	·060578	16·50745	·9981701	32
29	·0607582	·060870	16·42827	·9981525	31
30	·0610485	·061162	16·34985	·9981348	30
31	·0613389	·061454	16·27217	·9981170	29
32	·0616292	·061746	16·19522	·9980991	28
33	·0619196	·062038	16·11899	·9980811	27
34	·0622099	·062330	16·04348	·9980631	26
35	·0625002	·062622	15·96866	·9980450	25
36	·0627905	·062914	15·89454	·9980267	24
37	·0630808	·063206	15·82110	·9980084	23
38	·0633711	·063498	15·74833	·9979900	22
39	·0636614	·063790	15·67623	·9979716	21
40	·0639517	·064082	15·60478	·9979530	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 86.

3 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·0642420	·064375	15·53398	·9979343	19
42	·0645323	·064667	15·46381	·9979156	18
43	·0648226	·064959	15·39427	·9978968	17
44	·0651129	·065251	15·32535	·9978779	16
45	·0654031	·065543	15·25705	·9978589	15
46	·0656934	·065835	15·18934	·9978399	14
47	·0659836	·066127	15·12224	·9978207	13
48	·0662739	·066419	15·05572	·9978015	12
49	·0665641	·066712	14·98978	·9977821	11
50	·0668544	·067004	14·92441	·9977627	10
51	·0671446	·067296	14·85961	·9977433	9
52	·0674349	·067588	14·79537	·9977237	8
53	·0677251	·067880	14·73167	·9977040	7
54	·0680153	·068173	14·66852	·9976843	6
55	·0683055	·068465	14·60591	·9976645	5
56	·0685957	·068757	14·54383	·9976445	4
57	·0688859	·069049	14·48227	·9976245	3
58	·0691761	·069342	14·42123	·9976045	2
59	·0694663	·069634	14·36069	·9975843	1
60	·0697565	·069926	14·30066	·9975641	0
′	Cosine.	Cotan.	Tang.	Sine·	′

Deg. 86.

4 Deg

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0697565	·069926	14·30066	·9975641	60
1	·0700467	·070219	14·24113	·9975437	59
2	·0703368	·070511	14·18209	·9975233	58
3	·0706270	·070803	14·12353	·9975028	57
4	·0709171	·071096	14·06545	·9974822	56
5	·0712073	·071388	14·00785	·9974615	55
6	·0714974	·071680	13·95071	·9974408	54
7	·0717876	·071973	13·89404	·9974199	53
8	·0720777	·072265	13·83782	·9973990	52
9	·0723678	·072558	13·78206	·9973780	51
10	·0726580	·072850	13·72673	·9973569	50
11	·0729481	·073143	13·67185	·9973357	49
12	·0732382	·073435	13·61740	·9973145	48
13	·0735283	·073727	13·56339	·9972931	47
14	·0738184	·074020	13·50979	·9972717	46
15	·0741085	·074312	13·45662	·9972502	45
16	·0743986	·074605	13·40386	·9972286	44
17	·0746887	·074897	13·35151	·9972069	43
18	·0749787	·075190	13·29957	·9971851	42
19	·0752688	·075482	13·24803	·9971633	41
20	·0755589	·075775	13·19688	·9971413	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 85

4 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·0758489	·076068	13·14612	·9971193	39
22	·0761390	·076360	13·09575	·9970972	38
23	·0764290	·076653	13.04576	·9970750	37
24	·0767190	·076945	12·99616	·9970528	36
25	·0770091	·077238	12·94692	·9970304	35
26	·0772991	·077531	12·89805	·9970080	34
27	·0775891	·077823	12·84955	·9969854	33
28	·0778791	·078116	12·80141	·9969628	32
29	·0781691	·078409	12·75363	·9969401	31
30	·0784591	·078701	12·70620	·9969173	30
31	·0787491	·078994	12·65912	·9968945	29
32	·0790391	·079287	12·61239	·9968715	28
33	·0793290	·079579	12·56599	·9968485	27
34	·0796190	·079872	12·51994	·9968254	26
35	·0799090	.080165	12·47422	·9968022	25
36	·0801989	·080458	12·42883	·9967789	24
37	·0804889	·080750	12·38376	·9967555	23
38	·0807788	·081043	12·33902	·9967321	22
39	·0810687	·081336	12·29460	·9967085	21
10	·0813587	·081629	12·25050	·9966849	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 85

4 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·0816486	·081922	12·20671	·9966612	19
42	·0819385	·082215	12·16323	·9966374	18
43	·082 2284	·082507	12·12006	·9966135	17
44	·0825183	·082800	12·07719	·9965895	16
45	·0828082	·083093	12·03462	·9965655	15
46	·0830981	·083386	11·99234	·9965414	14
47	·0833880	·083679	11·95037	·9965172	13
48	·0836778	·083972	11·90868	·9964929	12
49	·0839677	·084265	11·86728	·9964685	11
50	·0842576	·084558	11·82616	·9964440	10
51	·0845474	·084851	11·78533	·9964195	9
52	·0848373	·085144	11·74477	·9963948	8
53	·0851271	·085437	11·70450	·9963701	7
54	·0854169	·085730	11·66449	·9963453	6
55	·0857067	·086023	11·62476	·9963204	5
56	·0859966	·086316	11·58529	·9962954	4
57	·0862864	·086609	11·54609	·9962704	3
58	·0865762	·086902	11·50715	·9962452	2
59	·0868660	·087195	11·46847	·9962200	1
60	·0871557	·087488	11·43005	·9961947	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 85.

5 Deg

5 Deg.

5 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0871557	·087488	11·43005	·9961947	60	21	·0932395	·093647	10·67834	·9956437	39	41	·0990303	·099519	10·04828	·9950844	19
1	·0874455	·087781	11·39188	·9961693	59	22	·0935291	·093940	10·64499	·9956165	38	42	·0993197	·099813	10·01871	·9950556	18
2	·0877353	·088074	11·35397	·9961438	58	23	·0938187	·094234	10·61184	·9955892	37	43	·0996092	·100107	9·989305	·9950266	17
3	·0880251	·088368	11·31630	·9961183	57	24	·0941083	·094527	10·57889	·9955620	36	44	·0998986	·100400	9·960072	·9949976	16
4	·0883148	·088661	11·27888	·9960926	56	25	·0943979	·094821	10·54615	·9955345	35	45	·1001881	·100694	9·931008	·9949685	15
5	·0886046	·088954	11·24171	·9960669	55	26	·0946875	·095114	10·51360	·9955070	34	46	·1004775	·100988	9·902112	·9949393	14
6	·0888943	·089247	11·20478	·9960411	54	27	·0949771	·095408	10·48126	·9954794	33	47	·1007669	·101282	9·873382	·9949101	13
7	·0891840	·089540	11·16808	·9960152	53	28	·0952666	·095701	10·44911	·9954517	32	48	·1010563	·101576	9·844816	·9948807	12
8	·0894738	089834	11·13163	·9959892	52	29	·0955562	·095995	10·41715	·9954240	31	49	·1013457	·101870	9·816414	·9948513	11
9	·0897635	·090127	11·09541	·9959631	51	30	·0958458	·096289	10·38539	·9953962	30	50	·1016351	·102164	9·788173	·9948217	10
10	·0900532	·090420	11·05943	·9959370	50	31	·0961353	·096582	10·35382	·9953683	29	51	·1019245	·102458	9·760092	·9947921	9
11	·0903429	·090713	11·02367	·9959107	49	32	·0964248	·096876	10·32244	·9953403	28	52	·1022138	·102752	9·732171	·9947625	8
12	·0906326	·091007	10·98815	·9958844	48	33	·0967144	·097169	10·29125	·9953122	27	53	·1025032	·103046	9·704407	·9947327	7
13	·0909223	·091300	10·95285	·9958580	47	34	·0970039	·097463	10·26024	·9952840	26	54	·1027925	·103339	9·676800	·9947028	6
14	·0912119	·091593	10·91777	·9958315	46	35	·0972934	·097757	10·22942	·9952557	25	55	·1030819	·103634	9·649347	·9946729	5
15	·0915016	·091887	10·88292	·9958049	45	36	·0975829	·098050	10·19878	·9952274	24	56	·1033712	·103928	9·622048	·9946428	4
16	·0917913	·092180	10·84828	·9957783	44	37	·0978724	·098344	10·16833	·9951990	23	57	·1036605	·104222	9·594902	·9946127	3
17	·0920809	·092473	10·81387	·9957515	43	38	·0981619	·098638	10·13805	·9951705	22	58	·1039499	·104516	9·567906	·9945825	2
18	·0923706	·092767	10·77967	·9957247	42	39	·0984514	·098932	10·10795	·9951419	21	59	·1042392	·104810	9·541061	·9945523	1
19	·0926602	·093060	10·74568	·9956978	41	40	·0987408	·099225	10·07803	·9951132	20	60	·1045285	·105104	9·514364	·9945219	0
20	·0929499	·093354	10·71191	·9956708	40												
′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 84.

Deg. 84.

Deg. 84.

6 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1045285	·105104	9·514364	·9945219	60
1	·1048178	·105398	9·487814	·9944914	59
2	·1051070	·105692	9·461411	·9944609	58
3	·1053963	·105986	9·435153	·9944303	57
4	·1056856	·106280	9·409038	·9943996	56
5	·1059748	·106575	9·383066	·9943688	55
6	·1062641	·106869	9·357235	·9943379	54
7	·1065533	·107163	9·331545	·9943070	53
8	·1068425	·107457	9·305993	·9942760	52
9	·1071318	·107751	9·280580	·9942448	51
10	·1074210	·108046	9·255303	·9942136	50
11	·1077102	·108340	9·230162	·9941823	49
12	·1079994	·108634	9·205156	·9941510	48
13	·1082885	·108929	9·180283	·9941195	47
14	·1085777	·109223	9·155543	·9940880	46
15	·1088669	·109517	9·130934	·9940563	45
16	·1091560	·109812	9·106456	·9940246	44
17	·1094452	·110106	9·082107	·9939928	43
18	·1097343	·110401	9·057886	·9939610	42
19	·1100234	·110695	9·033793	·9939290	41
20	·1103126	·110989	9·009826	·9938969	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 83.

6 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·1106017	·111284	8·985984	·9938648	39
22	·1108908	·111578	8·962266	·9938326	38
23	·1111799	·111873	8·938672	·9938003	37
24	·1114689	·112168	8·915200	·9937679	36
25	·1117580	·112462	8·891850	·9937355	35
26	·1120471	·112757	8·868620	·9937029	34
27	·1123361	·113051	8·845510	·9936703	33
28	·1126252	·113346	8·822518	·9936375	32
29	·1129142	·113641	8·799644	·9936047	31
30	·1132032	·113935	8·776887	·9935719	30
31	·1134922	·114230	8·754246	·9935389	29
32	·1137812	·114525	8·731719	·9935058	28
33	·1140702	·114819	8·709307	·9934727	27
34	·1143592	·115114	8·687008	·9934395	26
35	·1146482	·115409	8·664822	·9934062	25
36	·1149372	·115703	8·642747	·9933728	24
37	·1152261	·115998	8·620783	·9933393	23
38	·1155151	·116293	8·598929	·9933057	22
39	·1158040	·116588	8·577183	·9932721	21
40	·1160929	·116883	8·555546	·9932381	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 83.

6 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·1163818	·117178	8·534017	·9932045	19
42	·1166707	·117473	8·512594	·9931706	18
43	·1169596	·117767	8·491277	·9931367	17
44	·1172485	·118062	8·470065	·9931026	16
45	·1175374	·118357	8·448957	·9930685	15
46	·1178263	·118652	8·427953	·9930342	14
47	·1181151	·118947	8·407051	·9929999	13
48	·1184040	·119242	8·386251	·9929655	12
49	·1186928	·119537	8·365553	·9929310	11
50	·1189816	·119832	8·344955	·9928965	10
51	·1192704	·120127	8·324457	·9928618	9
52	·1195593	·120423	8·304058	·9928271	8
53	·1198481	·120718	8·283757	·9927922	7
54	·1201368	·121013	8·263554	·9927573	6
55	·1204256	·121308	8·243418	·9927224	5
56	·1207144	·121603	8·223438	·9926873	4
57	·1210031	·121898	8·203523	·9926521	3
58	·1212919	·122194	8·183704	·9926169	2
59	·1215806	·122489	8·163978	·9925816	1
60	·1218693	·122784	8·144346	·9925162	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 83.

7 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1218693	·122784	8·144346	·9925462	60
1	·1221581	·123079	8·124807	·9925107	59
2	·1224468	·123375	8·105359	·9924751	58
3	·1227355	·123670	8·086004	·9924394	57
4	·1230241	·123965	8·066739	·9924037	56
5	·1233128	·124261	8·047564	·9923679	55
6	·1236015	·124556	8·028479	·9923319	54
7	·1238901	·124852	8·009483	·9922959	53
8	·1241788	·125147	7 990575	·9922599	52
9	·1244674	·125442	7·971755	·9922237	51
10	·1247560	·125738	7·953022	·9921874	50
11	·1250446	·126033	7·934375	·9921511	49
12	·1253332	·126329	7·915815	·9921147	48
13	·1256218	·126624	7·897339	·9920782	47
14	·1259104	·126920	7·878948	·9920416	46
15	·1261990	·127216	7·860642	·9920049	45
16	·1264875	·127511	7·842419	·9919682	44
17	·1267761	·127807	7·824279	·9919314	43
18	·1270646	·128103	7·806221	·9918944	42
19	·1273531	·128398	7·788245	·9918574	41
20	·1276416	·128694	7·770350	·9918204	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 82.

7 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·1279302	·128990	7·752536	·9917832	39
22	·1282186	·129285	7·734802	·9917459	38
23	·1285071	·129581	7·717148	·9917086	37
24	·1287956	·129877	7·699573	·9916712	36
25	·1290841	·130173	7·682076	·9916337	35
26	·1293725	·130469	7·664658	·9915961	34
27	·1296609	·130764	7·647317	·9915584	33
28	·1299494	·131060	7·630053	·9915206	32
29	·1302378	·131356	7·612865	·9914828	31
30	·1305262	·131652	7·595754	·9914449	30
31	·1308146	·131948	7·578717	·9914069	29
32	·1311030	·132244	7·561756	·9913688	28
33	·1313913	·132540	7·544869	·9913306	27
34	·1316797	·132836	7·528057	·9912923	26
35	·1319681	·133132	7·511317	9912540	25
36	·1322564	·133428	7·494651	·9912155	24
37	·1325447	·133724	7·478057	·9911770	23
38	·1328330	·134020	7·461535	·9911384	22
39	·1331213	·134316	7·445085	·9910997	21
40	·1334096	·134612	7·428706	·9910610	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 82.

7 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·1336979	·134909	7·412397	·9910221	19
42	·1339862	·135205	7·396159	·9909832	18
43	·1342744	·135501	7·379990	·9909442	17
44	·1345627	·135797	7·363891	·9909051	16
45	·1348509	·136094	7·347861	·9908659	15
46	·1351392	·136390	7·331898	·9908266	14
47	·1354274	·136686	7·316004	·9907873	13
48	·1357156	·136983	7·300178	·9907478	12
49	·1360038	·137279	7·284418	·9907083	11
50	·1362919	·137575	7·268725	·9906687	10
51	·1365801	·137872	7·253098	·9906290	9
52	·1368683	·138168	7·237537	·9905893	8
53	·1371564	·138465	7·222042	·9905494	7
54	·1374445	·138761	7·206611	·9905095	6
55	·1377327	·139058	7·191245	·9904694	5
56	·1380208	·139354	7·175943	·9904293	4
57	·1383089	·139651	7·160705	·9903891	3
58	·1385970	·139947	7·145530	·9903489	2
59	·1388850	·140244	7·130419	·9903085	1
60	·1391731	·140540	7·115369	·9902681	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 82.

8 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1391731	·140540	7·115369	·9902681	60
1	·1394612	·140837	7·100382	·9902275	59
2	·1397492	·141134	7·085457	·9901869	58
3	·1400372	·141430	7·070593	·9901462	57
4	·1403252	·141727	7·055790	·9901055	56
5	·1406132	·142024	7·041048	·9900646	55
6	·1409012	·142321	7·026366	·9900237	54
7	·1411892	·142617	7·011744	·9899826	53
8	·1414772	·142914	6·997180	·9899415	52
9	·1417651	·143211	6·982678	·9899003	51
10	·1420531	·143508	6·968233	·9898590	50
11	·1423410	·143805	6·953847	·9898177	49
12	·1426289	·144102	6·939519	·9897762	48
13	·1429168	·144399	6·925248	·9897347	47
14	·1432047	·144696	6·911035	·9896931	46
15	·1434926	·144993	6·896879	·9896514	45
16	·1437805	·145290	6·882780	·9896096	44
17	·1440684	·145587	6·868737	·9895677	43
18	·1443562	·145884	6·854750	·9895258	42
19	·1446440	·146181	6·840819	·9891838	41
20	·1449319	·146478	6·826943	·9894416	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg 81

8 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·1452197	·146775	6·813122	·9893994	39
22	·1455075	·147072	6·799356	·9893572	38
23	·1457953	·147369	6·785644	·9893148	37
24	·1460830	·147667	6·771980	·9892723	36
25	·1463708	·147964	6·758382	·9892298	35
26	·1466585	·148261	6·744831	·9891872	34
27	·1469463	·148559	6·731334	·9891445	33
28	·1472340	·148856	6·717889	·9891017	32
29	·1475217	·149153	6·704496	·9890588	31
30	·1478094	·149451	6·691156	·9890159	30
31	·1480971	·149748	6·677867	·9889728	29
32	·1483848	·150045	6·664630	·9889297	28
33	·1486724	·150343	6·651444	·9888865	27
34	·1489601	·150640	6·638310	·9888432	26
35	·1492477	·150938	6·625225	·9887998	25
36	·1495353	·151235	6·612191	·9887564	24
37	·1498230	·151533	6·599208	·9887128	23
38	·1501106	·151830	6·586273	·9886692	22
39	·1503981	·152128	6·573389	·9886255	21
40	·1506857	·152426	6·560553	·9885817	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 81.

8 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·1509733	·152723	6·547767	·9885378	19
42	·1512608	·153021	6·535029	·9884939	18
43	·1515484	·153319	6·522339	·9884498	17
44	·1518359	·153617	6 509698	·9884057	16
45	·1521234	·153914	6·497104	·9883615	15
46	·1524109	·154212	6·484558	·9883172	14
47	·1526984	·154510	6·472059	·9882728	13
48	·1529858	·154808	6·459607	·9882284	12
49	·1532733	·155106	6·447201	·9881838	11
50	·1535607	·155404	6·434842	·9881392	10
51	·1538482	·155701	6·422530	·9880945	9
52	·1541356	·155999	6·410263	·9880497	8
53	·1544230	·156297	6·398042	·9880048	7
54	·1547104	·156595	6·385866	·9879599	6
55	·1549978	·156893	6·373735	·9879148	5
56	·1552851	·157191	6·361650	·9878697	4
57	·1555725	·157490	6·349609	·9878245	3
58	·1558598	·157788	6·337612	·9877792	2
59	·1561472	·158086	6·325660	·9877338	1
60	·1564345	·158384	6·313751	·9876883	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 81

9 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1561345	·158384	6·313751	·9876883	60
1	·1567218	·158682	6·301886	·9876428	59
2	·1570091	·158980	6·290065	·9875972	58
3	·1572963	·159279	6·278286	·9875514	57
4	·1575836	·159577	6·266551	·9875057	56
5	·1578708	·159875	6·254858	·9874598	55
6	·1581581	·160174	6·243208	·9874138	54
7	·1584453	·160472	6·231600	·9873678	53
8	·1587325	·160770	6·220034	·9873216	52
9	·1590197	·161069	6·208510	·9872754	51
10	·1593069	·161367	6·197027	·9872291	50
11	·1595940	·161666	6·185586	·9871827	49
12	·1598812	·161964	6·174186	·9871363	48
13	·1601683	·162263	6·162827	·9870897	47
14	·1604555	·162561	6·151508	·9870431	46
15	·1607426	·162860	6·140230	·9869964	45
16	·1610297	·163159	6·128992	·9869496	44
17	·1613167	·163457	6·117794	·9869027	43
18	·1616038	·163756	6·106636	·9868557	42
19	·1618909	·164055	6·095517	·9868087	41
20	·1621779	·164353	6·084438	·9867615	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 80.

9 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·1624650	·164652	6·073397	·9867143	39
22	·1627520	·164951	6·062396	·9866670	38
23	·1630390	·165250	6·051434	·9866196	37
24	·1633260	·165548	6·040510	·9865722	36
25	·1636129	·165847	6·029624	·9865246	35
26	·1638999	·166146	6·018777	·9864770	34
27	·1641868	·166445	6·007967	·9864293	33
28	·1644738	·166744	5·997195	·9863815	32
29	·1647607	·167043	5·986461	·9863336	31
30	·1650476	·167342	5·975764	·9862856	30
31	·1653345	·167641	5·965104	·9862375	29
32	·1656214	·167940	5·954481	·9861894	28
33	·1659082	·168239	5·943895	·9861412	27
34	·1661951	·168539	5·933345	·9860929	26
35	·1664819	·168838	5·922832	·9860445	25
36	·1667687	·169137	5·912355	·9859960	24
37	·1670556	·169436	5·901913	·9859475	23
38	·1673423	·169735	5·891508	·9858988	22
39	·1676291	·170035	5·881138	·9858501	21
40	·1679159	·170334	5·870804	·9858013	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 80.

9 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·1682026	·170633	5·860505	·9857524	19
42	·1684894	·170933	5·850241	·9857035	18
43	·1687761	·171232	5·840011	·9856544	17
44	·1690628	·171532	5·829817	·9856053	16
45	·1693495	·171831	5·819657	·9855561	15
46	·1696362	·172130	5·809531	·9855068	14
47	·1699228	·172430	5·799440	·9854574	13
48	·1702095	·172730	5·789382	·9854079	12
49	·1704961	·173029	5·779358	·9853583	11
50	·1707828	·173329	5·769368	·9853087	10
51	·1710694	·173628	5·759412	·9852590	9
52	·1713560	·173928	5·749488	·9852092	8
53	·1716425	·174228	5·739598	·9851593	7
54	·1719291	·174527	5·729741	·9851093	6
55	·1722156	·174827	5·719917	·9850593	5
56	·1725022	·175127	5·710125	·9850091	4
57	·1727887	·175427	5·700366	·9849589	3
58	·1730752	·175727	5·690639	·9849086	2
59	·1733617	·176027	5·680944	·9848582	1
60	·1736482	·176327	5·671281	·9848078	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 80.

10 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1736482	·176327	5·671281	·9848078	60
1	·1739346	·176626	5·661650	·9847572	59
2	·1742211	·176926	5·652051	·9847066	58
3	·1745075	·177226	5·642483	·9846558	57
4	·1747939	·177527	5·632947	·9846050	56
5	·1750803	·177827	5 623442	·9845542	55
6	·1753667	·178127	5·613968	·9845032	54
7	·1756531	·178427	5·604524	·9844521	53
8	·1759395	·178727	5·595112	·9844010	52
9	·1762258	·179027	5·585730	·9843498	51
10	·1765121	·179327	5·576378	·9842985	50
11	·1767984	·179628	5·567057	·9842471	49
12	·1770847	·179928	5·557766	·9841956	48
13	·1773710	·180228	5·548505	·9841441	47
14	·1776573	·180529	5·539274	·9840924	46
15	·1779435	·180829	5·530072	·9840407	45
16	·1782298	·181129	5·520900	·9839889	44
17	·1785160	·181430	5·511757	·9839370	43
18	·1788022	·181730	5·502644	·9838850	42
19	·1790884	·182031	5·493560	·9838330	41
20	·1793746	·182331	5·484505	·9837808	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 79

10 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·1796607	·182632	5·475478	·9837286	39
22	·1799469	·182933	5·466481	·9836763	38
23	·1802330	·183233	5 457512	·9836239	37
24	·1805191	·183534	5·448571	·9835715	36
25	·1808052	·183835	5·439659	·9835189	35
26	·1810913	·184135	5·430775	·9834663	34
27	·1813774	·184436	5·421918	·9834136	33
28	·1816635	·184737	5·413090	·9833608	32
29	·1819495	·185038	5·404290	·9833079	31
30	·1822355	·185339	5·395517	·9832549	30
31	·1825215	·185639	5·386771	·9832019	29
32	·1828075	·185940	5·378053	·9831487	28
33	·1830935	·186241	5·369363	·9830955	27
34	·1833795	·186542	5·360699	·9830422	26
35	·1836654	·186843	5·352062	·9829888	25
36	·1839514	·187144	5·343452	·9829353	24
37	·1842373	·187446	5·334869	·9828818	23
38	·1845232	·187747	5·326313	·9828282	22
39	·1848091	·188048	5·317783	·9827744	21
40	·1850949	·188349	5·309279	·9827206	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 79.

10 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·1853808	·188650	5·300801	·9826668	19
42	·1856666	·188952	5·292350	·9826128	18
43	·1859524	·189253	5·283925	·9825587	17
44	·1862382	·189554	5·275525	·9825046	16
45	·1865240	·189855	5·267151	·9824504	15
46	·1868098	·190157	5·258803	·9823961	14
47	·1870956	·190458	5·250480	·9823417	13
48	·1873813	·190760	5·242183	·9822873	12
49	·1876670	·191061	5·233911	·9822327	11
50	·1879528	·191363	5·225664	·9821781	10
51	·1882385	·191664	5·217442	·9821234	9
52	·1885241	·191966	5·209245	·9820686	8
53	·1888098	·192268	5·201073	·9820137	7
54	·1890954	·192569	5·192926	·9819587	6
55	·1893811	·192871	5·184803	·9819037	5
56	·1896667	·193173	5·176705	·9818485	4
57	·1899523	·193474	5·168631	·9817933	3
58	·1902379	·193776	5·160581	·9817380	2
59	·1905234	·194078	5·152555	·9816826	1
60	·1908090	·194380	5·144554	·9816272	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 79.

11 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1908090	·194380	5·144554	·9816272	60
1	·1910945	·194682	5·136576	·9815716	59
2	·1913801	·194984	5·128622	·9815160	58
3	·1916656	·195286	5·120692	·9814603	57
4	·1919510	·195588	5·112785	·9814045	56
5	·1922365	·195890	5·104902	·9813486	55
6	·1925220	·196192	5·097042	·9812927	54
7	·1928074	·196494	5·089206	·9812366	53
8	·1930928	·196796	5·081392	·9811805	52
9	·1933782	·197098	5·073602	·9811243	51
10	·1936636	·197400	5·065835	·9810680	50
11	·1939490	·197703	5·058090	·9810116	49
12	·1942344	·198005	5·050369	·9809552	48
13	·1945197	·198307	5·042670	·9808986	47
14	·1948050	·198610	5·034993	·9808420	46
15	·1950903	·198912	5·027339	·9807853	45
16	·1953756	·199214	5·019707	·9807285	44
17	·1956609	·199517	5·012098	·9806716	43
18	·1959461	·199819	5·004511	·9806147	42
19	·1962314	·200122	4·996945	·9805576	41
20	·1965166	·200424	4·989402	·9805005	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 78.

11 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·1968018	·200727	4·981881	·9804433	39
22	·1970870	·201030	4·974381	·9803860	38
23	·1973722	·201332	4·966903	·9803286	37
24	·1976573	·201635	4·959447	·9802712	36
25	·1979425	·201938	4·952012	·9802136	35
26	·1982276	·202240	4·944599	·9801560	34
27	·1985127	·202543	4·937206	·9800983	33
28	·1987978	·202846	4·929835	·9800405	32
29	·1990829	·203149	4·922485	·9799827	31
30	·1993679	·203452	4·915157	·9799247	30
31	·1996530	·203755	4·907849	·9798667	29
32	·1999380	·204058	4·900562	·9798086	28
33	·2002230	·204361	4·893295	·9797504	27
34	·2005080	·204664	4·886049	·9796921	26
35	·2007930	·204967	4·878824	·9796337	25
36	·2010779	·205270	4·871620	·9795752	24
37	·2013629	·205573	4·864435	·9795167	23
38	·2016478	·205876	4·857271	·9794581	22
39	·2019327	·206180	4·850128	·9793994	21
40	·2022176	·206483	4·843004	·9793406	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 78.

11 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·2025024	·206786	4·835901	·9792818	19
42	·2027873	·207090	4·828817	·9792228	18
43	·2030721	·207393	4·821753	·9791638	17
44	·2033569	·207696	4·814709	·9791047	16
45	·2036418	·208000	4·807685	·9790455	15
46	·2039265	·208303	4·800680	·9789862	14
47	·2042113	·208607	4·793695	·9789268	13
48	·2044961	·208910	4·786730	·9788674	12
49	·2047808	·209214	4·779783	·9788079	11
50	·2050655	·209518	4·772856	·9787483	10
51	·2053502	·209821	4·765949	·9786886	9
52	·2056349	·210125	4·759060	·9786288	8
53	·2059195	·210429	4·752190	·9785689	7
54	·2062042	·210733	4·745340	·9785090	6
55	·2064888	·211036	4·738508	·9784490	5
56	·2067734	·211340	4·731695	·9783889	4
57	·2070580	·211644	4·724901	·9783287	3
58	·2073426	·211948	4·718125	·9782684	2
59	·2076272	·212252	4·711368	·9782080	1
60	·2079117	·212556	4·704630	·9781476	0
′	Cosine.	Cot ng.	Tang.	Sine.	′

Deg. 78.

12 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2079117	·212556	4·704630	·9781476	60
1	·2081962	·212860	4·697910	·9780871	59
2	·2084807	·213164	4·691208	·9780265	58
3	·2087652	·213468	4·684524	·9779658	57
4	·2090497	·213773	4·677859	·9779050	56
5	·2093341	·214077	4·671212	·9778441	55
6	·2096186	·214381	4·664583	·9777832	54
7	·2099030	·214685	4·657972	·9777222	53
8	·2101874	·214990	4·651378	·9776611	52
9	·2104718	·215294	4·644803	·9775999	51
10	·2107561	·215598	4·638245	·9775386	50
11	·2110405	·215903	4·631705	·9774773	49
12	·2113248	·216207	4·625183	·9774159	48
13	·2116091	·216512	4·618678	·9773544	47
14	·2118934	·216816	4·612190	·9772928	46
15	·2121777	·217121	4·605720	·9772311	45
16	·2124619	·217425	4·599268	·9771693	44
17	·2127462	·217730	4·592832	·9771075	43
18	·2130304	·218035	4·586414	·9770456	42
19	·2133146	·218340	4·580012	·9769836	41
20	·2135988	·218644	4·573628	·9769215	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 77.

12 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·2138829	·218949	4·567261	·9768593	39
22	·2141671	·219254	4·560911	·9767970	38
23	·2144512	·219559	4·554577	·9767347	37
24	·2147353	·219864	4·548260	·9766723	36
25	·2150194	·220169	4·541960	·9766098	35
26	·2153035	·220474	4·535677	·9765472	34
27	·2155876	·220779	4·529410	·9764845	33
28	·2158716	·221084	4·523160	·9764217	32
29	·2161556	·221389	4·516926	·9763589	31
30	·2164396	·221694	4·510708	·9762960	30
31	·2167236	·221999	4·504507	·9762330	29
32	·2170076	·222305	4·498322	·9761699	28
33	·2172915	·222610	4·492153	·9761067	27
34	·2175754	·222915	4·486000	·9760435	26
35	·2178593	·223221	4·479863	·9759802	25
36	·2181432	·223526	4·473742	·9759168	24
37	·2184271	·223831	4·467637	·9758533	23
38	·2187110	·224137	4·461548	·9757897	22
39	·2189948	·224442	4·455475	·9757260	21
40	·2192786	·224748	4·449418	·9756623	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 77.

12 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·2195624	·225054	4·443376	·9755985	19
42	·2198462	·225359	4·437350	·9755345	18
43	·2201300	·225665	4·431339	·9754706	17
44	·2204137	·225971	4·425343	·9754065	16
45	·2206974	·226276	4·419364	·9753423	15
46	·2209811	·226582	4·413399	·9752781	14
47	·2212648	·226888	4·407450	·9752138	13
48	·2215485	·227194	4·401516	·9751494	12
49	·2218321	·227500	4·395597	·9750849	11
50	·2221158	·227806	4·389694	·9750203	10
51	·2223994	·228112	4·383805	·9749556	9
52	·2226830	·228418	4·377931	·9748909	8
53	·2229666	·228724	4·372073	·9748261	7
54	·2232501	·229030	4·366229	·9747612	6
55	·2235337	·229336	4·360400	·9746962	5
56	·2238172	·229642	4·354586	·9746311	4
57	·2241007	229949	4·348786	·9745660	3
58	·2243842	·230255	4·343001	·9745008	2
59	·2246676	·230561	4·337231	·9744355	1
60	·2249511	·230868	4·331475	·9743701	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 77.

13 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2249511	·230868	4·331475	·9743701	60
1	·2252345	·231174	4·325734	·9743046	59
2	·2255179	·231481	4·320007	·9742390	58
3	·2258013	·231787	4·314295	·9741734	57
4	·2260846	·232094	4·308597	·9741077	56
5	·2263680	·232400	4·302913	·9740419	55
6	·2266513	·232707	4·297244	·9739760	54
7	·2269346	·233014	4·291588	·9739100	53
8	·2272179	·233320	4·285947	·9738439	52
9	2275012	·233627	4·280319	·9737778	51
10	·2277844	·233934	4·274706	·9737116	50
11	·2280677	·234241	4·269107	·9736453	49
12	·2283509	·234547	4·263521	·9735789	48
13	·2286341	·234854	4·257950	·9735124	47
14	·2289172	·235161	4·252392	·9734458	46
15	·2292004	·235468	4·246848	·9733792	45
16	·2294835	·235775	4·241317	·9733125	44
17	·2297666	·236082	4·235800	·9732457	43
18	·2300497	·236390	4·230297	·9731789	42
19	·2303328	·236697	4·224808	·9731119	41
20	·2306159	·237004	4·219331	·9730449	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 76.

13 Deg.

′	Sine.	Tang.	Cotang.	Cosine	′
21	·2308989	·237311	4·213869	·9729777	39
22	·2311819	·237618	4·208419	·9729105	38
23	·2314649	·237926	4·202983	·9728432	37
24	·2317479	·238233	4·197560	·9727759	36
25	·2320309	·238541	4·192151	·9727084	35
26	·2323138	·238848	4·186754	·9726409	34
27	·2325967	·239156	4·181371	·9725733	33
28	·2328796	·239463	4·176001	·9725056	32
29	·2331625	·239771	4·170644	·9724378	31
30	·2334454	·240078	4·165299	·9723699	30
31	·2337282	·240386	4·159968	·9723020	29
32	·2340110	·240694	4·154650	·9722339	28
33	·2342938	·241001	4·149344	·9721658	27
34	·2345766	·241309	4·144051	·9720976	26
35	·2348594	·241617	4·138771	·9720294	25
36	·2351421	·241925	4·133504	·9719610	24
37	·2354248	·242233	4·128249	·9718926	23
38	·2357075	·242541	4·123007	·9718240	22
39	·2359902	·242849	4·117778	·9717554	21
40	·2362729	·243157	4·112561	·9716867	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 76.

13 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·2365555	·243465	4·107356	·9716180	19
42	·2368381	·243773	4·102164	·9715491	18
43	·2371207	·244081	4·096985	·9714802	17
44	·2374033	·244390	4·091817	·9714112	16
45	·2376859	·244698	4·086662	·9713421	15
46	·2379684	·245006	4·081519	·9712729	14
47	·2382510	·245315	4·076389	·9712036	13
48	·2385335	·245623	4·071270	·9711343	12
49	·2388159	·245932	4·066164	·9710649	11
50	·2390984	·246240	4·061070	·9709953	10
51	·2393808	·246549	4·055987	·9709258	9
52	·2396633	·246857	4·050917	·9708561	8
53	·2399457	·247166	4·045859	·9707863	7
54	·2402280	·247475	4·040812	·9707165	6
55	·2405104	·247783	4·035777	·9706466	5
56	·2407927	·248092	4·030755	·9705766	4
57	·2410751	·248401	4·025744	·9705065	3
58	·2413574	·248710	4·020744	·9704363	2
59	·2416396	·249019	4·015757	·9703660	1
60	·2419219	·249328	4·010780	·9702957	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 76

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
	14 Deg.						14 Deg.						14 Deg.				
0	2419219	·249328	4.010780	·9702957	60	21	·2478445	·255826	3·908901	·9687998	39	41	·2534766	·262034	3·816295	·9673415	19
1	·2422041	·249637	4·005816	·9702253	59	22	·2481263	·256136	3·904171	·9687277	38	42	·2537579	·262345	3·811773	·9672678	18
2	·2424863	·249946	4·000863	·9701548	58	23	·2484081	·256446	3·899451	·9686555	37	43	·2540393	·262656	3·807260	·9671939	17
3	·2427685	·250255	3·995922	·9700842	57	24	·2486899	·256756	3·894742	·9685832	36	44	·2543206	·262967	3·802758	·9671200	16
4	·2430507	·250564	3·990992	·9700135	56	25	·2489716	·257066	3·890044	·9685108	35	45	·2546019	·263278	3·798266	·9670459	15
5	·2433329	·250873	3·986073	·9699428	55	26	·2492533	·257376	3·885357	·9684383	34	46	·2548832	·263589	3·793783	·9669718	14
6	·2436150	·251182	3·981166	·9698720	54	27	·2495350	·257686	3·880680	·9683658	33	47	·2551645	·263900	3·789310	·9668977	13
7	·2438971	·251491	3·976271	·9698011	53	28	·2498167	·257997	3 876014	·9682931	32	48	·2554458	·264211	3·784848	.9668234	12
8	·2441792	·251801	3·971386	·9697301	52	29	·2500984	·258307	3·871358	·9682204	31	49	·2557270	·264522	3·780395	·9667490	11
9	·2444613	·252110	3·966513	·9696591	51	30	·2503800	·258617	3·866713	·9681476	30	50	·2560082	·264833	3·775951	·9666746	10
10	·2447433	·252420	3·961651	·9695879	50	31	·2506616	·258928	3·862078	·9680748	29	51	·2562894	·265145	3·771518	·9666001	9
11	·2450254	·252729	3·956801	·9695167	49	32	·2509432	·259238	3·857453	·9680018	28	52	·2565705	·265456	3·767094	·9665255	8
12	·2453074	·253038	3·951961	·9694453	48	33	·2512248	·259548	3·852839	·9679288	27	53	·2568517	·265768	3·762680	·9664508	7
13	·2455894	·253348	3·947133	·9693740	47	34	·2515063	·259859	3·848235	·9678557	26	54	·2571328	·266079	3·758276	·9663761	6
14	·2458713	·253658	3·942315	·9693025	46	35	·2517879	·260169	3·843642	·9677825	25	55	·2574139	·266390	3·753881	·9663012	5
15	·2461533	·253967	3·937509	·9692309	45	36	·2520694	·260480	3·839059	·9677092	24	56	·2576950	·266702	3·749496	·9662263	4
16	·2464352	·254277	3·932714	·9691593	44	37	·2523508	·260791	3·834486	·9676358	23	57	·2579760	·267014	3·745120	·9661513	3
17	·2467171	·254587	3·927929	·9690875	43	38	·2526323	·261101	3·829923	·9675624	22	58	·2582570	·267325	3·740754	·9660762	2
18	·2469990	·254896	3·923156	·9690157	42	39	·2529137	·261412	3·825370	·9674888	21	59	·2585381	·267637	3·736398	·9660011	1
19	·2472809	·255206	3·918393	·9689438	41	40	·2531952	·261723	3·820828	·9674152	20	60	·2588190	·267949	3·732050	·9659258	0
20	·2475627	·255516	3·913642	·9688719	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
	Deg. 75.						Deg. 75.						Deg. 75.				

15 Deg. 15 Deg. 15 Deg

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2588190	·267949	3·732050	·9659258	60	21	·2647147	·274507	3·642891	·9643268	39	41	·2703204	·280773	3·561590	·9627704	19
1	·2591000	·268261	3·727713	·9658505	59	22	·2649952	·274820	3·638744	·9642497	38	42	·2706004	·281087	3·557613	·9626917	18
2	·2593810	·268572	3·723384	·9657751	58	23	·2652757	·275133	3·634606	·9641726	37	43	·2708805	·281401	3·553644	·9626130	17
3	·2596619	·268884	3·719065	·9656996	57	24	·2655561	·275445	3·630477	·9640954	36	44	·2711605	·281715	3·549684	·9625342	16
4	·2599428	·269196	3·714756	·9656240	56	25	·2658366	·275758	3·626356	·9640181	35	45	·2714404	·282029	3·545732	·9624552	15
5	2602237	·269508	3·710455	·9655484	55	26	·2661170	·276071	3·622244	·9639407	34	46	·2717204	·282343	3·541788	·9623762	14
6	·2605045	·269820	3·706164	·9654726	54	27	·2663973	·276385	3·618141	·9638633	33	47	·2720003	·282657	3·537852	·9622972	13
7	·2607853	·270132	3·701883	·9653968	53	28	·2666777	·276698	3·614046	·9637858	32	48	·2722802	·282971	3·533925	·9622180	12
8	·2610662	·270444	3·697610	·9653209	52	29	·2669581	·277011	3·609960	·9637081	31	49	·2725601	·283285	3·530005	·9621387	11
9	·2613469	·270757	3·693346	·9652449	51	30	·2672384	·277324	3·605883	·9636305	30	50	·2728400	·283599	3·526093	·9620594	10
10	·2616277	·271069	3·689092	·9651689	50	31	·2675187	·277637	3·601814	·9635527	29	51	·2731198	·283914	3·522190	·9619800	9
11	·2619085	·271381	3·684847	·9650927	49	32	·2677989	·277951	3·597754	·9634748	28	52	·2733997	·284228	3·518294	·9619005	8
12	·2621892	·271694	3·680611	·9650165	48	33	·2680792	·278264	3·593702	·9633969	27	53	·2736794	·284543	3·514407	·9618210	7
13	·2624699	·272006	3·676384	·9649402	47	34	·2683594	·278578	3·589659	·9633189	26	54	·2739592	·284857	3·510527	·9617413	6
14	·2627506	·272318	3·672166	·9648638	46	35	·2686396	·278891	3·585624	·9632408	25	55	·2742390	·285172	3·506655	·9616616	5
15	·2630312	·272631	3·667957	·9647873	45	36	·2689198	·279205	3·581597	·9631626	24	56	·2745187	·285486	3·502791	·9615818	4
16	·2633118	·272943	3·663757	·9647108	44	37	·2692000	·279518	3·577579	·9630843	23	57	·2747984	·285801	3·498935	·9615019	3
17	·2635925	·273256	3·659566	·9646341	43	38	·2694801	·279832	3·573569	·9630060	22	58	·2750781	·286115	3·495087	·9614219	2
18	·2638730	·273569	3·655384	·9645574	42	39	·2697602	·280145	3·569568	·9629275	21	59	·2753577	·286430	3·491247	·9613418	1
19	·2641536	·273881	3·651211	·9644806	41	40	·2700403	·280459	3·565574	·9628490	20	60	·2756374	·286745	3·487414	·9612617	0
20	·2644342	·274194	3·647046	·9644037	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 74. Deg. 74. Deg. 74.

16 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2756374	·286745	3·487414	·9612617	60
1	·2759170	·287060	3·483589	·9611815	59
2	·2761965	·287375	3·479772	·9611012	58
3	·2764761	·287690	3·475963	·9610208	57
4	·2767556	·288005	3·472161	·9609403	56
5	·2770352	·288320	3·468367	·9608598	55
6	·2773147	·288635	3·464581	·9607792	54
7	·2775941	·288950	3·460802	·9606984	53
8	·2778736	·289265	3·457031	·9606177	52
9	·2781530	·289580	3·453267	·9605368	51
10	·2784324	·289896	3·449512	·9604558	50
11	·2787118	·290211	3·445763	·9603748	49
12	·2789911	·290526	3·442022	·9602937	48
13	·2792704	·290842	3·438289	·9602125	47
14	·2795497	·291157	3·434563	·9601312	46
15	·2798290	·291473	3·430844	·9600499	45
16	·2801083	·291789	3·427133	·9599684	44
17	·2803875	·292104	3·423429	·9598869	43
18	·2806667	·292420	3·419733	·9598053	42
19	·2809459	·292736	3·416044	·9597236	41
20	·2812251	·293052	3·412362	·9596418	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 73.

16 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·2815042	·293368	3·408688	·9595600	39
22	·2817833	·293683	3·405021	·9594781	38
23	·2820624	·293999	3·401361	·9593961	37
24	·2823415	·294316	3·397708	·9593140	36
25	·2826205	·294632	3·394063	·9592318	35
26	·2828995	·294948	3·390424	·9591496	34
27	·2831785	·295264	3·386793	·9590672	33
28	·2834575	·295580	3·383169	·9589848	32
29	·2837364	·295897	3·379553	·9589023	31
30	·2840153	·296213	3·375943	·9588197	30
31	·2842942	·296529	3·372340	·9587371	29
32	·2845731	·296846	3·368745	·9586543	28
33	·2848520	·297163	3 365156	·9585715	27
34	·2851308	·297479	3·361575	·9584886	26
35	·2854096	·297796	3·358000	·9584056	25
36	·2856884	·298112	3·354433	·9583226	24
37	·2859671	·298429	3·350872	·9582394	23
38	·2862458	·298746	3·347319	·9581562	22
39	·2865246	·299063	3·343772	·9580729	21
40	·2868032	·299380	3·340232	·9579895	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 73

16 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·2870819	·299697	3·336699	·9579060	19
42	·2873605	·300014	3·333173	·9578225	18
43	·2876391	·300331	3·329654	·9577389	17
44	·2879177	·300648	3·326141	·9576552	16
45	·2881963	·300965	3·322636	·9575714	15
46	·2884748	·301283	3·319137	·9574875	14
47	·2887533	·301600	3·315645	·9574035	13
48	·2890318	·301917	3·312159	·9573195	12
49	·2893103	·302235	3·308681	·9572354	11
50	·2895887	·302552	3·305209	·9571512	10
51	·2898671	·302870	3·301743	·9570669	9
52	·2901455	·303187	3·298285	·9569825	8
53	·2904239	·303505	3·294833	·9568981	7
54	·2907022	·303823	3·291387	·9568136	6
55	·2909805	·304141	3·287948	·9567290	5
56	·2912588	·304458	3·284516	·9566443	4
57	·2915371	·304776	3·281090	·9565595	3
58	·2918153	·305094	3·277671	·9564747	2
59	·2920935	·305412	3·274258	·9563898	1
60	·2923717	·305730	3·270852	·9563048	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 73.

17 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2923717	·305730	3·270852	·9563048	60
1	·2926499	·306048	3·267452	·9562197	59
2	·2929280	·306367	3·264059	·9561345	58
3	·2932061	·306685	3·260672	·9560492	57
4	·2934842	·307003	3·257292	·9559639	56
5	·2937623	·307321	3·253918	·9558785	55
6	·2940403	·307640	3·250550	·9557930	54
7	·2943183	·307958	3·247189	·9557074	53
8	·2945963	·308277	3·243834	·9556218	52
9	·2948743	·308595	3·240486	·9555361	51
10	·2951522	·308914	3·237143	·9554502	50
11	·2954302	·309233	3·233807	·9553643	49
12	·2957081	·309551	3·230478	·9552784	48
13	·2959859	·309870	3·227154	·9551923	47
14	·2962638	·310189	3·223837	·9551062	46
15	·2965416	·310508	3·220526	·9550199	45
16	·2968194	·310827	3·217221	·9549336	44
17	·2970971	·311146	3·213922	·9548473	43
18	·2973749	·311465	3·210630	·9547608	42
19	·2976526	·311784	3·207344	·9546743	41
20	·2979303	·312103	3·204063	·9545876	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 72.

17 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·2982079	·312422	3·200789	·9545009	39
22	·2984856	·312742	3·197521	·9544141	38
23	·2987632	·313061	3·194259	·9543273	37
24	·2990408	·313381	3·191003	·9542403	36
25	·2993184	·313700	3·187754	·9541533	35
26	·2995959	·314020	3·184510	·9540662	34
27	·2998734	·314339	3·181272	·9539790	33
28	·3001509	·314659	3·178040	·9538917	32
29	·3004284	·314979	3·174814	·9538044	31
30	·3007058	·315298	3·171594	·9537170	30
31	·3009832	·315618	3 168380	·9536294	29
32	·3012606	·315938	3·165172	·9535418	28
33	·3015380	·316258	3·161970	·9534542	27
34	·3018153	·316578	3·158774	·9533664	26
35	·3020926	·316898	3·155584	·9532786	25
36	·3023699	·317218	3·152399	·9531907	24
37	·3026471	·317538	3·149220	·9531027	23
38	·3029244	·317859	3·146047	·9530146	22
39	·3032016	·318179	3·142880	·9529264	21
40	·3034788	·318499	3·139719	·9528382	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 72.

17 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·3037559	·318820	3·136563	·9527499	19
42	·3040331	·319140	3·133414	·9526615	18
43	·3043102	·319461	3·130270	·9525730	17
44	·3045872	·319781	3·127131	·9524844	16
45	·3048643	·320102	3·123999	·9523958	15
46	·3051413	·320423	3·120872	·9523071	14
47	·3054183	·320744	3·117750	·9522183	13
48	·3056953	·321064	3·114635	·9521294	12
49	·3059723	·321385	3·111525	·9520404	11
50	·3062492	·321706	3·108421	·9519514	10
51	·3065261	·322027	3·105322	·9518623	9
52	·3068030	·322348	3·102229	·9517731	8
53	·3070798	·322670	3·099141	·9516838	7
54	·3073566	·322991	3·096059	·9515944	6
55	·3076334	·323312	3·092983	·9515050	5
56	·3079102	·323633	3·089912	·9514154	4
57	·3081869	·323955	3·086846	·9513258	3
58	·3084636	·324276	3·083786	·9512361	2
59	·3087403	·324598	3·080732	·9511464	1
60	·3090170	·324919	3·077683	·9510565	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 72.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
	18 Deg.						18 Deg.						18 Deg.				
0	·3090170	·324919	3·077683	·9510565	60	21	·3148209	·331686	3·014892	·9491511	39	41	·3203374	·338157	2·957205	·9473035	19
1	·3092936	·325241	3·074640	·9509666	59	22	·3150969	·332009	3·011960	·9490595	38	42	·3206130	·338481	2·954372	·9472103	18
2	·3095702	·325563	3·071602	·9508766	58	23	·3153730	·332332	3·009033	·9489678	37	43	·3208885	·338805	2·951545	·9471170	17
3	·3098468	·325884	3·068569	·9507865	57	24	·3156490	·332655	3·006110	·9488760	36	44	·3211640	·339129	2·948722	·9470236	16
4	·3101234	·326206	3·065542	·9506963	56	25	·3159250	·332978	3·003193	·9487842	35	45	·3214395	·339454	2·945905	·9469301	15
5	·3103999	·326528	3·062520	·9506061	55	26	·3162010	·333302	3·000282	·9486922	34	46	·3217149	·339778	2·943092	·9468366	14
6	·3106764	·326850	3·059503	·9505157	54	27	·3164770	·333625	2·997375	·9486002	33	47	·3219903	·340103	2·940284	·9467430	13
7	·3109529	·327172	3·056492	·9504253	53	28	·3167529	·333948	2·994473	·9485081	32	48	·3222657	·340427	2·937480	·9466493	12
8	·3112294	·327494	3·053487	·9503348	52	29	·3170288	·334271	2·991576	·9484159	31	49	·3225411	·340752	2·934682	·9465555	11
9	·3115058	·327816	3·050486	·9502443	51	30	·3173047	·334595	2·988685	·9483237	30	50	·3228164	·341077	2·931888	·9464616	10
10	·3117822	·328138	3·047491	·9501536	50	31	·3175805	·334918	2·985798	·9482313	29	51	·3230917	·341401	2·929099	·9463677	9
11	·3120586	·328461	3·044501	·9500629	49	32	·3178563	·335242	2·982916	·9481389	28	52	·3233670	·341726	2·926315	·9462736	8
12	·3123349	·328783	3·041517	·9499721	48	33	·3181321	·335566	2·980040	·9480464	27	53	·3236422	·342051	2·923535	·9461795	7
13	·3126112	·329105	3·038538	·9498812	47	34	·3184079	·335889	2·977168	·9479538	26	54	·3239174	·342376	2·920761	·9460854	6
14	·3128875	·329428	3·035564	·9497902	46	35	·3186836	·336213	2·974301	·9478612	25	55	·3241926	·342701	2·917990	·9459911	5
15	·3131638	·329750	3·032595	·9496991	45	36	·3189593	·336537	2·971439	·9477684	24	56	·3244678	·343026	2·915225	·9458968	4
16	·3134400	·330073	3·029632	·9496080	44	37	·3192350	·336861	2·968583	·9476756	23	57	·3247429	·343351	2·912464	·9458023	3
17	3137163	·330395	3·026673	·9495168	43	38	·3195106	·337185	2·965731	·9475827	22	58	·3250180	·343677	2·909708	·9457078	2
18	·3139925	·330718	3·023720	·9494255	42	39	·3197863	·337509	2·962884	·9474897	21	59	·3252931	·344002	2·906957	·9456132	1
19	·3142686	·331041	3·020772	·9493341	41	40	·3200619	·337833	2·960042	·9473966	20	60	·3255682	·344327	2·904210	·9455186	0
20	·3145448	·331363	3·017830	·9492426	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
	Deg. 71						Deg. 71.						Deg. 71.				

19 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3255682	·344327	2·904210	·9455186	60
1	·3258432	·344653	2·901468	·9454238	59
2	·3261182	·344978	2·898731	·9453290	58
3	·3263932	·345304	2·895998	·9452341	57
4	·3266681	·345629	2·893270	·9451391	56
5	·3269430	·345955	2·890546	·9450441	55
6	·3272179	·346281	2·887827	·9449489	54
7	·3274928	·346606	2·885113	·9448537	53
8	·3277676	·346932	2·882403	·9447584	52
9	·3280424	·347258	2·879697	·9446630	51
10	·3283172	·347584	2·876997	·9445675	50
11	·3285919	·347910	2·874300	·9444720	49
12	·3288666	·348236	2·871608	·9443764	48
13	·3291413	·348563	2·868921	·9442807	47
14	·3294160	·348889	2·866238	·9441849	46
15	·3296906	·349215	2·863560	·9440890	45
16	·3299653	·349542	2·860886	·9439931	44
17	·3302398	·349868	2·858216	·9438971	43
18	·3305144	·350195	2·855551	·9438010	42
19	·3307889	·350521	2·852891	·9437048	41
20	·3310634	·350848	2·850234	·9436085	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 70.

19 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·3313379	·351175	2·847583	·9435122	39
22	·3316123	·351501	2·844935	·9434157	38
23	·3318867	·351828	2·842292	·9433192	37
24	·3321611	·352155	2·839653	·9432227	36
25	·3324355	·352482	2·837019	·9431260	35
26	·3327098	·352809	2·834389	·9430293	34
27	·3329841	·353136	2·831763	·9429324	33
28	·3332584	·353464	2·829142	·9428355	32
29	·3335326	·353791	2·826525	·9427386	31
30	·3338069	·354118	2·823912	·9426415	30
31	·3340810	·354446	2·821304	·9425444	29
32	·3343552	·354773	2·818700	·9424471	28
33	·3346293	·355101	2·816100	·9423498	27
34	·3349034	·355428	2·813504	·9422525	26
35	·3351775	·355756	2·810913	·9421550	25
36	·3354516	·356084	2·808326	·9420575	24
37	·3357256	·356411	2·805743	·9419598	23
38	·3359996	·356739	2·803164	·9418621	22
39	·3362735	·357067	2·800590	·9417644	21
40	·3365475	·357395	2·798019	·9416665	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 70.

19 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·3368214	·357723	2·795453	·9415686	19
42	·3370953	·358051	2·792891	·9414705	18
43	·3373691	·358380	2·790333	·9413724	17
44	·3376429	·358708	2·787780	·9412743	16
45	·3379167	·359036	2·785230	·9411760	15
46	·3381905	·359365	2·782685	·9410777	14
47	·3384642	·359693	2·780144	·9409793	13
48	·3387379	·360022	2·777606	·9408808	12
49	·3390116	·360350	2·775073	·9407822	11
50	·3392852	·360679	2·772544	·9406835	10
51	·3395589	·361008	2·770019	·9405848	9
52	·3398325	·361337	2·767499	·9404860	8
53	·3401060	·361666	2·764982	·9403871	7
54	·3403796	·361994	2·762469	·9402881	6
55	·3406531	·362324	2·759960	·9401891	5
56	·3409265	·362653	2·757456	·9400899	4
57	·3412000	·362982	2·754955	·9399907	3
58	·3414734	·363311	2·752458	·9398914	2
59	·3417468	·363640	2·749966	·9397921	1
60	·3420201	·363970	2·747477	·9396926	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 70

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3420201	·363970	2·747477	·9396926	60	21	·3477540	·370903	2·696118	·9375858	39	41	·3532027	·377536	2·648753	·9355468	19
1	·3422935	·364299	2·744992	·9395931	59	22	·3480267	·371234	2·693714	·9374846	38	42	·3534748	·377868	2·646423	·9354440	18
2	·3425668	·364629	2·742512	·9394935	58	23	·3482994	·371565	2·691314	·9373833	37	43	·3537469	·378201	2·644096	·9353412	17
3	·3428400	·364958	2·740035	·9393938	57	24	·3485720	·371896	2·688919	·9372820	36	44	·3540190	·378533	2·641774	·9352382	16
4	·3431133	·365288	2·737562	·9392940	56	25	·3488447	·372227	2·686526	·9371806	35	45	·3542910	·378866	2·639454	·9351352	15
5	·3433865	·365618	2·735093	·9391942	55	26	·3491173	·372559	2·684138	·9370790	34	46	·3545630	·379198	2·637139	·9350321	14
6	·3436597	·365948	2·732628	·9390943	54	27	·3493898	·372890	2·681753	·9369774	33	47	·3548350	·379531	2·634827	·9349289	13
7	·3439329	·366277	2·730167	·9389943	53	28	·3496624	·373221	2·679372	·9368758	32	48	·3551070	·379864	2·632518	·9348257	12
8	·3442060	·366607	2·727710	·9388942	52	29	·3499349	·373553	2·676995	·9367740	31	49	·3553789	·380197	2·630213	·9347223	11
9	·3444791	·366937	2·725256	·9387940	51	30	·3502074	·373884	2·674621	·9366722	30	50	·3556508	·380530	2·627912	·9346189	10
10	·3447521	·367268	2·722807	·9386938	50	31	·3504798	·374216	2·672251	·9365703	29	51	·3559226	·380863	2·625614	·9345154	9
11	·3450252	·367598	2·720362	·9385934	49	32	·3507523	·374547	2·669885	·9364683	28	52	·3561944	·381196	2·623319	·9344119	8
12	·3452982	·367928	2·717920	·9384930	48	33	·3510246	·374879	2·667522	·9363662	27	53	·3564662	·381529	2·621028	·9343082	7
13	·3455712	·368258	2·715482	·9383925	47	34	·3512970	·375211	2·665163	·9362641	26	54	·3567380	·381862	2·618741	·9342045	6
14	·3458441	·368589	2·713048	·9382920	46	35	·3515693	·375543	2·662808	·9361618	25	55	·3570097	·382196	2·616457	·9341007	5
15	·3461171	·368919	2·710618	·9381913	45	36	·3518416	·375875	2·660456	·9360595	24	56	·3572814	·382529	2·614176	·9339968	4
16	·3463900	·369250	2·708192	·9380906	44	37	·3521139	·376207	2·658108	·9359571	23	57	·3575531	·382863	2·611899	·9338928	3
17	·3466628	·369580	2·705769	·9379898	43	38	·3523862	·376539	2·655764	·9358547	22	58	·3578248	·383196	2·609625	·9337888	2
18	·3469357	·369911	2·703351	·9378889	42	39	·3526584	·376871	2·653423	·9357521	21	59	·3580964	·383530	2·607355	·9336846	1
19	·3472085	·370242	2·700936	·9377880	41	40	·3529306	·377203	2·651086	·9356495	20	60	·3583679	·383864	2·605089	·9335804	0
20	·3474812	·370572	2·698525	·9376869	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

20 Deg. 20 Deg. 20 Deg.

Deg. 69. Deg. 69. Deg. 69.

21 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3583679	·383864	2·605089	·9335804	60
1	·3586395	·384197	2·602825	·9334761	59
2	·3589110	·384531	2·600565	9333718	58
3	·3591825	·384865	2·598309	·9332673	57
4	·3594540	·385199	2·596056	·9331628	56
5	·3597254	·385533	2·593806	·9330582	55
6	·3599968	·385867	2·591560	·9329535	54
7	·3602682	·386202	2·589317	·9328488	53
8	·3605395	·386536	2·587078	·9327439	52
9	·3608108	·386870	2·584842	·9326390	51
10	·3610821	·387205	2·582609	·9325340	50
11	·3613534	·387539	2·580380	·9324290	49
12	·3616246	·387874	2·578153	·9323238	48
13	·3618958	·388209	2·575931	·9322186	47
14	·3621669	·388543	2·573711	·9321133	46
15	·3624380	·388878	2·571495	·9320079	45
16	·3627091	·389213	2·569283	·9319024	44
17	·3629802	·389548	2·567073	·9317969	43
18	·3632512	·389883	2·564867	·9316912	42
19	·3635222	·390218	2·562664	·9315855	41
20	·3637932	·390554	2·560464	·9314797	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 68.

21 Deg.

′	Sine.	Tang.	Cotang.	Cosine	′
21	·3640641	·390889	2·558268	·9313739	39
22	·3643351	·391224	2·556075	·9312679	38
23	·3646059	·391560	2·553885	·9311619	37
24	·3648768	·391895	2·551699	·9310558	36
25	·3651476	·392231	2·549516	·9309496	35
26	·3654184	·392567	2·547335	·9308434	34
27	·3656891	·392902	2·545159	·9307370	33
28	·3659599	·393238	2·542985	·9306306	32
29	·3662306	·393574	2·540815	·9305241	31
30	·3665012	·393910	2·538647	·9304176	30
31	·3667719	·394246	2·536483	·9303109	29
32	·3670425	·394582	2·534323	·9302042	28
33	·3673130	·394918	2·532165	·9300974	27
34	3675836	·395255	2·530011	·9299905	26
35	·3678541	·395591	2·527859	·9298835	25
36	·3681246	·395928	2·525711	·9297765	24
37	·3683950	·396264	2·523566	·9296694	23
38	·3686654	·396601	2·521424	·9295622	22
39	·3689358	·396937	2·519286	·9294549	21
40	·3692061	·397274	2·517150	·9293475	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 68.

21 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·3694765	·397611	2·515018	·9292491	19
42	·3697468	·397948	2·512889	·9291326	18
43	·3700170	·398285	2·510762	·9290250	17
44	·3702872	·398622	2·508639	·9289173	16
45	·3705574	·398959	2·506519	·9288096	15
46	·3708276	·399296	2·504403	·9287017	14
47	·3710977	·399634	2·502289	·9285938	13
48	·3713678	·399971	2·500178	·9284858	12
49	·3716379	·400308	2·498070	·9283778	11
50	·3719079	·400646	2·495966	·9282696	10
51	·3721780	·400984	2·493864	·9281614	9
52	·3724479	·401321	2·491766	·9280531	8
53	·3727179	·401659	2·489670	·9279447	7
54	·3729878	·401997	2·487578	·9278363	6
55	·3732577	·402335	2·485488	·9277277	5
56	·3735275	·402673	2·483402	·9276191	4
57	·3737973	·403011	2·481319	·9275104	3
58	·3740671	·403349	2·479238	·9274016	2
59	·3743369	·403687	2·477161	·9272928	1
60	·3746066	·404026	2·475086	·9271839	0
′	Cosine.	Cotang.	Tang	Sine.	′

Deg. 68.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
	2 Deg.						22 Deg.						22 Deg.				
0	·3746066	·404026	2·475086	·9271839	60	21	·3802634	·411149	2·432204	·9248782	39	41	·3856377	·417967	2·392531	·9226503	19
1	·3748763	·404364	2·473015	·9270748	59	22	·3805324	·411489	2·430193	·9247676	38	42	·3859060	·418309	2·390576	·9225381	18
2	·3751459	·404703	2·470947	·9269658	58	23	·3808014	·411830	2·428186	·9246568	37	43	·3861744	·418650	2·388625	·9224258	17
3	·3754156	·405041	2·468881	·9268566	57	24	·3810704	·412170	2·426181	·9245460	36	44	·3864427	·418992	2·386675	·9223134	16
4	·3756852	·405380	2·466819	·9267474	56	25	·3813393	·412510	2·424180	·9244351	35	45	·3867110	·419334	2·384729	·9222010	15
5	·3759547	·405719	2·464759	·9266380	55	26	·3816082	·412851	2·422181	·9243242	34	46	·3869792	·419676	2·382785	·9220884	14
6	·3762243	·406057	2·462703	·9265286	54	27	·3818770	·413191	2·420185	·9242131	33	47	·3872474	·420019	2·380844	·9219758	13
7	·3764938	·406396	2·460649	·9264192	53	28	·3821459	·413532	2·418191	·9241020	32	48	·3875156	·420361	2·378906	·9218632	12
8	·3767632	·406735	2·458598	·9263096	52	29	·3824147	·413872	2·416201	·9239908	31	49	·3877837	·420703	2·376970	·9217504	11
9	·3770327	·407074	2·456551	·9262000	51	30	·3826834	·414213	2·414213	·9238795	30	50	·3880518	·421046	2·375037	·9216375	10
10	·3773021	·407413	2·454506	·9260902	50	31	·3829522	·414554	2·412228	·9237682	29	51	·3883199	·421388	2·373106	·9215246	9
11	·3775714	·407753	2·452464	·9259805	49	32	·3832209	·414895	2·410246	·9236567	28	52	·3885880	·421731	2·371179	·9214116	8
12	·3778408	·408092	2·450425	·9258706	48	33	·3834895	·415236	2·408267	·9235452	27	53	·3888560	·422073	2·369254	·9212986	7
13	·3781101	·408431	2·448389	·9257606	47	34	·3837582	·415577	2·406290	·9234336	26	54	·3891240	·422416	2·367331	·9211854	6
14	·3783794	·408771	2·446355	·9256506	46	35	·3840268	·415918	2·404316	·9233220	25	55	·3893919	·422759	2·365411	·9210722	5
15	·3786[illegible]	·409110	2·444325	·9255405	45	36	·3842953	·416259	2·402345	·9232102	24	56	·3896598	·423102	2·363494	·9209589	4
17	·3466[illegible]	·09450	2·442298	·9254303	44	37	·3845639	·416601	2·400377	·9230984	23	57	·3899277	·423445	2·361580	·9208455	3
18	·34693[illegible]	·790	2·440273	·9253201	43	38	·3848324	·416942	2·398411	·9229865	22	58	·3901955	·423788	2·359668	·9207320	2
19	·3472[illegible]	[illegible]	·438251	·9252097	42	39	·3851008	·417284	2·396449	·9228745	21	59	·3904633	·424131	2·357759	·9206185	1
20	·3474[illegible]53	·410469	2·436233	·9250993	41	40	·3853693	·417625	2·394488	·9227624	20	60	·3907311	·424474	2·355852	·9205049	0
	[illegible]944	·410809	2·434217	·9249888	40												
′	Cos[illegible]e.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
	Deg. 67.								Deg. 67.							Deg. 67	

21 Deg.						23 Deg.						23 Deg.					
′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·35836		2·355852	·9205049	60	21	·3963468	·431703	2·316407	·9181009	39	41	·4016814	·438622	2·279865	·9157795	19
1	·358639	816	2·353948	·9203912	59	22	·3966139	·432048	2·314557	·9179855	38	42	·4019478	·438969	2·278063	·9156626	18
2	·3589116	5161	2·352046	·9202774	58	23	·3968809	·432393	2·312709	·9178701	37	43	·4022141	·439316	2·276264	·9155456	17
3	·359082	5505	2·350148	·9201635	57	24	·3971479	·432738	2·310863	·9177546	36	44	·4024804	·439663	2·274467	·9154286	16
4	·3918019	·425848	2·348251	·9200496	56	25	·3974148	·433084	2·309020	·9176391	35	45	·4027467	·440010	2·272672	·9153115	15
5	·3920695	·426192	2·346358	·9199356	55	26	·3976818	·433429	2·307180	·9175234	34	46	·4030129	·440357	2·270880	·9151943	14
6	·3923371	·426536	2·344467	·9198215	54	27	·3979486	·433775	2·305342	·9174077	33	47	·4032791	·440705	2·269090	·9150770	13
7	·3926047	·426880	2·342578	·9197073	53	28	·3982155	·434120	2·303506	·9172919	32	48	·4035453	·441052	2·267303	·9149597	12
8	·3928722	·427223	2·340692	·9195931	52	29	·3984823	·434466	2·301673	·9171760	31	49	·4038114	·441400	2·265518	·9148422	11
9	·3931397	·427568	2·338809	·9194788	51	30	·3987491	·434812	2·299842	·9170601	30	50	·4040775	·441747	2·263735	·9147247	10
10	·3934071	·427912	2·336928	·9193644	50	31	·3990158	·435158	2·298014	·9169440	29	51	·4043436	·442095	2·261955	·9146072	9
11	·3936745	·428256	2·335050	·9192499	49	32	·3992825	·435504	2·296188	·9168279	28	52	·4046096	·442443	2·260177	·9144895	8
12	·3939419	·428600	2·333174	·9191353	48	33	·3995492	·435850	2·294365	·9167118	27	53	·4048756	·442791	2·258401	·9143718	7
13	·3942093	·428944	2·331301	·9190207	47	34	·3998158	·436196	2·292544	·9165955	26	54	·4051416	·443139	2·256628	·9142540	6
14	·3944766	·429289	2·329431	·9189060	46	35	·4000825	·436542	2·290725	·9164791	25	55	·4054075	·443487	2·254857	·9141361	5
15	·3947439	·429633	2·327563	·9187912	45	36	·4003490	·436889	2·288909	·9163627	24	56	·4056734	·443835	2·253088	·9140181	4
16	·3950111	·429978	2·325697	·9186763	44	37	·4006156	·437235	2·287095	·9162462	23	57	·4059393	·444183	2·251322	·9139001	3
17	·3952783	·430323	2·323834	·9185614	43	38	·4008821	·437582	2·285284	·9161297	22	58	·4062051	·444531	2·249558	·9137819	2
18	·3955455	·430668	2·321974	·9184464	42	39	·4011486	·437928	2·283475	·9160130	21	59	·4064709	·444880	2·247796	·9136637	1
19	·3958127	·431012	2·320116	·9183313	41	40	·4014150	·438275	2·281669	·9158963	20	60	·4067366	·445228	2·246036	·9135455	0
20	·3960798	·431357	2·318260	·9182161	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
	Deg. 66.						Deg. 66.						Deg. 66.				

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
	24 Deg.						24 Deg.						24 Deg.				
0	·4067366	·445228	2·246036	·9135455	60	21	·4123096	·452568	2·209611	·9110438	39	41	·4176028	·459596	2·175822	·9086297	19
1	·4070024	·445577	2·244279	·9134271	59	22	·4125745	·452918	2·207901	·9109238	38	42	·4178671	·459948	2·174155	·9085082	18
2	·4072681	·445926	2·242524	·9133087	58	23	·4128395	·453269	2·206193	·9108038	37	43	·4181313	·460301	2·172491	·9083866	17
3	·4075337	·446274	2·240772	·9131902	57	24	·4131044	·453620	2·204487	·9106837	36	44	·4183956	·460653	2·170828	·9082649	16
4	·4077993	·446623	2·239021	·9130716	56	25	·4133693	·453970	2·202784	·9105635	35	45	·4186597	·461006	2·169167	·9081432	15
5	·4080649	·446972	2·237273	·9129529	55	26	·4136342	·454321	2·201083	·9104432	34	46	·4189239	·461359	2·167509	·9080214	14
6	·4083305	·447321	2·235528	·9128342	54	27	·4138990	·454672	2·199384	·9103228	33	47	·4191880	·461711	2·165852	·9078995	13
7	·4085960	·447670	2·233784	·9127154	53	28	·4141638	·455023	2·197687	·9102024	32	48	·4194521	·462064	2·164198	·9077775	12
8	·4088615	·448020	2·232043	·9125965	52	29	·4144285	·455375	2·195992	·9100819	31	49	·4197161	·462417	2·162546	·9076554	11
9	·4091269	·448369	2·230304	·9124775	51	30	·4146932	·455726	2·194299	·9099613	30	50	·4199801	·462771	2·160895	·9075333	10
10	·4093923	·448718	2·228567	·9123584	50	31	·4149579	·456077	2·192609	·9098406	29	51	·4202441	·463124	2·159247	·9074111	9
11	·4096577	·449068	2·226833	·9122393	49	32	·4152226	·456429	2·190921	·9097199	28	52	·4205080	·463477	2·157601	·9072888	8
12	·4099230	·449417	2·225100	·9121201	48	33	·4154872	·456780	2·189234	·9095990	27	53	·4207719	·463831	2·155957	·9071665	7
13	·4101883	·449767	2·223370	·9120008	47	34	·4157517	·457132	2·187551	·9094781	26	54	·4210358	·464184	2·154315	·9070440	6
14	·4104536	·450117	2·221643	·9118815	46	35	·4160163	·457483	2·185869	·9093572	25	55	·4212996	·464538	2·152675	·9069215	5
15	·4107189	·450467	2·219917	·9117620	45	36	4162808	·457835	2·184189	·9092361	24	56	·4215634	·464891	2·151037	·9067989	4
16	·4109841	·450817	2·218194	·9116425	44	37	·4165453	·458187	2·182511	·9091150	23	57	·4218272	·465245	2·149402	·9066762	3
17	·4112492	·451167	2·216473	·9115229	43	38	·4168097	·458539	2·180836	·9089938	22	58	·4220909	·465599	2·147768	·9065535	2
18	·4115144	·451517	2·214754	·9114033	42	39	·4170741	·458891	2·179163	·9088725	21	59	·4223546	·465953	2·146136	·9064307	1
19	·4117795	·451867	2·213037	·9112835	41	40	·4173385	·459243	2·177492	·9087511	20	60	·4226183	·466307	2·144506	·9063078	0
20	·4120445	·452217	2·211323	·9111637	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
	Deg. 65.						Deg. 65.						Deg. 65				

25 Deg

25 Deg.

25 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4226183	·466307	2·144506	·9063078	60	21	·4281467	·473765	2·110747	·9037093	39	41	·4333970	·480909	2·079394	·9012031	19
1	·4228819	·466661	2·142879	·9061848	59	22	·4284095	·474122	2·109161	·9035847	38	42	·4336591	·481267	2·077846	·9010770	18
2	·4231455	·467016	2·141253	·9060618	58	23	·4286723	·474478	2·107577	·9034600	37	43	·4339212	·481625	2·076300	·9009508	17
3	·4234090	·467370	2·139630	·9059386	57	24	·4289351	·474834	2·105995	·9033353	36	44	·4341832	·481984	2·074756	·9008246	16
4	·4236725	·467725	2·138008	·9058154	56	25	·4291979	·475191	2·104415	·9032105	35	45	·4344453	·482342	2·073214	·9006982	15
5	·4239360	·468079	2·136389	·9056922	55	26	·4294606	·475548	2·102836	·9030856	34	46	·4347072	·482701	2·071674	·9005718	14
6	·4241994	·468434	2·134771	·9055688	54	27	·4297233	·475904	2·101260	·9029606	33	47	·4349692	·483060	2·070135	·9004453	13
7	·4244628	·468789	2·133155	·9054454	53	28	·4299859	·476261	2·099686	·9028356	32	48	·4352311	·483418	2·068599	·9003188	12
8	·4247262	·469143	2·131542	·9053219	52	29	·4302485	·476618	2·098114	·9027105	31	49	·4354930	·483777	2·067064	·9001921	11
9	·4249895	·469498	2·129930	·9051983	51	30	·4305111	·476975	2·096543	·9025853	30	50	·4357548	·484136	2·065531	·9000654	10
10	·4252528	·469853	2·128321	·9050746	50	31	·4307736	·477332	2·094975	·9024600	29	51	·4360166	·484495	2·064000	·8999386	9
11	·4255161	·470209	2·126713	·9049509	49	32	·4310361	·477689	2·093408	·9023347	28	52	·4362784	·484855	2·062471	·8998117	8
12	·4257793	·470564	2·125108	·9048271	48	33	·4312986	·478047	2·091843	·9022092	27	53	·4365401	·485214	2·060944	·8996848	7
13	·4260425	·470919	2·123504	·9047032	47	34	·4315610	·478404	2·090280	·9020838	26	54	·4368018	·485573	2·059418	·8995578	6
14	·4263056	·471275	2·121903	·9045792	46	35	·4318234	·478762	2·088720	·9019582	25	55	·4370634	·485933	2·057895	·8994307	5
15	·4265687	·471630	2·120303	·9044551	45	36	·4320857	·479119	2·087161	·9018325	24	56	·4373251	·486293	2·056373	·8993035	4
16	·4268318	·471986	2·118705	·9043310	44	37	·4323481	·479477	2·085603	·9017068	23	57	·4375866	·486652	2·054853	·8991763	3
17	·4270949	·472342	2·117110	·9042068	43	38	·4326103	·479835	2·084048	·9015810	22	58	·4378482	·487012	2·053334	·8990489	2
18	·4273579	·472697	2·115516	·9040825	42	39	·4328726	·480193	2·082495	·9014551	21	59	·4381097	·487372	2·051818	·8989215	1
19	·4276208	·473053	2·113924	·9039582	41	40	·4331348	·480551	2·080943	·9013292	20	60	·4383711	·487732	2·050303	·8987940	0
20	·4278838	·473409	2·112334	·9038338	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine	′

Deg. 64.

Deg. 64.

Deg. 64.

26 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4383711	·487732	2·050303	·8987940	60
1	·4386326	·488092	2·048791	·8986665	59
2	·4388940	·488453	2·047280	·8985389	58
3	·4391553	·488813	2·045770	·8984112	57
4	·4394166	·489173	2·044263	·8982834	56
5	·4396779	·489534	2·042757	·8981555	55
6	·4399392	·489894	2·041254	·8980276	54
7	·4402004	·490255	2·039751	·8978996	53
8	·4404615	·490616	2·038251	·8977715	52
9	·4407227	·490977	2·036753	·8976433	51
10	·4409838	·491338	2·035256	·8975151	50
11	·4412448	·491699	2·033761	·8973868	49
12	·4415059	·492061	2·032268	·8972584	48
13	·4417668	·492422	2·030776	·8971299	47
14	·4420278	·492783	2·029287	·8970014	46
15	·4422887	·493145	2·027799	·8968727	45
16	·4425496	·493507	2·026313	·8967440	44
17	·4428104	·493868	2·024828	·8966153	43
18	·4430712	·494230	2·023346	·8964864	42
19	·4433319	·494592	2·021865	·8963575	41
20	·4435927	·494954	2·020386	·8962285	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 63.

26 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·4438534	·495317	2·018908	·8960994	39
22	·4441140	·495679	2·017433	·8959703	38
23	·4443746	·496041	2·015959	·8958411	37
24	·4446352	·496404	2·014486	·8957118	36
25	·4448957	·496766	2·013016	·8955824	35
26	·4451562	·497129	2·011547	·8954529	34
27	·4454167	·497492	2·010080	·8953234	33
28	·4456771	·497855	2·008615	·8951938	32
29	·4459375	·498218	2·007151	·8950641	31
30	·4461978	·498581	2·005689	·8949344	30
31	·4464581	·498944	2·004229	·8948045	29
32	·4467184	·499308	2·002771	·8946746	28
33	·4469786	·499671	2·001314	·8945446	27
34	·4472388	·500035	1·999859	·8944146	26
35	·4474990	·500398	1·998405	·8942844	25
36	·4477591	·500762	1·996953	·8941542	24
37	·4480192	501126	1·995503	·8940240	23
38	·4482792	·501490	1·994055	·8938936	22
39	·4485392	·501854	1·992608	·8937632	21
40	·4487992	·502218	1·991163	·8936326	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 63.

26 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·4490591	·502583	1·989720	·8935021	19
42	·4493190	·502947	1·988278	·8933714	18
43	·4495789	·503312	1·986838	·8932406	17
44	·4498387	·503676	1·985400	·8931098	16
45	·4500984	·504041	1·983963	·8929789	15
46	·4503582	·504406	1·982528	·8928480	14
47	·4506179	·504771	1·981095	·8927169	13
48	·4508775	·505136	1·979663	·8925858	12
49	·4511372	·505501	1·978233	·8924546	11
50	·4513967	·505866	1·976805	·8923234	10
51	·4516563	·506232	1·975378	·8921920	9
52	·4519158	·506597	1·973953	·8920606	8
53	·4521753	·506963	1·972529	·8919291	7
54	·4524347	·507329	1·971107	·8917975	6
55	·4526941	·507694	1·969687	·8916659	5
56	·4529535	·508060	1·968268	·8915342	4
57	·4532128	·508426	1·966851	·8914024	3
58	·4534721	·508792	1·965436	·8912705	2
59	·4537313	·509159	1·964022	·8911385	1
60	·4539905	509525	1·962610	·8910065	0
′	Cosine.	Cotang.	Tang.	Sine	′

Deg. 63

27 Deg. 27 Deg. 27 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4539905	·509525	1·962610	·8910065	60	21	·4594248	·517244	1·933323	·8882166	39	41	·4645845	·524640	1·906066	·8855288	19
1	·4542497	·509891	1·961200	·8908744	59	22	·4596832	·517612	1·931945	·8880830	38	42	·4648420	·525011	1.904719	·8853936	18
2	·4545088	·510258	1·959791	·8907423	58	23	·4599415	·517981	1·930569	·8879492	37	43	·4650996	·525382	1.903373	·8852584	17
3	·4547679	·510625	1·958383	·8906100	57	24	·4601998	·518350	1·929195	·8878154	36	44	·4653571	·525754	1·902029	·8851230	16
4	·4550269	·510991	1·956978	·8904777	56	25	·4604580	·518719	1·927822	·8876815	35	45	·4656145	·526125	1 900687	·8849876	15
5	·4552859	·511358	1·955573	·8903453	55	26	·4607162	·519089	1·926451	·8875475	34	46	·4658719	·526496	1·899346	·8848522	14
6	·4555449	·511725	1·954171	·8902128	54	27	·4609744	·519458	1·925081	·8874134	33	47	·4661293	·526868	1·898006	·8847166	13
7	·4558038	·512093	1·952770	·8900803	53	28	·4612325	·519827	1·923713	·8872793	32	48	·4663866	·527240	1·896668	·8845810	12
8	·4560627	·512460	1·951371	·8899476	52	29	·4614906	·520197	1·922347	·8871451	31	49	·4666439	·527612	1·895332	·8844453	11
9	·4563216	·512827	1·949973	·8898149	51	30	·4617486	·520567	1·920982	·8870108	30	50	·4669012	·527983	1·893997	·8843095	10
10	·4565804	·513195	1·948577	·8896822	50	31	·4620066	·520936	1·919618	·8868765	29	51	·4671584	·528356	1·892663	·8841736	9
11	·4568392	·513562	1·947182	·8895493	49	32	·4622646	·521306	1·918256	·8867420	28	52	·4674156	·528728	1·891331	·8840377	8
12	·4570979	·513930	1·945789	·8894164	48	33	·4625225	·521676	1·916896	·8866075	27	53	·4676727	·529100	1·890000	·8839017	7
13	·4573566	·514298	1·944398	·8892834	47	34	·4627804	·522046	1·915537	·8864730	26	54	·4679298	·529472	1·888671	·8837656	6
14	·4576153	·514665	1·943008	·8891503	46	35	·4630382	·522417	1·914179	·8863383	25	55	·4681869	·529845	1·887343	·8836295	5
15	·4578739	·515033	1·941620	·8890171	45	36	·4632960	·522787	1·912823	·8862036	24	56	·4684439	·530217	1·886017	·8834933	4
16	·4581325	·515401	1·940233	·8888839	44	37	·4635538	·523157	1·911469	·8860688	23	57	·4687009	·530590	1·884692	·8833569	3
17	·4583910	·515770	1·938848	·8887506	43	38	·4638115	·523528	1·910116	·8859339	22	58	·4689578	·530963	1·883369	·8832206	2
18	·4586496	·516138	1·937464	·8886172	42	39	·4640692	·523899	1·908764	·8857989	21	59	·4692147	·531336	1·882047	·8830841	1
19	·4589080	·516506	1·936082	·8884838	41	10	·4643269	·524269	1·907414	·8856639	20	60	·4694716	·531709	1·880726	·8829476	0
20	·4591665	·516875	1·934702	·8883503	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine	′

Deg. 62. Deg. 62. Deg. 62.

28 Deg. 28 Deg. 28 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4694716	·531709	1·880726	·8829476	60	21	·4748564	·539570	1·853325	·8800633	39	41	·4799683	·547106	1·827799	·8772858	19
1	·4697284	·532082	1·879407	·8828110	59	22	·4751124	·539946	1·852035	·8799251	38	42	·4802235	·547484	1·826537	·8771462	18
2	·4699852	·532455	1·878089	·8826743	58	23	·4753683	·540322	1·850747	·8797869	37	43	·4804786	·547862	1·825276	·8770064	17
3	·4702419	·532829	1·876773	·8825376	57	24	·4756242	·540698	1·849461	·8796486	36	44	·4807337	·548240	1·824017	·8768666	16
4	·4704986	·533202	1·875458	·8824007	56	25	·4758801	·541074	1·848176	·8795102	35	45	·4809888	·548618	1·822759	·8767268	15
5	·4707553	533576	1·874145	·8822638	55	26	·4761359	·541450	1·846892	·8793717	34	46	·4812438	·548997	1·821502	·8765868	14
6	·4710119	·533950	1·872833	·8821269	54	27	·4763917	·541826	1·845609	·8792332	33	47	·4814987	·549375	1·820247	·8764468	13
7	·4712685	·534324	1·871523	·8819898	53	28	·4766474	·542202	1·844328	·8790946	32	48	·4817537	·549754	1·818993	·8763067	12
8	·4715250	·534698	1·870214	·8818527	52	29	·4769031	·542579	1·843049	·8789559	31	49	·4820086	·550133	1·817740	·8761665	11
9	·4717815	·535072	1·868906	·8817155	51	30	·4771588	·542955	1·841770	·8788171	30	50	·4822634	·550512	1·816489	·8760263	10
10	·4720380	·535446	1·867600	·8815782	50	31	·4774144	·543332	1·840494	·8786783	29	51	·4825182	·550891	1·815239	·8758859	9
11	·4722944	·535820	1·866295	·8814409	49	32	·4776700	·543709	1·839218	·8785394	28	52	·4827730	·551270	1·813990	·8757455	8
12	·4725508	·536195	1·864992	·8813035	48	33	·4779255	·544086	1·837944	·8784004	27	53	·4830277	·551650	1·812743	·8756051	7
13	·4728071	·536569	1·863690	·8811660	47	34	·4781810	·544463	1·836671	·8782613	26	54	·4832824	·552029	1·811496	·8754645	6
14	·4730634	·536944	1·862389	·8810284	46	35	·4784364	·544840	1·835399	·8781222	25	55	·4835370	·552409	1·810252	·8753239	5
15	·4733197	·537319	1·861090	·8808907	45	36	·4786919	·545217	1·834129	·8779830	24	56	·4837916	·552789	1·809008	·8751832	4
16	·4735759	·537694	1·859792	·8807530	44	37	·4789472	·545595	1·832861	·8778437	23	57	·4840462	·553168	1·807766	·8750425	3
17	·4738321	·538069	1·858496	·8806152	43	38	·4792026	·545972	1·831593	·8777043	22	58	·4843007	·553548	1·806525	·8749016	2
18	·4740882	·538444	1·857201	·8804774	42	39	·4794579	·546350	1·830327	·8775649	21	59	·4845552	·553928	1·805286	·8747607	1
19	·4743443	·538819	1·855908	·8803394	41	40	·4797131	·546728	1·829062	·8774254	20	60	·4848096	·554309	1·804047	·8746197	0
20	·4746004	·539195	1·854615	·8802014	40												
′	Cosine.	Cotang.	Tang	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 61. Deg. 61. Deg. 61

29 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4848096	·554309	1·804047	·8746197	60
1	·4850640	·554689	1·802810	·8744786	59
2	·4853184	·555069	1·801575	·8743375	58
3	·4855727	·555450	1·800340	·8741963	57
4	·4858270	·555831	1·799107	·8740550	56
5	·4860812	·556211	1·797875	·8739137	55
6	·4863354	·556592	1·796645	·8737722	54
7	·4865895	·556973	1·795416	·8736307	53
8	·4868436	·557355	1·794188	·8734891	52
9	·4870977	·557736	1·792961	·8733475	51
10	·4873517	·558117	1·791736	·8732058	50
11	·4876057	·558499	1·790512	·8730640	49
12	·4878597	·558881	1·789289	·8729221	48
13	·4881136	·559262	1·788067	·8727801	47
14	·4883674	·559644	1·786847	·8726381	46
15	·4886212	·560026	1·785628	·8724960	45
16	·4888750	·560409	1·784410	·8723538	44
17	·4891288	·560791	1·783194	·8722116	43
18	·4893825	·561173	1·781979	·8720693	42
19	·4896361	·561556	1·780765	·8719269	41
20	·4898897	·561939	1·779552	·8717844	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 60.

29 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·4901433	·562321	1·778340	·8716419	39
22	·4903968	·562704	1·777130	·8714993	38
23	·4906503	·563087	1·775921	·8713566	37
24	·4909038	·563471	1·774714	·8712138	36
25	·4911572	·563854	1·773507	·8710710	35
26	·4914105	·564237	1·772302	·8709281	34
27	·4916638	·564621	1·771098	·8707851	33
28	.4919171	·565005	1·769895	·8706420	32
29	·4921704	·565388	1·768694	·8704989	31
30	·4924236	·565772	1·767494	·8703557	30
31	·4926767	·566156	1·766295	·8702124	29
32	·4929298	·566541	1·765097	·8700691	28
33	·4931829	·566925	1·763900	·8699256	27
34	·4934359	·567309	1·762705	·8697821	26
35	·4936889	·567694	1·761511	·8696386	25
36	·4939419	·568079	1·760318	·8694949	24
37	·4941948	·568463	1·759126	·8693512	23
38	·4944476	·568848	1·757936	·8692074	22
39	·4947005	·569233	1·756747	·8690636	21
40	·4949532	·569619	1·755559	·8689196	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 60.

29 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·4952060	·570004	1·754372	·8687756	19
42	·4954587	·570389	1·753186	·8686315	18
43	·4957113	·570775	1·752002	·8684874	17
44	·4959639	·571161	1·750819	·8683431	16
45	·4962165	·571547	1·749637	·8681988	15
46	·4964690	·571933	1·748456	·8680544	14
47	·4967215	·572319	1·747276	·8679100	13
48	·4969740	·572705	1·746098	·8677655	12
49	·4972264	·573091	1·744921	·8676209	11
50	·4974787	·573478	1·743745	·8674762	10
51	·4977310	·573864	1·742570	·8673314	9
52	·4979833	.574251	1·741396	·8671866	8
53	·4982355	·574638	1·740224	·8670417	7
54	·4984877	·575025	1·739053	·8668967	6
55	·4987399	·575412	1·737883	·8667517	5
56	·4989920	·575799	1·736714	·8666066	4
57	·4992441	·576187	1·735546	·8664614	3
58	·4994961	·576574	1·734380	·8663161	2
59	·4997481	·576962	1·733214	·8661708	1
60	·5000000	·577350	1·732050	·8660254	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 60.

30 Deg. 30 Deg. 30 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5000000	·577350	1·732050	·8660254	60	21	·5052809	·585524	1·707871	·8629549	39	41	·5102928	·593363	1·685308	·8600007	19
1	·5002519	·577738	1·730887	·8658799	59	22	·5055319	·585914	1·706732	·8628079	38	42	·5105429	·593756	1·684191	·8598523	18
2	·5005037	·578126	1·729726	·8657344	58	23	·5057828	·586305	1·705595	·8626608	37	43	·5107930	·594150	1·683076	·8597037	17
3	·5007556	·578514	1·728565	·8655887	57	24	·5060338	·586696	1·704458	·8625137	36	44	·5110431	·594543	1·681962	·8595551	16
4	·5010073	·578902	1·727406	·8654430	56	25	·5062846	·587087	1·703323	·8623664	35	45	·5112931	·594937	1·680848	·8594064	15
5	·5012591	·579291	1·726247	·8652973	55	26	·5065355	·587478	1·702189	·8622191	34	46	·5115431	·595331	1·679736	·8592576	14
6	·5015107	·579679	1·725090	·8651514	54	27	·5067863	·587870	1·701055	·8620717	33	47	·5117930	·595725	1·678625	·8591088	13
7	·5017624	·580068	1·723934	·8650055	53	28	·5070370	·588261	1·699923	·8619243	32	48	·5120429	·596119	1.677515	·8589599	12
8	·5020140	·580457	1·722779	·8648595	52	29	·5072877	·588653	1·698792	·8617768	31	49	·5122927	·596514	1·676406	·8588109	11
9	·5022655	·580846	1·721626	·8647134	51	30	·5075384	·589045	1·697663	.8616292	30	50	·5125425	·596908	1·675298	·8586619	10
10	·5025170	·581235	1·720473	·8645673	50	31	·5077890	·589436	1·696534	·8614815	29	51	·5127923	·597303	1·674192	·8585127	9
11	·5027685	·581624	1·719322	·8644211	49	32	·5080396	·589828	1·695406	·8613337	28	52	·5130420	·597697	1·673086	·8583635	8
12	·5030199	·582013	1·718172	·8642748	48	33	·5082901	·590221	1·694280	·8611859	27	53	·5132916	·598092	1·671981	·8582143	7
13	·5032713	·582403	1·717023	·8641284	47	34	·5085406	·590613	1·693155	·8610380	26	54	·5135413	·598487	1·670878	·8580649	6
14	·5035227	·582793	1·715875	·8639820	46	35	·5087910	·591005	1·692030	·8608901	25	55	·5137908	·598882	1·669775	·8579155	5
15	·5037740	·583182	1·714728	·8638355	45	36	·5090414	·591398	1·690907	·8607420	24	56	·5140404	·599278	1·668674	·8577660	4
16	·5040252	·583572	1·713582	·8636889	44	37	·5092918	·591791	1·689785	·8605939	23	57	·5142899	·599673	1·667574	·8576164	3
17	·5042765	·583962	1·712438	·8635423	43	38	·5095421	·592183	1·688664	·8604457	22	58	·5145393	·600069	1·666474	·8574668	2
18	·5045276	·584352	1·711294	·8633956	42	39	·5097924	·592576	1·687544	·8602975	21	59	·5147887	·600464	1·665376	·8573171	1
19	·5047788	·584743	1·710152	·8632488	41	40	·5100426	·592969	1·686426	·8601491	20	60	·5150381	·600860	1·664279	·8571673	0
20	·5050298	·585133	1·709011	·8631019	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 59. Deg. 59 Deg. 59.

31 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′
0	·5150381	·600860	1·664279	·8571673	60	21
1	·5152874	·601256	1·663183	·8570174	59	22
2	·5155367	·601652	1·662088	·8568675	58	23
3	·5157859	·602049	1·660994	·8567175	57	24
4	·5160351	·602445	1·659901	·8565674	56	25
5	·5162842	·602841	1·658809	·8564173	55	26
6	·5165333	·603238	1·657718	·8562671	54	27
7	·5167824	·603635	1·656629	·8561168	53	28
8	·5170314	·604032	1·655540	·8559664	52	29
9	·5172804	·604429	1·654452	·8558160	51	30
10	·5175293	·604826	1·653366	·8556655	50	31
11	·5177782	·605224	1·652280	·8555149	49	32
12	·5180270	·605621	1·651196	·8553643	48	33
13	·5182758	·606019	1·650112	·8552135	47	34
14	·5185246	·606417	1·649030	·8550627	46	35
15	·5187733	·606814	1·647949	·8549119	45	36
16	·5190219	·607213	1·646868	·8547609	44	37
17	·5192705	·607611	1·645789	·8546099	43	38
18	·5195191	·608009	1·644711	·8544588	42	39
19	·5197676	·608408	1·643633	·8543077	41	40
20	·5200161	·608806	1·642557	·8541564	40	
′	Cosine.	Cotang.	Tang.	Sine.	′	′

Deg. 58·

31 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′
0	·5202646	·609205	1·641482	·8540051	39	41
1	·5205130	·609604	1·640408	·8538538	38	42
2	·5207613	·610003	1·639335	·8537023	37	43
3	·5210096	·610402	1·638263	·8535508	36	44
4	·5212579	·610801	1·637191	·8533992	35	45
5	·5215061	·611201	1·636121	·8532475	34	46
6	·5217543	·611601	1·635052	·8530958	33	47
7	·5220024	·612000	1·633984	·8529440	32	48
8	·5222505	·612400	1·632917	·8527921	31	49
9	·5224986	·612800	1·631851	·8526402	30	50
10	·5227466	·613201	1·630786	·8524881	29	51
11	·5229945	·613601	1·629722	·8523360	28	52
12	·5232424	·614001	1·628659	·8521839	27	53
13	·5234903	·614402	1·627597	·8520316	26	54
14	·5237381	·614803	1·626536	·8518793	25	55
15	·5239859	·615204	1·625476	·8517269	24	56
16	·5242336	·615605	1·624417	·8515745	23	57
17	·5244813	·616006	1 623359	·8514219	22	58
18	·5247290	·616407	1·622302	·8512693	21	59
19	·5249766	·616809	1·621246	·8511167	20	60
′	Cosine	Cotang.	Tang.	Sine.	′	′

Deg. 58.

31 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5252241	·617210	1·620192	·8509639	19
1	·5254717	·617612	1·619138	·8508111	18
2	·5257191	·618014	1·618085	·8506582	17
3	·5259665	·618416	1·617033	·8505053	16
4	·5262139	·618818	1·615982	·8503522	15
5	·5264613	·619221	1·614932	·8501991	14
6	·5267085	·619623	1·613882	·8500459	13
7	·5269558	·620026	1·612834	·8498927	12
8	·5272030	·620429	1·611787	·8497394	11
9	·5274502	·620832	1·610741	·8495860	10
10	·5276973	·621235	1·609696	·8494325	9
11	·5279443	·621638	1·608652	·8492790	8
12	·5281914	·622041	1·607609	·8491254	7
13	·5284383	·622445	1·606567	·8489717	6
14	·5286853	·622848	1·605526	·8488179	5
15	·5289322	·623252	1·604485	·8486641	4
16	·5291790	·623656	1·603446	·8485102	3
17	·5294258	·624060	1·602408	·8483562	2
18	·5296726	·624465	1·601370	·8482022	1
19	·5299193	·624869	1·600334	·8480481	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 58.

32 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5299193	·624869	1·600334	·8180481	60
1	·5301659	·625273	1·599299	·8178939	59
2	·5304125	·625678	1·598264	·8477397	58
3	·5306591	·626083	1·597231	·8475853	57
4	·5309057	·626488	1·596198	·8474309	56
5	·5311521	·626893	1·595167	·8472765	55
6	·5313986	·627298	1·594136	·8471219	54
7	·5316450	·627704	1·593107	·8469673	53
8	·5318913	·628109	1·592078	·8468126	52
9	·5321376	·628515	1·591050	·8466579	51
10	·5323839	·628921	1·590023	·8465030	50
11	·5326301	·629327	1·588997	8463481	49
12	·5328763	·629733	1·587973	·8461932	48
13	·5331224	·630139	1·586949	·8460381	47
14	·5333685	·630546	1·585926	·8458830	46
15	·5336145	·630953	1·584904	·8457278	45
16	·5338605	·631359	1·583883	8455726	44
17	·5341065	·631766	1·582862	8454172	43
18	·5343523	·632173	1·581843	·8452618	42
19	·5345982	·632581	1·580825	·8451064	41
20	·5348440	632988	1·579807	8449508	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 57.

32 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·5350898	·633395	1·578791	·8447952	39
22	·5353355	·633803	1·577776	·8446395	38
23	·5355812	·634211	1·576761	·8444838	37
24	·5358268	·634619	1·575747	·8443279	36
25	·5360724	·635027	1·574735	·8441720	35
26	·5363179	·635435	1·573723	·8440161	34
27	·5365634	·635844	1·572712	·8438600	33
28	·5368089	·636252	1·571702	·8437039	32
29	·5370543	·636661	1·570693	·8435477	31
30	·5372996	·637070	1·569685	·8433914	30
31	·5375449	·637479	1·568678	·8432351	29
32	·5377902	·637888	1·567672	·8430787	28
33	·5380354	·638297	1·566666	·8429222	27
34	·5382806	·638707	1·565662	·8427657	26
35	·5385257	·639116	1·564659	·8426091	25
36	·5387708	·639526	1·563656	·8424524	24
37	·5390158	·639936	1·562654	·8422956	23
38	·5392608	·640346	1·561654	·8421388	22
39	·5395058	·640756	1·560654	·8419819	21
40	·5397507	·641167	1·559655	·8418249	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 57.

32 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·5399955	·641577	1·558657	·8416679	19
42	·5402403	·641988	1·557660	·8415108	18
43	·5404851	·642399	1·556663	·8413536	17
44	·5407298	·642810	1·555668	·8411963	16
45	·5409745	·643221	1·554674	·8410390	15
46	·5412191	·643632	1·553680	·8408816	14
47	·5414637	·644044	1·552688	·8407241	13
48	5417082	·644456	1·551696	·8405666	12
49	.5419527	·644867	1·550705	·8404090	11
50	·5421971	·645279	1·549715	·8402513	10
51	·5424415	·645691	1·548726	·8400936	9
52	·5426859	·646104	1·547738	·8399357	8
53	·5429302	·646516	1·546751	·8397778	7
54	·5431744	·646929	1·545764	·8396199	6
55	·5434187	·647341	1·544779	·8394618	5
56	·5436628	·647754	1·543794	·8393037	4
57	·5439069	·648167	1·542810	·8391455	3
58	·5441510	·648580	1·541828	·8389873	2
59	·5443951	·648994	1·540846	·8388290	1
60	·5446390	·649407	1·539865	·8386706	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 57.

33 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5446390	·649407	1·539865	·8386706	60
1	·5448830	·649821	1·538884	·8385121	59
2	·5451269	·650235	1·537905	·8383536	58
3	·5453707	·650649	1·536927	·8381950	57
4	·5456145	·651063	1·535949	·8380363	56
5	·5458583	·651477	1·534972	·8378775	55
6	·5461020	·651891	1·533996	·8377187	54
7	·5463456	·652306	1·533021	·8375598	53
8	·5465892	·652721	1·532047	·8374009	52
9	·5468328	·653136	1·531074	·8372418	51
10	·5470763	·653551	1·530102	·8370827	50
11	·5473198	·653966	1·529130	·8369236	49
12	·5475632	·654381	1·528160	·8367643	48
13	·5478066	·654797	1·527190	·8366050	47
14	·5480499	·655212	1·526221	·8364456	46
15	·5482932	·655628	1·525253	·8362862	45
16	·5485365	·656044	1·524286	·8361266	44
17	·5487797	·656460	1·523320	·8359670	43
18	·5490228	·656877	1·522354	·8358074	42
19	·5492659	·657293	1·521389	·8356476	41
20	·5495090	·657710	1·520426	·8354878	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 56.

33 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·5497520	·658127	1·519463	·8353279	39
22	·5499950	·658544	1·518501	·8351680	38
23	·5502379	·658961	1·517540	·8350080	37
24	·5504807	·659378	1·516579	·8348479	36
25	·5507236	·659796	1·515620	·8346877	35
26	·5509663	·660213	1·514661	·8345275	34
27	·5512091	·660631	1·513703	·8343672	33
28	·5514518	·661049	1·512746	·8342068	32
29	·5516944	·661467	1·511790	·8340463	31
30	·5519370	·661885	1·510835	·8338858	30
31	·5521795	·662304	1·509880	·8337252	29
32	·5524220	·662722	1·508927	·8335646	28
33	·5526645	·663141	1·507974	·8334038	27
34	·5529069	·663560	1·507022	·8332430	26
35	·5531492	·663979	1·506071	8330822	25
36	·5533915	·664398	1·505121	·8329212	24
37	·5536338	·664817	1·504171	·8327602	23
38	·5538760	·665237	1·503222	·8325991	22
39	·5541182	·665657	1·502275	·8324380	21
40	·5543603	·666076	1·501328	·8322768	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 56.

33 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·5546024	·666496	1·500382	·8321155	19
42	·5548444	·666917	1·499436	·8319541	18
43	·5550864	·667337	1·498492	·8317927	17
44	·5553283	·667758	1·497548	·8316312	16
45	·5555702	·668178	1·496605	·8314696	15
46	·5558121	·668599	1·495663	·8313080	14
47	5560539	·669020	1·494722	·8311463	13
48	·5562956	·669441	1·493782	·8309845	12
49	·5565373	·669863	1·492842	·8308226	11
50	·5567790	·670284	1·491903	·8306607	10
51	·5570206	·670706	1·490965	·8304987	9
52	·5572621	·671128	1·490028	·8303366	8
53	·5575036	·671550	1·489092	·8301745	7
54	·5577451	·671972	1·488157	·8300123	6
55	·5579865	·672394	1·487222	·8298500	5
56	·5582279	·672816	1·486288	·8296877	4
57	·5584692	·673239	1·485355	·8295252	3
58	·5587105	·673662	1·484423	·8293628	2
59	·5589517	·674085	1·483491	·8292002	1
60	·5591929	·674508	1·482561	·8290376	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 56.

34 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5591929	·674508	1·482561	·8290376	60
1	·5594340	·674931	1·481631	·8288749	59
2	·5596751	·675355	1·480702	·8287121	58
3	·5599162	·675779	1·479773	·8285493	57
4	·5601572	·676202	1·478846	·8283864	56
5	·5603981	·676626	1·477919	·8282234	55
6	·5606390	·677050	1·476993	·8280603	54
7	·5608798	·677475	1·476068	·8278972	53
8	·5611206	·677899	1·475144	·8277340	52
9	·5613614	·678324	1·474221	·8275708	51
10	·5616021	·678749	1·473298	·8274074	50
11	·5618428	·679174	1·472376	·8272440	49
12	·5620834	·679599	1·471455	·8270806	48
13	·5623239	·680024	1·470535	·8269170	47
14	·5625645	·680450	1·469615	·8267534	46
15	·5628049	·680875	1·468696	·8265897	45
16	·5630453	·681301	1·467778	·8264260	44
17	·5632857	·681727	1·466861	·8262622	43
18	·5635260	·682153	1·465945	·8260983	42
19	·5637663	·682580	1·465029	·8259343	41
20	·5640066	·683006	1·464114	·8257703	40
′	Cosine.	Cotang.	Tang.	Sine.	′

34 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·5642467	·683433	1·463200	·8256062	39
22	·5644869	·683860	1·462287	·8254420	38
23	·5647270	·684287	1·461374	·8252778	37
24	·5649670	·684714	1·460463	·8251135	36
25	·5652070	·685141	1·459552	·8249491	35
26	·5654469	·685569	1·458642	·8247847	34
27	·5656868	·685996	1·457732	·8246202	33
28	·5659267	·686424	1·456824	·8244556	32
29	·5661665	·686852	1·455916	·8242909	31
30	·5664062	·687281	1·455009	·8241262	30
31	·5666459	·687709	1·454102	·8239614	29
32	·5668856	·688137	1·453197	·8237965	28
33	·5671252	·688566	1·452292	·8236316	27
34	·5673648	·688995	1·451388	·8234666	26
35	·5676043	·689424	1·450485	·8233015	25
36	·5678437	·689853	1·449582	·8231364	24
37	·5680832	·690283	1·448680	·8229712	23
38	·5683225	·690712	1·447779	·8228059	22
39	·5685619	·691142	1·446879	·8226405	21
40	·5688011	·691572	1·445980	·8224751	20
′	Cosine.	Cotang.	Tang.	Sine.	′

34 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·5690403	·692002	1·445081	·8223096	19
42	·5692795	·692432	1·444183	·8221440	18
43	·5695187	·692863	1·443286	·8219784	17
44	·5697577	·693293	1·442389	·8218127	16
45	·5699968	·693724	1·441494	·8216469	15
46	·5702357	·694155	1·440599	·8214811	14
47	·5704747	·694586	1·439704	·8213152	13
48	·5707136	·695018	1·438811	·8211492	12
49	·5709524	·695449	1·437918	·8209832	11
50	·5711912	·695881	1·437026	·8208170	10
51	·5714299	·696313	1·436135	·8206509	9
52	·5716686	·696745	1·435245	·8204846	8
53	·5719073	·697177	1·434355	·8203183	7
54	·5721459	·697609	1·433466	·8201519	6
55	·5723844	.698042	1·432578	·8199854	5
56	·5726229	·698474	1·431690	·8198189	4
57	·5728614	·698907	1.430803	·8196523	3
58	·5730998	·699340	1.429917	·8194856	2
59	·5733381	·699774	1.429032	·8193189	1
60	·5735764	·700207	1.428148	·8191520	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 55.

Deg. 55.

Deg. 55.

35 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5735764	·700207	1·428148	·8191520	60
1	·5738147	·700641	1·427264	·8189852	59
2	·5740529	·701074	1·426381	·8188182	58
3	·5742911	·701508	1·425498	·8186512	57
4	·5745292	·701943	1·424617	·8184841	56
5	·5747672	·702377	1·423736	·8183169	55
6	·5750053	·702811	1·422856	·8181497	54
7	·5752432	·703246	1·421976	·8179824	53
8	·5754811	·703681	1·421097	·8178151	52
9	·5757190	·704116	1·420220	·8176476	51
10	·5759568	·704551	1·419342	·8174801	50
11	·5761946	·704986	1·418466	·8173125	49
12	·5764323	·705422	1·417590	·8171449	48
13	·5766700	·705858	1·416715	·8169772	47
14	·5769076	·706294	1·415840	·8168094	46
15	·5771452	·706730	1·414967	·8166416	45
16	·5773827	·707166	1·414094	·8164736	44
17	·5776202	·707602	1·413222	·8163056	43
18	·5778576	·708039	1·412350	·8161376	42
19	·5780950	·708476	1·411479	8159695	41
20	·5783323	·708913	1·410609	8158013	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 54.

35 Deg.

′	Sine.	Tang.	Cotang.	Cosine	′
21	·5785696	·709350	1·409740	·8156330	39
22	·5788069	·709787	1·408871	·8154647	38
23	·5790440	·710225	1·408003	·8152963	37
24	·5792812	·710663	1·407136	·8151278	36
25	.5795183	·711100	1·406270	·8149593	35
26	·5797553	·711539	1·405404	·8147906	34
27	·5799923	·711977	1·404539	·8146220	33
28	·5802292	·712415	1·403674	·8144532	32
29	·5804661	·712854	1·402811	·8142844	31
30	·5807030	·713293	1·401948	·8141155	30
31	·5809397	·713732	1·401086	·8139466	29
32	·5811765	·714171	1·400224	·8137775	28
33	·5814132	·714610	1·399363	·8136084	27
34	·5816498	·715050	1·398503	·8134393	26
35	·5818864	·715489	1·397644	·8132701	25
36	·5821230	·715929	1·396785	·8131008	24
37	·5823595	·716369	1·395927	·8129314	23
38	·5825959	·716810	1·395069	·8127620	22
39	·5828323	·717250	1·394213	·8125925	21
40	·5830687	·717691	1 393357	·8124229	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 54.

35 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·5833050	·718131	1·392501	·8122532	19
42	·5835412	·718572	1·391647	·8120835	18
43	·5837774	·719014	1·390793	·8119137	17
44	·5840136	·719455	1·389940	·8117439	16
45	·5842497	·719897	1·389087	·8115740	15
46	·5844857	·720338	1·388235	·8114040	14
47	·5847217	·720780	1·387384	·8112339	13
48	·5849577	·721222	1·386534	·8110638	12
49	·5851936	·721665	1·385684	·8108936	11
50	·5854294	·722107	1·384835	·8107234	10
51	·5856652	·722550	1·383986	·8105530	9
52	·5859010	·722993	1·383139	·8103826	8
53	·5861367	·723436	1·382292	·8102122	7
54	·5863724	·723879	1·381445	·8100416	6
55	·5866080	·724322	1·380600	·8098710	5
56	·5868435	·724766	1·379755	·8097004	4
57	·5870790	·725210	1·378910	·8095296	3
58	·5873145	·725654	1·378067	·8093588	2
59	·5875499	·726098	1·377224	·8091879	1
60	·5877853	·726542	1·376381	·8090170	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 54.

36 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5877853	·726542	1·376381	·8090170	60
1	·5880206	·726987	1·375540	·8088460	59
2	·5882558	·727431	1·374699	·8086749	58
3	·5884910	·727876	1·373859	·8085037	57
4	·5887262	·728321	1·373019	·8083325	56
5	5889613	·728767	1·372180	·8081612	55
6	·5891964	·729212	1·371342	·8079899	54
7	·5894314	·729658	1·370504	·8078185	53
8	·5896663	·730104	1·369667	·8076470	52
9	·5899012	·730550	1·368831	·8074754	51
10	·5901361	·730996	1·367995	·8073038	50
11	·5903709	·731442	1·367161	·8071321	49
12	·5906057	·731889	1·366326	·8069603	48
13	·5908404	·732336	1·365493	·8067885	47
14	·5910750	·732783	1·364660	·8066166	46
15	·5913096	·733230	1·363827	·8064446	45
16	·5915442	·733677	1·362996	·8062726	44
17	·5917787	·734125	1·362165	·8061005	43
18	·5920132	·734573	1·361335	·8059283	42
19	·5922476	·735021	1·360505	·8057560	41
20	·5924819	·735469	1·359676	·8055837	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 53.

36 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·5927163	·735917	1·358848	·8054113	39
22	·5929505	·736366	1·358020	·8052389	38
23	·5931847	·736814	1·357193	·8050664	37
24	·5934189	·737263	1·356367	·8048938	36
25	·5936530	·737712	1·355541	·8047211	35
26	·5938871	·738162	1·354716	·8045484	34
27	·5941211	·738611	1·353891	·8043756	33
28	·5943550	·739061	1·353068	·8042028	32
29	·5945889	·739511	1·352244	·8040299	31
30	·5948228	·739961	1·351422	·8038569	30
31	·5950566	·740411	1·350600	·8036838	29
32	·5952904	·740861	1·349779	·8035107	28
33	·5955241	·741312	1·348958	·8033375	27
34	·5957577	·741763	1·348139	·8031642	26
35	·5959913	·742214	1·347319	·8029909	25
36	·5962249	·742665	1·346501	·8028175	24
37	·5964584	·743117	1.345683	·8026440	23
38	·5966918	·743568	1.344865	·8024705	22
39	·5969252	·744020	1.344049	·8022969	21
40	·5971586	·744472	1.343233	·8021232	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 53.

36 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·5973919	·744924	1·342417	·8019495	19
42	·5976251	·745377	1·341602	·8017756	18
43	·5978583	·745829	1·340788	·8016018	17
44	·5980915	·746282	1·339975	·8014278	16
45	·5983246	·746735	1·339162	·8012538	15
46	·5985577	·747188	1·338350	·8010797	14
47	·5987906	·747642	1·337538	·8009056	13
48	·5990236	·748095	1·336727	·8007314	12
49	·5992565	·748549	1·335917	·8005571	11
50	·5994893	·749003	1·335107	·8003827	10
51	·5997221	·749457	1·334298	·8002083	9
52	·5999549	·749911	1·333490	·8000338	8
53	·6001876	·750366	1·332682	·7998593	7
54	·6004202	·750821	1·331875	·7996847	6
55	·6006528	·751276	1·331068	·7995100	5
56	·6008854	·751731	1·330262	·7993352	4
57	·6011179	·752186	1·329457	·7991604	3
58	·6013503	·752642	1·328652	·7989855	2
59	·6015827	·753098	1·327848	·7988105	1
60	·6018150	·753554	1·327044	·7986355	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 53.

37 Deg.					37 Deg.					37 Deg.				
′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6018150	·753554	1·327044	·7986355	60	21	·6066824	·763175	1·310314	·7949444	39	41	·6112969	·772423	1·294627	·7914014	19
1	·6020473	·754010	1·326242	·7984604	59	22	·6069136	·763636	1·309523	·7947678	38	42	·6115270	·772887	1·293848	·7912235	18
2	·6022795	·754466	1·325439	·7982853	58	23	·6071447	·764096	1·308734	·7945913	37	43	·6117572	·773352	1·293071	·7910456	17
3	.6025117	·754923	1·324638	·7981100	57	24	·6073758	·764557	1·307945	·7944146	36	44	·6119873	·773817	1·292294	·7908676	16
4	·6027439	·755379	1·323837	·7979347	56	25	·6076069	·765018	1·307157	·7942379	35	45	·6122173	·774282	1·291517	·7906896	15
5	·6029760	·755836	1·323036	·7977594	55	26	·6078379	·765480	1·306369	·7940611	34	46	·6124473	·774748	1·290742	·7905115	14
6	·6032080	·756294	1·322237	·7975839	54	27	·6080689	·765941	1·305582	·7938843	33	47	·6126772	·775213	1·289966	·7903333	13
7	·6034400	·756751	1·321437	·7974084	53	28	·6082998	·766403	1·304796	·7937074	32	48	·6129071	·775679	1·289192	·7901550	12
8	·6036719	·757209	1·320639	·7972329	52	29	·6085306	·766864	1·304010	·7935304	31	49	·6131369	·776145	1·288418	·7899767	11
9	·6039038	·757666	1·319841	·7970572	51	30	·6087614	·767327	1·303225	·7933533	30	50	·6133666	·776611	1·287644	·7897983	10
10	·6041356	758124	1·319044	·7968815	50	31	·6089922	·767789	1·302440	·7931762	29	51	·6135964	·777078	1·286871	·7896198	9
11	·6043674	·758582	1·318247	·7967058	49	32	·6092229	·768251	1·301656	·7929990	28	52	·6138260	·777544	1·286099	·7894413	8
12	·6045991	·759041	1·317451	·7965299	48	33	·6094535	·768714	1·300873	·7928218	27	53	·6140556	·778011	1·285327	·7892627	7
13	·6048308	·759499	1·316655	·7963540	47	34	·6096841	·769177	1·300090	·7926445	26	54	·6142852	·778478	1·284556	·7890841	6
14	·6050624	·759958	1·315861	·7961780	46	35	·6099147	·769640	1·299308	·7924671	25	55	·6145147	·778946	1·283786	·7889054	5
15	·6052940	·760417	1.315066	·7960020	45	36	·6101452	·770103	1·298526	·7922896	24	56	·6147442	·779413	1·283016	·7887266	4
16	·6055255	·760876	1·314273	·7958259	44	37	·6103756	·770567	1·297745	·7921121	23	57	·6149736	·779881	1·282246	·7885477	3
17	·6057570	·761336	1.313480	·7956497	43	38	·6106060	·771030	1·296964	·7919345	22	58	·6152029	·780349	1·281477	·7883688	2
18	·6059884	·761795	1.312687	·7954735	42	39	·6108363	·771494	1·296185	·7917569	21	59	·6154322	·780817	1·280709	·7881898	1
19	·6062198	·762255	1.311895	.7952972	41	40	·6110666	·771958	1·295405	·7915792	20	60	·6156615	·781285	1·279941	·7880108	0
20	·6064511	·762715	1.311104	·7951208	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
	Deg. 52.						Deg. 52.						Deg. 52.				

38 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6156615	·781285	1·279941	·7880108	60
1	·6158907	·781754	1·279174	·7878316	59
2	·6161198	·782222	1·278407	·7876524	58
3	·6163489	·782691	1·277641	·7874732	57
4	·6165780	·783161	1·276876	·7872939	56
5	·6168069	·783630	1·276111	·7871145	55
6	·6170359	·784100	1·275347	·7869350	54
7	·6172648	·784570	1·274583	·7867555	53
8	·6174936	·785040	1·273820	·7865759	52
9	·6177224	·785510	1·273057	·7863963	51
10	·6179511	·785980	1·272295	·7862165	50
11	·6181798	·786451	1·271534	·7860367	49
12	·6184084	·786922	1·270773	·7858569	48
13	·6186370	·787393	1·270013	·7856770	47
14	·6188655	·787864	1·269253	·7854970	46
15	·6190939	·788336	1·268494	·7853169	45
16	·6193224	788808	1·267735	·7851368	44
17	·6195507	·789280	1·266977	·7849566	43
18	·6197790	·789752	1·266219	·7847764	42
19	·6200073	·790224	1·265462	·7845961	41
20	·6202355	·790697	1·264706	·7844157	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 51

38 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·6204636	·791170	1·263950	·7842352	39
22	·6206917	·791643	1·263195	·7840547	38
23	·6209198	·792116	1·262440	·7838741	37
24	·6211478	·792590	1·261686	·7836935	36
25	·6213757	·793064	1·260932	·7835127	35
26	·6216036	·793537	1·260179	·7833320	34
27	·6218314	·794012	1·259426	·7831511	33
28	·6220592	·794486	1·258674	·7829702	32
29	·6222870	·794961	1·257923	·7827892	31
30	·6225146	·795435	1·257172	·7826082	30
31	·6227423	·795911	1·256421	·7824270	29
32	·6229698	·796386	1·255672	·7822459	28
33	·6231974	·796861	1·254922	·7820646	27
34	·6234248	·797337	1·254174	·7818833	26
35	·6236522	·797813	1·253426	·7817019	25
36	·6238796	·798289	1·252678	·7815205	24
37	·6241069	·798765	1·251931	·7813390	23
38	·6243342	·799242	1·251184	·7811574	22
39	·6245614	·799719	1·250438	·7809757	21
40	·6247885	·800196	1·249693	·7807940	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 51.

38 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·6250156	·800673	1·248948	·7806123	19
42	·6252427	·801151	1·248204	·7804304	18
43	·6254696	·801628	1·247460	·7802485	17
44	·6256966	·802106	1·246716	·7800665	16
45	·6259235	·802584	1·245974	·7798845	15
46	·6261503	·803063	1·245232	·7797024	14
47	·6263771	·803541	1·244490	·7795202	13
48	·6266038	·804020	1·243749	·7793380	12
49	·6268305	·804499	1·243008	·7791557	11
50	·6270571	·804979	1·242268	·7789733	10
51	·6272837	805458	1·241529	·7787909	9
52	·6275102	·805938	1·240790	·7786084	8
53	·6277366	·806418	1·240051	·7784258	7
54	·6279631	·806898	1·239313	·7782431	6
55	·6281894	·807378	1·238576	·7780604	5
56	·6284157	·807859	1·237839	·7778777	4
57	·6286420	·808340	1·237103	·7776949	3
58	·6288682	·808821	1·236367	·7775120	2
59	·6290943	·809302	1·235631	·7773290	1
60	·6293204	·809784	1·234897	·7771460	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 5

39 Deg.					39 Deg.					39 Deg.					
′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6293204	·809784	1·234897	·7771460	60	21	·6340559	·819948	1·219588	·7732872	39	41	·6385440	·829724	1·205219	·7695853	19
1	·6295464	·810265	1·234162	·7769629	59	22	·6342808	·820435	1·218865	·7731027	38	42	·6387678	·830216	1·204505	·7693996	18
2	·6297724	·810747	1·233429	·7767797	58	23	·6345057	·820922	1·218142	·7729182	37	43	·6389916	·830707	1·203793	·7692137	17
3	·6299983	·811230	1·232696	·7765965	57	24	·6347305	·821409	1·217419	·7727336	36	44	·6392153	·831199	1·203081	·7690278	16
4	·6302242	·811712	1·231963	·7764132	56	25	·6349553	·821896	1·216698	·7725489	35	45	·6394390	·831691	1·202369	·7688418	15
5	·6304500	·812195	1·231231	·7762298	55	26	·6351800	·822384	1·215976	·7723642	34	46	·6396626	·832183	1·201658	·7686558	14
6	·6306758	·812678	1·230499	·7760464	54	27	·6354046	·822871	1·215256	·7721794	33	47	·6398862	·832675	1·200947	·7684697	13
7	·6309015	·813161	1·229768	·7758629	53	28	·6356292	·823359	1·214535	·7719945	32	48	·6401097	·833168	1·200237	·7682835	12
8	·6311272	·813644	1·229038	·7756794	52	29	·6358537	·823847	1·213816	·7718096	31	49	·6403332	·833661	1·199527	·7680973	11
9	·6313528	·814128	1·228308	·7754957	51	30	·6360782	·824336	1·213097	·7716246	30	50	·6405566	·834154	1·198818	·7679110	10
10	·6315784	·814611	1·227578	·7753121	50	31	·6363026	·824825	1·212378	·7714395	29	51	·6407799	·834648	1·198109	·7677246	9
11	·6318039	·815095	1·226849	·7751283	49	32	·6365270	·825314	1·211660	·7712544	28	52	·6410032	·835141	1·197401	·7675382	8
12	·6320293	·815580	1·226121	·7749445	48	33	·6367513	·825803	1·210942	·7710692	27	53	·6412264	·835635	1·196693	·7673517	7
13	·6322547	·816064	1·225393	·7747606	47	34	·6369756	·826292	1·210225	·7708840	26	54	·6414496	·836129	1·195986	·7671652	6
14	·6324800	·816549	1·224665	·7745767	46	35	·6371998	·826782	1·209508	·7706986	25	55	·6416728	·836624	1·195279	·7669785	5
15	·6327053	·817034	1·223938	·7743926	45	36	·6374240	·827271	1·208792	·7705132	24	56	·6418958	·837118	1·194573	·7667918	4
16	·6329306	·817519	1·223212	·7742086	44	37	·6376481	·827762	1·208076	·7703278	23	57	·6421189	·837613	1·193867	·7666051	3
17	·6331557	·818004	1·222486	·7740244	43	38	·6378721	·828252	1·207361	·7701423	22	58	·6423418	·838108	1·193162	·7664183	2
18	·6333809	·818490	1·221761	·7738402	42	39	·6380961	·828742	1·206646	·7699567	21	59	·6425647	·838604	1·192457	·7662314	1
19	·6336059	·818976	1·221036	·7736559	41	40	·6383201	·829233	1·205932	·7697710	20	60	·6427876	·839099	1·191753	·7660444	0
20	·6338310	·819462	1·220312	·7734716	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
	Deg 50.						Deg. 50.						Deg. 50.				

40 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6427876	·839099	1·191753	·7660444	60
1	·6430104	·839595	1·191049	·7658574	59
2	·6432332	·840091	1.190346	·7656704	58
3	·6434559	·840587	1·189643	·7654832	57
4	·6436785	·841084	1·188941	·7652960	56
5	·6439011	·841581	1·188239	·7651087	55
6	·6441236	·842078	1·187538	·7649214	54
7	·6443461	·842575	1·186837	·7647340	53
8	·6445685	·843073	1·186136	·7645465	52
9	·6447909	·843570	1·185437	·7643590	51
10	·6450132	·844068	1·184737	·7641714	50
11	·6452355	·844567	1·184038	·7639838	49
12	·6454577	·845065	1·183340	·7637960	48
13	·6456798	·845564	1·182642	·7636082	47
14	·6459019	·846063	1·181944	·7634204	46
15	·6461240	·846562	1·181247	·7632325	45
16	·6463460	·847062	1·180551	·7630445	44
17	·6465679	·847561	1·179855	·7628564	43
18	·6467898	·848061	1·179159	·7626683	42
19	·6470116	·848561	1·178464	·7624802	41
20	·6472334	·849062	1·177769	·7622919	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 49.

40 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·6474551	·849563	1·177075	·7621036	39
22	·6476767	·850064	1·176382	·7619152	38
23	·6478984	·850565	1·175688	·7617268	37
24	·6481199	·851066	1·174996	·7615383	36
25	·6483414	·851568	1·174303	·7613497	35
26	·6485628	·852070	1·173612	·7611611	34
27	·6487842	·852572	1·172920	·7609724	33
28	·6490056	·853075	1·172229	·7607837	32
29	·6492268	·853577	1·171539	·7605949	31
30	·6494480	·854080	1·170849	·7604060	30
31	·6496692	·854583	1·170160	·7602170	29
32	·6498903	·855087	1·169471	·7600280	28
33	·6501114	·855591	1·168782	·7598389	27
34	·6503324	·856095	1·168094	·7596498	26
35	·6505533	·856599	1·167407	·7594606	25
36	·6507742	·857103	1·166720	·7592713	24
37	·6509951	·857608	1·166033	·7590820	23
38	·6512158	·858113	1·165347	·7588926	22
39	·6514366	·858618	1·164661	·7587031	21
40	·6516572	·859124	1·163976	·7585136	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 49

40 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·6518778	·859629	1·163291	·7583240	19
42	·6520984	·860135	1·162607	·7581343	18
43	6523189	·860641	1·161923	·7579446	17
44	·6525394	·861148	1·161240	·7577548	16
45	·6527598	·861655	1·160557	·7575650	15
46	·6529801	·862162	1·159874	·7573751	14
47	·6532004	862669	1·159192	7571851	13
48	·6534206	·863176	1·158511	·7569951	12
49	·6536408	·863684	1·157830	·7568050	11
50	·6538609	·864192	1·157149	·7566148	10
51	·6540810	·864700	1·156469	·7564246	9
52	·6543010	·865209	1·155789	·7562343	8
53	·6545209	·865718	1·155110	·7560439	7
54	·6547408	·866227	1·154431	·7558535	6
55	·6549607	·866736	1·153753	·7556630	5
56	·6551804	·867246	1·153075	·7554724	4
57	·6554002	·867755	1·152397	·7552818	3
58	·6556198	·868265	1·151721	·7550911	2
59	·6558395	·868776	1·151044	·7549004	1
60	·6560590	·869286	1·150368	·7547096	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 49.

41 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6560590	·869286	1·150368	·7547096	60
1	·6562785	·869797	1·149692	·7545187	59
2	·6564980	·870308	1·149017	·7543278	58
3	6567174	·870820	1·148342	·7541368	57
4	·6569367	·871331	1·147668	·7539457	56
5	·6571560	·871843	1·146994	·7537546	55
6	·6573752	·872355	1·146321	·7535634	54
7	·6575944	·872868	1·145648	·7533721	53
8	·6578135	·873380	1·144976	7531808	52
9	·6580326	·873893	1·144304	·7529894	51
10	·6582516	·874406	1·143632	·7527980	50
11	·6584706	·874920	1·142961	·7526065	49
12	·6586895	·875433	1·142290	·7524149	48
13	·6589083	·875947	1·141620	·7522233	47
14	·6591271	·876462	1·140950	·7520316	46
15	·6593458	·876976	1·140281	·7518398	45
16	·6595645	·877491	1·139612	·7516480	44
17	·6597831	·878006	1·138944	·7514561	43
18	·6600017	·878521	1·138276	·7512641	42
19	·6602202	·879037	1·137608	·7510721	41
20	·6604386	·879552	1·136941	·7508800	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 48.

41 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·6606570	·880068	1·136274	·7506879	39
22	·6608754	·880585	1·135608	·7504957	38
23	·6610936	·881101	1·134942	·7503034	37
24	·6613119	·881618	1·134277	·7501111	36
25	·6615300	·882135	1·133612	·7499187	35
26	·6617482	·882653	1·132947	·7497262	34
27	·6619662	·883170	1·132283	·7495337	33
28	·6621842	·883688	1·131620	·7493411	32
29	·6624022	·884206	1·130957	·7491484	31
30	·6626200	·884725	1·130294	·7489557	30
31	·6628379	·885244	1·129632	·7487629	29
32	·6630557	·885763	1·128970	·7485701	28
33	·6632734	·886282	1·128308	·7483772	27
34	·6634910	·886801	1·127647	·7481842	26
35	·6637087	·887321	1·126987	·7479912	25
36	·6639262	·887841	1·126327	·7477981	24
37	·6641437	·888361	1·125667	·7476049	23
38	·6643612	·888882	1·125008	·7474117	22
39	·6645785	·889403	1·124349	·7472184	21
40	·6647959	·889924	1·123690	·7470251	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 48.

41 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·6650131	·890445	1·123032	·7468317	19
42	·6652304	·890967	1·122375	·7466382	18
43	·6654475	·891489	1·121718	·7464446	17
44	·6656646	·892011	1·121061	·7462510	16
45	·6658817	·892534	1·120405	·7460574	15
46	·6660987	·893056	1·119749	·7458636	14
47	·6663156	·893579	1·119094	·7456699	13
48	·6665325	894103	1·118439	·7454760	12
49	·6667493	.894626	1·117784	·7452821	11
50	·6669661	·895150	1·117130	·7450881	10
51	·6671828	·895674	1·116476	·7448941	9
52	·6673994	·896199	1·115823	·7446999	8
53	·6676160	·896723	1·115170	·7445058	7
54	·6678326	·897248	1·114518	·7443115	6
55	·6680490	·897773	1·113866	·7441173	5
56	·6682655	·898299	1·113214	·7439229	4
57	·6684818	·898825	1·112563	·7437285	3
58	·6686981	·899351	1·111912	·7435340	2
59	·6689144	·899877	1·111262	·7433394	1
60	·6691306	·900404	1·110612	·7431448	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 48.

42 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′
0	·6691306	·900404	1·110612	·7431448	60	21
1	·6693468	·900930	1·109963	·7429502	59	22
2	·6695628	·901458	1·109314	·7427554	58	23
3	·6697789	·901985	1·108665	·7425606	57	24
4	·6699948	·902513	1·108017	·7423658	56	25
5	·6702108	·903041	1·107369	·7421708	55	26
6	·6704266	·903569	1·106721	·7419758	54	27
7	·6706424	904097	1·106075	·7417808	53	28
8	·6708582	·904626	1·105428	·7415857	52	29
9	·6710739	·905155	1·104782	·7413905	51	30
10	·6712895	·905685	1·104136	·7411953	50	31
11	·6715051	·906214	1·103491	·7410000	49	32
12	·6717206	·906744	1·102846	·7408046	48	33
13	·6719361	·907274	1·102201	·7406092	47	34
14	·6721515	·907805	1·101557	·7404137	46	35
15	·6723668	·908336	1·100914	·7402181	45	36
16	·6725821	·908867	1·100270	·7400225	44	37
17	·6727973	·909398	1·099628	·7398268	43	38
18	·6730125	·909920	1·098985	·7396311	42	39
19	·6732276	·910461	1·098343	·7394353	41	40
20	·6734427	·910994	1·097702	·7392394	40	
	Cosine.	Cotang.	Tang.	Sine.	′	′

Deg. 47.

42 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′
0	·6736577	·911526	1·097060	·7390435	39	41
1	·6738727	·912059	1·096420	·7388475	38	42
2	·6740876	·912592	1·095779	·7386515	37	43
3	·6743024	·913125	1·095139	·7384553	36	44
4	·6745172	·913659	1·094500	·7382592	35	45
5	·6747319	·914192	1·093861	·7380629	34	46
6	·6749466	·914727	1·093222	·7378666	33	47
7	·6751612	·915261	1·092584	·7376703	32	48
8	·6753757	·915796	1·091946	·7374738	31	49
9	·6755902	·916331	1·091308	·7372773	30	50
10	·6758046	·916866	1·090671	·7370808	29	51
11	·6760190	·917402	1·090034	·7368842	28	52
12	·6762333	·917937	1·089398	·7366875	27	53
13	·6764476	·918474	1·088762	·7364908	26	54
14	·6766618	·919010	1·088126	·7362940	25	55
15	·6768760	·919547	1·087491	·7360971	24	56
16	·6770901	·920084	1·086857	·7359002	23	57
17	·6773041	·920621	1·086222	·7357032	22	58
18	·6775181	·921159	1·085588	·7355061	21	59
19	·6777320	·921696	1·084955	·7353090	20	60
	Cosine.	Cotang.	Tang.	Sine.	′	′

Deg. 47.

42 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·6779459	·922235	1.084322	·7351118	19
42	·6781597	·922773	1·083689	·7349146	18
43	·6783734	·923312	1·083057	·7347173	17
44	·6785871	·923851	1·082425	·7345199	16
45	·6788007	·924390	1·081793	·7343225	15
46	·6790143	·924930	1·081162	·7341250	14
47	·6792278	·925470	1·080532	·7339275	13
48	·6794413	·926010	1·079901	·7337299	12
49	·6796547	·926550	1·079271	·7335322	11
50	·6798681	·927091	1·078642	·7333345	10
51	·6800813	·927632	1·078013	·7331367	9
52	·6802946	·928173	1·077384	·7329388	8
53	·6805078	·928715	1·076756	·7327409	7
54	·6807209	·929257	1·076128	·7325429	6
55	·6809339	·929799	1·075500	·7323449	5
56	·6811469	·930342	1·074873	·7321467	4
57	·6813599	·930884	1·074246	·7319486	3
58	·6815728	·931428	1·073620	·7317503	2
59	·6817856	·931971	1·072994	·7315521	1
60	·6819984	·932515	1·072368	·7313537	0
	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 47.

43 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6819984	·932515	1·072368	·7313537	60
1	·6822111	·933059	1·071743	·7311553	59
2	·6824237	·933603	1·071118	·7309568	58
3	·6826363	·934147	1·070494	·7307583	57
4	·6828489	·934692	1·069870	·7305597	56
5	·6830613	·935238	1·069246	·7303610	55
6	·6832738	·935783	1·068623	·7301623	54
7	·6834861	·936329	1·068000	·7299635	53
8	·6836984	·936875	1·067377	·7297646	52
9	·6839107	·937421	1·066755	·7295657	51
10	·6841229	·937968	1·066134	·7293668	50
11	·6843350	·938515	1·065512	·7291677	49
12	·6845471	·939062	1·064891	·7289686	48
13	·6847591	·939610	1·064271	·7287695	47
14	·6849711	·940157	1·063651	·7285703	46
15	·6851830	·940706	1·063031	·7283710	45
16	·6853948	·941254	1·062411	·7281716	44
17	·6856066	·941803	1·061792	·7279722	43
18	·6858184	·942352	1·061174	·7277728	42
19	·6860300	·942901	1·060556	·7275732	41
20	·6862416	·943451	1·059938	·7273736	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 46.

43 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·6864532	·944001	1·059320	·7271740	39
22	·6866647	·944551	1·058703	·7269743	38
23	·6868761	·945102	1·058086	·7267745	37
24	·6870875	·945653	1·057470	·7265747	36
25	·6872988	·946204	1·056854	·7263748	35
26	·6875101	·946755	1·056238	·7261748	34
27	·6877213	·947307	1·055623	·7259748	33
28	·6879325	·947859	1·055008	·7257747	32
29	·6881435	·948411	1·054394	·7255746	31
30	·6883546	·948964	1·053780	·7253744	30
31	·6885655	·949517	1·053166	·7251741	29
32	·6887765	·950070	1·052553	·7249738	28
33	·6889873	·950624	1·051940	·7247734	27
34	·6891981	·951178	1·051327	·7245729	26
35	·6894089	·951732	1·050715	·7243724	25
36	·6896195	·952287	1·050103	·7241719	24
37	·6898302	·952842	1·049492	·7239712	23
38	·6900407	·953397	1·048880	·7237705	22
39	·6902512	·953952	1·048270	·7235698	21
40	·6904617	·954508	1·047659	·7233690	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 46.

43 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·6906721	·955064	1·047049	·7231681	19
42	·6908824	·955620	1·046440	·7229671	18
43	·6910927	·956177	1·045831	·7227661	17
44	·6913029	·956734	1·045222	·7225651	16
45	·6915131	·957291	1·044613	·7223640	15
46	·6917232	·957849	1·044005	·7221628	14
47	·6919332	·958407	1·043397	·7219615	13
48	·6921432	·958965	1·042790	·7217602	12
49	·6923531	·959524	1·042183	·7215589	11
50	·6925630	·960082	1·041576	·7213574	10
51	·6927728	·960642	1·040970	·7211559	9
52	·6929825	·961201	1·040364	·7209544	8
53	·6931922	·961761	1·039758	·7207528	7
54	·6934018	·962321	1·039153	·7205511	6
55	·6936114	·962881	1·038548	·7203494	5
56	·6938209	·963442	1·037944	·7201476	4
57	·6940304	·964003	1·037340	·7199457	3
58	·6942398	·964565	1·036736	·7197438	2
59	·6944491	·965126	1·036133	·7195418	1
60	·6946584	·965688	1·035530	·7193398	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 46.

44 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6946584	·965688	1·035530	·7193398	60
1	·6948676	·966251	1·034927	·7191377	59
2	·6950767	·966813	1·034325	·7189355	58
3	6952858	·967376	1·033723	·7187333	57
4	·6954949	·967939	1·033122	·7185310	56
5	·6957039	·968503	1·032520	·7183287	55
6	·6959128	·969067	1·031919	·7181263	54
7	·6961217	·969631	1·031319	·7179238	53
8	·6963305	·970196	1·030719	·7177213	52
9	·6965392	·970761	1·030119	·7175187	51
10	·6967479	·971326	1·029520	·7173161	50
11	·6969565	·971891	1·028921	·7171134	49
12	·6971651	·972457	1·028322	·7169106	48
13	·6973736	·973023	1·027724	·7167078	47
14	·6975821	·973590	1·027126	·7165049	46
15	·6977905	·974156	1·026528	·7163019	45
16	·6979988	·974724	1·025931	·7160989	44
17	·6982071	·975291	1·025334	·7158959	43
18	·6984153	·975859	1·024738	·7156927	42
19	·6986234	·976427	1·024141	·7154895	41
20	·6988315	·976995	1·023546	·7152863	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 45.

44 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·6990396	·977564	1·022950	·7150830	39
22	·6992476	·978133	1·022355	·7148796	38
23	·6994555	·978702	1·021760	·7146762	37
24	·6996633	·979272	1·021166	·7144727	36
25	·6998711	·979842	1·020572	·7142691	35
26	·7000789	·980412	1·019978	·7140655	34
27	·7002866	·980983	1·019385	·7138618	33
28	·7004942	·981554	1·018792	·7136581	32
29	·7007018	·982125	1·018199	·7134543	31
30	·7009093	·982697	1·017607	·7132504	30
31	·7011167	·983269	1·017015	·7130465	29
32	·7013241	·983841	1·016423	·7128426	28
33	·7015314	·984414	1·015832	·7126385	27
34	·7017387	·984987	1·015241	·7124344	26
35	·7019459	·985560	1·014651	·7122303	25
36	·7021531	·986133	1·014061	·7120260	24
37	·7023601	.986707	1·013471	·7118218	23
38	·7025672	.987282	1·012881	·7116174	22
39	·7027741	.987856	1·012292	·7114130	21
40	·7029811	.988431	1·011703	·7112086	20
′	Cosine.	Cotang.	Tang.	Sine.	′

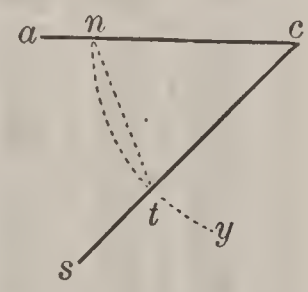
Deg. 45.

44 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·7031879	·989006	1·011115	·7110041	19
42	·7033947	·989582	1·010527	·7107995	18
43	·7036014	·990158	1·009939	·7105948	17
44	·7038081	·990734	1·009352	·7103901	16
45	·7040147	·991311	1·008764	·7101854	15
46	·7042213	·991888	1·008178	·7099806	14
47	·7044278	·992465	1·007591	·7097757	13
48	·7046342	·993042	1·007005	·7095707	12
49	·7048406	·993620	1·006420	·7093657	11
50	·7050469	·994199	1·005834	·7091607	10
51	·7052532	·994777	1·005249	·7089556	9
52	·7054594	·995356	1·004665	·7087504	8
53	·7056655	·995935	1·004080	·7085451	7
54	·7058716	·996515	1·003496	·7083398	6
55	·7060776	·997095	1·002913	·7081345	5
56	·7062835	·997675	1·002329	·7079291	4
57	·7064894	·998256	1·001746	·7077236	3
58	·7066953	·998837	1·001164	·7075180	2
59	·7069011	·999418	1·000581	·7073124	1
60	·7071068	1·00000	1·000000	·7071068	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 45

The table of chords, below, furnishes the means of laying down angles on paper more accurately than by an ordinary protractor. To do this, after having drawn and measured the first side (say $a\,c$) of the figure that is to be plotted; from its end c as a center, describe an arc $n\,y$ of a circle of sufficient extent to subtend the angle at that point. The rad $c\,n$ with which the arc is described should be as great as convenience will permit; and it is to be assumed as unity or 1; and must be decimally divided, and subdivided, to be used as a scale for laying down the chords taken from the table, in which their lengths are given in parts of said rad 1. Having described the arc, find in the table the length of the chord $n\,t$ corresponding to the angle $a\,c\,t$. Let us suppose this angle to be 45°; then we find that the tabular chord is .7654 of our rad 1. Therefore from n we lay off the chord $n\,t$, equal to .7654 of our radius-scale; and the line $c\,s$ drawn through the point t will form the reqd angle $a\,c\,t$ of 45°. And so at each angle. The degree of accuracy attained will evidently depend on the length of the rad, and the neatness of the drafting. The method becomes preferable to the common protractor in proportion as the lengths of the sides of the angles exceed the rad of the protractor. With a protractor of 4 to 6 ins rad, and with sides of angles not much exceeding the same limits, the protractor will usually be preferable. The dividers in boxes of instruments are rarely fit for accurate arcs of more than about 6 ins diam. In practice it is not necessary to actually describe the whole arc, but merely the portion near t, as well as can be judged by eye. We thus avoid much use of the India-rubber, and dulling of the pencil-point. For larger radii we may dispense with the dividers, and use a straight strip of paper with the length of the rad marked on one edge; and by laying it from c toward s, and at the same time placing another strip (with one edge divided to a radius-scale) from n toward t, we can by trial find their exact point of intersection at the required point t. In such matters, practice and some ingenuity are very essential to satisfactory results. We cannot devote more space to the subject.



CHORDS TO A RADIUS 1.

M.	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	M.
0′	.0000	.0175	.0349	.0524	.0698	.0872	.1047	.1221	.1395	.1569	.1743	0′
2	.0006	.0180	.0355	.0529	.0704	.0878	.1053	.1227	.1401	.1575	.1749	2
4	.0012	.0186	.0361	.0535	.0710	.0884	.1058	.1233	.1407	.1581	.1755	4
6	.0017	.0192	.0366	.0541	.0715	.0890	.1064	.1238	.1413	.1587	.1761	6
8	.0023	.0198	.0372	.0547	.0721	.0896	.1070	.1244	.1418	.1592	.1766	8
10	.0029	.0204	.0378	.0553	.0727	.0901	.1076	.1250	.1424	.1598	.1772	10
12	.0035	.0209	.0384	.0558	.0733	.0907	.1082	.1256	.1430	.1604	.1778	12
14	.0041	.0215	.0390	.0564	.0739	.0913	.1087	.1262	.1436	.1610	.1784	14
16	.0047	.0221	.0396	.0570	.0745	.0919	.1093	.1267	.1442	.1616	.1789	16
18	.0052	.0227	.0401	.0576	.0750	.0925	.1099	.1273	.1447	.1621	.1795	18
20	.0058	.0233	.0407	.0582	.0756	.0931	.1105	.1279	.1453	.1627	.1801	20
22	.0064	.0239	.0413	.0588	.0762	.0936	.1111	.1285	.1459	.1633	.1807	22
24	.0070	.0244	.0419	.0593	.0768	.0942	.1116	.1291	.1465	.1639	.1813	24
26	.0076	.0250	.0425	.0599	.0774	.0948	.1122	.1296	.1471	.1645	.1818	26
28	.0081	.0256	.0430	.0605	.0779	.0954	.1128	.1302	.1476	.1650	.1824	28
30	.0087	.0262	.0436	.0611	.0785	.0960	.1134	.1308	.1482	.1656	.1830	30
32	.0093	.0268	.0442	.0617	.0791	.0965	.1140	.1314	.1488	.1662	.1836	32
34	.0099	.0273	.0448	.0622	.0797	.0971	.1145	.1320	.1494	.1668	.1842	34
36	.0105	.0279	.0454	.0628	.0803	.0977	.1151	.1325	.1500	.1674	.1847	36
38	.0111	.0285	.0460	.0634	.0808	.0983	.1157	.1331	.1505	.1679	.1853	38
40	.0116	.0291	.0465	.0640	.0814	.0989	.1163	.1337	.1511	.1685	.1859	40
42	.0122	.0297	.0471	.0646	.0820	.0994	.1169	.1343	.1517	.1691	.1865	42
44	.0128	.0303	.0477	.0651	.0826	.1000	.1175	.1349	.1523	.1697	.1871	44
46	.0134	.0308	.0483	.0657	.0832	.1006	.1180	.1355	.1529	.1703	.1876	46
48	.0140	.0314	.0489	.0663	.0838	.1012	.1186	.1360	.1534	.1708	.1882	48
50	.0145	.0320	.0494	.0669	.0843	.1018	.1192	.1366	.1540	.1714	.1888	50
52	.0151	.0326	.0500	.0675	.0849	.1023	.1198	.1372	.1546	.1720	.1894	52
54	.0157	.0332	.0506	.0681	.0855	.1029	.1204	.1378	.1552	.1726	.1900	54
56	.0163	.0337	.0512	.0686	.0861	.1035	.1209	.1384	.1558	.1732	.1905	56
58	.0169	.0343	.0518	.0692	.0867	.1041	.1215	.1389	.1563	.1737	.1911	58
60	.0175	.0349	.0524	.0698	.0872	.1047	.1221	.1395	.1569	.1743	.1917	60

Table of Chords, in parts of a rad 1; for protracting—Continued

M.	11°	12°	13°	14°	15°	16°	17°	18°	19°	20°	M.
0	.1917	.2091	.2264	.2437	.2611	.2783	.2956	.3129	.3301	.3473	0
2	.1923	.2096	.2270	.2443	.2616	.2789	.2962	.3134	.3307	.3479	2
4	.1928	.2102	.2276	.2449	.2622	.2795	.2968	.3140	.3312	.3484	4
6	.1934	.2108	.2281	.2455	.2628	.2801	.2973	.3146	.3318	.3490	6
8	.1940	.2114	.2287	.2460	.2634	.2807	.2979	.3152	.3324	.3496	8
10	.1946	.2119	.2293	.2466	.2639	.2812	.2985	.3157	.3330	.3502	10
12	.1952	.2125	.2299	.2472	.2645	.2818	.2991	.3163	.3335	.3507	12
14	.1957	.2131	.2305	.2478	.2651	.2824	.2996	.3169	.3341	.3513	14
16	.1963	.2137	.2310	.2484	.2657	.2830	.3002	.3175	.3347	.3519	16
18	.1969	.2143	.2316	.2489	.2662	.2835	.3008	.3180	.3353	.3525	18
20	.1975	.2148	.2322	.2495	.2668	.2841	.3014	.3186	.3358	.3530	20
22	.1981	.2154	.2328	.2501	.2674	.2847	.3019	.3192	.3364	.3536	22
24	.1986	.2160	.2333	.2507	.2680	.2853	.3025	.3198	.3370	.3542	24
26	.1992	.2166	.2339	.2512	.2685	.2858	.3031	.3203	.3376	.3547	26
28	.1998	.2172	.2345	.2518	.2691	.2864	.3037	.3209	.3381	.3553	28
30	.2004	.2177	.2351	.2524	.2697	.2870	.3042	.3215	.3387	.3559	30
32	.2010	.2183	.2357	.2530	.2703	.2876	.3048	.3221	.3393	.3565	32
34	.2015	.2189	.2362	.2536	.2709	.2881	.3054	.3226	.3398	.3570	34
36	.2021	.2195	.2368	.2541	.2714	.2887	.3060	.3232	.3404	.3576	36
38	.2027	.2200	.2374	.2547	.2720	.2893	.3065	.3238	.3410	.3582	38
40	.2033	.2206	.2380	.2553	.2726	.2899	.3071	.3244	.3416	.3587	40
42	.2038	.2212	.2385	.2559	.2732	.2904	.3077	.3249	.3424	.3593	42
44	.2044	.2218	.2391	.2564	.2737	.2910	.3083	.3255	.3427	.3599	44
46	.2050	.2224	.2397	.2570	.2743	.2916	.3088	.3261	.3433	.3605	46
48	.2056	.2229	.2403	.2576	.2749	.2922	.3094	.3267	.3439	.3610	48
50	.2062	.2235	.2409	.2582	.2755	.2927	.3100	.3272	.3444	.3616	50
52	.2067	.2241	.2414	.2587	.2760	.2933	.3106	.3278	.3450	.3622	52
54	.2073	.2247	.2420	.2593	.2766	.2939	.3111	.3284	.3456	.3628	54
56	.2079	.2253	.2426	.2599	.2772	.2945	.3117	.3289	.3462	.3633	56
58	.2085	.2258	.2432	.2605	.2778	.2950	.3123	.3295	.3467	.3639	58
60	.2091	.2264	.2437	.2611	.2783	.2956	.3129	.3301	.3473	.3645	60

M.	21°	22°	23°	24°	25°	26°	27°	28°	29°	30°	M.
0	.3645	.3816	.3987	.4158	.4329	.4499	.4669	.4838	.5008	.5176	0
2	.3650	.3822	.3993	.4164	.4334	.4505	.4675	.4844	.5013	.5182	2
4	.3656	.3828	.3999	.4170	.4340	.4510	.4680	.4850	.5019	.5188	4
6	.3662	.3833	.4004	.4175	.4346	.4516	.4686	.4855	.5024	.5193	6
8	.3668	.3839	.4010	.4181	.4352	.4522	.4692	.4861	.5030	.5199	8
10	.3673	.3845	.4016	.4187	.4357	.4527	.4697	.4867	.5036	.5204	10
12	.3679	.3850	.4022	.4192	.4363	.4533	.4703	.4872	.5041	.5240	12
14	.3685	.3856	.4027	.4198	.4369	.4539	.4708	.4878	.5047	.5246	14
16	.3690	.3862	.4033	.4204	.4374	.4544	.4714	.4884	.5053	.5221	16
18	.3696	.3868	.4039	.4209	.4380	.4550	.4720	.4889	.5058	.5227	18
20	.3702	.3873	.4044	.4215	.4386	.4556	.4725	.4895	.5064	.5233	20
22	.3708	.3879	.4050	.4221	.4391	.4561	.4731	.4901	.5070	.5238	22
24	.3713	.3885	.4056	.4226	.4397	.4567	.4737	.4906	.5075	.5244	24
26	.3719	.3890	.4061	.4232	.4403	.4573	.4742	.4912	.5081	.5249	26
28	.3725	.3896	.4067	.4238	.4408	.4578	.4748	.4917	.5086	.5255	28
30	.3730	.3902	.4073	.4244	.4414	.4584	.4754	.4923	.5092	.5261	30
32	.3736	.3908	.4079	.4249	.4420	.4590	.4759	.4929	.5098	.5266	32
34	.3742	.3913	.4084	.4255	.4425	.4595	.4765	.4934	.5103	.5272	34
36	.3718	.3919	.4090	.4261	.4431	.4601	.4771	.4940	.5109	.5277	36
38	.3753	.3925	.4096	.4266	.4437	.4607	.4776	.4946	.5115	.5283	38
40	.3759	.3930	.4104	.4272	.4442	.4612	.4782	.4951	.5120	.5289	40
42	.3765	.3936	.4107	.4278	.4448	.4618	.4788	.4957	.5126	.5294	42
44	.3770	.3942	.4113	.4283	.4454	.4624	.4793	.4963	.5131	.5300	44
46	.3776	.3947	.4118	.4289	.4459	.4629	.4799	.4968	.5137	.5306	46
48	.3782	.3953	.4124	.4295	.4465	.4635	.4805	.4974	.5143	.5311	48
50	.3788	.3959	.4130	.4300	.4471	.4641	.4810	.4979	.5148	.5317	50
52	.3793	.3965	.4135	.4306	.4476	.4646	.4816	.4985	.5154	.5322	52
54	.3799	.3970	.4141	.4312	.4482	.4652	.4822	.4991	.5160	.5328	54
56	.3805	.3976	.4147	.4317	.4488	.4658	.4827	.4996	.5165	.5334	56
58	.3810	.3982	.4153	.4323	.4493	.4663	.4833	.5002	.5171	.5339	58
60	.3816	.3987	.4158	.4329	.4499	.4669	.4838	.5008	.5176	.5345	60

Table of chords, in parts of a rad 1; for protracting—Continued.

M.	31°	32°	33°	34°	35°	36°	37°	38°	39°	40°	M.
0′	.5345	.5513	.5680	.5847	.6014	.6180	.6346	.6511	.6676	.6840	0
2	.5350	.5518	.5686	.5853	.6020	.6186	.6352	.6517	.6682	.6846	2
4	.5356	.5524	.5691	.5859	.6025	.6191	.6357	.6522	.6687	.6851	4
6	.5362	.5530	.5697	.5864	.6031	.6197	.6363	.6528	.6693	.6857	6
8	.5367	.5535	.5703	.5870	.6036	.6202	.6368	.6533	.6698	.6862	8
10	.5373	.5541	.5708	.5875	.6042	.6208	.6374	.6539	.6704	.6868	10
12	.5378	.5546	.5714	.5881	.6047	.6214	.6379	.6544	.6709	.6873	12
14	.5384	.5552	.5719	.5886	.6053	.6219	.6385	.6550	.6715	.6879	14
16	.5390	.5557	.5725	.5892	.6058	.6225	.6390	.6555	.6720	.6884	16
18	.5395	.5563	.5730	.5897	.6064	.6230	.6396	.6561	.6725	.6890	18
20	.5401	.5569	.5736	.5903	.6070	.6236	.6401	.6566	.6731	.6895	20
22	.5406	.5574	.5742	.5909	.6075	.6241	.6407	.6572	.6736	.6901	22
24	.5412	.5580	.5747	.5914	.6081	.6247	.6412	.6577	.6742	.6906	24
26	.5418	.5585	.5753	.5920	.6086	.6252	.6418	.6583	.6747	.6911	26
28	.5423	.5591	.5758	.5925	.6092	.6258	.6423	.6588	.6753	.6917	28
30	.5429	.5597	.5764	.5931	.6097	.6263	.6429	.6594	.6758	.6922	30
32	.5434	.5602	.5769	.5936	.6103	.6269	.6434	.6599	.6764	.6928	32
34	.5440	.5608	.5775	.5942	.6108	.6274	.6440	.6605	.6769	.6933	34
36	.5446	.5613	.5781	.5947	.6114	.6280	.6445	6610	.6775	.6939	36
38	.5451	.5619	.5786	.5953	.6119	.6285	.6451	.6616	.6780	.6944	38
40	.5457	.5625	.5792	.5959	.6125	.6291	.6456	.6621	.6786	.6950	40
42	.5462	.5630	.5797	.5964	.6130	.6296	.6462	.6627	.6791	.6955	42
44	.5468	.5636	.5803	.5970	.6136	.6302	.6467	.6632	.6797	.6961	44
46	.5474	.5641	.5808	.5975	.6142	.6307	.6473	.6638	.6802	.6966	46
48	.5479	.5647	.5814	.5981	.6147	.6313	.6478	.6643	.6808	.6971	48
50	.5485	.5652	.5820	.5986	.6153	.6318	.6484	.6649	.6813	.6977	50
52	.5490	.5658	.5825	.5992	.6158	.6324	.6489	.6654	.6819	.6982	52
54	.5496	.5664	.5831	.5997	.6164	.6330	.6495	.6660	.6824	.6988	54
56	.5502	.5669	.5836	.6003	.6169	.6335	.6500	.6665	.6829	.6993	56
58	.5507	.5675	.5842	.6009	.6175	.6341	.6506	.6671	.6835	.6999	58
60	.5513	.5680	.5847	.6014	.6180	.6346	.6511	.6676	.6840	.7004	60

M.	41°	42°	43°	44°	45°	46°	47°	48°	49°	50°	M.
0′	.7004	.7167	.7330	.7492	.7654	.7815	.7975	.8135	.8294	.8452	0′
2	.7010	.7173	.7335	.7498	.7659	.7820	.7980	.8140	.8299	.8458	2
4	.7015	.7178	.7341	.7503	.7664	.7825	.7986	.8145	.8304	.8463	4
6	.7020	.7184	.7346	.7508	.7670	.7831	.7991	.8151	.8310	.8468	6
8	.7026	.7189	.7352	.7514	.7675	.7836	.7996	.8156	.8315	.8473	8
10	.7031	.7195	.7357	.7519	.7681	.7841	.8002	.8161	.8320	.8479	10
12	.7037	.7200	.7362	.7524	.7686	.7847	.8007	.8167	.8326	.8484	12
14	.7042	.7205	.7368	.7530	.7691	.7852	.8012	.8172	.8331	.8489	14
16	.7048	.7211	.7373	.7535	.7697	.7857	.8018	.8177	.8336	.8495	16
18	.7053	.7216	.7379	.7541	.7702	.7863	.8023	.8183	.8341	.8500	18
20	.7059	.7222	.7384	.7546	.7707	.7868	.8028	.8188	.8347	.8505	20
22	.7064	.7227	.7390	.7551	.7713	.7873	.8034	.8193	.8352	.8510	22
24	.7069	.7232	.7395	.7557	.7718	.7879	.8039	.8198	.8357	.8516	24
26	.7075	.7238	.7400	.7562	.7723	.7884	.8044	.8204	.8363	.8521	26
28	.7080	.7243	.7406	.7568	.7729	.7890	.8050	.8209	.8368	.8526	28
30	.7086	.7249	.7411	.7573	.7734	.7895	.8055	.8214	.8373	.8531	30
32	.7091	.7254	.7417	.7578	.7740	.7900	.8060	.8220	.8378	.8537	32
34	.7097	.7260	.7422	.7584	.7745	.7906	.8066	.8225	.8384	.8542	34
36	.7102	.7265	.7427	.7589	.7750	.7911	.8071	.8230	.8389	.8547	36
38	.7108	.7270	.7433	.7595	.7756	.7916	.8076	.8236	.8394	.8552	38
40	.7113	.7276	.7438	.7600	.7761	.7922	.8082	.8241	.8400	.8558	40
42	.7118	.7281	.7443	.7605	.7766	.7927	.8087	.8246	.8405	.8563	42
44	.7124	.7287	.7449	.7611	.7772	.7932	.8092	.8251	.8410	.8568	44
46	.7129	.7292	.7454	.7616	.7777	.7938	.8098	.8257	.8415	.8573	46
48	.7135	.7298	.7460	.7621	.7782	.7943	.8103	.8262	.8421	.8579	48
50	.7140	.7303	.7465	.7627	.7788	.7948	.8108	.8267	.8426	.8584	50
52	.7146	.7308	.7471	.7632	.7793	.7954	.8113	.8273	.8431	.8589	52
54	.7151	.7314	.7476	.7638	.7799	.7959	.8119	.8278	.8437	.8594	54
56	.7156	.7319	.7481	.7643	.7804	.7964	.8124	.8283	.8442	.8600	56
58	.7162	.7325	.7487	.7648	.7809	.7970	.8129	.8289	.8447	.8605	58
60	.7167	.7330	.7492	.7654	.7815	.7975	.8135	.8294	.8452	.8610	60

Table of chords, in parts of a rad 1; for protracting — Continued.

M.	51°	52°	53°	54°	55°	56°	57°	58°	59°	60°	M.
0′	.8610	.8767	.8924	.9080	.9235	.9389	.9543	.9696	.9848	1.0000	0′
2	.8615	.8773	.8929	.9085	.9240	.9395	.9548	.9701	.9854	1.0005	2
4	.8621	.8778	.8934	.9090	.9245	.9400	.9553	.9706	.9859	1.0010	4
6	.8626	.8783	.8940	.9095	.9250	.9405	.9559	.9711	.9864	1.0015	6
8	.8631	.8788	.8945	.9101	.9256	.9410	.9564	.9717	.9869	1.0020	8
10	.8636	.8794	.8950	.9106	.9261	.9415	.9569	.9722	.9874	1.0025	10
12	.8642	8799	.8955	.9111	.9266	.9420	.9574	.9727	.9879	1.0030	12
14	.8647	.8804	.8960	.9116	.9271	.9425	.9579	.9732	.9884	1.0035	14
16	.8652	.8809	.8966	.9121	.9276	.9430	.9584	.9737	.9889	1.0040	16
18	.8657	.8814	.8971	.9126	.9281	.9436	.9589	.9742	.9894	1.0045	18
20	.8663	.8820	.8976	.9132	.9287	.9441	.9594	.9747	.9899	1.0050	20
22	.8668	.8825	.8981	.9137	.9292	.9446	.9599	.9752	.9904	1.0055	22
24	.8673	.8830	.8986	.9142	.9297	.9451	.9604	.9757	.9909	1.0060	24
26	.8678	.8835	.8992	.9147	.9302	.9456	.9610	.9762	.9914	1.0065	26
28	.8684	.8841	.8997	.9152	.9307	.9461	.9615	.9767	.9919	1.0070	28
30	.8689	.8846	.9002	.9157	.9312	.9466	.9620	.9772	.9924	1.0075	30
32	.8694	.8851	.9007	.9163	.9317	.9472	.9625	.9778	.9929	1.0080	32
34	.8699	.8856	.9012	.9168	.9323	.9477	.9630	.9783	.9934	1.0086	34
36	.8705	.8861	.9018	.9173	.9328	.9482	.9635	.9788	.9939	1.0091	36
38	.8710	.8867	.9023	.9178	.9333	.9487	.9640	.9793	.9945	1.0096	38
40	.8715	.8872	.9028	.9183	.9338	.9492	.9645	.9798	.9950	1.0101	40
42	.8720	.8877	.9033	.9188	.9343	.9497	.9650	.9803	.9955	1.0106	42
44	.8726	.8882	.9038	.9194	.9348	.9502	.9655	.9808	.9960	1.0111	44
46	.8731	.8887	.9044	.9199	.9353	.9507	.9661	.9813	.9965	1.0116	46
48	.8736	.8893	.9049	.9204	.9359	.9512	.9666	.9818	.9970	1.0121	48
50	.8741	.8898	.9054	.9209	.9364	.9518	.9671	.9823	.9975	1.0126	50
52	.8747	.8903	.9059	.9214	.9369	.9523	.9676	.9828	.9980	1.0131	52
54	.8752	.8908	.9064	.9219	.9374	.9528	.9681	.9833	.9985	1.0136	54
56	.8757	.8914	.9069	.9225	.9379	.9533	.9686	.9838	.9990	1.0141	56
58	.8762	.8919	.9075	.9230	.9384	.9538	.9691	.9843	.9995	1.0146	58
60	.8767	.8924	.9080	.9235	.9389	.9543	.9696	.9848	1.0000	1.0151	60

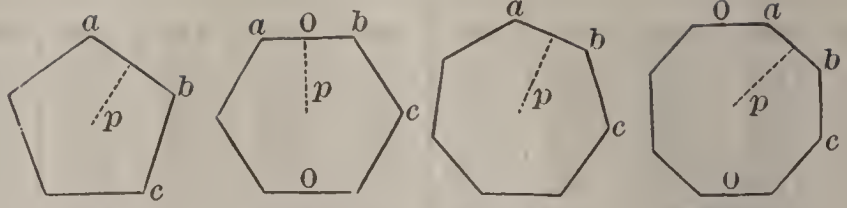
M.	61°	62°	63°	64°	65°	66°	67°	68°	69°	70°	M.
0′	1.0151	1.0301	1.0450	1.0598	1.0746	1.0893	1.1039	1.1184	1.1328	1.1472	0′
2	1.0156	1.0306	1.0455	1.0603	1.0751	1.0898	1.1044	1.1189	1.1333	1.1476	2
4	1.0161	1.0311	1.0460	1.0608	1.0756	1.0903	1.1048	1.1194	1.1338	1.1481	4
6	1.0166	1.0316	1.0465	1.0613	1.0761	1.0907	1.1053	1.1198	1.1342	1.1486	6
8	1.0171	1.0321	1.0470	1.0618	1.0766	1.0912	1.1058	1.1203	1.1347	1.1491	8
10	1.0176	1.0326	1.0475	1.0623	1.0771	1.0917	1.1063	1.1208	1.1352	1.1495	10
12	1.0181	1.0331	1.0480	1.0628	1.0775	1.0922	1.1068	1.1213	1.1357	1.1500	12
14	1.0186	1.0336	1.0485	1.0633	1.0780	1.0927	1.1073	1.1218	1.1362	1.1505	14
16	1.0191	1.0341	1.0490	1.0638	1.0785	1.0932	1.1078	1.1222	1.1366	1.1510	16
18	1.0196	1.0346	1.0495	1.0643	1.0790	1.0937	1.1082	1.1227	1.1371	1.1514	18
20	1.0201	1.0351	1.0500	1.0648	1.0795	1.0942	1.1087	1.1232	1.1376	1.1519	20
22	1.0206	1.0356	1.0504	1.0653	1.0800	1.0946	1.1092	1.1237	1.1381	1.1524	22
24	1.0211	1.0361	1.0509	1.0658	1.0805	1.0951	1.1097	1.1242	1.1386	1.1529	24
26	1.0216	1.0366	1.0514	1.0662	1.0810	1.0956	1.1102	1.1246	1.1390	1.1533	26
28	1.0221	1.0370	1.0519	1.0667	1.0815	1.0961	1.1107	1.1251	1.1395	1.1538	28
30	1.0226	1.0375	1.0524	1.0672	1.0820	1.0966	1.1111	1.1256	1.1400	1.1543	30
32	1.0231	1.0380	1.0529	1.0677	1.0824	1.0971	1.1116	1.1261	1.1405	1.1548	32
34	1.0236	1.0385	1.0534	1.0682	1.0829	1.0976	1.1121	1.1266	1.1409	1.1552	34
36	1.0241	1.0390	1.0539	1.0687	1.0834	1.0980	1.1126	1.1271	1.1414	1.1557	36
38	1.0246	1.0395	1.0544	1.0692	1.0839	1.0985	1.1131	1.1275	1.1419	1.1562	38
40	1.0251	1.0400	1.0549	1.0697	1.0844	1.0990	1.1136	1.1280	1.1424	1.1567	40
42	1.0256	1.0405	1.0554	1.0702	1.0849	1.0995	1.1140	1.1285	1.1429	1.1571	42
44	1.0261	1.0410	1.0559	1.0707	1.0854	1.1000	1.1145	1.1290	1.1433	1.1576	44
46	1.0266	1.0415	1.0564	1.0712	1.0859	1.1005	1.1150	1.1295	1.1438	1.1581	46
48	1.0271	1.0420	1.0569	1.0717	1.0863	1.1010	1.1155	1.1299	1.1443	1.1586	48
50	1.0276	1.0425	1.0574	1.0721	1.0868	1.1014	1.1160	1.1304	1.1448	1.1590	50
52	1.0281	1.0430	1.0579	1.0726	1.0873	1.1019	1.1165	1.1309	1.1452	1.1595	52
54	1.0286	1.0435	1.0584	1.0731	1.0878	1.1024	1.1169	1.1314	1.1457	1.1600	54
56	1.0291	1.0440	1.0589	1.0736	1.0883	1.1029	1.1174	1.1319	1.1462	1.1605	56
58	1.0296	1.0445	1.0593	1.0741	1.0888	1.1034	1.1179	1.1323	1.1467	1.1609	58
60	1.0301	1.0450	1.0598	1.0746	1.0893	1.1039	1.1184	1.1328	1.1472	1.1614	60

Table of Chords, in parts of a rad 1; for protracting—Continued.

M.	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°	M.
0	1.1614	1.1756	1.1896	1.2036	1.2175	1.2313	1.2450	1.2586	1.2722	1.2856	0′
2	1 1619	1.1760	1.1901	1.2041	1.2180	1.2318	1.2455	1.2591	1.2726	1.2860	2
4	1.1624	1.1765	1.1906	1.2046	1.2184	1.2322	1.2459	1.2595	1.2731	1.2865	4
6	1.1628	1.1770	1.1910	1.2050	1.2189	1.2327	1.2464	1.2600	1.2735	1.2869	6
8	1.1633	1.1775	1.1915	1.2055	1.2194	1.2332	1.2468	1.2604	1.2740	1.2874	8
10	1.1638	1.1779	1.1920	1.2060	1.2198	1.2336	1.2473	1.2609	1.2744	1 2878	10
12	1.1642	1.1784	1.1924	1.2064	1.2203	1.2341	1.2478	1.2614	1.2748	1.2882	12
14	1.1647	1.1789	1.1929	1.2069	1.2208	1.2345	1.2482	1.2618	1.2753	1.2887	14
16	1.1652	1.1793	1.1934	1.2073	1.2212	1.2350	1.2487	1.2623	1.2757	1.2891	16
18	1.1657	1.1798	1.1938	1.2078	1.2217	1.2354	1.2491	1.2627	1.2762	1.2896	18
20	1.1661	1.1803	1.1943	1.2083	1.2221	1.2359	1.2496	1.2632	1.2766	1.2900	20
22	1.1666	1.1807	1.1948	1.2087	1.2226	1.2364	1.2500	1.2636	1.2771	1.2905	22
24	1.1671	1.1812	1.1952	1.2092	1.2231	1 2368	1.2505	1.2641	1.2775	1.2909	24
26	1.1676	1.1817	1.1957	1.2097	1.2235	1.2373	1.2509	1.2645	1 2780	1.2914	26
28	1.1680	1.1821	1 1962	1.2101	1.2240	1.2377	1.2514	1.2650	1.2784	1.2918	28
30	1.1685	1.1826	1.1966	1.2106	1.2244	1.2382	1.2518	1.2654	1.2789	1.2922	30
32	1.1690	1.1831	1.1971	1.2111	1.2249	1.2386	1.2523	1.2659	1.2793	1.2927	32
34	1.1694	1.1836	1.1976	1.2115	1.2254	1.2391	1.2528	1.2663	1.2798	1.2931	34
36	1.1699	1.1840	1.1980	1.2120	1.2258	1.2396	1.2532	1.2668	1.2802	1.2936	36
38	1.1704	1.1845	1.1985	1.2124	1.2263	1.2400	1.2537	1.2672	1.2807	1.2940	38
40	1.1709	1.1850	1.1990	1.2129	1.2267	1.2405	1.2541	1.2677	1.2811	1.2945	40
42	1.1713	1.1854	1.1994	1.2134	1.2272	1.2409	1.2546	1.2681	1.2816	1.2949	42
44	1.1718	1.1859	1.1999	1.2138	1.2277	1.2414	1.2550	1.2686	1.2820	1.2954	44
46	1.1723	1.1864	1.2004	1.2143	1.2281	1.2418	1.2555	1.2690	1.2825	1.2958	46
48	1.1727	1.1868	1.2008	1.2148	1.2286	1.2423	1.2559	1.2695	1.2829	1.2962	48
50	1.1732	1.1873	1.2013	1.2152	1.2290	1.2428	1.2564	1.2699	1.2833	1.2967	50
52	1.1737	1.1878	1.2018	1.2157	1.2295	1.2432	1.2568	1.2704	1.2838	1.2971	52
54	1 1742	1.1882	1.2022	1.2161	1.2299	1.2437	1.2573	1.2708	1.2842	1.2976	54
56	1 1746	1.1887	1.2027	1.2166	1.2304	1.2441	1.2577	1.2713	1.2847	1 2980	56
58	1.1751	1.1892	1.2032	1.2171	1.2309	1.2446	1.2582	1.2717	1.2851	1.2985	58
60	1.1756	1.1896	1.2036	1.2175	1.2313	1.2450	1.2586	1.2722	1.2856	1.2989	60

M.	81°	82°	83°	84°	85°	86°	87°	88°	89°	M.
0′	1.2989	1.3121	1.3252	1.3383	1.3512	1.3640	1.3767	1.3893	1.4018	0′
2	1.2993	1.3126	1.3257	1.3387	1.3516	1.3644	1.3771	1.3897	1.4022	2
4	1.2998	1.3130	1.3261	1.3391	1.3520	1.3648	1.3776	1.3902	1.4026	4
6	1.3002	1.3134	1.3265	1.3396	1.3525	1 3653	1.3780	1.3906	1.4031	6
8	1.3007	1.3139	1.3270	1.3400	1.3529	1.3657	1.3784	1.3910	1 4035	8
10	1.3011	1.3143	1.3274	1.3404	1.3533	1.3661	1.3788	1.3914	1.4039	10
12	1.3015	1.3147	1.3279	1.3409	1.3538	1.3665	1.3792	1.3918	1.4043	12
14	1.3020	1.3152	1.3283	1.3413	1.3542	1.3670	1.3797	1.3922	1.4047	14
16	1.3024	1 3156	1.3287	1.3417	1.3546	1.3674	1.3801	1.3927	1.4051	16
18	1.3029	1.3161	1.3292	1 3421	1.3550	1.3678	1.3805	1 3931	1.4055	18
20	1.3033	1.3165	1.3296	1.3426	1.3555	1.3682	1.3809	1.3935	1.4060	20
22	1.3038	1.3169	1.3300	1.3430	1.3559	1.3687	1.3813	1.3939	1.4064	22
24	1.3042	1.3174	1.3305	1.3434	1.3563	1.3691	1.3818	1.3943	1.4068	24
26	1.3046	1.3178	1.3309	1.3439	1.3567	1.3695	1.3822	1.3947	1.4072	26
28	1.3051	1.3183	1.3313	1.3443	1.3572	1.3699	1.3826	1.3952	1.4076	28
30	1.3055	1.3187	1.3318	1.3447	1.3576	1.3704	1.3830	1.3956	1.4080	30
32	1.3060	1.3191	1.3322	1.3452	1.3580	1.3708	1.3834	1.3960	1.4084	32
34	1 3064	1.3196	1.3326	1.3456	1.3585	1.3712	1.3839	1.3964	1.4089	34
36	1.3068	1.3200	1.3331	1.3460	1.3589	1.3716	1.3843	1.3968	1.4093	36
38	1.3073	1.3204	1.3335	1.3465	1.3593	1.3721	1.3847	1.3972	1.4097	38
40	1.3077	1.3209	1.3339	1.3469	1.3597	1.3725	1.3851	1.3977	1.4101	40
42	1.3082	1.3213	1.3344	1.3473	1.3602	1.3729	1.3855	1.3981	1.4105	42
44	1.3086	1.3218	1.3348	1.3477	1.3606	1.3733	1.3860	1.3985	1.4109	44
46	1.3090	1.3222	1.3352	1.3482	1.3610	1.3738	1.3864	1.3989	1.4113	46
48	1.3095	1.3226	1.3357	1.3486	1.3614	1.3742	1.3868	1.3993	1.4117	48
50	1.3099	1.3231	1.3361	1.3490	1.3619	1.3746	1.3872	1.3997	1.4122	50
52	1.3104	1.3235	1.3365	1.3495	1 3623	1.3750	1.3876	1.4002	1.4126	52
54	1.3108	1.3239	1.3370	1.3499	1.3627	1.3754	1.3881	1.4006	1.4130	54
56	1.3112	1.3244	1.3374	1.3503	1.3631	1.3759	1.3885	1.4010	1.4134	56
58	1.3117	1.3248	1.3378	1.3508	1.3636	1.3763	1.3889	1.4014	1.4138	58
60	1.3121	1.3252	1.3383	1.3512	1.3640	1.3767	1.3893	1.4018	1.4142	60

POLYGONS.



Pentagon. 5 *sides.* **Hexagon.** 6 *sides.* **Heptagon.** 7 *sides.* **Octagon.** 8 *sides.*

Any straight-sided fig is called a polygon. If all the sides and angles are equal, it is a **regular** polygon; if not, it is **irregular.** Of course the number of polygons is infinite.

Table of Regular Polygons.

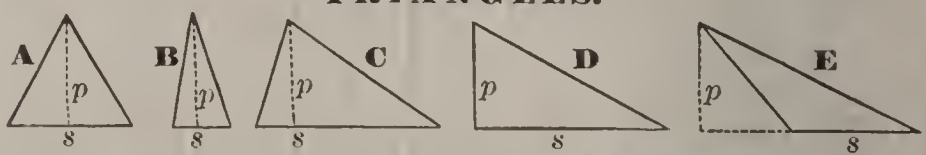
Number of Sides.	Name of Polygon.	**Area** = (square of one side) mult by	**Radius** of circumscribing circle = side mult by	**Interior angle** *a b c* contained between two sides.	**Angle at cen,** subteuded by a side.
3	Equilateral triangle.	.433013	.577350	60°	120°
4	Square.	1.000000	.707107	90°	90°
5	Pentagon.	1.720477	.850651	108°	72°
6	Hexagon.	2.598076	1.000000	120°	60°
7	Heptagon.	3.633912	1.152382	128° 34.2857′	51° 25.7143′
8	Octagon.	4.828427	1.306563	135°	45°
9	Nonagon.	6.181824	1.461902	140°	40°
10	Decagon.	7.694209	1.618034	144°	36°
11	Undecagon.	9.365640	1.774733	147° 16.3636′	32° 43.6364′
12	Dodecagon	11.196152	1.931854	150°	30°

Area of any regular polygon = length of one side, *a b* × perp *p* drawn from cen of fig to cen of side × half the number of sides.

Sum of interior angles, a b c, etc, **of any polygon, regular or irregular** = 180° × (number of sides — 2).

Angle at cen subtended by a side, in any regular polygon = 360° ÷ number of sides.

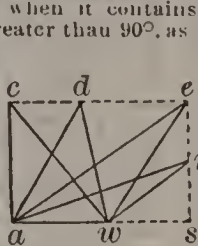
TRIANGLES.



We speak here of **plane** triangles only; or those having **straight** sides.

A triangle is equilateral when all its sides are equal, as A; **isosceles** when only two sides are equal, as B; **scalene** when all the sides are unequal, as C, D, and E; **acute-angled** when all its angles are acute, or each less than 90°, as A, B, and C; **right-angled** when it contains a right angle, as D; **obtuse-angled** when it contains an obtuse angle, or one greater than 90°, as E.

All the three angles of any triangle are equal to two right angles, or 180°; therefore, if we know two of them, we can find the third by subtracting their sum from 180°. **All triangles which have equal bases,** and equal perp heights, have also equal areas; thus the areas of *a w c*, *a w d*, and *a w e*, are equal to each other. **The area of any triangle** is equal to half that of any parallelogram which has an equal base, and an equal perp height. **The areas of triangles which have equal bases,** but diff perp heights, are to each other as, or in proportion to, their perp heights; thus the triangle *a w n*, with a perp height *s n*, equal to but one-half that (*s e*) of the three other triangles, but with the same base *a w*, has also but half the area of either of those others.



Area of any triangle, Figs A, B, C, D, E, = half the base, S, × the height, or perp dist *p* to the opposite angle. Any side may be taken as the base of a triangle; but the perp height must always be measured from the side so assumed; to do which, the side must sometimes be prolonged, as in Fig E; but the prolongation is not to be considered as a part of the base.

Area of any equilateral triangle = .433013 × square of one side.

To find area, having the three sides.

Add them together; div the sum by 2; from the half sum, subtract each side separately; mult the half sum and the three remainders continuously together; take the sq rt of the prod.

Ex.—The three sides = 20, 30, 40 ft. Here $20 + 30 + 40 = 90$; and $\frac{90}{2} = 45$. And $45 - 20 = 25$, $45 - 30 = 15$; and $45 - 40 = 5$. And $45 \times 25 \times 15 \times 5 = 84375$; and the sq rt of 84375 is 290.47 sq ft, area reqd.

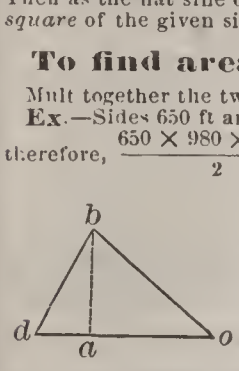
To find area, having one side and the 2 angles at its ends.

Add the 2 angles together: take the sum from 180°; the rem will be the angle opp the given side. Find the nat sine of this angle; also find the nat sines of the other angles, and mult them together. Then as the nat sine of the single angle, is to the prod of the nat sines of the other 2 angles, so is the *square* of the given side to *double* the reqd area.

To find area, having two sides, and the included angle.

Mult together the two sides, and the nat sine of the included angle; div by 2.

Ex.—Sides 650 ft and 980 ft; included angle 69° 20'. By the table we find the nat sine .9356; therefore, $\frac{650 \times 980 \times .9356}{2} = 297988.6$ square ft area.



To find area, having the three angles and the perp height, *a b*.

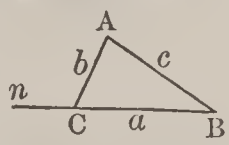
Find the nat sines of the three angles; mult together the sines of the angles *d* and *o*; div the sine of the angle *b* by the prod; mult the quot by the square of the perp height *a b*; div by 2.

To find any side, as *d o*, having the three angles, *d*, *b* and *o*, and the area.

(Sine of *d* × sine of *o*) : sine of *b* :: **twice** the area : **square** of *d* o. See Rem 2, p 119.

The perp height of an equilateral triangle is equal to one side × .866025. Hence one of its sides is equal to the perp height div by .866025 or to perp height × 1.1547. Or, **to find a side,** mult the sq rt of its area by 1.51967. The side of an equilateral triangle, mult by .658037 = side of a square of the same area; or mult by .742517 it gives the diam of a circle of the same area.

The following apply to any plane triangle, whether oblique or right-angled:



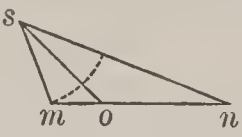
1. The three angles amount to 180°, or two right angles.

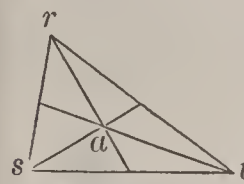
2. Any exterior angle, as A C *n*, is equal to the two interior and opposite ones, A and B.

3. The greater side is opposite the greater angle.

4. The sides are as the sines of the opposite angles. Thus, the side *a* is to the side *b* as the sine of A is to the sine of B.

5. If any angle as *s* be bisected by a line *s* o, the two parts *m o*, o *n* of the opposite side *m n* will be to each other as the other two sides *s m*, *s n*; or, *m* o : o *n* :: *s m* : *s n*.

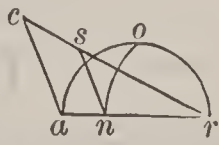




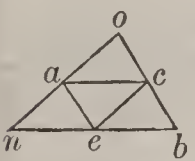
6. If lines be drawn from each angle *r s t* to the center of the opposite side, they will cross each other at one point, *a*, and the short part of each of the lines will be the third part of the whole line. Also, *a* is the **cen of grav** of the triangle.

7. If lines be drawn bisecting the three angles, they will meet at a point perpendicularly equidistant from each side, and consequently the **center of the greatest circle** that can be drawn in the triangle.

8. If a line *s n* be drawn parallel to any side c *a*, the two triangles *r s n*, *r c a*, will be similar.

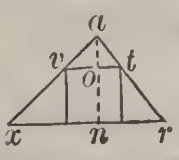


9. To divide any triangle *a c r* into two equal parts by a line *s n* parallel to any one of its sides c *a*. On either one of the other sides, as *a r*, as a diam, describe a semicircle *a o r*; and find its middle o. From *r* (opposite c *a*), with radius *r* o, describe the arc o *n*. From *n* draw *n s*, parallel to c *a*.



10. To find the greatest parallelogram that can be drawn in any given triangle o *n b*. Bisect the three sides at *a c e*, and join *a c*, *a e*, *e c*. Then either *a c b c*, *a e c* o, or *a c e n*, each equal to half the triangle, will be the reqd parallelogram. Any of these parallelograms can plainly be converted into a rectangle of equal area, and the greatest that can be drawn in the triangle.

10½. If a line *a c* bisects any two sides o *b*, o *n*, of a triangle, it will be parallel to the third side *n b*, and half as long as it.



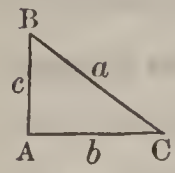
11. To find the greatest square that can be drawn in any triangle *a x r*. From an angle as *a* draw a perp *a n* to the opposite side *x r*, and find its length. Then o *n*, or a side *v t* of the square will $= \frac{x\,r \times a\,n}{x\,r + a\,n}$

Rem.—If the triangle is such that two or three such perps can be drawn, then two or three equal squares may be found.

Right-angled Triangles.

All the foregoing apply also to right-angled triangles; but what follow apply to them only.

Call the right angle A, and the others B and C; and call the sides respectively opposite to them a, b, and c. Then is



$$a = \frac{c}{\text{Sine C}} = c \times \text{Sec B} = \frac{b}{\text{Cosine C}} = b \times \text{Sec C} = \sqrt{b^2 + c^2}.$$

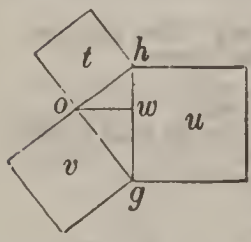
$b = a \times \text{Sine B} = a \times \text{Cos C} = c \times \text{Cot C} = c \times \text{Tang B}.$
$c = a \times \text{Sine C} = a \times \text{Cos B} = b \times \text{Tang C}.$

Also Sine of $C = \frac{c}{a}$; Cos $C = \frac{b}{a}$; Tang $C = \frac{c}{b}$.

And Sine of $B = \frac{b}{a}$; Cos $B = \frac{c}{a}$; Tang $B = \frac{b}{c}$.

And Sine of A or $90° = 1$. Cos A $= 0$. Tang A $=$ infinity. Sec A $=$ infinity.

1. If from the right angle o a line o w be drawn perp to the hypothenuse or long side $h\ g$, then the two small triangles $o\ w\ h$, o $w\ g$, and the large one o $h\ g$, will be similar. Or $g\ w : w\ o :: w\ o : w\ h$; and $g\ w \times w\ h = w\ o^2$.



2. A line drawn from the right angle to the **center** of the long side will be half as long as said side.

3. If on the three sides o h, o g, $g\ h$ we draw three squares t, u, v, or three circles, or triangles, or any other three figs that are similar, then the area of the largest one is equal to the sum of the areas of the two others.

4. In a triangle whose sides are as 3, 4, and 5 (as are those of the triangle A B C), the angles are very approximately 90°; 53° 7′ 48.38″; and 36° 52′ 11.62″. Their Sines, 1.; .8; and .6. Their Tangs, infinity; 1.3333; and .75.

5. One whose sides are as 7, 7, and 9 9, has very approx one angle of 90° and two of 45° each, near enough for all practical purposes.

PLANE TRIGONOMETRY.

Plane trigonometry teaches how to find certain unknown parts of plane, or straight-sided triangles, by means of other parts which are known; and thus enables us to measure inaccessible distances, &c. A triangle consists of six parts, namely, three sides, and three angles; and if we know any three of these, (except the three angles, and in the ambiguous case under "Case 2,") we can find the other three. The following four cases include the whole subject; the student should commit them to memory.

Case 1. Having any two angles, and one side, to find the other sides and angle.

Add the two angles together; and subtract their sum from 180°; the rem will be the third angle. And for the sides, as

Sine of the angle opp the given side : Sine of the angle opp the reqd side :: given side : reqd side.

Use the side thus found, as the given one; and in the same manner find the third side.

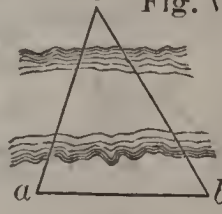


Fig. W.

Case 2. Having two sides, $b\ a$, $a\ c$, Fig X. and the angle $a\ b\ c$, opposite to one of them, to find the other side and angles.

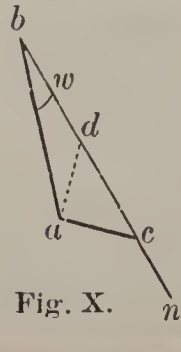


Fig. X.

Side a c opp the given angle $a\ b\ c$: The other given side $b\ a$ :: Sine of the given angle $a\ b$ c : Sine of angle $b\ d\ a$ or $b\ c\ a$ opposite the other given side $b\ a$.

Having found the sine, take out the corresponding angle from the table of nat sines, but, in doing so, if the side a c opp the given angle is shorter than the other given side $b\ a$, bear in mind that an angle and its supplement have the same sine. Thus, in Fig X, the sine, as found above, is opp the angle b c a in the table. But a c, if *shorter than* $b\ a$, can evidently be laid off in the opp direction, $a\ d$, in which case $b\ d\ a$ is the supplement of b c a. If $a\ c$ is as long as, or longer than, $b\ a$, there can be no doubt; for in that case it cannot be drawn toward b, but only toward n and the angle $b\ c\ a$ will be found at once in the table, opp the sine as found above.

When the two angles, *a b* c, *b* c *a*, have been found, find the remaining side by Case 1.

For the remaining angle, *b a* e, add together the angle *a b* c first given, and the one, *b* c *a*, found as above. Deduct their sum from 180°.

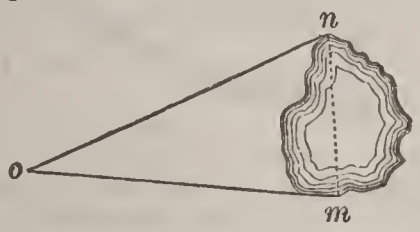
Case 3. Having two sides, and the angle included between them.

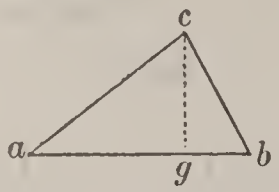
Take the angle from 180°; the rem will be the sum of the two unknown angles. Div this sum by 2; and find the nat tang of the quot. Then as

The sum of the two given sides : Their diff :: Tang of half the sum of the two unknown angles . Tang of half their diff.

Take from the table of nat tang, the angle opposite this last tang. Add this angle to the half sum of the two unknown angles, and it will give the angle opp the longest given side; and subtract it from the same half sum, for the angle opp the shortest given side. Having thus found the angles, find the third side by Case 1.

As a practical example of the use of Case 3, we can ascertain the dist *n m* across a deep pond, by measuring two lines *n* o and *m* o; and the angle *n o m*. From these data we may calculate *n m*; or by drawing the two sides, and the angle on paper, by a scale, we can afterward measure *n m* on the drawing.





Case 4. Having the three sides,

To find the three angles: upon one side *a b* as a base, draw (or suppose to be drawn) a perp e *g* from the opposite angle *c*. Find the diff between the other two sides, *a c* and *c b*; also their sum. Then, as

The base : Sum of the other two sides :: Diff of other two sides . Diff of the two parts *a g* and *b g*, of the base.

Add *half* this diff of the parts, to *half* the base *a b*; the sum will be the longest part *a g*; which taken from the whole base, gives the shortest part *g b*. By this means we get in each of the small triangles *a c g* and *c g b*, two sides, (namely, *a* e and *a g*; and *c b* and *g b*;) and an angle (namely, the right angle *c g a*, or *c g b*) opposite to one of the given sides. Therefore, use Case 2 for finding the angles *a* and *b*. When that is done, take their sum from 180°, for the angle *a* e *b*.

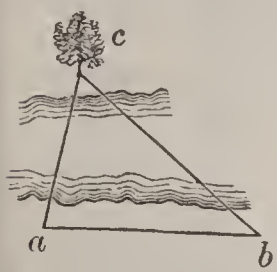


Fig. 1.

Or, 2d mode: call **half** the sum of the three sides, **s**; and call the two sides which form either angle, **m** and **n**. Then the nat sine of **half** that angle will be equal to $\sqrt{\frac{(s-m)\times(s-n)}{m\times n}}$.

Ex. 1. To find the dist from *a* to an inaccessible object c.

Measure a line *a b*; and from its ends measure the angles e *a b* and e *b a*. Thus having found one side and two angles of the triangle *a b c*, calculate *a c* by means of Case 1. Or if extreme accuracy is not reqd, draw the line *a b* on paper to any convenient scale; then by means of a protractor lay off the angles e *a b*, e *b a*; and draw *a* e and e *b*; then measure *a* e by the same scale.

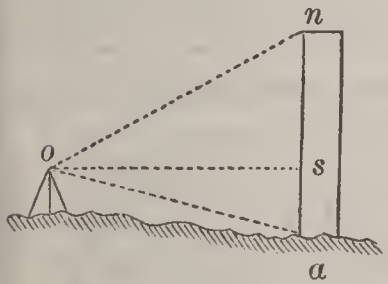


Fig. 2.

Ex. 2. To find the height of a *vertical* object, *n a*.

Place the instrument for measuring angles, at any convenient spot *o*; also meas the dist o *a*; or if o *a* cannot be actually measd in consequence of some obstacle, calculate it by the same process as *a c* in Fig 1. Then, first directing the instrument horizontally,* as o *s*, measure the angle of depression, *s o a*, say 12°; also the augle *s o n*, say 30°. These two angles added together, give the angle *a o n*, 42°. Now, in the small triangle o *s a* we have the angle o *s a* equal to 90°, because *a n* is vert, and *o s* hor; and since the three angles of any triangle are equal to 180°, if we subtract the angles o *s a* (90°), and *s o a* (12°) from 180°, the rem (78°) will be the angle o *a s* or *o a n*. Therefore, in the triangle o *n a*, we have one side *o a*; and two angles *a o n*, and *o a n*, to calculate the side *a n* by Case 1.

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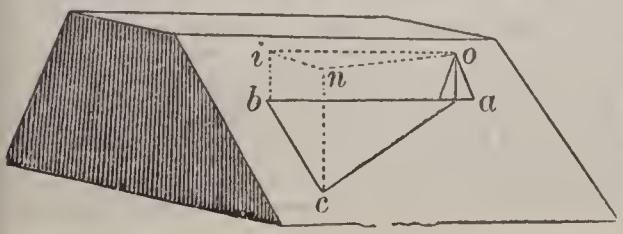
* **Angles and dists on sloping ground must be measured horizontally.** The graduated hor circle of the instrument evidently measures the angle between two objects horizontally, no matter how much higher one of them may be than the other; one perhaps requiring the telescope of the instrument to be directed upward toward it; and the other downward. If, therefore, the sides of triangles lying upon sloping ground, are not also measd hor, there can be no accordance between the two. Thus



Rem. If, as in Fig 3, it should be necessary to ascertain the *vert* height $a\,n$ from a point o, entirely above it, then both the angles measd at o, namely, $s\,o\,n$, and $s\,o\,a$, will be angles of depression, or *below* the hor line $o\,s$ assumed to measure them from. In this case we have the side $o\,a$ as before; the angle $n\,o\,a = s\,o\,a - s\,o\,n$; and the angle $o\,a\,n = 180° - \big(o\,s\,a\ (90°)$, and $s\,o\,a$; $\big)$ to calculate $a\,n$ by Case 1.

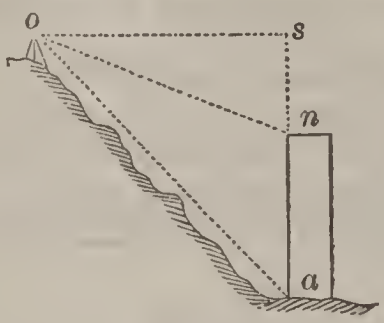


Fig. 3.

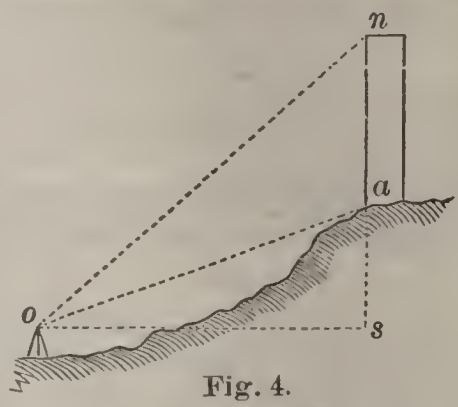


Fig. 4.

Or If, as in Fig 4, the observations are to be taken from a point o, entirely below the object $a\,n$, then both the angles $s\,o\,a$, $s\,o\,n$, will be angles of elevation, or *above* the assumed hor line $o\,s$. Here we have in the triangle $o\,n\,a$, the given side $o\,a$ as before; the angle $a\,o\,n = s\,o\,n - s\,o\,a$; and the angle $o\,n\,a = 180° - \big(o\,s\,n\ (90°)$, and $n\,o\,s$, $\big)$ to calculate $a\,n$ by Case 1.

If the object $a\,n$, as in Fig 5, instead of being *vert.* is *inclined;* and instead of its vert height, we wish to find its length $a\,n$, we must first ascertain its angle $y\,t\,i$ of inclination to the horizon; to which angle each of the angles $o\,s\,n$ will be equal. To find this angle $y\,t\,i$, suspend a plumb-line $i\,y$, of any convenient known length, from the object $a\,n$: and measure also $y\,t$ horizontally. Then say as

$y\,t : i\,y :: 1$: nat tang of angle $y\,t\,i$.

From the table of nat tangs take out the angle $y\,t\,i$ found opposite this nat tang; and use it for the angles $o\,s\,n$ or $o\,s\,a$; instead of the 90° of Figs 3 and 4. Also when the object inclines, the side $a\,o$ of the triangle must be measd in line, or in range with the inclination. If the object, as the rock $a\,n$, Fig 6, is curved or irregular, a pole $a\,s$ may be planted sloping in the direction $a\,n$; and

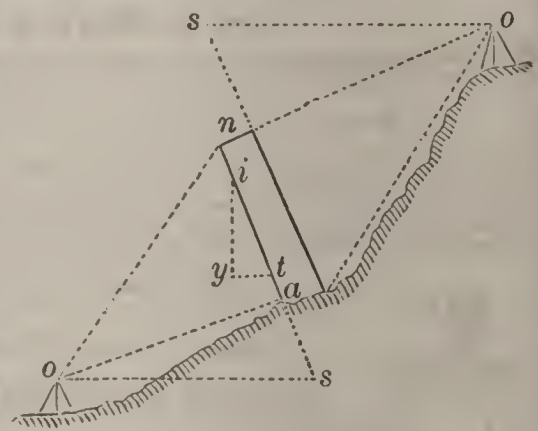


Fig. 5.

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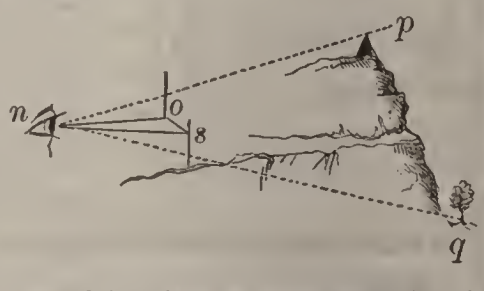
in the triangle $a\,b\,c$, upon sloping ground, the instrument at o, measures the hor angle $i\,o\,n$: and not the angle $b\,a\,c$. Therefore, the side which corresponds with this hor angle $i\,o\,n$, is the hor dist $i\,n$; and not the sloping dist $b\,c$. In other words, when sides and angles are on sloping ground, we do not seek their *actual* measures; but their *hor* ones. This remark applies to all surveying for farms, railroads, triangulations of countries, &c. &c; and the want of a strict attention to it, is one cause of the small errors, almost unavoidable, (and fortunately, of but trifling consequence in practice), which occur in all ordinary field operations. See p 176.

When a sextant is used, angles between objects at diff altitudes, as p and q, may be measd hor, by first planting two vert rods o and s, in range with the objects; and then taking the hor angle $o\,n\,s$, subtended by the rods.



Angles may be measd without any inst, thus: Measure 100 ft toward each object, and drive stakes; measure the dist across from one stake to the other. Half this dist will be the sine of *half* the angle to a rad of 100; and if we move the decimal point two places to the left, we get the *nat sine* of this one half of the angle to a rad of 1, as in the tables. Thus, suppose the dist to be 80.64 feet; then 40.32 is the sine of half the angle; and .4032 will be the nat sine, opposite to which in the table of nat sines we find the angle 23° 47'; which mult by 2 gives 47° 34', the reqd angle. If obstacles prevent measuring *toward* the objects, we may measure directly *from* them; because, when two lines intersect, the opposite angles are equal. A rough measurement may be made by sticking three pins vert, and a few ins apart, into a small piece of board, nailed hor to the top of a post. The pins would occupy the positions $n\,o\,s$, of the last figure. Pencil-lines may then be drawn, connecting the pin-holes; and the angle be measd with a protractor. By nailing a piece of board vert to a tree, and then drawing upon it a short hor line, by means of a pocket carpenters' spirit-level, vert angles of elevation and depression may be taken roughly in the same way. In this way the writer has at times availed himself of the outer door of a house, by opening it until it pointed toward some mountain-peak, the dist of which he knew approximately; but of the height of

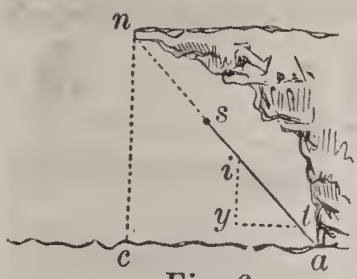


Fig. 6.

its angle $y\,t\,i$ of inclination with the horizon found as before; in which case the dist $a\,n$ is calculated. Or if the *vert* height $c\,n$ is sought, the point c may first be found by sighting upward along a plumb-line held above the head.

Ex. 3. To find the approximate height, $s\,x$, of a mountain,

Of which, perhaps, only the very summit, x, is visible above interposing forests, or other obstacles; but the dist, $m\,i$, of which is known. In this case, first direct the instrument hor, as $m\,h$; and then measure the angle $i\,m\,x$. Then in the triangle $i\,m\,x$ we have one side $m\,i$; the measd angle $i\,m\,x$, and the angle $m\,i\,x$ (90°), to find $i\,x$ by Case 1. But to this $i\,x$ we must add $i\,o$, equal to the height $y\,m$ of the instrument above the ground; and also $o\,s$. Now, $o\,s$ is *apparently* due entirely to the curvature of the earth, which is equal to very nearly 8 ins, or .667 ft in one mile; and increases as the squares of the dists; being 4 times 8 ins in 2 miles; 9 times 8 ins in 3 miles, &c. But this is somewhat diminished by the refraction of the atmosphere; which varies with temperature, moisture, &c; but always tends to make the object x appear higher than it actually is. At an average, this deceptive elevation amounts to about $\frac{1}{7}$th part of the curvature of the earth; and like the latter, it varies with the squares of the dists. Consequently if we subtract $\frac{1}{7}$ part from 8 ins, or .667 ft, we have at once the combined effect of curvature and refraction for one mile, equal to 6.857 ins, or .5714 ft; and for other dists, as shown in the following table, by the use of which we avoid the necessity of making *separate* allowances for curvature and refraction.

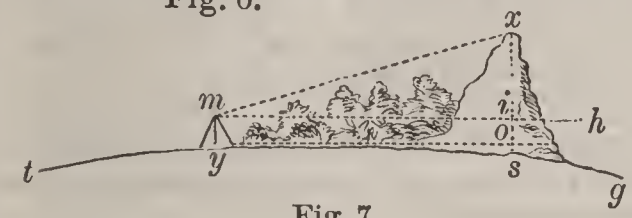


Fig. 7.

Table of allowances to be added for curvature of the earth; and for refraction; combined.

Dist. in yards.	Allow. feet.	Dist. in miles.	Allow. feet.	Dist. in miles.	Allow. feet.	Dist. in miles.	Allow. feet.
100	.002	¼	.036	6	20.6	20	229
150	.004	½	.143	7	28.0	22	277
200	.007	¾	.321	8	36.6	25	357
300	.017	1	.572	9	46.3	30	514
400	.030	1¼	.893	10	57.2	35	700
500	.046	1½	1.29	11	69.2	40	915
600	.066	1¾	1.75	12	82.3	45	1158
700	.090	2	2.29	13	96.6	50	1429
800	.118	2½	3.57	14	112	55	1729
900	.149	3	5.14	15	129	60	2058
1000	.185	3½	7.00	16	146	70	2801
1200	.266	4	9.15	17	165	80	3659
1500	.415	4½	11.6	18	185	90	4631
2000	.738	5	14.3	19	206	100	5717

Hence, if a person whose eye is 5.14 ft, or 112 ft above the sea, sees an object just at the sea's horizon, that object will be about 3 miles, or 14 miles distant from him.

A horizontal line is not a level one, for a straight line cannot be a level one. The curve of the earth, as exemplified in an expanse of quiet water, is level. In Fig 7, if we suppose the curved line $t\,y\,s\,g$ to represent the surface of the sea, then the points $t\,y\,s$ and g are on a level with each other. They need not be equidistant from the center of the earth, for the sea at the poles is about 13 miles nearer it than at the equator; yet its surface is everywhere on a level. **Up, and down,** refer to sea level. **Level** means parallel to the curvature of the sea; and **horizontal** means tangential to a level.

Ex. 4. If the inaccessible vert height $c\,d$, Fig 8,

Is so situated that we cannot reach it at all, then place the instrument for measuring angles, at any convenient spot n; and in range between n and d, plant two staffs, whose tops o and i shall range precisely, with n, though they need not be on the same level or hor line with it. Measure $n\,o$: also from n measure the angles $o\,n\,d$ and $o\,n\,c$. Then move the instrument to the precise spot previously

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which he had no idea. For allowance for curvature and refraction see above Table. **A triangle whose sides are as 3, 4, and 5,** is right angled; and one whose sides are as 7; 7; and 9. 9; contains 1 right angle; and 2 angles of 45° each. As it is frequently necessary to lay down angles of 45° and 90° on the ground, these proportions may be used for the purpose, by shaping a portion of a tape-line or chain into such a triangle, and driving a stake at each angle. See p. 58.

occupied by the top o of the staff; and from o measure the angles $i\,o\,d$ and $d\,o\,c$. This being done, subtract the angle $i\,o\,c$ from 180°; the rem will be the angle $c\,o\,n$. Consequently in the triangle $n\,o\,c$, we have one side $n\,o$, and two angles, $c\,n\,o$ and $c\,o\,n$, to find by Case I the side $o\,c$. Again, take the angle $i\,o\,d$ from 180°; the remainder will be the angle $n\,o\,d$, so that in the triangle $d\,n\,o$ we have one side $n\,o$, and the two angles $d\,n\,o$ and $n\,o\,d$, to find by Case 1 the side $o\,d$. Finally, in the triangle $c\,o\,d$, we have two sides $c\,o$ and $o\,d$, and their included angle $c\,o\,d$, to find $c\,d$, the reqd vert height.

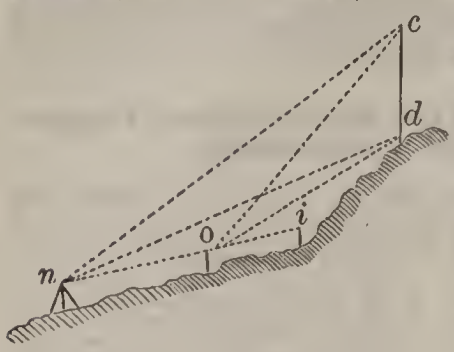


Fig. 8.

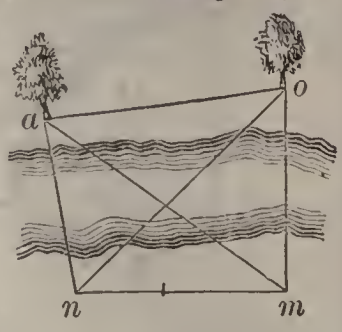


Fig. 9.

Rem. If $c\,d$ were in a valley, or on a hill, and the observations reqd to be made from either higher or lower ground, the operation would be precisely the same.

Ex. 5. See Ex 10.

To find the dist $a\,o$, Fig 9, between two entirely inaccessible objects,

Measure a side $n\,m$; at n measure the angles $a\,n\,m$ and $o\,n\,m$; also at m measure the angles $o\,m\,n$, and $a\,m\,n$. This being done, we have in the triangle $a\,n\,m$, one side $n\,m$, Fig 9, and the angles $a\,n\,m$, and $n\,m\,a$; hence, by Case 1, we can calculate the side $a\,n$. Again, in the triangle $o\,m\,n$ we have one side $n\,m$, and the two angles $o\,m\,n$, and $m\,n\,o$; hence, by Case 1, we can calculate the side $n\,o$. This being done, we have in the triangle $a\,n\,o$, two sides $a\,n$, and $n\,o$; and their included angle $a\,n\,o$; hence, by Case 3, we can calculate the side $a\,o$, which is the reqd dist. It is plain that in this manner we may obtain also the position or direction of the inaccessible line $a\,o$; for we can calculate the angle $n\,a\,o$; and can therefrom deduce that of $a\,o$; and thus be enabled to run a line parallel to it, if required. By drawing $n\,m$ on paper by a scale, and laying down the four measd angles, the dist $a\,o$ may be measd upon the drawing by the same scale.

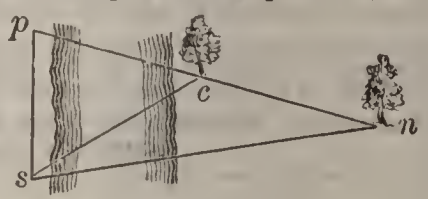


Fig. 10.

If the position of the inaccessible dist $c\,n$, Fig 10, be such that we can place a stake p in line with it, we may proceed thus: Place the instrument at any suitable point s, and take the angles $p\,s\,c$ and $c\,s\,n$. Also find the angle $c\,p\,s$, and measure the dist $p\,s$. Then in the triangle $p\,s\,c$ find $s\,c$ by Case 1; again, the exterior angle $n\,c\,s$, being equal to the two interior and opposite angles $c\,p\,s$, and $p\,s\,c$, we have in the triangle $c\,s\,n$, one side and two angles to find $c\,n$ by Case 1.

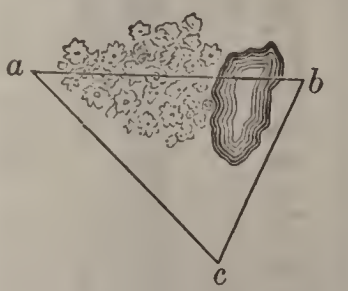


Fig. 11.

Ex. 6. To find a dist $a\,b$, Fig 11. of which the ends only are accessible.

From a and b, measure any two lines $a\,c$, $b\,c$, meeting at c; also measure the angle $a\,c\,b$. Then in the triangle $a\,b\,c$ we have two sides, and the included angle, to find the third side $a\,b$ by Case 3.

Ex. 7. To find the vert height $o\,m$, of a hill, above a given point i.

Place the instrument at i; measure $a\,m$. Directing the instrument hor, as $a\,n$, take the angle $n\,a\,m$. Then, since $a\,n\,m$ is 90° Fig 12, we have one side $a\,m$, and two angles, $n\,a\,m$ and $a\,n\,m$, to find $n\,m$ by Case 1. Add $n\,o$, equal to $a\,i$, the height of the instrument. Also, if the hill is a long one, add for curvature of the earth, and for refraction, as explained in Example 3, Fig 7. Or the instrument may be placed at the top of the hill; and an angle of depression measured; instead of the angle of elevation $n\,a\,m$.

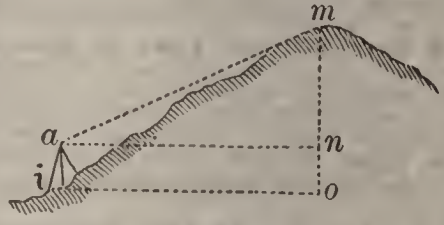


Fig. 12.

Rem. 1. It is plain, that if the height $o\,m$ be previously known, and we wish to ascertain the dist from its summit m to any point i, the same measurement as before, of the angle $n\,a\,m$, will enable us to calculate $a\,m$ by Case 1. So in Ex. 2, if the height $n\,a$ be known, the angles measd in that example, will enable us to compute the dist $a\,o$; so also in Figs 3, 4, 5, and 7; in all of which the process is so plain as to require no further explanation.

Rem. 2. The height of a vert object **by means of its shadow.** Plant one end of a straight stick vert in the ground; and measure its shadow; also measure the length of the shadow of the object. Then, as the length of the shadow of the stick is to the length of the stick above

ground, so is the length of the shadow of the object, to its height. If the object is inclined, the stick must be equally inclined.

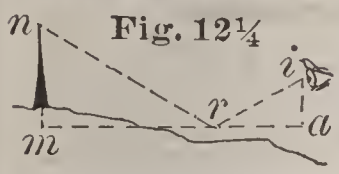


Fig. 12¼

Rem. 3. Or the height of a vert object *m n*, Fig 12¼, whose distance *r m* is known, may be found **by its reflection** in a vessel of water, or in a piece of looking glass placed perfectly horizontal at *r*; for as *r a* is to the height *a i* of the eye above the reflector *r*, so is *r m* to the height *m n* of the object above *r*.

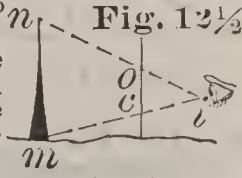


Fig. 12½

Rem. 4. Or let *o c*, Fig 12½, **be a planted pole,** or a rod held vert by an assistant. Then stand at a proper dist back from it, and keeping the eyes steady, let marks be made at o and c, where the lines of sight *i n* and *i m* strike the rod. Then as *i c* is to *c o*, so is *i m* to *m n*.

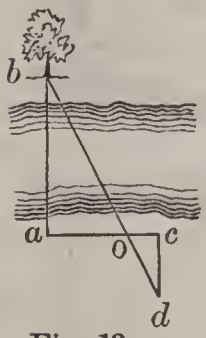


Fig. 13.

The following examples may be regarded as substitutes for strict trigonometry: and will at times be useful, in case a table of sines, &c, is not at hand for making trigonometrical calculations.

Ex. 8. To find the dist *a b*, of which one end only is accessible.

Drive a stake at any convenient point *a*; from *a* lay off any angle *b a c*. In the line *a c*, at any convenient point c, drive a stake; and from c lay off an angle *a c d*, equal to the angle *b a c*. In the line *c d*, at any convenient point, as *d*, drive a stake. Then, standing at *d*, and looking at *b*, place a stake *o* in range with *d b*; and at the same time in the line *a c*. Measure *a o*, *o c*, and *c d*; then, from the principle of similar triangles, as

$$o\,c : c\,d :: a\,o : a\,b.$$

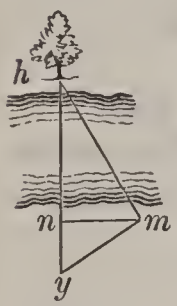


Fig. 14.

Or thus:

Fig 14, *n h* being the dist, place a stake at *n*; and lay off the angle *h n m* 90°. At any convenient dist *n m*, place a stake *m*. Make the angle *h m y* = 90°; and place a stake at *y*, in range with *h n*. Measure *n y* and *n m*; then, from the principle of similar triangles, as

$$n\,y : n\,m :: n\,m : n\,h.$$

Or thus, Fig 14. Lay off the angle *h n m* = 90°, placing a stake *m*, at any convenient dist *n m*. Measure *n m*. Also measure the angle *n m h*. Find nat tang of *n m h* by Table Mult this nat tang by *n m*. The prod will be *n h*.

Or thus. Lay off angle *h n m* = 90°. From *m* measure the angle *n m h*, and lay off angle *n m y* equal to it, placing a stake at *y* in range with *h n*. Then is *n y* = *n h*.

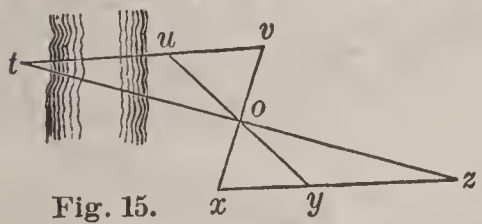


Fig. 15.

Or thus, without measuring any angle;

t u being the dist. Make *u v* of any convenient length, in range with *t u*. Measure any *v o*; and *o x* equal to it, in range. Measure *u o*; and *o y* equal to it in range. Place a stake *z* in range with both *x y*, and *t o*. Then will *y z* be both equal to *t u*, and parallel to it.

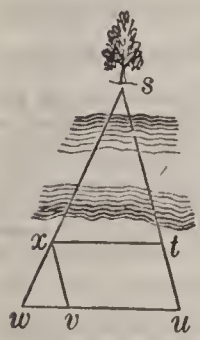


Fig. 16.

Or thus, without measuring any angle.

Drive two stakes *t* and *u*, in range with the object *s*. From *t* lay off any convenient dist *t x*, in any direction. From *u* lay off *u w* parallel to *t x*, placing *w* in range with *x s*. Make *u v* equal to *t x*. Measure *w v*, *v x*, and *x t*. Then, as

$$w\,v : v\,x :: x\,t : t\,s.$$

Or thus. At *a* lay off angle *o a c* = 5° 43′. Lay off *o c* at right angles to *a o*. Measure *o c*. Then *a o* = 10 *o c*, too long only 1 part in 935.6, or 5.643 feet in a mile, or .1069 foot (full 1¼ inches) in 100 feet.

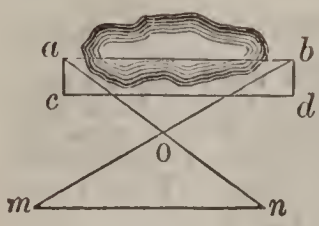


Fig. 17.

Ex. 9. To find the dist $a\,b$, of which the ends only are accessible.

From a lay off the angle $b\,a\,c$; and from b, the angle $a\,b\,d$, each 90°. Make $a\,c$ and $b\,d$ equal to each other. Then, $c\,d = a\,b$. Or $a\,b$ may be considered as the dist across the river in Figs 15, 13, or 14; and be ascertained in the same way. Or measure any dist, Fig 17, $a\,o$; and make $o\,n$ in line and equal to it. Also measure $b\,o$; and make $o\,m$ in line and equal to it. Then will $m\,n$ be both parallel to $a\,b$, and equal to it.

Ex. 10. See Ex. 4. To find the entirely inaccessible dist $y\,z$, and also its direction.

At any two convenient points a and b, from each of which y and z can be seen, drive stakes. Then we have the four corners of a four-sided figure, in which are given the directions of three of its sides, and of its two diags. These data enable us to lay out on the ground, the small four-sided fig $a\,c\,o\,i$, exactly similar to the large one. Thus, in the line $a\,b$ place a stake c: and make $c\,o$ parallel to $b\,z$; o being at the same time in range of the diag $a\,z$. Also, from c make $c\,i$ parallel to $b\,y$; i being at the same time in range of $a\,y$. Then will $i\,o$ be in the same direction as $y\,z$, or parallel to it. Measure $a\,c$, $a\,b$, and $i\,o$; then evidently, from the principle of similar figures, as

$$a\,c : a\,b :: i\,o : y\,z.$$

If $y\,z$ were a visible line such as a fence or road, we could from a divide it into any required portions. Thus, if we wish to place a stake halfway between y and z, first place one halfway between i and o; then standing at a, by means of signals, place a person in range on $y\,z$. Or, to find along $a\,b$, a point t perp to $y\,z$ at y, first make $o\,i\,s = 90°$; and measure $a\,s$. Then, as

$$o\,i : a\,s :: y\,z : a\,t.$$

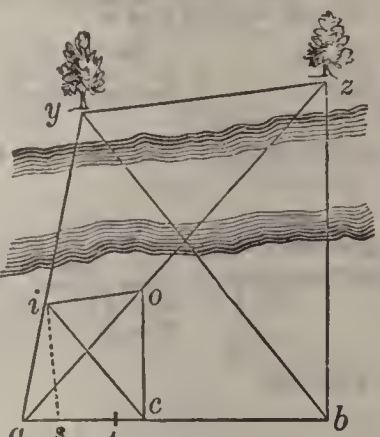


Fig. 18.

Ex. 11. To find the position of a point, n, Fig 19,

By means of two angles a n b *and* b n c, *taken from it to the three objects* a b c, *whose positions and dists apart are known.*

The use of this problem is more frequent in marine than in land surveying. It is chiefly employed for determining the position n of a boat from which soundings are being taken along a coast. As the boat moves from point to point to take fresh soundings, it becomes necessary to make a fresh observation at each point, in order to define its position on the chart. An observation consists in the measurement by a sextant of the two angles $a\,n\,b$, $b\,n\,c$, to the signals $a\,b\,c$, previously arranged on the shore. When practicable, this method should be rejected; and the observations taken to the boat at the same instant, by two observers on shore, at two of the stations. The boat to show a signal at the proper moment. The most expeditious mode of fixing the point n upon the map, is to draw three lines, forming the two angles, and extended indefinitely, on a piece of transparent paper. Place the paper upon the map, and move it about until the three lines pass through the three stations; then prick through the point n wherever it happens to come.

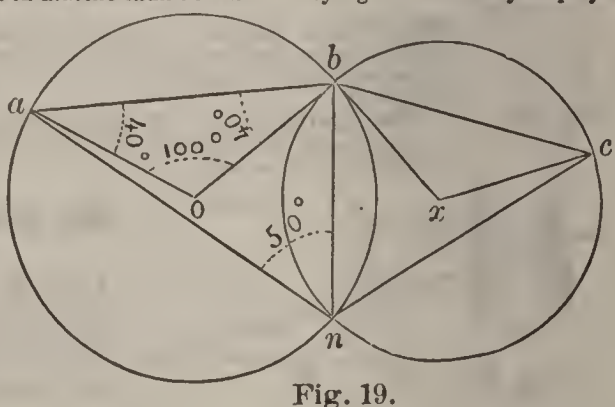


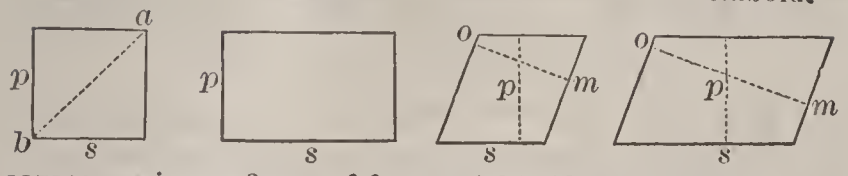
Fig. 19.

Instead of the transparent paper, an instrument called a *station pointer* may be used when there are many points to be fixed.

But the position of the point n can be found more correctly by describing two circles, as in Fig 19, each of which shall pass through n and two of the station points. The question is to find the centers o and x of two such circles. This is very simple. We know that the angle $a\,o\,b$ at the center of a circle is twice as great as any angle $a\,n\,b$ at the circumf of the same circle, when both are subtended by the same chord $a\,b$. Consequently, if the angle $a\,n\,b$, observed from the boat, is say, 50°, the angle $a\,o\,b$ must be 100°. And, since the three angles of every plane triangle are equal to 180°, the two angles $o\,a\,b$ and $o\,b\,a$ are together equal to $180° - 100° = 80°$. And, since the two sides $a\,o$ and $b\,o$ are equal (being radii of the same circle), therefore, the angles $o\,a\,b$ and $o\,b\,a$ are equal; and each equal to $\frac{80°}{2} = 40°$. Consequently, on the map we have only to lay down at a and b, two angles of 40°; the point o of intersection will be the center of the circle $a\,b\,n$. Proceed in the same way with the angle $b\,n\,c$, to find the center x. Then the interseetion of the two circles at n will be the point sought.

PARALLELOGRAMS.

Square. Rectangle. Rhombus. Rhomboid.



A PARALLELOGRAM is any figure of four straight sides, the opposite ones of which are parallel. There are but four, as in the above figs. The rhombus, like the rhombohedron, Fig 3, p 155, is sometimes called "rhomb." In the square and rhombus all the four sides are equal; in the rectangle and rhomboid only the opposite ones are equal. In any parallelogram the four angles amount to four right angles, or 360°; and any two diagonally opposite angles are equal to each other; hence, having one angle given, the other three can readily be found. In a square, or a rhombus, a diag divides each of two angles into two equal parts; but in the two other parallelograms it does not.

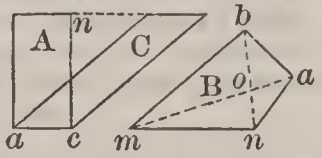
To find the area of any parallelogram.

Multiply any side, as S, by the perp height, or dist p to the opposite side. **Or,** multiply together two sides and nat sine of their included angle.

The diag a b of any square is equal to one side mult by 1.41421; and a side is equal to diagonal $\div$ 1.41421; **or,** to diag mult by .707107.

The side of a square equal in area to a given circle, is equal to diam × .886227.

The side of the greatest square, *that can be inscribed in a given circle,* is equal to diam × .707107.

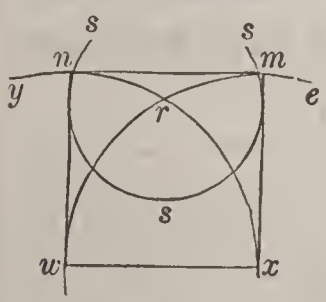


The side of a square mult by 1.51967 gives the **side of an equilateral triangle** of the same area. **All parallelograms** as A and C, **which have equal bases,** $a\ c$, and equal perp heights $n\ c$, have also equal areas; and the area of each is twice that of a triangle having the same base, and perp height. **The area of a square inscribed in a circle** is equal to twice the square of the rad.

In every parallelogram, the 4 squares drawn on its sides have a united area equal to that of the two squares drawn on its 2 diags. **If a larger square** be drawn on the diag $a\ b$ of a smaller square, its area will be twice that of said smaller square. **Either diag of any parallelogram** divides it into two equal triangles, and the 2 diags div it into 4 triangles of equal areas. **The two diags of any parallelogram** divide each other into two equal parts. **Any line drawn through the center of a diag** divides the parallelogram into two equal parts.

Remark 1.—The area of any fig whatever as B **that is enclosed by four stralght lines,** may be found thus: Mult together the two diags $a\ m$, $n\ b$; and the nat sine of the least angle $a\ o\ b$; or $n\ o\ m$, formed by their intersection. Div the product by 2. This is useful in land surveying, when obstacles, as is often the case, make it difficult to measure the sides of the fig or field; while it may be easy to measure the diags; and after finding their point of intersection o, to measure the required angle. **But if the fig is to be drawn,** the **parts** $o\ a$, $o\ b$, $o\ n$, $o\ m$ of the diags must also be measd.

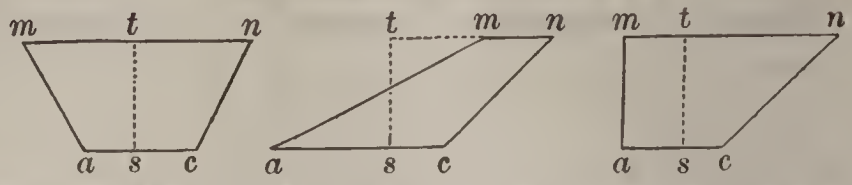
Rem. 2.—The sides of a parallelogram, triangle, and many other figs may be found, when only the area and angles are given, thus: Assume some particular one of its sides to be of the length 1; and calculate what its area would be if that were the case. Then as the sq rt of the area thus found is to this side 1, so is the sq rt of the actual given area, to the corresponding actual side of the fig.



On a given line $w\ x$, to draw a square, $w\ x\ n\ m$.

From w and x, with rad $w\ x$, describe the arcs $x\ r\ y$ and $w\ r\ e$. From their intersection r, and with rad equal to ½ of $w\ x$, describe $s\ s\ s$. From w and x draw $w\ n$ and $x\ m$ tangential to $s\ s\ s$, and ending at the other arcs; join $n\ m$.

TRAPEZOIDS.

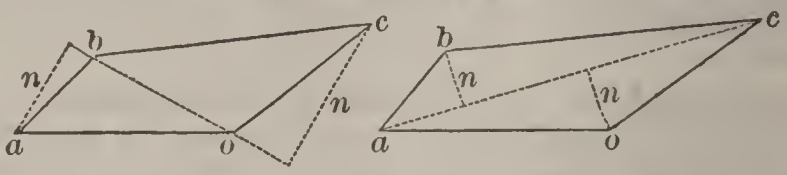


A trapezoid $a\ c\ n\ m$, is any figure with four straight sides, only two of which, as ac and $n\ m$, are parallel.

To find the area of any trapezoid.

Add together the two parallel sides, $a\ c$ and $m\ n$; mult the sum by the perp dist $s\ t$ between them; div the prod by 2. See the following rules for trapeziums, which are all equally applicable to trapezoids; also see Remarks after Parallelograms.

TRAPEZIUMS.



A trapezium $a\ b\ c\ o$, is any fig with four straight sides, of which no two are parallel.

To find the area of any trapezium, having given the diag $b\ o$, or $a\ c$, between either pair of opposite angles; and also the two perps, n, n, from the other two angles.

Add together these two perps; mult the sum by the diag; div the prod by 2.

Having the four sides; and either pair of opposite angles, as $a\ b\ c$, $a\ o\ c$; or $b\ a\ o$, and $b\ c\ o$.

Consider the trapezium as divided into two triangles, in each of which are given two sides and the included angle. Find the area of each of these triangles as directed under the preceding head "Triangles," and add them together.

Having the four angles, and either pair of opposite sides.

Begin with one of the sides, and the two angles at its ends. If the sum of these two angles exceeds 180°, subtract each of them from 180°, and make use of the rems instead of the angles themselves. Then consider this side and its two adjacent angles (or the two rems, as the case may be) as those of a triangle; and find its area as directed for that case under the preceding head "Triangle." Do the same with the other given side, and its two adjacent angles, (or their rems, as the case may be.) Subtract the least of the areas thus found, from the greatest; the rem will be the reqd area.

Having three sides; and the two included angles.

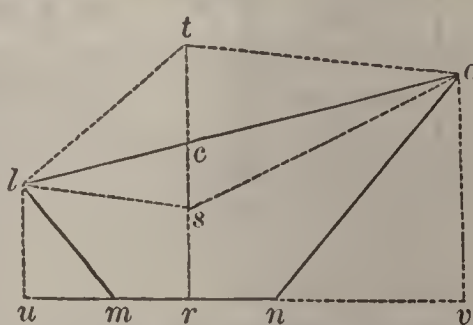
Mult together the middle side, and one of the adjacent sides; mult the prod by the nat sine of their included angle; call the result a. Do the same with the middle side and its other adjacent side, and the nat sine of the other included angle; call the result b. Add the two angles together; find the diff between their sum and 180°, whether greater or less; find the nat sine of this diff; mult together the two given sides which are *opposite* one another; mult the prod by the nat sine just found; call the result c. Add together the results a and b; then, if the sum of the two given angles is *less* than 180°, subtract c from the sum of a and b; *half* the rem will be the area of the trapezium. But if the sum of the two given angles be *greater* than 180°, add together the three results a, b, and c: *half* their sum will be the area.

Having the two diagonals, and either angle formed by their intersection.

See Remarks after Parallelograms, p 119.

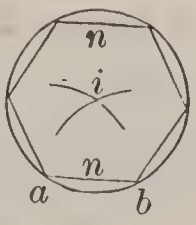
In railroad measurements

Of excavation and embaukment, the trapezium $l\ m\ n\ o$ frequently occurs; as well as the two 5-sided figures $l\ m\ n\ o\ t$ and $l\ m\ n\ o\ s$; in all of which $m\ n$ represents the roadway; $r\ s$, r c, and $r\ t$ the center-depths or heights; $l\ u$ and o v the side-depths or heights, as given by the level; $l\ m$ and $n\ o$ the side-slopes.



The same general rule for area applies to all three of these figs; namely, mult the extreme hor width $u\ v$ by *half* the center depth $r\ s$, r c, or $r\ t$, as the case may be. Also mult one *fourth* of the width of roadway $m\ n$, by the *sum of* the two side-depths $l\ u$ and $o\ v$. Add the two prods together; the sum is the reqd area. This rule applies whether the two side-slopes $m\ l$ and $n\ o$ have the same angle of inclination or not. **In railroad work, etc.,** the mid-way hor width, center depth, and side depths (see page 161) of a prismoid are respectively = The half sums of the corresponding end ones, and thus can be found without actual measurement.

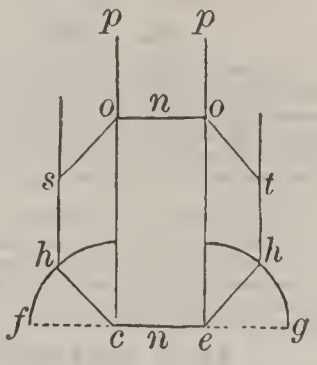
To draw a hexagon, each side of which shall be equal to a given line, *a b*.



From *a* and *b*, with rad *a b*, describe the two arcs; from their intersection, *i*, with the same rad, describe a circle; around the circumf of which, step off the same rad.

Side of a hexagon = *n n* × .57735.

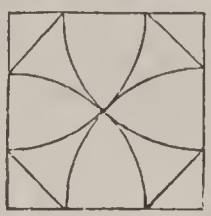
To draw an octagon, with each side equal to a given line, *c e*.



From *c* and *e* draw two perps, *c p*, *e p*. Also prolong *c e* toward *f* and *g*; and from *c* and *e*, with rad equal *c e*, draw the two quadrants; and find their centers *h h*; join *c h*, and *e h*; draw *h s* and *h t* parallel to *c p*; and make each of them equal to *c e*; make c o, and *e* o, each equal to *h h*; join o o, o *s*, and o *t*.

Side of an octagon = *n n* × .41421354.

To draw an octagon in a given square.

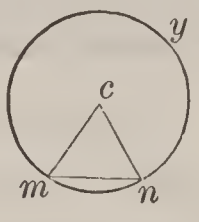


From each corner of the square, and with a rad equal to half its diag, describe the four arcs; and join the points at which they cut the sides of the square.

To draw any regular polygon, with each side equal to *m n*.

Div 360 degrees by the number of sides; take the quot from 180°; div the rem by 2. This will give the angle *c m n*, or *c n m*. At *m* and *n* lay down these angles by a protractor: the sides of these angles will meet at a point, c, from which describe the circle *m n y*; and around its circumf step off dists equal to *m n*.

In any circle, *m n y*, to draw any regular polygon.



Div 360° by the number of sides; the quot will be the angle *m c n*, at the center. Lay off this angle by a protractor; and its chord *m n* will be one side; which step off around the circumf.

To reduce any polygon, as *a b c d e f a*, to a triangle of the same area.

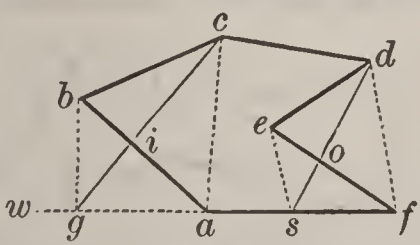


Fig. 1.

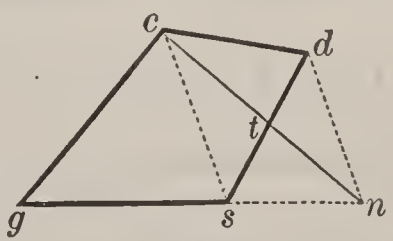
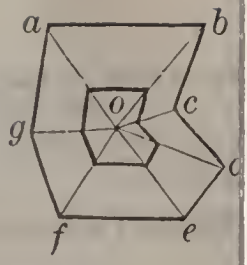


Fig. 2.

If we produce the side *f a* toward *w*; and draw *b g* parallel to *a c*, and join *g* e, we get equal triangles *a c b*, and *a c g*, both on the same base *a c*; and both of the same perp height, inasmuch as they are between the two parallels *a c* and *g b*. But the part *a c i* forms a portion of both these triangles, or in other words, *is common to both*. Therefore, if it be taken away from both triangles, the remaining parts, *i* e *b* of one of them, and *i g a* of the other, are also equal. Therefore, if the part *i c b* be left off from the polygon, and the part *i g a* be taken into it, the polygon *g f e d c i g* will have the same area as *a f e d c b a*; but it will have but five sides, while the other has six. Again, if *e s* be drawn parallel to *d f*, and *d s* joined, we have upon the same base *e s*, and between the same parallels *e s* and *d f*, the two equal triangles *e s d*, and *e s f*, with the part *e o s* common to both; and consequently the remaining part *e o d* of one, and *o s f* of the other, are equal. Therefore, if *o s f* be left off from the polygon, and *e o d* be taken into it, the new polygon *g s d c g*, Fig 2, will have the same area as *g f e d c g*; but it has but four sides, while the other has five. Finally, if *g s*, Fig 2, be extended toward *n*; and *d n* drawn parallel to *e s*; and *c n* joined, we have on the same base *c s*, and between the same parallels *c s* and *d n*, the two equal triangles *c s n*, and *c s d*, with the part *c s t* common to both. Therefore, if we leave out *c d t*, and take in *s t n*, we have the triangle *g n c* equal to the polygon *g s d c g*, Fig 2; or to *a f e d c b a*, Fig 1.

This simple method is applicable to polygons of any number of sides.



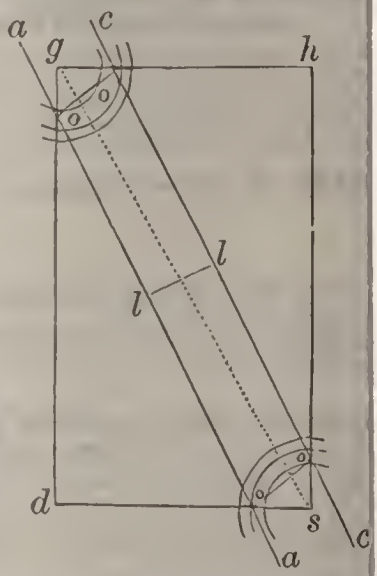
To reduce a large fig, *a b c d e f g*, to a smaller similar one.

From any interior point o, which had better be near the center, draw lines to all the angles *a*, *b*, c, &c. Join these lines by others parallel to the sides of the fig. If it should be reqd to enlarge a small fig, draw, from any point *o* within it, lines extending beyond its angles; and join these lines by others parallel to the sides of the small fig.

To reduce a map to one on a smaller scale.

The best method is by dividing the large map into squares by faint lines, with a very soft lead pencil, and then drawing the reduced map upon a sheet of smaller squares. A pair of proportional dividers will assist much in fixing points intermediate of the sides of the squares. If the large map would be injured by drawing, and rubbing out the squares, threads may be stretched across it to form the squares.

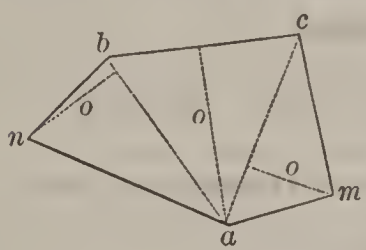
Maps, plans, and drawings of all kinds, are now copied, reduced, enlarged, and multiplied, cheaply and expeditiously, by photography. Most of the newer illustrations in this work are from electrotypes made by the "wax process" of Struthers, Servoss & Co., 34 New Chambers St, New York, and American Bank Note Co, 78 Trinity Place, New York.



In a rectangular fig, *g h s d*,

Representing an open panel, to find the points o o o o in its sides; and at equal dists from the angles *g*, and *s*; for inserting a diag piece o o o o, of a given width *l l*, measured at right angles to its length. From *g* and *s* as centers, describe several concentric arcs, as in the Fig. Draw upon transparent paper, two parallel lines *a a*, c c, at a distance apart equal to *l l*; and placing these lines on top of the panel, move them about until it is shown by the arcs that the four dists *g* o, *g* o, *s* o, *s* o, are equal. Instead of the transparent paper, a strip of common paper, of the width *l l* may be used.

Rem. Many problems which would otherwise be very difficult, may be thus solved with an accuracy sufficient for practical purposes, **by means of transparent paper.**

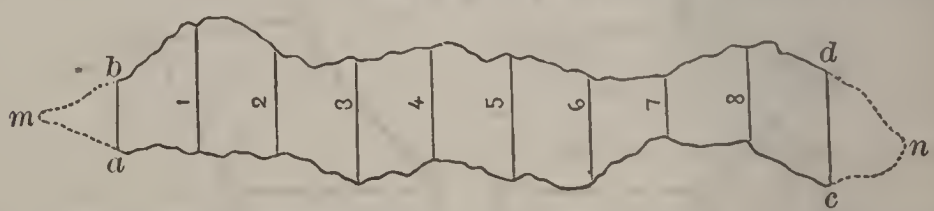


To find the area of any irregular polygon, *a n b c m*.

Div it into triangles, as *a n b*, *a m* c, and *a b* c; in each of which find the perp dist o, between its base *a b*, *a* c, or *b* c; and the opposite angle *n*, *m*, or *a*; mult each base by its perp dist add all the prods together; div by 2.

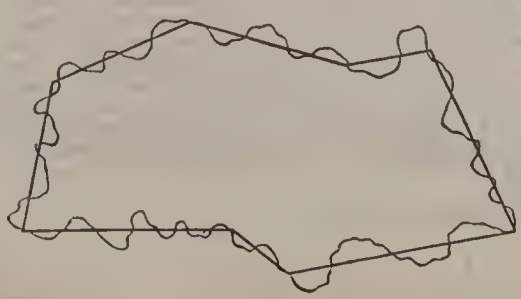
To find approx the area of a long irregular fig, as *a b c d*.

Between its ends *a b*, c *d*



space off *equal* dists, (the shorter they are the more accurate will be the result,) through whic draw the intermediate parallel lines 1, 2, 3, &c, across the breadth of the fig. Measure the length of these intermediate lines; add them together; to the sum add *half* the sum of the two end breadth *a b* and c *d*. Mult the entire sum by one of the equal spaces between the parallel lines. The pro will be the area. This rule answers as well if either one or both the ends terminate in points, as at *m* and *n*. In the last of these cases, both *a b* and c *d* will be included in the intermediate lines; an half the two end breadths will be 0, or nothing.

To find the area of a fig whose outline is extremely irregular.



Draw lines around it which shall enclose within them (as nearly as can be judged by eye) as muc space not belonging to the fig, as they exclude spac belonging to it. The area of the simplified fig thu formed, being in this manner rendered equal to tha of the complicated one, may be calculated by dividin it into triangles, &c. By using a piece of fine thread the proper position for the new boundary lines ma be found, before drawing them in. Small irregula areas may be found from a drawing, by laying upo it a piece of transparent paper carefully ruled int small squares, each of a given area, say 10, 20, o 100 sq ft each; and by first counting the whol squares, and then adding the fractions of squares

CIRCLES.

A circle is the area included within a curved line of such a character that every point in it is equally distant from a certain point within it, called its center. The curved line itself is called the circumference, or periphery of the circle; or very commonly it is called the circle.

To find the circumference.

Mult diam by 3.1416, which gives too much by only .148 of an inch in a mile. **Or,** as 113 is to 355 so is diam to circumf; too great 1 inch in 186 miles. **Or,** mult diam by $3\frac{1}{7}$; too great by about 1 part in 2485. **Or,** mult area by 12.566, and take sq root of prod. **Or,** use tables pp. 125 &c. The Greek letter π, also **p**, is used by writers to denote this 3.1416; and $\mathbf{p}^2 = 9.86960$.

To find the diam.

Div the circumf by 3.1416; or, as 355 is to 113, so is circumf to diam; or, mult the circumf. by 7; and div the prod by 22, which gives the diam too small by only about one part in 2485; or, mult the area by 1.2732; and take the sq rt of the prod; or use tables of circles, pp 125, &c.

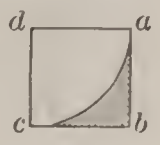
The diam is to the circumf more exactly as 1 to 3.14159265.

To find the area of a circle.

Square the diam; mult this square by .7854; or more accurately by .78539816; or square the circumf; mult this square by .07958; or more accurately by .07957747; or mult half the diam by half the circumf; or refer to the following table of areas of circles. Also area = sq of rad × 3.1416.

The area of a circle is to the area of any circumscribed straight-sided fig, as the circumf of the circle is to the circumf or periphery of the fig. The area of a square inscribed in a circle, is equal to twice the square of the rad. Of a circle in a square, = square × .7854.

It is convenient to remember, in rounding off a square corner *a b c*, by a quarter of a circle, that the shaded area *a b c* is equal to about $\frac{1}{5}$ part (correctly .2146) of the whole square *a b c d*.



For tables of circumferences and areas of circles, see pages 125 to 140.

To find the diam of a circle equal in area to a given square.

Mult one side of the square by 1.12838.

To find the rad of a circle to circumscribe a given square.

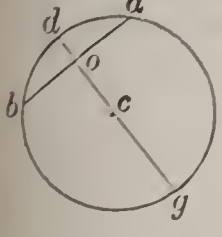
Mult one side by .7071; or take ½ the diag.

To find the side of a square equal in area to a given circle.

Mult the diam by .88623.

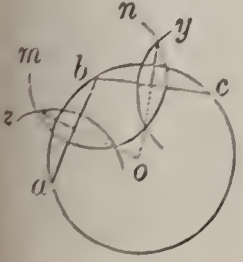
To find the side of the greatest square in a given circle.

Mult diam by .7071. The area of the greatest square that can be inscribed in a circle is equal to twice the square of the rad. The diam × by 1.3468 gives the side of an equilateral triangle of equal area.



To find the center *c*, of a given circle.

Draw any chord *a b*; and from the middle of it *o*, draw at right angles to it, a diam *d g*; find the center c of this diam.

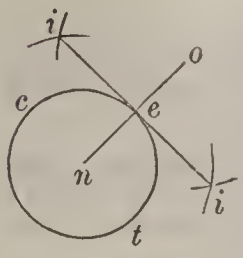


To describe a circle through any three points, *a b c*, not in a straight line.

Join the points by the lines *a b*, *b c*; from the centers of these lines draw the dotted perps meeting, as at *o*, which will be the center of the circle. Or from *b*, with any convenient rad. draw the arc *m n*; and from *a* and c, with the same rad, draw arcs *y* and *z*; then two lines drawn through the intersections of these arcs, will meet at the center *o*.

To describe a circle to touch the three angles of a triangle is plainly the same as this.

To inscribe a circle in a triangle draw two lines bisecting any two of the angles. Where these lines meet is the center of the circle.

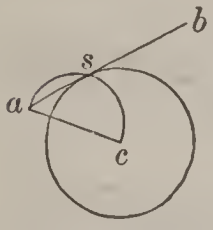


To draw a tangent, *i e i*, to a circle, from any given point, *e*, in its circumf.

Through the center *n*, and the given point *e*, draw *n* o; make *e* o equal to *e n*; from *n* and *o*, with any rad greater than half of o *n*, describe the two pairs of arc *i i*; join their intersections *i i*.

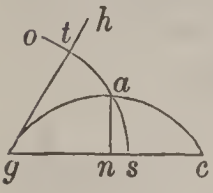
Here, and in the following three figs, the *tangents* are *ordinary* or *geometrical* ones; and may end where we please. But the *trigonometrical* tangent of a given *angle*, must end in a secant, as in the top fig of p 59.

Or from *e* lay off two equal distances *e c*, *e t*; and draw *i i* parallel to c *t*.



To draw a tang, *a s b*, to a circle, from a point, *a*, which is outside of the circle.

Draw *a* c, and on it describe a semicircle; through the intersection, *s*, draw *a s b*. Here c is the center of the circle.

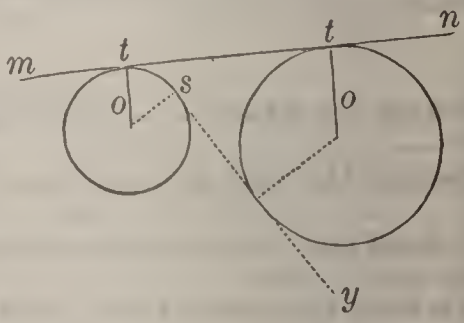


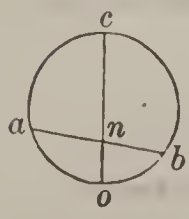
To draw a tang, *g h*, from a circular arc, *g a c*, Of which *n a* is the rise. With rad *g a*, describe an arc, *s a* o. Make *t a* equal to *s a*. Through *t* draw *g h*.

To draw a tang to two circles.

First draw the line *m n*, just touching the two circles; this gives the *direction* of the tang. Then from the centers of the circles draw the radii. o o, perp to *m n*. The points *t t* are the tang points. If the tang is in the position of the dotted line, *s y*, the operation is the same.

Rem. This empirical method is at least *as accurate* as the scientific ones, especially if a correct triangular ruler is used for the radii.





If any two chords, as a b, o c, cross each other, then as *o n* : *n b* : : *a n* : *n c*. Hence, *n b* × *a n* = *o n* × *n c*. That is, the product of the two parts of one of the lines, is = the product of the two parts of the other line.

TABLE 1 OF CIRCLES.

Diameters in units and eighths, &c.

Circumferences or areas intermediate of those in this table, may be found by simple arithmetical proportion. No errors.

Diam.	Circumf.	Area.	Diam.	Circumf.	Area.	Diam.	Circumf.	Area.	Diam.	Circumf.	Area.
1-64	.019087	.00019	3. ½	10.9956	9.6211	10⅛	31.8086	80.516	19¼	60.4757	291.04
1-32	.098175	.00077	9-16	11.1919	9.9678	¼	32.2013	82.516	⅜	60.8684	294.83
3-64	.147262	.00173	⅝	11.3883	10.321	⅜	32.5940	84.541	½	61.2611	298.65
1-16	.196350	.00307	11-16	11.5846	10.680	½	32.9867	86.590	⅝	61.6538	302.49
3-32	.294524	.00690	¾	11.7810	11.045	⅝	33.3794	88.664	¾	62.0465	306.35
⅛	.392699	.01227	13-16	11.9773	11.416	¾	33.7721	90.763	⅞	62.4392	310.24
5-32	.490874	.01917	⅞	12.1737	11.793	⅞	34.1648	92.886	20.	62.8319	314.16
3-16	589049	.02761	15-16	12.3700	12.177	11.	34.5575	95.033	⅛	63.2246	318.10
7-32	.687223	.03758	**4.**	12.5664	12.566	⅛	34.9502	97.205	¼	63.6173	322.06
¼	.785398	.04909	1-16	12.7627	12.962	¼	35.3429	99.402	⅜	64.0100	326.05
9-32	.883573	.06213	⅛	12 9591	13.361	⅜	35.7356	101.62	½	64.4026	330.06
5-16	.981748	.07670	3-16	13.1554	13.772	½	36.1283	103.87	⅝	64.7953	334.10
11-32	1.07992	.09281	¼	13.3518	14.186	⅝	36.5210	106.14	¾	65.1880	338.16
⅜	1.17810	.11045	5-16	13.5481	14.607	¾	36.9137	108.43	⅞	65.5807	342.25
13-32	1.27627	.12962	⅜	13.7445	15.033	⅞	37.3064	110.75	21.	65.9734	346.36
7-16	1.37445	.15033	7-16	13 9408	15.466	12.	37.6991	113.10	⅛	66.3661	350.50
15-32	1.47262	.17257	½	14.1372	15.904	⅛	38.0918	115.47	¼	66.7588	354.66
½	1.57080	.19635	9-16	14.3335	16.349	¼	38.4845	117.86	⅜	67.1515	358.84
17-32	1.66897	.22165	⅝	14.5299	16.800	⅜	38.8772	120.28	½	67.5442	363.05
9-16	1.76715	.24850	11-16	14.7262	17.257	½	39.2699	122.72	⅝	67.9369	367.28
19-32	1.86532	.27688	¾	14.9226	17.721	⅝	39.6626	125.19	¾	68.3296	371.54
⅝	1.96350	.30680	13-16	15.1189	18.190	¾	40.0553	127.68	⅞	68.7223	375.83
21-32	2.06167	.33824	⅛	15.3153	18.665	⅞	40.4480	130.19	22.	69.1150	380.13
11-16	2.15984	.37122	15-16	15.5116	19.147	13.	40.8407	132.73	⅛	69.5077	384.46
23-32	2.25802	.40574	**5.**	15.7080	19.635	⅛	41.2334	135.30	¼	69.9004	388.82
¾	2.35619	.44179	1-16	15.9043	20.129	¼	41.6261	137.89	⅜	70.2931	393.20
25-32	2.45437	.47937	⅛	16.1007	20.629	⅜	42.0188	140.50	½	70.6858	397.61
13-16	2.55254	.51849	3-16	16.2970	21.135	½	42.4115	143.14	⅝	71.0785	402.04
27-32	2.65072	.55914	¼	16.4934	21.648	⅝	42.8042	145.80	¾	71.4712	406.49
⅞	2.74889	.60132	5-16	16.6897	22.166	¾	43.1969	148.49	⅞	71.8639	410.97
29-32	2.84707	.64504	⅜	16.8861	22.691	⅞	43.5896	151.20	23.	72.2566	415.48
15-16	2.94524	.69029	7-16	17.0824	23.221	14.	43.9823	153.94	⅛	72.6493	420.00
31-32	3.04342	.73708	½	17.2788	23.758	⅛	44.3750	156.70	¼	73.0420	424.56
1.	3.14159	.78540	9-16	17.4751	24.301	¼	44.7677	159.48	⅜	73.4347	429.13
1-16	3.33794	.88664	⅝	17.6715	24.850	⅜	45.1604	162.30	½	73.8274	433.74
⅛	3.53429	.99402	11-16	17.8678	25.406	½	45.5531	165.13	⅝	74.2201	438.36
3-16	3.73064	1.1075	¾	18.0642	25.967	⅝	45.9458	167.99	¾	74.6128	443.01
¼	3.92699	1.2272	13-16	18.2605	26.535	¾	46.3385	170.87	⅞	75.0055	447.69
5-16	4.12334	1.3530	⅞	18.4569	27.109	⅞	46.7312	173.78	24.	75.3982	452 39
⅜	4 31969	1.4849	15-16	18.6532	27.688	15.	47.1239	176.71	⅛	75.7909	457.11
7-16	4.51604	1.6230	19.8496	28.274	⅛	47.5166	179.67	¼	76.1836	461.86
½	4.71239	1.7671	**6.**	19.2423	29.465	¼	47.9093	182.65	⅜	76.5763	466.64
9-16	4.90874	1.9175	⅛	19.6350	30.680	⅜	48.3020	185.66	½	76.9690	471.44
⅝	5.10509	2.0739	¼	20.0277	31.919	½	48.6947	188.69	⅝	77.3617	476.26
11-16	5.30144	2.2365	⅜	20.4204	33.183	⅝	49.0874	191.75	¾	77.7544	481.11
¾	5.49779	2.4053	½	20.8131	34.472	¾	49.4801	194.83	⅞	78 1471	485.98
13-16	5.69414	2.5802	⅝	21.2058	35.785	⅞	49.8728	197.93	25.	78.5398	490.87
⅞	5.89049	2.7612	¾	21.5984	37.122	16.	50.2655	201.06	⅛	78.9325	495.79
15-16	6.08684	2.9483	⅞	21.9911	38.485	⅛	50.6582	204.22	¼	79.3252	500.74
2.	6.28319	3.1416	7.	22.3838	39.871	¼	51.0509	207.39	⅜	79.7179	505.71
1-16	6.47953	3.3410	⅛	22.7765	41.282	⅜	51.4436	210.60	½	80.1106	510.71
⅛	6.67588	3.5466	¼	23.1692	42.718	½	51.8363	213.82	⅝	80.5033	515.72
3-16	6 87223	3.7583	⅜	23.5619	44.179	⅝	52.2290	217.08	¾	80.8960	520.77
¼	7.06858	3.9761	½	23.9546	45 664	¾	52.6217	220.35	⅞	81.2887	525.84
5 16	7.26493	4.2000	⅝	24.3473	47.173	⅞	53.0144	223.65	26.	81.6814	530.93
⅜	7.46128	4.4301	¾	24.7400	48.707	17.	53.4071	226.98	⅛	82.0741	536.05
7-16	7.65763	4.6664	⅞	25.1327	50.265	⅛	53.7998	230.33	¼	82.4668	541.19
½	7.85398	4 9087	8.	25.5254	51.849	¼	54.1925	233.71	⅜	82.8595	546.35
9-16	8.05033	5.1572	⅛	25 9181	53.456	⅜	54.5852	237.10	½	83.2522	551.55
⅝	8.24668	5.4119	¼	26.3108	55.088	½	54.9779	240.53	⅝	83.6449	556.76
11-16	8.44303	5.6727	⅜	26.7035	56.745	⅝	55.3706	243.98	¾	84.0376	562.00
¾	8.63938	5.9396	½	27.0962	58.426	¾	55.7633	247.45	⅞	84.4303	567.27
13-16	8.83573	6.2126	⅝	27.4889	60.132	⅞	56.1560	250.95	27.	84.8230	572.56
⅞	9.03208	6.4918	¾	27.8816	61.862	18.	56.5487	254.47	⅛	85.2157	577.87
15-16	9.22843	6.7771	9.	28.2743	63.617	⅛	56.9414	258.02	¼	85.6084	583.21
3.	9.42478	7.0686	⅛	28.6670	65.397	¼	57.3341	261.59	⅜	86.0011	588.57
1-16	9.62113	7.3662	¼	29.0597	67.201	⅜	57.7268	265.18	½	86.3938	593.96
⅛	9.81748	7.6699	⅜	29.4524	69.029	½	58.1195	268.80	⅝	86.7865	599.37
3-16	10.0138	7.9798	½	29.8451	70.882	⅝	58.5122	272.45	¾	87.1792	604.81
¼	10.2102	8.2958	⅝	30.2378	72.760	¾	58.9049	276.12	⅞	87.5719	610.27
5-16	10.4065	8.6179	¾	30.6305	74 662	⅞	59.2976	279.81	28.	87.9646	615.75
⅜	10.6029	8 9462	⅞	31.0232	76.589	19.	59.6903	283.53	⅛	88.3573	621.26
7-16	10.7992	9.2806	10.	31.4159	78.540	⅛	60.0830	287.27	¼	88.7500	626.80

TABLE 1 OF CIRCLES—(Continued).
Diameters in units and eighths, &c.

Diam.	Circumf.	Area.	Diam.	Circumf.	Area.	Diam.	Circumf.	Area.	Diam.	Circumf.	Area.
28⅜	89.1427	632.36	38.	119 381	1134.1	47⅝	149.618	1781.4	57¼	179.856	2574.2
½	89.5354	637.94	⅛	119.773	1141.6	¾	150.011	1790.8	⅜	180.249	2585.4
⅝	89.9281	643.55	¼	120.166	1149.1	⅞	150.404	1800.1	½	180.642	2596.7
¾	90.3208	649.18	⅜	120.559	1156.6	48.	150.796	1809.6	⅝	181.034	2608.0
⅞	90.7135	654.84	½	120.951	1164.2	⅛	151.189	1819.0	¾	181.427	2619.4
29.	91.1062	660.52	⅝	121.344	1171.7	¼	151.582	1828.5	⅞	181.820	2630.7
⅛	91.4989	666.23	¾	121.737	1179.3	⅜	151.975	1837.9	58.	182.212	2642.1
¼	91.8916	671.96	⅞	122.129	1186.9	½	152.367	1847.5	⅛	182.605	2653.5
⅜	92.2843	677.71	39.	122.522	1194.6	⅝	152.760	1857.0	¼	182.998	2664.9
½	92.6770	683.49	⅛	122.915	1202.3	¾	153.153	1866.5	⅜	183.390	2676.4
⅝	93.0697	689.30	¼	123.308	1210.0	⅞	153.545	1876.1	½	183.783	2687.8
¾	93.4624	695.13	⅜	123.700	1217.7	49.	153.938	1885.7	⅝	184.176	2699.3
⅞	93.8551	700.98	½	124.093	1225.4	⅛	154.331	1895.4	¾	184.569	2710.9
30.	94.2478	706.86	⅝	124.486	1233.2	¼	154.723	1905.0	⅞	184.961	2722.4
⅛	94.6405	712.76	¾	124.878	1241.0	⅜	155.116	1914.7	59.	185.354	2734.0
¼	95.0332	718.69	⅞	125.271	1248.8	½	155.509	1924.4	⅛	185.747	2745.6
⅜	95.4259	724.64	40.	125.664	1256.6	⅝	155.902	1934.2	¼	186.139	2757.2
½	95.8186	730.62	⅛	126.056	1264.5	¾	156.294	1943.9	⅜	186.532	2768.8
⅝	96.2113	736.62	¼	126.449	1272.4	⅞	156.687	1953.7	½	186.925	2780.5
¾	96.6040	742.64	⅜	126.812	1280.3	50.	157.080	1963.5	⅝	187.317	2792.2
⅞	96.9967	748.69	½	127.235	1288.2	⅛	157.472	1973.3	¾	187.710	2803.9
31.	97.3894	754.77	⅝	127.627	1296.2	¼	157.865	1983.2	⅞	188.103	2815.7
⅛	97.7821	760.87	¾	128.020	1304.2	⅜	158.258	1993.1	60.	188.496	2827.4
¼	98.1718	766.99	⅞	128.413	1312.2	½	158.650	2003.0	⅛	188.888	2839.2
⅜	98.5675	773.14	41.	128.805	1320.3	⅝	159.043	2012.9	¼	189.281	2851.0
½	98.9602	779.31	⅛	129 198	1328.3	¾	159.436	2022.8	⅜	189.674	2862 9
⅝	99.3529	785.51	¼	129.591	1336.4	⅞	159.829	2032.8	½	190.066	2874.8
¾	99.7456	791.73	⅜	129.983	1344.5	51.	160.221	2042.8	⅝	190.459	2886.6
⅞	100.138	797.98	½	130.376	1352.7	⅛	160.614	2052.8	¾	190.852	2898.6
32.	100.531	804.25	⅝	130.769	1360.8	¼	161.007	2062.9	⅞	191.244	2910.5
⅛	100.924	810.54	¾	131.161	1369.0	⅜	161.399	2073.0	61.	191.637	2922.5
¼	101.316	816.86	⅞	131.554	1377.2	½	161.792	2083.1	⅛	192.030	2934.5
⅜	101.709	823.21	42.	131.947	1385.4	⅝	162.185	2093.2	¼	192.423	2946.5
½	102.102	829.58	⅛	132.340	1393.7	¾	162.577	2103.3	⅜	192.815	2958.5
⅝	102.494	835.97	¼	132.732	1402.0	⅞	162.970	2113.5	½	193.208	2970.6
¾	102.887	842.39	⅜	133.125	1410.3	52.	163.363	2123.7	⅝	193.601	2982.7
⅞	103.280	848.83	½	133.518	1418.6	⅛	163.756	2133.9	¾	193.993	2994.8
33.	103.673	855.30	⅝	133.910	1427.0	¼	164.148	2144.2	⅞	194.386	3006.9
⅛	104.065	861.79	¾	134.303	1435.4	⅜	164.541	2154.5	62.	194.779	3019.1
¼	104.458	868.31	⅞	134.696	1443.8	½	164.934	2164.8	⅛	195.171	3031.3
⅜	104.851	874.85	43.	135.088	1452.2	⅝	165.326	2175.1	¼	195.564	3043.5
½	105.243	881.41	⅛	135.481	1460.7	¾	165.719	2185.4	⅜	195.957	3055.7
⅝	105.636	888.00	¼	135.874	1469.1	⅞	166.112	2195.8	½	196.350	3068.0
¾	106.029	894.62	⅜	136.267	1477.6	53.	166.504	2206.2	⅝	196.742	3080.3
⅞	106.421	901.26	½	136.659	1486.2	⅛	166.897	2216.6	¾	197.135	3092.6
34.	106.814	907.92	⅝	137.052	1494.7	¼	167.290	2227.0	⅞	197.528	3104.9
⅛	107.207	914.61	¾	137.415	1503.3	⅜	167.683	2237.5	63.	197.920	3117.2
¼	107.600	921.32	⅞	137.837	1511.9	½	168.075	2248.0	⅛	198.313	3129.6
⅜	107.992	928.06	44.	138.230	1520.5	⅝	168.468	2258.5	¼	198.706	3142.0
½	108.385	934.82	⅛	138.623	1529.2	¾	168.861	2269.1	⅜	199.098	3154.5
⅝	108.778	941.61	¼	139.015	1537.9	⅞	169.253	2279.6	½	199.491	3166.9
¾	109.170	948.42	⅜	139.408	1546.6	54.	169.616	2290.2	⅝	199.884	3179.4
⅞	109.563	955.25	½	139.801	1555.3	⅛	170.039	2300.8	¾	200.277	3191.9
35.	109.956	962.11	⅝	140.194	1564.0	¼	170.431	2311.5	⅞	200.669	3204.4
⅛	110.348	969.00	¾	140 586	1572.8	⅜	170.824	2322.1	64.	201.062	3217.0
¼	110.741	975.91	⅞	140.979	1581.6	½	171.217	2332.8	⅛	201.455	3229.6
⅜	111.134	982.84	45.	141.372	1590.4	⅝	171.609	2343.5	¼	201.847	3242.2
½	111.527	989.80	⅛	141.764	1599.3	¾	172.002	2354.3	⅜	202.240	3254.3
⅝	111.919	996.78	¼	142.157	1608.2	⅞	172.395	2365.0	½	202.633	3267.5
¾	112.312	1003.8	⅜	142.550	1617.0	55.	172.788	2375.8	⅝	203.025	3280.1
⅞	112.705	1010.8	½	142.942	1626.0	⅛	173.180	2386.6	¾	203.418	3292.8
36.	113.097	1017.9	⅝	143.335	1634.9	¼	173.573	2397.5	⅞	203.811	3305.6
⅛	113.490	1025.0	¾	143.728	1643.9	⅜	173.966	2408.3	65.	204.204	3318.3
¼	113.883	1032 1	⅞	144.121	1652.9	½	174.358	2419.2	⅛	204.596	3331.1
⅜	114.275	1039.2	46.	144.513	1661.9	⅝	174.751	2430.1	¼	204.989	3343.9
½	114.668	1046.3	⅛	144.906	1670.9	¾	175.144	2441.1	⅜	205.382	3356.7
⅝	115.061	1053.5	¼	145.299	1680.0	⅞	175.536	2452.0	½	205.774	3369.6
¾	115.454	1060.7	⅜	145.691	1689.1	56.	175.929	2463.0	⅝	206.167	3382.4
⅞	115.846	1068 0	½	146.084	1698.2	⅛	176.322	2474.0	¾	206.560	3395.3
37.	116.239	1075.2	⅝	146.477	1707.4	¼	176.715	2485.0	⅞	206.952	3408.2
⅛	116.632	1082.5	¾	146.869	1716.5	⅜	177.107	2496.1	66.	207.345	3421.2
¼	117.024	1089.8	⅞	147.262	1725.7	½	177.500	2507.2	⅛	207.738	3431.2
⅜	117.417	1097.1	47.	147.655	1734.9	⅝	177.893	2518.3	¼	208.131	3447.2
½	117.810	1104.5	⅛	148.048	1744.2	¾	178.285	2529.4	⅜	208.523	3460.2
⅝	118.202	1111.8	¼	148.440	1753.5	⅞	178.678	2540.6	½	208.916	3473.2
¾	118.596	1119.2	⅜	148.833	1762.7	57.	179.071	2551.8	⅝	209.309	3486.3
⅞	118.988	1126.7	½	149.226	1772.1	⅛	179.463	2563.0	¾	209.701	3499.4

TABLE 1 OF CIRCLES—(Continued).
Diameters in units and eighths, &c.

Diam.	Circumf.	Area.	Diam.	Circumf.	Area.	Diam.	Circumf.	Area.	Diam.	Circumf.	Area.
⅛	210.094	3512.5	75¼	236.405	4447.4	83⅝	262.716	5492.4	92.	289.027	6647.6
67.	210.487	3525.7	⅜	236.798	4462.2	¾	263.108	5508.8	⅛	289.419	6665.7
⅛	210.879	3538.8	½	237.190	4477.0	⅞	263.501	5525.3	¼	289.812	6683.8
¼	211.272	3552.0	⅝	237.583	4491.8	84.	263.894	5541.8	⅜	290.205	6701.9
⅜	211.665	3565.2	¾	237.976	4506.7	⅛	264.286	5558.3	½	290.597	6720.1
½	212.058	3578.5	⅞	238.368	4521.5	¼	264.679	5574.8	⅝	290.990	6738.2
⅝	212.450	3591.7	76.	238.761	4536.5	⅜	265.072	5591.4	¾	291.383	6756.4
¾	212.843	3605.0	⅛	239.154	4551.4	½	265.465	5607.9	⅞	291.775	6774.7
⅞	213.236	3618.3	¼	239.546	4566.4	⅝	265.857	5624.5	93.	292.168	6792.9
68.	213.628	3631.7	⅜	239.939	4581.3	¾	266.250	5641.2	⅛	292.561	6811.2
⅛	214.021	3645.0	½	240.332	4596.3	⅞	266.643	5657.8	¼	292.954	6829.5
¼	214.414	3658.4	⅝	240.725	4611.4	85.	267.035	5674.5	⅜	293.346	6847.8
⅜	214.806	3671.8	¾	241.117	4626.4	⅛	267.428	5691.2	½	293.739	6866.1
½	215.199	3685.3	⅞	241.510	4641.5	¼	267.821	5707.9	⅝	294.132	6884.5
⅝	215 592	3698.7	77.	241.903	4656.6	⅜	268.213	5724.7	¾	294.524	6902.9
¾	215.984	3712.2	⅛	242.295	4671.8	½	268.606	5741.5	⅞	294.917	6921.3
⅞	216.377	3725.7	¼	242.688	4686.9	⅝	268.999	5758.3	94.	295.310	6939.8
69.	216.770	3739.3	⅜	243.081	4702.1	¾	269.392	5775.1	⅛	295.702	6958.2
⅛	217.163	3752.8	½	243.473	4717.3	⅞	269.784	5791.9	¼	296.095	6976.7
¼	217.555	3766.4	⅝	243.866	4732.5	86.	270.177	5808 8	⅜	296.488	6995.3
⅜	217.948	3780.0	¾	244.259	4747.8	⅛	270.570	5825.7	½	296.881	7013.8
½	218.341	3793.7	⅞	244.652	4763.1	¼	270.962	5842.6	⅝	297.273	7032.4
⅝	218.733	3807.3	78.	245.044	4778.4	⅜	271.355	5859.6	¾	297.666	7051.0
¾	219.126	3821.0	⅛	245.437	4793.7	½	271.748	5876.5	⅞	298.059	7069.6
⅞	219.519	3834.7	¼	245.830	4809.0	⅝	272.140	5893.5	95.	298.451	7088.2
70.	219.911	3848.5	⅜	246.222	4824.4	¾	272.533	5910.6	⅛	298.844	7106.9
⅛	220.304	3862.2	½	246.615	4839.8	⅞	272.926	5927.6	¼	299.237	7125.6
¼	220.697	3876.0	⅝	247.008	4855.2	87.	273.319	5944.7	⅜	299.629	7144.3
⅜	221.090	3889.8	¾	247.400	4870.7	⅛	273.711	5961.8	½	300.022	7163.0
½	221.482	3903.6	⅞	247.793	4886.2	¼	274.104	5978.9	⅝	300.415	7181.8
⅝	221.875	3917.5	79.	248.186	4901.7	⅜	274.497	5996.0	¾	300.807	7200.6
¾	222.268	3931.4	⅛	248.579	4917.2	½	274.889	6013.2	⅞	301.200	7219.4
⅞	222.660	3945.3	¼	248.971	4932.7	⅝	275.282	6030.4	96.	301.593	7238.2
71.	223.053	3959.2	⅜	249.364	4948.3	¾	275.675	6047.6	⅛	301.986	7257.1
⅛	223.446	3973.1	½	249.757	4963.9	⅞	276.067	6064.9	¼	302.378	7276 0
¼	223.838	3987.1	⅝	250.149	4979.5	88.	276.460	6082.1	⅜	302.771	7294.9
⅜	224.231	4001.1	¾	250.542	4995.2	⅛	276.853	6099.4	½	303.164	7313.8
½	224.624	4015.2	⅞	250.935	5010.9	¼	277.246	6116.7	⅝	303.556	7332.8
⅝	225.017	4029.2	80.	251.327	5026.5	⅜	277.638	6134.1	¾	303.949	7351.8
¾	225.409	4043.3	⅛	251.720	5042.3	½	278.031	6151.4	⅞	304.342	7370.8
⅞	225.802	4057.4	¼	252.113	5058.0	⅝	278.424	6168.8	97.	304.734	7389.8
72.	226.195	4071.5	⅜	252.506	5073.8	¾	278.816	6186.2	⅛	305.127	7408.9
⅛	226.587	4085.7	½	252.898	5089.6	⅞	279.209	6203.7	¼	305 520	7428.0
¼	226.980	4099.8	⅝	253.291	5105.4	89.	279.602	6221.1	⅜	305.913	7447.1
⅜	227.373	4114.0	¾	253.684	5121.2	⅛	279.994	6238.6	½	306.305	7466.2
½	227.765	4128.2	⅞	254.076	5137.1	¼	280.387	6256.1	⅝	306.698	7485.3
⅝	228.158	4142.5	81.	254.469	5153.0	⅜	280.780	6273.7	¾	307.091	7504.5
¾	228.551	4156.8	⅛	254.862	5168.9	½	281.173	6291.2	⅞	307.483	7523.7
⅞	228 944	4171.1	¼	255.254	5184.9	⅝	281.565	6308.8	98.	307.876	7543.0
73.	229.336	4185.4	⅜	255.647	5200.8	¾	281.958	6326.4	⅛	308.269	7562.2
⅛	229.729	4199.7	½	256.040	5216.8	⅞	282.351	6344.1	¼	308.661	7581.5
¼	230.122	4214 1	⅝	256.433	5232.8	90.	282.743	6361.7	⅜	309.054	7600.8
⅜	230.514	4228.5	¾	256.825	5248.9	⅛	283.136	6379.4	½	309.447	7620.1
½	230.907	4242.9	⅞	257.218	5264.9	¼	283.529	6397.1	⅝	309.840	7639.5
⅝	231.300	4257.4	82.	257.611	5281.0	⅜	283.921	6414.9	¾	310.232	7658.9
¾	231.692	4271.8	⅛	258.003	5297.1	½	284.314	6432.6	⅞	310.625	7678.3
⅞	232.085	4286.3	¼	258.396	5313.3	⅝	284.707	6450.4	99.	311.018	7697.7
74.	232.478	4300.8	⅜	258.789	5329.4	¾	285.100	6468.2	⅛	311.410	7717.1
⅛	232.871	4315.4	½	259.181	5345.6	⅞	285.492	6486.0	¼	311.803	7736.6
¼	233 263	4329.9	⅝	259.574	5361.8	91.	285.885	6503.9	⅜	312.196	7756.1
⅜	233.656	4344.5	¾	259.967	5378.1	⅛	286.278	6521.8	½	312.588	7775.6
½	234.049	4359.2	⅞	260.359	5394.3	¼	286.670	6539.7	⅝	312.981	7795.2
⅝	234.441	4373.8	83.	260.752	5410.6	⅜	287.063	6557.6	¾	313.374	7814.8
¾	234.834	4388.5	⅛	261.145	5426.9	½	287.456	6575.5	⅞	313.767	7834.4
⅞	235.227	4403.1	¼	261.538	5443.3	⅝	287.848	6593.5	100.	314.159	7854.0
75.	235.619	4417.9	⅜	261.930	5459 6	¾	288.241	6611.5			
⅛	236.012	4432.6	½	262.323	5476.0	⅞	288.634	6629.6			

TABLE 2 OF CIRCLES.

Diameters in units and tenths.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
0.1	.314159	.007854	6.3	19.79203	31.17245	12.5	39.26991	122.7185
.2	.628319	.031416	.4	20.10619	32.16991	.6	39.58407	124.6898
.3	.942478	.070686	.5	20.42035	33.18307	.7	39.89823	126.6769
.4	1.256637	.125664	.6	20.73451	34.21194	.8	40.21239	128.6796
.5	1.570796	.196350	.7	21.04867	35.25652	.9	40.52655	130.6981
.6	1.884956	.282743	.8	21.36283	36.31681	13.0	40.84070	132.7323
.7	2.199115	.384845	.9	21.67699	37.39281	.1	41.15486	134.7822
.8	2.513274	.502655	7.0	21.99115	38.48451	.2	41.46902	136.8478
.9	2.827433	.636173	.1	22.30531	39.59192	.3	41.78318	138.9291
1.0	3.141593	.785398	.2	22.61947	40.71504	.4	42.09734	141.0261
.1	3.455752	.950332	.3	22.93363	41.85387	.5	42.41150	143.1388
.2	3.769911	1.13097	.4	23.24779	43.00840	.6	42.72566	145.2672
.3	4.084070	1.32732	.5	23.56194	44.17865	.7	43.03982	147.4114
.4	4.398230	1.53938	.6	23.87610	45.36460	.8	43.35398	149.5712
.5	4.712389	1.76715	.7	24.19026	46.56626	.9	43.66814	151.7468
.6	5.026548	2.01062	.8	24.50442	47.78362	14.0	43.98230	153.9380
.7	5.340708	2.26980	.9	24.81858	49.01670	.1	44.29646	156.1450
.8	5.654867	2.54469	8.0	25.13274	50.26548	.2	44.61062	158.3677
.9	5.969026	2.83529	.1	25.44690	51.52997	.3	44.92477	160.6061
2.0	6.283185	3.14159	.2	25.76106	52.81017	.4	45.23893	162.8602
.1	6.597345	3.46361	.3	26.07522	54.10608	.5	45 55309	165.1300
.2	6.911504	3.80133	.4	26.38938	55.41769	.6	45.86725	167.4155
.3	7.225663	4.15476	.5	26.70354	56.74502	.7	46.18141	169.7167
.4	7.539822	4.52389	.6	27.01770	58.08805	.8	46.49557	172.0336
.5	7.853982	4.90874	.7	27.33186	59.44679	.9	46.80973	174.3662
.6	8.168141	5.30929	.8	27.64602	60.82123	15.0	47.12389	176.7146
.7	8.482300	5.72555	.9	27.96017	62.21139	.1	47.43805	179.0786
.8	8.796459	6.15752	9.0	28.27433	63.61725	.2	47.75221	181.4584
.9	9.110619	6.60520	.1	28.58849	65.03882	.3	48.06637	183.8539
3.0	9.424778	7.06858	.2	28.90265	66.47610	.4	48.38053	186.2650
.1	9.738937	7.54768	.3	29.21681	67.92909	.5	48.69469	188.6919
.2	10.05310	8.04248	.4	29.53097	69.39778	.6	49.00885	191.1345
.3	10.36726	8.55299	.5	29.84513	70.88218	.7	49.32300	193.5928
.4	10.68142	9.07920	.6	30.15929	72.38229	.8	49.63716	196.0668
.5	10.99557	9.62113	.7	30.47345	73.89811	.9	49.95132	198.5565
.6	11.30973	10.17876	.8	30.78761	75.42964	16.0	50.26548	201.0619
.7	11.62389	10.75210	.9	31.10177	76.97687	.1	50.57964	203.5831
.8	11.93805	11.34115	10.0	31.41593	78.53982	.2	50.89380	206.1199
.9	12.25221	11.94591	.1	31.73009	80.11847	.3	51.20796	208.6724
4.0	12.56637	12.56637	.2	32.04425	81.71282	.4	51.52212	211.2407
.1	12.88053	13.20254	.3	32.35840	83.32289	.5	51.83628	213.8246
.2	13.19469	13.85442	.4	32.67256	84.94867	.6	52.15044	216.4243
.3	13.50885	14.52201	.5	32.98672	86.59015	.7	52.46460	219.0397
.4	13.82301	15.20531	.6	33.30088	88.24734	.8	52.77876	221.6708
.5	14.13717	15.90431	.7	33.61504	89.92024	.9	53.09292	224.3176
.6	14.45133	16.61903	.8	33.92920	91.60884	17.0	53.40708	226.9801
.7	14.76549	17.34945	.9	34.24336	93.31316	.1	53.72123	229.6583
.8	15.07964	18.09557	11.0	34.55752	95.03318	.2	54.03539	232.3522
.9	15.39380	18.85741	.1	34.87168	96.76891	.3	54.34955	235.0618
5.0	15.70796	19.63495	.2	35.18584	98.52035	.4	54.66371	237.7871
.1	16.02212	20.42821	.3	35.50000	100.2875	.5	54.97787	240.5282
.2	16.33628	21.23717	.4	35.81416	102.0703	.6	55.29203	243.2849
.3	16.65044	22.06183	.5	36.12832	103.8689	.7	55.60619	246.0574
.4	16.96460	22.90221	.6	36.44247	105.6832	.8	55.92035	248.8456
.5	17.27876	23.75829	.7	36.75663	107.5132	.9	56.23451	251.6494
.6	17.59292	24.63009	.8	37.07079	109.3588	18.0	56.54867	254.4690
.7	17.90708	25.51759	.9	37.38495	111.2202	.1	56.86283	257.3043
.8	18.22124	26.42079	12.0	37.69911	113.0973	.2	57.17699	260.1553
.9	18.53540	27.33971	.1	38.01327	114.9901	.3	57.49115	263.0220
6.0	18.84956	28.27433	.2	38.32743	116.8987	.4	57.80530	265.9044
.1	19.16372	29.22467	.3	38.64159	118.8229	.5	58.11946	268.8025
.2	19.47787	30.19071	.4	38.95575	120.7628	.6	58.43362	271.7163

TABLE 2 OF CIRCLES—(Continued).

Diameters in units and tenths.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
18.7	58.74778	274.6459	24.9	78.22566	486.9547	31.1	97.70353	759.6450
.8	59.06194	277.5911	25.0	78.53982	490.8739	.2	98.01769	764.5380
.9	59.37610	280.5521	.1	78.85398	494.8087	.3	98.33185	769.4467
19.0	59.69026	283.5287	.2	79.16813	498.7592	.4	98.64601	774.3712
.1	60.00442	286.5211	.3	79.48229	502.7255	.5	98.96017	779.3113
.2	60.31858	289.5292	.4	79.79645	506.7075	.6	99.27433	784.2672
.3	60.63274	292.5530	.5	80.11061	510.7052	.7	99.58849	789.2388
.4	60.94690	295.5925	.6	80.42477	514.7185	.8	99.90265	794.2260
.5	61.26106	298.6477	.7	80.73893	518.7476	.9	100.2168	799.2290
.6	61.57522	301.7186	.8	81.05309	522.7924	32.0	100.5310	804.2477
.7	61.88938	304.8052	.9	81.36725	526.8529	.1	100.8451	809.2821
.8	62.20353	307.9075	26.0	81.68141	530.9292	.2	101.1593	814.3322
.9	62.51769	311.0255	.1	81.99557	535.0211	.3	101.4734	819.3980
20.0	62.83185	314.1593	.2	82.30973	539.1287	.4	101.7876	824.4796
.1	63.14601	317.3087	.3	82.62389	543.2521	.5	102.1018	829.5768
.2	63.46017	320.4739	.4	82.93805	547.3911	.6	102.4159	834.6898
.3	63.77433	323.6547	.5	83.25221	551.5459	.7	102.7301	839.8184
.4	64.08849	326.8513	.6	83.56636	555.7163	.8	103.0442	844.9628
.5	64.40265	330.0636	.7	83.88052	559.9025	.9	103.3584	850.1228
.6	64.71681	333.2916	.8	84.19468	564.1044	33.0	103.6726	855.2986
.7	65.03097	336.5353	.9	84.50884	568.3220	.1	103.9867	860.4901
.8	65.34513	339.7947	27.0	84.82300	572.5553	.2	104.3009	865.6973
.9	65.65929	343.0698	.1	85.13716	576.8043	.3	104.6150	870.9202
21.0	65.97345	346.3606	.2	85.45132	581.0690	.4	104.9292	876.1588
.1	66.28760	349.6671	.3	85.76548	585.3494	.5	105.2434	881.4131
.2	66.60176	352.9894	.4	86.07964	589.6455	.6	105.5575	886.6831
.3	66.91592	356.3273	.5	86.39380	593.9574	.7	105.8717	891.9688
.4	67.23008	359.6809	.6	86.70796	598.2849	.8	106.1858	897.2703
.5	67.54424	363.0503	.7	87.02212	602.6282	.9	106.5000	902.5874
.6	67.85840	366.4354	.8	87.33628	606.9871	34.0	106.8142	907.9203
.7	68.17256	369.8361	.9	87.65044	611.3618	.1	107.1283	913.2688
.8	68.48672	373.2526	28.0	87.96459	615.7522	.2	107.4425	918.6331
.9	68.80088	376.6848	.1	88.27875	620.1582	.3	107.7566	924.0131
22.0	69.11504	380.1327	.2	88.59291	624.5800	.4	108.0708	929.4088
.1	69.42920	383.5963	.3	88.90707	629.0175	.5	108.3849	934.8202
.2	69.74336	387.0756	.4	89.22123	633.4707	.6	108.6991	940.2473
.3	70.05752	390.5707	.5	89.53539	637.9397	.7	109.0133	945.6901
.4	70.37168	394.0814	.6	89.84955	642.4243	.8	109.3274	951.1486
.5	70.68583	397.6078	.7	90.16371	646.9246	.9	109.6416	956.6228
.6	70.99999	401.1500	.8	90.47787	651.4407	35.0	109.9557	962.1128
.7	71.31415	404.7078	.9	90.79203	655.9724	.1	110.2699	967.6184
.8	71.62831	408.2814	29.0	91.10619	660.5199	.2	110.5841	973.1397
.9	71.94247	411.8707	.1	91.42035	665.0830	.3	110.8982	978.6768
23.0	72.25663	415.4756	.2	91.73451	669.6619	.4	111.2124	984.2296
.1	72.57079	419.0963	.3	92.04866	674.2565	.5	111.5265	989.7980
.2	72.88495	422.7327	.4	92.36282	678.8668	.6	111.8407	995.3822
.3	73.19911	426.3848	.5	92.67698	683.4928	.7	112.1549	1000.9821
.4	73.51327	430.0526	.6	92.99114	688.1345	.8	112.4690	1006.5977
.5	73.82743	433.7361	.7	93.30530	692.7919	.9	112.7832	1012.2290
.6	74.14159	437.4354	.8	93.61946	697.4650	36.0	113.0973	1017.8760
.7	74.45575	441.1503	.9	93.93362	702.1538	.1	113.4115	1023.5387
.8	74.76991	444.8809	30.0	94.24778	706.8583	.2	113.7257	1029.2172
.9	75.08406	448.6273	.1	94.56194	711.5786	.3	114.0398	1034.9113
24.0	75.39822	452.3893	.2	94.87610	716.3145	.4	114.3540	1040.6212
.1	75.71238	456.1671	.3	95.19026	721.0662	.5	114.6681	1046.3467
.2	76.02654	459.9606	.4	95.50442	725.8336	.6	114.9823	1052.0880
.3	76.34070	463.7698	.5	95.81858	730.6166	.7	115.2965	1057.8449
.4	76.65486	467.5947	.6	96.13274	735.4154	.8	115.6106	1063.6176
.5	76.96902	471.4352	.7	96.44689	740.2299	.9	115.9248	1069.4060
.6	77.28318	475.2916	.8	96.76105	745.0601	37.0	116.2389	1075.2101
.7	77.59734	479.1636	.9	97.07521	749.9060	.1	116.5531	1081.0299
.8	77.91150	483.0513	31.0	97.38937	754.7676	.2	116.8672	1086.8654

TABLE 2 OF CIRCLES—(Continued).

Diameters in units and tenths.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
37.3	117.1814	1092.7166	43.5	136.6593	1486.1697	49.7	156.1372	1940.0041
.4	117.4956	1098.5835	.6	136.9734	1493.0105	.8	156.4513	1947.8189
.5	117.8097	1104.4662	.7	137.2876	1499.8670	.9	156.7655	1955.6493
.6	118.1239	1110.3645	.8	137.6018	1506.7393	50.0	157.0796	1963.4954
.7	118.4380	1116.2786	.9	137.9159	1513.6272	.1	157.3938	1971.3572
.8	118.7522	1122.2083	44.0	138.2301	1520.5308	.2	157.7080	1979.2348
.9	119.0664	1128.1538	.1	138.5442	1527.4502	.3	158.0221	1987.1280
38.0	119.3805	1134.1149	.2	138.8584	1534.3853	.4	158.3363	1995.0370
.1	119.6947	1140.0918	.3	139.1726	1541.3360	.5	158.6504	2002.9617
.2	120.0088	1146.0844	.4	139.4867	1548.3025	.6	158.9646	2010.9020
.3	120.3230	1152.0927	.5	139.8009	1555.2847	.7	159.2787	2018.8581
.4	120.6372	1158.1167	.6	140.1150	1562.2826	.8	159.5929	2026.8299
.5	120.9513	1164.1564	.7	140.4292	1569.2962	.9	159.9071	2034 8174
.6	121.2655	1170.2118	.8	140.7434	1576.3255	51.0	160.2212	2042.8206
.7	121.5796	1176.2830	.9	141.0575	1583.3706	.1	160.5354	2050.8395
.8	121.8938	1182.3698	45.0	141.3717	1590.4313	.2	160.8495	2058.8742
.9	122.2080	1188.4724	.1	141.6858	1597.5077	.3	161.1637	2066.9245
39.0	122.5221	1194.5906	.2	142.0000	1604.5999	.4	161.4779	2074 9905
.1	122.8363	1200.7246	.3	142.3141	1611.7077	.5	161.7920	2083.0723
.2	123.1504	1206.8742	.4	142.6283	1618.8313	.6	162.1062	2091.1697
.3	123.4646	1213.0396	.5	142.9425	1625.9705	.7	162.4203	2099.2829
.4	123.7788	1219.2207	.6	143.2566	1633.1255	.8	162.7345	2107.4118
.5	124.0929	1225.4175	.7	143.5708	1640.2962	.9	163.0487	2115.5563
.6	124.4071	1231.6300	.8	143.8849	1647.4826	52.0	163.3628	2123.7166
.7	124.7212	1237.8582	.9	144.1991	1654.6847	.1	163.6770	2131.8926
.8	125.0354	1244.1021	46.0	144.5133	1661.9025	.2	163.9911	2140.0843
.9	125.3495	1250.3617	.1	144.8274	1669.1360	.3	164.3053	2148.2917
40.0	125.6637	1256.6371	.2	145.1416	1676.3853	.4	164.6195	2156.5149
.1	125.9779	1262.9281	.3	145.4557	1683.6502	.5	164.9336	2164 7537
.2	126.2920	1269.2348	.4	145.7699	1690.9308	.6	165.2478	2173.0082
.3	126.6062	1275.5573	.5	146.0841	1698.2272	.7	165.5619	2181.2785
.4	126.9203	1281.8955	.6	146.3982	1705.5392	.8	165.8761	2189.5644
.5	127.2345	1288.2493	.7	146.7124	1712.8670	.9	166.1903	2197.8661
.6	127.5487	1294.6189	.8	147.0265	1720.2105	53.0	166.5044	2206.1834
.7	127.8628	1301.0042	.9	147.3407	1727.5697	.1	166.8186	2214.5165
.8	128.1770	1307.4052	47.0	147.6549	1734.9445	.2	167.1327	2222.8653
.9	128.4911	1313.8219	.1	147.9690	1742.3351	.3	167.4469	2231.2298
41.0	128.8053	1320.2543	.2	148.2832	1749.7414	.4	167.7610	2239.6100
.1	129.1195	1326.7024	.3	148.5973	1757.1635	.5	168.0752	2248.0059
.2	129.4336	1333.1663	.4	148.9115	1764.6012	.6	168.3894	2256.4175
.3	129.7478	1339.6458	.5	149.2257	1772.0546	.7	168.7035	2264.8448
.4	130.0619	1346.1410	.6	149.5398	1779.5237	.8	169.0177	2273 2879
.5	130.3761	1352.6520	.7	149.8540	1787.0086	.9	169.3318	2281.7466
.6	130.6903	1359.1786	.8	150.1681	1794.5091	54.0	169.6460	2290.2210
.7	131.0044	1365.7210	.9	150.4823	1802.0254	.1	169.9602	2298.7112
.8	131.3186	1372.2791	48.0	150.7964	1809.5574	.2	170.2743	2307.2171
.9	131.6327	1378.8529	.1	151.1106	1817.1050	.3	170.5885	2315.7386
42.0	131.9469	1385.4424	.2	151.4248	1824.6684	.4	170.9026	2324.2759
.1	132.2611	1392.0476	.3	151.7389	1832.2475	.5	171.2168	2332 8289
.2	132.5752	1398.6685	.4	152.0531	1839.8423	.6	171.5310	2341.3976
.3	132.8894	1405.3051	.5	152.3672	1847.4528	.7	171.8451	2349.9820
.4	133.2035	1411.9574	.6	152.6814	1855.0790	.8	172.1593	2358.5821
.5	133.5177	1418.6254	.7	152.9956	1862.7210	.9	172.4734	2367.1979
.6	133.8318	1425.3092	.8	153.3097	1870.3786	55.0	172.7876	2375.8294
.7	134.1460	1432.0086	.9	153.6239	1878.0519	.1	173.1018	2384.4767
.8	134.4602	1438.7238	49.0	153.9380	1885.7410	.2	173.4159	2393.1396
.9	134.7743	1445.4546	.1	154.2522	1893.4457	.3	173.7301	2401.8183
43.0	135.0885	1452.2012	.2	154 5664	1901.1662	.4	174.0442	2410.5126
.1	135.4026	1458.9635	.3	154.8805	1908.9024	.5	174.3584	2419.2227
.2	135.7168	1465.7415	.4	155.1947	1916.6543	.6	174.6726	2427.9485
.3	136.0310	1472.5352	.5	155.5088	1924.4218	.7	174.9867	2436 6899
.4	136.3451	1479.3446	.6	155.8230	1932.2051	.8	175.3009	2445.4471

TABLE 2 OF CIRCLES—(Continued).

Diameters in units and tenths.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
55.9	175.6150	2454.2200	62.1	195.0929	3028.8173	68.3	214.5708	3663.7960
56.0	175.9292	2463.0086	.2	195.4071	3038.5798	.4	214.8849	3674.5324
.1	176.2433	2471.8130	.3	195.7212	3048.3580	.5	215.1991	3685.2845
.2	176.5575	2480.6330	.4	196 0354	3058.1520	.6	215.5133	3696.0523
.3	176.8717	2489.4687	.5	196.3495	3067.9616	.7	215.8274	3706.8359
.4	177.1858	2498.3201	.6	196.6637	3077.7869	.8	216.1416	3717.6351
.5	177.5000	2507.1873	.7	196.9779	3087.6279	.9	216.4557	3728.4500
.6	177.8141	2516.0701	.8	197.2920	3097.4847	69.0	216.7699	3739.2807
.7	178.1283	2524.9687	.9	197.6062	3107.3571	.1	217.0841	3750.1270
.8	178.4425	2533.8830	63.0	197.9203	3117.2453	.2	217.3982	3760.9891
.9	178.7566	2542.8129	.1	198.2345	3127.1492	.3	217.7124	3771.8668
57.0	179.0708	2551.7586	.2	198.5487	3137.0688	.4	218.0265	3782.7603
.1	179.3849	2560.7200	.3	198.8628	3147.0040	.5	218.3407	3793.6695
.2	179.6991	2569.6971	.4	199.1770	3156.9550	.6	218.6548	3804.5944
.3	180.0133	2578.6899	.5	199.4911	3166.9217	.7	218.9690	3815.5350
.4	180.3274	2587.6985	.6	199.8053	3176.9042	.8	219.2832	3826.4913
.5	180 6416	2596.7227	.7	200.1195	3186.9023	.9	219.5973	3837.4633
.6	180.9557	2605.7626	.8	200.4336	3196.9161	70.0	219.9115	3848.4510
.7	181.2699	2614.8183	.9	200.7478	3206.9456	.1	220.2256	3859.4544
.8	181.5841	2623.8896	64.0	201.0619	3216.9909	.2	220.5398	3870.4736
.9	181.8982	2632.9767	.1	201.3761	3227.0518	.3	220.8540	3881.5084
58.0	182.2124	2642.0794	.2	201.6902	3237.1285	.4	221.1681	3892.5590
.1	182.5265	2651.1979	.3	202.0044	3247.2209	.5	221.4823	3903.6252
.2	182.8407	2660.3321	.4	202.3186	3257.3289	.6	221.7964	3914.7072
.3	183 1549	2669.4820	.5	202.6327	3267.4527	.7	222.1106	3925.8049
.4	183.4690	2678.6476	.6	202.9469	3277.5922	.8	222.4248	3936.9182
.5	183.7832	2687.8289	.7	203.2610	3287.7474	.9	222.7389	3948.0473
.6	184.0973	2697.0259	.8	203.5752	3297.9183	71.0	223.0531	3959.1921
.7	184.4115	2706.2386	.9	203.8894	3308.1049	.1	223.3672	3970.3526
.8	184.7256	2715.4670	65.0	204.2035	3318.3072	.2	223.6814	3981.5289
.9	185.0398	2724.7112	.1	204.5177	3328.5253	.3	223.9956	3992.7208
59.0	185.3540	2733.9710	.2	204.8318	3338.7590	.4	224.3097	4003.9284
.1	185.6681	2743.2466	.3	205.1460	3349.0085	.5	224.6239	4015.1518
.2	185.9823	2752.5378	.4	205.4602	3359.2736	.6	224.9380	4026.3908
.3	186.2964	2761.8448	.5	205.7743	3369.5545	.7	225.2522	4037.6456
.4	186.6106	2771.1675	.6	206.0885	3379.8510	.8	225.5664	4048.9160
.5	186.9248	2780.5058	.7	206.4026	3390.1633	.9	225.8805	4060.2022
.6	187.2389	2789.8599	.8	206 7168	3400.4913	72.0	226.1947	4071.5041
.7	187 5531	2799.2297	.9	207.0310	3410.8350	.1	226.5088	4082.8217
.8	187.8672	2808.6152	66.0	207.3451	3421.1944	.2	226.8230	4094.1550
.9	188.1814	2818.0165	.1	207.6593	3431.5695	.3	227.1371	4105.5040
60.0	188.4956	2827.4334	.2	207.9734	3441.9603	.4	227.4513	4116.8687
.1	188.8097	2836.8660	.3	208.2876	3452.3669	.5	227 7655	4128.2491
.2	189.1239	2846.3144	.4	208.6018	3462.7891	.6	228.0796	4139.6452
.3	189.4380	2855.7784	.5	208.9159	3473.2270	.7	228.3938	4151.0571
.4	189.7522	2865.2582	.6	209.2301	3483.6807	.8	228.7079	4162.4846
.5	190.0664	2874.7536	.7	209.5442	3494.1500	.9	229.0221	4173.9279
.6	190 3805	2884.2648	.8	209.8584	3504.6351	73.0	229.3363	4185.3868
.7	190.6947	2893.7917	.9	210.1725	3515.1359	.1	229.6504	4196.8615
.8	191.0088	2903.3343	67.0	210.4867	3525.6524	.2	229.9646	4208.3519
.9	191.3230	2912.8926	.1	210.8009	3536.1845	.3	230.2787	4219.8579
61.0	191.6372	2922.4666	.2	211.1150	3546.7324	.4	230.5929	4231.3797
.1	191.9513	2932.0563	.3	211.4292	3557.2960	.5	230.9071	4242.9172
.2	192.2655	2941.6617	.4	211.7433	3567.8754	.6	231.2212	4254.4704
.3	192.5796	2951 2828	.5	212.0575	3578.4704	.7	231.5354	4266.0394
.4	192.8938	2960.9197	.6	212.3717	3589.0811	.8	231.8495	4277.6240
.5	193.2079	2970.5722	.7	212.6858	3599.7075	.9	232.1637	4289.2243
.6	193.5221	2980.2405	.8	213.0000	3610.3497	74.0	232.4779	4300.8403
.7	193.8363	2989.9244	.9	213.3141	3621.0075	.1	232.7920	4312.4721
.8	194.1504	2999.6241	68.0	213.6283	3631.6811	.2	233.1062	4324.1195
.9	194.4646	3009.3395	.1	213.9425	3642.3704	.3	233.4203	4335.7827
62.0	194.7787	3019.0705	.2	214.2566	3653.0754	.4	233.7345	4347.4616

TABLE 2 OF CIRCLES—(Continued).

Diameters in units and tenths.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
74.5	234.0487	4359.1562	80.7	253.5265	5114.8977	86.9	273.0044	5931.0206
.6	234.3628	4370.8664	.8	253.8407	5127.5819	87.0	273.3186	5944.6787
.7	234.6770	4382.5924	.9	254.1548	5140.2818	.1	273.6327	5958.3525
.8	234.9911	4394.3341	81.0	254.4690	5152.9974	.2	273.9469	5972.0420
.9	235.3053	4406.0916	.1	254.7832	5165.7287	.3	274.2610	5985.7472
75.0	235.6194	4417.8647	.2	255.0973	5178.4757	.4	274.5752	5999.4681
.1	235.9336	4429.6535	.3	255.4115	5191.2384	.5	274.8894	6013.2047
.2	236.2478	4441.4580	.4	255.7256	5204.0168	.6	275.2035	6026.9570
.3	236.5619	4453.2783	.5	256.0398	5216.8110	.7	275.5177	6040 7250
.4	236.8761	4465.1142	.6	256.3540	5229.6208	.8	275.8318	6054.5088
.5	237.1902	4476.9659	.7	256.6681	5242.4463	.9	276.1460	6068.3082
.6	237.5044	4488 8332	.8	256.9823	5255.2876	88.0	276.4602	6082.1234
.7	237.8186	4500.7163	.9	257.2964	5268.1446	.1	276.7743	6095.9542
.8	238.1327	4512.6151	82.0	257.6106	5281.0173	.2	277.0885	6109 8008
.9	238.4469	4524.5296	.1	257.9248	5293.9056	.3	277.4026	6123.6631
76.0	238.7610	4536.4598	.2	258.2389	5306.8097	.4	277.7168	6137.5411
.1	239.0752	4548.4057	.3	258.5531	5319.7295	.5	278.0309	6151.4348
.2	239.3894	4560.3673	.4	258.8672	5332.6650	.6	278.3451	6165.3442
.3	239.7035	4572.3446	.5	259.1814	5345.6162	.7	278.6593	6179.2693
.4	240.0177	4584.3377	.6	259.4956	5358.5832	.8	278.9734	6193.2101
.5	240.3318	4596.3464	.7	259.8097	5371.5658	.9	279.2876	6207.1666
.6	240 6460	4608.3708	.8	260.1239	5384.5641	89.0	279.6017	6221.1389
.7	240.9602	4620.4110	.9	260.4380	5397.5782	.1	279.9159	6235.1268
.8	241.2743	4632.4669	83.0	260.7522	5410.6079	.2	280.2301	6249.1304
.9	241.5885	4644.5384	.1	261.0663	5423.6534	.3	280.5442	6263.1498
77.0	241.9026	4656.6257	.2	261.3805	5436.7146	.4	280.8584	6277.1849
.1	242.2168	4668.7287	.3	261.6947	5449.7915	.5	281.1725	6291 2356
.2	242.5310	4680.8474	.4	262.0088	5462.8840	.6	281.4867	6305.3021
.3	242.8451	4692.9818	.5	262.3230	5475.9923	.7	281.8009	6319.3843
.4	243.1593	4705.1319	.6	262.6371	5489.1163	.8	282.1150	6333.4822
.5	243.4734	4717.2977	.7	262.9513	5502.2561	.9	282.4292	6347.5958
.6	243.7876	4729.4792	.8	263.2655	5515.4115	90.0	282.7433	6361.7251
.7	244.1017	4741.6765	.9	263.5796	5528.5826	.1	283.0575	6375.8701
.8	244.4159	4753.8894	84.0	263.8938	5541.7694	.2	283.3717	6390.0309
.9	244.7301	4766.1181	.1	264.2079	5554.9720	.3	283.6858	6404.2073
78.0	245.0442	4778.3624	.2	264.5221	5568.1902	.4	284.0000	6418.3995
.1	245.3584	4790.6225	.3	264.8363	5581.4242	.5	284.3141	6432.6073
.2	245.6725	4802.8983	.4	265.1504	5594.6739	.6	284.6283	6446.8309
.3	245.9867	4815.1897	.5	265.4646	5607.9392	.7	284.9425	6461.0701
.4	246.3009	4827.4969	.6	265.7787	5621.2203	.8	285.2566	6475.3251
.5	246.6150	4839.8198	.7	266.0929	5634.5171	.9	285.5708	6489.5958
.6	246.9292	4852.1584	.8	266.4071	5647.8296	91.0	285.8849	6503.8822
.7	247.2433	4864.5128	.9	266.7212	5661.1578	.1	286.1991	6518.1843
.8	247.5575	4876.8828	85.0	267.0354	5674.5017	.2	286.5133	6532.5021
.9	247.8717	4889.2685	.1	267.3495	5687.8614	.3	286.8274	6546.8356
79.0	248.1858	4901.6699	.2	267.6637	5701.2367	.4	287.1416	6561.1848
.1	248.5000	4914.0871	.3	267.9779	5714.6277	.5	287.4557	6575.5498
.2	248.8141	4926.5199	.4	268.2920	5728.0345	.6	287.7699	6589.9304
.3	249.1283	4938.9685	.5	268.6062	5741.4569	.7	288.0840	6604.3268
.4	249.4425	4951.4328	.6	268.9203	5754.8951	.8	288.3982	6618.7388
.5	249.7566	4963.9127	.7	269.2345	5768.3490	.9	288.7124	6633.1666
.6	250.0708	4976.4084	.8	269.5486	5781.8185	92.0	289.0265	6647.6101
.7	250.3849	4988.9198	.9	269.8628	5795.3038	.1	289.3407	6662.0692
.8	250.6991	5001.4469	86.0	270.1770	5808.8048	.2	289.6548	6676.5441
.9	251.0133	5013.9897	.1	270.4911	5822.3215	.3	289.9690	6691.0347
80.0	251.3274	5026.5482	.2	270.8053	5835.8539	.4	290.2832	6705.5410
.1	251.6416	5039.1225	.3	271.1194	5849.4020	.5	290.5973	6720.0630
.2	251.9557	5051.7124	.4	271.4336	5862.9659	.6	290.9115	6734.6008
.3	252.2699	5064.3180	.5	271.7478	5876.5454	.7	291.2256	6749.1542
.4	252 5840	5076.9394	.6	272.0619	5890.1407	.8	291.5398	6763.7233
.5	252.8982	5089.5764	.7	272.3761	5903.7516	.9	291.8540	6778.3082
.6	253.2124	5102.2292	.8	272.6902	5917.3783	93.0	292.1681	6792.9087

TABLE 2 OF CIRCLES—(Continued).

Diameters in units and tenths.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
93.1	292.4823	6807.5250	**95**.5	300.0221	7163.0276	**97**.8	307.2478	7512.2078
.2	292.7964	6822.1569	.6	300.3363	7178.0366	.9	307.5619	7527.5780
.3	293.1106	6836.8046	.7	300.6504	7193.0612	**98**.0	307.8761	7542.9640
.4	293.4248	6851.4680	.8	300.9646	7208.1016	.1	308.1902	7558.3656
.5	293.7389	6866.1471	.9	301.2787	7223.1577	.2	308.5044	7573.7830
.6	294.0531	6880.8419	**96**.0	301.5929	7238.2295	.3	308.8186	7589.2161
.7	294.3672	6895.5524	.1	301.9071	7253.3170	.4	309.1327	7604.6648
.8	294.6814	6910.2786	.2	302.2212	7268.4202	.5	309.4469	7620.1293
.9	294.9956	6925.0205	.3	302.5354	7283.5391	.6	309.7610	7635.6095
94.0	295.3097	6939.7782	.4	302.8495	7298.6737	.7	310.0752	7651.1054
.1	295.6239	6954.5515	.5	303.1637	7313.8240	.8	310.3894	7666.6170
.2	295.9380	6969.3406	.6	303.4779	7328.9901	.9	310.7035	7682.1444
.3	296.2522	6984.1453	.7	303.7920	7344.1718	**99**.0	311.0177	7697.6874
.4	296.5663	6998.9658	.8	304.1062	7359.3693	.1	311.3318	7713.2461
.5	296.8805	7013.8019	.9	304.4203	7374.5824	.2	311.6460	7728.8206
.6	297.1947	7028.6538	**97**.0	304.7345	7389.8113	.3	311.9602	7744.4107
.7	297.5088	7043.5214	.1	305.0486	7405.0559	.4	312.2743	7760.0166
.8	297.8230	7058.4047	.2	305.3628	7420.3162	.5	312.5885	7775.6382
.9	298.1371	7073.3037	.3	305.6770	7435.5922	.6	312.9026	7791.2754
95.0	298.4513	7088.2184	.4	305.9911	7450.8839	.7	313.2168	7806.9284
.1	298.7655	7103.1488	.5	306.3053	7466.1913	.8	313.5309	7822.5971
.2	299.0796	7118.0950	.6	306.6194	7481.5144	.9	313.8451	7838.2815
.3	299.3938	7133.0568	.7	306.9336	7496.8532	**100**.0	314.1593	7853.9816
.4	299.7079	7148.0343						

Circumferences when the diameter has more than one place of decimals.

Diam.	Circ.	Diam.	Circ.	Diam.	Circ.	Diam.	Circ.	Diam.	Circ.
.1	.314159	.01	.031416	.001	.003142	.0001	.000314	.00001	.000031
.2	.628319	.02	.062832	.002	.006283	.0002	.000628	.00002	.000063
.3	.942478	.03	.094248	.003	.009425	.0003	.000942	.00003	.000094
.4	1.256637	.04	.125664	.004	.012566	.0004	.001257	.00004	.000126
.5	1 570796	.05	.157080	.005	.015708	.0005	.001571	.00005	.000157
.6	1.884956	.06	.188496	.006	.018850	.0006	.001885	.00006	.000188
.7	2.199115	.07	.219911	.007	.021991	.0007	.002199	.00007	.000220
.8	2.513274	.08	.251327	.008	.025133	.0008	.002513	.00008	.000251
.9	2.827433	.09	.282743	.009	.028274	.0009	.002827	.00009	.000283

Examples.

Diameter = 3.12699

Circumference =

			Sum of
Circ for dia of	3.1	=	9.738937
"	.02	=	.062832
"	.006	=	.018850
"	.0009	=	.002827
"	.00009	=	.000283
			9.823729

Circumfce = 9.823729

Diameter =

			Sum of
Dia for circ of	9.738937	=	3.1
	.084792		
"	.062832	=	.02
	.021960		
"	.018850	=	.006
	.003110		
"	.002827	=	.0009
	.000283		
"	.000283	=	.00009
			3.12699

TABLE 3 OF CIRCLES.

Diams in units and twelfths; as in feet and inches.

Dia.		Circumf.	Area.	Dia.		Circumf.	Area.	Dia.		Circumf.	Area.
Ft.	In.	Feet.	Sq. ft.	Ft.	In.	Feet.	Sq. ft.	Ft.	In.	Feet.	Sq. ft.
				5	0	15.70796	19.63495	10	0	31.41593	78.53982
0	1	.261799	.005454		1	15.96976	20.29491		1	31.67773	79.85427
	2	.523599	.021817		2	16.23156	20.96577		2	31.93953	81.17968
	3	.785398	.049087		3	16.49336	21.64754		3	32.20132	82.51585
	4	1.047198	.087266		4	16.75516	22.34021		4	32.46312	83.86307
	5	1.308997	.136354		5	17.01696	23.04380		5	32.72492	85.22115
	6	1.570796	.196350		6	17.27876	23.75829		6	32.98672	86.59015
	7	1.832596	.267254		7	17.54056	24.48370		7	33.24852	87.97005
	8	2.094395	.349066		8	17.80236	25.22001		8	33.51032	89.36086
	9	2.356195	.441786		9	18.06416	25.96723		9	33.77212	90.76258
	10	2.617994	.545415		10	18.32596	26.72535		10	34.03392	92.17520
	11	2.879793	.659953		11	18.58776	27.49439		11	34.29572	93.59874
1	0	3.14159	.785398	6	0	18.84956	28.27433	11	0	34.55752	95.03318
	1	3.40339	.921752		1	19.11136	29.06519		1	34.81932	96.47853
	2	3.66519	1.06901		2	19.37315	29.86695		2	35.08112	97.93479
	3	3.92699	1.22718		3	19.63495	30.67962		3	35.34292	99.40196
	4	4.18879	1.39626		4	19.89675	31.50319		4	35.60472	100.8800
	5	4.45059	1.57625		5	20.15855	32.33768		5	35.86652	102.3690
	6	4.71239	1.76715		6	20.42035	33.18307		6	36.12832	103.8689
	7	4.97419	1.96895		7	20.68215	34.03937		7	36.39011	105.3797
	8	5.23599	2.18166		8	20.94395	34.90659		8	36.65191	106.9014
	9	5.49779	2.40528		9	21.20575	35.78470		9	36.91371	108.4340
	10	5.75959	2.63981		10	21.46755	36.67373		10	37.17551	109.9776
	11	6.02139	2.88525		11	21.72935	37.57367		11	37.43731	111.5320
2	0	6.28319	3.14159	7	0	21.99115	38.48451	12	0	37.69911	113.0973
	1	6.54498	3.40885		1	22.25295	39.40626		1	37.96091	114.6736
	2	6.80678	3.68701		2	22.51475	40.33892		2	38.22271	116.2607
	3	7.06858	3.97608		3	22.77655	41.28249		3	38.48451	117.8588
	4	7.33038	4.27606		4	23.03835	42.23697		4	38.74631	119.4678
	5	7.59218	4.58694		5	23.30015	43.20235		5	39.00811	121.0877
	6	7.85398	4.90874		6	23.56194	44.17865		6	39.26991	122.7185
	7	8.11578	5.24144		7	23.82374	45.16585		7	39.53171	124.3602
	8	8.37758	5.58505		8	24.08554	46.16396		8	39.79351	126.0128
	9	8.63938	5.93957		9	24.34734	47.17298		9	40.05531	127.6763
	10	8.90118	6.30500		10	24.60914	48.19290		10	40.31711	129.3507
	11	9.16298	6.68134		11	24.87094	49.22374		11	40.57891	131.0360
3	0	9.42478	7.06858	8	0	25.13274	50.26548	13	0	40.84070	132.7323
	1	9.68658	7.46674		1	25.39454	51.31813		1	41.10250	134.4394
	2	9.94838	7.87580		2	25.65634	52.38169		2	41.36430	136.1575
	3	10.21018	8.29577		3	25.91814	53.45616		3	41.62610	137.8865
	4	10.47198	8.72665		4	26.17994	54.54154		4	41.88790	139.6263
	5	10.73377	9.16843		5	26.44174	55.63782		5	42.14970	141.3771
	6	10.99557	9.62113		6	26.70354	56.74502		6	42.41150	143.1388
	7	11.25737	10.08473		7	26.96534	57.86312		7	42.67330	144.9114
	8	11.51917	10.55924		8	27.22714	58.99213		8	42.93510	146.6949
	9	11.78097	11.04466		9	27.48894	60.13205		9	43.19690	148.4893
	10	12.04277	11.54099		10	27.75074	61.28287		10	43.45870	150 2947
	11	12.30457	12.04823		11	28.01253	62.44461		11	43.72050	152 1109
4	0	12.56637	12.56637	9	0	28.27433	63.61725	14	0	43.98230	153.9380
	1	12.82817	13.09542		1	28.53613	64.80080		1	44.24410	155.7761
	2	13.08997	13.63538		2	28.79793	65.99526		2	44.50590	157.6250
	3	13.35177	14.18625		3	29 05973	67.20063		3	44.76770	159.4849
	4	13.61357	14.74803		4	29.32153	68.41691		4	45.02949	161.3557
	5	13.87537	15.32072		5	29.58333	69.64409		5	45.29129	163.2374
	6	14.13717	15.90431		6	29.84513	70.88218		6	45.55309	165.1300
	7	14.39897	16.49882		7	30.10693	72.13119		7	45.81489	167.0335
	8	14.66077	17.10423		8	30.36873	73.39110		8	46.07669	168.9479
	9	14.92257	17.72055		9	30.63053	74.66191		9	46.33849	170.8732
	10	15.18436	18.34777		10	30.89233	75.94364		10	46.60029	172.8094
	11	15.44616	18.98591		11	31.15413	77.23627		11	46.86209	174.7565

TABLE 3 OF CIRCLES—(Continued).

Diams in units and twelfths; as in **feet and inches.**

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.
15 0	47.12389	176.7146	20 0	62.83185	314.1593	25 0	78.53982	490.8739
1	47.38569	178.6835	1	63.09365	316.7827	1	78.80162	494.1518
2	47.64749	180.6634	2	63.35545	319.4171	2	79.06342	497.4407
3	47.90929	182.6542	3	63.61725	322.0623	3	79.32521	500.7404
4	48.17109	184.6558	4	63.87905	324.7185	4	79.58701	504.0511
5	48.43289	186.6684	5	64.14085	327.3856	5	79.84881	507.3727
6	48.69469	188.6919	6	64.40265	330.0636	6	80.11061	510.7052
7	48.95649	190.7263	7	64.66445	332.7525	7	80.37241	514.0486
8	49.21828	192.7716	8	64.92625	335.4523	8	80.63421	517.4029
9	49.48008	194.8278	9	65.18805	338.1630	9	80.89601	520.7681
10	49.74188	196.8950	10	65.44985	340.8846	10	81.15781	524.1442
11	50.00368	198.9730	11	65.71165	343.6172	11	81.41961	527.5312
16 0	50.26548	201.0619	21 0	65.97345	346.3606	26 0	81.68141	530.9292
1	50.52728	203.1618	1	66.23525	349.1149	1	81.94321	534.3380
2	50.78908	205.2725	2	66.49704	351.8802	2	82.20501	537.7578
3	51.05088	207.3942	3	66.75884	354.6564	3	82.46681	541.1884
4	51.31268	209.5268	4	67.02064	357.4434	4	82.72861	544.6300
5	51.57448	211.6703	5	67.28244	360.2414	5	82.99041	548.0825
6	51.83628	213.8246	6	67.54424	363.0503	6	83.25221	551.5459
7	52.09808	215.9899	7	67.80604	365.8701	7	83.51400	555.0202
8	52.35988	218.1662	8	68.06784	368.7008	8	83.77580	558.5054
9	52.62168	220.3533	9	68.32964	371.5424	9	84.03760	562.0015
10	52.88348	222.5513	10	68.59144	374.3949	10	84.29940	565.5085
11	53.14528	224.7602	11	68.85324	377.2584	11	84.56120	569.0264
17 0	53.40708	226.9801	22 0	69.11504	380.1327	27 0	84.82300	572.5553
1	53.66887	229.2108	1	69.37684	383.0180	1	85.08480	576.0950
2	53.93067	231.4525	2	69.63864	385.9141	2	85.34660	579.6457
3	54.19247	233.7050	3	69.90044	388.8212	3	85.60840	583.2072
4	54.45427	235.9685	4	70.16224	391.7392	4	85.87020	586.7797
5	54.71607	238.2429	5	70.42404	394.6680	5	86.13200	590.3631
6	54.97787	240.5282	6	70.68583	397.6078	6	86.39380	593.9574
7	55.23967	242.8244	7	70.94763	400.5585	7	86.65560	597.5626
8	55.50147	245.1315	8	71.20943	403.5201	8	86.91740	601.1787
9	55.76327	247.4495	9	71.47123	406.4926	9	87.17920	604.8057
10	56.02507	249.7784	10	71.73303	409.4761	10	87.44100	608.4436
11	56.28687	252.1183	11	71.99483	412.4704	11	87.70279	612.0924
18 0	56.54867	254.4690	23 0	72.25663	415.4756	28 0	87.96459	615.7522
1	56.81047	256.8307	1	72.51843	418.4918	1	88.22639	619.4228
2	57.07227	259.2032	2	72.78023	421.5188	2	88.48819	623.1044
3	57.33407	261.5867	3	73.04203	424.5568	3	88.74999	626.7968
4	57.59587	263.9810	4	73.30383	427.6057	4	89.01179	630.5002
5	57.85766	266.3863	5	73.56563	430.6654	5	89.27359	634.2145
6	58.11946	268.8025	6	73.82743	433.7361	6	89.53539	637.9397
7	58.38126	271.2296	7	74.08923	436.8177	7	89.79719	641.6758
8	58.64306	273.6676	8	74.35103	439.9102	8	90.05899	645.4228
9	58.90486	276.1165	9	74.61283	443.0137	9	90.32079	649.1807
10	59.16666	278.5764	10	74.87462	446.1280	10	90.58259	652.9495
11	59.42846	281.0471	11	75.13642	449.2532	11	90.84439	656.7292
19 0	59.69026	283.5287	24 0	75.39822	452.3893	29 0	91.10619	660.5199
1	59.95206	286.0213	1	75.66002	455.5364	1	91.36799	664.3214
2	60.21386	288.5247	2	75.92182	458.6943	2	91.62979	668.1339
3	60.47566	291.0391	3	76.18362	461.8632	3	91.89159	671.9572
4	60.73746	293.5644	4	76.44542	465.0430	4	92.15338	675.7915
5	60.99926	296.1006	5	76.70722	468.2337	5	92.41518	679.6367
6	61.26106	298.6477	6	76.96902	471.4352	6	92.67698	683.4928
7	61.52286	301.2056	7	77.23082	474.6477	7	92.93878	687.3597
8	61.78466	303.7746	8	77.49262	477.8711	8	93.20058	691.2377
9	62.04645	306.3544	9	77.75442	481.1055	9	93.46238	695.1265
10	62.30825	308.9451	10	78.01622	484.3507	10	93.72418	699.0262
11	62.57005	311.5467	11	78.27802	487.6068	11	93.98598	702.9368

TABLE 3 OF CIRCLES—(Continued).

Diams in units and twelfths; as in feet and inches.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
Ft. In.	Feet.	Sq. ft.	Ft. In.	Feet.	Sq. ft.	Ft. In.	Feet.	Sq. ft.
30 0	94.24778	706.8583	**35** 0	109.9557	962.1128	**40** 0	125.6637	1256.6371
1	94.50958	710.7908	1	110.2175	966.6997	1	125.9255	1261.8785
2	94.77138	714.7341	2	110.4793	971.2975	2	126.1873	1267.1309
3	95.03318	718.6884	3	110.7411	975.9063	3	126.4491	1272.3941
4	95.29498	722.6536	4	111.0029	980.5260	4	126.7109	1277.6683
5	95.55678	726.6297	5	111.2647	985.1566	5	126.9727	1282.9534
6	95.81858	730.6166	6	111.5265	989.7980	6	127.2345	1288.2493
7	96.08038	734.6145	7	111.7883	994.4504	7	127.4963	1293.5562
8	96.34217	738.6233	8	112.0501	999.1137	8	127.7581	1298.8740
9	96.60397	742.6431	9	112.3119	1003.7879	9	128.0199	1304.2027
10	96.86577	746 6737	10	112.5737	1008.4731	10	128.2817	1309.5424
11	97.12757	750.7152	11	112.8355	1013.1691	11	128.5435	1314.8929
31 0	97.38937	754.7676	**36** 0	113.0973	1017.8760	**41** 0	128.8053	1320.2543
1	97.65117	758.8310	1	113.3591	1022.5939	1	129.0671	1325.6267
2	97.91297	762.9052	2	113.6209	1027.3226	2	129.3289	1331.0099
3	98.17477	766.9904	3	113.8827	1032.0623	3	129.5907	1336.4041
4	98.43657	771.0865	4	114.1445	1036.8128	4	129.8525	1341.8091
5	98.69837	775.1934	5	114.4063	1041.5743	5	130.1143	1347.2251
6	98.96017	779.3113	6	114.6681	1046.3467	6	130.3761	1352.6520
7	99.22197	783.4401	7	114.9299	1051.1300	7	130.6379	1358.0898
8	99.48377	787.5798	8	115.1917	1055.9242	8	130.8997	1363.5385
9	99.74557	791.7304	9	115.4535	1060.7293	9	131.1615	1368.9981
10	100.0074	795.8920	10	115.7153	1065.5453	10	131.4233	1374.4686
11	100.2692	800.0644	11	115.9771	1070.3723	11	131.6851	1379.9500
32 0	100.5310	804.2477	**37** 0	116.2389	1075.2101	**42** 0	131.9469	1385.4424
1	100.7928	808.4420	1	116.5007	1080.0588	1	132.2087	1390.9456
2	101.0546	812.6471	2	116.7625	1084.9185	2	132.4705	1396.4598
3	101.3164	816.8632	3	117.0243	1089.7890	3	132.7323	1401.9848
4	101.5782	821.0901	4	117.2861	1094.6705	4	132.9941	1407.5208
5	101.8400	825.3280	5	117.5479	1099.5629	5	133.2559	1413.0676
6	102.1018	829.5768	6	117.8097	1104.4662	6	133.5177	1418.6254
7	102.3636	833.8365	7	118.0715	1109.3804	7	133.7795	1424.1941
8	102.6254	838.1071	8	118.3333	1114.3055	8	134.0413	1429.7737
9	102.8872	842.3886	9	118.5951	1119.2415	9	134.3031	1435.3642
10	103.1490	846.6810	10	118.8569	1124.1884	10	134.5649	1440.9656
11	103.4108	850.9844	11	119.1187	1129.1462	11	134.8267	1446.5780
33 0	103.6726	855.2986	**38** 0	119.3805	1134.1149	**43** 0	135.0885	1452.2012
1	103.9344	859.6237	1	119.6423	1139.0946	1	135.3503	1457.8353
2	104.1962	863.9598	2	119.9041	1144.0851	2	135.6121	1463.4804
3	104.4580	868.3068	3	120.1659	1149.0866	3	135.8739	1469.1364
4	104.7198	872.6646	4	120.4277	1154.0990	4	136.1357	1474.8032
5	104.9816	877.0334	5	120.6895	1159.1222	5	136.3975	1480.4810
6	105.2434	881.4131	6	120.9513	1164.1564	6	136.6593	1486.1697
7	105.5052	885.8037	7	121.2131	1169.2015	7	136.9211	1491.8693
8	105.7670	890.2052	8	121.4749	1174.2575	8	137.1829	1497.5798
9	106.0288	894.6176	9	121.7367	1179.3244	9	137.4447	1503.3012
10	106.2906	899.0409	10	121.9985	1184.4022	10	137.7065	1509.0335
11	106 5524	903.4751	11	122.2603	1189.4910	11	137.9683	1514.7767
34 0	106.8142	907.9203	**39** 0	122.5221	1194.5906	**44** 0	138.2301	1520.5308
1	107.0759	912.3763	1	122.7839	1199.7011	1	138.4919	1526.2959
2	107.3377	916.8433	2	123.0457	1204.8226	2	138.7537	1532.0718
3	107.5995	921.3211	3	123.3075	1209.9550	3	139.0155	1537.8587
4	107.8613	925.8099	4	123.5693	1215.0982	4	139.2773	1543.6565
5	108.1231	930.3096	5	123.8311	1220.2524	5	139.5391	1549.4651
6	108.3849	934.8202	6	124.0929	1225.4175	6	139.8009	1555.2847
7	108.6467	939.3417	7	124.3547	1230.5935	7	140.0627	1561.1152
8	108.9085	943.8741	8	124.6165	1235.7804	8	140.3245	1566.9566
9	109.1703	948.4174	9	124.8783	1240.9782	9	110.5863	1572 8089
10	109.4321	952.9716	10	125.1401	1246.1869	10	140.8481	1578.6721
11	109.6939	957.5367	11	125.4019	1251.4065	11	141.1099	1584.5462

TABLE 3 OF CIRCLES—(Continued).

Diams in units and twelfths; as in **feet and inches.**

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.
45 0	141.3717	1590.4313	**50** 0	157.0796	1963.4954	**55** 0	172.7876	2375.8294
1	141.6335	1596.3272	1	157.3414	1970.0458	1	173.0494	2383.0344
2	141.8953	1602.2341	2	157.6032	1976.6072	2	173.3112	2390.2502
3	142.1571	1608.1518	3	157.8650	1983.1794	3	173.5730	2397.4770
4	142 4189	1614.0805	4	158.1268	1989.7626	4	173.8348	2404.7146
5	142.6807	1620.0201	5	158.3886	1996.3567	5	174.0966	2411.9632
6	142.9425	1625.9705	6	158.6504	2002.9617	6	174.3584	2419.2227
7	143.2043	1631.9319	7	158.9122	2009.5776	7	174.6202	2426.4931
8	143.4661	1637.9042	8	159.1740	2016.2044	8	174.8820	2433.7744
9	143.7279	1643.8874	9	159.4358	2022.8421	9	175.1438	2441.0666
10	143.9897	1649.8816	10	159.6976	2029.4907	10	175.4056	2448.3697
11	144.2515	1655.8866	11	159.9594	2036.1502	11	175.6674	2455.6837
46 0	144.5133	1661.9025	**51** 0	160.2212	2042.8206	**56** 0	175.9292	2463.0086
1	144.7751	1667.9294	1	160.4830	2049.5020	1	176.1910	2470.3445
2	145.0369	1673.9671	2	160.7448	2056.1942	2	176.4528	2477.6912
3	145.2987	1680.0158	3	161.0066	2062.8974	3	176.7146	2485.0489
4	145.5605	1686.0753	4	161.2684	2069.6114	4	176.9764	2492.4174
5	145.8223	1692.1458	5	161.5302	2076.3364	5	177.2382	2499.7969
6	146.0841	1698.2272	6	161.7920	2083.0723	6	177.5000	2507.1873
7	146.3459	1704.3195	7	162.0538	2089.8191	7	177.7618	2514.5886
8	146.6077	1710.4227	8	162.3156	2096.5768	8	178.0236	2522.0008
9	146.8695	1716.5368	9	162.5774	2103.3454	9	178,2854	2529.4239
10	147.1313	1722.6618	10	162.8392	2110.1249	10	178.5472	2536.8579
11	147.3931	1728.7977	11	163.1010	2116.9153	11	178.8090	2544.3028
47 0	147.6549	1734.9445	**52** 0	163.3628	2123.7166	**57** 0	179.0708	2551.7586
1	147.9167	1741.1023	1	163.6246	2130.5289	1	179.3326	2559.2254
2	148.1785	1747.2709	2	163.8864	2137.3520	2	179.5944	2566.7030
3	148.4403	1753.4505	3	164.1482	2144.1861	3	179.8562	2574.1916
4	148.7021	1759.6410	4	164.4100	2151.0310	4	180.1180	2581.6910
5	148.9639	1765.8423	5	164.6718	2157.8869	5	180.3798	2589.2014
6	149.2257	1772.0546	6	164.9336	2164.7537	6	180.6416	2596.7227
7	149.4875	1778.2778	7	165.1954	2171.6314	7	180.9034	2604.2549
8	149.7492	1784.5119	8	165.4572	2178.5200	8	181.1652	2611.7980
9	150.0110	1790.7569	9	165.7190	2185.4195	9	181.4270	2619.3520
10	150.2728	1797.0128	10	165.9808	2192.3299	10	181.6888	2626.9169
11	150.5346	1803.2796	11	166.2426	2199.2512	11	181.9506	2634.4927
48 0	150.7964	1809.5574	**53** 0	166.5044	2206.1834	**58** 0	182.2124	2642.0794
1	151.0582	1815.8460	1	166.7662	2213.1266	1	182.4742	2649.6771
2	151.3200	1822.1456	2	167.0280	2220.0806	2	182.7360	2657.2856
3	151.5818	1828.4560	3	167.2898	2227.0456	3	182.9978	2664.9051
4	151.8436	1834.7774	4	167.5516	2234.0214	4	183.2596	2672.5354
5	152.1054	1841.1096	5	167.8134	2241.0082	5	183.5214	2680.1767
6	152.3672	1847.4528	6	168.0752	2248.0059	6	183.7832	2687.8289
7	152.6290	1853.8069	7	168.3370	2255.0145	7	184.0450	2695.4920
8	152.8908	1860.1719	8	168.5988	2262.0340	8	184.3068	2703.1659
9	153.1526	1866.5478	9	168.8606	2269.0644	9	184.5686	2710.8508
10	153.4144	1872.9346	10	169.1224	2276.1057	10	184.8304	2718.5467
11	153.6762	1879.3324	11	169.3842	2283.1579	11	185.0922	2726.2534
49 0	153.9380	1885.7410	**54** 0	169.6460	2290.2210	**59** 0	185.3540	2733.9710
1	154.1998	1892.1605	1	169.9078	2297.2951	1	185.6158	2741.6995
2	154.4616	1898.5910	2	170.1696	2304.3800	2	185.8776	2749.4390
3	154.7234	1905.0323	3	170.4314	2311.4759	3	186.1394	2757.1893
4	154.9852	1911.4846	4	170.6932	2318.5826	4	186.4012	2764.9506
5	155 2470	1917.9478	5	170.9550	2325.7003	5	186.6630	2772.7228
6	155.5088	1924.4218	6	171.2168	2332.8289	6	186.9248	2780.5058
7	155.7706	1930.9068	7	171.4786	2339.9684	7	187.1866	2788.2998
8	156.0324	1937.4027	8	171.7404	2347.1188	8	187.4484	2796.1047
9	156.2942	1943.9095	9	172.0022	2354.2801	9	187.7102	2803.9205
10	156.5560	1950.4273	10	172.2640	2361.4523	10	187.9720	2811.7472
11	156.8178	1956.9559	11	172.5258	2368.6354	11	188.2338	2819.5849

TABLE 3 OF CIRCLES—(Continued).

Diams in units and twelfths; as in feet and inches.

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.
60 0	188.4956	2827.4334	65 0	204.2035	3318.3072	70 0	219.9115	3848.4510
1	188.7574	2835.2928	1	204.4653	3326.8212	1	220.1733	3857.6194
2	189.0192	2843.1632	2	204.7271	3335.3460	2	220.4351	3866.7988
3	189.2810	2851.0444	3	204.9889	3343.8818	3	220.6969	3875.9890
4	189.5428	2858.9366	4	205.2507	3352.4284	4	220.9587	3885.1902
5	189.8046	2866.8397	5	205.5125	3360.9860	5	221.2205	3894.4022
6	190.0664	2874.7536	6	205.7743	3369.5545	6	221.4823	3903.6252
7	190.3282	2882.6785	7	206.0361	3378.1339	7	221.7441	3912.8591
8	190.5900	2890.6143	8	206.2979	3386.7241	8	222.0059	3922.1039
9	190.8518	2898.5610	9	206.5597	3395.3253	9	222.2677	3931.3596
10	191.1136	2906.5186	10	206.8215	3403.9375	10	222.5295	3940.6262
11	191.3754	2914.4871	11	207.0833	3412.5605	11	222.7913	3949.9037
61 0	191.6372	2922.4666	66 0	207.3451	3421.1944	71 0	223.0531	3959.1921
1	191.8990	2930.4569	1	207.6069	3429.8392	1	223.3149	3968.4915
2	192.1608	2938.4581	2	207.8687	3438.4950	2	223.5767	3977.8017
3	192.4226	2946.4703	3	208.1305	3447.1616	3	223.8385	3987.1229
4	192.6843	2954.4934	4	208.3923	3455.8392	4	224.1003	3996.4549
5	192.9461	2962.5273	5	208.6541	3464.5277	5	224.3621	4005.7979
6	193.2079	2970.5722	6	208.9159	3473.2270	6	224.6239	4015.1518
7	193.4697	2978.6280	7	209.1777	3481.9373	7	224.8857	4024 5165
8	193.7315	2986.6947	8	209.4395	3490.6585	8	225.1475	4033.8922
9	193.9933	2994.7723	9	209.7013	3499.3906	9	225.4093	4043.2788
10	194.2551	3002.8608	10	209.9631	3508.1336	10	225.6711	4052.6763
11	194.5169	3010.9602	11	210.2249	3516.8875	11	225.9329	4062.0848
62 0	194.7787	3019.0705	67 0	210.4867	3525.6524	72 0	226.1947	4071.5041
1	195.0405	3027.1918	1	210.7485	3534.4281	1	226.4565	4080.9343
2	195.3023	3035.3239	2	211.0103	3543.2147	2	226.7183	4090.3755
3	195.5641	3043.4670	3	211.2721	3552.0123	3	226.9801	4099.8275
4	195.8259	3051.6209	4	211.5339	3560.8207	4	227.2419	4109.2905
5	196.0877	3059.7858	5	211.7957	3569.6401	5	227.5037	4118.7643
6	196.3495	3067.9616	6	212.0575	3578.4704	6	227.7655	4128.2491
7	196.6113	3076.1483	7	212.3193	3587.3116	7	228.0273	4137.7448
8	196.8731	3084.3459	8	212.5811	3596.1637	8	228.2891	4147.2514
9	197.1349	3092.5544	9	212.8429	3605.0267	9	228.5509	4156.7689
10	197.3967	3100.7738	10	213.1047	3613.9006	10	228.8127	4166 2973
11	197.6585	3109.0041	11	213.3665	3622.7854	11	229.0745	4175.8366
63 0	197.9203	3117.2453	68 0	213.6283	3631.6811	73 0	229.3363	4185.3868
1	198.1821	3125.4974	1	213.8901	3640.5877	1	229.5981	4191.9479
2	198.4439	3133.7605	2	214.1519	3649.5053	2	229 8599	4204.5200
3	198.7057	3142.0344	3	214.4137	3658.4337	3	230.1217	4214.1029
4	198.9675	3150.3193	4	214.6755	3667.3731	4	230.3835	4223 6968
5	199.2293	3158.6151	5	214.9373	3676.3234	5	230.6453	4233.3016
6	199.4911	3166.9217	6	215.1991	3685.2845	6	230.9071	4242.9172
7	199.7529	3175.2393	7	215.4609	3694.2566	7	231.1689	4252.5438
8	200.0147	3183.5678	8	215.7227	3703.2396	8	231.4307	4262.1813
9	200.2765	3191.9072	9	215.9845	3712.2335	9	231.6925	4271.8297
10	200.5383	3200.2575	10	216.2463	3721.2383	10	231.9543	4281.4890
11	200.8001	3208.6188	11	216.5081	3730.2540	11	232.2161	4291.1592
64 0	201.0619	3216.9909	69 0	216.7699	3739.2807	74 0	232.4779	4300.8403
1	201.3237	3225.3739	1	217.0317	3748.3182	1	232.7397	4310.5324
2	201.5855	3233.7679	2	217.2935	3757.3666	2	233.0015	4320.2353
3	201.8473	3242.1727	3	217.5553	3766.4260	3	233.2633	4329.9492
4	202.1091	3250.5885	4	217.8171	3775.4962	4	233.5251	4339.6739
5	202.3709	3259.0151	5	218.0789	3784.5774	5	233.7869	4349.4096
6	202.6327	3267.4527	6	218.3407	3793.6695	6	234.0487	4359.1562
7	202.8945	3275.9012	7	218.6025	3802.7725	7	234.3105	4368.9136
8	203.1563	3284.3606	8	218.8643	3811.8864	8	234.5723	4378.6820
9	203 4181	3292.8309	9	219.1261	3821.0112	9	234.8341	4388.4613
10	203.6799	3301.3121	10	219.3879	3830.1469	10	235.0959	4398.2515
11	203.9417	3309.8042	11	219.6497	3839.2935	11	235.3576	4408.0526

TABLE 3 OF CIRCLES—(Continued).

Diams in units and twelfths; as in **feet and inches.**

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.
75 0	235.6194	4417.8647	80 0	251.3274	5026.5482	85 0	267.0354	5674.5017
1	235.8812	4427.6876	1	251.5892	5037.0257	1	267.2972	5685.6337
2	236.1430	4437.5214	2	251.8510	5047.5140	2	267.5590	5696.7765
3	236.4048	4447.3662	3	252.1128	5058.0133	3	267.8208	5707.9302
4	236.6666	4457.2218	4	252.3746	5068.5234	4	268.0826	5719.0949
5	236.9284	4467.0884	5	252.6364	5079.0445	5	268.3444	5730.2705
6	237.1902	4476.9659	6	252.8982	5089.5764	6	268.6062	5741.4569
7	237.4520	4486.8543	7	253.1600	5100.1193	7	268.8680	5752.6543
8	237.7138	4496.7536	8	253.4218	5110.6731	8	269.1298	5763.8626
9	237.9756	4506.6637	9	253.6836	5121.2378	9	269.3916	5775.0818
10	238.2374	4516.5849	10	253.9454	5131.8134	10	269.6534	5786.3119
11	238.4992	4526.5169	11	254.2072	5142.3999	11	269.9152	5797.5529
76 0	238.7610	4536.4598	81 0	254.4690	5152.9974	86 0	270.1770	5808.8048
1	239.0228	4546.4136	1	254.7308	5163.6057	1	270.4388	5820.0676
2	239.2846	4556.3784	2	254.9926	5174.2249	2	270.7006	5831.3414
3	239.5464	4566.3540	3	255.2544	5184.8551	3	270.9624	5842.6260
4	239.8082	4576.3406	4	255.5162	5195.4961	4	271.2242	5853.9216
5	240.0700	4586.3380	5	255.7780	5206.1481	5	271.4860	5865.2280
6	240.3318	4596.3464	6	256.0398	5216.8110	6	271.7478	5876.5454
7	240.5936	4606.3657	7	256.3016	5227.4847	7	272.0096	5887.8737
8	240.8554	4616.3959	8	256.5634	5238.1694	8	272.2714	5899.2129
9	241.1172	4626.4370	9	256.8252	5248.8650	9	272.5332	5910.5630
10	241.3790	4636.4890	10	257.0870	5259.5715	10	272.7950	5921.9240
11	241.6408	4646.5519	11	257.3488	5270.2889	11	273.0568	5933.2959
77 0	241.9026	4656.6257	82 0	257.6106	5281.0173	87 0	273.3186	5944.6787
1	242.1644	4666.7104	1	257.8724	5291.7565	1	273.5804	5956.0724
2	242.4262	4676.8061	2	258.1342	5302.5066	2	273.8422	5967.4771
3	242.6880	4686.9126	3	258.3960	5313.2677	3	274.1040	5978.8926
4	242.9498	4697.0301	4	258.6578	5324.0396	4	274.3658	5990.3191
5	243.2116	4707.1584	5	258.9196	5334.8225	5	274.6276	6001.7564
6	243.4734	4717.2977	6	259.1814	5345.6162	6	274.8894	6013.2047
7	243.7352	4727.4479	7	259.4432	5356.4209	7	275.1512	6024.6639
8	243.9970	4737.6090	8	259.7050	5367.2365	8	275.4130	6036.1340
9	244.2588	4747.7810	9	259.9668	5378.0630	9	275.6748	6047.6149
10	244.5206	4757.9639	10	260.2286	5388.9004	10	275.9366	6059.1068
11	244.7824	4768.1577	11	260.4904	5399.7487	11	276.1984	6070.6097
78 0	245.0442	4778.3624	83 0	260.7522	5410.6079	88 0	276.4602	6082.1234
1	245.3060	4788.5781	1	261.0140	5421.4781	1	276.7220	6093.6480
2	245.5678	4798.8046	2	261.2758	5432.3591	2	276.9838	6105.1835
3	245.8296	4809.0420	3	261.5376	5443.2511	3	277.2456	6116.7300
4	246.0914	4819.2904	4	261.7994	5454.1539	4	277.5074	6128.2873
5	246.3532	4829.5497	5	262.0612	5465.0677	5	277.7692	6139.8556
6	246.6150	4839.8198	6	262.3230	5475.9923	6	278.0309	6151.4348
7	246.8768	4850.1009	7	262.5848	5486.9279	7	278.2927	6163.0248
8	247.1386	4860.3929	8	262.8466	5497.8744	8	278.5545	6174.6258
9	247.4004	4870 6958	9	263.1084	5508.8318	9	278.8163	6186.2377
10	247.6622	4881.0096	10	263.3702	5519.8001	10	279.0781	6197.8605
11	247.9240	4891.3343	11	263.6320	5530.7793	11	279.3399	6209.4942
79 0	248.1858	4901.6699	84 0	263.8938	5541.7694	89 0	279.6017	6221.1389
1	248.4476	4912.0165	1	264.1556	5552.7705	1	279.8635	6232.7944
2	248.7094	4922.3739	2	264.4174	5563.7824	2	280.1253	6244.4608
3	248.9712	4932.7423	3	264.6792	5574.8053	3	280.3871	6256.1382
4	249.2330	4943.1215	4	264.9410	5585.8390	4	280.6489	6267.8264
5	249.4948	4953.5117	5	265.2028	5596.8837	5	280.9107	6279.5256
6	249.7566	4963.9127	6	265.4646	5607.9392	6	281.1725	6291.2356
7	250.0184	4974.3247	7	265.7264	5619.0057	7	281.4343	6302.9566
8	250.2802	4984.7476	8	265.9882	5630.0831	8	281.6961	6314.6885
9	250.5420	4995.1814	9	266.2500	5641.1714	9	281.9579	6326.4313
10	250.8038	5005.6261	10	266.5118	5652.2706	10	282.2197	6338.1850
11	251.0656	5016.0817	11	266.7736	5663.3807	11	282.4815	6349.9496

TABLE 3 OF CIRCLES—(Continued).

Diams in units and twelfths; as in **feet and inches.**

Dia.	Circumf.	Area.	Dia.	Circumf.	Area.	Dia.	Circumf.	Area.
Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.	Ft.In.	Feet.	Sq. ft.
90 0	282.7433	6361.7251	93 5	293.4771	6853.9134	96 9	303.9491	7351.7686
1	283.0051	6373.5116	6	293.7389	6866.1471	10	304.2109	7364.4386
2	283.2669	6385.3089	7	294.0007	6878.3917	11	304.4727	7377.1195
3	283.5287	6397.1171	8	294.2625	6890.6472	97 0	304.7345	7389.8113
4	283.7905	6408.9363	9	294.5243	6902.9135	1	304.9963	7402.5140
5	284.0523	6420.7663	10	294.7861	6915.1908	2	305.2581	7415.2277
6	284.3141	6432.6073	11	295.0479	6927.4791	3	305.5199	7427.9522
7	284.5759	6444.4592	94 0	295.3097	6939.7782	4	305.7817	7440.6877
8	284.8377	6456.3220	1	295.5715	6952.0882	5	306.0435	7453.4340
9	285.0995	6468.1957	2	295.8333	6964.4091	6	306.3053	7466.1913
10	285.3613	6480.0803	3	296.0951	6976.7410	7	306.5671	7478.9595
11	285.6231	6491.9758	4	296.3569	6989.0837	8	306.8289	7491.7385
91 0	285.8849	6503.8822	5	296.6187	7001.4374	9	307.0907	7504.5285
1	286.1467	6515.7995	6	296.8805	7013.8019	10	307.3525	7517.3294
2	286.4085	6527.7278	7	297.1423	7026.1774	11	307.6143	7530.1412
3	286.6703	6539.6669	8	297.4041	7038.5638	98 0	307.8761	7542.9640
4	286.9321	6551.6169	9	297.6659	7050.9611	1	308.1379	7555.7976
5	287.1939	6563.5779	10	297.9277	7063.3693	2	308.3997	7568.6421
6	287.4557	6575.5498	11	298.1895	7075.7884	3	308.6615	7581.4976
7	287.7175	6587.5325	95 0	298.4513	7088.2184	4	308.9233	7594.3639
8	287.9793	6599.5262	1	298.7131	7100.6593	5	309.1851	7607.2412
9	288.2411	6611.5308	2	298.9749	7113.1112	6	309.4469	7620.1293
10	288.5029	6623.5463	3	299.2367	7125.5739	7	309.7087	7633.0284
11	288.7647	6635.5727	4	299.4985	7138.0476	8	309.9705	7645.9384
92 0	289.0265	6647.6101	5	299.7603	7150.5321	9	310.2323	7658.8593
1	289.2883	6659.6583	6	300.0221	7163.0276	10	310.4941	7671.7911
2	289.5501	6671.7174	7	300.2839	7175.5340	11	310.7559	7684.7338
3	289.8119	6683.7875	8	300.5457	7188.0513	99 0	311.0177	7697.6874
4	290.0737	6695.8684	9	300.8075	7200.5794	1	311.2795	7710.6519
5	290.3355	6707.9603	10	301.0693	7213.1185	2	311.5413	7723.6274
6	290.5973	6720.0630	11	301.3311	7225.6686	3	311.8031	7736.6137
7	290.8591	6732.1767	96 0	301.5929	7238.2295	4	312.0649	7749.6109
8	291.1209	6744.3013	1	301.8547	7250.8013	5	312.3267	7762.6191
9	291.3827	6756.4368	2	302.1165	7263.3840	6	312.5885	7775.6382
10	291.6445	6768.5832	3	302.3783	7275.9777	7	312.8503	7788.6681
11	291.9063	6780.7405	4	302.6401	7288.5822	8	313.1121	7801.7090
93 0	292.1681	6792.9087	5	302.9019	7301.1977	9	313.3739	7814.7608
1	292.4299	6805.0878	6	303.1637	7313.8240	10	313.6357	7827.8235
2	292.6917	6817.2779	7	303.4255	7326.4613	11	313.8975	7840.8971
3	292.9535	6829.4788	8	303.6873	7339.1095	100 0	314.1593	7853.9816
4	293.2153	6841.6907						

Circumferences in feet, when the diam contains fractions of an inch. See similar process, p 133.

Diam, Inch.	Circumf, foot	Diam, Inch.	Circumf, foot	Diam, Inch	Circumf, foot.	Diam, Inch.	Circumf, foot.	Diam, Inch.	Circumf, foot.
1-64	.004091	7-32	.057269	27-64	.110447	5-8	.163625	53-64	.216803
1-32	.008181	15-64	.061359	7-16	.114537	41-64	.167715	27-32	.220893
3-64	.012272	1/4	.065450	29-64	.118628	21-32	.171806	55-64	.224984
1-16	.016362	17-64	.069540	15-32	.122718	43-64	.175896	7-8	.229074
5-64	.020453	9-32	.073631	31-64	.126809	11-16	.179987	57-64	.233165
3-32	.024544	19-64	.077722	1/2	.130900	45-64	.184078	29-32	.237256
7-64	.028634	5-16	.081812	33-64	.134990	23-32	.188168	59-64	.241346
1/8	.032725	21-64	.085903	17-32	.139081	47-64	.192259	15-16	.245437
9-64	.036816	11-32	.089994	35-64	.143172	3/4	.196350	61-64	.249528
5-32	.040906	23-64	.094084	9-16	.147262	49-64	.200440	31-32	.253618
11-64	.044997	3/8	.098175	37-64	.151353	25-32	.204531	63-64	.257709
3-16	.049087	25-64	.102265	19-32	.155443	51-64	.208621	1	.261799
13-64	.053178	13-32	.106356	39-64	.159534	13-16	.212712		

CIRCULAR ARCS.

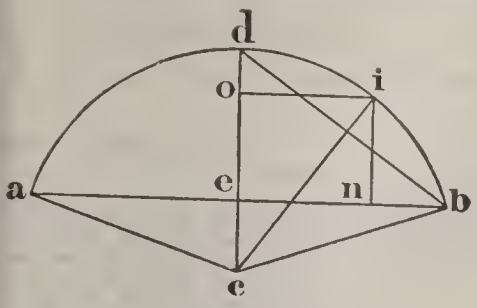


Fig. 1

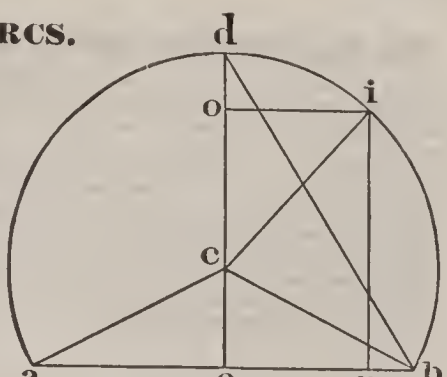


Fig. 2.

Rules for Fig. 1 apply to all arcs equal to, or less than, a semi-circle.
" " Fig. 2 " " " " or greater than, a semi-circle.

Chord, *a b*, of whole arc, *a d b*,

$= 2 \times \sqrt{\text{rad}^2 - (\text{rad} - \text{rise})^2}$. Fig 1.

$= 2 \times \sqrt{\text{rad}^2 - (\text{rise} - \text{rad})^2}$. Fig 2.

$= 2 \times \sqrt{\text{rise} \times [(2 \times \text{rad}) - \text{rise}]}$. Figs 1 and 2.

$= (8 \times d\,b\S) - (3 \times$ lgth of arc $a\,d\,b)$ ¶ Fig 1.
$= 2 \times$ rad $\times$ sine of $\frac{1}{2}\,a\,c\,b$. Figs 1 and 2.
$= 2 \times$ rise $\div$ tang of $a\,b\,d$.* Figs 1 and 2.
$= 2 \times d\,b\S \times$ cosine of $a\,b\,d$.* Figs 1 and 2.
$= 2 \times \sqrt{d\,b^2 - \text{rise}^2}$. Figs 1 and 2.§

For table of chords to radius 1, see p 105.

Length, *a d b*,

= rad $\times$ 6.2832 $\times$ number of degrees subtended by arc $a\,d\,b \div 360$. Figs 1, 2. If the arc contains fractions of a degree, see table, p. 57.
= chord $\times$ number in column of lengths, pp 143, 144, opp "rise $\div$ chord." Fig 1. For Fig 2, see p 144.
= rad $\times$ sum of numbers in columns of lengths, p 145, opp degs, mins, &c subtended by arc. Fig 1.
= (rad $\times$ 6.2832) $-$ lgth of *small* arc subtending ang $a\,c\,b$. Fig 2. For said lgth see preceding formula.
= approximately $[(8 \times d\,b\S) -$ chord $a\,b] \div 3$. Fig 1.**

Radius, *c a*, *c d*, *c i*, or *c b*,

$= [(\frac{1}{2}\,a\,b)^2 + \text{rise}^2] \div (2 \times \text{rise})$. Figs 1, 2.
$= d\,b^2\S \div (2 \times \text{rise})$. Figs 1, 2.
$= \frac{1}{2}\,a\,b \div$ sine of $\frac{1}{2}\,a\,c\,b$. Figs 1, 2.

= rise $\div$ (1 $-$ cosine of $\frac{1}{2}\,a\,c\,b$). Fig 1.
= rise $\div$ (1 $+$ cosine of $\frac{1}{2}\,a\,c\,b$†). Fig 2.
$= \frac{1}{2}\,d\,b \div$ sine of $\frac{1}{2}\,b\,c\,d$‖. Figs 1 and 2.§

For tables of radii to chords of 100 ft, and of 20 metres, see pp 726, 728.

Rise, or middle ordinate, *e d*,

$= \text{rad} - \sqrt{\text{rad}^2 - (\frac{1}{2}\,a\,b)^2}$. Fig 1.

$= \text{rad} + \sqrt{\text{rad}^2 - (\frac{1}{2}\,a\,b)^2}$. Fig 2.

= rad $\times$ (1 $-$ cosine of $b\,c\,d$‖). Fig 1.
= rad $\times$ (1 $+$ cosine of $b\,c\,d$‖)†. Fig 2.
$= d\,b^2\S \div (2 \times \text{rad})$. Figs 1, 2.

$= \frac{1}{2}\,a\,b \times$ tang of $a\,b\,d$.* Figs 1, 2.
$=$ approx $(\frac{1}{2}\,a\,b)^2 \div (2 \times \text{rad})$. Fig 1.
If rad $= a\,b$, add .067 of the result.
If rad $= 3\,a\,b$, add .007 "

For tables of mid ords, see pp 726-730.

Side ordinate, *n i*,

$= \sqrt{\text{rad}^2 - e\,n^2} - (\text{rad} - \text{rise})$. Fig 1.

$= \sqrt{\text{rad}^2 - e\,n^2} + (\text{rise} - \text{rad})$. Fig 2.

= approximately $\frac{a\,n \times n\,b}{2 \times \text{radius}}$. Fig 1.

For tables of side ordinates, see p 730.

Angle, *a c b*, subtended by arc, *a d b*,

An angle and its supplement (as $b\,c\,e$ and $b\,c\,d$, Fig 2) have the same *sine*, *cosine*, and *tangent*.

Caution. The following sines, &c, are those of only *half* $a\,c\,b$.

Sine of $\frac{1}{2}\,a\,c\,b = \frac{1}{2}\,a\,b \div$ rad. Figs 1, 2.
Cosine " = (rad $-$ rise) $\div$ rad. Fig 1.
= (rise $-$ rad) $\div$ rad. Fig 2.

Tang of $\frac{1}{2}\,a\,c\,b$ = $\frac{1}{2}\,a\,b \div$ (rad $-$ rise). Fig 1.
= $\frac{1}{2}\,a\,b \div$ (rise $-$ rad). Fig 2.
Versed sine " = rise $\div$ rad. Figs 1, 2.

For **arcas of segments** $a\,d\,b\,e$, see p 146. For **arcas of sectors** $a\,d\,b\,c$, see p 146. For **ccns of grav** of arcs, segments, and sectors, see p. 348.

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* $a\,b\,d$ is $= \frac{1}{4}$ of the angle, $a\,c\,b$, subtended by the *arc*. In Fig 2 the latter angle exceeds 180°.
† Strictly, this should read 1 *minus* cosine, but the cosines of angles between 90° and 270° must then be regarded as *minus* or *negative*. Our rule, therefore, amounts practically to the same thing.
‖ $b\,c\,d$ = half the angle $a\,c\,b$ subtended by the *arc*.

¶ If rise = .5 ch'd, add .036 of the result.
" = .4 " " .0196 "
" = .333 " " .0114 "
" = .3 " " .0083 "

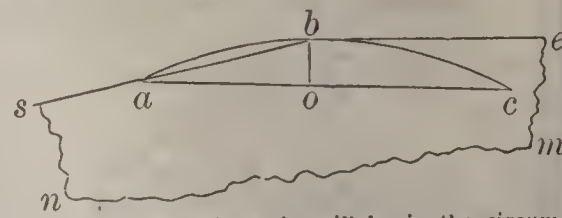
If rise = .25 ch'd, add .0044 of the result.
" = .2 " " .0021 "
" = .125 " " .00036 "
" = .1 " " .00015 "

§ $d\,b$ = chord of *half*-arc $d\,i\,b = \sqrt{\text{rise}^2 + (\frac{1}{2}\,a\,b)^2}$. Figs 1, 2.

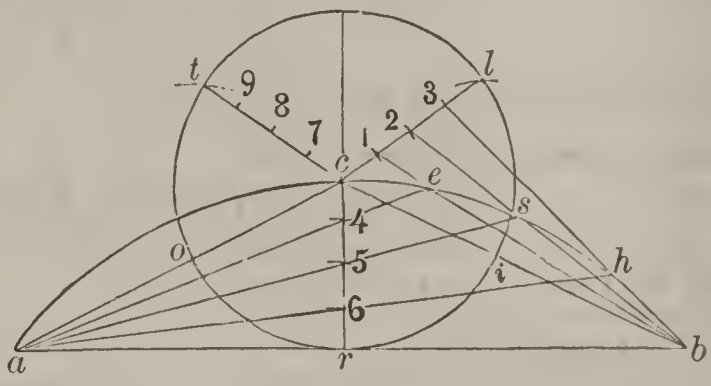
** If rise = .5 ch'd, add .012 of the result.
" = .4 " " .0065 "
" = .333 " " .0038 "
" = .3 " " .0028 "

If rise = .25 ch'd, add .0015 of the result.
" = .2 " " .0007 "
" = .125 " " .00012 "
" = .1 " " .00005 "

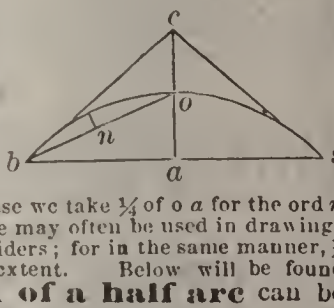
To describe the arc of a circle too large for the dividers.



Let *a c* he the chord; and *o b* the height of the reqd arc, as laid down *on the drawing*. On a separate strip of paper, *s e m n*, draw *a c*, *o b*, and *a b*: also *b e*, parallel to the chord *a c*. It is well to make *b s*, and *b e*, each a little longer than *a b*. Then cut off the paper carefully along the lines *s b* and *b e*, so as to leave remaining only the strip *s a b e m n*. Now, if the straight sides *s b* and *b e* be applied to the drawing, so that any parts of them shall touch at the same time the points *a* and *b*, or *b* and *c*, the point *b* on the strip will be in the circumf of the arc, and may be pricked off. Thus, any number of points in the arc may be found, and afterward united to form the curve.



Or thus: draw the span *a b*; the rise *r c*; and *a c*, *b c*. From *c* with rad *c r* describe a circle. Make each of the arcs *o t* and *i l* equal to *r o* or *r i*; and draw *c t*, *c l*. Div *c t*, *c l*, *c r*, each into half as many equal parts as the curve is to be divided into. Draw the lines *b* 1, *b* 2, *b* 3; and *a* 4, *a* 5, *a* 6, extended to meet the first ones at *e*, *s*, *h*. Then *e*, *s*, *h*, are points in one half the curve. Then for the other half, draw similar lines from *a* to 7, 8, 9; and others from *b* to meet them, as before. Trace the curve by hand.



Remark.—It may frequently be of use to remember, that in any arc *b o s*, not exceeding 29°, or in other words, *whose chord b s is at least sixteen times its rise*, the middle ordinate *a o*, will be one half of *a c*, quite near enough for many purposes; *b c* and *s c* being tangents to the arc.† And vice versa, if in such an arc we make *o c* equal *a o*, then will *c* be, very nearly, the point at which tangents from the ends of the arc will meet. Also the middle ordinate *n*, of the half arc *o b*, or *o s*, will be approximately ¼ of *a o*, the mid ord of the whole arc. Indeed, this last observation will apply near enough for many approximate uses even if the arc he as great as 45°; for if in that case we take ¼ of *o a* for the ord *n*, *n* will then be but 1 part in 103 too small; and therefore the principle may often be used in drawings, for finding points in a curve of too great rad to be drawn by the dividers; for in the same manner, ¼ of *n* will be the mid ord for the arc *n b* or *n o*; and so on to any extent. Below will be found **a table by which the rise or middle ord of a half arc** can be obtained with greater accuracy when required for more exact drawings.

CIRCULAR ARCS IN FREQUENT USE.

The fifth column is of use for finding points for drawing arcs too large for the beam-compass, on the principle given above. In even the largest office drawings it will not be necessary to use more than the first three decimals of the fifth column; and after the arc is subdivided into parts smaller than about 35° each, the first two decimals .25 will generally suffice. Original.

Rise in parts of chord.	Deg in whole arc.	For rad mult rise by	For length of arc mult chord by	For rise of half arc mult rise by	Rise in parts of chord.	Deg in whole arc.	For rad mult rise by	For length of arc mult chord by	For rise of half arc mult rise by
	° ′					° ′			
1-50	9 9.75	313.	1.00107	.2501	⅛	56 8.70	8.5	1.04116	.2538
1-45	10 10.75	253.625	1.00132	.2501	1-7	63 46.90	6.625	1.05356	.2549
1-40	11 26.98	200.5	1.00167	.2502	.155	68 53.63	5.70291	1.06288	.2557
1-35	13 4.92	153.625	1.00219	.2502	1-6	73 44.39	5.	1.07250	.2566
1-30	15 15.38	113.	1.00296	.2503	.18	79 11.73	4.35803	1.08428	.2576
1-25	18 17.74	78.625	1.00426	.2504	1-5	87 12.34	3.625	1.10347	.2593
1-20	22 50.54	50 5	1.00665	.2506	.207107	90	3.41422	1.11072	.2599
1-19	24 2.16	45.625	1.00737	.2507	.225	96 54.67	2.96913	1.12997	.2615
1-18	25 21.65	41.	1.00821	.2508	¼	106 15.61	2.5	1.15912	.2639
1-17	26 50.36	36.625	1.00920	.2509	.275	115 14.59	2.15289	1.19082	.2665
1-16	28 30.00	32.5	1.01038	.2510	.3	123 51.30	1.88889	1.22495	.2692
1-15	30 22.71	28.625	1.01181	.2511	⅓	134 45.62	1.625	1.27401	.2729
1-14	32 31.22	25.	1.01355	.2513	.365	144 30 98	1.43827	1.32413	.2766
1-13	34 59.08	21.625	1.01571	.2515	.4	154 38.35	1.28125	1.38322	.2808
1-12	37 50.96	18.5	1.01842	.2517	.425	161 27.52	1 19204	1.42764	.2838
1-11	41 13.16	15.625	1.02189	.2520	.45	167 56.93	1.11728	1.47377	.2868
1-10	45 14.38	13.	1.02646	.2525	.475	174 7.49	1.05402	1.52152	.2899
1-9	50 6.91	10.625	1.03260	.2530	.5	180	1.	1.57080	.2929

† At 29° *o c* thus found will be but about $\frac{1}{33}$ part too short.

Lengths of circular arcs. If arc exceeds a semicircle, see p 144.

Knowing its chord and height, divide the height by the chord. Find in the column of heights the number equal to this quotient. Take out the corresponding number from the column of lengths. Multiply this last number by the length of the given chord.

TABLE OF CIRCULAR ARCS.

No errors.

H'ghts.	Lengths.	H'ghts.	Lengths.	H'ghts.	Lengths.	H'ghts.	Lengths.	H'ghts.	Lengths.
.001	1.00002	.076	1.01533	.151	1.05973	.226	1.13108	.301	1.22636
002	1.00002	.077	1.01573	.152	1.06051	.227	1.13219	.302	1.22778
.003	1.00003	.078	1.01614	.153	1.06130	.228	1.13331	.303	1.22920
.004	1.00004	.079	1.01656	.154	1.06209	.229	1.13444	.304	1.23063
.005	1.00007	.080	1.01698	.155	1.06288	.230	1.13557	.305	1.23206
.006	1.00010	.081	1.01741	.156	1.06368	.231	1.13671	.306	1.23349
.007	1.00013	.082	1.01784	.157	1.06449	.232	1.13785	.307	1.23492
.008	1.00017	.083	1.01828	.158	1.06530	.233	1.13900	.308	1.23636
.009	1.00022	.084	1.01872	.159	1.06611	.234	1.14015	.309	1.23781
010	1.00027	.085	1.01916	.160	1.06693	.235	1.14131	.310	1.23926
.011	1.00032	.086	1.01961	.161	1.06775	.236	1.14247	.311	1.24070
012	1.00038	.087	1.02006	.162	1.06858	.237	1.14363	.312	1.24216
.013	1.00045	.088	1.02052	.163	1.06941	.238	1.14480	.313	1.24361
.014	1.00053	.089	1.02098	.164	1.07025	.239	1.14597	.314	1.24507
.015	1.00061	.090	1.02146	.165	1.07109	.240	1.14714	.315	1.24654
.016	1.00069	.091	1.02192	.166	1.07194	.241	1.14832	.316	1.24801
017	1.00078	.092	1.02240	.167	1.07279	.242	1.14951	.317	1.24948
.018	1.00087	.093	1.02289	.168	1.07365	.243	1.15070	.318	1.25095
.019	1.00097	.094	1.02339	.169	1.07451	.244	1.15189	.319	1.25243
.020	1.00107	.095	1.02389	.170	1.07537	.245	1.15308	.320	1.25391
.021	1.00117	.096	1.02440	.171	1.07624	.246	1.15428	.321	1.25540
.022	1.00128	097	1.02491	.172	1.07711	.247	1.15549	.322	1.25689
.023	1.00140	.098	1.02542	.173	1.07799	.248	1.15670	.323	1.25838
.024	1.00153	.099	1.02593	.174	1.07888	.249	1.15791	.324	1.25988
.025	1 00167	.100	1.02646	.175	1.07977	.250	1.15912	.325	1.26138
.026	1.00182	.101	1.02698	.176	1.08066	.251	1.16034	.326	1.26288
.027	1.00196	.102	1.02752	.177	1.08156	.252	1.16156	.327	1.26437
.028	1.00210	.103	1.02806	.178	1.08246	.253	1.16279	.328	1.26588
.029	1.00225	.104	1.02860	.179	1.08337	.254	1.16402	.329	1.26740
.030	1.00240	.105	1.02914	.180	1.08428	.255	1.16526	.330	1.26892
.031	1.00256	.106	1.02970	.181	1.08519	.256	1.16650	.331	1.27044
.032	1.00272	.107	1.03026	.182	1.08611	.257	1.16774	.332	1.27196
.033	1.00289	.108	1.03082	.183	1.08704	.258	1.16899	.333	1.27349
.034	1.00307	.109	1.03139	.184	1.08797	.259	1.17024	.334	1.27502
.035	1.00327	.110	1.03196	.185	1.08890	.260	1.17150	.335	1.27656
.036	1.00345	.111	1.03254	.186	1.08984	.261	1.17276	.336	1.27810
.037	1.00364	.112	1.03312	.187	1.09079	.262	1.17403	.337	1.27964
.038	1.00384	.113	1.03371	.188	1.09174	.263	1.17530	.338	1.28118
.039	1.00405	.114	1.03430	.189	1.09269	.264	1.17657	.339	1.28273
.040	1.00426	.115	1.03490	.190	1.09365	.265	1.17784	.340	1.28428
.041	1.00447	.116	1.03551	.191	1.09461	.266	1.17912	.341	1.28583
.042	1.00469	.117	1.03611	.192	1.09557	.267	1.18040	.342	1.28739
.043	1.00492	.118	1.03672	.193	1.09654	.268	1.18169	.343	1.28895
.044	1.00515	.119	1.03734	.194	1.09752	.269	1.18299	.344	1.29052
.045	1.00539	.120	1.03797	.195	1.09850	.270	1.18429	.345	1.29209
.046	1.00563	.121	1.03860	.196	1.09949	.271	1.18559	.346	1.29366
.047	1.00587	.122	1.03923	.197	1.10048	.272	1.18689	.347	1.29523
048	1.00612	.123	1.03987	.198	1.10147	.273	1.18820	.348	1.29681
.049	1 00638	.124	1.04051	.199	1.10247	.274	1.18951	.349	1.29839
.050	1.00665	.125	1 04116	.200	1.10347	.275	1.19082	.350	1.29997
.051	1.00692	.126	1.04181	.201	1.10447	.276	1.19214	.351	1.30156
.052	1 00720	.127	1.04247	.202	1.10548	.277	1.19346	.352	1.30315
053	1 00748	.128	1.04313	.203	1 10650	.278	1.19479	.353	1.30474
.054	1.00776	.129	1.04380	.204	1.10752	.279	1.19612	.354	1 30634
.055	1.00805	.130	1 04447	.205	1.10855	.280	1.19746	.355	1.30794
.056	1.00834	.131	1 04515	.206	1.10958	.281	1.19880	.356	1.30954
.057	1 00864	.132	1.04584	.207	1.11062	.282	1.20014	.357	1.31115
.058	1.00895	.133	1.04652	.208	1.11165	.283	1.20149	.358	1.31276
.059	1.00926	134	1.04722	.209	1.11269	.284	1.20284	.359	1.31437
.060	1.00957	.135	1.04792	.210	1.11374	.285	1.20419	.360	1.31599
.061	1.00989	.136	1.04862	.211	1.11479	.286	1.20555	.361	1.31761
.062	1.01021	.137	1.04932	.212	1.11584	.287	1.20691	.362	1.31923
.063	1.01054	.138	1.05003	.213	1.11690	.288	1.20827	.363	1.32086
.064	1.01088	.139	1.05075	.214	1.11796	.289	1.20964	.364	1 32249
.065	1.01123	.140	1.05147	.215	1.11904	.290	1.21102	.365	1.32413
.066	1.01158	.141	1.05220	.216	1.12011	.291	1.21239	.366	1.32577
.067	1.01193	.142	1.05293	.217	1.12118	.292	1.21377	.367	1.32741
.068	1.01228	.143	1.05367	.218	1.12225	.293	1.21515	.368	1.32905
.069	1 01264	.144	1.05441	.219	1.12334	.294	1.21654	.369	1.33069
.070	1.01302	.145	1 05516	.220	1.12444	.295	1.21794	.370	1.33234
.071	1.01338	.146	1.05591	.221	1.12554	.296	1.21933	.371	1.33399
.072	1.01376	.147	1.05667	.222	1.12664	.297	1.22073	.372	1.33564
.073	1.01414	.148	1.05743	.223	1.12774	.298	1.22213	373	1.33730
.074	1.01453	.149	1.05819	.224	1.12885	.299	1.22354	.374	1.33896
.075	1.01493	.150	1.05896	.225	1 12997	.300	1.22495	.375	1.34063

TABLE OF CIRCULAR ARCS—(CONTINUED.)

H'ghts.	Lengths.	H'ghts.	Lengths.	H'ghts.	Lengths.	H'ghts.	Lengths.	H'ghts.	Lengths.
.376	1.34229	.401	1.38496	.426	1.42945	.451	1.47565	.476	1.52346
.377	1.34393	.402	1.38671	.427	1.43127	.452	1.47753	.477	1.52541
.378	1.34563	.403	1.38846	.428	1.43309	.453	1.47942	.478	1.52736
.379	1.34731	.404	1.39021	.429	1.43491	.454	1.48131	.479	1.52931
.380	1.34899	.405	1.39196	.430	1.43673	.455	1.48320	.480	1.53126
.381	1.35068	.406	1.39372	.431	1.43856	.456	1.48509	.481	1.53322
.382	1.35237	.407	1.39548	.432	1.44039	.457	1.48699	.482	1.53518
.383	1.35406	.408	1.39724	.433	1.44222	.458	1.48889	.483	1.53714
.384	1.35575	.409	1.39900	.434	1.44405	.459	1.49079	.484	1.53910
.385	1.35744	.410	1.40077	.435	1.44589	.460	1.49269	.485	1.54106
.386	1.35914	.411	1.40254	.436	1.44773	.461	1.49460	.486	1.54302
.387	1.36084	.412	1.40432	.437	1.44957	.462	1.49651	.487	1.54499
.388	1.36254	.413	1.40610	.438	1.45142	.463	1.49842	.488	1.54696
.389	1.36425	.414	1.40788	.439	1.45327	.464	1.50033	.489	1.54893
.390	1.36596	.415	1.40966	.440	1.45512	.465	1.50224	.490	1.55091
.391	1.36767	.416	1.41145	.441	1.45697	.466	1.50416	.491	1.55289
.392	1.36939	.417	1.41324	.442	1.45883	.467	1.50608	.492	1.55487
.393	1.37111	.418	1.41503	.443	1.46069	.468	1.50800	.493	1.55685
.394	1.37283	.419	1.41682	.444	1.46255	.469	1.50992	.494	1.55884
.395	1.37455	.420	1.41861	.445	1.46441	.470	1.51185	.495	1.56083
.396	1.37628	.421	1.42041	.446	1.46628	.471	1.51378	.496	1.56282
.397	1.37801	.422	1.42221	.447	1.46815	.472	1.51571	.497	1.56481
.398	1.37974	.423	1.42402	.448	1.47002	.473	1.51764	.498	1.56681
.399	1.38148	.424	1.42583	.449	1.47189	.474	1.51958	.499	1.56881
.400	1.38322	.425	1.42764	.450	1.47377	.475	1.52152	.500	1.57080

If the arc is greater than a semicircle, then, as directed at top of p 147, find the diam of the circle. Then find its circumf. From diam take ht of arc. The rem will be ht of the smaller arc of the circle. By rule at top of p 143 find the length of this smaller arc. Subtract it from circumf.

The length of 1 degree of a circular arc is equal to .017453 292 520 × its radius.
" " " 1 minute " " " " " .000290 888 209 × " "
" " " 1 second " " " " " .000004 848 137 × " "

An arc of 1° of the **earth's great circle** is but 4.6356 feet longer than its chord. Its length is 69.16 land or statute miles. Earth's equatorial rad = 3962.5705 miles. Polar 3949.67.

An arc of 1°, rad 1 mile, is 92.1534 feet; a minute is 1.5359 feet; a second is .0256 of a foot; or very nearly 5-sixteenths of an inch. **Arc of 1°, rad 100 ft** = 1.74533 feet.

To find the length of a circular arc by the following table.

Knowing the rad of the circle, and the measure of the arc in deg, min, &c.

Rule. Add together the lengths in the table found respectively opposite to the deg, min &c, of the arc. Mult the sum by the rad of the circle.

Ex. In a circle of 12.43 feet rad, is an arc of 13 deg, 27 min, 8 sec. How long is the arc?

Here, opposite 13 deg in the table, we find, .2268928
" 27 min " " " .0078540
" 8 sec " " " .0000388

Sum = .2347856

And .2347856 × 12.43 or rad = 2.918385 feet, the reqd length of arc.

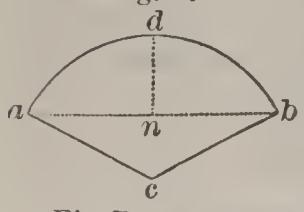
LENGTHS OF CIRCULAR ARCS TO RAD 1.

No errors.

Deg.	Length.	Deg.	Length.	Deg.	Length.	Min.	Length.	Sec.	Length.
1	.0174533	61	1.0646508	121	2.1118484	1	.0002909	1	.0000048
2	.0349066	62	1.0821041	122	2.1293017	2	.0005818	2	.0000097
3	.0523599	63	1.0995574	123	2.1467550	3	.0008727	3	.0000145
4	.0698132	64	1.1170107	124	2.1642083	4	.0011636	4	.0000194
5	.0872665	65	1.1344640	125	2.1816616	5	.0014544	5	.0000242
6	.1047198	66	1.1519173	126	2.1991149	6	.0017453	6	.0000291
7	.1221730	67	1.1693706	127	2.2165682	7	.0020362	7	.0000339
8	.1396263	68	1.1868239	128	2.2340214	8	.0023271	8	.0000388
9	.1570796	69	1.2042772	129	2.2514747	9	.0026180	9	.0000436
10	.1745329	70	1.2217305	130	2.2689280	10	.0029089	10	.0000485
11	.1919862	71	1.2391838	131	2.2863813	11	.0031998	11	.0000533
12	.2094395	72	1.2566371	132	2.3038346	12	.0034907	12	.0000582
13	.2268928	73	1.2740904	133	2.3212879	13	.0037815	13	.0000630
14	.2443461	74	1.2915436	134	2.3387412	14	.0040724	14	.0000679
15	.2617994	75	1.3089969	135	2.3561945	15	.0043633	15	.0000727
16	.2792527	76	1.3264502	136	2.3736478	16	.0046542	16	.0000776
17	.2967060	77	1.3439035	137	2.3911011	17	.0049451	17	.0000824
18	.3141593	78	1.3613568	138	2.4085544	18	.0052360	18	.0000873
19	.3316126	79	1.3788101	139	2.4260077	19	.0055269	19	.0000921
20	.3490659	80	1.3962634	140	2.4434610	20	.0058178	20	.0000970
21	.3665191	81	1.4137167	141	2.4609142	21	.0061087	21	.0001018
22	.3839724	82	1.4311700	142	2.4783675	22	.0063995	22	.0001067
23	.4014257	83	1.4486233	143	2.4958208	23	.0066904	23	.0001115
24	.4188790	84	1.4660766	144	2.5132741	24	.0069813	24	.0001164
25	.4363323	85	1.4835299	145	2.5307274	25	.0072722	25	.0001212
26	.4537856	86	1.5009832	146	2.5481807	26	.0075631	26	.0001261
27	.4712389	87	1.5184364	147	2.5656340	27	.0078540	27	.0001309
28	.4886922	88	1.5358897	148	2.5830873	28	.0081449	28	.0001357
29	.5061455	89	1.5533430	149	2.6005406	29	.0084358	29	.0001406
30	.5235988	90	1.5707963	150	2.6179939	30	.0087266	30	.0001454
31	.5410521	91	1.5882496	151	2.6354472	31	.0090175	31	.0001503
32	.5585054	92	1.6057029	152	2.6529005	32	.0093084	32	.0001551
33	.5759587	93	1.6231562	153	2.6703538	33	.0095993	33	.0001600
34	.5934119	94	1.6406095	154	2.6878070	34	.0098902	34	.0001648
35	.6108652	95	1.6580628	155	2.7052603	35	.0101811	35	.0001697
36	.6283185	96	1.6755161	156	2.7227136	36	.0104720	36	.0001745
37	.6457718	97	1.6929694	157	2.7401669	37	.0107629	37	.0001794
38	.6632251	98	1.7104227	158	2.7576202	38	.0110538	38	.0001842
39	.6806784	99	1.7278760	159	2.7750735	39	.0113446	39	.0001891
40	.6981317	100	1.7453293	160	2.7925268	40	.0116355	40	.0001939
41	.7155850	101	1.7627825	161	2.8099801	41	.0119264	41	.0001988
42	.7330383	102	1.7802358	162	2.8274334	42	.0122173	42	.0002036
43	.7504916	103	1.7976891	163	2.8448867	43	.0125082	43	.0002085
44	.7679449	104	1.8151424	164	2.8623400	44	.0127991	44	.0002133
45	.7853982	105	1.8325957	165	2.8797933	45	.0130900	45	.0002182
46	.8028515	106	1.8500490	166	2.8972466	46	.0133809	46	.0002230
47	.8203047	107	1.8675023	167	2.9146999	47	.0136717	47	.0002279
48	.8377580	108	1.8849556	168	2.9321531	48	.0139626	48	.0002327
49	.8552113	109	1.9024089	169	2.9496064	49	.0142535	49	.0002376
50	.8726646	110	1.9198622	170	2.9670597	50	.0145444	50	.0002424
51	.8901179	111	1.9373155	171	2.9845130	51	.0148353	51	.0002473
52	.9075712	112	1.9547688	172	3.0019663	52	.0151262	52	.0002521
53	.9250245	113	1.9722221	173	3.0194196	53	.0154171	53	.0002570
54	.9424778	114	1.9896753	174	3.0368729	54	.0157080	54	.0002618
55	.9599311	115	2.0071286	175	3.0543262	55	.0159989	55	.0002666
56	.9773844	116	2.0245819	176	3.0717795	56	.0162897	56	.0002715
57	.9948377	117	2.0420352	177	3.0892328	57	.0165806	57	.0002763
58	1.0122910	118	2.0594885	178	3.1066861	58	.0168715	58	.0002812
59	1.0297443	119	2.0769418	179	3.1241394	59	.0171624	59	.0002860
60	1.0471976	120	2.0943951	180	3.1415927	60	.0174533	60	.0002909

CIRCULAR SECTORS, RINGS, SEGMENTS, ETC.

Fig. A.



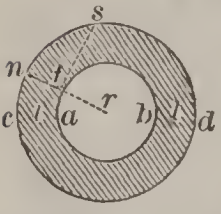
To find the area of a circular sector *a d b c*.

Rule 1. Mult the length of the arc *a d b*, by the rad e *a* or *c b*; and divide prod by 2.

Rem. Knowing the chord *a b*, and the height *n d*; or the rad. and the number of degs in the arc, the length of the arc itself can be found from table, p 143 or 145.

Rule 2. As 360° is to the number of degs in the angle *a c b*, so is the area of the entire circle to the area of the sector.

Fig. B.



To find the area of a circular ring.

Find the areas of the two circles, and take the least from the greatest. Or mult the sum of the two diams, *a b*, *c d*, by their diff; mult the prod by .7854. Or mult together the thickness e *a*; the mean diam *l l*; and 3.1416.

To find the rad of a circle which shall have the same area as a given circular ring *c s d a b*.

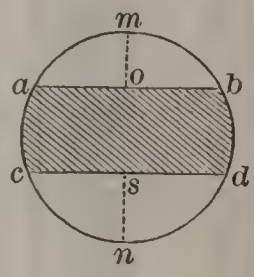
Draw any rad *n r* of the outer circle; and from where said rad cuts the inner circle at *t*, draw *t s* at right angles to it. Then will *t s* be the reqd rad.

To find the breadth of a circular ring.

Having its area, and the diam of its outer circle. Find the area of the whole circle. From it take the area of the ring. Mult the rem by 1.2732. Take the sq rt of the prod. This sq rt will be the diam of the inner circle. Take it from the diam of the outer one; and div the rem by 2, for the reqd breadth.

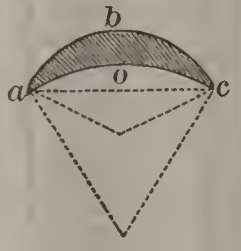
To find the area of a circular zone *a b c d*.

Knowing the diam of the circle; the chords *a b*, and *c d*; and the heights *o m*, and *s n*, of the segments *a m b*, and *c n d*. First find the area of the entire circle; then by means of the following table of circular segments, find the areas of the two segments *a m b*, and *c n d*; and subtract their sum from the area of the circle.



To find the area of a circular lune *a b c o*.

A lune is a crescent-shaped fig, comprised between two arcs of circles of diff diams. Having the chord *a c*, and the heights of the two segments *a o c*, and *a b c*, find the areas of those segments; take the least of these areas from the greatest; the rem is evidently the area of the lune.



To find the area of a circular segment, *a b c d*, Figs C, D, see table, pp 147, 148. **Also, Area of Segment** *a d b n* Fig A = Area of Sector *a d b c* — Area of Triangle *a b c*.

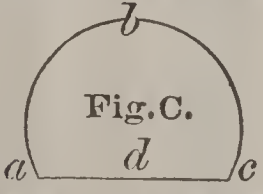
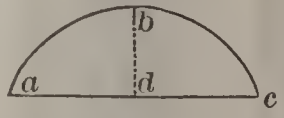


Fig. D.



Having the area of a segment reqd to be cut off from a given circle, to find its chord and height.

Div the area by the square of the diam of the circle; look for the quot in the col of areas in the table of areas, pp 147, 148; take out from the table the corresponding height: and mult it by the diam. The prod will be the reqd height. Then from the diam, take the actual height thus found; mult the rem by the actual height; take the sq rt of the prod; mult it by 2, for the reqd chord.

TABLE OF AREAS OF CIRCULAR SEGMENTS, Figs C, D.

If the segment exceeds a semicircle, its area is = area of circle − area of a segment whose rise is = (diam of circle − rise of given segment). **Diam of circle** = (square of half chord ÷ rise) + rise, whether the segment exceeds a semicircle or not.

Rise div by diam of circle.	Area = (square of diam) mult by	Rise div by diam of circle.	Area = (square of diam) mult by	Rise div by diam of circle.	Area = (square of diam) mult by	Rise div by diam of circle.	Area = (square of diam) mult by	Rise div by diam of circle.	Area = (square of diam mult by
.001	.000042	.064	.021168	.127	.057991	.190	.103900	.253	.156149
.002	.000119	.065	.021660	.128	.058658	.191	.104686	.254	.157019
.003	.000219	.066	.022155	.129	.059328	.192	.105472	.255	.157891
.004	.000337	.067	.022653	.130	.059999	.193	.106261	.256	.158763
.005	.000471	.068	.023155	.131	.060673	.194	.107051	.257	.159636
.006	.000619	.069	.023660	.132	.061349	.195	.107843	.258	.160511
.007	.000779	.070	.024168	.133	.062027	.196	.108636	.259	.161386
.008	.000952	.071	.024680	.134	.062707	.197	.109431	.260	.162263
.009	.001135	.072	.025196	.135	.063389	.198	.110227	.261	.163141
.010	.001329	.073	.025714	.136	.064074	.199	.111025	.262	.164020
.011	.001533	.074	.026236	.137	.064761	.200	.111824	.263	.164900
.012	.001746	.075	.026761	.138	.065449	.201	.112625	.264	.165781
.013	.001969	.076	.027290	.139	.066140	.202	.113427	.265	.166663
.014	.002199	.077	.027821	.140	.066833	.203	.114231	.266	.167546
.015	.002438	.078	.028356	.141	.067528	.204	.115036	.267	.168431
.016	.002685	.079	.028894	.142	.068225	.205	.115842	.268	.169316
.017	.002940	.080	.029435	.143	.068924	.206	.116651	.269	.170202
.018	.003202	.081	.029979	.144	.069626	.207	.117460	.270	.171090
.019	.003472	.082	.030526	.145	.070329	.208	.118271	.271	.171978
.020	.003749	.083	.031077	.146	.071034	.209	.119084	.272	.172868
.021	.004032	.084	.031630	.147	.071741	.210	.119898	.273	.173758
.022	.004322	.085	.032186	.148	.072450	.211	.120713	.274	.174650
.023	.004619	.086	.032746	.149	.073162	.212	.121530	.275	.175542
.024	.004922	.087	.033308	.150	.073875	.213	.122348	.276	.176436
.025	.005231	.088	.033873	.151	.074590	.214	.123167	.277	.177330
.026	.005546	.089	.034441	.152	.075307	.215	.123988	.278	.178226
.027	.005867	.090	.035012	.153	.076026	.216	.124811	.279	.179122
.028	.006194	.091	.035586	.154	.076747	.217	.125634	.280	.180020
.029	.006527	.092	.036162	155	.077470	.218	.126459	.281	.180918
.030	.006866	.093	.036742	.156	.078194	.219	.127286	.282	.181818
.031	.007209	.094	.037324	.157	.078921	.220	.128114	.283	.182718
.032	.007559	.095	.037909	.158	.079650	.221	.128943	.284	.183619
.033	.007913	.096	.038497	.159	.080380	.222	.129773	.285	.184522
.034	.008273	.097	.039087	.160	.081112	.223	.130605	.286	.185425
.035	.008638	.098	.039681	.161	.081847	.224	.131438	.287	.186329
.036	.009008	.099	.040277	.162	.082582	.225	.132273	.288	.187235
.037	.009383	.100	.040875	.163	.083320	.226	.133109	.289	.188141
.038	.009764	.101	.041477	.164	.084060	.227	.133946	.290	.189048
.039	.010148	.102	.042081	.165	.084801	.228	.134784	.291	.189956
.040	.010538	.103	.042687	.166	.085545	.229	.135624	.292	.190865
041	.010932	.104	.043296	.167	.086290	.230	.136465	.293	.191774
.042	.011331	.105	.043908	.168	.087037	.231	.137307	.294	.192685
.043	.011734	.106	.044523	.169	.087785	.232	.138151	.295	.193597
.044	.012142	.107	.045140	.170	.088536	.233	.138996	.296	.194509
.045	.012555	.108	.045759	.171	.089288	.234	.139842	.297	.195423
.046	.012971	.109	.046381	.172	.090042	.235	.140689	.298	.196337
.047	.013393	.110	.047006	.173	.090797	.236	.141538	.299	.197252
.048	.013818	.111	.047633	.174	.091555	.237	.142388	.300	.198168
.049	.014248	.112	.048262	.175	.092314	.238	.143239	.301	.199085
.050	.014681	.113	.048894	.176	.093074	.239	.144091	.302	.200003
.051	.015119	.114	.049529	.177	.093837	.240	.144945	.303	.200922
.052	.015561	.115	.050165	.178	.094601	.241	.145800	.304	.201841
.053	.016008	.116	.050805	.179	.095367	.242	.146656	.305	.202762
.054	.016458	.117	.051446	.180	.096135	.243	.147513	.306	.203683
.055	.016912	.118	.052090	.181	.096904	.244	.148371	.307	.204605
.056	.017369	.119	.052737	.182	.097675	.245	.149231	.308	.205528
.057	.017831	.120	.053385	.183	.098447	.246	.150091	.309	.206452
.058	.018297	.121	.054037	.184	.099221	.247	.150953	.310	.207376
.059	.018766	.122	.054690	.185	.099997	.248	.151816	.311	.208302
.060	.019239	.123	.055346	.186	.100774	.249	.152681	.312	.209228
.061	.019716	.124	.056004	.187	.101553	.250	.153546	.313	.210155
.062	.020197	.125	.056664	.188	.102334	.251	.154413	.314	.211083
.063	.020681	.126	.057327	.189	.103116	.252	.155281	.315	.212011

TABLE OF AREAS OF CIRCULAR SEGMENTS—(CONTINUED.)

Rise div by diam of circle.	Area = (square of diam) mult by	Rise div by diam of circle.	Area = (square of diam) mult by	Rise div by diam of circle.	Area = (square of diam) mult by	Rise div by diam of circle.	Area = (square of diam) mult by	Rise div by diam of circle.	Area = (square of diam mult by
.316	.212941	.353	.247845	·390	.283593	.427	.319959	.464	.356730
.317	.213871	.354	.248801	·391	.284569	.428	.320949	.465	.357728
.318	.214802	.355	.249758	·392	.285545	.429	.321938	.466	.358725
.319	.215734	.356	.250715	·393	.286521	.430	.322928	.467	.359723
.320	.216666	.357	.251673	·394	.287499	.431	.323919	.468	.360721
.321	.217600	.358	.252632	·395	.288476	.432	.324909	.469	.361719
.322	.218534	.359	.253591	·396	.289454	.433	.325900	.470	.362717
.323	.219469	.360	.254551	·397	.290432	.434	.326891	.471	.363715
.324	.220404	.361	.255511	·398	.291411	.435	.327883	.472	.364714
.325	.221341	.362	.256472	·399	.292390	.436	.328874	.473	.365712
.326	.222278	.363	.257433	·400	.293370	.437	.329866	.474	.366711
.327	.223216	.364	.258395	·401	.294350	.438	.330858	.475	.367710
.328	.224154	.365	.259358	·402	.295330	.439	.331851	.476	.368708
.329	.225094	.366	.260321	·403	.296311	.440	.332843	.477	.369707
.330	.226034	.367	.261285	·404	.297292	.441	.333836	.478	.370706
.331	.226974	.368	.262249	·405	.298274	.442	.334829	.479	.371705
.332	.227916	.369	.263214	·406	.299256	.443	.335823	.480	.372704
.333	.228858	.370	.264179	·407	.300238	.444	.336816	.481	.373704
.334	.229801	.371	.265145	·408	.301221	.445	.337810	.482	.374703
.335	.230745	.372	.266111	·409	.302204	.446	.338804	.483	.375702
.336	.231689	.373	.267078	·410	.303187	.447	.339799	.484	.376702
.337	.232634	.374	.268046	·411	.304171	.448	.340793	.485	.377701
.338	.233580	.375	.269014	·412	.305156	.449	.341788	.486	.378701
.339	.234526	.376	.269982	·413	.306140	.450	.342783	.487	.379701
.340	.235473	.377	.270951	·414	.307125	.451	.343778	.488	.380700
.341	.236421	.378	.271921	·415	.308110	.452	.344773	.489	.381700
.342	.237369	.379	.272891	·416	.309096	.453	.345768	.490	.382700
.343	.238319	.380	.273861	·417	.310082	.454	.346764	.491	.383700
.344	.239268	.381	.274832	·418	.311068	.455	.347760	.492	384699
.345	.240219	.382	.275804	·419	.312055	456	.348756	.493	.385699
.346	.241170	.383	.276776	.420	.313042	.457	.349752	.494	.386699
.347	.242122	.384	.277748	.421	.314029	.458	.350749	.495	387699
.348	.243074	.385	.278721	.422	.315017	.459	.351745	.496	.388699
.349	.244027	.386	.279695	.423	.316005	.460	.352742	.497	.389699
.350	.244980	.387	.280669	.424	.316993	.461	.353739	.498	.390699
.351	.245935	.388	.281643	.425	.317981	.462	.354736	.499	.391699
.352	.246890	.389	.282618	.426	.318970	.463	.355733	.500	.392699

THE ELLIPSE.

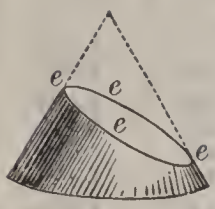


Fig. 1.

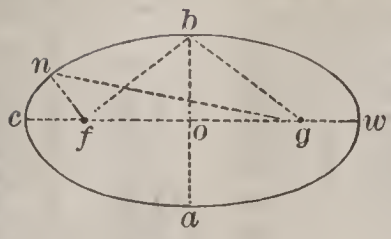


Fig. 2.

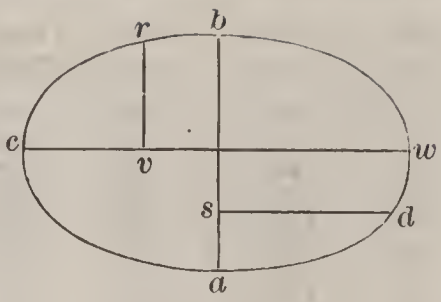


Fig. 3.

An ellipse is a curve, *e e e e*, Fig 1, formed by an oblique section of either a cone or a cylinder, passing through its curved surface, without cutting the base. Its nature is such that if two lines, as *n f* and *n g*, Fig. 2, be drawn from any point *n* in its periphery or circumf, to two certain points *f* and *g*, in its long diam *c w*, (and called the foci of the ellipse,) their sum will be equal to that of any other two lines, as *b f*, and *b g* drawn from any other point, as *b*, in the circumf, to the foci *f* and *g*; also the sum of any two such lines will be equal to the long diam *c w*. The line *c w* dividing the ellipse into two equal parts lengthwise, is called its transverse, or major axis, or long diam; and *a b*, which divides it equally at right-angles to *c w*, is called the conjugate, or minor axis, or short diam. To find the position of the foci of an ellipse, from either end, as *b*, of the short diam, measure off the dists *b f* and *b g*, Fig 2, each equal to *o c*, or one half the long diam.

The parameter of an ellipse is a certain length obtained thus; as the long diam : short diam : : short diam : parameter. Any line *r v*, or *s d*, Fig 3, drawn from the circumf, to, and at right angles to, either diam, is called an *ordinate;* and the parts *c v* and *v w*, *b s* and *s a*, of that diam, between the ord and the circumf, are called *abscissæ*, or *abscisses*.

To find the length of any ordinate, *r v* or *s d*, drawn to either diam, *c w* or *b a*.

Knowing the abscis, *c v* or *s a*, and the two diams, *c w*, *b a*;

$$c\,w^2 : b\,a^2 :: c\,v \times v\,w : r\,v^2. \qquad b\,a^2 : c\,w^2 :: b\,s \times s\,a : s\,d^2.$$

To find the circumf of an ellipse.

Mathematicians have furnished practical men with no simple working rule for this purpose. The so-called approximate rules do not deserve the name. They are as follows, D being the long diam: and *d* the short one.

Rule **1.** Circumf $= 3.1416 \dfrac{D+d}{2}$; Rule **2.** $3.1416 \sqrt{\left(\dfrac{D^2+d^2}{2}\right)}$; Rule **3.** $2.2215 \sqrt{D^2+d^2}$;

this is the same as Rule 2, but in a diff shape. Rule **4.** $2 \times \sqrt{D^2 + 1.4674\ d^2}$. Now, in an ellipse whose long and short diams are 10 and 2, the circumf is actually 21, very approximately; but rule 1 gives it $= 18.85$; rule 2, or 3, $= 22.65$; and rule 4, $= 20.51$. Again, if the diams be 10 and 6, the circumf actually $= 25.59$; but rule 4 gives 24.72. These examples show that none of the rules usually given are reliable. The following one by the writer, is sufficiently exact for ordinary purposes; not being in error probably more than 1 part in 1000. When D is not more than 5 times as long as *d*,

$$\text{Circumf} = 3.1416 \sqrt{\frac{D^2+d_2}{2} - \frac{(D-d)^2}{8.8}}$$

If D exceeds 5 times *d*, then instead of dividing $(D-d)^2$ by 8.8, div it by the number in this table.

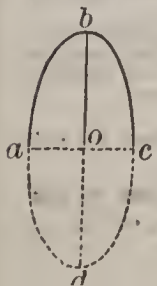
D =	Div by
6 *d*	9
7 *d*	9.2
8 *d*	9.3
9 *d*	9.35
10 *d*	9.4
12 *d*	9.5
14 *d*	9.6
16 *d*	9.68
18 *d*	9.75
20 *d*	9.8
25 *d*	9.87
30 *d*	9.92
40 *d*	9.98
50 *d*	10.04
60 *d*	10.10
70 *d*	10.17
80 *d*	10.23
100 *d*	10.35

The following rule originated with Mr. M. Arnold Pears, of New South Wales, Australia, and was by him kindly communicated to the author. Although not more accurate than our own, it is much neater.

$$\text{Circumf} = 3.1416\,d + 2(D-d) - \frac{d(D-d)}{\sqrt{(D+d) \times (D+2d)}}$$

The following table of semi-elliptic arcs

was prepared by our rule.



To use this table, div the height or rise of the arc, by its span or chord. The quot will be the height of an arc whose span is 1. Find this quot in the column of heights; and take out the corresponding number from the col. of leugths. Mult this number by the actual span. The prod will be the reqd length.

When the height becomes .500 of the chord (as at the end of the table) the ellipse becomes a circle. When the height exceeds .500 of the chord, as in *a b c*, then take *a o*, or half the chord, as the rise; and div this rise by the long diam *b d*, for the quot to be looked for in the col of heights; and to be mult by long diam. We thus get the arc *b a d*, which is evidently equal to *a b c*.

TABLE OF LENGTHS OF SEMI-ELLIPTIC ARCS. (Original.)

Height ÷ span.	Length = span × by	Height ÷ span.	Length = span × by	Height ÷ span.	Length = span × by	Height ÷ span.	Length = span × by
.005	1.000	.130	1.079	.255	1.219	.380	1.390
.01	1.001	.135	1.084	.260	1.226	.385	1.397
.015	1.002	.140	1.089	.265	1.233	.390	1.404
.02	1.003	.145	1.094	.270	1.239	.395	1.412
.025	1.004	.150	1.099	.275	1.245	.400	1.419
.03	1.006	.155	1.104	.280	1.252	.405	1.426
.035	1.008	.160	1.109	.285	1.259	.410	1.434
.04	1.011	.165	1.115	.290	1.265	.415	1.441
.045	1.014	.170	1.120	.295	1.272	.420	1.449
.05	1.017	.175	1.125	.300	1.279	.425	1.456
.055	1.020	.180	1.131	.305	1.285	.430	1.464
.06	1.023	.185	1.137	.310	1.292	.435	1.471
.065	1.026	.190	1.142	.315	1.298	.440	1.479
.07	1.029	.195	1.147	.320	1.305	.445	1.486
.075	1.032	.200	1.153	.325	1.312	.450	1.494
.08	1.036	.205	1.159	.330	1.319	.455	1.501
.085	1.039	.210	1.165	.335	1.325	.460	1.509
.09	1.043	.215	1.171	.340	1.332	.465	1.517
.095	1.046	.220	1.177	.345	1.339	.470	1.524
.100	1.051	.225	1.183	.350	1.346	.475	1.532
.105	1.055	.230	1.189	.355	1.353	.480	1.540
.110	1.059	.235	1.196	.360	1.361	.485	1.547
.115	1.064	.240	1.202	.365	1.368	.490	1.555
.120	1.069	.245	1.207	.370	1.375	.495	1.563
.125	1.074	.250	1.213	.375	1.382	.500	1.571

Area of an ellipse = prod of diams × .7854. Ex. D = 10; $d = 6$. Then $10 \times 6 \times .7854 = 47.124$ area. The area of an ellipse is a mean proportional between the areas of two circles, described on its two diams; therefore it may be found by mult together the areas of those two circles; and taking the sq rt of the prod. The area of an ellipse is therefore always greater than that of the circular section of the cylinder from which it may be supposed to be derived.

Diam of circ of same area as a given ellipse = $\sqrt{\text{Long diam} \times \text{short diam.}}$

To find the area of an elliptic segment whose base is parallel to either diam. Div the height of the segment, by that diam of which said height is a part. From the table of circular segments take out the tabular area opposite the quot. Mult together this area, the long diam, and the short diam.

To draw an ellipse. Having its long and short diams $a\,b$ and $c\,d$, Fig. 4.

Rule 1. From either end of the short diam, as c, lay off the dists $c\,f$, $c\,f'$, each equal to $e\,a$, or to one-half of the long diam. The points f, f' are the foci of the ellipse. Prepare a string, $f'\,n\,f$, or $f'\,g\,f$, with a loop at each end; the total length of string from end to end of loop, being equal to the long diam. Place pins at f and f'; and placing the loops over them, trace the curve by a pencil, which in every position, as at n, or g, keeps the string $f'\,n\,f$, or $f'\,g\,f$, equally stretched all the time.

Note. Owing to the difficulty of keeping the string equally stretched, this method is not as satisfactory as the following.

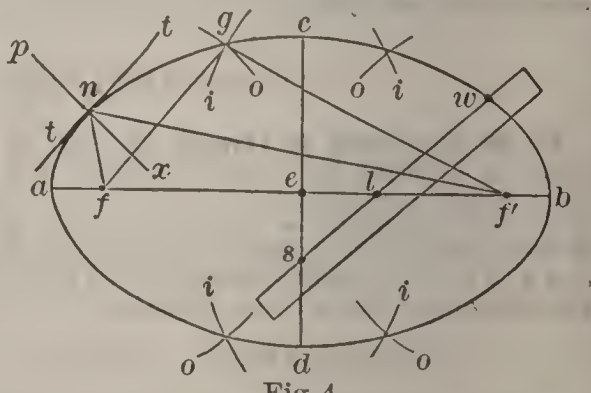


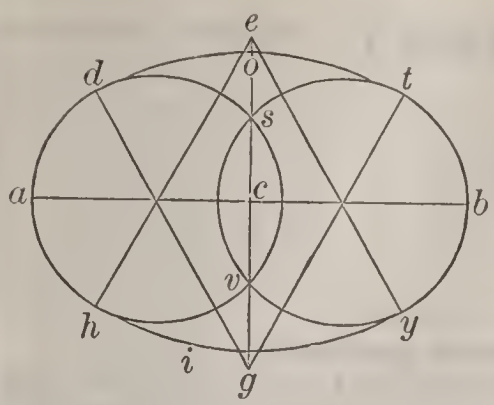
Fig. 4.

Rule 2. On the edge of a strip of paper $w\,s$, mark $w\,l$ equal to half the short diam; and $w\,s$ equal half the long diam. Then in whatever position this strip be placed, keeping l on the long diam, and s on the short diam, w will mark a point in the circumf of the ellipse. We may thus obtain as many such points as we please; and then draw the curve through them by hand.

Rule 3. From the two foci f and f', Fig 4, with a rad equal to any part whatever of the long diam, describe 4 short arcs, o o o o; also with a rad equal to the remaining part of the long diam, describe 4 other arcs, $i\,i\,i\,i$. The intersections of these four pairs of arcs, will give four points in the circumf. In this manner any number of such points may be found, and the curve be drawn by hand.

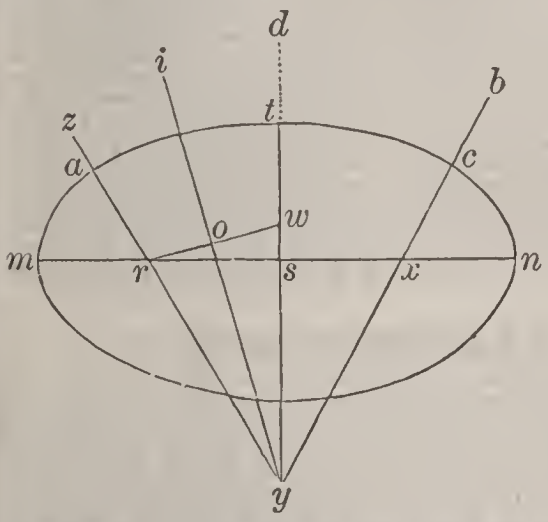
To draw a tangent $t\,t$, at any point n of an ellipse. Draw $n\,f$ and $n\,f'$, to the foci; bisect the angle $f\,n\,f'$ by the line $x\,p$; draw $t\,n\,t$ at right angles to $x\,p$.

To draw a joint $n\,p$, of an elliptic arch, from any point n, in the arch. Proceed as in the foregoing rule for a tangent, only omitting $t\,t$; $n\,p$ will be the reqd joint.



To draw an oval, or false ellipse.

When only the long diam *a b* is given, the following will give agreeable curves, of which the span *a b* will not exceed about three times the rise c o. On *a b* describe two intersecting circles of any rad; through their intersections *s*, *v*, draw *e g*; make *s g* and *v e* each equal to the diam of one of the circles. Through the centers of the circles, draw *e y*, *e h*, *g d*, *g t*. From *e* describe *h i y*; and from *g* describe *d o t*.



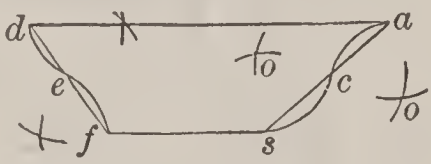
When the span, *m n*, and the rise, *s t*, are both given.

Make any *t w* and *m r*, equal to each other, but each less than *t s*. Draw *r w*; and through its center o draw the perp *i o y*. Draw *y r z*. Make *n x* equal *m r*, and draw *y x b*. From *x* and *r* describe *n c* and *m a*; and from *y* describe *a t c*. By making *s d* equal to *s y*, we obtain the center for the other side of the oval.

The beauty of the curve will depend upon what portion of *t s* is taken for *m r* and *t w*. When an oval is very flat, more than three centers are required for drawing a graceful curve; but the finding of these centers is quite as troublesome as to draw the correct ellipse.

On the given line, *a s*, to draw a cyma recta, *a c s*.

Find the center c, of *a s*. From *a*, *c*, and *s*, with one-half of *a s* as rad, draw the four small arcs at o, o. The intersections o, o, are the centers for drawing the cyma, with the same rad. By reversing the position of the arcs, we obtain the *cyma reversa*, or *ogee*, *d e f*.



THE PARABOLA.

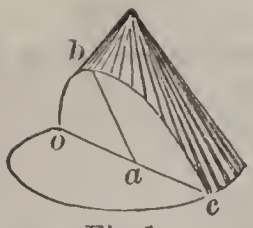


Fig. 1.

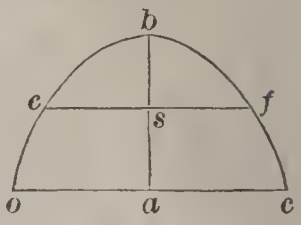


Fig. 2.

The common or conic parabola,

o b c, Fig 1, is a curve formed by cutting a cone in a direction *b a*, parallel to its side. The curved line o *b* c itself is called the *perimeter* of the parabola; the line o c is called its *base*; *b a* its height or *axis*; *b* its *apex* or *vertex*; any line *e s*, or o *a*, Fig 2, drawn from the curve, to, and at right angles to, the axis, is an *ordinate*; and the part *s b*, or *a b*, of the axis, between the ordinate and the apex *b*, is an *abscissa*. The *focus* of a parabola is that point in the axis, where the abscissa *b s*, is equal to one-half of the ord *e s*. The dist from the apex to the focus, is called the *focal dist*. The focus may be entirely beyond or outside of the curve itself. Its dist from the apex is found thus: square any ord, as o *a*; div this square by the abscissa *b a* of that ord; div the quot by 4. The nature of the parabola is such that its abscissas, as *b s*, *b a*, &c, are to each other as, or in proportion to, the squares of their respective ords *e s*, o *a*, &c; that is, as $b\,s : b\,a :: e\,s^2 : o\,a^2$; or $b\,s : e\,s^2 :: b\,a : o\,a^2$. If the square of any ord be divided by its abscissa, the quot will be a constant quantity; that is, it will be equal to the square of any other ord divided by its abscissa. This quot or constant quantity is also equal to a certain quantity called the *parameter* of the parabola. Therefore the parameter may be found by squaring *e s*, or o *a*, (one-*half* of the base,) and dividing said square by the height *b s*, or *b a*, as the case may be. If the square of any ord be divided by the parameter, the quot will be the abscissa of that ord.

To find the length of a parabolic curve.

The approximate rule given by various pocket-books, is as follows:

$$\text{Length} = 2 \times \sqrt{(\tfrac{1}{2}\text{ base})^2 + 1\tfrac{1}{3}\text{ times the (Height}^2)}$$

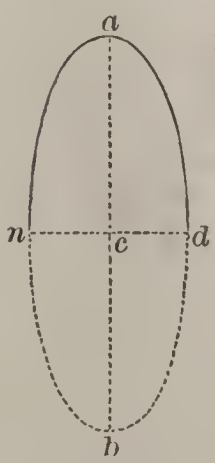


Fig. 4.

Where the height does not exceed 1-10th of the base, this rule may, for practical purposes, be called exact. With ht = ¼ base, it gives about ½ per cent too much: ht = ⅓ base, about 3⅓ per cent; ht = base, about 8½ per cent; ht = twice the base, about 12⅓ per cent; ht = 10 × base, or more, about 15½ per cent.

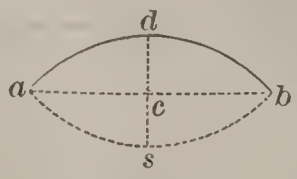


Fig. 3.

The following by the writer is correct within perhaps 1 part in 800, in all cases; and will therefore answer for many purposes.

Let *a d b*, Fig 3, or *n a d*, Fig 4, be the parabola, in which are given the base *a b* or *n d*; and the height c *d* or c *a*. Imagine the complete fig *a d b s*, or *n a d b*, to be drawn; and in *either* case, assume its *long* diam *a b* to be the chord or base; and one-half the short diam, or c *d*, to be the height, of a circular arc. Find the length of this circular arc, by means of the rule and table given for that purpose. Then div the chord or base *a b*, or *n d* of the parabola, by its height c *d* or c *a*. Look for the quot in the column of bases in the following table, and take from the table the corresponding multiplier. Mult the length of the circular arc by this; the prod will be the length of arc *a d b*, or *n a d*, as the case may be. For bases of parabolas less than .05 of the height, or greater than 10 times the height, the multiplier is 1, and is very approximate; or in other words, the parabola will be of almost exactly the same length as the circular arc.

Base	Multiplier	Base	Multiplier
.05	1.0001	.55	1.0284
.10	1.0004	.60	1.0340
.15	1.0009	.65	1.0398
.20	1.0017	.70	1.0457
.25	1.0026	.75	1.0518
.30	1.0038	.80	1.0580
.35	1.0052	.85	1.0643
.40	1.0068	.90	1.0708
.45	1.0086	.95	1.0774
.50	1.0106	1.00	1.0842

To find the area of a parabola *m a n b*.

Mult its base *m n*, Fig 5, by its height *a b*; and take ⅔ds of the prod. The area of any segment, as *u b v*, whose base *u v* is parallel to *m n*, is found in the same way, using *u v* and *s b*, instead of *m n* and *a b*.

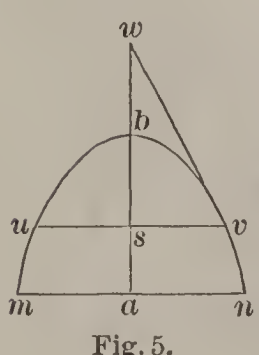


Fig. 5.

To find the area of a parabolic zone, or frustum, as *m n u v*.

Rule 1. First find by the preceding rule the area of the whole parabola *m b n*; then that of the segment *u b v*; and subtract the last from the first.

Rule 2. From the cube of *m n*, take the cube of *u v*; call the diff *c*. From the square of *m n*, take the square of *u v*; call the diff *s*. Div *c* by *s*. Mult the quot by ⅔ds of the height *a s*.

Table for Lengths of Parabolic Curves. See opp page. (Original.)

Base.	Mult.	Base.	Mult.	Base.	Mult.	Base.	Mult.
.05	1.000	1.10	.999	2 15	.949	3.20	.983
.10	1.001	1.15	.997	2.20	.951	3.30	.984
.15	1.002	1.20	.995	2.25	.954	3.40	.985
.20	1.004	1.25	.993	2.30	.956	3.50	.986
.25	1.006	1.30	.990	2.35	.958	3.60	.987
.30	1.007	1.35	.987	2.40	.960	3.70	.988
.35	1.007	1.40	.984	2.45	.962	3.80	.989
.40	1.008	1.45	.980	2.50	.963	3.90	.990
.45	1.009	1.50	.977	2.55	.965	4.00	.991
.50	1.010	1.55	.974	2.60	.967	4.25	.992
.55	1.010	1.60	.970	2.65	.969	4.50	.993
.60	1.010	1.65	.966	2.70	.970	4.75	.994
.65	1.011	1.70	.963	2.75	.972	5.00	.995
.70	1.011	1.75	.960	2.80	.973	5.25	.996
.75	1.010	1.80	.957	2.85	.975	5.50	.997
.80	1.009	1.85	.953	2.90	.976	5.75	.998
.85	1.008	1.90	.950	2.95	.978	6.00	.998
.90	1.006	1.95	.946	3.00	.979	7.00	.999
.95	1.004	2.00	.942	3 05	.980	8.00	1.000
1.00	1.002	2.05	.944	3.10	.981	10.00	1.000
1.05	1.001	2.10	.946	3.15	.982		

To draw a parabola, having base *c s* and height *e o*.

c o s, Fig 6. Make *o t* equal to the height *e* o. Draw *c t* and *s t*; and divide each of them into any number of equal parts; numbering them as in the Fig. Join 1, 1; 2, 2; 3, 3. &c; then draw the curve by hand. It will be observed that the intersections of the lines 1, 1; 2, 2, &c, do not give points in the curve; but a portion of each of those lines forms a tangent to the curve. By increasing the number of divisions on *c t* and *s t*, an almost perfect curve is formed, scarcely requiring to be touched up by hand. In practice it is best first to draw only the center portions of the two lines which cross each other just above o; and from them to work downward; actually drawing only that small portion of each successive lower line, which is necessary to indicate the curve.

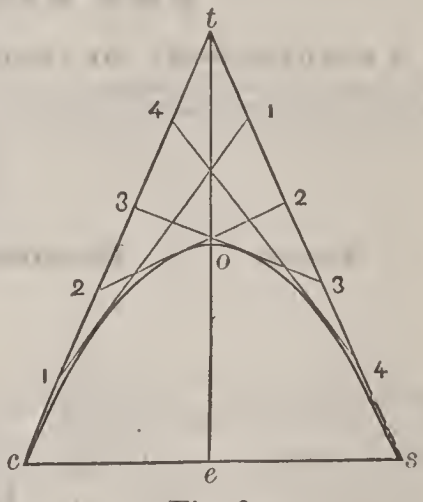


Fig. 6.

Or the parabola may be drawn thus:

Let *b* c, Fig 7, be the base; and *a d* the height. Draw the rectangle *b n m* c; div each half of the base into any number of equal parts, and number them from the center each way. Div *n b*, and *m c* into the same number of equal parts; and number them from the top, downward. From the points on *b* c draw vert lines; and from those at the sides draw lines to *d*. Then the intersections of lines 1, 1; 2, 2, &c, will form points in the parabola. As in the preceding case, it is not necessary to draw the entire lines; but merely portions of them, as shown between *d* and *c*.

Or a parabola may be drawn by first div the height *a b*, Fig 5, into any number of parts, either equal or unequal; and then calculating the ordinates *u s*, &c; thus, as the height *a b* : square of half base *a m* : : any abscess *b s* : square of its ord *u s*. Take the sq rt for *u s*.

Rem. — When the height of a parabola is not greater than 1-10th part its base, the curve coincides so *very* closely with that of a circular arc, that in the preparation of drawings for suspension bridges, &c., the circular arc may be employed; or if no great accuracy is reqd, the circle may be used even when the height is as great as one-eighth of the base.

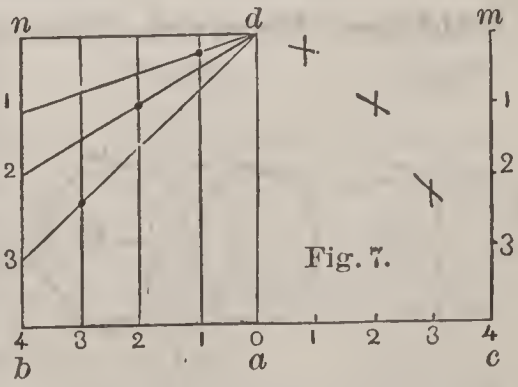


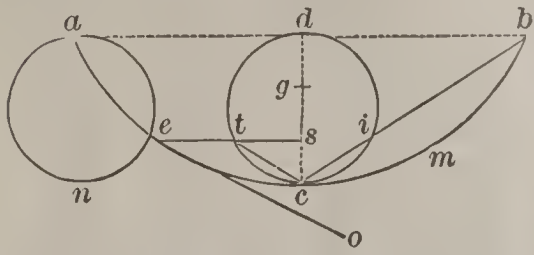
Fig. 7.

To draw a tangent *w v*, Fig. 5, **to a parabola, from any point** *v*.

Draw *v s* perp to axis *a b*; prolong *a b* until *b w* equals *s b*. Join *w v*.

The Cycloid,

$a\,c\,b$, is the curve described by a point a in the circumference of a circle, $a\,n$, during one complete revolution of the circle, rolled along a straight line $a\,b$; which is called the base of the cycloid.



The vertex of the cycloid is at c.

Base, $a\,b$, = circumference of generating circle $a\,n$
= diameter, $c\,d$, of generating circle $\times \pi = 3.1416 c d$.

Axis, or height, $c\,d = a\,n$.

Length, $a\,c\,b = 4\,c\,d$.

Area, $a\,c\,b\,d = 3 \times$ area of generating circle, $a\,n$

$$= 3\frac{c\,d^2\pi}{4} = c\,d^2 \times \tfrac{3}{4}\pi = c\,d^2 \times 2.3562.$$

Center of gravity at g. $\quad c\,g = \frac{5}{8} c\,d$.

To draw a tangent, $e\,o$, from any point e in a cycloid; draw $e\,s$ at right angles to the axis $c\,d$; on $c\,d$ describe the generating circle $d\,c\,t$; join $t\,c$; from e draw $e\,o$ parallel to $t\,c$. The cycloid is the **curve of quickest descent;** so that a body would fall from b to c along the curve $b\,m\,c$, in less time than along the inclined plane $b\,i\,c$, or any other line.

SOLIDS.

THE REGULAR BODIES.

A regular body, or regular polyhedron, is one which has all its sides, and its solid angles, respectively similar and equal to each other. There are but five such bodies, as follows:

Name.	Bounded by	Surface (= sum of surfaces of all the faces). Multiply the square of the length of one edge by	Volume. Multiply the cube of the length of one edge by
Tetrahedron	4 equilateral triangles.	1.7320	.1178
Hexahedron or cube	6 squares.	6.	1.
Octahedron	8 equilateral triangles.	3.4641	.4714
Dodecahedron	12 " pentagons.	20.6458	7.6631
Icosahedron	20 " triangles.	8.6602	2.1817

Guldinus' Theorem. To find the **volume of any body** (as the irregular mass $a\,b\,c\,m$, Fig A, or the ring $a\,b\,c\,m$, Fig B), generated by a complete or partial revolution of any figure (as $a\,b\,c\,a$) around one of its sides (as $a\,c$, Fig A), or around any other axis (as $x\,y$, Fig B).

Fig. A.

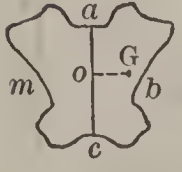
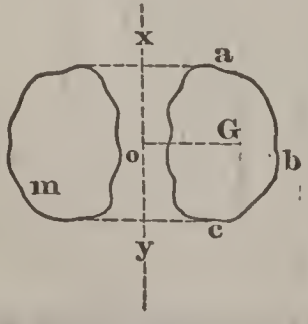


Fig. B.



Volume = surface $a\,b\,c\,a \times$ length of arc described by its center of gravity G.

If the revolution is complete, the arc described is = circumference = radius $o\,G^* \times 2\pi$ = radius $o\,G^* \times 6.283186$; and

Volume = surface $a\,b\,c\,a \times$ radius $o\,G^* \times 6.283186$.

If the revolution is incomplete,

complete revolution : incomplete revolution :: circumference found as above : arc described

*Measured perpendicularly to the axis of revolution.

PARALLELOPIPEDS.

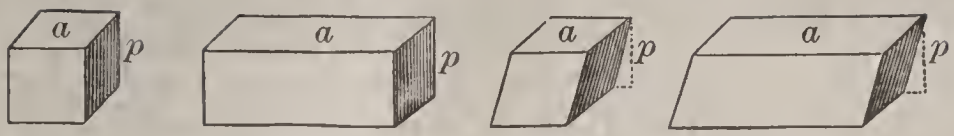


Fig. 1. Fig. 2. Fig. 3. Fig. 4.

A parallelopiped is any solid contained within six sides, all of which are parallelograms; and those of each opposite pair, parallel to each other. We show but four of them; corresponding to the four parallelograms; namely, the *cube*, Fig 1, which has all its sides equal squares, and all its angles right angles; the *right rectangular prism*, Fig 2, has all its angles right angles, each pair of opposite faces equal, but not all of its faces equal; the *Rhombohedron*, Fig 3, which has all its sides equal rhombuses, and which, like the rhombus, p 119, is sometimes called "rhomb"; the *Rhombic* prism, Fig 4; its faces, rhombuses, or rhomboids, each pair of opposite faces equal, but not all its faces equal. All parallelopipeds are prisms.

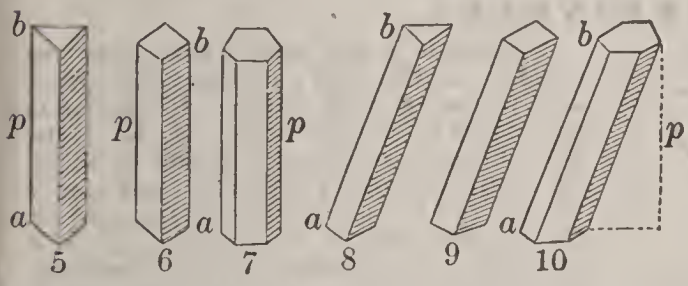
Volume of any parallelopiped = area of any face, as a, $\times$ *perpendicular* distance, p, to the opposite face.

Volume of a cube = cube of length of one edge,
= 1.90985 $\times$ volume of inscribed sphere,
= 1.27324 $\times$ " " cylinder,
= 3.81972 $\times$ " " cone.

Diagonal of a cube = diameter of circumscribing sphere,
= 1.7320508 $\times$ length of one edge of cube.

The diagonal of a rhomb, or of a rhombic prism, cannot be calculated by means of its sides and their angles.

PRISMS.

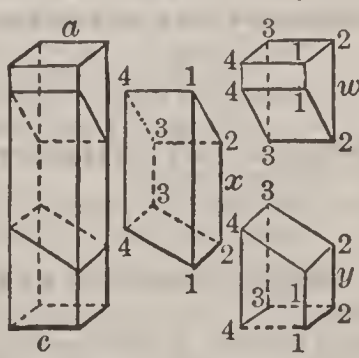


A prism is any solid whose two *ends* are parallel, similar, and equal; and whose *sides are parallelograms*, as Figs 5 to 10. Consequently the foregoing parallelopipeds are prisms. A *right* prism is one whose sides are perpendicular to its ends as 5, 6, 7; when not so, the prism is *oblique*, as 8, 9, 10. When all the sides of the figures which form the ends are equal, and the angles included between those sides are also equal, the prism is said to be *regular*: otherwise, *irregular*.

Volume of any prism (whether regular or irregular, right or oblique)
= area of one end $\times$ *perpendicular* distance, p, to the other end,
= area of cross section perpendicular to the sides $\times$ actual length, $a\,b$, Figs 5 to 10,
= 3 $\times$ volume of pyramid whose base and height are = those of the prism.

To find the volume of any frustum* of any prism.

Whose cross section, perpendicular to its sides, is either any triangle; any parallelogram; a square, (as in Fig 10¼) or a *regular* polygon of any number of sides; no matter how the two ends of the frustum may be inclined with regard to each other; or whether one, or neither of them, is parallel to the base of the original prism.



Figs. 10¼.

Volume of frustum = $\dfrac{\text{sum of lengths of } \textit{parallel} \text{ edges, } 1\,1 + 2\,2 + 3\,3 + 4\,4}{\text{number of such edges (4 in Fig 10¼)}} \times$ area of cross section *perpendicular* to such edges.

* Often misspelt "frustrum."

This rule may be used for ascertaining beforehand, the quantity of earth to be removed from a "borrow pit." The irregular surface of the ground is first staked out in squares; (the tape-line being stretched *horizontally*, when measuring off their sides). These squares should be of such a size that without material error each of them may be considered to be a plane surface, either horizontal or inclined. The depth of the horizontal bottom of the pit being determined on, and the levels being taken at every corner of the squares, we are thereby furnished with the lengths of the four parallel vertical edges of each of the resulting frustums of earth. In Figs 10¼ *y* may be supposed to represent one of these frustums.

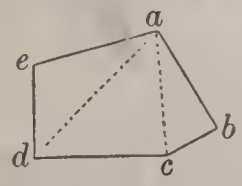


Fig. 10½.

If the frustum is that of an *irregular* 4-sided, or polygonal prism, first divide its cross section perpendicular to its sides, into triangles, by lines drawn from any one of its angles, as *a*, Fig 10½. Calculate the area of each of these triangles separately; then consider the entire frustum to be made up of so many triangular ones; calculate the volume of each of these by the preceding rule for triangular frustums; and add them together, for the volume of the entire frustum.

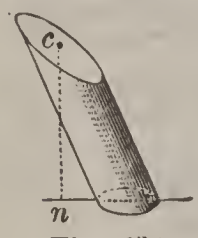


Fig. 10¾.

Volume of any frustum of any prism,

Or of a cylinder. Consider either end to be the base; and find its area. Also find the center of gravity *c* of the other end, and the *perpendicular* distance *n c*, from the base to said center of gravity. Then **Volume of frustum** = area of base $\times n\,c$, Fig 10¾.

The slant end, *c*, is an ellipse. Its area is greater than that of the circular end.

Surface of any prism, Figs 5 to 10, whether right or oblique, regular or irregular

$$= \left(\begin{matrix}\text{circumference measured}\\ \text{perpendicular to the sides}\end{matrix} \times \text{actual length, } a\,b\right) + \begin{matrix}\text{sum of the areas}\\ \text{of the two ends.}\end{matrix}$$

CYLINDERS.

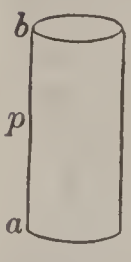


Fig. 11.

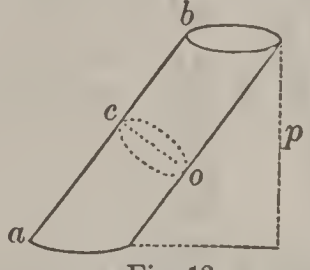


Fig. 12.

A cylinder is any solid whose ends are parallel, similar, and equal *curved* figures; and whose sections parallel to the ends are everywhere the same as the ends. Hence there are circular cylinders, elliptic cylinders (or cylindroids) and many others; but when not otherwise expressed, the circular one is understood. A *right* cylinder is one whose ends are perpendicular to its sides, as Fig. 11; when otherwise, it is *oblique*, as Fig 12. If the ends of a right circular cylinder be cut so as to make it oblique, it becomes an elliptic one; because then both its ends, and all sections parallel to them, are ellipses. An oblique circular cylinder seldom occurs; it may be conceived of by imagining the two ends of Fig 12 to be circles, united by straight lines forming its curved sides

A cylinder is a prism having an infinite number of sides.

Volume of any cylinder (whether circular or elliptic, &c, right or oblique)
= area of one end $\times$ *perpendicular* distance, *p*, to the other end,
$= \left\{\begin{matrix}\text{area of cross section}\\ \text{measured perp to the sides}\end{matrix}\right. \times$ actual length, *a b*, Figs 11 and 12,
= 3 $\times$ volume of a cone whose base and height are = those of the cylinder.

Surface of any cylinder (whether circular or elliptic, &c, right or oblique)

$$= \left(\begin{matrix}\text{circumference}\\ \text{measured perpendicularly}\\ \text{to the sides, as at } c\,o, \text{ Fig 12,}\end{matrix} \times \text{actual length, } a\,b\right) + \begin{matrix}\text{sum of the areas}\\ \text{of the two ends.}\end{matrix}$$

Right circular cylinder whose height = diameter.

Volume = 1½ $\times$ volume of inscribed sphere.
Curved surface = surface of inscribed sphere.
Area of one end = ¼ surface of inscribed sphere = ¼ curved surface.
Entire surface = 1½ $\times$ surface of inscribed sphere = 1½ $\times$ curved surface.

Contents for one foot in length. in Cub Ft, and in U. S. Gallons of 231 cub ins, or 7.4805 Galls to a Cub Ft. **A cub ft of water weighs** about 62½ lbs; and a gallon about 8⅓ lbs. **Diams 2, 8, or 10 times as great,** give 4, 9, or 100 times the content.
For the weight of water in pipes, see Table 2 page 246

No errors.

Diam. in Ins.	Diam. in decimals of a foot.	For 1 ft. in length. Cub. Feet. Also area in sq. ft.	For 1 ft. in length. Gallons of 231 Cub. Ins.	Diam. in Ins.	Diam. in decimals of a foot.	For 1 ft in length. Cub. Feet. Also area in sq. ft.	For 1 ft in length. Gallons of 231 Cub. Ins.	Diam. in Ins.	Diam. in decimals of a foot.	For 1 ft. in length. Cub. Feet. Also area in sq. ft.	For 1 ft. in length. Gallons of 231 Cub. Ins.
1/4	.0208	.0003	.0025	3/4	.5625	.2485	1 859	19.	1.583	1.969	14.73
5-16	.0260	.0005	.0040	7.	.5833	.2673	1.999	1/2	1.625	2.074	15.51
3/8	.0313	.0008	.0057	1/4	.6042	.2867	2.145	20.	1.667	2.182	16.32
7-16	.0365	.0010	.0078	1/2	.6250	.3068	2.295	1/2	1.708	2 292	17.15
1/2	0417	.0014	.0102	3/4	.6458	.3276	2.450	2 .	1.750	2 405	17.99
9 16	.0469	.0017	.0129	8.	.6667	.3491	2.611	1/2	1.792	2.521	18.86
5/8	.0521	.0021	.0159	1/4	.6875	.3712	2.777	22.	1.833	2.640	19 75
11-16	.0573	.0026	.0193	1/2	.7083	.3941	2.948	1/2	1.875	2.761	20.66
3/4	.0625	.0031	.0230	3/4	.7292	.4176	3.125	23.	1.917	2.885	21.58
13-16	.0677	.0036	.0269	9.	.7500	.4418	3.305	1/2	1.958	3 012	22.53
7/8	.0729	.0042	.0312	1/4	.7708	.4667	3 491	24.	2.000	3.142	23.50
15-16	.0781	.0048	.0359	1/2	.7917	.4922	3.682	25.	2.083	3.409	25.50
1.	.0833	.0055	.0408	3/4	.8125	.5185	3.879	26.	2.167	3.687	27.58
1/4	.1042	.0085	.0638	10.	.8333	.5454	4.080	27.	2.250	3.976	29.74
1/2	.1250	.0123	.0918	1/4	.8542	.5730	4.286	28.	2.333	4.276	31.99
3/4	.1458	.0167	.1249	1/2	.8750	.6013	4.498	29.	2.417	4.587	34 31
2.	.1667	.0218	.1632	3/4	.8958	.6303	4.715	30.	2.500	4.909	36.72
1/4	.1875	.0276	.2066	11.	.9167	.6600	4.937	31.	2.583	5.241	39.21
1/2	.2083	.0341	.2550	1/4	.9375	.6903	5.164	32.	2.667	5 585	41.78
3/4	.2292	.0412	.3085	1/2	.9583	.7213	5.396	33.	2.750	5.940	44.43
3.	.2500	.0491	.3672	3/4	.9792	.7530	5.633	34.	2.833	6.305	47.15
1/4	.2708	.0576	.4309	12.	1 Foot.	.7854	5.875	35.	2.917	6.681	49.98
1/2	.2917	.0668	.4998	1/2	1.042	.8522	6.375	36.	3 000	7.069	52.88
3/4	.3125	.0767	.5738	13.	1.083	.9218	6.895	37.	3.083	7.467	55.86
4.	.3333	.0873	.6528	1/2	1.125	.9940	7.436	38.	3.167	7.876	58.92
1/4	.3542	.0985	.7369	14.	1.167	1.069	7.997	39.	3.250	8.296	62.06
1/2	.3750	.1104	.8263	1/2	1.208	1.147	8.578	40.	3.333	8.727	65.28
3/4	.3958	.1231	.9206	15.	1.250	1.227	9.180	41.	3.417	9.168	68.58
5.	.4167	.1364	1.020	1/2	1.292	1.310	9.801	42.	3.500	9.621	71.97
1/4	.4375	.1503	1.125	16.	1.333	1.396	10.44	43.	3.583	10.085	75.44
1/2	.4583	.1650	1.234	1/2	1.375	1.485	11.11	44.	3.667	10.559	78.99
3/4	.4792	.1803	1.349	17.	1.417	1.576	11.79	45.	3.750	11.045	82.62
6	.5000	.1963	1.469	1/2	1.458	1.670	12.49	46.	3.833	11.541	86.33
1/4	.5208	.2131	1.594	18.	1.500	1.767	13.22	47.	3.917	12.048	90.13
1/2	.5417	.2304	1.724	1/2	1.542	1.867	13.96	48.	4.000	12.566	94.00

Table continued, but with the diams in feet.

Diam. Feet.	Cub. Feet.	U. S. Galls.	Diam. Feet.	Cub. Feet.	U. S. Galls.	Dia. Feet.	Cub. Feet.	U. S. Galls.	Dia. Feet.	Cub. Feet.	U. S. Galls.
4	12.57	94.0	7	38.49	287.9	12	113.1	846.1	24	452.4	3384
1/4	14.19	106.1	1/4	41.28	308.8	13	132.7	992.8	25	490.9	3672
1/2	15.90	119.0	1/2	44.18	330.5	14	153.9	1152.	26	530.9	3971
3/4	17.72	132.5	3/4	47.17	352.9	15	176.7	1322.	27	572.6	4283
5	19.64	146.9	8	50.27	376.0	16	201.1	1504.	28	615.8	4606
1/4	21.65	161.9	1/2	56.75	424.5	17	227.0	1698.	29	660.5	4941
1/2	23.76	177.7	9	63.62	475.9	18	254.5	1904.	30	706.9	5288
3/4	25.97	194.3	1/2	70.88	530.2	19	283.5	2121.	31	754.8	5646
6	28.27	211.5	10	78.54	587.6	20	314.2	2350.	32	804.3	6017
1/4	30.68	229.5	1/2	86.59	647.7	21	346.4	2591.	33	855.3	6398
1/2	33.18	248.2	11	95.03	710.9	22	380.1	2844.	34	907.9	6792
3/4	35.79	267.7	1/2	103.90	777.0	23	415.5	3108.	35	962.1	7197

CONTENTS AND LININGS OF WELLS.

For diams twice as great as those in the table, for the cub yds of digging, take out those opposite *one half* of the greater diam; and mult them by 4. Thus, for the cub yds in each foot of depth of a well 31 feet in diam, first take out from the table those opposite the diam of 15½ feet; namely, 6.989. Then 6.989 × 4 = 27.956 cub yds reqd for the 31 ft diam. But for the stone lining or walling, bricks or plastering, mult the tabular quantity opposite *half* the greater diam, by 2. Thus, the perches of stone walling for each foot of depth of a well of 31 ft diam, will be 2.073 × 2 = 4.146. If the wall is more or less than one foot thick, within usual moderate limits, it will generally be near enough for practice to assume that the number of perches, or of bricks, will increase or decrease in the same proportion.

The size of the bricks is taken at 8¼ × 4 × 2 inches; and to be laid dry, or without mortar. In practice an addition of about 5 per cent should be made for waste. The brick lining is supposed to be 1 brick thick, or 8¼ ins.

CAUTION.—Be careful to observe that the diams to be used for the digging, are greater than those for the walling, bricks, or plastering. No errors.

Diam. in Feet.	For each foot of depth. For this col use the Diameter of the Digging. Cub Yds. of Digging.	For these three cols use the diam in clear of the lining. Stone Lining 1 ft thick. Perches of 25 Cub Ft.	No. of Bricks in a Lining 1 Brick thick.	Square Yards of Plastering.
1.	.0291	.2513	57	.3491
¼	.0455	.2827	71	.4364
½	.0654	.3142	85	.5236
¾	.0891	.3456	99	.6109
2.	.1164	.3770	114	.6982
¼	.1473	.4084	128	.7855
½	.1818	.4398	142	.8727
¾	.2200	.4712	156	.9600
3.	.2618	.5027	170	1.047
¼	3073	.5341	184	1.135
½	.3563	.5655	198	1.222
¾	.4091	.5969	212	1.309
4.	.4654	.6283	227	1.396
¼	.5254	.6597	241	1.484
½	.5890	.6912	255	1.571
¾	.6563	.7226	269	1.658
5.	.7272	.7540	283	1.745
¼	.8018	.7854	297	1.833
½	.8799	.8168	311	1.920
¾	.9617	.8482	326	2.007
6.	1.047	.8796	340	2.095
¼	1.136	.9111	354	2.182
½	1 229	.9425	368	2.269
¾	1.325	.9739	382	2.356
7.	1.425	1.005	396	2.444
¼	1.529	1.037	410	2.531
½	1.636	1.068	425	2.618
¾	1.747	1.100	439	2.705
8.	1.862	1.131	453	2.793
¼	1.980	1.162	467	2.880
½	2.102	1.194	481	2.967
¾	2.227	1.225	495	3.054
9.	2.356	1.257	509	3.142
¼	2.489	1.288	523	3.229
½	2.625	1.319	538	3.316
¾	2.765	1.351	552	3.404
10.	2.909	1.382	566	3.491
¼	3.056	1.414	580	3.578
½	3.207	1.445	594	3.665
¾	3.362	1.477	608	3.753
11.	3.520	1.508	622	3.840
¼	3.682	1 539	637	3.927
½	3.847	1.571	651	4.014
¾	4.016	1.602	665	4.102
12.	4.189	1.634	679	4.189
¼	4.365	1.665	693	4.276
½	4.545	1.696	707	4.364
¾	4.729	1.728	721	4.451
13.	4.916	1.759	736	4.538
¼	5.107	1.791	750	4.625
½	5.301	1.822	764	4.713
¾	5.500	1.854	778	4.800
14.	5.701	1.885	792	4.887
¼	5.907	1.916	806	4.974
½	6.116	1.948	820	5.062
¾	6.329	1.979	834	5.149
15.	6.545	2.011	849	5.236
¼	6.765	2.042	863	5.323
½	6.989	2.073	877	5.411
¾	7.216	2.105	891	5.498
16.	7.447	2.136	905	5.585
¼	7.681	2.168	919	5 673
½	7.919	2.199	933	5.760
¾	8.161	2.231	948	5.847
17.	8.407	2.262	962	5.934
¼	8.656	2.293	976	6.022
½	8.908	2.325	990	6.109
¾	9.165	2.356	1004	6.196
18.	9.425	2.388	1018	6.283
¼	9.688	2.419	1032	6.371
½	9.956	2.450	1046	6.458
¾	10.23	2.482	1061	6.545
19.	10.50	2.513	1075	6.633
¼	10.78	2.545	1089	6.720
½	11.06	2.576	1103	6 807
¾	11.35	2.608	1117	6.894
20.	11.64	2.639	1131	6.982
¼	11.93	2.670	1145	7.069
½	12.22	2.702	1160	7.156
¾	12.52	2.733	1174	7.243
21.	12.83	2.765	1188	7.331
¼	13.14	2.796	1202	7.418
½	13.45	2.827	1216	7.505
¾	13.76	2.859	1230	7 593
22.	14.08	2.890	1244	7.680
¼	14.40	2.922	1259	7.767
½	14.73	2.953	1273	7.854
¾	15.06	2.985	1287	7.942
23.	15.39	3.016	1301	8.029
¼	15.72	3.047	1315	8.116
½	16.06	3.079	1329	8.203
¾	16.41	3.110	1343	8.291
24.	16.76	3.142	1357	8.378
¼	17.11	3.173	1372	8.465
½	17.46	3 204	1386	8.552
¾	17.82	3.236	1400	8.640
25.	18.18	3.267	1414	8.727

A cub yd = 202 U. S. galls.

If **perches** are named in a contract, it is necessary, in order to prevent fraud, to specify the number of cub feet contained in the perch; for stone-quarriers have one perch, stone-masons another, &c. Engineers, on this account, contract by the *cubic yard*. The perch should be done away with entirely; Perches of 25 cub ft × .926 = cub yds; and cub yds ÷ .926 = pers of 25 cub ft.

CIRCULAR CYLINDRIC UNGULAS.

I. When the cutting plane does not cut the base. Figs 13, 14.

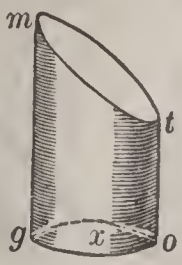


FIG. 13.

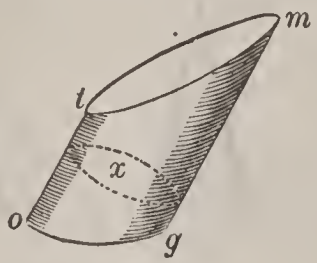


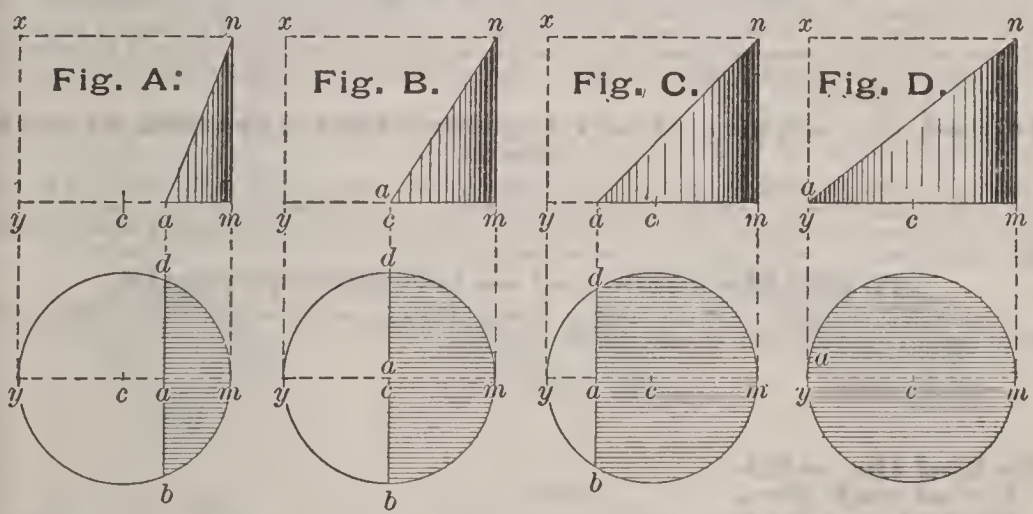
FIG. 14.

Volume of ungula $\Big\}$ = area of base $o\,g \times \frac{1}{2}$ sum of greatest & least perp heights, $o\,n$, $c\,m$,
$= \Big\{$ area of cross sec measd perp to sides, as x, Fig 14 $\times$ $\frac{1}{2}$ sum of greatest and least lengths, $g\,m$, $o\,t$, measd along the sides.

Area of *curved* surface $\Big\} = \Big\{$ circumf measd perp to sides, as at x, $\times$ half sum of greatest and least lengths, $g\,m$, $o\,t$, measd along the sides.

Add areas of ends if required.
For areas of sections perpendicular to the sides, see Circles, pp 123, &c.
For areas of sections oblique to the sides, see The Ellipse, pp 149, &c.

II. When the cutting plane touches the base. Figs A to D.



Volume

Fig **A** $= (\frac{2}{3}\,a\,b^3 - a\,c \times$ area $a\,d\,m\,b$* of base$) \dfrac{m\,n}{a\,m}$.

Fig **B** $= \frac{2}{3}\,c\,b^2 \times m\,n$.

Fig **C** $= (\frac{2}{3}\,a\,b^3 + a\,c \times$ area $a\,d\,m\,b$* of base$) \dfrac{m\,n}{a\,m}$.

Fig **D** $= \frac{1}{2}$ area of circle $y\,m$† $\times\, m\,n$
$= \frac{1}{2}$ volume of cylinder $x\,y\,m\,n$.

Curved surface

Fig **A** $= (a\,b \times m\,y - a\,c \times$ length of arc $d\,m\,b$‡$) \dfrac{m\,n}{a\,m}$.

Fig **B** $= m\,y \times m\,n$.

Fig **C** $= (a\,b \times m\,y + a\,c \times$ length of arc $d\,m\,b$‡$) \dfrac{m\,n}{a\,m}$.

Fig **D** $= \frac{1}{2}$ circumference of base† $m\,y \times m\,n$
$= \frac{1}{2}$ curved surface of cylinder $x\,y\,m\,n$.

For area of sloping plane, see p 150.

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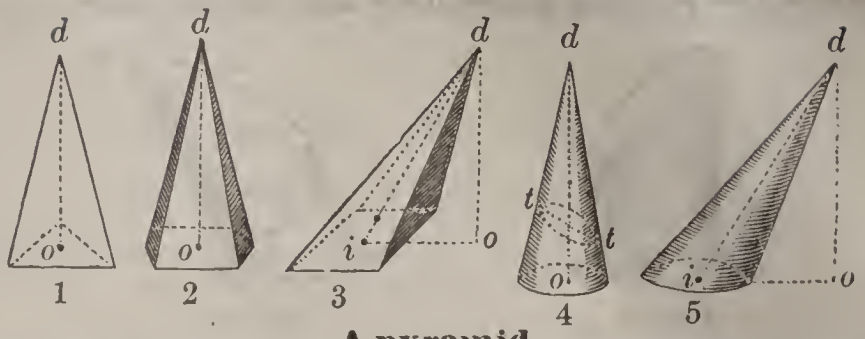
* For area of base (segment of circle), see pp 146 to 148.
† For circles, see pp 123, etc.
‡ For length of circular arc, see pp 141, etc.

PYRAMIDS AND CONES.



A pyramid,

Figs 1, 2, 3, is any solid which has for its base, a plane fig of any number of sides: and for its sides, plane triangles, all terminating at one point *d*, called its *apex*, or top. When the base is a regular fig, the pyramid is *regular;* otherwise *irregular*. For regular figs see Polygons, p. 110.

A cone,

Figs 4 and 5, is a solid, of which the base is a curved fig; and which may be considered as made or generated by a line, of which one end is stationary at a certain point *d*, called the apex or top, while the line is being carried around the circumf of the base, which may be a circle, ellipse, or other curve.

The axis of a pyramid, or cone, is a straight line *d o* in Figs 1, 2, 4; and *d i* in Figs 3 and 5, from the apex *d*, to the centre of the base. When the axis is perp to the base, as in Figs 1, 2, 4, the solid is said to be a *right* one; when otherwise, as Figs 3, 5, an *oblique* one. When the word cone is used alone, the right circular cone, Fig 4, is understood. If such a cone be cut, as at *t t*, obliquely to its base, the new base *t t* will be an ellipse; and the cone *d t t* becomes an oblique elliptic one. Fig 5 will represent either an oblique circular cone, or an oblique elliptic one, according as its base is a circle, or an ellipse.

To find the solidity of any pyramid, or cone.

Whether regular or irregular; right or oblique; mult the area of its base, by one-third of its *perp* height *d o*. Figs 1 to 5. Every pyramid, or cone, has one-third of the solidity of either a prism or a cylinder having the same area of base, and the same perp height; and one-half that of a hemisphere of the same base and height; in other words, a cone, hemisphere, and cylinder of the same base and height, have solidities as 1, 2, 3.

To find the surface of any regular right pyramid, or right cone.

Mult the circumf or outline of its base, by the *slant* height; take half the prod. This will give the surf of the sides; to which add that of the base if reqd. In the pyramid, this slant height must be measd from *d* to the middle of one of the equal sides, and not along one of the edges of the pyramid. Mathematicians have been unable to devise any measurement of the surf of an *oblique* cone.

To find the surface of an irregular pyramid.

Whether right or oblique, each side must be calculated as a separate triangle; and the several areas added together. Add the area of base if reqd.

To find the solidity of any frustum of any pyramid, or cone, when the base and top are parallel.

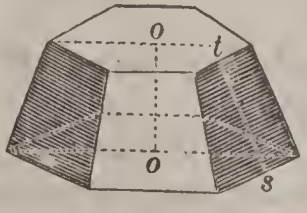


Fig. 6.

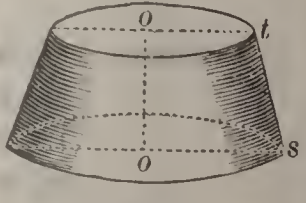


Fig. 7.

Rule 1. Whether regular or irregular, right or oblique, add together the areas of the base, and top, and the area of their mean proportional; mult the sum by one-third of the perp height o o, Figs 6 and 7, between the base and top.

Rem. For the area of the mean proportional, (which is *not* the area halfway between, and parallel with the ends,) mult together the areas of base and top; and take the sq rt of the prod.

Rule 2. This applies only to the right circular conical frust. Add together the squares of the two diams; and the prod of the two diams; mult together the sum, one-third the perp height o o, and .7854.

To find the surface of any frustum of a regular right pyramid, or cone, Figs 6 and 7, when the base and top are parallel.

Add together the circumfs of the two ends; mult the sum by the *slant* height *s t*; take half the prod. This gives the surf of the sides: to which add that of the ends when reqd. In the frustum of the pyramid, the slant height must be measd between the middles *s* and *t* of two corresponding sides of the base, and top. Fig. 6.

If the pyramid is not regular; or *if it is oblique*, then the surf of the sides must be obtained for each side separately as a trapezoid.

PRISMOIDS.

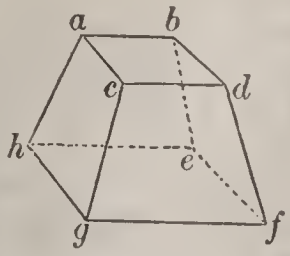


Fig. 1.

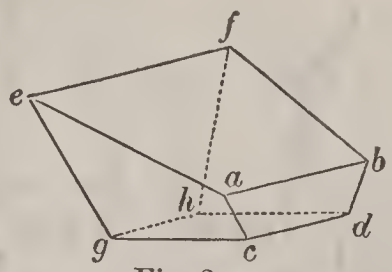


Fig. 2.

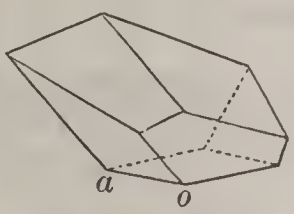
A prismoid is

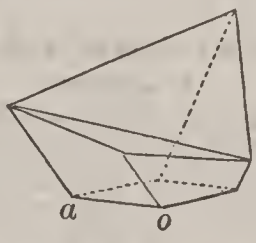
Any solid bounded by six plane surfaces, of which but two, as *a b c d* and *e f g h*, Figs 1 and 2, are *necessarily* parallel; and at least two other opposite ones not parallel.

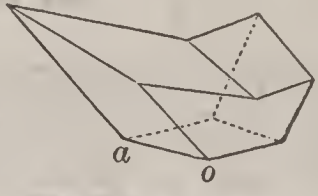
To find the solidity of any prismoid.

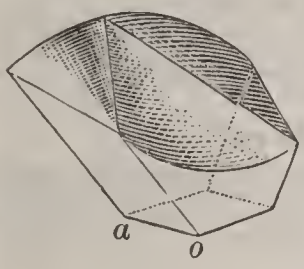
Add together the areas of the two parallel surfaces; and four times the area of the section taken halfway between them, and parallel to them: mult the sum by the perp dist between the two parallel sides; div the prod by 6. To find the areas referred to, see **Trapezoids, Trapeziums,** p 120.

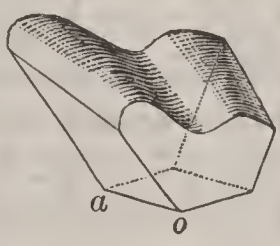
The foregoing rule is the well-known "*prismoidal formula;*" the very extended application of which to other solids than those which fall strictly within the definition of the prismoid, was first discovered and made known by Ellwood Morris, civ eng of Philadelphia, in 1840. It embraces *all* parallelopipeds, prisms, pyramids, cones, wedges, &c, whether regular or irregular, right or oblique; together with their frustums when cut parallel to their bases; indeed all solids whatever having two parallel faces, or sides, provided these two faces are united by surfaces, whether plane or curved, upon which, and through every point of which, a straight line may be drawn from one of the parallel faces to the other. The following six Figs represent a few such solids; they may be regarded as one-chain lengths of railroad cuttings; *a o* being the perp dist between the two parallel ends.

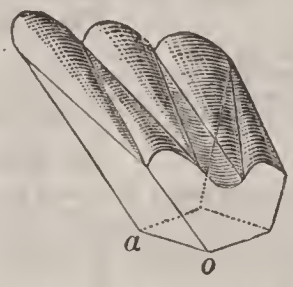


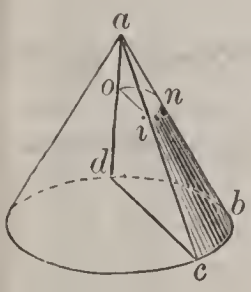


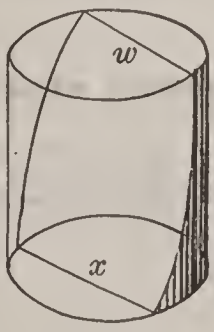












The prismoidal formula applies also to the sphere, hemisphere, and other spherical segments; also to any sections such as *a b c d*, and o *n i d b* c, of the cone, in which the sides *a d*, *a c*, or *o d*, *i c*, are straight; as they are only when the cutting plane *a d c* passes through the apex, or top *a*. **Also to the cylinder** when a plane **parallel to the sides** passes through both ends, but not if the plane *w x* is **oblique,** as in the fig, though never erring more than 1 in 142. **In this last case** we must imagine the plane to be extended until it cuts the side of the cylinder likewise extended; and then by page 159 find the solidity of the ungula thus formed. Then find the solidity of the small ungula above *w*, also thus formed, and subtract it from the large one.

WEDGES.

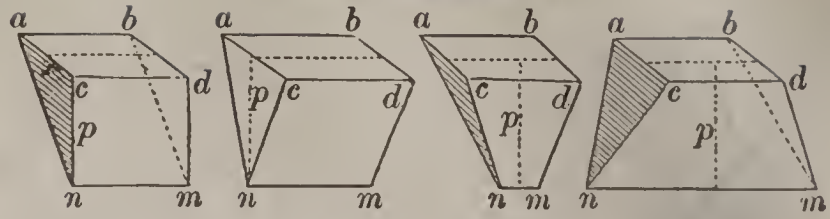


Fig. 8. Fig. 9. Fig. 10. Fig. 11.

A wedge

Is usually defined to be a solid, Figs 8 and 9, generated by a plane triangle, *a n* c, moving parallel to itself, in a straight line. This definition requires that the two triangular ends of the wedge should be parallel; but a wedge may be shaped as in Fig 10 or 11. We would therefore propose the following definition, which embraces all the figs; besides various modifications of them. A solid of five plane faces; one of which is a parallelogram *a b* c *d*, two opposite sides of which, as *a* c and *b d*, are united by means of two triangular faces *a* c *n*, and *b d m*, to an edge or line *n m*, parallel to the other opposite sides *a b* and c *d*. The parallelogram *a b* c *d* may be either rectangular, or not; the two triangular faces may be similar, or not; and the same with regard to the other two faces. The following rule applies equally to all.

To find the solidity of any wedge.

Add together the length of the edge *m n*, and twice the length *a b* or c *d* of the back; mult the sum by the *perp* height *p*, from the edge to the back; mult this prod by the breadth of the back, measd *perp* to its two sides *a b* and c *d*; div this last prod by 6.

SPHERES OR GLOBES.

A sphere

Is a solid generated by the revolution of a semicircle aronnd its diam. Any line passing entirely through a sphere, and through its center, is called its *axis*, or diam. Any circle described on the surface of a sphere, from the center of the sphere as the center of the circle, is called a *great circle* of that sphere; in other words, any entire circumf of a sphere is a great circle. A sphere has a greater content or solidity than any other solid with the same amount of surface; so that if the shape of a sphere be any way changed, its content will be reduced.

To find the solidity of a sphere.*

Cube its diam; mult said cube by .5236. Or cube its circumf; and mult by .01689. Or mult its surface by its diam; and div by 6. Or refer to the following table of spheres. The solidity of a sphere is ⅔ that of its circumscribing cylinder; or .5236 that of its circumscribing cube; or = 4 great circles × ⅙ diam; = cube of rad × 4.1888; = surface × ⅙ diam.

To find the surface of a sphere.*

Square its diam; mult said square by 3.1416. Or mult the diam by the circumf. Or square its circumf; and mult by .3183. The surf of a sphere is equal to 4 times the area of its great circle; or to the area of a circle whose diam is twice as great as that of the sphere; or to the curved surf of its circumscribing cylinder; or to sq of rad × 12.5664; or to solidity ÷ ⅙ diam.

For tables of surfaces and solidities of spheres, see pp 163 to 165.

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* If the diam is measured in inches, divide the surfaces in the table by 144, if it is required to reduce them to square feet; and divide the solidities by 1728, if required in cubic feet.

SPHERES. (ORIGINAL.)

Some errors of 1 in the last figure only.

Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.
1-64	.00077		13-32	18.190	7.2949	⅞	170.87	210.03	⅛	921.33	2629.6
1 32	.00307	.00002	7-16	18.666	7.5829	½	176.71	220.89	¼	934.83	2687.6
3-64	.00690	.00005	15-32	19.147	7.8783	⅝	182.66	232.13	⅜	948.43	2746.5
1 16	.01227	.00013	½	19.635	8.1813	¾	188.69	243.73	½	962.12	2806.2
3-32	.02761	.00043	17-32	20.129	8.4919	⅞	194.83	255.72	⅝	975.91	2866.8
⅛	.04909	.00102	9-16	20.629	8 8103	8.	201.06	268.08	¾	989.80	2928.2
5-32	.07670	.00200	19-32	21.135	9.1366	⅛	207.39	280.85	⅞	1003.8	2990.5
3-16	.11045	.00345	⅝	21.648	9.4708	¼	213.82	294.01	18.	1017.9	3053.6
7-32	.15033	.00548	21-32	22.166	9.8131	⅜	220.36	307.58	⅛	1032.1	3117.7
¼	.19635	.00818	11-16	22.691	10.164	½	226.98	321.56	¼	1046.4	3182.6
9-32	.24851	.01165	23-32	23.222	10.522	⅝	233.71	335.95	⅜	1060.8	3248.5
5-16	.30680	.01598	¾	23.758	10.889	¾	240.53	350.77	½	1075.2	3315.3
11-32	.37123	.02127	25-32	24.302	11.265	⅞	247.45	366.02	⅝	1089 8	3382.9
⅜	.44179	.02761	13-16	24.850	11.649	9.	254.47	381.70	¾	1104.5	3451.5
13-32	.51848	.03511	27-32	25.405	12.041	⅛	261.59	397.83	⅞	1119.3	3521.0
7-16	.60132	.04385	⅞	25.967	12.443	¼	268.81	414.41	19.	1134.1	3591.4
15-32	.69028	.05393	29-32	26.535	12.853	⅜	276.12	431.44	⅛	1149.1	3662.8
½	.78540	.06545	15-16	27.109	13.272	½	283.53	448.92	¼	1164.2	3735.0
17-32	.88664	.07850	31-32	27.688	13.700	⅝	291.04	466 87	⅜	1179.3	3808.2
9-16	.99403	.09319	**3.**	28.274	14.137	¾	298.65	485.31	½	1194.6	3882.5
19-32	1.1075	.10960	1-16	29.465	15.039	⅞	306.36	504.21	⅝	1210.0	3957.6
⅝	1.2272	.12783	⅛	30.680	15.979	10.	314.16	523.60	¾	1225.4	4033.7
21-32	1.3530	.14798	3-16	31.919	16.957	⅛	322.06	543 48	⅞	1241.0	4110.8
11-16	1.4849	.17014	¼	33.183	17.974	¼	330.06	563.86	20.	1256.7	4188.8
23-32	1.6230	.19442	5-16	34.472	19.031	⅜	338.16	584.74	⅛	1272.4	4267.8
¾	1.7671	.22089	⅜	35.784	20.129	½	346.36	606.13	¼	1288 3	4347.8
25-32	1.9175	.24967	7-16	37.122	21.268	⅝	354.66	628.04	⅜	1304.2	4428.8
13-16	2.0739	.28084	½	38.484	22.449	¾	363.05	650.46	½	1320.3	4510.9
27-32	2.2365	.31451	9-16	39.872	23.674	⅞	371.54	673.42	⅝	1336.4	4593.9
⅞	2.4053	.35077	⅝	41.283	24.942	11.	380.13	696 91	¾	1352.7	4677.9
29-32	2.5802	.38971	11-16	42.719	26.254	⅛	388.83	720.95	⅞	1369.0	4763.0
15-16	2.7611	.43143	¾	44.179	27.611	¼	397.61	745.51	21.	1385.5	4849.1
31-32	2.9483	.47603	13-16	45.664	29.016	⅜	406.49	770.64	⅛	1402.0	4936.2
1.	3.1416	.52360	⅞	47.173	30.466	½	415.48	796.33	¼	1418.6	5024.3
1-32	3.3410	.57424	15-16	48.708	31.965	⅝	424.56	822.58	⅜	1435.4	5113.5
1-16	3.5466	.62804	**4.**	50.265	33.510	¾	433.73	849.40	½	1452.2	5203.7
3-32	3.7583	.68511	1 16	51.848	35.106	⅞	443.01	876.79	⅝	1469.2	5295.1
⅛	3.9761	.74551	⅛	53.456	36.751	12.	452.39	904.78	¾	1486.2	5387.4
5-32	4.2000	.80939	3-16	55.089	38.448	⅛	461.87	933.34	⅞	1503.3	5480.8
3-16	4.4301	.87681	¼	56.745	40.195	¼	471.44	962.52	22.	1520.5	5575.3
7-32	4.6664	.94786	5-16	58.427	41.994	⅜	481.11	992.28	⅛	1537 9	5670.8
¼	4.9088	1.0227	⅜	60.133	43.847	½	490.87	1022.7	¼	1555.3	5767.6
9-32	5.1573	1.1013	7-16	61.863	45.752	⅝	500.73	1053.6	⅜	1572.8	5865.2
5-16	5.4119	1.1839	½	63.617	47.713	¾	510.71	1085.3	½	1590.4	5964.1
11-32	5.6728	1.2704	9-16	65.397	49.729	⅞	520.77	1117.5	⅝	1608.2	6064.1
⅜	5.9396	1.3611	⅝	67.201	51.801	13.	530.93	1150.3	¾	1626.0	6165.2
13-32	6.2126	1.4561	11-16	69.030	53.929	⅛	541.19	1183.8	⅞	1643.9	6267.3
7-16	6.4919	1.5553	¾	70.883	56.116	¼	551.55	1218.0	23.	1661.9	6370.6
15-32	6 7771	1.6590	13-16	72.759	58.359	⅜	562 00	1252.7	⅛	1680.0	6475.0
½	7.0686	1.7671	⅞	74.663	60.663	½	572.55	1288.3	¼	1698.2	6580.6
17-32	7.3663	1.8799	15-16	76.589	63.026	⅝	583.20	1324.4	⅜	1716.5	6687.3
9-16	7.6699	1.9974	**5.**	78.540	65.450	¾	593.95	1361.2	½	1735.0	6795.2
19-32	7.9798	2.1196	1-16	80.516	67.935	⅞	604.80	1398.6	⅝	1753.5	6904 2
⅝	8.2957	2.2468	⅛	82.516	70.482	14.	615.75	1436.8	¾	1772.1	7014.3
21-32	8.6180	2.3789	3-16	84.541	73.092	⅛	626.80	1475.6	⅞	1790.8	7125.6
11-16	8.9461	2.5161	¼	86.591	75.767	¼	637.95	1515.1	24.	1809.6	7238.2
23-32	9.2805	2.6586	5-16	88.664	78.505	⅜	649.17	1555.3	⅛	1828.5	7351.9
¾	9.6211	2.8062	⅜	90.763	81.308	½	660 52	1596.3	¼	1847.5	7466.7
25-32	9.9678	2.9592	7-16	92.887	84.178	⅝	671.95	1637.9	⅜	1866.6	7583.0
13-16	10.321	3.1177	½	95.033	87.113	¾	683.49	1680.3	½	1885.8	7700.1
27-32	10.680	3.2818	9-16	97.205	90.118	⅞	695.13	1723.3	⅝	1905.1	7818.6
⅞	11.044	3.4514	⅝	99.401	93.189	15.	706.85	1767.2	¾	1924.4	7938.3
29-32	11.416	3.6270	11-16	101.62	96.331	⅛	718.69	1811.7	⅞	1943.9	8059.2
15-16	11.793	3.8083	¾	103.87	99.541	¼	730.63	1857.0	25.	1963.5	8181.3
31-32	12.177	3.9956	13-16	106.14	102.82	⅜	742.65	1903.0	⅛	1983.2	8304.7
2.	12.566	4.1888	⅞	108.44	106.18	½	754.77	1949.8	¼	2002.9	8429.2
1-32	12.962	4.3882	15-16	110.75	109.60	⅝	767.00	1997.4	⅜	2022.9	8554.9
1-16	13.364	4.5939	**6.**	113.10	113.10	¾	779.32	2045.7	½	2042.8	8682.0
3-32	13.772	4.8060	⅛	117.87	120.31	⅞	791.73	2094.8	⅝	2062.9	8810.3
⅛	14.186	5.0243	¼	122.72	127.83	16.	804.25	2144.7	¾	2083.0	8939.9
5-32	14.607	5.2493	⅜	127 68	135.66	⅛	816.85	2195.3	⅞	2103.4	9070.6
3-16	15.033	5.4809	½	132.73	143.79	¼	829.57	2246.8	26.	2123.7	9202.8
7-32	15.466	5.7190	⅝	137.89	152.25	⅜	842.40	2299.1	⅛	2144.2	9336.2
¼	15.904	5.9641	¾	143.14	161.03	½	855.29	2352.1	¼	2164.7	9470.8
9-32	16.349	6 2161	⅞	148.49	170.14	⅝	868.31	2406.0	⅜	2185.5	9606.7
5-16	16.800	6.4751	**7.**	153.94	179.59	¾	881.42	2460.6	½	2206.2	9744.0
11-32	17.258	6 7412	⅛	159 49	189.39	⅞	894.62	2516.1	⅝	2227.1	9882.5
⅜	17.721	7.0144	¼	165.13	199.53	17.	907.93	2572.4	¾	2248.0	10022

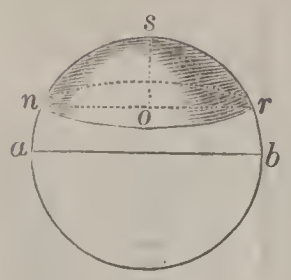
SPHERES—(CONTINUED.)

Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.
⅛	2269.1	10164	⅝	4214.1	25724	⅜	6756.5	52222	⅛	9896.0	92570
27.	2280.2	10300	¾	4243.0	25988	½	6792.9	52645	¼	9940.2	93190
⅛	2311.5	10450	⅞	4271.8	26254	⅝	6829.5	53071	⅜	9984.4	93812
¼	2332.8	10595	37.	4300.9	26522	¾	6866.1	53499	½	10029	94438
⅜	2354.3	10741	⅛	4330.0	26792	⅞	6902.9	53929	⅝	10073	95066
½	2375.8	10889	¼	4359.2	27063	47.	6939.9	54362	¾	10118	95697
⅝	2397.5	11038	⅜	4388.5	27337	⅛	6976.8	54797	⅞	10163	96330
¾	2419.2	11189	½	4417.9	27612	¼	7013.9	55234	57.	10207	96967
⅞	2441.1	11341	⅝	4447.5	27889	⅜	7050.9	55674	⅛	10252	97606
28.	2463.0	11494	¾	4477.1	28168	½	7088.3	56115	¼	10297	98248
⅛	2485.1	11649	⅞	4506.8	28449	⅝	7125.6	56559	⅜	10342	98893
¼	2507.2	11805	38.	4536.5	28731	¾	7163.1	57006	½	10387	99541
⅜	2529.5	11962	⅛	4566.5	29016	⅞	7200.7	57455	⅝	10432	100191
½	2551.8	12121	¼	4596.4	29302	48.	7238.3	57906	¾	10478	100845
⅝	2574.3	12281	⅜	4626.5	29590	⅛	7276.0	58360	⅞	10523	101501
¾	2596.7	12443	½	4656.7	29880	¼	7313.9	58815	58.	10568	102161
⅞	2619.4	12606	⅝	4686.9	30173	⅜	7351.9	59274	⅛	10614	102823
29.	2642.1	12770	¾	4717.3	30466	½	7389.9	59734	¼	10660	103488
⅛	2665.0	12936	⅞	4747.9	30762	⅝	7428.0	60197	⅜	10706	104155
¼	2687.8	13103	39.	4778.4	31059	¾	7466.3	60663	½	10751	104826
⅜	2710.9	13272	⅛	4809.0	31359	⅞	7504.5	61131	⅝	10798	105499
½	2734.0	13442	¼	4839.9	31661	49.	7543.1	61601	¾	10844	106175
⅝	2757.3	13614	⅜	4870.8	31964	⅛	7581.6	62074	⅞	10890	106854
¾	2780.5	13787	½	4901.7	32270	¼	7620.1	62549	59.	10936	107536
⅞	2804.0	13961	⅝	4932.7	32577	⅜	7658.9	63026	⅛	10983	108221
30.	2827.4	14137	¾	4964.0	32886	½	7697.7	63506	¼	11029	108909
⅛	2851.1	14315	⅞	4995.3	33197	⅝	7736.7	63989	⅜	11076	109600
¼	2874.8	14494	40.	5026.5	33510	¾	7775.7	64474	½	11122	110294
⅜	2898.7	14674	⅛	5058.1	33826	⅞	7814.8	64961	⅝	11169	110990
½	2922.5	14856	¼	5089.6	34143	50.	7854.0	65450	¾	11216	111690
⅝	2946.6	15039	⅜	5121.3	34462	⅛	7893.3	65941	⅞	11263	112392
¾	2970.6	15224	½	5153.1	34783	¼	7932.8	66436	60.	11310	113098
⅞	2994.9	15411	⅝	5184.9	35106	⅜	7972.2	66934	⅛	11357	113806
31.	3019.1	15599	¾	5216.8	35431	½	8011.8	67433	¼	11404	114518
⅛	3043.6	15788	⅞	5248.9	35758	⅝	8051.6	67935	⅜	11452	115232
¼	3068.0	15979	41.	5281.1	36087	¾	8091.4	68439	½	11499	115949
⅜	3092.7	16172	⅛	5313.3	36418	⅞	8131.3	68946	⅝	11547	116669
½	3117.3	16366	¼	5345.6	36751	51.	8171.2	69456	¾	11595	117392
⅝	3142.1	16561	⅜	5378.1	37086	⅛	8211.4	69967	⅞	11642	118118
¾	3166.9	16758	½	5410.7	37423	¼	8251.6	70482	61.	11690	118847
⅞	3192.0	16957	⅝	5443.3	37763	⅜	8292.0	70999	⅛	11738	119579
32.	3217.0	17157	¾	5476.0	38104	½	8332.3	71519	¼	11786	120315
⅛	3242.2	17359	⅞	5508.9	38448	⅝	8372.8	72040	⅜	11834	121053
¼	3267.4	17563	42.	5541.9	38792	¾	8413.4	72563	½	11882	121794
⅜	3292.9	17768	⅛	5574.9	39140	⅞	8454.1	73092	⅝	11931	122538
½	3318.3	17974	¼	5608.0	39490	52.	8494.8	73622	¾	11980	123286
⅝	3343.9	18182	⅜	5641.3	39841	⅛	8535.8	74154	⅞	12028	124036
¾	3369.6	18392	½	5674.5	40194	¼	8576.8	74689	62.	12076	124789
⅞	3395.4	18604	⅝	5708.0	40551	⅜	8617.8	75226	⅛	12126	125545
33.	3421.2	18817	¾	5741.5	40908	½	8658.9	75767	¼	12174	126305
⅛	3447.3	19032	⅞	5775.2	41268	⅝	8700.4	76309	⅜	12223	127067
¼	3473.3	19248	43.	5808.8	41630	¾	8741.7	76854	½	12272	127832
⅜	3499.5	19466	⅛	5842.7	41994	⅞	8783.2	77401	⅝	12322	128601
½	3525.7	19685	¼	5876.5	42360	53.	8824.8	77952	¾	12371	129373
⅝	3552.1	19907	⅜	5910.7	42729	⅛	8866.4	78505	⅞	12420	130147
¾	3578.5	20129	½	5944.7	43099	¼	8908.2	79060	63.	12469	130925
⅞	3605.1	20354	⅝	5978.9	43472	⅜	8950.1	79617	⅛	12519	131706
34.	3631.7	20580	¾	6013.2	43846	½	8992.0	80178	¼	12568	132490
⅛	3658.5	20808	⅞	6047.7	44224	⅝	9034.1	80741	⅜	12618	133277
¼	3685.3	21037	44.	6082.1	44602	¾	9076.4	81308	½	12668	134067
⅜	3712.3	21268	⅛	6116.8	44984	⅞	9118.5	81876	⅝	12718	134860
½	3739.3	21501	¼	6151.5	45367	54.	9160.8	82448	¾	12768	135657
⅝	3766.5	21736	⅜	6186.3	45753	⅛	9203.3	83021	⅞	12818	136456
¾	3793.7	21972	½	6221.2	46141	¼	9216.0	83598	64.	12868	137259
⅞	3821.1	22210	⅝	6256.1	46530	⅜	9288.5	84177	⅛	12918	138065
35.	3848.5	22449	¾	6291.2	46922	½	9331.2	84760	¼	12969	138874
⅛	3876.1	22691	⅞	6326.5	47317	⅝	9374.1	85344	⅜	13019	139686
¼	3903.7	22934	45.	6361.7	47713	¾	9417.2	85931	½	13070	140501
⅜	3931.5	23179	⅛	6397.2	48112	⅞	9460.2	86521	⅝	13121	141320
½	3959.2	23425	¼	6432.7	48513	55.	9503.2	87114	¾	13172	142142
⅝	3987.2	23674	⅜	6468.3	48916	⅛	9546.5	87709	⅞	13222	142966
¾	4015.2	23924	½	6503.9	49321	¼	9590.0	88307	65.	13273	143794
⅞	4043.3	24176	⅝	6539.7	49729	⅜	9633.3	88908	⅛	13324	144625
36.	4071.5	24429	¾	6575.5	50139	½	9676.8	89511	¼	13376	145460
⅛	4099.9	24685	⅞	6611.6	50551	⅝	9720.6	90117	⅜	13427	146297
¼	4128.3	24942	46.	6647.6	50965	¾	9764.4	90726	½	13478	147138
⅜	4156.9	25201	⅛	6683.7	51382	⅞	9808.1	91338	⅝	13530	147982
½	4185.5	25461	¼	6720.0	51801	56.	9852.0	91953	¾	13582	148828

SPHERES—(CONTINUED.)

Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.
	1/8	13633	149680		1/2	17437	216505		1/8	21708	300743		3/4	26446	404406
66.		13685	150533		5/8	17496	217597		1/4	21773	302100		7/8	26518	406060
	1/8	13737	151390		3/4	17554	218693		3/8	21839	303463	92.		26590	407721
	1/4	13789	152251		7/8	17613	219792		1/2	21904	304831		1/8	26663	409384
	3/8	13841	153114	75.		17672	220894		5/8	21970	306201		1/4	26735	411054
	1/2	13893	153980		1/8	17731	222001		3/4	22036	307576		3/8	26808	412726
	5/8	13946	154850		1/4	17790	223111		7/8	22102	308957		1/2	26880	414405
	3/4	13998	155724		3/8	17849	224224	84.		22167	310340		5/8	26953	416086
	7/8	14050	156600		1/2	17908	225341		1/8	22234	311728		3/4	27026	417774
67.		14103	157480		5/8	17968	226463		1/4	22300	313118		7/8	27099	419464
	1/8	14156	158363		3/4	18027	227588		3/8	22366	314514	93.		27172	421161
	1/4	14208	159250		7/8	18087	228716		1/2	22432	315915		1/8	27245	422862
	3/8	14261	160139	76.		18146	229848		5/8	22499	317318		1/4	27318	424567
	1/2	14314	161032		1/8	18206	230984		3/4	22565	318726		3/8	27391	426277
	5/8	14367	161927		1/4	18266	232124		7/8	22632	320140		1/2	27464	427991
	3/4	14420	162827		3/8	18326	233267	85.		22698	321556		5/8	27538	429710
	7/8	14474	163731		1/2	18386	234414		1/8	22765	322977		3/4	27612	431433
68.		14527	164637		5/8	18446	235566		1/4	22832	324402		7/8	27686	433160
	1/8	14580	165547		3/4	18506	236719		3/8	22899	325831	94.		27759	434894
	1/4	14634	166460		7/8	18566	237879		1/2	22966	327264		1/8	27833	436630
	3/8	14688	167376	77.		18626	239041		5/8	23034	328702		1/4	27907	438373
	1/2	14741	168295		1/8	18687	240206		3/4	23101	330142		3/8	27981	440118
	5/8	14795	169218		1/4	18748	241376		7/8	23168	331588		1/2	28055	441871
	3/4	14849	170145		3/8	18809	242551	86.		23235	333039		5/8	28130	443625
	7/8	14903	171074		1/2	18869	243728		1/8	23303	334492		3/4	28204	445387
69		14957	172007		5/8	18930	244908		1/4	23371	335951		7/8	28278	447151
	1/8	15012	172944		3/4	18992	246093		3/8	23439	337414	95.		28353	448920
	1/4	15066	173883		7/8	19053	247283		1/2	23506	338882		1/8	28428	450695
	3/8	15120	174828	78.		19114	248475		5/8	23575	340352		1/4	28503	452475
	1/2	15175	175774		1/8	19175	249672		3/4	23643	341829		3/8	28577	454259
	5/8	15230	176723		1/4	19237	250873		7/8	23711	343307		1/2	28652	456047
	3/4	15284	177677		3/8	19298	252077	87.		23779	344792		5/8	28727	457839
	7/8	15339	178635		1/2	19360	253284		1/8	23847	346281		3/4	28802	459638
70.		15394	179595		5/8	19422	254496		1/4	23916	347772		7/8	28878	461439
	1/8	15449	180559		3/4	19483	255713		3/8	23984	349269	96.		28953	463248
	1/4	15504	181525		7/8	19545	256932		1/2	24053	350771		1/8	29028	465059
	3/8	15560	182497	79.		19607	258155		5/8	24122	352277		1/4	29104	466875
	1/2	15615	183471		1/8	19669	259383		3/4	24191	353785		3/8	29180	468697
	5/8	15670	184449		1/4	19732	260613		7/8	24260	355301		1/2	29255	470524
	3/4	15726	185430		3/8	19794	261848	88.		24328	356819		5/8	29331	472354
	7/8	15782	186414		1/2	19856	263088		1/8	24398	358342		3/4	29407	474189
71.		15837	187402		5/8	19919	264330		1/4	24467	359869		7/8	29483	476029
	1/8	15893	188394		3/4	19981	265577		3/8	24536	361400	97.		29559	477874
	1/4	15949	189389		7/8	20044	266829		1/2	24606	362935		1/8	29636	479725
	3/8	16005	190387	80.		20106	268083		5/8	24676	364476		1/4	29712	481579
	1/2	16061	191389		1/8	20170	269342		3/4	24745	366019		3/8	29788	483438
	5/8	16117	192395		1/4	20232	270604		7/8	24815	367568		1/2	29865	485302
	3/4	16174	193404		3/8	20296	271871	89.		24885	369122		5/8	29942	487171
	7/8	16230	194417		1/2	20358	273141		1/8	24955	370678		3/4	30018	489045
72.		16286	195433		5/8	20422	274416		1/4	25025	372240		7/8	30095	490924
	1/8	16343	196453		3/4	20485	275694		3/8	25095	373806	98.		30172	492808
	1/4	16400	197476		7/8	20549	276977		1/2	25165	375378		1/8	30249	494695
	3/8	16456	198502	81.		20612	278263		5/8	25236	376954		1/4	30326	496588
	1/2	16513	199532		1/8	20676	279553		3/4	25306	378531		3/8	30404	498486
	5/8	16570	200566		1/4	20740	280847		7/8	25376	380115		1/2	30481	500388
	3/4	16628	201604		3/8	20804	282145	90.		25447	381704		5/8	30558	502296
	7/8	16685	202645		1/2	20867	283447		1/8	25518	383297		3/4	30636	504208
73.		16742	203689		5/8	20932	284754		1/4	25589	384894		7/8	30713	506125
	1/8	16799	204737		3/4	20996	286064		3/8	25660	386496	99.		30791	508047
	1/4	16857	205789		7/8	21060	287378		1/2	25730	388102		1/8	30869	509975
	3/8	16914	206844	82.		21124	288696		5/8	25802	389711		1/4	30947	511906
	1/2	16972	207903		1/8	21189	290019		3/4	25873	391327		3/8	31025	513843
	5/8	17030	208966		1/4	21253	291345		7/8	25944	392945		1/2	31103	515785
	3/4	17088	210032		3/8	21318	292674	91.		26016	394570		5/8	31181	517730
	7/8	17146	211102		1/2	21382	294010		1/8	26087	396197		3/4	31259	519682
74.		17204	212175		5/8	21448	295347		1/4	26159	397831		7/8	31338	521638
	1/8	17262	213252		3/4	21512	296691		3/8	26230	399468	100.		31416	523598
	1/4	17320	214333		7/8	21578	298036		1/2	26302	401109				
	3/8	17379	215417	83.		21642	299388		5/8	26374	402756				

To find the solidity of a spherical segment.



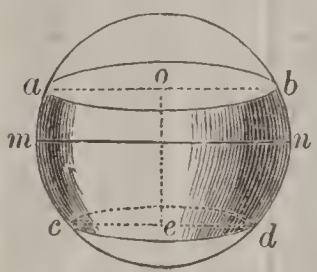
Rule 1. Square the rad o *n*, of its base; mult this square by 3; to the prod add the square of its height *o s*; mult the sum by the height *o s*; and mult this last prod by .5236.

Rule 2. Mult the diam *a b of the sphere* by 3; from the prod take twice the height *o s* of the segment; mult the rem by the square of the height *o s*; and mult this prod by .5236.

The solidity of a sphere being ⅔ds that of its circumscribing cylinder, if we add to any solidity in the table, its half, we obtain that of a cylinder of the same diam as the sphere, and whose height equals its diam.

To find the curved surface of a spherical segment.

Rule 1. Mult the diam *a b* of the sphere from which the segment is cut, by 3.1416; mult the prod by the height o *s* of the seg. Add area of base if reqd. Rem. Having the diam *n r* of the seg, and its height o *s*, the diam *a b* of the sphere may be found thus: Div the square of half the diam *n r*, by its height o *s*; to the quot add the height *o s*. Rule 2. The *curved* surf of either a segment, last Fig, or of a zone, (next Fig,) bears the same proportion to the surf of the whole sphere, that the height of the seg or zone bears to the diam of the sphere. Therefore, first find the surf of the whole sphere, either by rule or from the preceding table; mult it by the height of the seg or zone; div the prod by diam of sphere. Rule 3. Mult the circumf of the sphere by the height o *s* of the seg.

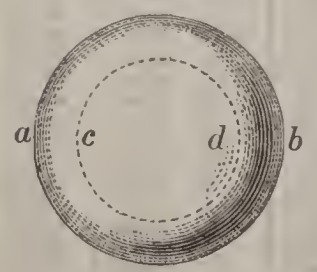


To find the solidity of a spherical zone.

Add together the square of the rad *e d*, the square of rad o *b*, and ¼d of the square of the perp height *e o*; mult the sum by 1.5708; and mult this prod by the height *e* o.

To find the curved surface of a spherical zone.

Rule 1. Mult together the diam *m n* of the sphere; the height *e* o of the zone, and the number 3.1416. Or see preceding Rule 2 for surf of segments. Rule 2. Mult the circumf of the sphere, by the height of the zone.



To find the solidity of a hollow spherical shell.

Take from the foregoing table the solidities of two spheres having the diams *a b*, and *c d*. Subtract the least from the greatest. Here *a c* or *b d* is the thickness of the shell.

THE ELLIPSOID, OR SPHEROID,

Is a solid generated by the revolution of an ellipse around either its long or its short diam. When around the long (or **transverse**) diam, as at *a*, Fig 1, it is an **oblong** or **prolate** spheroid; when around the short (or **conjugate**) one, as at *m*, in Fig 2, it is **oblate.**

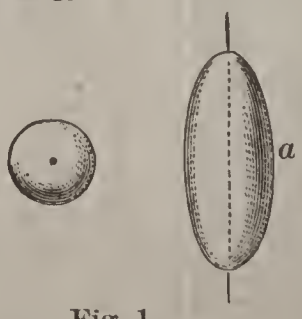


Fig. 1.

Fig. 2.

For the solidity in either case, mult the fixed diam or axis by the square of the revolving one; and mult the prod by .5236.

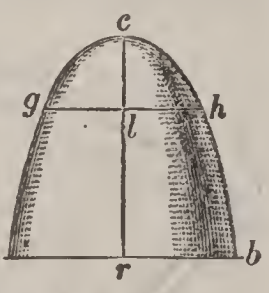
THE PARABOLOID, OR PARABOLIC CONOID,

next Fig, is a solid generated by the revolution of a parabola $a\ c\ b$, around its axis, $c\ r$.

For its solidity mult the area of its base, by half its height, $r\ c$. Or mult together the square of the rad $a\ r$ of the base; the height $r\ c$; and the number 1.5708.

For the solidity of a frustum,

$a\ b\ g\ h$, the ends of which are perp to the axis $r\ c$; add together the squares of the two diams $a\ b$ and $g\ h$; mult the sum by the height $r\ l$; mult the prod by the decimal .3927.



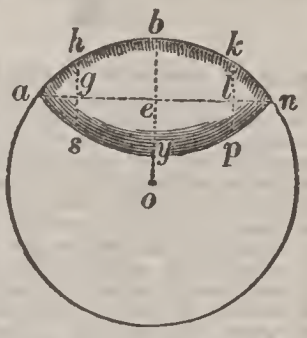
To find the surface of a paraboloid,

Mult the rad $a\ r$ of its base, by 6.2832; div the prod by 12 times the square of the height $r\ c$; call the quot p. Then add together the square of the rad $a\ r$, and 4 times the square of the height $r\ c$. Cube the sum; take the sq rt of this cube; from the sq rt subtract the cube of the rad $a\ r$. Mult the rem by p.

Either the solidity, or the surface of a frustum, $a\ b\ g\ h$, when $g\ h$ is parallel to $a\ b$, may be found by calculating for the whole paraboloid, and for the upper portion $c\ g\ h$, as two separate paraboloids, and taking their diff.

THE CIRCULAR SPINDLE,

Is a solid $a\ b\ n\ y$ generated by the revolution of a circular segment $a\ b\ n\ e\ a$, around its chord $a\ n$ as an axis.



To find its solidity.

Rule 1. First find the area of $a\ b\ e$, or half the generating circular segment. Then to the square of $a\ e$, add the square of $b\ e$; div the sum by $b\ y$; from the quot take $b\ e$: mult the rem by the area of $a\ e\ b$; call the prod p. Cube $a\ e$; div the cube by 3; from the quot take p. Mult the rem by 12.5664.

Rule 2. When the dist $o\ e$ is known, from the center of the circle to the cen of the spindle, then mult that dist $o\ e$, by the area of $a\ b\ e$; call the prod p; cube $a\ e$: div the cube by 3; from the quot take p; mult the rem by 12.5664.

To find its surface.

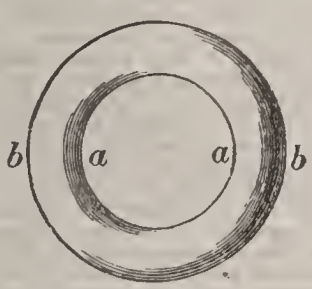
Rule 1. First find the length of the circular arc $a\ b\ n$; and mult it by the dist $o\ e$ from the center of the circle to the center of the spindle. Call the prod p. Next mult the length $a\ n$ of the spindle, by the rad $o\ b$ of the circle. From the prod take p; mult the rem by 6.2832.

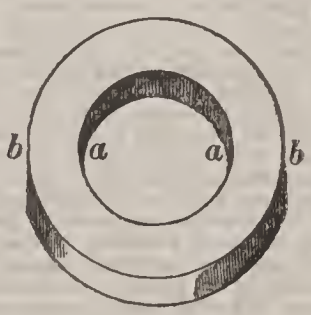
Rule 2. First find the length of the arc $a\ b\ n$. Square $a\ e$; also square $b\ e$; add these squares together; div their sum by $b\ y$; call the quot s; and mult it by $a\ n$; call the prod p. Next from s take $b\ e$: mult the rem by the length of the arc $a\ b\ n$. Subtract the prod from p; mult the rem by 6.2832.

To find the solidity of a middle zone of a circular spindle,

As $h\ s\ k\ p$

$$\left(\left(a\,e^2-\frac{g\,e^2}{3}\right)\times g\,e\right)-\left(o\,e\times \text{area of } g\,h\,l\,k\right)\times 6.2832.$$





CIRCULAR RINGS.

Volume = area of cross section of bar of which ring is made $\times$ $\frac{1}{2}$ sum of inner and outer diameters, $a\,a$ and $b\,b$ $\times$ 3.141593.

Surface = circumference of bar of which ring is made $\times$ $\frac{1}{2}$ sum of inner and outer diameters, $a\,a$ and $b\,b$ $\times$ 3.141593.

LAND SURVEYING.

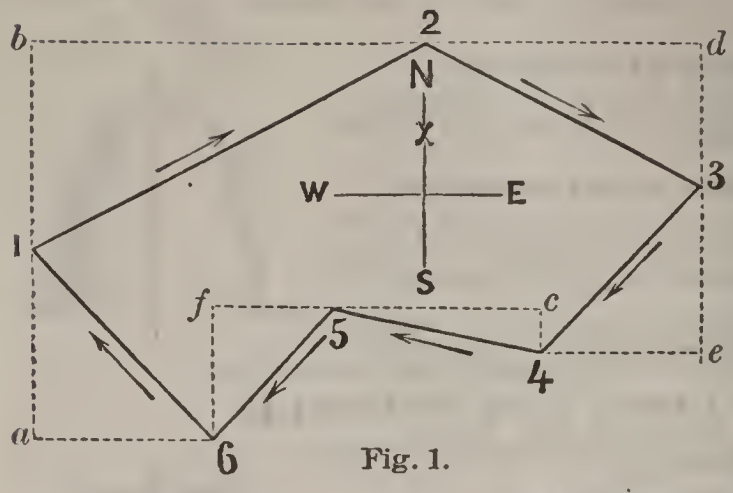


Fig. 1.

In surveying a tract of ground, the sides which compose its outline are designated by numbers in the order in which they occur. That end of each side which first presents itself in the course of the survey, may be called its *near* end; and the other its *far* end. The number of each side is placed at its far end. Thus, in Fig 1, the survey being supposed to commence at the corner 6, and to follow the direction of the arrows, the first side is 6, 1; and its number is placed at its far end at 1; and so of the rest. Let N S be a meridian line, that is, a north and south line; and E W an east and west line. Then in any side which runs northwardly, whether due north, or northeast, as side 2; or northwest, as sides 5 and 1, the dist in a due north direction between its near end and its far end, is called its *northing;* thus, *a* 1 is the northing of side 1; 1 *b* the northing of side 2; 4 *c* of side 5. In like manner, if any side runs in a southwardly direction, whether due south, or southeastwardly, as side 3; or southwestwardly, as sides 4 and 6, the corresponding dist in a due south direction between its near end and its far end, is called its *southing;* thus, *d* 3 is the southing of side 3; 3 *e* of side 4; *f* 6 of side 6. Both northings and southings are included in the general term *Difference of Latitude* of a side; or more commonly, but erroneously, its *latitude*. The dist due east, or due west, between the near and the far end of *any* side, is in like manner called the *easting*, or *westing* of that side, as the case may be; thus, 6 *a* is the westing of side 1; 5 *f* of side 6; *c* 5 of side 5; *e* 4 of side 4; and *b* 2 is the easting of side 2; 2 *d* of side 3. Both eastings and westings are included in the general term *Departure* of a side; implying that the side *departs* so far from a north or south direction. We may employ the directions (or courses, or bearings, as they are usually called) of sides, as verbs; and say that a side norths, wests, southeasts, &c. We shall call the northings, southings, &c, the Ns, Ss, Es, and Ws; the latitudes, lats; and the departures, deps. The Traverse Table consists of the lats, (or Ns and Ss;) and the deps, (or Es and Ws,) corresponding to diff angles or courses, for a side whose length is 1; therefore, to obtain the actual lat and dep of any given side, those taken from the table must be mult by the length of the side. Beyond 44°, the lats and deps of this table must be read upward from the bottom of the page. The angles in the table are those which the course or bearing of any side would make with a meridian line drawn through either end of said side; but it is self-evident that what would be N from one end, would be S from the other; and so of E and W; in other words, the *angle* is the same at both ends; but the *direction* is reversed.

Perfect accuracy is unattainable in any operation involving the measurements of angles and dists. That work is accurate enough, which cannot be made more so without an expenditure more than commensurate with the object to be gained. The writer conceives that the accuracy essential to constitute practically fair surveying is purely a matter of dollars and cents. In the purchase and sale of tracts of land, such as farms, &c, an uncertainty of about 1 part in 200 respecting the content, and consequently respecting the price, probably never prevents a transfer; and on this principle we assume that a survey which proves itself within that limit, may *ordinarily* be regarded as accurate enough. There is no great difficulty in attaining this limit, which, if exceeded, is the result of bad work. Many circumstances combine to render trifling errors absolutely unavoidable;* they always become apparent when we come to *work out* the field notes; and since the map or plot of the survey, and the calculations for ascertaining the content, should be consistent within themselves, we do what is usually called *correcting* the errors, but what in fact is simply *humoring them in*, no matter how scientific the process may appear. We distribute them all around the survey. Two methods are used for this purpose, both based upon precisely the same principle; one of them mechanical, by means of drawing; the other more exact, but much more troublesome, by calculation. We shall describe both; but will state now, that by proportioning the scale of the plot in the following manner, the mechanical method becomes, in the hands of a correct draftsman, sufficiently exact for all ordinary purposes. Add all the sides in feet together; and div the sum by their number, for the average length. Div this average by 8; the quot will be the proper scale in feet per inch. In other words, take about 8 ins to represent an average side.† We shall take it for granted, that an engineer does not consider it accurate work to

* A 100 ft measuring-chain may vary its length 5 feet per mile, between winter and summer, by mere change of temperature; and by this alone we shall make a difference of about $1\frac{1}{5}$ acres in a lot 1 mile square, which contains 640 acres. Even this error amounts to 1 acre in 533. Not one farmer in a hundred would dream of paying for a scrupulously correct survey and plot of his property.

† It will seldom happen that precisely this quotient will be adopted for a scale. For instance, if the quot should be 738 feet, or 83 feet, per inch, we should adopt the most convenient near number, if smaller the better, as 700, or 80; or rather more than 8 inches to a side. With a scale so proportioned, and with good drawing instruments, the error in *protracting* (excluding of course errors of the field work) will rarely exceed about $\frac{1}{400}$ part of the periphery of the plot; and the area may be found mechanically by dividing the plot into triangles, within $\frac{1}{200}$ part of the truth. This remark applies particularly to such plots as may be protracted, and computed within a period so short as not to allow the paper to contract or expand appreciably by atmospheric changes. A larger scale will insure proportionally greater accuracy. The young assistant should practise plotting from perfectly accurate data; as, for instance, from the example given in the table, p. 175 or,

measure his angles to the nearest quarter of a degree, which is the usual practice among land-surveyors. They can, by means of the engineer's transit, now in universal use on our public works, be readily measured within a minute or two; and being thus much more accurate than the compass courses, (which cannot be read off so closely, and which are moreover subject to many sources of error,) they serve to correct the latter in the office. The noting of the courses, however, should not be confined to the nearest quarters of a degree, but should be read as closely as the observer can guess at the minutes. The back courses also should be taken at every corner, as an additional check, and for the detection of local attraction. It is well in taking the compass bearings, to adopt as a rule, always to point the *north* of the compass-box toward the object whose bearing is to be taken, and to read off from the *north* end of the needle. A person who uses indifferently the N and the S of the box, and of the needle, will be very liable to make mistakes. It is best to measure the *least* angle (shown by dotted arcs, Fig 2,) at the corners; whether it be exterior, as that at corner 5; or interior, as all the others; because it is always less than 180°; so that there is less danger of reading it off incorrectly, than if it exceeded 180°; taking it for granted that the transit instrument is graduated from the same zero to 180° each way; if it is graduated from zero to 360° the precaution is useless. When the small angle is exterior, subtract it from 360° for the interior one.

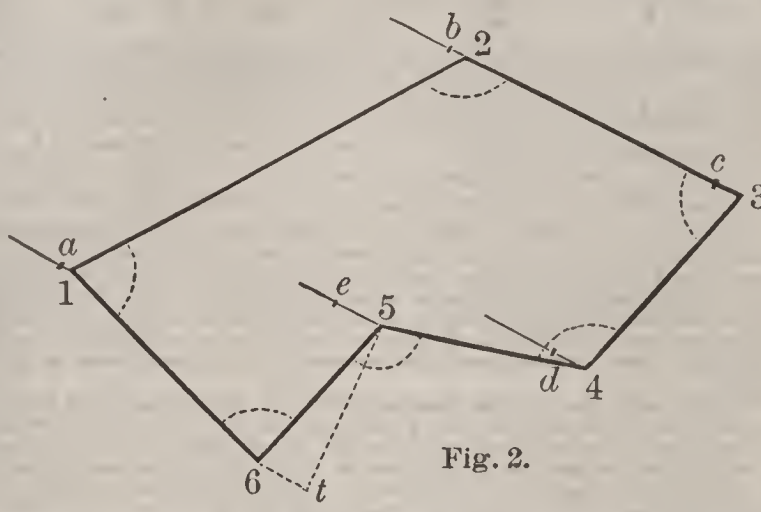


Fig. 2.

Supposing the field work to be finished, and that we require a plot from which the contents may be obtained mechanically, by dividing it into triangles, (the bases and heights of which may be measured by scale, and their areas calculated one by one,) a protraction of it may be made at once from the field notes, either by using the *angles*, or by first correcting the *bearings* by means of the angles, and then using them. The last is the best, because in the first the protractor must be moved to each angle; whereas in the last it will remain stationary while all the bearings are being pricked off. Every movement of it increases the liability to errors. The manner of correcting the bearings is explained on the next page.

In either case the protracted plot will certainly not close precisely; not only in consequence of errors in the field work, but also in the protracting itself. Thus the last side, No 6, Fig 2, instead of closing in at corner 6, will end somewhere else, say, for instance, at *t*; the dist *t* 6 being the *closing error*, which, however, as represented in Fig 2, is more than ten times as great, proportionally to the size of the survey, as would be allowable in practice. Now to humor in this error, rule through every corner a short line parallel to *t* 6; and, in all cases, in the direction *from t* (wherever it may be) *to* the starting point 6. Add all the sides together; and measure *t* 6 by the scale of the plot. Then beginning at corner 1, at the far end of side 1, say, as the

Sum of all the sides	:	Total closing error *t* 6	::	Side 1	:	Error for side 1.

Lay off this error from 1 to *a*. Then at corner 2, say, as the

Sum of all the sides	:	Total closing error *t* 6	::	Sum of sides 1 and 2	:	Error for side 2.

Which error lay off from 2 to *b*; and so at each of the corners; always using, as the third term, the sum of the sides between the starting point and the given corner. Finally, join the points *a*, *b*, c, *d*, *e*, 6; and the plot is finished.

The correction has evidently changed the length of every side; lengthening some and shortening others. It has also changed the angles. The new lengths and angles may with tolerable accuracy be found by means of the scale and protractor; and be marked on the plot instead of the old ones.

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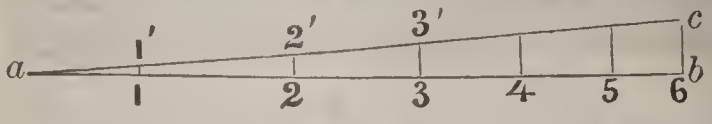
from those to be found in books on surveying. This is the only way in which he can learn what is meant by accurate work. His semicircular protractor should be about 9 to 12 ins in diam and graduated to 10 min. His straight edge and triangle should be of metal; we prefer German silver, which does not rust as steel does; and they should be made with *scrupulous accuracy* by a skilful instrument-maker. A very fine needle, with a sealing-wax head, should be used for pricking off dists and angles; it must be held vertically; and the eye of the draftsman must be directly over it. The lead pencil should be hard (Faber's No. 4 is good for protracting), and must be kept to a sharp point by rubbing on a fine file, after using a knife for removing the wood. The scale should be at least as long as the longest side of the plot, and should be made at the edge of a strip of the same paper as the plot is drawn on. This will obviate to a considerable extent, errors arising from contraction and expansion. Unfortunately, a sheet of paper does not contract and expand in the same proportion lengthwise and crosswise, thus preventing the paper scale from being a perfect corrective. In plots of common farm surveys, &c, however, the errors from this source may be neglected. For such plots as may be protracted, divided, and computed within a time too short to admit of appreciable change, the ordinary scales of wood, ivory or metal may be used; but satisfactory accuracy cannot be obtained with them on plots requiring several days, if the air be meanwhile alternately moist and dry, or subject to considerable variations in temperature. What is called *parchment* paper is worse in this respect than good ordinary drawing-paper.

With the foregoing precautions we may work from a drawing, with as much accuracy as is usually attained in the field work.

When the plot has many sides, this calculating the error for each of them becomes tedious; and since, in a well-performed survey and protraction, the entire error will be but a very small quantity, (it should not *exceed* about $\frac{1}{400}$ part of the periphery,) it may usually be divided among the sides by merely placing about ¼, ½, and ¾ of it at corners about ¼, ½, and ¾ way around the plot; and at intermediate corners proportion it by eye. Or calculation may be avoided entirely by drawing a line *a b* of a length equal to the united lengths of all the sides; dividing it into distances *a*, 1; 1, 2; &c, equal to the respective sides. Make *b c* equal to the entire closing error; join *a c*; and draw 1, 1'; 2, 2', &c, which will give the error at each corner.



When the plot is thus completed, it may be divided by fine pencil lines into triangles, whose bases and heights may be measured by the scale, in order to compute the contents. With care in both the survey and the drawing, the error should not *exceed* about $\frac{1}{200}$ part of the true area. At least two distinct sets of triangles should be drawn and computed, as a guard against mistakes; and if the two sets differ in calculated contents more than about $\frac{1}{200}$ part, they have not been as carefully prepared as they should have been. The closing error due to imperfect field-work, may be accurately calculated, as we shall show, and laid down on the paper before beginning the plot; thus furnishing a perfect test of the accuracy of the protraction work, which, if correctly done, will not close at the point of beginning, but at the point which indicates the error. But this *calculation* of the error, by a little additional trouble, furnishes data also for dividing it by calculation among the diff sides; besides the means of drawing the plot *correctly* at once, without the use of a protractor: thus enabling us to make the subsequent measurements and computations of the triangles with more certainty.

We shall now describe this process, but would recommend that even when it is employed, and especially in complicated surveys, a rough plot should first be made and corrected, by the first of the two mechanical methods already alluded to. It will prove to be of great service in using the method by calculation, inasmuch as it furnishes an eye check to vexatious mistakes which are otherwise apt to occur; for, although the principles involved are extremely simple, and easily remembered when once understood, yet the continual changes in the directions of the sides will, without great care, cause us to use Ns instead of Ss; Es instead of Ws, &c.

We suppose, then, that such a rough plot has been prepared, and that the angles, bearings, and distances, as taken from the field book, are figured upon it in *lead pencil*.

Add together the *interior* angles formed at all the corners: call their sum *a*. Mult the number of sides by 180°; from the prod subtract 360°: if the remainder is equal to the sum *a*, it is a proof that the angles have been correctly measured.* This, however, will rarely if ever occur; there will always be some discrepancy; but if the field work has been performed with moderate care, this will not *exceed* about two min for each angle. In this case div it *in equal parts* among all the angles, adding or subtracting, as the case may be, unless it amounts to less than a min to each angle, when it may be entirely disregarded in common farm surveys. The corrected angles may then be marked on the plot in ink, and the pencilled figures erased. We will suppose the corrected ones to be as shown in Fig 3.

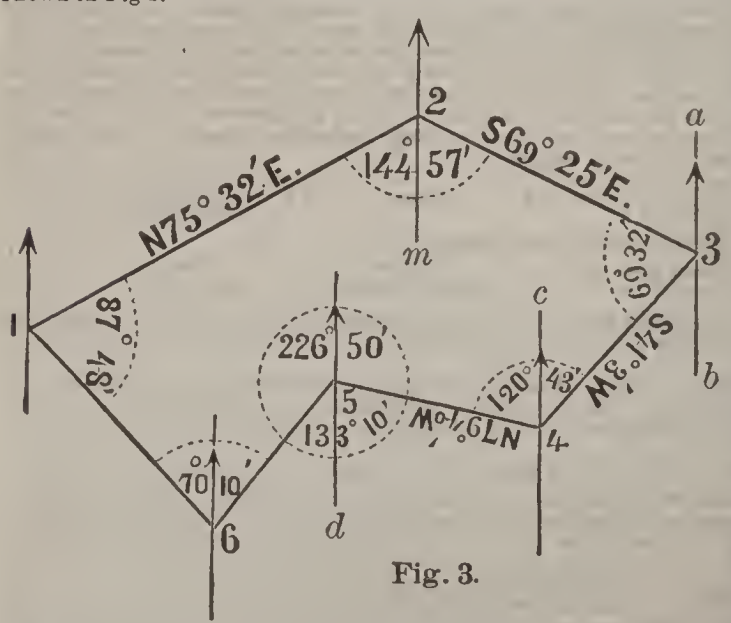


Fig. 3.

Next, by means of these corrected angles, correct the bearings also, thus. Fig 3; Select some side (the longer the better) from the two ends of which the bearing and the reverse bearing agreed; thus showing that that bearing was probably not influenced by local attraction. Let side 2 be the one so selected: assume its bearing, N 75° 32' E, as taken on the ground, to be correct; through either end of it, as at its far end 2, draw the short meridian line: parallel to which draw others through every corner. Now, having the bearing of side 2, N 75° 32' E, and requiring that of side 3, it is plain that the reverse bearing from corner 2 is S 75° 32' W; and that therefore the angle 1, 2, *m*, is 75° 32'. Therefore, if we take 75° 32' from the entire corrected angle 1, 2, 3, or 144° 57', the rem 69° 25' will be the angle *m* 23; consequently the bearing of side 3 *must* be S 69° 25' E. For finding the bearing of side 4, we now have the angle 23 *a* of the reverse bearing of side 3, also equal to 69° 25'; and if we add this to the entire corrected angle 234, or to 69° 32', we have the angle *a* 34 = 69° 25' + 69° 32' = 138° 57'; which taken from 180°, leaves the angle *b* 34 = 41° 3'; consequently the bearing of side 4 *must* be S 41° 3' W. For the bearing of side 5 we now have the angle 34 *c* = 41° 3', which taken from the corrected angle 345, or 120° 43', leaves the angle *c* 45 = 79° 40'; consequently the bearing of side 5 *must* be N 79° 40' W. At corner 5, for the bearing of side 6, we have the angle 45 *d* = 79° 40', which taken from 133° 10', leaves the angle *d* 56 = 53° 30'; consequently the bearing of side 6 *must* be S 53° 30' W. And so with each of the sides, nothing but

* Because in every straight-lined figure the sum of all its interior angles is equal to twice as many right angles as the figure has sides, minus 4 right angles, or 360°.

careful observation is necessary to see how the several angles are to be employed at each corner. Rules are sometimes given for this purpose, but unless frequently used, they are soon forgotten. The plot mechanically prepared obviates the necessity for such rules, inasmuch as the principle of proceeding thereby becomes merely a matter of sight, and tends greatly to prevent error from using the wrong bearings; while the protractor will at once detect any serious mistakes as to the angles, and thus prevent their being carried farther along. After having obtained all the corrected bearings, they may be figured on the plot instead of those taken in the field. They will, however, require a still further correction after a while, since they will be affected by the adjustment of the closing error.

We now proceed to calculate the closing error *t* 6 of Fig 2, which is done on the principle that in a correct survey the northings will be equal to the southings, and the eastings to the westings. Prepare a table of 7 columns, as below, and in the first 3 cols place the numbers of the sides, and their corrected courses; also the dists or lengths of the sides, as measured on the rough plot, if such a one has been prepared; but if not, then as measured on the ground. Let them be as follows:

Side.	Bearing.	Dist. Ft.	Latitudes.		Departures.	
			N.	S.	E.	W.
1	N 16° 40′ W	1060	1015.5			304.
2	N 75° 32′ E	1202	300.3		1163.9	
3	S 69° 25′ E	1110		390.2	1039.2	
4	S 41° 3′ W	850		641.		558.2
5	N 79° 40′ W	802	143.9			789.
6	S 53° 30′ W	705		419.3		566.7
			1459.7	1450.5	2203.1	2317.9
			1450.5			2203.1
			9.2 Error in Lat.		Error in Dep.	14.8

Now find the N, S, E, W, of the several sides, and place them in the corresponding four columns, thus: By means of the Traverse Table find out the lat and dep for the angle of each course. Mult each of them by the length of the side; and place the prod in the corresponding col of N, S, E, W. Thus, for side 1, which is 1060 feet long, the latitude from the traverse table for 16° 40′ is .9580; and the departure is .2868: and .9580 × 1060 = 1015.5 lat; which, since the side norths, we put in the N col. Again, .2868 × 1060 = 304 dep; which, since the side wests, we put in the W col. Proceed thus with all. Add up the four cols; find the diff between the N and S cols; and also between the E and W ones. In this instance we find that the Ns are 9.2 feet greater than the Ss; and that the Ws are 14.8 ft greater than the Es; in other words, there is a closing error which would cause a *correct* protraction of our first three cols, to terminate 9.2 feet too far north of the starting point; and 14.8 feet too far west of it. So that by placing this error upon the paper before beginning to protract, we should have a test for the accuracy of the protracting work; but, as before remarked, a little more trouble will now enable us to div the error proportionally among all the Ns, Ss, Es, and Ws, and thereby give us data for drawing the plot correctly at once, without using a protractor at all.

To divide the errors, prepare a table precisely the same as the foregoing, except that the hor spaces are farther apart; and that the addings up of the *old* N, S, E, W columns are omitted. The additions here noticed are made subsequently.

The new table is on the next page.

Remark. **The bearing and the reverse bearing** from the two ends of a line will not read precisely the same angle; and the difference varies with the latitude and with the length of the line, but not in the same proportion with either. It is, however, generally too small to be detected by the needle, being, according to Gummere, only three quarters of a minute in a line one mile long in lat 40°. In higher lats it is more, and in lower ones less. It is caused by the fact that meridians or north and south lines are not truly parallel to each other; but would if extended meet at the poles.

Hence the only bearing that can be run in a straight line, with strict accuracy, is a true N and S one; except on the very equator, where alone a due E and W one will also be straight. But **a true curved E and W line** may be found anywhere with sufficient accuracy for the surveyor's purposes thus. Having first by means of the N star p 177, or otherwise got a true N and S bearing at the starting point, lay off from it 90°, for a true E and W bearing *at that point.* This E and W bearing will be tangent to the true E and W curve. Run this tangent carefully; and at intervals (say at the end of each mile) lay off from it (towards the N if in N lat, or vice versa) an offset whose length in *feet* is equal to the proper one from the following table, multiplied by the *square* of the distance in *miles* from the starting point. These offsets will mark points in the true E and W curve.

Latitude N or S.

5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°
Offsets in ft one mile from starting point.												
.058	.118	.179	.243	.311	.385	.467	.559	.667	.795	.952	1.15	1.43

Or, any offset in ft = .6666 × Total Dist in miles^2 × Nat Tang of Lat.

A rhumb line is any one that crosses a meridian obliquely, that is, is neither due N and S, nor E and W.

Side.	Bearing.	Dist. Ft.	Latitudes.		Departures.	
			N.	S.	E.	W.
1	N 16° 40′ W	1060	1015.5 1.7			304.0 2.7
			1013.8...	....	.....	... 301.3
2	N 75° 32′ E	1202	300.3 1.9		1163.9 3.1	
			298.4	....	... 1167.0	
3	S 69° 25′ E	1110		390.2 1.8	1039.2 2.9	
				392 ...	... 1042.1	
4	S 41° 3′ W	850		641.0 1.3		558.2 2.2
				642.3...	.....	.. 556.0
5	N 79° 40′ W	802	143.9 1.3			789.0 2.1
			142.6...	.....	.....	... 786.9
6	S 53° 30′ W	705		419.3 1.1		566.7 1.8
				420.4...	.....	564 9
		5729 Sum of Sides.	1454.8 Cor'd Ns.	1454.7 Cor'd Ss.	2209.1 Cor'd Es.	2209.1 Cor'd Ws.

Now we have already found by the old table that the Ns and the Ws are too long; consequently they must be shortened; while the Ss, and Es, must be lengthened; all in the following proportions: As the

Sum of all the sides : Any given side :: Total err of lat or dep : Err of lat. or dep, of given side.

Thus, commencing with the lat of side 1, we have, as

Sum of all the sides. 5729 : Side 1. 1060 :: Total lat err. 9.2 : Lat err of side 1. 1.7

Now as the lat of side 1 is north, it must be shortened; hence it becomes $= 1015.5 - 1.7 = 1013.8$, as figured out in the new table. Again we have for the departure of side 1,

Sum of all the sides. 5729 : Side 1. 1060 :: Total dep err. 14.8 : Dep err of side 1. 2.7

Now as the dep of side 1 is west, it must be shortened; hence it becomes $304 - 2.7 = 301.3$, as figured out in the new table.

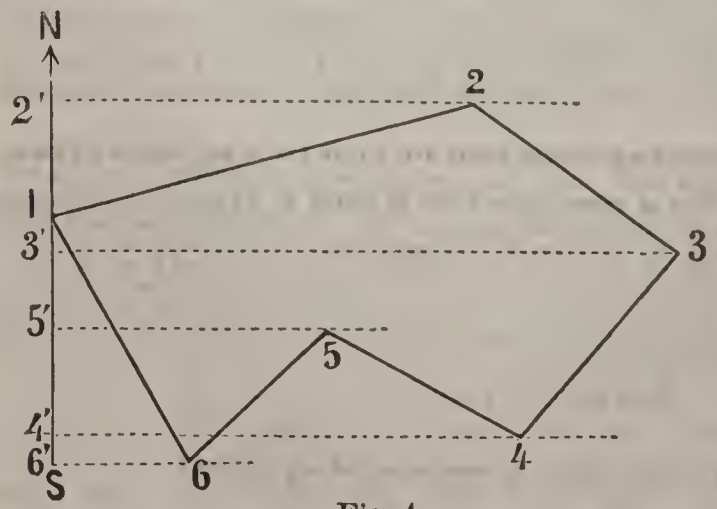


Fig. 4.

Proceeding thus with each side, we obtain all the corrected lats and deps as shown in the new table; where they are connected with their respective sides by dotted lines; but in practice it is better to cross out the original ones when the calculation is finished and proved. If we now add up the 4 cols of corrected N, S, E, W, we find that the Ns = the Ss; and the Es = the Ws; thus proving that the work is right. There is, it is true, a discrepancy of .1 of a ft between the Ns, and the Ss; but this is owing to our carrying out the corrections to only one decimal place; and is too small to be regarded. Discrepancies of 3 or 4 tenths of a foot will sometimes occur from this cause; but may be neglected. The corrected lats and deps must evidently change the bearing and distance of every side; but without knowing either of these, we can now plot the survey by means of the corrected

lats and deps alone. The principle is self-evident, explaining itself. First draw a meridian line N S, Fig 4; and upon it fix on a point 1, to represent the *extreme west* corner* of the survey.

Then from the point 1, prick off by scale, northward, the dist 1, 2 = the corrected northing 298.4 of side 2, taken from the last table; from 2′ southward prick off the dist 2′, 3′, the corrected southing 392 of side 3; from 3′ southward prick off 3′, 4′, = southing 642.3 of side 4; from 4′ northward prick off 4′, 5′ = northing 142.6 of side 5; from 5′ prick off southward 5′, 6′ = southing of side 6.† Then from the points 2′, 3′, 4′, 5′, 6′, draw indefinite lines due eastward, or at right angles to the meridian line. Make by scale, 2′, 2 = corrected departure of side 2; and join 1, 2. Make 3′, 3 = dep of side 2 + dep of side 3; and join 2, 3; make 4′, 4 = 3′, 3 — dep of side 4; and join 3, 4; make 5′, 5 = 4′, 4 — dep of side 5; and join 4, 5; make 6′, 6 = 5′, 5 — dep of side 6; and join 5, 6 ‡ Finally join 6, 1; and the plot is complete. If scrupulous accuracy is not required, the contents may be found by the mechanical method of triangles; the bearings, by the protractor; and the lengths of the sides, by the scale; all with an approximation sufficient for ordinary purposes; and perhaps quite as close as by the method by calculation, when, as is customary, the bearings are taken only to the nearest quarter of a degree. We have already said that with a scale of feet per inch = $\frac{\text{average length of sides}}{8}$, the error of area need not *exceed* the $\frac{1}{200}$th part.

But if it is required to calculate the area of the corrected survey with rigorous exactness, it may be done on the following principle, (see Fig 5.) If a meridian line N S be supposed to be drawn through the extreme west corner 1 of a survey; and lines (called *middle distances*) drawn (as the dotted ones in the Fig) at right angles to said meridian, from the *center* of each side of the survey; then if each of the middle dists of such sides as have northings, be mult by the corrected northing of its corresponding side; and if each of the middle dists of such sides as have southings, be mult by the corrected southing of its corresponding side; if we add all the north prods into one sum; and all the south prods into another sum; and subtract the least of these sums from the greatest, the rem will be the area

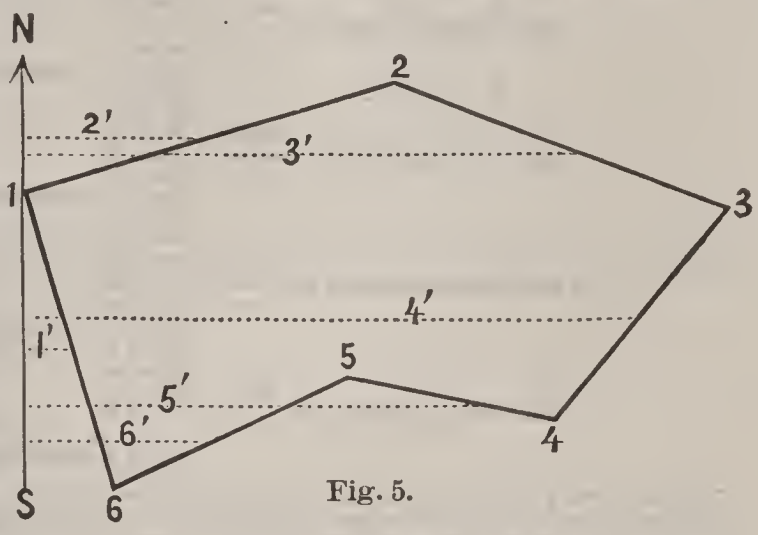


Fig. 5.

* The extreme east corner would answer as well, with a slight change in the subsequent operations, as will become evident.

† Instead of pricking off these northings and southings in succession, *from each other*, it will be more correct in practice to prepare first a table showing how far each of the points 2′, 3′, &c, is north or south from 1. This being done, the points can be pricked off north or south from 1, without *moving the scale each time;* and of course with greater accuracy. Such a table is readily formed. Rule it as below; and in the first three columns place the numbers of the sides (starting with side 2 from point 1;) and their respective corrected northings and southings. The formation of the 4th and 5th cols by means of the 3d and 4th ones, explains itself. Its accuracy is proved by the final result being 0.

Side.	N. lat.	S. lat.	Dist N or S from Point 1. N.	S.
2	298.4		298.4	
3		392.		93.6
4		642.3		735.9
5	142.6			593.3
6		420.4		1013.7
1	1013.8		000.0	000.0

‡ A similar table should be prepared beforehand for the dists of the points 2, 3, 4, &c, east from the meridian line. It is done in the same manner, but requires one col less, as all the dists are on the same side of the mer line. Thus, starting from point 1, with side 2:

Side.	E. dep.	W. dep.	Dist east from meridian line.
2	1167.0		1167.0
3	1042.1		2209.1
4		556.0	1653.1
5		786.9	866.2
6		564.9	301.3
1		301.3	000.0

This work like also proves itself by the final result being 0.

of the survey.* The corrected northings and southings we have already found; as also the eastings and westings. The middle dists are found by means of the latter, by employing their *halves*; adding half eastings, and subtracting half westings. Thus it is evident that the middle dist 2′ of side 2, is equal to half the easting of side 2. To this add the other half easting of side 2, and a half easting of side 3; and the sum is plainly equal to the middle dist 3′ of side 3. To this add the other half easting of side 3, and subtract a half westing of side 4, for the middle dist 4′ of side 4. From this subtract the other half westing of side 4, and a half westing of side 5, for the middle dist 5′ of side 5; and so on. The actual calculation may be made thus:

Half easting of side 2 = $\frac{1167}{2}$ = 583.5 E = mid dist of side 2.
583.5 E

Half easting of side 3 = $\frac{1042.1}{2}$ = 1167.0 E
521.0 E
1688.0 E = mid dist of side 3.
521.0 E

Half westing of side 4 = $\frac{556}{2}$ = 2209.0 E
278.0 W
1931.0 E = mid dist of side 4.
278.0 W

Half westing of side 5 = $\frac{786.9}{2}$ = 1653.0 E
393.5 W
1259.5 E = mid dist of side 5.
393.5 W

Half westing of side 6 = $\frac{564.9}{2}$ = 866.0 E
282.4 W
583.6 E = mid dist of side 6.
282.4 W

Half westing of side 1 = $\frac{301.3}{2}$ = 301.2 E
150.6 W
150.6 E = mid dist of side 1.

The work always proves itself by the last two results being equal.

Next make a table like the following, in the first 4 cols of which place the numbers of the sides, the middle dists, the northings, and southings. Mult each middle dist by its corresponding northing or southing, and place the products in their proper col. Add up each col; subtract the least from the

Side.	Middle dist.	Northing.	Southing.	North prod.	South prod.
1	150.6	1013.8		152678	
2	583.5	298.4		174116	
3	1688		392		661696
4	1931		642.3		1240281
5	1259.5	142.6		179605	
6	583.6		420.4		245345
				506399	2147322
					506399

43560)1640923(37.67 Acres.

* *Proof.* To illustrate the principle upon which this rule is based, let $a\,b$, $b\,c$, and $c\,a$, Fig 6, represent in order the 3 sides of the triangular plot of a survey, with a meridian line $d\,f$ drawn through the extreme west corner, a. Let lines $b\,d$ and $c\,f$ be drawn from each corner, perp to the meridian line; also from the middle of each side draw lines $w\,e$, $m\,n$, $s\,o$, also perp to meridian; and representing the middle dists of the sides. Then since the sides are regarded in the order $a\,b$, $b\,c$, $c\,a$, it is plain that $a\,d$ represents the northing of the side $a\,b$; $f\,a$ the northing of $c\,a$; and $d\,f$ the southing of $b\,c$. Now if we mult the *northing* $a\,d$ of the side $a\,b$, by its mid dist $e\,w$, the prod is the area of the triangle $a\,b\,d$. In like manner the *northing* $f\,a$ of the side $c\,a$, mult by its mid dist $s\,o$, gives the area of the triangle $a\,c\,f$. Again, the *southing* $d\,f$ of the side $b\,c$, mult by its mid dist $m\,n$, gives the area of the entire fig $d\,b\,c\,f\,d$. If from this area we subtract the areas of the two triangles $a\,b\,d$, and $a\,c\,f$, the rem is evidently the area of the plot $a\,b\,c$. So with any other plot, however complicated.

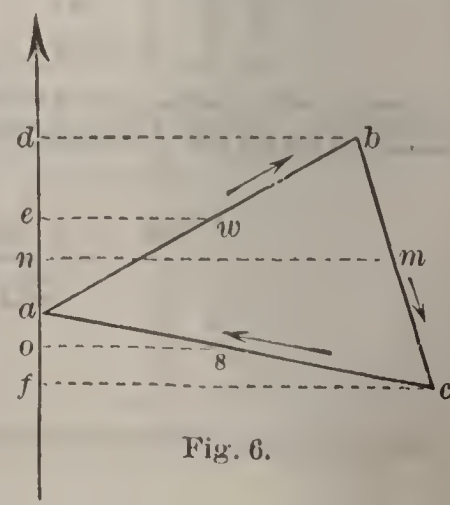


Fig. 6.

greatest. The rem will be the area of the survey in sq ft; which, div by 43560, (the number of sq ft in an acre,) will be the area in acres; in this instance, 37.67 ac.

It now remains only to calculate the corrected bearings and lengths of the sides of the survey, all of which are necessarily changed by the adoption of the corrected lats and deps. To find the bearing of any side, div its departure (E or W) by its lat (N or S); in the table of nat tang, find the quot; the angle opposite it is the reqd angle of bearing. Thus, for the course of side 1, we have $\frac{301.3\ \text{W}}{1013.8\ \text{N}} = .2972 =$ nat tang; opposite which in the table is the reqd angle, 16° 33′; the bearing, therefore, is N 16° 33′ W.

Again: for the dist or length of any side, from the table of nat cosines take the cos opposite to the angle of the corrected bearing; divide the corrected lat (N or S) of the side by the cos. Thus, for the dist of side 1, we find opposite 16° 33′, the cos .9586. And

Lat. Cos.
1013.8 ÷ .9586 = 1057.6 the reqd dist.

The following table contains all the corrections of the foregoing survey; consequently, if the bearings and dists are correctly plotted, they will close perfectly. The young assistant is advised to practise doing this, as well as dividing the plot into triangles, and computing the content. In this manner he will soon learn what degree of care is necessary to insure accurate results.

Side.	Bearing.	Dist. Ft.
1	N 16° 33′ W	1057.6
2	N 75° 39′ E	1204.0
3	S 69° 23′ E	1113.3
4	S 40° 53′ W	849.6
5	N 79° 44′ W	800.1
6	S 53° 21′ W	704.3

The following **hints** may often be of service.

1st. Avoid taking bearings and dists along a circuitous boundary line like *a b c*, Fig 7; but run the straight line *a c*; and at right angles to it, measure offsets to the crooked line. **2d.** Wishing to survey a straight line from *a* to c, but being unable to direct the instrument precisely toward c, on account of intervening woods, or other obstacles; first run a trial line, as *a m*, as nearly in the proper direction as can be guessed at. Measure *m* c, and say, as *a m* is to *m* c, so is 100 ft to ? Lay off *a o* equal to 100 ft, and *o s* equal to ?; and run the final line *a s* c. Or, if *m* c is quite small, calculate offsets like *o s* for every 100 ft along *a m*, and thus avoid the necessity for running a second line. **3d.** When c is visible from *a*, but the intervening ground difficult to measure along, on account of marshes, &c, extend the side *y a* to good ground at *t*: then, making the angle *y t d* equal to *y a* c, run the line *t n* to that point *d* at which the angle *n d* c is found by trial to be equal to the angle *a t d*. It will rarely be necessary to make more than one trial for this point *d*; for, suppose it to be made at *x*, see where it strikes *a* c at *i*; measure *i* c, and continue from *x*, making *x d* = *i* c. **4th.** In case of a very irregular piece of land, or a lake, Fig 8, surround it by straight lines. Survey these, and at right angles to them, measure offsets to the crooked boundary. **5th.** Surveying a straight line from *w* toward *y*, Fig 9

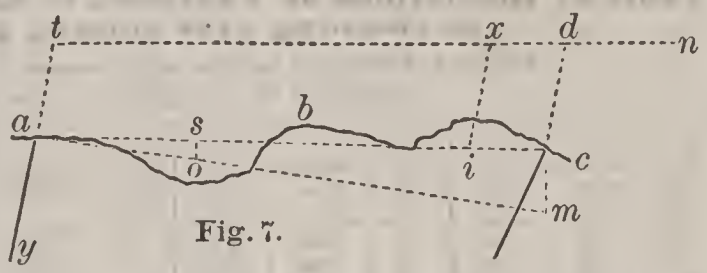


Fig. 7.

Fig. 8.

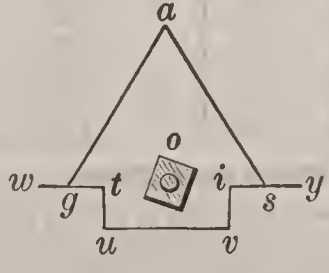


Fig. 9.

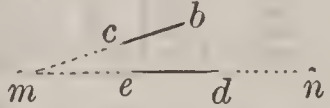


Fig. 10.

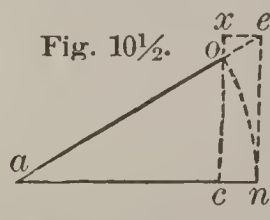
an **obstacle**, *o*, is met. To pass it, lay off a right angle *w t u*; measure any *t u*; make *t u v* = 90°; measure *u v*; make *u v i* = 90°; make *v i* = *t u*; make *v i y* = 90°. Then is *t i* = *u v*; and *i y* is in the straight line. Or, with less trouble, at *g* make *t g a* = 60°; measure any *g a*; make *g a s* = 60°; and *a s* = *g a*: make *a s i* = 60°. Then is *g s* = *g a* or *a s*; and *i s*, continued toward *y*, is in the straight line. **6th.** Being between two objects, *m* and *n*, and wishing to place myself in range with them, I lay a straight rod *c b* on the ground, and point it to one of the objects *m*; then going to the end c, I find that it does not point to the other object. By successive trials, I find the position *e d*, in which it points to both objects, and consequently is in range with them. If no rod

is at hand, two stones will answer, or two chain-pins. A plumb-line (a pebble tied to a piece of thread) will add to the accuracy of ranging the rod, or stones, &c.

THE FOLLOWING TABLE

gives deductions or additions to be made every 100 ft as actually chained along sloping ground, in order to reduce the sloping measurements to horizontal ones. Even when it is so nearly level that the eye cannot detect the slope, an over-measurement of an inch or two in 100 ft may readily occur. It is plain, that, if we measure all the undulations of the ground, we shall get greater totals than if we measure hor, as is *supposed* always to be done; but since few surveyors pretend to measure hor until the slope becomes apparent to the eye, their lines are usually too long by from one to two ins in 100 feet. To counteract this to some extent, chains are frequently made from one to two ins longer than 100 feet; and for ordinary purposes the precaution is a good one. When greater accuracy is required the chainmen should be attended by a third person, with a rod and slope-level, for taking the inclinations of the ground. These deductions being made, the remainder will be the actual hor dist.

Fig. 10½.



For example, in Fig 10½, each 100 ft *a* o measured up or down the slope *a e* plainly corresponds to the shorter horizontal distance *a c*; the difference or deduction being *c n*. Taking *a* o as Rad. then *a c* is the cosine, and *c n* the versed sine of the angle *e a n* of the slope. Therefore *a* o multiplied by the nat. cosine of the angle *e a n* gives the reduced hor dist *a c*; which taken from *a o* gives the **deduction** *c n* of our table.

But if while chaining along the slope *a e* we wish to drive stakes that shall correspond with **hor dists** *a n* of 100 ft. it is evident that we must **add** *c n* to each 100 ft *a o*, as shown at *x e*; and the stake must be driven at *e*, instead of at *o*. **Observe that x e = c n must be measured horizontally.**

When the ground is very sloping, all this calculation may be avoided where great accuracy is not required, by actually holding the chain horizontal, as nearly as can be judged by eye, and finding, by means of a plumb-line, where its raised end would strike the ground. A whole chain at a time cannot be measured in this way; but shorter distances must be taken as the ground requires; at times, on very steep ground, not more than 5 or 10 feet. See note, p 113.

Table of Deductions or Additions to be made per 100 feet, in chaining over sloping ground.

IN ORDER TO REDUCE THE INCLINED MEASUREMENTS TO HORIZONTAL ONES

See pp 354, 723, 724, and 725 for other tables.

Slope in Deg.	Deduct Feet.	Rise in 100 ft hor.	Slope in Deg.	Deduct Feet.	Rise in 100 ft hor.	Slope in Deg.	Deduct Feet.	Rise in 100 ft hor.	Slope in Deg.	Deduct Feet.	Rise in 100 ft hor.
¼	.001	.436	¼	.420	9.189	¼	1.596	18 08	¼	3.521	27.26
½	.004	.873	½	.460	9.629	½	1.675	18.53	½	3.637	27.73
¾	.009	1.309	¾	.503	10.07	¾	1.755	18.99	¾	3.754	28.20
1	.015	1.746	6	.548	10.51	11	1.837	19.44	16	3.874	28.67
¼	.024	2.182	¼	.594	10.95	¼	1.921	19.89	¼	3.995	29.15
½	.034	2.619	½	.643	11.39	½	2.008	20.35	½	4.118	29.62
¾	.047	3.055	¾	.693	11.84	¾	2.095	20.80	¾	4 243	30.10
2	.061	3.492	7	.745	12.28	12	2.185	21.26	17	4.370	30.57
¼	.077	3.929	¼	.800	12.72	¼	2.277	21.71	¼	4.498	31.05
½	.095	4.366	½	.856	13.17	½	2.370	22.17	½	4.628	31.53
¾	.115	4.803	¾	.913	13.61	¾	2.466	22.63	¾	4.760	32.01
3	.137	5.241	8	.973	14.05	13	2.563	23.09	18	4.894	32.49
¼	.161	5.678	¼	1.035	14.50	¼	2.662	23 55	¼	5.030	32.98
½	.187	6.116	½	1.098	14.95	½	2.763	24.01	½	5.168	33.46
¾	.214	6.554	¾	1.164	15.39	¾	2.866	24.47	¾	5.307	33 95
4	.244	6.993	9	1.231	15.84	14	2.970	24.93	19	5.448	34.43
¼	.275	7.431	¼	1.300	16.29	¼	3.077	25.40	¼	5.591	34.92
½	.308	7.870	½	1.371	16.73	½	3.185	25.86	½	5.736	35.41
¾	.343	8.309	¾	1.444	17.18	¾	3.295	26.33	¾	5.882	35.90
5	.381	8.749	10	1.519	17.63	15	3 407	26.79	20	6.031	36.40

Chain and Pins.

Engineers have abandoned the Gunter's chain of 66 ft, div into 100 links of 7.92 ins in length; and use one of 100 ft, with links 1 ft long: and calculate areas in sq ft: which, div by 43560, reduces them to acres and decimal parts, instead of roods and perches. Both the chain and the chain pins, Fig. 11, should be of good strong steel; and there should be a stout leather bag for carrying them. To bear hammering into hard ground, the pins may be of this shape and size, 11 or 12 ins long, ⅛ inch thick, ⅝ wide, head 2⅛ wide, with a circular hole of 1⅛ diam. Each pin should have a strip of bright red flannel tied to its top, that it may be readily found among grass, &c. by the hind chainman. The length of the chain should be tested every few days; and the target-rod may be used for this purpose. While locating, it is well to have the chain one or two ins longer than 100 feet. Steel wire, No 11 or 12, is a good size for a 100 ft chain. This is scant one-eighth inch diam.

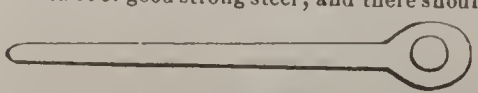


Fig. 11.

Nat Sines of Polar Dists of Polaris or N. Star.

Year.	Sine.	Year.	Sine.	Year.	Sine.	Year.	Sine.	Year.	Sine.	Year.	Sine.
1880	.0232	1883	.0229	1886	.0227	1889	.0224	1892	.0221	1895	.0218
1881	.0231	1884	.0229	1887	.0226	1890	.0223	1893	.0220	1896	.0217
1882	.0230	1885	.0228	1888	.0225	1891	.0222	1894	.0219	1897	.0216

Nat Secants of North Latitudes.

Lat.	Sec.	Lat.	Sec.	Lat.	Sec.	Lat.	Sec.	Lat.	Sec.	Lat.	Sec.
0°	1.000	24°	1.095	1/4°	1.196	1/2°	1.296	3/4°	1.433	52°	1.624
2	1.001	1/2	1.099	1/2	1.199	3/4	1.301	46	1.440	1/4	1 633
4	1.002	25	1.103	3/4	1.203	40	1.305	1/4	1.446	1/2	1.643
5	1.004	1/2	1.108	34	1.206	1/4	1.310	1/2	1.453	3/4	1.652
6	1.006	26	1.113	1/4	1.210	1/2	1.315	3/4	1.460	53	1.662
7	1.008	1/2	1.117	1/2	1.213	3/4	1.320	47	1.466	1/4	1.671
8	1.010	27	1.122	3/4	1.217	41	1.325	1/4	1.473	1/2	1.681
9	1.013	1/2	1.127	35	1.221	1/4	1.330	1/2	1.480	3/4	1.691
10	1.016	28	1.133	1/4	1.225	1/2	1.335	3/4	1.487	54	1.701
11	1.019	1/2	1.138	1/2	1.228	3/4	1.340	48	1.495	1/4	1.712
12	1.022	29	1.143	3/4	1.232	42	1.346	1/4	1.502	1/2	1.722
13	1.026	1/2	1.149	36	1.236	1/4	1.351	1/2	1.509	3/4	1.733
14	1.031	30	1.155	1/4	1.240	1/2	1.356	3/4	1.517	55	1.743
15	1.035	1/4	1.158	1/2	1.244	3/4	1.362	49	1.524	1/4	1.754
16	1.040	1/2	1.161	3/4	1.248	43	1.367	1/4	1.532	1/2	1.766
17	1.046	3/4	1.164	37	1.252	1/4	1.373	1/2	1.540	3/4	1.777
18	1.052	31	1.167	1/4	1.256	1/2	1.379	3/4	1.548	56	1.788
19	1.058	1/4	1.170	1/2	1.261	3/4	1.384	50	1.556	1/4	1.800
20	1.064	1/2	1.173	3/4	1.265	44	1.390	1/4	1.564	1/2	1.812
21	1.071	3/4	1.176	38	1.269	1/4	1.396	1/2	1.572	3/4	1.824
1/2	1.075	32	1.179	1/4	1.273	1/2	1.402	3/4	1.581	57	1.836
22	1.079	1/4	1.182	1/2	1.278	3/4	1.408	51	1.589	1/4	1.849
1/2	1.082	1/2	1.186	3/4	1.282	45	1.414	1/4	1.598	1/2	1.861
23	1.086	3/4	1.189	39	1.287	1/4	1.420	1/2	1.606	3/4	1.874
1/2	1.090	33	1.192	1/4	1.291	1/2	1.427	3/4	1.615	58	1.887

To find a Meridian Line (a true North and South line) by means of the North Star. (Polaris.)

The north star appears to describe a small circle, *n n'*, &c, Fig 14, around the true north point, or north pole, as a center. The rad of this circle is estimated by the angle between the star and the pole, as measured from the earth; and is called the **polar dist** of the star. This polar dist becomes $19\frac{3}{10}$ seconds, or very nearly 1/3 of a minute less every year. On Jan 1, 1885, it is, approximately, 1° 17′ 58″. On the first of 1890, it will be about 1° 16′ 41″, &c. When, in its revolution, the star is farthest east or west from the pole, as at *n'* or *n''*, it is said to be at its **greatest E or W elongation.** Then its apparent motion for several min is nearly vert, and consequently affords the best opportunity for an observation in the simple manner here described. The arrows in Fig 14 show the direction in which the stars appear to move from east to west when the spectator faces the north.

The latitude of the place must be known approximately. Taking it at the closest one in our foregoing table, the error in the position of the meridian will not exceed half a min of azimuth in lat 57°, or one quarter min in lat 40°; and still less in lower lats.

About 3 ft above ground fix firmly, perfectly level, and as nearly east and west as may be, a smooth narrow piece of board, about 3 ft long, to serve as a kind of table. Also prepare another piece *a a*, about a foot long; and fasten to it, at right angles, a compass-sight, or a strip of thin metal, with a straight slit, (shown by a black line in the fig.) about 6 ins long and $\frac{1}{16}$ inch wide. This piece of board is to be slid along the table, as the observer follows the motions of the star toward the east or west; looking at it through the vert slit. Plant a stout pole, about 20 ft long, firmly in the ground, with its top as nearly north as possible from the middle of the table. Its top should lean 2 or 3 ft toward either the east or the west; and a plumb-line must be suspended from its top, with a bob weighing one or two lbs, which may swing in a bucket of water placed on the ground. This is to prevent the line from being so easily moved by slight currents of air; and for further steadiness, the pole itself should be well braced from within, a

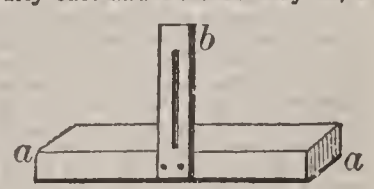


Fig. 12.

few feet below its top. The proper dist *a o*, of the pole *p* o, from the table *t a*, may be found thus: Make an angle *n m s*, equal to equal, by any convenient scale, approximately to the lat of the place. Open a pair of dividers the height *t a* of the table; and draw *t a*. Then take, by the same scale, the height, *p* o, of the pole above ground; and place it upon the sketch, so that the top *p* shall be by scale a ft or two above *m n*. Then *a* o, by the same scale, will be about the dist reqd; probably from 3 to 5 yards. A deviation of a ft or so from this will not be important.

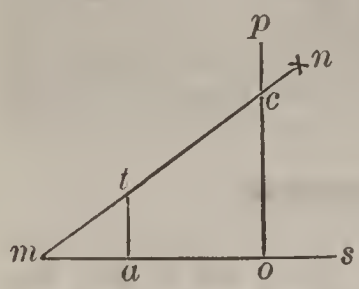


Fig. 13.

The correct clock time at any place, for the elongation, may be found within a few min from the following table.

Instead of a pole and plumb-line, the writer would suggest a planed, straight-edged board planted vert and braced; its *side* toward the observer.

The observer should be at his station at least ¼ of an hour in advance of the time. Placing the board *a a*, upon the table and in range with the plumb-line and star, he will watch both of them through the vert slit; sliding the board along the table, so as to keep the slit in the range as long as the star continues to move toward the east or west, as the case may be. An assistant must hold a candle, or lantern, on a pole near the plumb-line, to enable the observer to see the latter. As the star approaches its elongation, it will appear to move nearly vert for several min, so that it can be seen without moving the slit. When certain by this that the star has reached its elongation, confine the sliding board to the table by sticking a few tacks around its edges. Then let a third person, with another candle, go off some dist, (a hundred yards or more if convenient,) in a direction toward the star; and then drive a stake as directed by the observer, who will take care that it is exactly in range with the slit and plumb-line. Another stake must then be driven exactly under either the slit or the plumb-line. Having thus placed the two stakes in the *range* of the elongation, defer the remainder of the operation until morning. From the tables given above take out the sine of the polar dist, and also the secant of the lat. Mult these together. The prod will be the nat sine of an angle called the **azimuth** of the star. Find the sine in table, p 60, &c. and the angle which corresponds to it. This azimuth angle will be between 1° 20′ and 2° 30′, according to lat. Place an instrument over the S stake, sight to the N one, and lay off this angle to the E if the elong was W, or vice versa, and drive a stake to mark it. This last direction is true N and S. It might be supposed that after driving the first two stakes, a true meridian could be had by merely laying off the polar dist, by means of a compass or transit; but this is not so. Place the compass over the south stake, and take sight to the north one. If, then, the north end of the needle points east of the line, the variation of the compass is east, and vice versa.

Times by a correct clock of Elongations of the N. Star.

Deduced from U. S. Coast Survey table, calculated for April 1, 1883, to April 1, 1884, and for lat 38° N; but will answer within about 5 minutes for any lat up to 60° N, and until 1890.

Times of Eastern Elongations.

Day of Month.	Apr.	May.	June.	July.	Aug.	Sep.
	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	6 41 A M	4 43 A M	2 41 A M	12 43 A M	10 37 P M	8 36 P M
7	6 18 "	4 20 "	2 18 "	12 20 "	10 14 "	8 12 "
13	5 54 "	3 56 "	1 54 "	11 52 P M	9 50 "	7 48 "
19	5 30 "	3 32 "	1 30 "	11 29 "	9 27 "	7 25 "
25	5 7 "	3 9 "	1 7 "	11 5 "	9 3 "	7 1 "

Times of Western Elongations.

Day of Month.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.
	H. M.	H. M.	H. M.	H. M.	H. M.	H M.
1	6 31 A M	4 29 A M	2 32 A M	12 30 A M	10 24 P M	8 30 P M
7	6 8 "	4 6 "	2 8 "	12 6 "	10 00 "	8 6 "
13	5 44 "	3 42 "	1 44 "	11 39 P M	9 36 "	7 43 "
19	5 21 "	3 19 "	1 21 "	11 15 "	9 13 "	7 19 "
25	4 57 "	2 55 "	12 57 "	10 51 "	8 49 "	6 55 "

For days of the month intermediate of those in the table, it will be near enough to make the time 4 min earlier each succeeding day.

During nearly all of the four months, March, April, September, and October, the elongations take place in daylight; so that this method cannot then be used. Nor can it be used at any time in places south of about 4° north of the equator, because there the north star is not visible. But during that time a meridian may be found by recollecting that when the north star *n* is *on the meridian*, or, in other words, is *truly* north from us, the star **Alioth,** *a*, is very nearly vertically *above* it, if the north star *n* is on the meridian *below* the pole: or below it, as at *a'''*, if the north star, *n'''* is on the meridian *above* the pole. When the north star *n'* is at its east elongation, Alioth is horizontally west of it, as at *a'*; and when the north star, *n''*, is at its west elongation, Alioth is horizontally east of it, as at *a''*. All that is necessary, therefore, is to prepare an arrangement of table, pole plumb-line, &c, precisely as before; except that the plumb-line must be nearer the observer, as he will have to watch Alioth above the north star. Watch through the movable slit until Alioth is on the same vert line with the north star. Then put in two stakes as before, and they will be nearly in a true north and south line. But to be more exact, either lay off (to the E if Alioth is above

Polaris, or to the W if below) an azimuth angle of 11 minutes; or else do not drive the stakes when the two stars are in a vert line, but note the time, and then wait 24.5 minutes. Then take the range to Polaris alone. This last range will be true N and S within 2 or 3 minutes depending on lat. until 1890. **A transit** with illuminated cross-wires can plainly be used instead of the plumbline, &c, but is more troublesome except to an expert. A very correct method adapted to both N and S lats, is to take the two ranges to any N or S circumpolar star on the same night; when it is at any **two equal altitudes.** Half way between them will be true N and S.

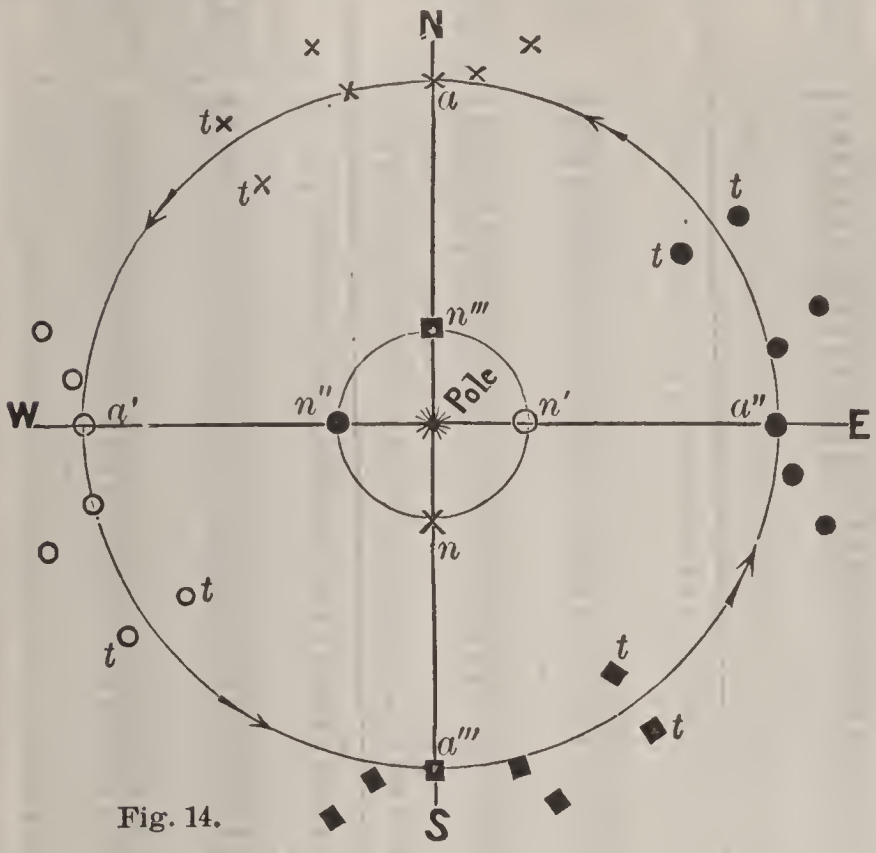


Fig. 14.

There can be no difficulty in finding Alioth, as it is one of the 7 bright stars in the fine coustellation so well known as the Great Bear, or the Wagon and Horses. Alioth is the horse nearest to the fore-wheels of the wagon. The two hind-wheels *t t* are known to every schoolboy as the "Pointers," because they point nearly in the direction to the North Star. The relative positions of these 7 stars, as shown in Fig 14, are tolerably correct.

Traverse Table for a Distance = 1.

	Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.	
0°0′	1.0000	.0000	90°0′	2°0′	.9994	.0349	88°0′	4°0′	.9976	.0698	86°0′
2	1.0000	.0006	58	2	.9994	.0355	58	2	.9975	.0703	58
4	1.0000	.0012	56	4	.9993	.0361	56	4	.9975	.0709	56
6	1.0000	.0017	54	6	.9993	.0366	54	6	.9974	.0715	54
8	1.0000	.0023	52	8	.9993	.0372	52	8	.9974	.0721	52
10	1.0000	.0029	50	10	.9993	.0378	50	10	.9974	.0727	50
12	1.0000	.0035	48	12	.9993	.0384	48	12	.9973	.0732	48
14	1.0000	.0041	46	14	.9992	.0390	46	14	.9973	.0738	46
16	1.0000	.0047	44	16	.9992	.0396	44	16	.9972	.0744	44
18	1.0000	.0052	42	18	.9992	.0401	42	18	.9972	.0750	42
20	1.0000	.0058	40	20	.9992	.0407	40	20	.9971	.0756	40
22	1.0000	.0064	38	22	.9991	.0413	38	22	.9971	.0761	38
24	1.0000	.0070	36	24	.9991	.0419	36	24	.9971	.0767	36
26	1.0000	.0076	34	26	.9991	.0425	34	26	.9970	.0773	34
28	1.0000	.0081	32	28	.9991	.0430	32	28	.9970	.0779	32
30	1.0000	.0087	30	30	.9990	.0436	30	30	.9969	.0785	30
32	1.0000	.0093	28	32	.9990	.0442	28	32	.9969	.0790	28
34	1.0000	.0099	26	34	.9990	.0448	26	34	.9968	.0796	26
36	.9999	.0105	24	36	.9990	.0454	24	36	.9968	.0802	24
38	.9999	.0111	22	38	.9989	.0459	22	38	.9967	.0808	22
40	.9999	.0116	20	40	.9989	.0465	20	40	.9967	.0814	20
42	.9999	.0122	18	42	.9989	.0471	18	42	.9966	.0819	18
44	.9999	.0128	16	44	.9989	.0477	16	44	.9966	.0825	16
46	.9999	.0134	14	46	.9988	.0483	14	46	.9965	.0831	14
48	.9999	.0140	12	48	.9988	.0488	12	48	.9965	.0837	12
50	.9999	.0145	10	50	.9988	.0494	10	50	.9964	.0843	10
52	.9999	.0151	8	52	.9987	.0500	8	52	.9964	.0848	8
54	.9999	.0157	6	54	.9987	.0506	6	54	.9963	.0854	6
56	.9999	.0163	4	56	.9987	.0512	4	56	.9963	.0860	4
58	.9999	.0169	2	58	.9987	.0518	2	58	.9962	.0866	2
1°0	.9998	.0175	89°0	3°0′	.9986	.0523	87°0′	5°0′	.9962	.0872	85°0′
2	.9998	.0180	58	2	.9986	.0529	58	2	.9961	.0877	58
4	.9998	.0186	56	4	.9986	.0535	56	4	.9961	.0883	56
6	.9998	.0192	54	6	.9985	.0541	54	6	.9960	.0889	54
8	.9998	.0198	52	8	.9985	.0547	52	8	.9960	.0895	52
10	.9998	.0204	50	10	.9985	.0552	50	10	.9959	.0901	50
12	.9998	.0209	48	12	.9984	.0558	48	12	.9959	.0906	48
14	.9998	.0215	46	14	.9984	.0564	46	14	.9958	.0912	46
16	.9998	.0221	44	16	.9984	.0570	44	16	.9958	.0918	44
18	.9997	.0227	42	18	.9983	.0576	42	18	.9957	.0924	42
20	.9997	.0233	40	20	.9983	.0581	40	20	.9957	.0929	40
22	.9997	.0239	38	22	.9983	.0587	38	22	.9956	.0935	38
24	.9997	.0244	36	24	.9982	.0593	36	24	.9956	.0941	36
26	.9997	.0250	34	26	.9982	.0599	34	26	.9955	.0947	34
28	.9997	.0256	32	28	.9982	.0605	32	28	.9955	.0953	32
30	.9997	.0262	30	30	.9981	.0610	30	30	.9954	.0958	30
32	.9996	.0268	28	32	.9981	.0616	28	32	.9953	.0964	28
34	.9996	.0273	26	34	.9981	.0622	26	34	.9953	.0970	26
36	.9996	.0279	24	36	.9980	.0628	24	36	.9952	.0976	24
38	.9996	.0285	22	38	.9980	.0634	22	38	.9952	.0982	22
40	.9996	.0291	20	40	.9980	.0640	20	40	.9951	.0987	20
42	.9996	.0297	18	42	.9979	.0645	18	42	.9951	.0993	18
44	.9995	.0302	16	44	.9979	.0651	16	44	.9950	.0999	16
46	.9995	.0308	14	46	.9978	.0657	14	46	.9949	.1005	14
48	.9995	.0314	12	48	.9978	.0663	12	48	.9949	.1011	12
50	.9995	.0320	10	50	.9978	.0669	10	50	.9948	.1016	10
52	.9995	.0326	8	52	.9977	.0674	8	52	.9948	.1022	8
54	.9995	.0332	6	54	.9977	.0680	6	54	.9947	.1028	6
56	.9994	.0337	4	56	.9976	.0686	4	56	.9946	.1034	4
58	.9994	.0343	2	58	.9976	.0692	2	58	.9946	.1039	2
2°0′	.9994	.0349	88°0′	4°0′	.9976	.0698	86°0′	6°0′	.9945	.1045	84°0′
	Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.	

Traverse Table for a Distance = 1. (CONTINUED.)

	Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.	
6°0	.9945	.1045	84°0′	8°0′	.9903	.1392	82°0′	10°0′	.9848	.1736	80°0′
2	.9945	.1051	58	2	.9902	.1397	58	2	.9847	.1742	58
4	.9944	.1057	56	4	.9901	.1403	56	4	.9846	.1748	56
6	.9943	.1063	54	6	.9900	.1409	54	6	.9845	.1754	54
8	.9943	.1068	52	8	.9899	.1415	52	8	.9844	.1759	52
10	.9942	.1074	50	10	.9899	.1421	50	10	.9843	.1765	50
12	.9942	.1080	48	12	.9898	.1426	48	12	.9842	.1771	48
14	.9941	.1086	46	14	.9897	.1432	46	14	.9841	.1777	46
16	.9940	.1092	44	16	.9896	.1438	44	16	.9840	.1782	44
18	.9940	.1097	42	18	.9895	.1444	42	18	.9839	.1788	42
20	.9939	.1103	40	20	.9894	.1449	40	20	.9838	.1794	40
22	.9938	.1109	38	22	.9894	.1455	38	22	.9837	.1799	38
24	.9938	.1115	36	24	.9893	.1461	36	24	.9836	.1805	36
26	.9937	.1120	34	26	.9892	.1467	34	26	.9835	.1811	34
28	.9936	.1126	32	28	.9891	.1472	32	28	.9834	.1817	32
30	.9936	.1132	30	30	.9890	.1478	30	30	.9833	.1822	30
32	.9935	.1138	28	32	.9889	.1484	28	32	.9831	.1828	28
34	.9934	.1144	26	34	.9888	.1490	26	34	.9830	.1834	26
36	.9934	.1149	24	36	.9888	.1495	24	36	.9829	.1840	24
38	.9933	.1155	22	38	.9887	.1501	22	38	.9828	.1845	22
40	.9932	.1161	20	40	.9886	.1507	20	40	.9827	.1851	20
42	.9932	.1167	18	42	.9885	.1513	18	42	.9826	.1857	18
44	.9931	.1172	16	44	.9884	.1518	16	44	.9825	.1862	16
46	.9930	.1178	14	46	.9883	.1524	14	46	.9824	.1868	14
48	.9930	.1184	12	48	.9882	.1530	12	48	.9823	.1874	12
50	.9929	.1190	10	50	.9881	.1536	10	50	.9822	.1880	10
52	.9928	.1196	8	52	.9880	.1541	8	52	.9821	.1885	8
54	.9928	.1201	6	54	.9880	.1547	6	54	.9820	.1891	6
56	.9927	.1207	4	56	.9879	.1553	4	56	.9818	.1897	4
58	.9926	.1213	2	58	.9878	.1559	2	58	.9817	.1902	2
7°0′	.9925	.1219	83°0′	9°0	.9877	.1564	81°0′	11°0′	.9816	.1908	79°0′
2	.9925	.1224	58	2	.9876	.1570	58	2	.9815	.1914	58
4	.9924	.1230	56	4	.9875	.1576	56	4	.9814	.1920	56
6	.9923	.1236	54	6	.9874	.1582	54	6	.9813	.1925	54
8	.9923	.1242	52	8	.9873	.1587	52	8	.9812	.1931	52
10	.9922	.1248	50	10	.9872	.1593	50	10	.9811	.1937	50
12	.9921	.1253	48	12	.9871	.1599	48	12	.9810	.1942	48
14	.9920	.1259	46	14	.9870	.1605	46	14	.9808	.1948	46
16	.9920	.1265	44	16	.9869	.1610	44	16	.9807	1954	44
18	.9919	.1271	42	18	.9869	.1616	42	18	.9806	.1959	42
20	.9918	.1276	40	20	.9868	.1622	40	20	.9805	.1965	40
22	.9917	.1282	38	22	.9867	.1628	38	22	.9804	.1971	38
24	.9917	.1288	36	24	.9866	.1633	36	24	.9803	.1977	36
26	.9916	.1294	34	26	.9865	.1639	34	26	.9802	.1982	34
28	.9915	.1299	32	28	.9864	.1645	32	28	.9800	.1988	32
30	.9914	.1305	30	30	.9863	.1650	30	30	.9799	.1994	30
32	.9914	.1311	28	32	.9862	.1656	28	32	.9798	.1999	28
34	.9913	.1317	26	34	.9861	.1662	26	34	.9797	.2005	26
36	.9912	.1323	24	36	.9860	.1668	24	36	.9796	.2011	24
38	.9911	.1328	22	38	.9859	.1673	22	38	.9795	.2016	22
40	.9911	.1334	20	40	.9858	.1679	20	40	.9793	.2022	20
42	.9910	.1340	18	42	.9857	.1685	18	42	.9792	.2028	18
44	.9909	.1346	16	44	.9856	.1691	16	44	.9791	.2034	16
46	.9908	.1351	14	46	.9855	.1696	14	46	.9790	.2039	14
48	.9907	.1357	12	48	.9854	.1702	12	48	.9789	.2045	12
50	.9907	.1363	10	50	.9853	.1708	10	50	.9787	.2051	10
52	.9906	.1369	8	52	.9852	.1714	8	52	.9786	.2056	8
54	.9905	.1374	6	54	.9851	.1719	6	54	.9785	.2062	6
56	.9904	.1380	4	56	.9850	.1725	4	56	.9784	.2068	4
58	.9903	.1386	2	58	.9849	.1731	2	58	.9783	.2073	2
8°0′	.9903	.1392	82°0′	10°0′	.9848	.1736	80°0′	12°0′	.9781	.2079	78°0′
	Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.	

Traverse Table for a Distance = 1. (CONTINUED.)

	Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.	
12°0′	.9781	.2079	78°0′	14°0	.9703	.2419	76°0′	16°0′	.9613	.2756	74°0′
2	.9780	.2084	58	2	.9702	.2425	58	2	.9611	.2762	58
4	.9779	.2090	56	4	.9700	.2431	56	4	.9609	.2768	56
6	.9778	.2096	54	6	.9699	.2436	54	6	.9608	.2773	54
8	.9777	.2102	52	8	.9697	.2442	52	8	.9606	.2779	52
10	.9775	.2108	50	10	.9696	.2447	50	10	.9605	.2784	50
12	.9774	.2113	48	12	.9694	.2453	48	12	.9603	.2790	48
14	.9773	.2119	46	14	.9693	.2459	46	14	.9601	.2795	46
16	.9772	.2125	44	16	.9692	.2464	44	16	.9600	.2801	44
18	.9770	.2130	42	18	.9690	.2470	42	18	.9598	.2807	42
20	.9769	.2136	40	20	.9689	.2476	40	20	.9596	.2812	40
22	.9768	.2142	38	22	.9687	.2481	38	22	.9595	.2818	38
24	.9767	.2147	36	24	.9686	.2487	36	24	.9593	.2823	36
26	.9765	.2153	34	26	.9684	.2493	34	26	.9591	.2829	34
28	.9764	.2159	32	28	.9683	.2498	32	28	.9590	.2835	32
30	.9763	.2164	30	30	.9681	.2504	30	30	.9588	.2840	30
32	.9762	.2170	28	32	.9680	.2509	28	32	.9587	.2846	28
34	.9760	.2176	26	34	.9679	.2515	26	34	.9585	.2851	26
36	.9759	.2181	24	36	.9677	.2521	24	36	.9583	.2857	24
38	.9758	.2187	22	38	.9676	.2526	22	38	.9582	.2862	22
40	.9757	.2193	20	40	.9674	.2532	20	40	.9580	.2868	20
42	.9755	.2198	18	42	.9673	.2538	18	42	.9578	.2874	18
44	.9754	.2204	16	44	.9671	.2543	16	44	.9577	.2879	16
46	.9753	.2210	14	46	.9670	.2549	14	46	.9575	.2885	14
48	.9751	.2215	12	48	.9668	.2554	12	48	.9573	.2890	12
50	.9750	.2221	10	50	.9667	.2560	10	50	.9572	.2896	10
52	.9749	.2227	8	52	.9665	.2566	8	52	.9570	.2901	8
54	.9748	.2232	6	54	.9664	.2571	6	54	.9568	.2907	6
56	.9746	.2238	4	56	.9662	.2577	4	56	.9566	.2913	4
58	.9745	.2244	2	58	.9661	.2583	2	58	.9565	.2918	2
13°0′	.9744	.2250	77°0′	15°0′	.9659	.2588	75°0′	17°0	.9563	.2924	73°0′
2	.9742	.2255	58	2	.9658	.2594	58	2	.9561	.2929	58
4	.9741	.2261	56	4	.9656	.2599	56	4	.9560	.2935	56
6	.9740	.2267	54	6	.9655	.2605	54	6	.9558	.2940	54
8	.9738	.2272	52	8	.9653	.2611	52	8	.9556	.2946	52
10	.9737	.2278	50	10	.9652	.2616	50	10	.9555	.2952	50
12	.9736	.2284	48	12	.9650	.2622	48	12	.9553	.2957	48
14	.9734	.2289	46	14	.9649	.2628	46	14	.9551	.2963	46
16	.9733	.2295	44	16	.9647	.2633	44	16	.9549	.2968	44
18	.9732	.2300	42	18	.9646	.2639	42	18	.9548	.2974	42
20	.9730	.2306	40	20	.9644	.2644	40	20	.9546	.2979	40
22	.9729	.2312	38	22	.9642	.2650	38	22	.9544	.2985	38
24	.9728	.2317	36	24	.9641	.2656	36	24	.9542	.2990	36
26	.9726	.2323	34	26	.9639	.2661	34	26	.9541	.2996	34
28	.9725	.2329	32	28	.9638	.2667	32	28	.9539	.3002	32
30	.9724	.2334	30	30	.9636	.2672	30	30	.9537	.3007	30
32	.9722	.2340	28	32	.9635	.2678	28	32	.9535	.3013	28
34	.9721	.2346	26	34	.9633	.2684	26	34	.9534	.3018	26
36	.9720	.2351	24	36	.9632	.2689	24	36	.9532	.3024	24
38	.9718	.2357	22	38	.9630	.2695	22	38	.9530	.3029	22
40	.9717	.2363	20	40	.9628	.2700	20	40	.9528	.3035	20
42	.9715	.2368	18	42	.9627	.2706	18	42	.9527	.3040	18
44	.9714	.2374	16	44	.9625	.2712	16	44	.9525	.3046	16
46	.9713	.2380	14	46	.9624	.2717	14	46	.9523	.3051	14
48	.9711	.2385	12	48	.9622	.2723	12	48	.9521	.3057	12
50	.9710	.2391	10	50	.9621	.2728	10	50	.9520	.3062	10
52	.9709	.2397	8	52	.9619	.2734	8	52	.9518	.3068	8
54	.9707	.2402	6	54	.9617	.2740	6	54	.9516	.3074	6
56	.9706	.2408	4	56	.9616	.2745	4	56	.9514	.3079	4
58	.9704	.2414	2	58	.9614	.2751	2	58	.9512	.3085	2
14°0′	.9703	.2419	76°0′	16°0′	.9613	.2756	74°0′	18°0′	.9511	.3090	72°0′
	Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.	

Traverse Table for a Distance = 1. (CONTINUED.)

	Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.	
18°0′	.9511	.3090	72°0′	20°0′	.9397	.3420	70°0	22°0	.9272	.3746	68°0
2	.9509	.3096	58	2	.9395	.3426	58	2	.9270	.3751	58
4	.9507	.3101	56	4	.9393	.3431	56	4	.9267	.3757	56
6	.9505	.3107	54	6	.9391	.3437	54	6	.9265	.3762	54
8	.9503	.3112	52	8	.9389	.3442	52	8	.9263	.3768	52
10	.9502	.3118	50	10	.9387	.3448	50	10	.9261	.3773	50
12	.9500	.3123	48	12	.9385	.3453	48	12	.9259	.3778	48
14	.9498	.3129	46	14	.9383	.3458	46	14	.9257	.3784	46
16	.9496	.3134	44	16	.9381	.3464	44	16	.9254	.3789	44
18	.9494	.3140	42	18	.9379	.3469	42	18	.9252	.3795	42
20	.9492	.3145	40	20	.9377	.3475	40	20	.9250	.3800	40
22	.9491	.3151	38	22	.9375	.3480	38	22	.9248	.3805	38
24	.9489	.3156	36	24	.9373	.3486	36	24	.9245	.3811	36
26	.9487	.3162	34	26	.9371	.3491	34	26	.9243	.3816	34
28	.9485	.3168	32	28	.9369	.3497	32	28	.9241	.3821	32
30	.9483	.3173	30	30	.9367	.3502	30	30	.9239	.3827	30
32	.9481	.3179	28	32	.9365	.3508	28	32	.9237	.3832	28
34	.9480	.3184	26	34	.9363	.3513	26	34	.9234	.3838	26
36	.9478	.3190	24	36	.9361	.3518	24	36	.9232	.3843	24
38	.9476	.3195	22	38	.9359	.3524	22	38	.9230	.3848	22
40	.9474	.3201	20	40	.9356	.3529	20	40	.9228	.3854	20
42	.9472	.3206	18	42	.9354	.3535	18	42	.9225	.3859	18
44	.9470	.3212	16	44	.9352	.3540	16	44	.9223	.3864	16
46	.9468	.3217	14	46	.9350	.3546	14	46	.9221	.3870	14
48	.9466	.3223	12	48	.9348	.3551	12	48	.9219	.3875	12
50	.9465	.3228	10	50	.9346	.3557	10	50	.9216	.3881	10
52	.9463	.3234	8	52	.9344	.3562	8	52	.9214	.3886	8
54	.9461	.3239	6	54	.9342	.3567	6	54	.9212	.3891	6
56	.9459	.3245	4	56	.9340	.3573	4	56	.9210	.3897	4
58	.9457	.3250	2	58	.9338	.3578	2	58	.9207	.3902	2
19°0′	.9455	.3256	71°0′	21°0′	.9336	.3584	69°0	23°0′	.9205	.3907	67°0′
2	.9453	.3261	58	2	.9334	.3589	58	2	.9203	.3913	58
4	.9451	.3267	56	4	.9332	.3595	56	4	.9200	.3918	56
6	.9449	.3272	54	6	.9330	.3600	54	6	.9198	.3923	54
8	.9448	.3278	52	8	.9327	.3605	52	8	.9196	.3929	52
10	.9446	.3283	50	10	.9325	.3611	50	10	.9194	.3934	50
12	.9444	.3289	48	12	.9323	.3616	48	12	.9191	.3939	48
14	.9442	.3294	46	14	.9321	.3622	46	14	.9189	.3945	46
16	.9440	.3300	44	16	.9319	.3627	44	16	.9187	.3950	44
18	.9438	.3305	42	18	.9317	.3633	42	18	.9184	.3955	42
20	.9436	.3311	40	20	.9315	.3638	40	20	.9182	.3961	40
22	.9434	.3316	38	22	.9313	.3643	38	22	.9180	.3966	38
24	.9432	.3322	36	24	.9311	.3649	36	24	.9178	.3971	36
26	.9430	.3327	34	26	.9308	.3654	34	26	.9175	.3977	34
28	.9428	.3333	32	28	.9306	.3660	32	28	.9173	.3982	32
30	.9426	.3338	30	30	.9304	.3665	30	30	.9171	.3987	30
32	.9424	.3344	28	32	.9302	.3670	28	32	.9168	.3993	28
34	.9423	.3349	26	34	.9300	.3676	26	34	.9166	.3998	26
36	.9421	.3355	24	36	.9298	.3681	24	36	.9164	.4003	24
38	.9419	.3360	22	38	.9296	.3687	22	38	.9161	.4009	22
40	.9417	.3365	20	40	.9293	.3692	20	40	.9159	.4014	20
42	.9415	.3371	18	42	.9291	.3697	18	42	.9157	.4019	18
44	.9413	.3376	16	44	.9289	.3703	16	44	.9154	.4025	16
46	.9411	.3382	14	46	.9287	.3708	14	46	.9152	.4030	14
48	.9409	.3387	12	48	.9285	.3714	12	48	.9150	.4035	12
50	.9407	.3393	10	50	.9283	.3719	10	50	.9147	.4041	10
52	.9405	.3398	8	52	.9281	.3724	8	52	.9145	.4046	8
54	.9403	.3404	6	54	.9278	.3730	6	54	.9143	.4051	6
56	.9401	.3409	4	56	.9276	.3735	4	56	.9140	.4057	4
58	.9399	.3415	2	58	.9274	.3741	2	58	.9138	.4062	2
20°0′	.9397	.3420	70°0′	22°0′	.9272	.3746	68°0′	24°0′	.9135	.4067	66°0
	Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.	

Traverse Table for a Distance = 1. (CONTINUED.)

	Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.	
24 0′	.9135	.4067	66°0′	26°0′	.8988	.4384	64°0′	28°0′	.8829	.4695	62°0′
2	.9133	.4073	58	2	.8985	.4389	58	2	.8827	.4700	58
4	.9131	.4078	56	4	.8983	.4394	56	4	.8824	.4705	56
6	.9128	.4083	54	6	.8980	.4399	54	6	.8821	.4710	54
8	.9126	.4089	52	8	.8978	.4405	52	8	.8819	.4715	52
10	.9124	.4094	50	10	.8975	.4410	50	10	.8816	.4720	50
12	.9121	.4099	48	12	.8973	.4415	48	12	.8813	.4726	48
14	.9119	.4105	46	14	.8970	.4420	46	14	.8810	.4731	46
16	.9116	.4110	44	16	.8967	.4425	44	16	.8808	.4736	44
18	.9114	.4115	42	18	.8965	.4431	42	18	.8805	.4741	42
20	.9112	.4120	40	20	.8962	.4436	40	20	.8802	.4746	40
22	.9109	.4126	38	22	.8960	.4441	38	22	.8799	.4751	38
24	.9107	.4131	36	21	.8957	.4446	36	24	.8796	.4756	36
26	.9104	.4136	34	26	.8955	.4452	34	26	.8794	.4761	34
28	.9102	.4142	32	28	.8952	.4457	32	28	.8791	.4766	32
30	.9100	.4147	30	30	.8949	.4462	30	30	.8788	.4772	30
32	.9097	.4152	28	32	.8947	.4467	28	32	.8785	.4777	28
34	.9095	.4158	26	34	.8944	.4472	26	34	.8783	.4782	26
36	.9092	.4163	24	36	.8942	.4478	24	36	.8780	.4787	24
38	.9090	.4168	22	38	.8939	.4483	22	38	.8777	.4792	22
40	.9088	.4173	20	40	.8936	.4488	20	40	.8774	.4797	20
42	.9085	.4179	18	42	.8934	.4493	18	42	.8771	.4802	18
44	.9083	.4184	16	44	.8931	.4498	16	44	.8769	.4807	16
46	.9080	.4189	14	46	.8928	.4504	14	46	.8766	.4812	14
48	.9078	.4195	12	48	.8926	.4509	12	48	.8763	.4818	12
50	.9075	.4200	10	50	.8923	.4514	10	50	.8760	.4823	10
52	.9073	.4205	8	52	.8921	.4519	8	52	.8757	.4828	8
54	.9070	.4210	6	54	.8918	.4524	6	54	.8755	.4833	6
56	.9068	.4216	4	56	.8915	.4530	4	56	.8752	.4838	4
58	.9066	.4221	2	58	.8913	.4535	2	58	.8749	.4843	2
25°0′	.9063	.4226	65°0	27°0	.8910	.4540	63°0′	29°0′	.8746	.4848	61°0′
2	.9061	.4231	58	2	.8907	.4545	58	2	.8743	.4853	58
4	.9058	.4237	56	4	.8905	.4550	56	4	.8741	.4858	56
6	.9056	.4242	54	6	.8902	.4555	54	6	.8738	.4863	54
8	.9053	.4247	52	8	.8899	.4561	52	8	.8735	.4868	52
10	.9051	.4253	50	10	.8897	.4566	50	10	.8732	.4874	50
12	.9048	.4258	48	12	.8894	.4571	48	12	.8729	.4879	48
14	.9046	.4263	46	14	.8892	.4576	46	14	.8726	.4884	46
16	.9043	.4268	44	16	.8889	.4581	44	16	.8724	.4889	44
18	.9041	.4274	42	18	.8886	.4586	42	18	.8721	.4894	42
20	.9038	.4279	40	20	.8884	.4592	40	20	.8718	.4899	40
22	.9036	.4284	38	22	.8881	.4597	38	22	.8715	.4904	38
24	.9033	.4289	36	24	.8878	.4602	36	24	.8712	.4909	36
26	.9031	.4295	34	26	.8875	.4607	34	26	.8709	.4914	34
28	.9028	.4300	32	28	.8873	.4612	32	28	.8706	.4919	32
30	.9026	.4305	30	30	.8870	.4617	30	30	.8704	.4924	30
32	.9023	.4310	28	32	.8867	.4623	28	32	.8701	.4929	28
34	.9021	.4316	26	34	.8865	.4628	26	34	.8698	.4934	26
36	.9018	.4321	24	36	.8862	.4633	24	36	.8695	.4939	24
38	.9016	.4326	22	38	.8859	.4638	22	38	.8692	.4944	22
40	.9013	.4331	20	40	.8857	.4643	20	40	.8689	.4950	20
42	.9011	.4337	18	42	.8854	.4648	18	42	.8686	.4955	18
44	.9008	.4342	16	44	.8851	.4654	16	44	.8683	.4960	16
46	.9006	.4347	14	46	.8849	.4659	14	46	.8681	.4965	14
48	.9003	.4352	12	48	.8846	.4664	12	48	.8678	.4970	12
50	.9001	.4358	10	50	.8843	.4669	10	50	.8675	.4975	10
52	.8998	.4363	8	52	.8840	.4674	8	52	.8672	.4980	8
54	.8996	.4368	6	54	.8838	.4679	6	54	.8669	.4985	6
56	.8993	.4373	4	56	.8835	.4684	4	56	.8666	.4990	4
58	.8990	.4378	2	58	.8832	.4690	2	58	.8663	.4995	2
26°0′	.8988	.4384	64°0′	28°0′	.8829	.4695	62°0′	30°0′	.8660	.5000	60°0′
	Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.	

Traverse Table for a Distance = 1. (CONTINUED.)

	Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.	
30°0′	.8660	.5000	60°0′	32°0′	·8480	·5299	58°0′	34°0′	.8290	.5592	56°0′
2	.8657	.5005	58	2	·8477	·5304	58	2	.8287	.5597	58
4	.8654	.5010	56	4	·8474	·5309	56	4	.8284	.5602	56
6	.8652	.5015	54	6	·8471	·5314	54	6	.8281	.5606	54
8	.8649	.5020	52	8	·8468	·5319	52	8	.8277	.5611	52
10	.8646	.5025	50	10	·8465	·5324	50	10	.8274	.5616	50
12	.8643	.5030	48	12	·8462	·5329	48	12	.8271	.5621	48
14	.8640	.5035	46	14	·8459	·5334	46	14	.8268	.5626	46
16	.8637	.5040	44	16	·8456	·5339	44	16	.8264	.5630	44
18	.8634	.5045	42	18	·8453	·5344	42	18	.8261	.5635	42
20	.8631	.5050	40	20	·8450	·5348	40	20	.8258	.5640	40
22	.8628	.5055	38	22	·8446	·5353	38	22	.8254	.5645	38
24	.8625	.5060	36	24	·8443	·5358	36	24	.8251	.5650	36
26	.8622	.5065	34	26	·8440	·5363	34	26	.8248	.5654	34
28	.8619	.5070	32	28	·8437	·5368	32	28	.8245	.5659	32
30	.8616	.5075	30	30	·8434	·5373	30	30	.8241	.5664	30
32	.8613	.5080	28	32	·8431	·5378	28	32	.8238	.5669	28
34	.8610	.5085	26	34	·8428	·5383	26	34	.8235	.5674	26
36	.8607	.5090	24	36	·8425	·5388	24	36	.8231	.5678	24
38	.8604	.5095	22	38	·8421	·5393	22	38	.8228	.5683	22
40	.8601	.5100	20	40	·8418	·5398	20	40	.8225	.5688	20
42	.8599	.5105	18	42	·8415	·5402	18	42	.8221	.5693	18
44	.8596	.5110	16	44	·8412	·5407	16	44	.8218	.5698	16
46	.8593	.5115	14	46	·8409	·5412	14	46	.8215	.5702	14
48	.8590	.5120	12	48	·8406	·5417	12	48	.8211	.5707	12
50	.8587	.5125	10	50	·8403	·5422	10	50	.8208	.5712	10
52	.8584	.5130	8	52	·8399	·5427	8	52	.8205	.5717	8
54	.8581	.5135	6	54	·8396	·5432	6	54	.8202	.5721	6
56	.8578	.5140	4	56	·8393	·5437	4	56	.8198	.5726	4
58	.8575	.5145	2	58	·8390	·5442	2	58	.8195	.5731	2
31°0′	.8572	.5150	59°0′	33°0′	·8387	·5446	57°0′	35°0′	.8192	.5736	55°0′
2	.8569	.5155	58	2	·8384	·5451	58	2	.8188	.5741	58
4	.8566	.5160	56	4	·8380	·5456	56	4	.8185	.5745	56
6	.8563	.5165	54	6	·8377	·5461	54	6	.8181	.5750	54
8	.8560	.5170	52	8	·8374	·5466	52	8	.8178	.5755	52
10	.8557	.5175	50	10	·8371	·5471	50	10	.8175	.5760	50
12	.8554	.5180	48	12	·8368	·5476	48	12	.8171	.5764	48
14	.8551	.5185	46	14	·8364	·5480	46	14	.8168	.5769	46
16	.8548	.5190	44	16	·8361	·5485	44	16	.8165	.5774	44
18	.8545	.5195	42	18	·8358	·5490	42	18	.8161	.5779	42
20	.8542	.5200	40	20	·8355	·5495	40	20	.8158	.5783	40
22	.8539	.5205	38	22	·8352	·5500	38	22	.8155	.5788	38
24	.8536	.5210	36	24	·8348	·5505	36	24	.8151	.5793	36
26	.8532	.5215	34	26	·8345	·5510	34	26	.8148	.5798	34
28	.8529	.5220	32	28	·8342	.5515	32	28	.8145	.5802	32
30	.8526	.5225	30	30	·8339	.5519	30	30	.8141	.5807	30
32	.8523	.5230	28	32	·8336	.5524	28	32	.8138	.5812	28
34	.8520	.5235	26	34	·8332	.5529	26	34	.8134	.5816	26
36	.8517	.5240	24	36	·8329	.5534	24	36	.8131	.5821	24
38	.8514	.5245	22	38	·8326	.5539	22	38	.8128	.5826	22
40	.8511	.5250	20	40	·8323	.5544	20	40	.8124	.5831	20
42	.8508	.5255	18	42	·8320	.5548	18	42	.8121	.5835	18
44	.8505	.5260	16	44	·8316	.5553	16	44	.8117	.5840	16
46	.8502	.5265	14	46	·8313	.5558	14	46	.8114	.5845	14
48	.8499	.5270	12	48	·8310	.5563	12	48	.8111	.5850	12
50	.8496	.5275	10	50	·8307	.5568	10	50	.8107	.5854	10
52	.8493	.5279	8	52	·8303	.5573	8	52	.8104	.5859	8
54	.8490	.5284	6	54	·8300	.5577	6	54	.8100	.5864	6
56	.8487	.5289	4	56	·8297	.5582	4	56	.8097	.5868	4
58	.8484	.5294	2	58	·8294	.5587	2	58	.8094	.5873	2
32°0′	.8480	.5299	58°0′	34°0′	·8290	.5592	56°0′	36°0′	.8090	.5878	54°0′
	Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.	

Traverse Table for a Distance = 1. (CONTINUED.)

	Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.	
36°0′	.8090	.5878	54°0′	38°0′	.7880	.6157	52°0	40°0′	.7660	.6428	50°0
2	.8087	.5883	58	2	.7877	.6161	58	2	.7657	.6432	58
4	.8083	.5887	56	4	.7873	.6166	56	4	.7653	.6437	56
6	.8080	.5892	54	6	.7869	.6170	54	6	.7649	.6441	54
8	.8076	.5897	52	8	.7866	.6175	52	8	.7645	.6446	52
10	.8073	.5901	50	10	.7862	.6180	50	10	.7642	.6450	50
12	.8070	.5906	48	12	.7859	.6184	48	12	.7638	.6455	48
14	.8066	.5911	46	14	.7855	.6189	46	14	.7634	.6459	46
16	.8063	.5915	44	16	.7851	.6193	44	16	.7630	.6463	44
18	.8059	.5920	42	18	.7848	.6198	42	18	.7627	.6468	42
20	.8056	.5925	40	20	.7844	.6202	40	20	.7623	.6472	40
22	.8052	.5930	38	22	.7841	.6207	38	22	.7619	.6477	38
24	.8049	.5934	36	24	.7837	.6211	36	24	.7615	.6481	36
26	.8045	.5939	34	26	.7833	.6216	34	26	.7612	.6486	34
28	.8042	.5944	32	28	.7830	.6221	32	28	.7608	.6490	32
30	.8039	.5948	30	30	.7826	.6225	30	30	.7604	.6494	30
32	.8035	.5953	28	32	.7822	.6230	28	32	.7600	.6499	28
34	.8032	.5958	26	34	.7819	.6234	26	34	.7596	.6503	26
36	.8028	.5962	24	36	.7815	.6239	24	36	.7593	.6508	24
38	.8025	.5967	22	38	.7812	.6243	22	38	.7589	.6512	22
40	.8021	.5972	20	40	.7808	.6248	20	40	.7585	.6517	20
42	.8018	.5976	18	42	.7804	.6252	18	42	.7581	.6521	18
44	.8014	.5981	16	44	.7801	.6257	16	44	.7578	.6525	16
46	.8011	.5986	14	46	.7797	.6262	14	46	.7574	.6530	14
48	.8007	.5990	12	48	.7793	.6266	12	48	.7570	.6534	12
50	.8004	.5995	10	50	.7790	.6271	10	50	.7566	.6539	10
52	.8000	.6000	8	52	.7786	.6275	8	52	.7562	.6543	8
54	.7997	.6004	6	54	.7782	.6280	6	54	.7559	.6547	6
56	.7993	.6009	4	56	.7779	.6284	4	56	.7555	.6552	4
58	.7990	.6014	2	58	.7775	.6289	2	58	.7551	.6556	2
37°0′	.7986	.6018	53°0′	39°0′	.7771	.6293	51°0′	41°0′	.7547	.6561	49°0
2	.7983	.6023	58	2	.7768	.6298	58	2	.7543	.6565	58
4	.7979	.6027	56	4	.7764	.6302	56	4	.7539	.6569	56
6	.7976	.6032	54	6	.7760	.6307	54	6	.7536	.6574	54
8	.7972	.6037	52	8	.7757	.6311	52	8	.7532	.6578	52
10	.7969	.6041	50	10	.7753	.6316	50	10	.7528	.6583	50
12	.7965	.6046	48	12	.7749	.6320	48	12	.7524	.6587	48
14	.7962	.6051	46	14	.7746	.6325	46	14	.7520	.6591	46
16	.7958	.6055	44	16	.7742	.6329	44	16	.7516	.6596	44
18	.7955	.6060	42	18	.7738	.6334	42	18	.7513	.6600	42
20	.7951	.6065	40	20	.7735	.6338	40	20	.7509	.6604	40
22	.7948	.6069	38	22	.7731	.6343	38	22	.7505	.6609	38
24	.7944	.6074	36	24	.7727	.6347	36	24	.7501	.6613	36
26	.7941	.6078	34	26	.7724	.6352	34	26	.7497	.6617	34
28	.7937	.6083	32	28	.7720	.6356	32	28	.7493	.6622	32
30	.7934	.6088	30	30	.7716	.6361	30	30	.7490	.6626	30
32	.7930	.6092	28	32	.7713	.6365	28	32	.7486	.6631	28
34	.7926	.6097	26	34	.7709	.6370	26	34	.7482	.6635	26
36	.7923	.6101	24	36	.7705	.6374	24	36	.7478	.6639	24
38	.7919	.6106	22	38	.7701	.6379	22	38	.7474	.6644	22
40	.7916	.6111	20	40	.7698	.6383	20	40	.7470	.6648	20
42	.7912	.6115	18	42	.7694	.6388	18	42	.7466	.6652	18
44	.7909	.6120	16	44	.7690	.6392	16	44	.7463	.6657	16
46	.7905	.6124	14	46	.7687	.6397	14	46	.7459	.6661	14
48	.7902	.6129	12	48	.7683	.6401	12	48	.7455	.6665	12
50	.7898	.6134	10	50	.7679	.6406	10	50	.7451	.6670	10
52	.7894	.6138	8	52	.7675	.6410	8	52	.7447	.6674	8
54	.7891	.6143	6	54	.7672	.6414	6	54	.7443	.6678	6
56	.7887	.6147	4	56	.7668	.6419	4	56	.7439	.6683	4
58	.7884	.6152	2	58	.7664	.6423	2	58	.7435	.6687	2
38°0′	.7880	.6157	52°0′	40°0′	.7660	.6428	50°0′	42°0′	.7431	.6691	48°0′
	Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.	

Traverse Table for a Distance = 1. (Concluded.)

	Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.	
42°0	.7431	.6691	48°0′	43°0′	.7314	.6820	47°0′	44°0′	.7193	.6947	46°0′
2	.7428	.6696	58	2	.7310	.6824	58	2	.7189	.6951	58
4	.7424	.6700	56	4	.7306	.6828	56	4	.7185	.6955	56
6	.7420	.6704	54	6	.7302	.6833	54	6	.7181	.6959	54
8	.7416	.6709	52	8	.7298	.6837	52	8	.7177	.6963	52
10	.7412	.6713	50	10	.7294	.6841	50	10	.7173	.6967	50
12	.7408	.6717	48	12	.7290	.6845	48	12	.7169	.6972	48
14	.7404	.6722	46	14	.7286	.6850	46	14	.7165	.6976	46
16	.7400	.6726	44	16	.7282	.6854	44	16	.7161	.6980	44
18	.7396	.6730	42	18	.7278	.6858	42	18	.7157	.6984	42
20	.7392	.6734	40	20	.7274	.6862	40	20	.7153	.6988	40
22	.7388	.6739	38	22	.7270	.6867	38	22	.7149	.6992	38
24	.7385	.6743	36	24	.7266	.6871	36	24	.7145	.6997	36
26	.7381	.6747	34	26	.7262	.6875	34	26	.7141	.7001	34
28	.7377	.6752	32	28	.7258	.6879	32	28	.7137	.7005	32
30	.7373	.6756	30	30	.7254	.6884	30	30	.7133	.7009	30
32	.7369	.6760	28	32	.7250	.6888	28	32	.7128	.7013	28
34	.7365	.6764	26	34	.7246	.6892	26	34	.7124	.7017	26
36	.7361	.6769	24	36	.7242	.6896	24	36	.7120	.7021	24
38	.7357	.6773	22	38	.7238	.6900	22	38	.7116	.7026	22
40	.7353	.6777	20	40	.7234	.6905	20	40	.7112	.7030	20
42	.7349	.6782	18	42	.7230	.6909	18	42	.7108	.7034	18
44	.7345	.6786	16	44	.7226	.6913	16	44	.7104	.7038	16
46	.7341	.6790	14	46	.7222	.6917	14	46	.7100	.7042	14
48	.7337	.6794	12	48	.7218	.6921	12	48	.7096	.7046	12
50	.7333	.6799	10	50	.7214	.6926	10	50	.7092	.7050	10
52	.7329	.6803	8	52	.7210	.6930	8	52	.7088	.7055	8
54	.7325	.6807	6	54	.7206	.6934	6	54	.7083	.7059	6
56	.7321	.6811	4	56	.7201	.6938	4	56	.7079	.7063	4
58	.7318	.6816	2	58	.7197	.6942	2	58	.7075	.7067	2
43°0′	.7314	.6820	47°0′	44°0′	.7193	.6947	46°0′	45°0′	.7071	.7071	45°0′
	Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.	

When the angle exceeds 45°, the lats and deps are read upward from the bottom.

Rem.—Since these lats and deps are for a dist 1, we may proceed as follows for greater dists. Thus, let the dist be 856.1. Add together 800 times, 50 times, 6 times, and $\frac{1}{10}$ time the corresponding lats and deps of the table.

Ex.—What is the lat and dep for 856.1 feet; the angle being 43°?

Here for 43° we have from the table, lat .7314; dep .6820.

Hence,	.7314 × 800 = 585.12;	and .6820 × 800 =	545.60
	.7314 × 50 = 36.57;	and .6820 × 50 =	34.10
	.7314 × 6 = 4.39;	and .6820 × 6 =	4.09
	.7314 × .1 = .07;	and .6820 × .1 =	.07
	Lat 626.15		Dep 583.86

These multiplications may be made mentally. Or we may, with a little more trouble, mult the lat and dep of the table by the given dist. Thus,

.7314 × 856.1 = 626.15 lat; and .6820 × 856.1 = 583.86 dep.*

* Inasmuch as the engineer but rarely needs a traverse table, we have thought it best to give a correct one, rather than the common one for ¼ degrees. The first involves more trouble in using it; but the last is entirely unfit for other than the rude calculations for common surveying with compass courses taken to the nearest ¼ degree.

To divide a scale of one mile into feet, first cut off one-sixth of it; then divide the remainder into four equal parts. Each of these parts will be 1100 feet.

THE ENGINEER'S TRANSIT.

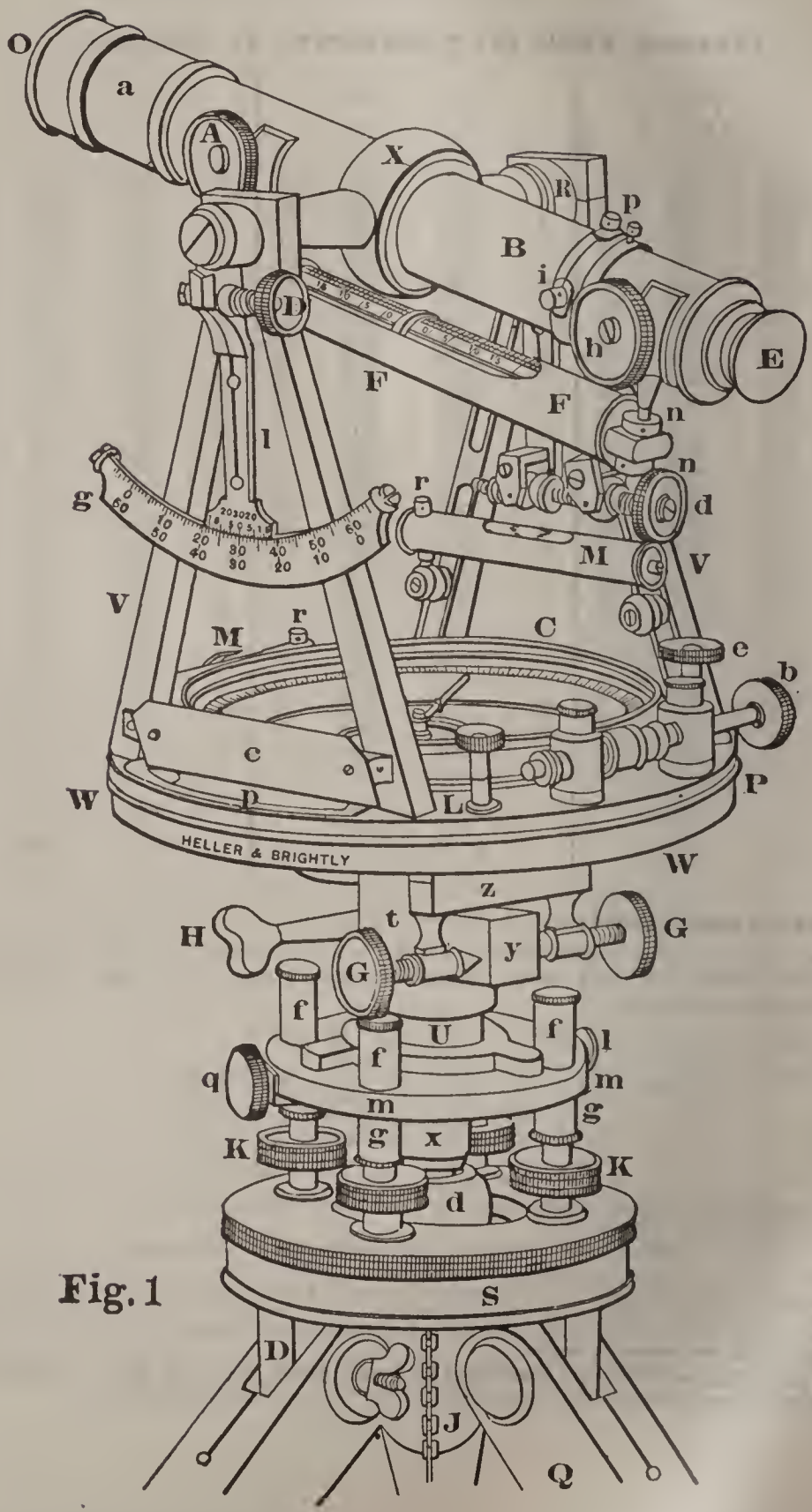


Fig. 1

THE details of the transit, like those of the level, are differently arranged by diff makers, and to suit particular purposes. We describe it in its modern form, as made by Heller and Brightly, of Philada. Without the **long bubble-tube** F F, Fig 1, under the telescope, and the **graduated arc** *g*, it is their **plain transit.** With these appendages, or rather with a graduated *circle* in place of the *arc*, it becomes virtually a **Complete Theodolite.***

B D D, Fig 2, is the **tripod-head.** The screw-threads at *v* receive the screw of a wooden tripod-head-cover when the instrument is out of use. S S A is the **lower parallel plate.** After the transit has been set very nearly over the center of a stake, the **shifting-plate,** *d d c c*, enables us, by slightly loosening the **levelling-screws** K, to shift the upper parts horizontally a trifle, and thus bring the plumb-bob exactly over the center with less trouble than by the older method of pushing one or two of the legs further into the ground, or spreading them more or less. The screws, K, are then tightened, thereby pushing upward the **upper parallel plate** *m m m x x*, and with it the half-ball *b*, thus pressing *c c* tightly up against the under side of S. The plumb-line passes

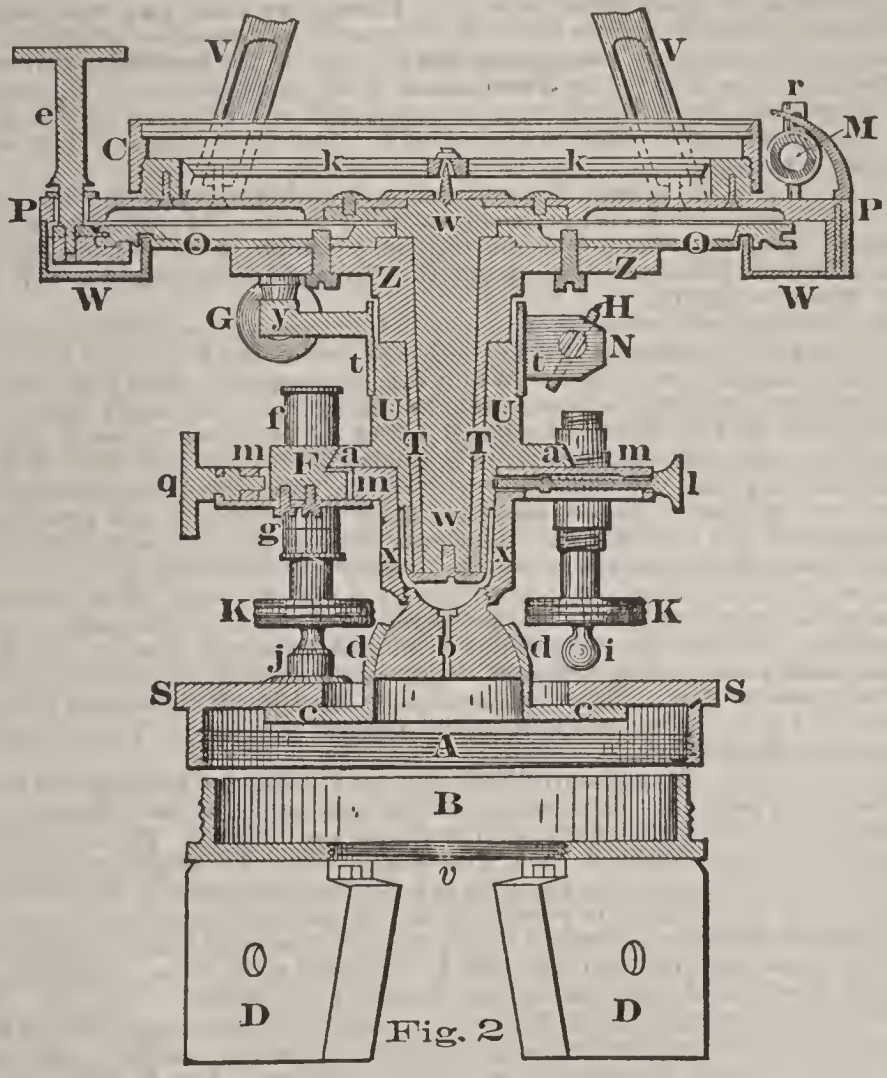


Fig. 2

through the vert hole in *b*. Screw-caps, *f*, *g*, protect the levelling-screws from dust, &c. The feet, *i*, of the screws, work in loose sockets, *j*, made flat at bottom, to preserve S from being indented. The parts thus far described are generally left attached to the legs at all times. Fig 1 shows the method of attachment.

To set the upper parts upon the parallel plates. Place the lower end of U U in *x x*, holding the instrument so that the three blocks on *m m* (of which the one shown at F is movable) may enter the three corresponding

* The price of a first-class plain transit with shifting-plate and plumb-bob, by Heller & Brightly, in 1886, is $185. One with vert arc *g* and long bubble-tube F F, $220.

recesses in *a*, thus allowing *a* to bear fully on *m*, upon which the upper parts then rest. (The inner end of the spring-catch, *l*, in the meantime enters a groove around U, just below *a*, and prevents the upper parts from falling off, if the instrument is now carried over the shoulder.) Revolve the upper parts horizontally a trifle, in either direction, until they are stopped by the striking of a small lug on *a* against one of the blocks F. The recesses in *a* are now clear of the blocks. Tighten *q*, thereby pushing inward the movable block F, which clamps the bevelled flange *a* between it and the two fixed blocks on *m m*, and confines the spindle U to the fixed parallel plates. It remains so clamped while the instrument is being used.

To remove the upper parts from the parallel plates. Loosen *q*, bring the recesses in *a* opposite the blocks F. Hold back *l*, and lift the upper parts, which are then held together by the broad head of the screw inserted into the foot of the spindle *w*.

T T is the **outer revolving spindle,** cast in one with the **supporting-plate** Z Z, to which is fastened the **graduated limb** O O. The limb extends beyond the compass-box, and thus admits of larger graduations than would otherwise be obtainable. *w w* is the **inner revolving spindle.** At its top it has a broad flange, to which is fastened the **vernier plate** P P. To the latter are fastened the **compass-box** C, one of the **bubble-tubes** M M (the one shown in Fig 2), the **dust-box** W W, the standards V V, supporting the telescope, &c. Each bubble-tube is supported and adjusted by two capstan-screws, one at each end. One is shown at *r*. The bent strip curving over the tube protects the glass.

The clamp-screw, H, presses the split collar, *t t*, tightly against the fixed spindle, U, but not against Z or T. The set-screws, G G, working in nuts that are cast in one with Z, hold between them the tongue, *y*, which projects from *t t*, and the graduated limb is thus held fast, except that by moving the screws, G, it may be made to revolve slightly.

In Fig 1, the tangent-screw, *b*, is seen passing through two towers, in which it works. One of the towers is fast to the lower one of the two small pieces at the foot of the clamp-screw *e*, Fig 2. When *e* is tightened, it draws the two small pieces together, confining between them an edge of the graduated limb, which is thus made fast to the above-mentioned tower. The other tower is fast to the vernier-plate; and the tangent-screw, *b*, holds the towers at a fixed dist apart. The clamping of *e* thus prevents the vernier-plate from revolving over the graduated limb, except that it may be moved slightly by turning *b*, and thus changing the dist apart of the towers. In Heller and Brightly's instruments, the screw, *b*, is provided with means for taking up its "wear," or "lost-motion."

There are two verniers. One is shown at *p*, Fig 1. Both may be read, and their mean taken, when great accuracy is required. Ivory reflectors, *c*, facilitate their reading. Before the instrument is moved from one place to another, the **compass-needle,** *k*, Fig 2, should always be pressed up against the glass cover of the compass-box by means of the upright milled-head screw seen on the vernier-plate in Fig 1, just to the right of the nearest standard. The pivot-point is thus protected from injury.

R, Fig 1, is a ring with a clamp (the latter not shown) for holding the telescope in any required position. It is best to let the *eye*-end, *e*, of the telescope revolve *downward*, as otherwise the shade on O, if in use, may fall off. The tangent-screw, *d*, moves a vert arm attached to R, and is thus used for slightly changing the elevation of the telescope. In the arm is a slit like that seen in the vernier-arm *l*. When 0° of the vernier is placed at 30° on the arc, *g*, and the index of the opposite arm is placed over a small notch on the horizontal brace (not seen in our figs) of the standards, the two slits will be opposite each other, and may be used for laying off offsets, &c, at right-angles to the line of sight.

One end, R, of the telescope axis rests in a movable box, under which is a screw. By means of the screw, the box may be raised or lowered, and the axis thus adjusted for very slight derangements of the standards. For E, B, O, and A, see *Level*, p 201. *a* is a dust-guard for the object-slide.

Stadia Hairs. Immediately behind the capstan-screw, *p*, Fig 1, is seen a smaller one. This and a similar one on the opposite side of the telescope, work in a ring inside the telescope, and hold the ring in position. Across the ring are stretched two additional horizontal hairs, called stadia hairs, placed at such a distance apart, vertically, that they will subtend say 10 divisions of a graduated rod placed 100 ft from the instrument, 15 divisions at 150 ft, &c. They are thus used for measuring hor and sloping distances.

The long bubble-tube. F F, Fig 1, enables us to use the transit as a level, although it is not so well adapted as the latter to this purpose.

To adjust a plain Transit.

When either a level or a transit is purchased, it is a good precaution (but one which the writer has never seen alluded to) to first screw the object-glass firmly home to its place; and then make a short continuous scratch upon the ring of the glass, and upon its slide; so as to be able to see at any time when at work, that the glass is always in the same position with regard to the slide. For if, after all the adjustments are completed, the position of the glass should become changed, (as it is apt to be if unscrewed, and afterward not screwed up to the same precise spot,) the adjustments may thereby become materially deranged; especially if the object-glass is eccentric, or not truly ground, which is often the case. Such scratches should be prepared by the maker. In making adjustments, as well as when using a transit or level, be careful that the eye-glass and object-glass are so drawn out that there shall be no parallax. The eye-glass must first be drawn out so as to obtain perfect distinctness of the cross-hairs; it must not be disturbed afterward; but the object-glass must be moved for different distances.

First, to ascertain that the bubble-tubes, M M, are placed parallel to the vernier-plate, *and that therefore when both bubbles are in the centers of their tubes the axis of the inst is vert.* By means of the four levelling-screws, K, bring both bubbles to the centers of their tubes in one position of the inst; then turn the upper parts of the inst *half-way* round. If the bubbles do not remain in the center, correct *half* the error by means of the two capstan-screws *r r;* and the other half by the levelling-screws K. Repeat the trial until both bubbles remain in the center while the inst is being turned entirely around on its spindle.

Second, to see that the standards have suffered no derangement; that is, that they are of equal height and perpendicular to the vernier-plate, as they always are when they leave the maker's hands. Level the inst perfectly; then direct the intersection of the hairs to some point of a high object (as the top of a steeple) near by; clamp the inst by means of screws H and *e*, and lower the telescope until the intersection strikes some point of a low object. (If there is none such drive a stake or chain-pin, &c, in the line.) Then unclamp either H or *e*, and turn the upper parts of the inst half-way round; fix the intersection again upon the high point; clamp; lower the telescope to the low point. If the intersection still strikes the low point, the standards are in order. If not, correct *one-quarter* of the diff (same principle as in Fig 4) by means of the adjusting-block and screw at the end, R, of the telescope axis, Fig 1, and repeat the trial *de novo*, resetting the stake or chain-pin at each trial. If the inst has no adjusting-block for the axis, it should be returned to the maker for correction of any derangement of the standards.

A transit may be used *for running straight lines*, even if the standards become slightly bent, by the process described at the end of the fourth adjustment.

Third, to see that the cross-hairs are truly vert and hor when the inst is level. When the telescope *inverts*, the cross-hairs are nearer the eye-end than when it shows objects erect. The maker takes care to place the cross-hairs at right-angles to each other in their ring, or diaphragm; and generally he so places the ring in the telescope, that when levelled, they shall be vert and hor. Sometimes, however, this is neglected; or the ring may by accident become turned a little. To be certain that one hair is vert, (in which case the other must, by construction, be hor,) after having adjusted the bubble-tubes, level the instrument carefully, and take sight with the telescope at a plumb-line, or other vert straight edge. If the vert hair coincides with this object, it is, *so far*, in adjustment; but if not, then loosen *slightly* only *two adjacent* screws of the four, *p p i i*, Fig 1; and with a knife, key, or other small instrument, tap very gently against the screw-heads, so as to turn the ring a little in the telescope; persevering until the hair becomes truly vertical. When this is done, tighten the screws. In the absence of a plumb-line, or vert straight edge, sight the cross-hair at a very small distinct point; and see if the hair still cuts that point, when the telescope is raised or lowered by revolving it on its axis.

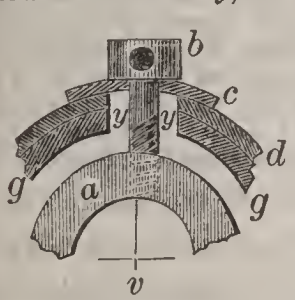


Fig. 3.

The mode of performing the foregoing will be readily understood from this Fig, which represents a section across the top part of the telescope, and at the cross-hairs. The hair-ring, or diaphragm, *a;* vert hair, *v;* telescope tube, *g;* ring outside of telescope tube, *d;* *b* is one of the four capstan-headed screws which hold the hair-ring, *a*, in its place, and also serve to adjust it. The lower ends of these screws work in the thickness of the hair-ring; so that when they are loosened somewhat, they do not lose their hold on the ring. Small loose

washers, *c*, are placed under the heads *b* of the screws. A space *y y* is left around each screw where it passes through the telescope tube, to allow the screws and ring together to be moved a little sideways when the screws *b* are slightly loosened.

Fourth, to see that the vertical hair is in the line of collimation. Plant the tripod firmly upon the ground, as at *a*. Level the inst; clamp it; and direct the vert hair by means of tangent-screws G (figs. 1 and 2) upon some convenient object *b*; or if there is none such, drive a thin stake, or a chain-pin. Then revolving the telescope vert on its axis, observe some object, as *c*, where the vert hair now strikes; or if there is none, place a second pin. Unclamp the instrument by the clamp-screw H; and turn the whole upper part of it around until the vert hair again strikes *b*. Clamp again; and again revolve the telescope vert on its axis. If the vert hair now strikes *c*, as it did before, it shows that *c* is really at *o*; and that *b*, *a*, *c*, *are in the same straight line*; and therefore this adjustment is in order. If not, observe where it does strike, say at *m*, (the dist *a m* being taken equal to *a c*,) and place a pin there also. Measure *m c*; and place a pin at *v*, in the line *m c*, making *m v* = one-fourth of *m c*. Also put a pin at *o*, half-way between *m* and *c*, or in range with *a* and *b*. By means of the two hor screws that move the ring carrying the cross-hairs, adjust the vert hair until it cuts *v*. Now repeat the *entire* operation; and persevere until the telescope, after being directed to *b*, shall strike the same object *o*, *both times*, when revolved on its axis. See whether the movement of the ring in this 4th adjustment has disturbed the verticality of the hair. If it has, repeat the 3d adjustment. Then repeat the 4th, if necessary; and so on until both adjustments are found to be right at the same time. Thus a straight line may be run, even if the hairs are out of adjustment; but with somewhat more trouble. For at each station, as at *a*, two back-sights, and two fore-sights, *a c* and *a m*, may be taken, as when making the adjustment; and the point *o*, half-way between *c* and *m*, will be in the straight line. The inst may then be moved to *o*, and the two back-sights be taken to *a*; and so on.

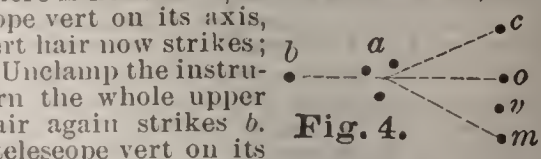


Fig. 4.

Angles measured by the transit, whether vert or hor, will evidently not be affected by the hairs being out of adjustment, provided either that the vert hair is truly vert, or that we use the *intersection* of the hairs when measuring.

The foregoing are all the adjustments needed, unless the transit is required for levelling, in which case the following one must be attended to:

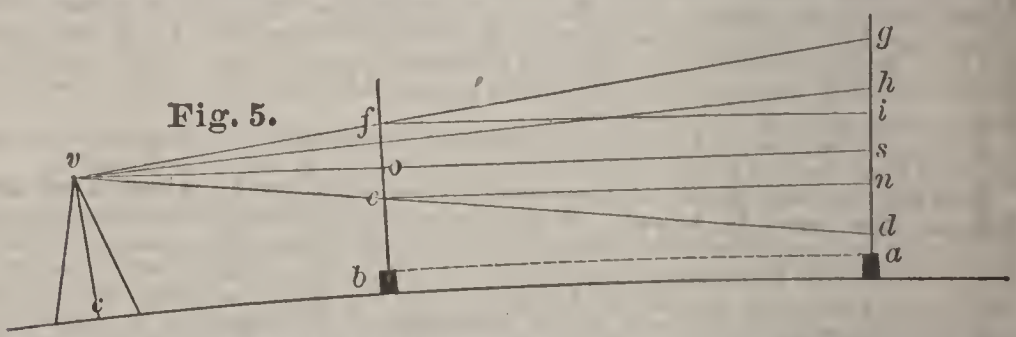


Fig. 5.

To adjust the long bubble-tube, F F, Fig. 1, we first place the line of sight of the telescope hor, and then make the bubble-tube hor, so that the two are parallel. Drive two pegs, *a* and *b* Fig. 5, with their tops at precisely the same level (see Rem. p. 193) and at least about 100 ft. apart; 300 or more will be better. Plant the inst firmly, in range with them, as at *c*, making *b c* an aliquot part of *a b*, and as short as will permit focusing on a rod at *b*. The inst need not be leveled. Suppose the line of sight to cut *e* and *d*. Take the readings *b e* and *a d*. Their diff is $b\,e - a\,d = a\,n - a\,d = d\,n$; and $a\,b : a\,c :: d\,n : d\,s$; *s* being the height of the target at *a* when the readings (*a s*, *b o*) on the two stakes are equal. $a\,s = a\,d + d\,s = a\,d + \frac{d\,n \times a\,c}{a\,b}$. If the reading on *a* *exceeds* that on *b* (as when the line of sight is *v f g*) the diff of readings is $= a\,g - b\,f = a\,g - a\,i = g\,i$; and $a\,s = a\,g - g\,s = a\,g - \frac{g\,i \times a\,c}{a\,b}$. Sight to *s*, bring the bubble to the cen of its tube by means of the two small nuts *n n* at one end of the tube, Fig. 1, and assume that the telescope and tube are parallel.* The zeros of

* This neglects a small error due to the curvature of the earth; for a *hor* line at *v* is *v h*, tangential to the curved (or "level") surface of still water at *v*, whereas *v s* is tangential to water surf at a point midway between *a* and *b*. Hence if the telescope at *v* points to *s* it will not be parallel to the level bubble-tube. To allow for this, and for the refraction by the air, which *diminishes* the error, raise the target on *a* to a point *h* above *s*. *h s* = .0000000205 × square of *a c* in ft; but when *a c* is 650 ft. *h s* is only about one tenth of an inch and barely covers the apparent thickness of the cross-hair in the telescope.

the vert circle, and of its vernier, may now be adjusted, if they require it, by loosening the vernier screws and then moving the vernier until the two coincide.

Rem. If no level is at hand for levelling the two pegs *a* and *b*, it may be done by the transit itself, thus: Carefully level the two short bubbles, by means of the levelling-screws K. Drive a peg *m*, from 100 to 300 feet from the instrument *o*. Then placing a target-rod on *m*, clamp the target tight at whatever height, as *e v*, the hor hair happens to cut it; it being of no importance whether the telescope is level or not; although it might as well be as nearly so as can conveniently be guessed at. Clamp the telescope in its position by the clamp-ring R, Fig 1. Revolve the inst a considerable way round: say nearly or quite half way. Place another peg *n*, *at precisely the same dist* from the instrument that *m* is; and continue to drive it until the hor hair cuts the target placed on it, and still kept clamped to the rod, at the same height as when it was on *m*. When this is done, the tops of the two pegs are on a level with each other, and are ready to be used as before directed.

Fig. 6.

When a transit is intended to be used for surveying farms, &c, or for retracing lines of old surveys, it is very useful to set the compass so as to allow for the "variation" during the interval between the two surveys. For this purpose a **"variation-vernier"** is added to such transits; and also to the compass.

When the graduations of a transit are figured, or numbered, so as to read both ways from zero, thus,

the vernier also is made double; that is, it also is graduated and numbered from its zero both ways. In this case, if the angle is measured from zero toward the right hand, the reading must be made from the right hand half of the vernier; and vice versa. If the figuring is single, or only in one direction, from zero to 360°, then only the single vernier is necessary, as the angles are then measured only in the direction that the figuring counts. Engineers differ in their preferences for various manners of figuring the graduations. The writer prefers from zero each way to 180°, with two double verniers.

To replace cross-hairs in a level, or transit. Take out the tube from the eye end of the telescope. Looking in, notice which side of the cross-hair diaphragm is turned toward the eye end. Then loosen the four screws which hold the diaphragm, so as to let the latter fall out of the telescope. Fasten on new hairs with beeswax, varnish, glue, or gum-arabic water, &c. This requires care. Then, to return the diaphragm to its place, press firmly into one of the screw-holes on the circumf of the diaphragm itself, the end of a piece of stick, long enough to reach easily into the telescope as far as to where the diaphragm belongs. By this stick, as a handle, insert the diaphragm edgewise to its place in the telescope, and hold it there until two *opposite* screws are put in place and screwed. Then draw the stick out of the hole in the diaphragm; and with it turn the diaphragm until the same side presents itself toward the eye end as before; then put in the other two screws.

The so-called cross-hairs are actually spider-web, so fine as to be barely visible to the naked eye. Heller & Brightly use very fine platina wire, which is much better. Human hair is entirely too coarse.

To replace a spirit-level, or bubble-glass. Detach the level from the instrument; draw off its sliding ends; push out the broken glass vial, and the cement which held it; insert the new one, with the proper side up (the upper side is always marked with a file by the maker); wrapping some paper around its ends, if it fits loosely. Finally, put a little putty, or melted beeswax over the ends of the vial, to secure it against moving in its tube.

In purchasing instruments, especially when they are to be used far from a maker, it is advisable to provide extras of such parts as may be easily broken or lost; such as glass compass-covers, and needles; adjusting pins; level vials; magnifiers, &c.

Theodolite adjustments are performed like those of the level and transit.

1st. That of the cross-hairs; the same as in the level.

2d. The long bubble-tube of the telescope; also as in the level.

3d. The two short bubble-tubes; as in the transit.

4th The vernier of the vert limb; as in the transit with a vert circle.

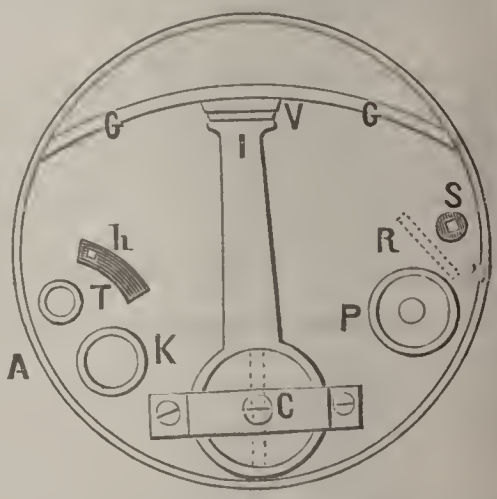
5th. To see that the vert hair travels vertically; as in the fourth adjustment of the transit. In some theodolites, no adjustment is provided for this; but in large ones it is provided for by screws under the feet of the standards.

Sometimes a second telescope is added; it is placed below the hor limb, and is

called a *watcher*. It has its own clamp, and tangent-screw. Its use is to ascertain whether the zero of that limb has moved during the measurement of hor angles. When, previously to beginning the measurement, the zero and upper telescope are directed toward the first object, point the lower telescope to any small distant object, and then clamp it. During the subsequent measurement, look through it, from time to time, to be sure that it still strikes that object; thus proving that no slipping has occurred.

THE BOX OR POCKET SEXTANT.

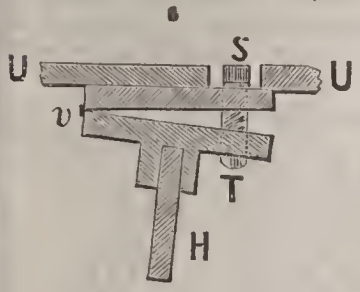
The portability of the pocket sextant, and the fact that it reads to single minutes, render it at times very useful to the engineer.* By it, angles can be measured while in a boat, or on horseback; and in many situations which preclude the use of a transit. It is useful for obtaining latitudes, by aid of an artificial horizon. When closed, it resembles a cylindrical brass box, about 3 inches in diameter, and 1½ inches deep. This box is in two parts; by unscrewing which, then inverting one part, and then screwing them together again, the lower part becomes a handle for holding the instrument. Looking down upon its top when thus arranged, we see, as in this figure, a movable arm 1 C, called the **index,** which turns on a center at C, and carries the vernier V at its other end. G G is the graduated arc or **limb.** It actually subtends about 73°, but is divided into about 146°. Its zero is at one end. Its graduations are not shown in the Fig.



Attached to the index is a small movable lens, (not shown in the figure,) likewise revolving around C, for reading the fine divisions of the limb. When measuring an angle, the index is moved by turning the milled-head P of a pinion, which works in a rack placed within the box. The eye is applied to a circular hole at the side of the box, near A. A small telescope, about 3 inches long, accompanies the instrument: but may generally be dispensed with. When so, the eye-hole at A should be partially closed by a slide which has a very small eye-hole in it; and which is moved by the pin *h*, moving in the curved slot. Another slide, at the side of the box, carries a dark glass for covering the eye-hole when observing the sun. When the telescope is used, it is fastened on by the milled-head screw T. The top part shown in our figure, can be separated from the cylindrical part, by removing 3 or 4 small screws around its edge; and the interior can then be examined, and cleaned if necessary. Like nautical, and other sextants, this one has two principal glasses, both of them mirrors. One, the **index-glass,** is attached to the underside of the index, at C; its upper edge being indicated by the two dotted lines. The other, the **horizon-glass,** (because, when measuring the vert angles of celestial bodies, it is directed toward the horizon,) is also within the box; the position of its upper edge being shown by the dotted lines at R. The horizon-glass is silvered only half-way down; so that one of the observed objects may be seen directly through its lower half, while the image of the other object is seen in the upper half, reflected from the index-glass. That the instrument may be in adjustment, ready for use, these two glasses must be at right angles to the plane of the instrument: that is, to the under side of the top of the box, to which they are attached; and must also be parallel to each other, when the zeros of the vernier and of the limb coincide. The index-glass is already permanently fixed by the maker, and requires no other adjustment. But the horizon-glass has two adjustments, which are made by a key like that of a watch, and having a milled-head K. It is screwed into the top of the box, so as to be always at hand for use. When needed, it is unscrewed. This key fits upon two small square-heads, (like that for

* Price, with telescope, about $50. Made by Stackpole & Bro., 41 Fulton St., New York.

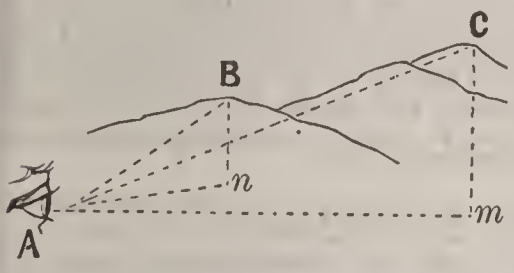
winding a watch;) one of which is shown at S; while the other is near it, but on the SIDE of the box. These squares are the heads of two small screws. If the horizon glass H should, as in this sketch, (where it is shown endwise,) not be at right angles to the top U U of the box, it is brought right by turning the square-head S of the screw S T; and if, after being so far rectified, it still is not parallel to the index-glass when the zeros coincide, it is moved a little backward or forward by the square head at the side.



To adjust a box sextant. bring the two zeros to coincide precisely; then look through the eye-hole, and the lower or unsilvered part of the horizon-glass, at some distant object. If the instrument is in adjustment, the object thus seen directly, will coincide precisely with its reflected image, seen at the same time, at the same spot. But if it is not in adjustment, the two will appear separated either hor or vert, or both, thus, * *; in which case apply the key K to the square-head S; and by turning it slightly in whichever direction may be necessary, *still looking at the object and its image*, bring the two into a hor position, or on a level with each other, thus, * *. Then apply the key to the square-head in the side of the box; and by turning it slightly, bring the two to coincide perfectly The instrument is then adjusted.

In some instruments, the hor glass has a *hinge* at *v*, to allow it play while being adjusted by the single screw S T; but others dispense with this hinge, and use *two* screws like S on top of the box, in addition to the one in the side.

If a sextant is used for measuring vert angles by means of an **artificial horizon,** the actual altitude will be but one-half of that read off on the limb; because we then read at once both the actual and the reflected angle. The great objection to the sextant for engineering purposes, is that it does not measure angles horizontally, as the transit does; unless when the observer, and the two objects happen to be in the same hor plane. Thus an observer with a sextant at A, if measuring the angle subtended by the mountain-peaks B and C, must hold the graduated plane of the sextant in the plane of A B C; and must actually measure the angle B A C; whereas what he wants is the hor angle *n* A *m*. This is greater than B A C, because the dists A *n* and A *m* are shorter than A B and A C. The transit gives the hor angle *n* A *m*, because its graduated plane is first fixed hor by the levelling-screws: and the subsequent measurement of the angle is not affected by his directing merely the line of sight upward, to any extent, in order to fix it upon B and C. For more on this subject; and for a method of partially obviating this objection to the sextant, see the note to Example 2, Case 4, of "Trigonometry," page 113.



The nautical sextant. used on ships, is constructed on the same principle as the box sextant; and its adjustments are very similar. In it, also, the index-glass is permanently fixed by the maker; and the horizon-glass has the two adjustments of the box sextant. It also has its dark glasses for looking at the sun; and a small sight-hole, to be used when the telescope is dispensed with.

THE COMPASS.

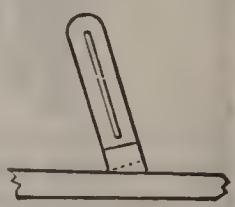
To adjust a Compass.

The first adjustment is that of the bubbles. Plant firmly; and level the instrument, in any position; that is, bring the bubbles to the centers of their tubes. Then turn the instrument half-way round. If the bubbles then remain at the centers, they are in adjustment; but if not, correct *one-half* the diff in each bubble, by means of the adjusting-screws of the tubes. Level the instrument again; turn it half round; and if the bubbles still do not remain at the center, the adjusting-screws must be again moved a little, so as to rectify half the remaining diff. Gener-

ally several trials must be thus made, until the bubbles will remain at the center, while the compass is being turned entirely around.

Second adjustment. Level the compass, and then see that the needle is hor; and if not, make it so by means of the small piece of wire which is wrapped around it; sliding the wire toward the high end. A needle thus horizontally adjusted at one place, will not remain so if removed far north or south from that place. If carried to the north, the north end will dip down; and if to the south, the south end will do so. The sliding wire is intended to counteract this.

Third adjustment. This is always fixed right at first by the maker; that is, the sights, or slits for sighting through, are placed at right angles to the compass plate; so that when the latter is levelled by the bubbles, the sights are vert. To test whether they are so, hang up a plumb-line; and having levelled the compass, take sight at the line, and see if the slits coincide with it. If one or both slits should prove to be out of plumb, as shown to an exaggerated extent in this sketch, it should be unscrewed from the compass, and a portion of its foot on the high side be filed or ground off, as per the dotted line; or as a temporary expedient, a small wedge may be placed under the low side, so as to raise it.



Fourth adjustment, to straighten the needle, if it should become bent. The compass being levelled, and the needle hor, and loose on its pivot, see whether its two ends continue to point to exactly opposite graduations, (that is, graduations 180° apart;) while the compass is turned completely around. If it does, the needle is straight; and its pin is in the center of the graduated circle; but if it does not, then one or both of these require adjusting. First level the compass. Then turn it until some graduation (say 90°) comes precisely to the north end of the needle. If the south end does not then point precisely to the opposite 90° division, lift off the needle, and bend the *pivot-point* until it does; remembering that every time said point is bent, the compass must be turned a hairsbreadth so as to keep the *north* end of the needle at its 90° mark. Then turn the compass half-way round, or until the *opposite* 90° mark comes precisely to the north end of the needle. Make a fine pencil mark where the *south* end of the needle now points. Then take off the needle, and bend it until its south end points *half-way between* its 90° mark and the pencil mark, while its north end is kept at 90° by moving the compass round a hairsbreadth. The needle will then be straight, and must not be altered in making the following adjustment, although it will not yet cut opposite degrees.

Fifth adjustment, of the pivot-pin. After being certain that the needle is straight, turn the compass around until a part is arrived at where the two ends of the needle happen to cut opposite degrees. Then turn the compass *quarter way* around, or through 90°. If the needle then cuts opposite degrees, the pivot-point is already in adjustment; but if the needle does not so cut, bend the *pivot-point* until it does. Repeat, if necessary, until the needle cuts opposite degrees while being turned entirely around.

Care and nicety of observation are necessary in making these adjustments properly; because the entire error to be rectified is, in itself, a minute quantity; and the novice is very apt to increase his trouble by not knowing how to use his **magnifier,** when looking at the end of the needle and the corresponding graduations. The magnifier must always be held with its *center directly over* the point to be examined: and it must be held parallel to the graduated circle. Otherwise annoying errors of several minutes will be made in a single observation; and the accumulation of two or three such errors, arising from a cause unknown to him, may compel him to abandon the adjustments in despair. This suggestion applies also to the reading of angles taken by the transit, &c; although the errors are not then likely to be so great as in the case of the compass. In purchasing a magnifier for a compass, see that no part of it, as hinges, or rivets, are made of iron; for such would change the direction of the needle.

If the sight-slits of a compass are not fixed by the maker in line with the two opposite zeros, the engineer cannot remedy the defect. This can be ascertained by passing a piece of fine thread through the slits, and observing whether it stands precisely over the zeros.

Variation of the Compass.*

The numerous disturbing influences to which the compass is subject, render its

* For full information on this subject see that useful little book "Magnetic Variation in the U S." by **J B Stone, C E,** 1878. It is invaluable in retracing old lines.

indications of bearings or courses very unreliable. The daily variation itself sometimes amounts to ¼ of a degree; and always to at least several minutes. It is almost incessantly changing the direction of the needle, to one side or the other, at the rate of 1 or 2 minutes per hour, especially in summer. Local attraction, from iron in the soil, ferruginous gravel, trap rocks, &c, is another source of inaccuracy; as are also the annual and the secular variations. Electricity, either atmospheric, or excited by rubbing the glass cover, sometimes gives trouble. It may be removed by touching the glass with the moist tongue, or finger. It is plain that none of these causes (except the last) will affect the measurement of *angles* by the compass.

In 1885 the line of no variation enters the U. S. near the N end of Lake Superior; passes through the Straits of Mackinaw; 40 m W of Detroit, Mich; 50 m E of Columbus, O; and reaches the Atlantic about half way between Charleston, S C, and Wilmington, N C. A compass placed anywhere in the vicinity of that line, will point nearly due north and south. To the eastward of this line, the variation is westward; and vice versa; becoming greater, the farther the place is from the line; until in some parts of Maine and along the Pacific coast it is as great as 18° to 21°. This line is moving westward, at an average rate of about 3 or 4 minutes per year.

The needle, if of *soft* metal, sometimes loses part of its magnetism, and consequently does not work well. It may be restored by simply drawing the north pole of a common magnet (either straight or horseshoe) about a dozen times, from the center to the end of the south half of the needle: and the south pole, in the same way, along the north half; pressing the magnet gently upon the needle. After each stroke, remove the magnet several inches from the needle, while bringing it back to the center for making another stroke. Each half of the needle in turn, while being thus operated on, should be held flat upon a smooth hard surface. Sluggish action of the needle is, however, more generally produced by the dulling or other injury of the point of the pivot. Remagnetizing will throw the needle out of balance; which must be counteracted by the sliding wire.

In order to prevent mistakes by reading sometimes from one end, and sometimes from the other end of the needle, it is best to ALWAYS point the N of the compass-box toward the object whose bearing is to be taken; and to read off from the north end of the needle. This is also more accurate.

CONTOUR LINES.

A CONTOUR LINE is a curved hor one, every point in which represents the same level; thus each of the contour lines 88c, 91c, 94c, &c, Fig 1, indicates that every point in the ground through which it is traced is at the same level; and that that level or height is everywhere 88, 91, or 94 ft above a certain other level or height called *datum;* to which all others are referred.

Frequently the level of the starting point of a survey is taken as being 0, or zero, or datum; and if we are sure of meeting with no points lower than it, this answers every purpose. But if there is a probability of many lower points, it is better to assume the starting point to be so far above a certain supposed datum, that none of these lower points shall become minus quantities, or *below* said supposed datum or zero. The only object in this is to avoid the liability to error which arises when some of the levels are +, or plus; and some —, or minus. Hence we may assume the level of the starting point to be 10, 100, 1000, &c, ft above datum, according to circumstances.

The vert dists between each two contour lines are supposed to be equal; and in railroad surveys through well-known districts, where the engineer knows that his actual line of survey will not require to be much changed, the dist may be 1 or 2 ft only; and the lines need not be laid down for widths greater than 100 or 200 ft on each side of his center-stakes. But in regions of which the topography is comparatively unknown; and where consequently unexpected obstacles may occur which require the line to be materially changed for a considerable dist back, the observations should extend to greater widths; and for expedition the vertical dists apart may be increased to 3, 5, or even 10 ft, depending on the character of the country, &c. Also, when a survey is made for a topographical map of a State, or of a county, vert dists of 5 or 10 ft will generally suffice.

Let the line A B, Fig 1, starting from 0, represent three stations (S 1, S 2, S 3,) of the center line of a railroad survey; and let the numbers 100, 103, 101, 104, along that line denote the heights at the stakes above datum, as determined by levelling. Then the use of the contour lines is to show in the office what would be the effect of changing the surveyed center line A B, by moving any part of it to the right or

left hand.* Thus, if it should be moved 100 ft to the left, the starting point O would be on ground about 6 ft higher than at present; inasmuch as its level would then be about 106 ft above datum, instead of 100. Station 1 would be about 7 ft higher, or 110 ft instead of 103 Station 2 would be about 7 ft higher, or 108 ft instead of 101. If the line be thrown to the right, it will plainly be on lower ground.

The field observations for contour lines are sometimes made with the spirit-level; but more frequently by a slope-man, with a straight 12-ft gradnated rod, and a slope instrument, or clinometer. At each station he lays his rod upon the ground, as

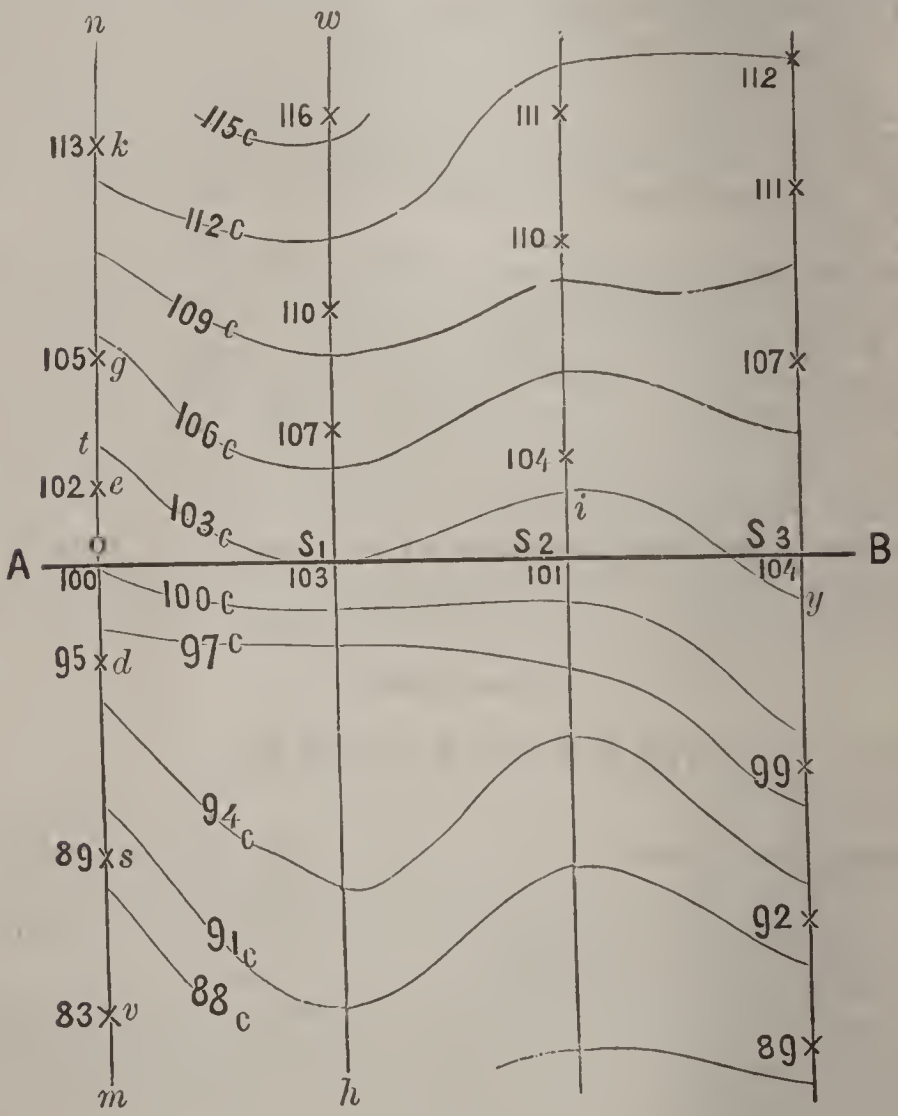


Fig. 1.

nearly at right angles to the center line A B as he can judge by eye; and placing the slope instrument upon it, he takes the angle of the slope of the ground to the nearest ¼ of a degree. He also observes how far beyond the rod the slope continues the same; and with the rod he measures the dist. Then laying down the rod at that point also, he takes the next slope, and measures its length; and so on as far as may be judged necessary. His notes are entered in his field-book as shown in Fig 2; the angles of the slopes being written above the lines, and their lengths below; and should be accompanied by such remarks as the locality suggests; such as woods, rocks, marsh, sand, field, garden, across small run, &c, &c.

* In thus using the words right and left we are supposed to have our backs turned to the starting point of the survey. **In a river, the right bank** or shore is that which is on the right hand as we **descend** it, that is, in speaking of its right or left bank, we are supposed to have our backs turned towards its head, or origin; and so with a survey.

It is not absolutely necessary to represent the slopes roughly in the field-book, as in Fig 2; for by using the sign + to signify "up;" —"down;" and = "level," the slopes may be written in a straight line, as in Fig 2½.

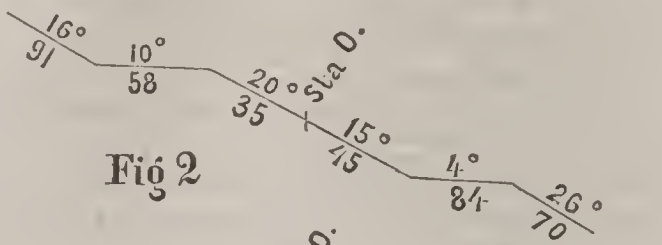


Fig 2

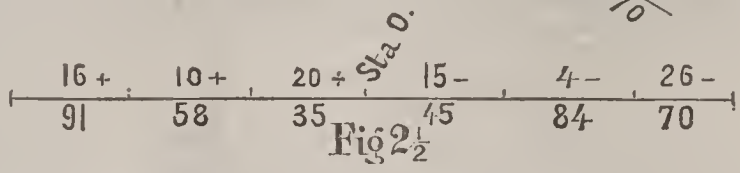


Fig 2½

The notes having been taken, the preparation of the contour lines by means of them, is of course office-work; and is usually done at the same time as the drawing of the map, &c. The field observations at each station are then separately drawn by protractor and scale, as shown in Fig 3 for the starting point O. The scale should not be less than about $\frac{1}{12}$ inch to a ft, if anything like accuracy is aimed at. Suppose that at said station the slopes to the right, taken in their order, are, as in Fig 2, 15°, 4°, and 26°; and those to the left, 20°, 10°, and 16°; and their lengths as in the same Fig. Draw a hor line *h o*, Fig 3; and consider the center of it to be the station-stake. From this point as a center, lay off these angles with a protractor, as shown on the arcs in Fig 3. Then beginning say on the right hand, with a parallel ruler draw the first dist *a c*, at its proper slope of 15°; and of its proper length, 45 ft, by scale. Then the same with *c y* and *y t*. Do the same with those on the left hand. We then have a *cross-section* of the ground at Sta O. Then on the map, as in Fig 1, draw a line as *m n*, or *h w*, at right angles to the line of road, and passing through the station-stake. On this line lay down the *hor* dists *a d*, *d s*, *s v*, *a e*, *e g*, *g k*, marking them with a small star, as is done and lettered in Fig 1, at Sta O.

When extreme accuracy is pretended to, these hor dists must be found by measure on Fig 3; but as a general rule it will be near enough, when the slopes do not exceed 10°, to assume them to be the same as the sloping dists measured in the field. Next ascertain how high each of the points *c y t l n i* is above datum. Thus, measure by scale the vert dist *d c*. Suppose it is found to be 5 ft; or in other words, that *c* is 5 ft *below* station-stake O. Then since the level at stake O is 100 ft above datum, that at *c* must be 5 ft *less*, or 100 — 5 = 95 ft above datum; which may be marked in light lead-pencil figures on the map, as at *d*, Fig 1. Next for the point *y*, suppose we find *s y* to be 11 ft, or *y* to be 11 ft *below* stake O; then its height above datum must be 100 — 11 = 89; which also write in pencil, as at *s*. Proceed in the same way with *t*. Next going to the left hand of the station-stake, we find *e l* to be say 2 ft; but *l* is *above* the level of the station-stake, therefore its height above datum is

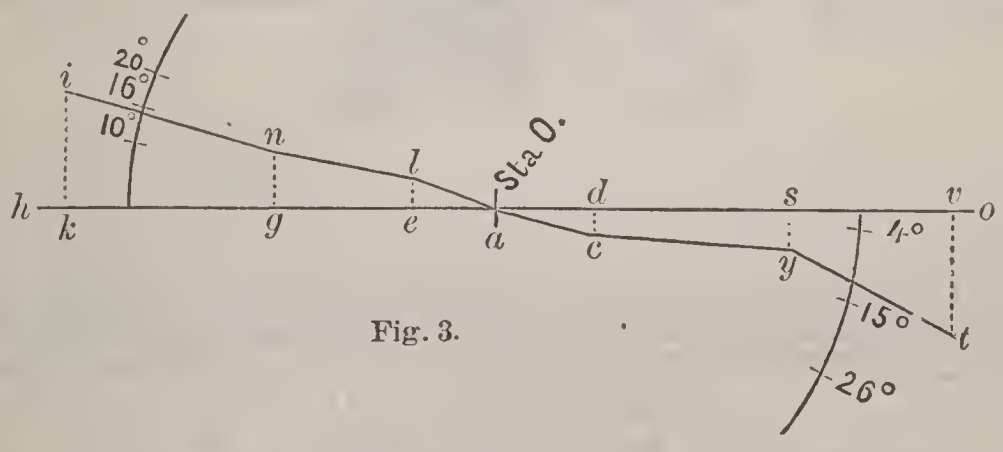


Fig. 3.

100 + 2 = 102 ft, as figured at *e* on the map. Let *n g* be 5 ft; then is *n*, 100 + 5 = 105 ft above datum, as marked at *g*; and so on at each station. When this has been done at several stations, we may draw in the contour lines of that portion by hand thus: Suppose they are to represent vert heights of 3 ft. Beginning at Station O (of which the height above datum is 100 ft) to lay down a contour line 103 ft above datum, we see at once that the height of 103 ft must be at *t*, or at ⅓ the dist from *e* to *g*. Make a light lead-pencil dot at *t*; and then go to the next Station 1. Here we see that the height of 103 ft coincides with the station-stake itself; place a dot there, and go to Sta 2. The level at this stake is 101; therefore the contour for 103

ft must evidently be 2 ft higher, or at i, $\frac{2}{5}$ of the dist from Sta 2 to +104; therefore make a dot at i. Then go to Sta 3. Here the level being 104 above datum, the contour of 103 must be at y, or $\frac{1}{5}$ of the dist from Sta 3 to +99; put a dot at y. Finally draw by hand a curving line through t, S 1, i, and y; and the contour line of 103 ft is done. All the others are prepared in the same way, one by one. The level of each must be figured upon it at short intervals along the map, as at 103 c, 106 c, &c.

Or, instead of first placing the + points on the map, to denote the slope dists actually measured upon the ground, we may at once, and with less trouble, find and show those only which represent the points t, S 1, i, y, &c, of the contours themselves. Thus, say that at any given station-stake, Fig 4, the level is 104; that the cross-section $c\ s$ of the ground has been prepared as before; and that we want the hor dists from the stake, to contour lines for 94, 97, 100 ft, &c, 3 ft apart vert.

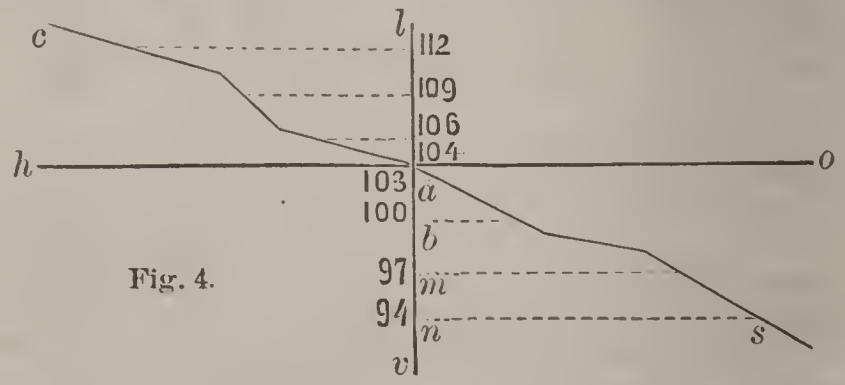


Fig. 4.

Draw a vert line $v\ l$, through the station-stake, and on it by scale mark levels of 94, 97, 100, &c. ft. This is readily done, inasmuch as we have the level 104 of the stake already given. Through these levels draw the hor lines a, b, m, n, &c. to the ground-slopes. Then these lines, measured by the scale, plainly give the required dists.

When the ground is very irregular transversely, the cross-sections must be taken in the field nearer together than 100 ft. The preparation of contour lines will be greatly facilitated by the use of paper ruled into small squares of not less than about $\frac{1}{12}$ inch to a side, for drawing the cross-sections upon.

When the ground is very steep, it is usual to shade such portions of the map to represent hill-side. The closer together the contours come, the steeper of course is the ground between them; and the shading should be proportionally darker at such portions. But for *working* maps it is best to omit the shading.

In surveys of wide districts, the transit instrument with a graduated vertical circle or arc, g, p. 188, is used for measuring the angles of slope, instead of the common slope-instrument.*

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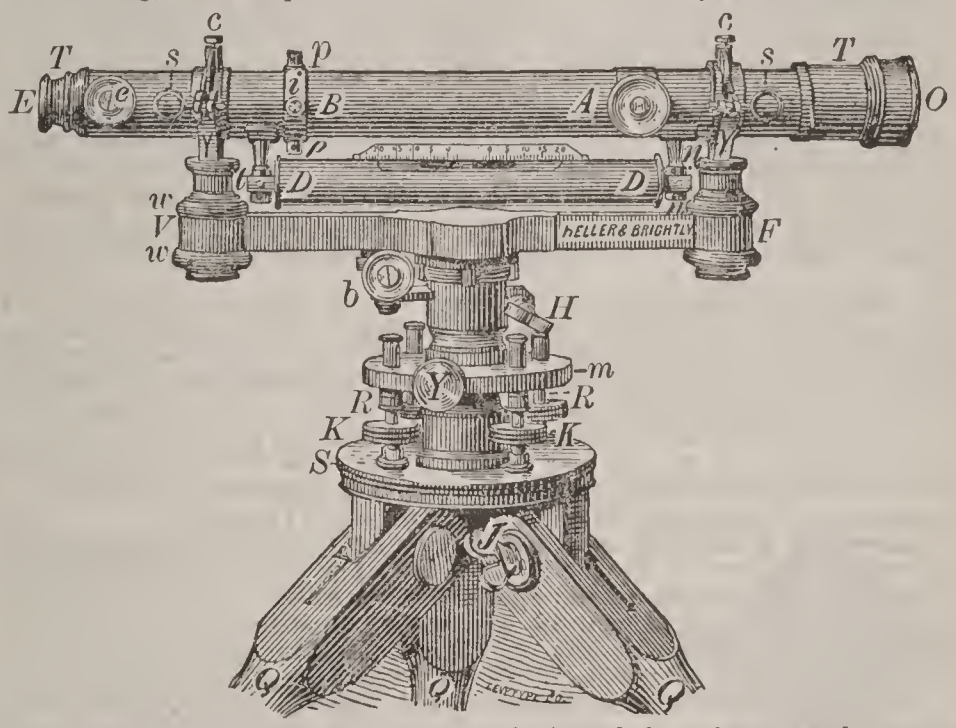
* The preparing of contour lines is a slow and tedious office-work; and the writer considers them of but little value in many cases; as when they are taken for only about 100 ft on each side of the line, with reference to slight changes of direction. He conceives that, ordinarily, every useful purpose is fulfilled if the leveller or the topographer enters into his field-book at each station, notes similar to the following:

Sta 60.....	— 3. 1 R.	+ 2. 1 L.
61.....	+ 2. 2 R.	— 1. 3 L.
62.....	= 1. R.	+ 4. 2 L.
63.....	"	"

Which means that at station 60, the slope of the ground on the right, as nearly as he can judge by eye, or by his hand-level, is about 3 ft downward, for 1 chain, or 100 ft; and on the left, about 2 ft upward in 1 chain. At 61, 2 ft. up, in 2 chains to the right; and 1 ft down in 3 chains to the left. At 62, level for 1 chain to the right; and ascending 4 ft in 2 chains to the left. At 63, the same as at 62. At some spots it will be well to add a sketch of a cross-section, like Fig 2; only, instead of the angles, use ft of rise or fall, to indicate the slopes, as judged by eye, or by a hand-level. By this method, the result at every station will be somewhat in error; but these small errors will balance each other so nearly that the total may be regarded as sufficiently correct for all the purposes of a preliminary estimate of the cost of a road. When the final stakes for guiding the workmen are placed, the slopes should be carefully taken, in order to calculate the quantity of excavation accurately for payment.

THE LEVEL.

ALTHOUGH the levels of different makers vary somewhat in their details, still their principal parts will be understood from the following figure.* The telescope T T rests upon two supports YY, called Ys; out of which it can be lifted, first removing the pins *s s* which confine the semicircular *clips c c*, and then opening the clips. The pins should be tied to the Ys, by pieces of string, to prevent their being lost. The *slide* of the *object-glass* O, is moved backward or forward by a rack and pinion, by means of the milled head A. The slide of the *eye-glass* E, is moved in the same way by the milled head *e*. A cylindrical tube of brass, called a *shade*, is usually furnished with each level. It is intended to be slid on to the object-end O of the telescope, to prevent the glare of the sun upon the object-glass, when the sun is low. At B is an outer ring encircling the telescope, and carrying 4 small capstan-headed screws; two of which, *p p*, are at top and bottom; while the other two, of which *i* is one, are at the sides, and at right angles to *p p*. Inside of this outer ring is another, inside of the telescope, and which has stretched across it two spider-webs, usually called the CROSS-HAIRS. These are much finer than they appear to be, being considerably magnified. They are at right angles to each other; and, in levelling, one is kept vert, and the other hor. They are liable at times to be



thrown out of this position by a partial revolution of the telescope, when carrying the level, or when setting the tripod down suddenly upon the ground; but since, in levelling, the *intersection* of the hairs is directed to the target-rod, this derangement does not affect the accuracy of the work. Still it is well to keep them nearly vert and hor, by keeping the BUBBLE-TUBE D D as nearly directly over the bar V F as can be judged by eye. This enables the leveller to see that the rod-man holds his rod nearly vert, which is absolutely essential for correct levelling. If perfect verticality is desired, as is sometimes the case, when staking out work, it may be obtained (*if the instrument is in perfect adjustment, and levelled*) by sighting at a plumb-line, or other vert object, and then turning the telescope a little in its Ys, so as to bring the hair to correspond. When this is done, a short continuous scratch may be made on the telescope and Y, to save that trouble in future. Heller & Brightly, however, provide their levels with a small projection inside of the Ys, and a corresponding stop on the telescope, the contact of which insures the verticality of the hair. Should the hairs be broken by accident, they may be replaced as directed hereafter.

The small holes around the heads of the 4 small capstan-screws *p*, *i*, just referred to, are for admitting the end of a small steel pin, or lever, for turning them. If first the upper screw *p* be loosened, and then the lower one tightened, the interior ring will be lowered, and the horizontal hair with it. But on looking through the tele-

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*__The price of a first-class level,__ by Heller & Brightly, is $145. It is bad economy to buy inferior instruments.

scope they will *appear* to be raised. If first the lower one be loosened, and the upper one tightened, the hor hair will be actually raised, but apparently lowered. This is because the glasses in the eye-piece E reverse the apparent position of objects *inside* of the telescope; which effect is obviated, as regards *exterior* objects, by means of the object-glass O. This must be remembered when adjusting the cross-hairs; for if a hair appears to strike too high, it must be raised still higher; if it appears to be already too far to the right or left, it must be actually moved still more in the same direction.

This remark, however, does not apply to telescopes which make objects appear inverted.

There is no danger of injuring the hairs by these motions, inasmuch as the four screws act against the ring only, and do not come in contact with the hairs themselves.

Under the telescope is the BUBBLE-TUBE D D. One end of this tube can be raised or lowered slightly by means of the two capstan-headed nuts *n n*, one of which must be loosened before the other is tightened. On top of the bubble-tube are scratches for showing when the bubble is central in the tube. Frequently these scratches, or marks, are made on a strip of brass placed above the tube, as in our fig. There are several of them, to allow for the lengthening or shortening of the bubble by changes of temperature. At the other end of the bubble-tube are two small capstan-screws, placed on opposite sides horizontally. The circular head of one of them is shown near *t*. By means of these two screws, that end of the tube can be slightly moved hor, or to right or left. Under the bubble-tube is the BAR V F; at one end of which, as at V, are two large capstan-nuts *w w*, which operate upon a stout interior screw which forms a prolongation of the Y. The holes in these nuts are larger than the others, as they require a larger lever for turning them. If the lower nut is loosened and the upper one tightened, the Y above is raised; and that end of the telescope becomes farther removed from the bar; and vice versa. Some makers place a similar screw and nuts under both Ys; while others dispense with the nuts entirely, and substitute beneath one end of the bar a large circular milled head, to be turned by the fingers. This, however, is exposed to accidental alteration, which should be avoided.

When the portions above *m* are put upon *m*, and fastened by the screw Y, all the upper part may be swung round hor, in either direction, by loosening the **clamp-screw** H; or such motion may be prevented by tightening that screw. It frequently happens, after the telescope has been sighted very nearly upon an object, and then clamped by H, that we wish to bring the cross-hairs to coincide more precisely with the object than we can readily do by turning the telescope *by hand;* and in this case we use the **tangent-screw** *b*, by means of which a slight but steady motion may be given after the instrument is clamped. For fuller remarks on the clamp and tangent-screws, see "Transit."

The **parallel plates** *m* and S are operated by four **levelling-screws;** three of which are seen in the figure, at K K. The screws work in sockets R; which, as well as the screws, extend above the upper plate. When the instrument is placed on the ground for levelling, it is well to set it so that the lower parallel plate S shall be as nearly horizontal as can be roughly judged by eye; in order to avoid much turning of the levelling screws K K in making the upper plate *m* hor. The lower plate S, and the brass parts below it, are together called the **tripod-head;** and, in connection with three wooden legs Q Q Q, constitute the tripod. In the figure are seen the heads of wing-nuts J which confine the legs to the tripod-head. Under the center of the tripod-head should always be placed a small ring, from which a plumb-bob may be suspended. This is not needed in ordinary levelling, but becomes useful when ranging center-stakes, &c.

To adjust a Level.

This is a quite simple operation, but requires a little patience. Be careful to avoid *straining* any of the screws. The large Y nuts *w w* sometimes require some force to *start* them; but it should be applied by pressure, and not by blows. Before beginning to adjust, attend to the object-glass, as directed in the first sentence under "To adjust a plain transit," p. 191.

Three adjustments are necessary; *and must be made in the following order:*

First, that of the cross-hairs: to secure that their intersection shall continue to strike the same point of a distant object, while the telescope is being turned round a complete revolution in its Ys. This is called adjusting the **line of collimation,** or sometimes, the *line of sight;* but it is not strictly the line of sight until all the adjustments are finished; for until then, the line of collimation will not serve for taking levelling sights. **If cross-hairs break,** see p 193.

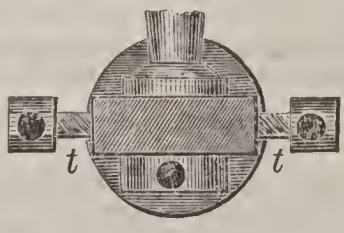
Second, that of the bubble-tube D D, to place it parallel to the line

of collimation. previously adjusted; so that when the bubble stands at the centre of its tube, indicating that it is level, we know that our sight through the telescope is hor. To replace **broken bubble tube,** see p 193.

Third, that of the Ys, by which the telescope and bubble-tube are supported; so that the bubble-tube, and line of sight, shall be perp to the vert axis of the instrument; so as to *remain* hor while the telescope is pointed to objects in diff directions, as when taking back and fore sights.

To make the first adjustment, or that of the cross-hairs, plant the tripod *firmly* upon the ground. In this adjustment it is not necessary to level the instrument. Open the clips of the Ys; unclamp; draw out the eye-glass E, until the cross-hairs are seen *perfectly clear;* sight the telescope toward some clear distant point of an object; or still better, toward some straight line, whether vert or not. Move the object-glass O, by means of the milled head A, so that the object shall be clearly seen, **without parallax,** that is, without any apparent dancing about of the cross-hairs, if the eye is moved a little up or down or sideways. To secure this, the object-glass alone is moved to suit different distances; the eye-glass is not to be changed after it is once properly fixed upon the cross-hairs. The neglect of parallax is a source of frequent errors in levelling. Clamp; and, by means of the tangent-screw *b,* bring either one of the cross-hairs to coincide *precisely* with the object. Then gently, and without jarring, revolve the telescope half-way round in its Ys. When this is done, if the hair still coincides precisely with the object, it is in adjustment; and we proceed to try the other hair. But if it does not coincide, then by means of the 4 screws *p, i,* move the ring which carries the hairs, so as to rectify, as nearly as can be judged by eye, only *one-half* of the error; remembering that the ring must be moved in the direction opposite to what *appears* to be the right one; unless the telescope is an inverting one. Then turn the telescope back again to its former position; and again by the tangent-screw bring the cross-hair to coincide with the object. Then again turn the telescope half-way round as before. The hair will now be found to be more nearly in its right place, but, in all probability, not precisely so; inasmuch as it is difficult to estimate one-half the error accurately by eye. Therefore a little more alteration of the ring must be made; and it may be necessary to repeat the operation several times, before the adjustment is perfect. Afterward treat the other hair in precisely the same manner. When both are adjusted, their intersection will strike the same precise spot while the telescope is being turned *entirely* round in its Ys. This must be tried before the adjustment can be pronounced perfect; because at times the adjustment of the second hair, slightly deranges that of the first one; especially if both were much *out* in the beginning.

To make the second adjustment, or to place the bubble-tube parallel to the line of collimation. This consists of two distinct adjustments, one vert, and one hor. The first of these is effected by means of the two nuts *n n* on the vert screw at one end of the tube; and the second by the two hor screws at the other end, *t,* of the tube. Looking at the bubble-tube endwise, from *t* in the foregoing Fig, its two hor adjusting-screws *t t* are seen as in this sketch. The larger capstan-headed nut *below,* has nothing to do with the adjustments; it merely holds the end of the tube in its place.



To make the vert adjustment of the bubble-tube, by means of the two nuts *nn.* Place the telescope over a diagonal pair of the levelling-screws K K; and clamp it there. Open the clips of the Ys; and by means of the levelling-screws bring the bubble to the center of its tube. Lift the telescope gently out of the Ys, turn it end for end, and put it back again in its reversed position. This being done, if the bubble still remains at the center of its tube, this adjustment is in order; but if it moves toward one end, that end is too high, and must be lowered; or else the other end must be raised. First, correct *half* the error by means of the levelling-screws K K, and then the remaining half by means of the two small capstan-headed nuts *n n.* To *raise* the end *n,* first loosen the upper nut and then tighten the lower one; to do which, turn each nut so that the *near* side moves toward your *right.* To *lower* it, first loosen the lower nut, then tighten the upper one, moving the *near* side of each nut toward your *left.* Having thus brought the bubble to the middle again, again lift the telescope out of its Ys; turn it end for end, and replace it. The bubble will now settle nearer the center than it did before, but will probably require still further adjustment. If so, correct *half* the remaining error by the levelling-screws, and half by the nuts, as before; and so continue to repeat the operation until the bubble remains at the center in both positions. For another method, see "To adjust the long bubble-tube," p 192.

Horizontal adjustment of bubble tube; to see that its axis is in the same plane with that of the telescope, as it usually is in new instruments. It is not easily de-

ranged, except by blows. Have the bubble-tube, as nearly as may be, directly under the telescope, or over the center of the bar V F. Bring the telescope over two of the levelling-screws K K; clamp it there; center the bubble with said screws; turn the telescope in its Ys, say about ¼ inch, bringing the bubble-tube out from over the center of the bar, first on one side, then on the other. If the bubble stays centered while so swung out, this adjustment is correct. If it runs toward *opposite* ends of its tube when swung out on opposite sides of the center, move the end *t* of the tube by the two horizontal screws *t t* until the bubble stays centered when the tube is swung out on either side. If the bubble runs toward the *same* end of its tube *on both sides*, the tube is not truly cylindrical, but slightly conical,* so that if the telescope is turned in its Ys the bubble will leave the center, even when the horizontal adjustment is correct. It is known to be correct, in such tubes, if the bubble runs *the same distance* from the center when swung out the same distance on each side.

Having made the horizontal adjustment, turn the telescope back in its Ys until the bubble-tube is over the bar. Repeat the *vertical* adjustment (p 203), which may have become deranged in making this horizontal one. Persevere until both adjustments are found to be correct at the same time.

To make the third adjustment. or to adjust the heights of the Ys, so as to make the line of collimation parallel to the bar V F, or perp to the vert axis of the instrument. The other adjustments being made, fasten down the clips of the Ys. Make the instrument nearly level by means of all four of the levelling-screws K. Place the telescope over two of the levelling-screws which stand diagonally; and leave it there unclamped. Then bring the bubble to the center of its tube, by the two levelling-screws. Swing the upper part of the instrument half-way around, so that the telescope shall again stand over the same two screws; but end for end. This done, if the bubble leaves the center, bring it *half-way* back by the large capstan nuts *w*, *w*; and the other half by the two levelling-screws. Remember that to raise the Y, and the end of the bubble over *w*, *w*, the lower *w* must be loosened; and the upper one tightened; and vice versa. Now place the telescope over the other diagonal pair of levelling-screws: and repeat the whole operation with them. Having completed it, again try with the first pair; and so keep on until the bubble remains at the center of its tube, in every position of the telescope.

Correct levelling may be performed even if all the foregoing adjustments are out of order; provided each fore-sight be taken *at precisely the same distance from the instrument as the back-sight is.* But a good leveller will keep his instrument always in adjustment; and will test the adjustments at least once a day when at work. As much, however, depends upon the rodman, or target-man, as upon the leveller. A rodman who is careless about holding the rod vert, or about reading the sights correctly, should be discharged without mercy.

The levelling-screws in many instruments become very hard to turn if dirty. Clean with water and a tooth-brush. Use no oil on field instruments.

Forms for level note-books. When the distance is short, so as not to require two sets of books, the following is perhaps as good as any.

No. of Station.	Back sights	Fore sights.	Diff.	Level.	Grade.	Cut.	Fill.

But on public works generally the original field-books have only the first five co's After the grades have been determined by means of the profile drawn from these, the results are placed in another book, which has only the first col and the last four. In both cases, the right-hand page is reserved for memoranda. The writer considers it best, both with the level and with the transit, to consider the term "STATION" to apply to the whole dist between two consecutive stakes; and that its number shall be that written on the last stake. Thus, with the transit, Station 6 means the dist from stake 5 to stake 6; that it has a bearing or course of so and so; and its length is so and so. And with the level, Station 6 also means the dist from stake 5 to stake 6; the back-sight for that dist being taken at stake 5, and the fore-sight on stake 6; and that the level, grade, cut, or fill is that at stake 6. The starting-point of the survey, whether a stake, or any thing else, we call and mark simply 0.

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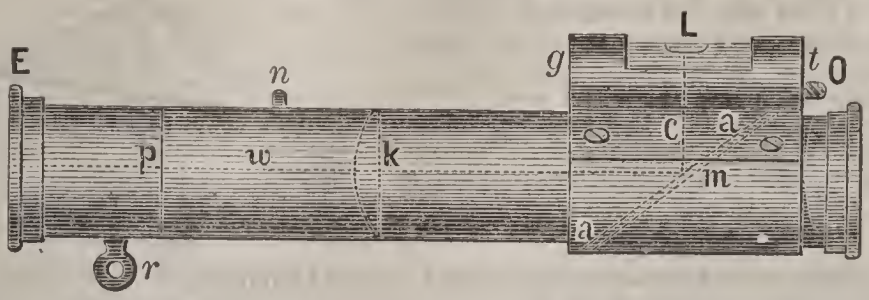
* This defect can be remedied only by removing the tube and inserting a correctly-shaped one, and this is best done by an instrument-maker; but correct work can be done in spite of it, thus: Make all the adjustments as nearly correct as possible. Level the instrument. By turning the telescope in its Ys, make the vertical hair coincide with a plumb-line or other vertical line, and make a short continuous knife-scratch on the collar nearest the object-glass, and on the adjoining Y. Lift the telescope out of its Ys, turn it end for end, replace it in its Ys; again bring the upright hair vertical, and make on the other Y a scratch coinciding with that on the collar. Then, in levelling or in adjusting, always see that the scratch on the collar coincides with that on the adjoining Y when the bubble-tube is under the telescope.

THE HAND-LEVEL.

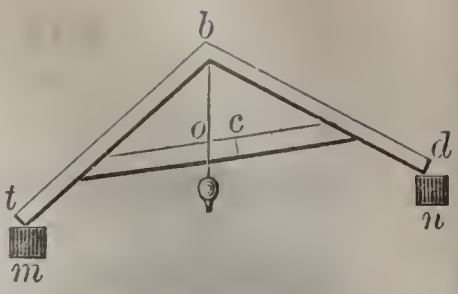


THIS very useful little instrument, as arranged by Professor Locke, of Cincinnati, is but about five or six inches long. Simply holding it in one hand, and looking through it in any direction, we can ascertain at once, approximately, what objects are at the same level with the eye. E is the eye end: and O the object end. L is a small level, enclosed in a kind of brass boxing *t g*, the bottom of which is open, with a corresponding opening under it, through the top of the main tube E O. Immediately at the bottom of the small level L, is a cross-wire, stretched across said opening, and carried by a small plate, which, for adjusting the wire, can be pushed backward a trifle by tightening the screw *t*, or pushed forward by a small spring within the boxing, near *g*, when the screw *t* is loosened. At *m* is a small semicircular mirror *a a*, silvered on the back *m*. This is placed at an angle of 45°, and occupies one-half the width of the tube E O. Through the forementioned openings, the images of the cross-wire and of the level-bubble are reflected down on the unsilvered face *a a* of the mirror, and thence to the eye, as shown by the single dotted lines *c* and *w*: and when the instrument is adjusted, and held level, the wire will appear to be at the center of the bubble. At *k* is one-half of a plano-convex lens, at the inner end of a short tube *k p*, which may be moved backward or forward by a pin *n*, projecting through a short slit in the main tube. By this means the image of the cross-wire is rendered distinct; and the half lens must be moved until, when viewing an object, the wire shall show no parallax; but appear steady against the object when the eye is slightly moved up or down. At each end of the tube E O is a circular piece of plain glass for excluding dust.

To adjust the hand-level, first fix two precisely level marks, say from 50 feet to 100 yards apart. This being done, rest the instrument against one of the level marks, and take sight at the other. If, then, the wire does not appear to be precisely at the center of the bubble, move it slightly backward or forward, as the case may be, by the screw *t*, until it does so appear. The two level marks may be fixed by means of the hand-level itself, even if it is entirely out of adjustment, thus: First, by the pin *n* arrange the half lens *k*, so as to show the wire distinctly and without parallax. Then holding the level steadily, at any selected object, as *a*, so that the wire appears to cut the center of the bubble, see where it cuts any other convenient object, as *b*. Then go to *b*, and from it, in like manner, sight back toward *a*. If the instrument is in adjustment, the wire will cut *a*; but if not, it will strike either above it or below it, as at *c*. In either case, make a mark *m*, half-way between *c* and *a*. Then *b* and *m* will be the two level marks required. With care, these adjustments, when once made, will remain in order for years. The instrument generally has a small ring *r*, for hanging it around the neck: it is not adapted to very accurate work, but admirably so for exploring a route. The height of a bare hill can be found by beginning at the foot, and sighting ahead at any little chance object which the cross-wire may strike, as a pebble, twig, &c; then going forward, stand at that object, and fix the wire on another one still farther on, and so to the top. At each observation we plainly rise a height equal to that of the eye, say 5¼ feet, or whatever it may be. Whether going up or down it, if the hill is covered with grass, bushes, &c, a target rod must be used for the fore-sights; and the constant height of the eye may be regarded as the back-sight at each station. An attachment may be made for screwing the level to a small ball and socket on top of a cane, or of a longer stick, for occasional use, when rather more accuracy is desired.*

* Price, about $10 or $12; with about $3 more for attachable ball and socket.

To adjust a builder's plumb-level, *t b d*; stand it upon any two supports *m* and *n*, and mark where the plumb-line cuts at *o*. Then reverse it, placing the foot *t* upon *n*, and *d* upon *m*, and mark where the line now cuts at *c*. Half-way between *o* and *c* make the permanent mark. Whenever the line cuts this, the feet *t* and *d* are on a level.



To adjust a slope-instrument, or clinometer. As usually made, the bubble-tube is attached to the movable bar by a screw near each end, and the head of one of the screws conceals a small slot in the bar, which allows a slight vert motion to the screw when loose, and with it to that end of the tube. Therefore, in order to adjust the bubble, this screw is first loosened a little, and then moved up or down a trifle, as may be reqd. It is then tightened again.

LEVELLING BY THE BAROMETER.

1. Many circumstances combine to render the results of this kind of levelling unreliable where great accuracy is required. This fact was most conclusively proved by the observations made by Captain T. J. Cram, of the U. S. Coast Survey. See Report of U. S. C. S., vol. for 1854. It is difficult to read off from an aneroid (the kind of barom generally employed for engineering purposes) to within from two to five or six ft, depending on its size. The moisture or dryness of the air affects the results; also winds, the vicinity of mountains, and the daily atmospheric tides, which cause incessant and irregular fluctuations in the barom. A barom hanging quietly in a room will often vary $\frac{1}{10}$ of an inch within a few hours, corresponding to a diff of elevation of nearly 100 ft. No formula can possibly be devised that shall embrace these sources of error. The variations dependent upon temperature, latitude, &c, are in some measure provided for; so that with *very delicate* instruments, a *skilful* observer may measure the diff of altitude of two points close together, such as the bottom and top of a steeple, with a tolerable confidence that he is within two or three feet of the truth. But if as short an interval as even a few hours elapses between his two observations, such changes may occur in the condition of the atmosphere that he *may* make the top of the steeple to be lower than its bottom; or at least, cannot feel by any means certain that he is not ten or twenty ft in error; and this may occur without any *perceptible* change in the atmosphere. Whenever practicable, therefore, there should be a person at each station, to observe at both points at the same time. Single observations at points many miles apart, and made on different days, and in different states of the atmosphere, are of little value. In such cases the mean of many observations, extending over several days, weeks, or months, and made when the air is apparently undisturbed, will give tolerable approximations to the truth. In the tropics the range of the atmospheric pres is much less than in other regions, seldom exceeding $\frac{1}{2}$ inch at any one spot; also more regular in time, and, therefore, less productive of error. Still, the barometer, especially either the aneroid, or Bourdon's metallic, may be rendered highly useful to the civil engineer, in cases where great accuracy is not demanded. By hurrying from point to point, and especially by repeating, he can form a judgment as to which of two summits is the lowest. Or a careful observer, keeping some miles ahead of a surveying party, may materially lessen their labors, especially in a rough country, by selecting the general route for them in advance. The accounts of the agreement within a few inches, in the measurements of high mountains, by diff observers, at diff periods; and those of ascertaining accurately the grades of a railroad, by means of an aneroid, while riding in a car, will be believed by those only who are ignorant of the subject. Such results can happen only by chance.

When possible, the observations at different places should be taken at the same time of day, as some check upon the effects of the daily atmospheric tides; and in very important cases, a memorandum should be made of the year, month, day, and hour, as well as of the state of the weather, direction of the wind, latitude of the place, &c, to be referred to an expert, if necessary.

The effects of latitude are not included in any of our formulas. When reqd they may be found in the table page 209. Several other corrections must be made when great accuracy is aimed at; but they require extensive tables.

In rapid railroad exploring, however, such refinements may be neglected, inasmuch as no approach to such accuracy is to be expected; but on the contrary, errors of from 1 to 10 or more feet in 100 of height, will frequently occur.

As a very rough average we may assume that the barometer falls $\frac{1}{10}$ inch for every 90 feet that we ascend above the level of the sea, up to 1000 ft. But in fact its rate of fall decreases continually as we rise; so that at one mile high it falls $\frac{1}{10}$ inch for about 106 ft rise. Table 2 shows the true rate.

To ascertain the diff of height between two points.

Rule 1. Take readings of the barom and therm (Fah) **in the shade** at both stations. Add together the two readings of the barom, and div their sum by 2, for their mean; which call *b*. Do the same with the two readings of the thermom, and call the mean *t*. Subtract the least reading of the barom from the greatest; and call the diff *d*. Then mult together this diff *d*; the number from the next Table No. 1, opposite *t*; and the constant number 30. Div the prod by *b*. Or

$$\text{Height in feet} = \frac{\text{Diff } (d) \text{ of barom} \times \text{Tabular number opposite mean } (t) \text{ of thermom} \times \text{Constant } 30.}{\text{mean } (b) \text{ of barom.}}$$

Example. Reading of the barom at lower station, 26.64 ins; and at the upper sta 20.82 ins. Thermom at lowest sta, 70°; at upper sta, 40°. What is the diff in height of the two stations? Here,

Barom, 26.64		Therm, 70°
" 20.82	Also,	" 40°
2)47.46		2)110
23.73 mean of bar, or *b*.		55° mean of therm, or *t*.

The tabular number opposite 55°, is 917.2.

Again, 26.64 (Bar.) — 20.82 (Bar.) = 5.82, diff of bar; or *d*. Hence,

$$\text{Height in feet} = \frac{d,\ 5.82 \times \text{Tab No. } 917.2 \times \text{Con. } 30}{23.73 \text{ (or } b)} = \frac{160143.12}{23.73} = 6748.5 \text{ ft; answer.}$$

Then correct for latitude, if more accuracy is reqd, by rule on next page.

The screw at the back of an aneroid is for adjusting the index by a standard barom. After this has been done it must by no means be meddled with. In some instruments specially made to order with that intention, this screw may be used also for turning the index back, after having risen to an elevation so great that the index has reached the extreme limit of the graduated arc. After thus turning it back, the indications of the index at greater heights must be added to that attained when it was turned back.

TABLE 1. For Rule 1.

Mean of Ther.	No.	Mean of Ther.	No.	Mean of Ther.	No.	Mean of Ther.	No.
0°	801.1	30°	864.4	60°	927.7	90°	991.0
1	803.2	31	866.5	61	929.8	91	993.1
2	805.3	32	868.6	62	931.9	92	995.2
3	807.4	33	870.7	63	934.0	93	997.3
4	809.5	34	872.8	64	936.1	94	999.4
5	811.7	35	874.9	65	938.2	95	1001.6
6	813.8	36	877.0	66	940.3	96	1003.7
7	815.9	37	879.2	67	942.4	97	1005.8
8	818.0	38	881.3	68	944.5	98	1007.9
9	820.1	39	883.4	69	946.7	99	1010.0
10	822.2	40	885.4	70	948.8	100	1012.1
11	824.3	41	887.5	71	950.9	101	1014.2
12	826.4	42	889.6	72	953.0	102	1016.3
13	828.5	43	891.7	73	955.1	103	1018.4
14	830.6	44	893.8	74	957.2	104	1020.5
15	832.8	45	896.0	75	959.3	105	1022.7
16	834.9	46	898.1	76	961.4	106	1024.8
17	837.0	47	900.2	77	963.5	107	1026.9
18	839.1	48	902.3	78	965.6	108	1029.0
19	841.2	49	904.5	79	967.7	109	1031.1
20	843.3	50	906.6	80	969.9	110	1033.2
21	845.4	51	908.7	81	972.0	111	1035.3
22	847.5	52	910.8	82	974.1	112	1037.4
23	849.6	53	913.0	83	976.2	113	1039.5
24	851.8	54	915.1	84	978.3	114	1041.6
25	853.9	55	917.2	85	980.4	115	1043.8
26	856.0	56	919.3	86	982.6	116	1045.9
27	858.1	57	921.4	87	984.7	117	1048.0
28	860.2	58	923.5	88	986.8	118	1050.1
29	862.3	59	925.6	89	988.9	119	1052.2

Rule 2. **Belville's short approx rule** is the one best adapted to rapid field use, namely, add together the two readings of the barom only. Also find the diff between said two readings; then, **as the sum of the two readings is to their diff, so is 55000 feet to the reqd altitude.**

Ex. Same as before. Here the sum = 26.64 + 20.82 = 47.46 ins; and the diff = 26.64 — 20.82 = 5.82 ins.

Sum. Diff. Feet. Feet.
Hence, as 47.46 : 5.82 : : 55000 : 6744.6 reqd alt; instead of the 6748.5 of Rule 1.

Correction for latitude. The lat of the place affects the result to some extent; but is usually omitted when great accuracy is not reqd. To apply the correction, first find the altitude by the rule, as before. Then divide it by the number in the following table opp the lat of the place. *Add* the quotient to the alt if the lat is *less* than 45°; or *subtract* it if the lat is more than 45°. If the two places are in diff lats, use their mean.

Table of corrections for latitude.

Lat.		Lat.		Lat.		Lat.		Lat.		Lat.	
0°	352	14°	399	28°	630	42°	3367	54°	1140	68°	490
2	354	16	416	30	705	44	10101	56	941	70	460
4	356	18	436	32	804	45	0	58	804	72	436
6	360	20	460	34	941	46	10101	60	705	74	416
8	367	22	490	36	1140	48	3367	62	630	76	399
10	375	24	527	38	1458	50	2028	64	572	78	386
12	386	26	572	40	2028	52	1458	66	527	80	375

Levelling by Barometer; or by the boiling point.

Rule 3. The following table, No. 2, enables us to measure heights either by means of boiling water, or by the barom. The third column shows the approximate altitude above sea-level corresponding to diff heights, or readings of the barom; and to the diff degrees of Fahrenheit's thermom, at which water boils in the open air. Thus when the barom, under undisturbed conditions of the atmosphere, stands at 24.08 inches, or when pure rain or distilled water boils at the temp of 201° Fah; the place is about 5764 ft above the level of the sea, as shown by the table. It is therefore very easy to find the *diff* of altitude of two places. Thus: take out from table No 2, the altitudes opposite to the two boiling temperatures; or to the two barom readings. Subtract the one opposite the lower reading, from that opposite the upper reading. The rem will be the reqd height, as a rough approximation. To correct this, add together the two therm readings; and div the sum by 2, for their mean. From table for temperature, p 211, take out the number opposite this mean. Mult the approximate height just found, by this tabular number. Then correct for lat if reqd.

Ex. The same as preceding; namely, barom at lower sta, 26.64; and at upper sta, 20.82. Thermom at lower sta, 70° Fah; and at the upper one, 40°. What is the diff of height of the two stations?

Alt.
Here the tabular altitudes are, for 20.82..... 9579
and for 26.64..... 3115

6464 ft, approx height.

To correct this, we have $\frac{70° + 40°}{2} = \frac{110°}{2} = 55°$ mean; and in table p 211, opp to 55°, we find 1.048. Therefore 6464 × 1.048 = 6774 ft, the reqd height.

This is about 26 ft more than by Rule 1; or nearly .4 of a ft in each 100 ft.

At 70° Fah, pure water will boil at 1° less of temp, for an average of about 550 ft of elevation above sea-level, up to a height of ½ a mile. At the height of 1 mile, 1° of boiling temp will correspond to about 560 ft of elevation. In table p 210 the mean of the temps at the two stations is assumed to be 32° Fah: at which no correction for temp is necessary in using the table; hence the tabular number opposite 32°, in table p 211, is 1.

This diff produced in the temp of the *boiling point*, by change of elevation, must not be confounded with that of the *atmosphere*, due to the same cause. The air becomes cooler as we ascend above sea-level, at the rate (very roughly) of about 1° Fah for every 200 ft near sea-level, to 350 ft at the height of 1 mile. See "Air," p 215.

The following table, No. 2, (so far as it relates to the barom.) was deduced by the writer from the standard work on the barom by Lieut.-Col. R. S. Williamson, U. S. army.*

* Published by permission of Government in 1868 by Van Nostrand, N. Y.

TABLE 2.

Levelling by Barometer; or by the boiling point.

Assumed temp in the shade 32° Fah. If not 32°, mult barom alt as per Table, p 211.

Boil point in deg Fah.	Barom. Ins.	Altitude above sea level Feet.	Boil point in deg Fah.	Barom. Ins.	Altitude above sea level Feet.	Boil point in deg Fah.	Barom. Ins.	Altitude above sea level Feet.	Boil point in deg Fah.	Barom. Ins.	Altitude above sea level Feet.
181°	16.79	15221	.3	19.66	11083	.6	22.93	7048	.9	26 59	3164
.1	16.83	15159	.4	19.70	11029	.7	22.98	6991	206	26·64	3115
.2	16.86	15112	.5	19.74	10976	.8	23.02	6945	.1	26 69	3066
.3	16.90	15050	.6	19 78	10923	.9	23.07	6888	.2	26.75	3007
.4	16 93	15003	.7	19.82	10870	199	23.11	6843	.3	26·80	2958
.5	16 97	14941	.8	19.87	10804	.1	23.16	6786	.4	26·86	2899
.6	17.00	14895	.9	19.92	10738	.2	23.21	6729	.5	26.91	2850
.7	17 04	14833	192	19.96	10685	.3	23.26	6673	.6	26.97	2792
.8	17.08	14772	.1	20.00	10633	.4	23.31	6617	.7	27.02	2743
.9	17.12	14710	.2	20.05	10567	.5	23.36	6560	.8	27.08	2685
185	17.16	14649	.3	20.10	10502	.6	23.40	6516	.9	27.13	2637
.1	17.20	14588	.4	20.14	10450	.7	23 45	6460	207	27.18	2589
.2	17.23	14543	.5	20.18	10398	.8	23.49	6415	.1	27.23	2540
.3	17.27	14482	.6	20.22	10346	.9	23 54	6359	.2	27.29	2483
.4	17.31	14421	.7	20.27	10281	200	23 59	6304	.3	27.34	2435
.5	17.35	14361	.8	20.31	10230	.1	23 64	6248	.4	27.40	2377
.6	17.38	14315	.9	20.35	10178	.2	23.69	6193	.5	27.45	2329
.7	17.42	14255	193	20.39	10127	.3	23.74	6137	.6	27.51	2272
.8	17.46	14195	.1	20.43	10075	.4	23.79	6082	.7	27.56	2224
.9	17.50	14135	.2	20.48	10011	.5	23 84	6027	.8	27.62	2167
186	17.54	14075	.3	20.53	9947	.6	23.89	5972	.9	27.67	2120
.1	17.58	14015	.4	20.57	9896	.7	23.94	5917	208	27.73	2063
.2	17.62	13956	.5	20.61	9845	.8	23.98	5874	.1	27.78	2016
.3	17.66	13896	.6	20.65	9794	.9	24.03	5819	.2	27.84	1959
.4	17.70	13837	.7	20.69	9743	201	24.08	5764	.3	27.89	1912
.5	17.74	13778	.8	20.73	9693	.1	24.13	5710	.4	27.95	1856
.6	17.78	13718	.9	20.77	9642	.2	24.18	5656	.5	28.00	1809
.7	17.82	13660	194	20.82	9579	.3	24.23	5602	.6	28.06	1753
.8	17.86	13601	.1	20.87	9516	.4	24.28	5547	.7	28.11	1706
.9	17.90	13542	.2	20.91	9466	.5	24.33	5494	.8	28.17	1650
187	17.93	13498	.3	20.96	9403	.6	24.38	5440	.9	28.23	1595
.1	17.97	13440	.4	21.00	9353	.7	24.43	5386	209	28.29	1539
.2	18.00	13396	.5	21.05	9291	.8	24.48	5332	.1	28.35	1483
.3	18.04	13338	.6	21.09	9241	.9	24.53	5279	.2	28.40	1437
.4	18.08	13280	.7	21.14	9179	202	24.58	5225	.3	28.45	1391
.5	18.12	13222	.8	21.18	9130	.1	24.63	5172	.4	28.51	1336
.6	18 16	13164	.9	21.22	9080	.2	24.68	5119	.5	28.56	1290
.7	18.20	13106	195	21.26	9031	.3	24.73	5066	.6	28.62	1235
.8	18.24	13049	.1	21.31	8969	.4	24.78	5013	.7	28.67	1189
.9	18.28	12991	.2	21.35	8920	.5	24.83	4960	.8	28.73	1134
188	18.32	12934	.3	21.40	8859	.6	24.88	4907	.9	28.79	1079
.1	18.36	12877	.4	21.44	8810	.7	24.93	4855	210	28.85	1025
.2	18.40	12820	.5	21.49	8749	.8	24.98	4802	.1	28.91	970
.3	18.44	12763	.6	21.53	8700	.9	25.03	4750	.2	28.97	916
.4	18.48	12706	.7	21.58	8639	203	25.08	4697	.3	29.03	862
.5	18.52	12649	.8	21.62	8590	.1	25.13	4645	.4	29.09	808
.6	18.56	12593	.9	21.67	8530	.2	25.18	4593	.5	29.15	754
.7	18.60	12536	196	21.71	8481	.3	25.23	4541	.6	29.20	709
.8	18.64	12480	.1	21.76	8421	.4	25.28	4489	.7	29.25	664
.9	18.68	12424	.2	21.81	8361	.5	25.33	4437	.8	29.31	610
189	18.72	12367	.3	21.86	8301	.6	25.38	4386	.9	29.36	565
.1	18.76	12311	.4	21.90	8253	.7	25.43	4334	211	29.42	512
.2	18.80	12256	.5	21.95	8193	.8	25.49	4272	.1	29.48	458
.3	18.84	12200	.6	21 99	8145	.9	25.54	4221	.2	29.54	405
.4	18.88	12144	.7	22.04	8086	204	25.59	4169	.3	29.60	352
.5	18.92	12089	.8	22.08	8038	.1	25.64	4118	.4	29.65	308
.6	18.96	12033	.9	22.13	7979	.2	25.70	4057	.5	29.71	255
.7	19.00	11978	197	22.17	7932	.3	25.76	3996	.6	29.77	202
.8	19.04	11923	.1	22.22	7873	.4	25.81	3945	.7	29 83	149
.9	19.08	11868	.2	22.27	7814	.5	25.86	3894	.8	29.88	105
190	19.13	11799	.3	22.32	7755	.6	25.91	3844	.9	29.94	52
.1	19.17	11745	.4	22.36	7708	.7	25.96	3793	212	30.00	sea lev=0
.2	19.21	11690	.5	22.41	7649	.8	26.01	3742	Below sea level.		
.3	19.25	11635	.6	22.45	7602	.9	26.06	3692	.1	30.06	— 52
.4	19.29	11581	.7	22.50	7544	205	26.11	3642	.2	30.12	—104
.5	19.33	11527	.8	22.54	7498	.1	26.17	3582	.3	30.18	—156
.6	19.37	11472	.9	22.59	7439	.2	26.22	3532	.4	30.24	—209
.7	19.41	11418	198	22.64	7381	.3	26.28	3472	.5	30.30	—261
.8	19.45	11364	.1	22.69	7324	.4	26.33	3422	.6	30.35	—304
.9	19.49	11310	.2	22.74	7266	.5	26.38	3372	.7	30.41	—356
191	19.54	11243	.3	22.79	7208	.6	26.43	3322	.8	30.47	—408
.1	19.58	11190	.4	22.84	7151	.7	26.48	3273	.9	30.53	—459
.2	19.62	11136	.5	22.89	7093	.8	26.54	3213	213	30.59	—511

Corrections for temperature; to be used in connection with Rule 3, when greater accuracy is necessary. Also in connection with Table 2 when the temp is not 32°.

Mean temp in the shade.	Mult by	Mean temp in the shade.	Mult by	Mean temp in the shade.	Mult by	Mean temp in the shade.	Mult by
Zero.	.933	28°	.992	56°	1.050	84°	1.108
2°	.937	30	.996	58	1.054	86	1.112
4	.942	32	1.000	60	1.058	88	1.117
6	.946	34	1.004	62	1.062	90	1.121
8	.950	36	1.008	64	1.066	92	1.125
10	.954	38	1.012	66	1.071	94	1.129
12	.958	40	1.016	68	1.075	96	1.133
14	.962	42	1.020	70	1.079	98	1.138
16	.967	44	1.024	72	1.083	100	1.142
18	.971	46	1.028	74	1.087	102	1.146
20	.975	48	1.032	76	1.091	104	1.150
22	.979	50	1.036	78	1.096	106	1.154
24	.983	52	1.041	80	1.100	108	1.158
26	.987	54	1.046	82	1.104	110	1.163

SOUND.

Its velocity, in quiet open air, has been experimentally determined to be very approximately 1090 ft per sec, when the temp is at freezing point, or 32° Fah; and that it increases about 1¼ ft per sec for every degree above 32°, or, by some authorities, 1 ft for every 2°. The first would make it

1090 ft at 32°	1150 ft at 80°
1100 " 40°	1162½ " 90°
1112½ " 50°	1175 " 100°
1125 " 60°	1187½ " 110°
1137½ " 70°	1200 " 120°

At 32° sound would travel a mile in 4.84 sec; at 80°, in 4.6; and at 120°, in 4.4 sec.

If the air is calm, fogs, or rain do not appreciably affect the result; but winds do. There is some reason to believe that very loud sounds travel somewhat faster than low ones. The watchword of sentinels has been heard across still water, on a calm night, 10½ miles; and a cannon 20 miles. Separate sounds, at intervals of $\frac{1}{16}$ of a sec, cannot be distinguished, but appear to be connected. The dists at which a speaker can be understood, in front, on one side, and behind him, are about as 4, 3, and 1.

Dr. Charles M. Cresson informs the writer that, by repeated trials, he found that in a Philadelphia gas main 20 ins diam. and 16000 ft long, laid and covered in the earth, but empty of gas and having one horizontal bend of 90°, and of 40 ft radius, the sound of a pistol-shot travelled 16000 ft in precisely 16 sec, or 1000 ft per sec. The arrival of the sound was barely audible; but was rendered very apparent to the eye by its blowing off a diaphragm of tissue-paper placed over the end of the main.

Two boats anchored some dist apart may serve as a base line for triangulating objects along the coast; the dist between them being first found by firing guns on board one of them.

In water the vel is about 4708 ft per sec, or about 4 times that in air. **In woods,** it is from 10 to 16 times; and **in metals,** from 4 to 16 times greater than in air, according to some authorities.

Approximate expansion of solids by heat: and their melting points by Fahrenheit's thermometer.‡

	For 1 degree.		For 180 degrees.*		Melting point in Deg.‡
	1 part in	⅛ inch in	1 part in	⅛ inch in	
Fire-brick	365220	3804 ft.	2029	21.14 ft.	
Granite { from	187560	1954	1042	10.85	
Granite { to	228060	2375	1267	13.20	
Glass rod	221400	2306	1230	12.81	
Glass tube	214200	2231	1190	12.40	
" crown	211500	2203	1175	12.24	
" plate	209700	2184	1165	12.13	
Platina	208800	2175	1160	12.08	4593
Marble, granular, white, dry	173000	1802	961	10.00	
" " " moist.	128000	1333	711	7.41	
" " black, compact	405000	4219	2250	23 44	
Antimony	166500	1722	925	9.63	955
Cast iron	162000‖	1688	900	9.38	1920 to
Slate	173000	1802	961	10.00	2800
Steel	151200	1575	840	8.75	2370 to
" blistered	159840	1665	888	9.25	2550
" untempered	167400	1744	930	9.69	
" tempered yellow	131400	1369	730	7.60	
" hardened	146880	1530	816	8.50	
" annealed	147600	1537	820	8.54	
Iron, rolled	149940‖	1562	833	8.68	3000 to
" soft, forged	147420	1536	819	8.53	3500
" wire	146340	1524	813	8 55	
Bismuth	129600	1350	720	7.50	506
Gold, annealed	123120	1282	684	7.12	2016
Copper average	104400	1088	580	6.04	2000
Sandstone†	103320	1076	574	5 98	
Brass average	97740	1018	543	5.66	1873
" wire	94140	981	523	5.45	
Silver	95040	990	528	5.50	1861
Tin average	87840	915	488	5.08	444
Lead average	63180	658	351	3.66	612
Pewter	78840	821	438	4.56	
Zinc (most of all metals)	61920	645	344	3.58	680 to 772
White pine	440530	4588	2447	25.49	

Heat of a common wood-fire, variously estimated from 800 to 1140 deg. That of a **charcoal** one about 2200°; **coal** about 2400°.

Each 12° to 15° of heat produce in **wrot iron** an expansion equal to that produced by a tension of about 1 ton per sq inch of section; varying with the quality of the iron.

For temperature, expansion, conducting power, &c, of **air,** see p 215.

* By adding $\frac{1}{10}$ part to the lengths in the two cols under 180°, we get the lengths corresponding to a number of degrees $\frac{1}{11}$ less than 180°; or to 163°.63 deg, which may be taken as about the *extremes* of temp in the colder portions of the United States. In the Middle States the extremes rarely reach 135°, or ¼ part less than 180°.

No dependence whatever is to be placed on results obtained by Wedgewood's pyrometer.

† The table shows that the contraction and **expansion of stone** will cause open joints in winter; and crushing of the mortar in summer, at the ends of long coping-stones.

‡ **The melting points are quite uncertain.** We give the mean of the best authorities. Assuming that with a change of temp of about 163°, wrought iron will alter its length 1 part in 916; this in a mile amounts to 5.764 ft, or about 5 ft 9⅛ ins, and in 100 ft to .109 of a foot; or 1⅜ ins; so that a diff of 5 ft, or more, can readily result from measuring a mile in winter and in summer with the same chain; and a 25 ft rail will change its length full ⅜ of an inch.

‖ Hence **wrought expands** by heat about one eleventh part more than cast; whereas under tension within elastic limits cast stretches twice as much as wrt.

THERMOMETERS.

Below about 35° below zero of Fah. the mercurial thermometer and barometer become too irregular to be depended on. **Mercury begins to freeze** at about −40° Fah. Below −40° pure alcohol is used.

To change degrees of Fahrenheit to the corresponding degrees of Centigrade; take a Fah reading 32° *lower* than the given one: mult this lower reading by 5; divide the prod by 9. Thus: +14° Fah = (14 −32) × 5 ÷ 9 = −18 × 5 ÷ 9 = −10° Cent. Again, −13° Fah = (−13 −32) × 5 ÷ 9 = −45 × 5 ÷ 9 = −25° Cent.

To change Fah to Réau; take a Fah reading 32° *lower* than the given one; mult this lower reading by 4; div the prod by 9. Thus: +14° Fah = (14 −32) × 4 ÷ 9 = −18 × 4 ÷ 9 = −8° Réau. Again, −13° Fah = (−13 −32) × 4 ÷ 9 = −45 × 4 ÷ 9 = −20° Réau.

To change Cent to Fah; mult the Cent reading by 9; divide by 5. Take a Fah reading 32° *higher* than the quot. Thus: +10° Cent = (10 × 9 ÷ 5) +32 = 18 +32 = +50° Fah. Again, −20° Cent = (−20 × 9 ÷ 5) +32 = −4° Fah.

To change Cent to Réau; mult by 4; div by 5. Thus: +10° Cent = 10 × 4 ÷ 5 = +8° Réau. Again, −10° Cent = −10 × 4 ÷ 5 = −8° Réau.

To change Réau to Fah; mult by 9; div by 4. Take a Fah reading 32° *higher*. Thus: +16° Réau = (16 × 9 ÷ 4) + 32 = 36 + 32 = +68° Fah. Again, −8° Réau = (−8 × 9 ÷ 4) +32 = −18 +32 = +14° Fah.

To change Réau to Cent; mult by 5; div by 4. Thus: +8° Réau = +8° × 5 ÷ 4 = +10° Cent. Again, −8° Réau = −8 × 5 ÷ 4 = −10° Cent.

TABLE 1. Fahrenheit compared with Centigrade and Réaumur. In *this* table the Cent and Réau readings are given to the nearest decimal.

F.	C.	R.	F.	C.	R.	F.	C.	R.	F.	C.	R.	F.	C.	R.
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
212	100	80.0	158	70.0	56.0	104	40 0	32.0	50	10.0	8.0	−3	−19.4	−15.6
211	99.4	79.6	157	69.4	55.6	103	39.4	31.6	49	9.4	7.6	−4	−20.0	−16.0
210	98.9	79.1	156	68.9	55.1	102	38.9	31.1	48	8.9	7.1	−5	−20.6	−16.4
209	98 3	78.7	155	68 3	54.7	101	38 3	30.7	47	8.3	6.7	−6	−21.1	−16.9
208	97.8	78.2	154	67.8	54.2	100	37 8	30 2	46	7.8	6.2	−7	−21.7	−17.3
207	97.2	77.8	153	67.2	53.8	99	37.2	29.8	45	7.2	5.8	−8	−22.2	−17.8
206	96.7	77.3	152	66.7	53.3	98	36.7	29.3	44	6.7	5.3	−9	−22.8	−18.2
205	96.1	76.9	151	66.1	52.9	97	36.1	28.9	43	6.1	4.9	−10	−23.3	−18.7
204	95.6	76.4	150	65.6	52.4	96	35.6	28.4	42	5.6	4.4	−11	−23.9	−19.1
203	95.0	76 0	149	65.0	52.0	95	35.0	28.0	41	5.0	4.0	−12	−24.4	−19.6
202	94.4	75.6	148	64.4	51.6	94	34.4	27.6	40	4.4	3.6	−13	−25 0	−20.0
201	93.9	75.1	147	63.9	51.1	93	33.9	27.1	39	3.9	3.1	−14	−25.6	−20 4
200	93.3	74.7	146	63.3	50.7	92	33.3	26.7	38	3.3	2.7	−15	−26.1	−20.9
199	92.8	74.2	145	62.8	50.2	91	32.8	26.2	37	2.8	2.2	−16	−26.7	−21.3
198	92.2	73.8	144	62 2	49.8	90	32.2	25.8	36	2.2	1.8	−17	−27.2	−21.8
197	91.7	73.3	143	61.7	49.3	89	31.7	25.3	35	1.7	1.3	−18	−27.8	−22 2
196	91.1	72.9	142	61.1	48.9	88	31.1	24.9	34	1.1	0.9	−19	−28.3	−22.7
195	90.6	72.4	141	60.6	48.4	87	30.6	24.4	33	0.6	0.4	−20	−28.9	−23.1
194	90.0	72.0	140	60.0	48.0	86	30.0	24.0	32	0.0	0.0	−21	−29.4	−23.6
193	89.4	71.6	139	59.4	47.6	85	29.4	23.6	31	−0.6	−0.4	−22	−30.0	−24.0
192	88.9	71.1	138	58.9	47.1	84	28.9	23.1	30	−1.1	−0.9	−23	−30.6	−24.4
191	88.3	70.7	137	58.3	46.7	83	28.3	22.7	29	−1.7	−1.3	−24	−31.1	−24 9
190	87 8	70.2	136	57.8	46.2	82	27.8	22.2	28	−2.2	−1.8	−25	−31.7	−25.3
189	87.2	69.8	135	57.2	45.8	81	27.2	21.8	27	−2.8	−2.2	−26	−32.2	−25 8
188	86.7	69.3	134	56.7	45.3	80	26.7	21.3	26	−3.3	−2.7	−27	−32.8	−26.2
187	86 1	68.9	133	56.1	44.9	79	26.1	20.9	25	−3.9	−3.1	−28	−33.3	−26.7
186	85.6	68.4	132	55.6	44.4	78	25.6	20.4	24	−4.4	−3.6	−29	−33.9	−27.1
185	85.0	68.0	131	55.0	44.0	77	25.0	20.0	23	−5.0	−4.0	−30	−34.4	−27.6
184	84.4	67.6	130	54.4	43.6	76	24.4	19 6	22	−5.6	−4.4	−31	−35.0	−28.0
183	83.9	67.1	129	53.9	43.1	75	23.9	19.1	21	−6.1	−4.9	−32	−35.6	−28.4
182	83.3	66.7	128	53.3	42.7	74	23.3	18.7	20	−6.7	−5.3	−33	−36.1	−28.9
181	82.8	66.2	127	52.8	42.2	73	22.8	18.2	19	−7.2	−5.8	−34	−36.7	−29 3
180	82.2	65.8	126	52.2	41.8	72	22 2	17.8	18	−7.8	−6.2	−35	−37.2	−29.8
179	81.7	65.3	125	51.7	41.3	71	21.7	17.3	17	−8.3	−6.7	−36	−37.8	−30.2
178	81.1	64.9	124	51.1	40.9	70	21.1	16.9	16	−8.9	−7.1	−37	−38.3	−30.7
177	80.6	64.4	123	50.6	40.4	69	20.6	16.4	15	−9.4	−7.6	−38	−38.9	−31.1
176	80.0	64.0	122	50.0	40.0	68	20.0	16.0	14	−10·0	−8.0	−39	−39.4	−31.6
175	79.4	63.6	121	49.4	39.6	67	19.4	15 6	13	−10.6	−8.4	−40	−40.0	−32.0
174	78.9	63.1	120	48.9	39.1	66	18.9	15.1	12	−11.1	−8.9	−41	−40.6	−32.4
173	78.3	62.7	119	48.3	38.7	65	18.3	14.7	11	−11.7	−9.3	−42	−41.1	−32.9
172	77.8	62.2	118	47.8	38.2	64	17.8	14.2	10	−12.2	−9.8	−43	−41.7	−33.3
171	77.2	61.8	117	47.2	37.8	63	17.2	13.8	9	−12.8	−10.2	−44	−42.2	33.8
170	76.7	61.3	116	46.7	37.3	62	16.7	13.3	8	−13.3	−10.7	−45	−42.8	−34 2
169	76.1	60.9	115	46.1	36.9	61	16.1	12.9	7	−13.9	−11.1	−46	−43.3	−34.7
168	75.6	60 4	114	45 6	36.4	60	15.6	12 4	6	−14.4	−11.6	−47	−43.9	−35.1
167	75.0	60.0	113	45.0	36.0	59	15.0	12.0	5	−15 0	−12.0	−48	−44.4	−35.6
166	74.4	59.6	112	44.4	35 6	58	14.4	11.6	4	−15.6	−12.4	−49	−45.0	−36.0
165	73 9	59.1	111	43.9	35.1	57	13.9	11.1	3	−16.1	−12.9	−50	−45 6	−36.4
164	73.3	58.7	110	43.3	34.7	56	13.3	10.7	2	−16.7	−13.3	−51	−46.1	−36.9
163	72.8	58.2	109	42 8	34.2	55	12.8	10.2	1	−17.2	−13.8	−52	−46.7	−37.3
162	72.2	57.8	108	42.2	33 8	54	12.2	9.8	0	−17.8	−14.2	−53	−47.2	−37.8
161	71.7	57.3	107	41.7	33.3	53	11.7	9.3	−1	−18.3	−14.7	−54	−47.8	−38.2
160	71.1	56.9	106	41.1	32.9	52	11.1	8.9	−2	−18.9	−15.1	−55	−48.3	−38.7
159	70.6	56.4	105	40.6	32.4	51	10.6	8.4						

TABLE 2. Centigrade compared with Fahrenheit and Réaumur.

C.	F.	R.	C.	F.	R.	C.	F.	R.	C.	F.	R.
°	° Exact.	° Exact.	°	° Exact.	° Exact.	°	° Exact.	° Exact.	°	° Exact.	° Exact.
100	212.0	80.0	62	143.6	49.6	24	75.2	19.2	—14	6.8	—11.2
99	210.2	79.2	61	141.8	48.8	23	73.4	18.4	—15	5.0	—12.0
98	208.4	78.4	60	140.0	48.0	22	71.6	17.6	—16	3.2	—12.8
97	206.6	77.6	59	138.2	47.2	21	69 8	16.8	—17	1.4	—13.6
96	204 8	76 8	58	136.4	46.4	20	68.0	16.0	—18	—0.4	—14.4
95	203.0	76.0	57	134.6	45.6	19	66.2	15.2	—19	—2.2	—15.2
94	201.2	75.2	56	132.8	44.8	18	64.4	14.4	—20	—4.0	—16.0
93	199.4	74 4	55	131.0	44.0	17	62.6	13.6	—21	—5.8	—16.8
92	197.6	73.6	54	129.2	43 2	16	60.8	12.8	—22	—7.6	—17.6
91	195.8	72.8	53	127.4	42.4	15	59 0	12.0	—23	—9.4	- 18.4
90	194.0	72.0	52	125.6	41.6	14	57.2	11.2	—24	—11.2	—19.2
89	192.2	71.2	51	123.8	40.8	13	55.4	10.4	—25	—13.0	—20.0
88	190.4	70.4	50	122.0	40.0	12	53.6	9.6	—26	—14.8	—20 8
87	188.6	69.6	49	120.2	39.2	11	51.8	8.8	—27	—16.6	—21.6
86	186.8	68 8	48	118.4	38.4	10	50.0	8.0	—28	—18.4	—22.4
85	185.0	68.0	47	116.6	37.6	9	48.2	7.2	—29	—20.2	—23.2
84	183.2	67.2	46	114.8	36.8	8	46.4	6.4	—30	—22.0	—24.0
83	181.4	66.4	45	113.0	36.0	7	44 6	5.6	—31	—23.8	—24.8
82	179.6	65.6	44	111.2	35.2	6	42.8	4.8	—32	—25.6	—25 6
81	177 8	64 8	43	109.4	34.4	5	41.0	4.0	—33	—27.4	—26 4
80	176.0	64.0	42	107 6	33.6	4	39.2	3.2	—34	—29.2	—27.2
79	174.2	63.2	41	105.8	32.8	3	37.4	2.4	—35	—31.0	—28.0
78	172.4	62.4	40	104.0	32.0	2	35.6	1.6	—36	—32.8	—28.8
77	170 6	61.6	39	102.2	31.2	1	33.8	0.8	—37	—34 6	—29.6
76	168.8	60.8	38	100.4	30.4	0	32.0	0.0	—38	—36.4	—30.4
75	167.0	60.0	37	98.6	29.6	—1	30.2	—0.8	—39	—38.2	—31.2
74	165.2	59.2	36	96.8	28.8	—2	28.4	—1.6	—40	—40.0	—32.0
73	163.4	58.4	35	95.0	28.0	—3	26.6	—2.4	—41	—41.8	—32.8
72	161 6	57.6	34	93.2	27.2	—4	24.8	—3.2	—42	—43.6	—33.6
71	159 8	56.8	33	91.4	26.4	—5	23.0	—4.0	—43	—45.4	—34.4
70	158.0	56.0	32	89.6	25.6	—6	21.2	—4.8	—44	—47.2	—35.2
69	156.2	55.2	31	87.8	24 8	—7	19.4	—5.6	—45	—49.0	—36.0
68	154.4	54.4	30	86.0	24.0	—8	17.6	—6.4	—46	—50.8	—36.8
67	152.6	53.6	29	84.2	23.2	—9	15.8	—7.2	—47	—52.6	—37.6
66	150.8	52 8	28	82.4	22.4	—10	14.0	—8.0	—48	—54.4	—38.4
65	149.0	52.0	27	80.6	21.6	—11	12.2	—8.8	—49	—56.2	—39.2
64	147 2	51.2	26	78.8	20.8	—12	10.4	—9.6	—50	—58.0	—40 0
63	145.4	50.4	25	77.0	20.0	—13	8.6	—10.4			

TABLE 3. Réaumur compared with Fahrenheit and Centigrade.

R.	F.	C.	R.	F.	C.	R.	F.	C.	R.	F.	C.
°	° Exact.	° Exact.	°	° Exact.	° Exact.	°	° Exact.	° Exact.	°	° Exact.	° Exact.
80	212.00	100.00	49	142.25	61.25	19	74.75	23.75	—11	7.25	—13.75
79	209.75	98.75	48	140.00	60.00	18	72.50	22.50	—12	5.00	—15.00
78	207.50	97.50	47	137.75	58.75	17	70.25	21.25	—13	2.75	—16.25
77	205.25	96.25	46	135 50	57.50	16	68.00	20.00	—14	0.50	—17.50
76	203.00	95.00	45	133.25	56.25	15	65.75	18.75	—15	—1.75	—18.75
75	200.75	93.75	44	131.00	55 00	14	63.50	17.50	—16	—4.00	—20.00
74	198.50	92.50	43	128.75	53.75	13	61.25	16.25	—17	—6.25	—21.25
73	196 25	91.25	42	126.50	52 50	12	59 00	15.00	—18	—8.50	—22.50
72	194.00	90.00	41	124.25	51.25	11	56.75	13.75	—19	—10.75	—23.75
71	191.75	88.75	40	122.00	50.00	10	54 50	12.50	—20	—13.00	- 25.00
70	189.50	87.50	39	119 75	48.75	9	52.25	11.25	—21	—15.25	—26.25
69	187.25	86.25	38	117.50	47.50	8	50.00	10.00	—22	—17.50	—27.50
68	185.00	85 00	37	115.25	46.25	7	47.75	8.75	—23	—19.75	—28.75
67	182.75	83.75	36	113.00	45.00	6	45.50	7.50	—24	—22.00	—30.00
66	180.50	82.50	35	110.75	43.75	5	43.25	6.25	—25	—24.25	—31.25
65	178 25	81.25	34	108.50	42.50	4	41.00	5.00	—26	—26.50	—32.50
64	176.00	80 00	33	106.25	41.25	3	38.75	3.75	—27	—28.75	—33.75
63	173.75	78.75	32	104.00	40 00	2	36.50	2.50	- 28	—31.00	—35 00
62	171.50	77.50	31	101.75	38.75	1	34.25	1.25	—29	—33.25	—36.25
61	169.25	76.25	30	99.50	37 50	0	32.00	0.00	—30	—35.50	—37.50
60	167.00	75.00	29	97.25	36.25	—1	29.75	—1.25	—31	—37.75	—38.75
59	164 75	73.75	28	95.00	35.00	—2	27.50	—2.50	—32	—40.00	—40 00
58	162.50	72.50	27	92.75	33 75	—3	25.25	—3.75	—33	—42.25	—41.25
57	160 25	71.25	26	90.50	32 50	—4	23.00	- 5.00	—34	—44.50	—42.50
56	158.00	70.00	25	88.25	31.25	—5	20.75	—6.25	—35	—46.75	—43.75
55	155.75	68.75	24	86.00	30.00	—6	18 50	—7.50	—36	—49 00	—45 00
54	153.50	67.50	23	83.75	28.75	—7	16.25	—8.75	—37	—51.25	—46.25
53	151.25	66.25	22	81.50	27.50	- 8	14.00	—10.00	—38	—53.50	—47.50
52	149.00	65.00	21	79.25	26.25	—9	11.75	—11.25	—39	—55.75	—48 75
51	146.75	63.75	20	77.00	25.00	—10	9.50	—12 50	—40	—58.00	—50.00
50	144.50	62.50									

AIR.—ATMOSPHERE.

The atmosphere is known to extend to at least 45 miles above the earth. **Its composition** is about .79 measures of nitrogen gas and .21 of oxygen gas; or about .77 nitrogen, .23 oxygen, by weight. It generally contains, however, a trace of water; carbonic acid, and carburetted hydrogen gases; and still less ammonia.

When the barometer is at 30 inches, and the temperature 60° Fah, **air weighs** about one-815th part as much as water; or 535 grains = 1.224 commercial ounces = .0765 commercial lb. per cubic foot. Or 13.072 cubic feet weigh 1 lb; and a cubic yard, 2.066 lbs. Or a cube of 30.82 feet on each edge, 1 ton. When colder it weighs more per cubic foot, and vice versa, at the rate of about a grain per degree of Fah. The average weight of the entire atmospheric column, (at least 45 miles high,) at sea level, is 14¾ lbs avoirdupois per square inch; or 2124 lbs per square foot* = weight of a column of water 34 feet, or of mercury 30 inches, high. This is what is usually called the "pressure of the air." At ¼ mile above sea level it is but 14.02 lbs per square inch; at ½ mile, 13.33; at ¾ mile, 12.66; at 1 mile, 12.02; at 1¼ mile, 11.42; at 1½ mile, 10.88; and at 2 miles, 9.80 lbs. Therefore, a pump in a high region will not lift water to as great a height as in a low one. The pressure of air, like that of water, is, at any given point, equal in all directions.

It is often stated that the **temperature of the atmosphere** lowers or becomes colder, at the rate of 1° Fah for each 300 feet of ascent **above the earth's surface;** but this is liable to many exceptions, and varies much with local causes. Actual observation in balloons seems to show that up to the first 1000 feet, about 200 feet to 1°, is nearer the truth; at 2000 feet, 250; at 4000 feet, 300; and at a mile, 350.

In breathing, a grown person at rest requires from .25 to .35 of a cubic foot of air per minute; which, when breathed, vitiates from 3½ to 5 cubic feet. When walking or hard at work, he breathes and vitiates two or three times as much. About 5 cubic feet of fresh air per person per minute is required for the perfect ventilation of rooms in winter; 8 in summer. Hospitals 40 to 80.

Beneath the general level of the surface of the earth in temperate regions, a tolerably uniform **temperature** of about 50° to 60° Fah exists at the depth of about 50 to 60 feet; and increases about 1° for each additional 50 to 60 feet; all subject, however, to considerable deviations from many local causes. In the Rose Bridge colliery, England, at the depth of 2424 feet, the temperature of the coal is 93½° Fah; and at the bottom of a boring 4169 feet deep, near Berlin, the temperature is 119°.

The air is a very slow conductor of heat; hence hollow walls serve to retain the heat in dwellings; besides keeping them dry. **It rushes into a vacuum** near sea level with a velocity of about 1157 feet per second; or 13⅓ miles per minute; or about as fast as sound ordinarily travels through quiet air. See Sound, p 211.

Like all other elastic fluids, it expands equally with equal increases of temperature. Every increase of 5° Fah, expands the bulk of any of them slightly more than 1 per cent of that which it has at 0° Fah; or 500° about doubles its bulk at zero. The bulk of any of them diminishes inversely in proportion to the total **pressure** to which it is subjected. Thus, if we have a cylinder open at top, and 1 foot deep, full of air at its natural pressure of about 15 lbs per square inch; if by means of a piston we apply an additional pressure of 15 lbs per square inch, making 30 lbs in all, or twice as much as the natural pressure, then the air will be compressed into 6 inches of depth of the cylinder, or one-half of what it occupied before. Or if we apply 45 lbs additional, making 60 lbs in all, or 4 times the natural pressure, then the air will be compressed into ¼ of the depth of the cylinder. Experiment shows that this holds good with air at least up to pressures of about 750 lbs per square inch, or 50 times its natural pressure; the air in this case occupying one-fiftieth of its natural bulk. In like manner the bulk will increase as the total pressure is diminished; so that if we remove our additional 45 lbs per square inch, the air in the cylinder will regain its original bulk, and will precisely fill the cylinder. Substances which follow these laws, are said to be **perfectly elastic.** Under a pressure of about 5½ tons per square inch, air would become as dense, or would weigh as much per cubic foot, as water. Since the air at the surface of the earth is pressed 14¾ lbs per square inch by the atmosphere above it, and since this is equal to the weight of a column of water 1 inch square and 34 feet high, it follows that at the depths of 34, 68, 102 feet, &c, **below water,** air will be com-

* = 1.033 kilogrammes per square centimetre.

pressed into ½, ⅓, ¼, &c, of its bulk at the surface; because at those depths it is exposed to pressures equal to 2, 3, 4, &c, times 14¾ lbs per square inch, inasmuch as the pressure of the atmosphere on the surface, is in each case to be added to that of the water. The pressure of the water *alone* at those depths would be but 1, 2, 3, &c, times 14¾ lbs per square inch.

In a diving-bell, men, after some experience, can readily work for several hours at a depth of 51 feet; or under a pressure of 2½ atmospheres; or 37½ lbs per square inch. But at 90 feet deep; or under 3.64 atmospheres; or nearly 55 lbs per square inch, they can work for but about an hour, without serious suffering from paralysis; or even danger of death. Still at the St. Louis bridge some work was done at a depth of 110½ feet; pressure 63.7 lbs per square inch.

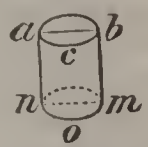
The dew point is that temperature (varying) at which the air deposits its vapor.

The greatest heat of the air in the sun probably never exceeds 145° Fah; nor the greatest cold —74° at night. About 130° above, and 40° below zero, are the extremes in the U. S. east of the Mississippi; and 65° below in the N. W.; all at common ground level. It is stated, however, that —81° has been observed in N. E. Siberia; and +101° Fah in the shade in Paris; and +153° in the sun at Greenwich Observatory, both in July, 1881. It has frequently exceeded +100° Fah in the shade in Philadelphia during recent years.

WIND.

The relation between the velocity of wind, and its pressure against an obstacle placed either at right angles to its course, or inclined to it, has not been well determined; and still less so its pressure against curved surfaces. The pressure against a large surface is probably proportionally greater than against a small one. It is generally supposed to vary nearly as the squares of the velocities; and when the obstacle is at right angles to its direction, the pressure in lbs per square foot of exposed surface is considered to be equal to the square of the velocity in miles per hour, divided by 200. On this basis, which is probably quite defective, the following table, as given by Smeaton, is prepared.

Vel. in Miles per Hour.	Vel. in Ft. per Sec.	Pres. in Lbs. per Sq. Ft.	Remarks.
1	1.467	.005	Hardly perceptible.
2	2.933	.020	Pleasant.
3	4.400	.045	
4	5.867	.080	
5	7.33	.125	
10	14.67	.5	
12½	18.33	.781	Fresh breeze.
15	22.	1.125	
20	29.33	2.	
25	36.67	3.125	Brisk wind.
30	44.	4.5	Strong wind.
40	58.67	8.	High wind.
50	73.33	12.5	Storm.
60	88.	18.	Violent storm.
80	117.3	32.	Hurricane.
100	146.7	50.	Violent hurricane, uprooting large trees.



The pres against a semicylindrical surface *a c b n o m* is about half that against the flat surf *a b n m*.

Tredgold recommends to allow 40 lbs per sq ft of roof for the pres of wind against it; but as roofs are constructed with a slope, and consequently do not receive the full force of the wind, this is plainly too much.* Moreover, only one-half of a roof is usually exposed, even thus partially, to the wind. Probably the force in such cases varies approximately as the sines of the angles of slopes. According to observations in Liverpool, in 1860, a wind of 38 miles per hour, produced a pres of 14 lbs per sq ft against an object perp to it; and one of 70 miles per hour, (the severest gale on record at that city,) 42 lbs per sq foot. These would make the pres per sq ft, more nearly equal to the square of the vel in miles per hour, div by 100; or nearly twice as great as given in Smeaton's table. We should ourselves give the preference to the Liverpool observations. A very violent gale in Scotland, registered by an excellent anemometer, or wind-gauge, 45 lbs per sq ft. It is stated that as high as 55 lbs has been observed at Glasgow. High winds often *lift* roofs.

The gauge at Girard College, Philada, broke under a strain of 42 lbs per sq ft; a tornado passing at the moment, within ¼ mile.

By inversion of Smeaton's rule, if the force in lbs per sq ft, be mult by 200, the sq rt of the prod will give the vel in miles per hour. Smeaton's rule is used by the U. S. Signal Service.

* The writer thinks 8 lbs per sq foot of *ordinary double-sloping roofs*, or 16 lbs for *shed-roofs*, sufficient allowance for pres of wind.

WATER.

PURE water, as boiled and distilled, **is composed of** the two gases, hydrogen and oxygen; in the proportions of 2 measures hydrogen to 1 of oxygen; or 1 weight of hydrogen to 8 of oxygen. Ordinarily, however, it contains several foreign ingredients, as carbonic and other acids; and soluble mineral, or organic substances. When it contains much lime, it is said to be *hard;* and will not make a good lather with soap. **The air** in its ordinary state **contains** about 4 grains of water per cubic foot.

The average **pressure of the air at sea level, will balance a column of water** 34 feet high; or about 30 inches of mercury. At its boiling point of 212° Fah, its bulk is about one twenty-third greater than at 70°.

Its weight per cubic foot is taken at 62½ lbs, or 1000 ounces avoir; but 62⅔ lbs would be nearer the truth, as per table below. It is about 815 times heavier than air, when both are at the temperature of 62°; and the barometer at 30 inches. With barometer at 30 inches the weight of perfectly pure water is as follows. At about 39° it has its maximum density of 62.425 lbs per cubic foot.

Temp, Fah.	Lbs per Cub Ft.	Temp, Fah.	Lbs per Cub Ft.
32°	62.417	70°	62.302
40°	62.423	80°	62.218
50°	62.409	90°	62.119
60°	62.367	212°	59.7

Weight of sea water 64.02 to 64.27 lbs per cubic foot, or say 1.6 to 1.9 lbs more than fresh.

Water has its **maximum density** when its temperature is a little above 39° Fah; or about 7° above the freezing point. By best authorities 39.2°. From about 39° it expands either by cold, or by heat. When the temperature of 32° reduces it to **ice,** its weight is but about 57.2 lbs. per cubic foot; and its specific gravity about .9175, according to the investigations of L. Dufour. Hence, as ice, it has expanded one-twelfth of its original bulk as water; and the **sudden expansive force** exerted at the moment of freezing, is sufficiently great to split iron water-pipes; being probably not less than 30000 lbs per square inch. Instances have occurred of its splitting cast tubular posts of iron bridges, and of ordinary buildings, when full of rain water from exposure. It also loosens and throws down masses of rock, through the joints of which rain or spring water has found its way. Retaining-walls also are sometimes overthrown, or at least bulged, by the freezing of water which has settled between their backs and the earth filling which they sustain; and walls which are not founded at a sufficient depth, are often lifted upward by the same process.

It is said that **in a glass tube ¼ inch in diameter,** water will not freeze until the temperature is reduced to 23°; and in tubes of less than $\frac{1}{200}$ inch, to 3° or 4°. Neither will it freeze until considerably colder than 32° in rapid running streams. **Anchor ice,** sometimes found at depths as great as 25 feet, consists of an aggregation of small crystals or needles of ice frozen at the surface of rapid open water; and probably carried below by the force of the stream. It does not form under frozen water. For **crushing strength of ice,** see page 437.

Since ice floats in water; and a floating body displaces a weight of the liquid equal to its own weight, it follows that a cubic foot of floating ice weighing 57.2 lbs, must displace 57.2 lbs of water. But 57.2 lbs of water, one foot square, is 11 inches deep: therefore, floating ice of a cubical or parallelopipedal shape, will have $\frac{1}{12}$ of its volume under water; and only $\frac{1}{12}$ above; and a square foot of ice of any thickness, will require a weight equal to $\frac{1}{11}$ of its own weight to sink it to the surface of the water. In practice, however, this must be regarded merely as a close approximation, since the weight of ice is somewhat affected by enclosed air-bubbles.

Pure water is usually assumed to **boil** at 212° Fah in the open air, at the level of the sea; the barometer being at 30 inches; and at about 1° less for every 520 feet above sea level, for heights within 1 mile. In fact, its boiling point varies like its freezing point, with its purity, the density of the air, the material of the vessel, &c. In a metallic vessel, it may boil at 210°; and in a glass one, at from 212° to 220°; and it is stated that if all air be previously extracted, it requires 275°. For **leveling by the boiling point,** see page 209.

It evaporates at all temperatures; **dissolves** more substances than any other agent: and has a greater capacity for heat than any other known substance.

It is compressed at the rate of about one-21740th, (or about $\frac{1}{1000}$ of an inch in $18\frac{1}{10}$ feet,) by each atmosphere or pressure of 15 lbs per square inch. When the pressure is removed, its elasticity restores its original bulk.

Effect on metals. The lime contained in many waters, forms deposits in metallic water-pipes, and in channels of earthenware, or of masonry; especially if the current be slow. Some other substances do the same; obstructing the flow of the water to such an extent, that it is always expedient to use pipes of diameters larger than would otherwise be necessary. The lime also forms very hard **incrustations at the bottoms of boilers;** very much impairing their efficiency; and rendering them more liable to burst. Such water is unfit for locomotives. We have seen it stated that the Southwestern R R Co, England, prevent this lime deposit, along their limestone sections, by dissolving 1 ounce of sal-ammoniac to 90 gallons of water. The salt of sea water forms similar deposits in boilers; as also does mud, and other impurities.

Water, either when very pure, as rain water; or when it contains carbonic acid, (as most water does,) **produces carbonate of lead in lead pipes;** and as this is an active poison, such pipes should not be used for such waters. Tinned lead pipes may be substituted for them. If, however, sulphate of lime also be present, as is very frequently the case, this effect is not always produced; and several other substances usually found in spring and river water, also diminish it to a greater or less degree. **Fresh water corrodes wrought iron more rapidly than cast;** but the reverse appears to be the case with **sea water;** although it also affects wrought iron very quickly; so that thick flakes may be detached from it with ease. The corrosion of iron or steel by sea water increases with the carbon. Cast-iron cannons from a vessel which had been sunk in the fresh water of the Delaware River for more than 40 years, were perfectly free from rust. Gen. Pasley, who had examined the metals found in the ships Royal George, and Edgar, the first of which had remained sunk in the sea for 62 years, and the last for 133 years, "stated that the cast iron had generally become quite soft; and in some cases resembled plumbago. Some of the shot when exposed to the air became hot; and burst into many pieces. The wrought iron was not so much injured, *except when in contact with copper, or brass gun-metal.* Neither of these last was much affected, except when in contact with iron. Some of the wrought iron was reworked by a blacksmith, and pronounced superior to modern iron." "Mr. Cottam stated that some of the guns had been carefully removed in their soft state, to the Tower of London; and in time (within 4 years) *resumed their original hardness.* Brass cannons from the Mary Rose, which had been sunk in the sea for 292 years, were considerably honeycombed in spots only; (perhaps where iron had been in contact with them.) The old cannons, of wrought-iron bars hooped together, were corroded about ¼ inch deep; but had probably been protected by mud. The cast-iron shot became redhot on exposure to the air; and fell to pieces like dry clay!"

"Unprotected parts of cast-iron sluice-valves, on the sea gates of the Caledonian canal, were converted into a soft plumbaginous substance, to a depth of ¾ of an inch, within 4 years; but where they had been coated with common Swedish tar, they were entirely uninjured. This softening effect on cast iron appears to be as rapid even when the water is but slightly brackish; and that only at intervals. It also takes place on cast iron imbedded in salt earth. Some water pipes thus laid near the Liverpool docks, at the expiration of 20 years were soft enough to be cut by a knife; while the same kind, on higher ground beyond the influence of the sea water, were as good as new at the end of 50 years."

Observation has, however, shown that **the rapidity of this action depends much on the quality of the iron:** that which is dark-colored, and contains much carbon mechanically combined with it, corrodes most rapidly: while hard white, or light-gray castings remain secure for a long time. Some cast-iron sea-piles of this character, showed no deterioration in 40 years. See note, page 645.

Contact with brass or copper is said to induce a galvanic action which greatly hastens decay in either fresh or salt water. Some muskets were recovered from a wreck which had been submerged in sea water for 70 years near New York. The brass parts were in perfect condition; but the iron parts had entirely disappeared. **Galvanizing** (coating with zinc) acts as a preservative to the iron, but at the expense of the zinc, which soon disappears. The iron then corrodes. If iron be well heated, and then coated with **hot coal-tar,** it will resist the action of either salt or fresh water for many years. It is very important that the tar be perfectly *purified.* See p 291. Such a coating, or one of paint, will not prevent **barnacles and other shells** from attaching themselves to the iron. Asphaltum, if pure, answers as well as coal-tar.

Copper and bronze are very little affected by sea water.

No galvanic action has been detected where brass ferules are inserted into the water-pipes in Philadelphia.

The most prejudicial exposure for iron, as well as for wood, is that to alternate wet and dry. At some dangerous spots in Long Island Sound, it has been the practice to drive round bars of rolled iron about 4 inches diameter, for supporting signals. These wear away most rapidly between high and low water; at the rate of about an inch in depth in 20 years; in which time the 4-inch bar becomes reduced to a 2-inch one. along that portion of it. Under fresh water especially, or under ground, a thin coating of such tar, so applied, (see preceding paragraph,) will protect iron, such as water-pipes, &c, for a long time. See page 291. The sulphuric acid contained in the water from coal mines corrodes iron pipes rapidly. **In the fresh water of canals,** iron boats have continued in service from 20 to 40 years. **Wood remains sound** for centuries under either fresh or salt water, if not exposed to be worn away by the action of currents; or to be destroyed by marine insects.

Sea water differs a little in weight, at different places; but at the same place it is appreciably the same at all depths; and may be generally assumed at about 64 lbs; or $1\frac{3}{4}$ lbs per cubic foot, more than fresh. The additional $1\frac{3}{4}$ lbs, or one 36.6th of its entire weight, is chiefly common salt. **Sea water freezes** at 27° Fah: the ice is fresh.

A teaspoonful of powdered alum, well stirred into a bucket of **dirty water,** will generally purify it sufficiently within a few hours to be drinkable. If a hole 3 or 4 feet deep be dug in the sand of the sea-shore, the infiltrating water will usually be sufficiently fresh for washing with soap; or even for drinking. It is also stated that water may be preserved sweet for many years by placing in the containing vessel 1 ounce of black oxide of manganese for each gallon of water.

It is said that water kept in zinc tanks; or flowing through iron tubes galvanized inside, rapidly becomes poisoned by soluble salts of zinc formed thereby; and it is recommended to coat zinc surfaces with asphalt varnish to prevent this. Yet, in the city of Hartford, Conn, service pipes of iron, galvanized inside and out, were adopted in 1855, at the recommendation of the water commissioners; and have been in use ever since. They are likewise used in Philadelphia and other cities to a considerable extent. In many hotels and other buildings in Boston, the "Seamless Drawn Brass Tube" of the American Tube Works at Boston, has for many years been in use for service pipe; and has given great satisfaction. It is stated that the softest water may be kept in brass vessels for years without any deleterious result.

The action of lead upon some waters (even pure ones) is highly poisonous. The subject, however, is a complicated one. An injurious ingredient may be attended by another which neutralizes its action. Organic matter, whether vegetable or animal, is injurious. Carbonic acid, when not in excess, is harmless. See near bottom of page 419.

Ice may be so impure that its water is dangerous to drink.

The popular notion that hot water freezes more quickly than cold, with air at the same temperature, is erroneous.

TIDES.

The tides are those well-known rises and falls of the surface of the sea and of some rivers, caused by the attraction of the sun and moon. There are two rises, floods, or high tides; and two falls, ebbs, or low tides, every 24 hours and 50 minutes (**a lunar day**); making the average of 6 hours $12\frac{1}{2}$ minutes between high and low water. These intervals are, however, **subject to great variations;** as are also the heights of the tides; and this not only at different places, but at the same place. These irregularities are owing to the shape of the coast line, the depth of water, winds, and other causes. *Usually* at new and full moon, or rather a day or two after, (or twice in each lunar month, at intervals of two weeks,) the tides rise higher, and fall lower than at other times; and these are called **spring tides.** Also, one or two days after the moon is in her *quarters*, twice in a lunar month, they both rise and fall less than at other times; and are then called **neap tides.** From neap to spring they rise and fall more daily; and vice versa. The **time of high water** at any place, is generally two or three hours after the moon has passed over either the upper or lower meridian; and is called the **establishment** of that place; because, when this time is established, the time of high water on any other day may be found from it in most cases. The total height of spring tides is *generally* from $1\frac{1}{2}$ to 2 times as great as that of neaps. The great **tidal wave** is merely an *undulation*, unattended by any current, or progressive motion of the particles of water. Each successive high tide occurs about 24 minutes later than the preceding one; and so with the low tides.

RAIN.

The quantity that falls annually in any one place, varies greatly from year to year; the extremes being frequently greater than 2 to 1. In making calculations for collecting water in reservoirs, whether for feeding canals, or for supplying cities, we cannot safely assume more than the minimum fall observed for many years; or rather, somewhat less. And from even this must be deducted the amount (a quite considerable one) lost by evaporation and leakage after it has been collected. The following table shows in some cases, the average annual falls; and in others, the least and the greatest ones observed at several places; including snow water. It is highly probable that most of the results are merely approximate. See Evaporation, p 222.

	Inches per an.
Augusta, Georgia	23
Albany, N. York	31 to 51
Arkansas	41
Bath, Maine	30 to 50
Baltimore, Md	40
Boston, Mass	25 to 46
Charleston, S. C	40 to 76!
Canada	36
Carlisle, Penna	34
Detroit, Michigan	30
Frankford, Penna	33 to 54
Fort Gaston, California, in 9 months.	129
Fort Yuma, Cal	3¼
Port (not Fort) Orford, Oregon	69
Fort Pike, Louisiana	72
Fort Pierce, E. Florida	63
Fort Conrad, New Mexico	6¾
Fort Kent, Maine	36½
Fort Preble, "	45¼
Fort Constitution, N. Hamp	35½
Fort Adams, Rhode Island	52½
Fort Hamilton, N. York Harbor	43⅝
Fort Niagara, N. Y	31¾
Fort Monroe, Virg	51
Fort Kearney, Nebraska	28
Fort Laramie, Nebraska	20
Fort Worth, Texas	41
Fort McIntosh, "	18¾
Fort Dallas, Oregon	14½
Key West, Florida	30 to 39
Lebanon, Penna	34 to 45
Michigan	35
Monterey, Cal	12¼
Marietta, Ohio	35 to 54
New Orleans, Louisiana	51
New Fane, Vermont	36 to 74!
New England, average	47
Natchez, Miss	37 to 58
New York State, average	36¼
Ohio "	36
Philadelphia, Penna	34 to 61
" av for 54 years, to 1884	45.2 *
Pennsylvania, average	41
Savannah, Georgia	30 to 60
Stow, Mass	33 to 49
St. Louis, Mo	42
Washington, D. C	41
West Chester, Penna	39 to 54
Williamstown, Mass	26 to 39

The greatest fall recorded in one day in Philadelphia, was 7.3 inches, Aug. 13, 1873. The greatest in any month, was 15.8 inches, Aug., 1867. In July, 1842, 6 inches fell in 2 hours. It has not reached 9 inches per month, more than 7 or 8 times, in 25 years. During a tremendous rain at Norristown, Pennsylvania, in 1865, the writer saw evidence that at least 9 inches fell within 5 hours. At Genoa, Italy, on one occasion, 32 inches fell in 24 hours; at Geneva, Switzerland, 6 inches in 3 hours; at Marseilles, France, 13 inches in 14 hours; in Chicago, Sept., 1878, .97 inch in 7 minutes.

Near London, England, the mean total fall for many years is 23 inches. On one occasion, 6 inches fell in 1½ hours! In the mountain districts of the English lakes, the fall is enormous; reaching in some years to 180 or 240 inches; or from 15 to 20 feet! while, in the adjacent neighborhood, it is but 40 to 60 inches. At Liverpool, the average is 34 inches; at Edinburgh, 30; Glasgow, 22; Ireland, 36; Madras. 47; Calcutta, 60; maximum for 16 years, 82; Delhi, 21; Gibraltar, 30; Adelaide, Australia, 23; West Indies, 36 to 96; Rome, 39. On the Khassya hills north of Calcutta, 500 inches, or 41 feet 8 inches, have fallen in the 6 rainy months! In other mountainous districts of India, annual falls of 10 to 20 feet are common.

It requires a quite heavy rain, for 24 hours, to yield the depth of an inch; still, inasmuch as at rare intervals falls of as much as from 1 to 3, or even 6 inches *per hour* occur, these latter depths must be considered in planning **sewers, culverts,** etc. See Art. 23, p 279*c*.

As a general rule, more rain falls in warm than in cold countries; and more in elevated regions than in low ones. Local peculiar-

* In 1869, during which occurred the greatest drought known in Philadelphia for at least 50 years, it was 48.84 inches.

ities, however, sometimes reverse this; and also cause great differences in the amounts in places quite near each other; as in the English lake districts just alluded to. It is sometimes difficult to account for these variations. In some lagoons in New Granada, South America, the writer has known three or four heavy rains to occur weekly for some months, during which not a drop fell on hills about 1000 feet high, within 10 miles' distance, and within full sight. At another locality, almost a dead-level plain, fully three-quarters of the rains that fell for 2 years, at a spot 2 miles from his residence, occurred in the morning; while those which fell about 3 miles from it, in an opposite direction, were in the afternoon.

"The returns of several rain gauges in the Longdendale district, England, for 1847, gave the rainfalls at different altitudes above the sea, as follows:"

At 500 feet altitude	.....	46.6 ins.		At 1750 feet altitude	.....	56.5 ins.	
800 "	.....	50.5 "		1800 "	.....	62.1 "	
1700 "	.....	52.1 "					

"The annual average fall at Edinburgh, 200 feet above the sea, in three successive years, was 30 inches. In the Pentland hills, a few miles south, and 700 feet above sea, 37.4 inches: and at Carlops, similarly situated near the last, but 900 feet above sea, 49.2 inches."

There are probably but few places in the United States, where an annual fall of 2 feet may not be safely relied on; and since, as an average, about half of it may be collected into reservoirs, **a square mile of drainage** (= 27878400 square feet) **should yield** annually 27878400 cubic feet; equal to 76379 cubic feet per day. Allowing 4 cubic feet, or 30 gallons, per day for each person; and making no deduction for evaporation and filtration, this would supply a population of 19095 persons; or a square of 38¼ feet on a side, would in like manner suffice for one person. From two-tenths to eight-tenths of all the water annually resulting from rain and snow, passes off into the neighboring rivulets; and thence into the larger streams and rivers; or may be collected into reservoirs. Under ordinary circumstances of locality, about one-half may usually be thus secured. The difference is owing chiefly to the distance which the water may have to run; the rates of absorption of various soils; the rate of descent of the sides of the valleys leading to the streams; the season of the year, &c, &c. See p 222.

An inch of rain amounts to 3630 cubic feet; or 27155 U. S. gallons; or 101.3 tons per acre; or to 2323200 cubic feet; or 17378743 U. S. gallons; or 64821 tons per square mile at 62½ lbs per cubic foot.

The most destructive rains are usually those which fall upon snow, under which the ground is frozen, so as not to absorb water.

SNOW.

Trials at different times by the writer, showed **the weight of freshly fallen snow** to vary from about 5 to 12 lbs per cubic foot; apparently depending chiefly upon the degree of humidity of the air through which it had passed. On one occasion when mingled snow and hail had fallen to the depth of 6 inches, he found its weight to be 31 lbs per cubic foot. It was very dry and incoherent. A cubic foot of heavy snow may, by a gentle sprinkling of water, be converted into about half a cubic foot of slush, weighing 20 lbs; which will not **slide or run off** from a shingled roof sloping 30°, if the weather is cold. A cubic block of snow saturated with water until it weighed 45 lbs per cubic foot, just slid on a rough board inclined at 45°; on a smoothly planed one at 30°; and on slate at 18°; all approximate. A prism of snow, saturated to 52 lbs per cubic foot; one inch square, and 4 inches high, **bore a weight** of 7 lbs; which at first compressed it about one-quarter part of its length. European engineers consider 6 lbs per square foot of roof, to be sufficient **allowance for the weight of snow;** and 8 lbs for the pressure of wind; total, 14 lbs. The writer thinks that in the U. S. the allowance for snow should not be taken at *less* than 12 lbs; or the total for snow and wind, at 20 lbs. There is no danger that snow on a roof will become saturated to the extent just alluded to; because a rain that would supply the necessary quantity of water, would also by its violence wash away the snow; but we entertain no doubt whatever that the united pressures from snow and wind, in our Northern States, do actually at times reach, and even surpass, 20 lbs per square foot of roof. See Table 4, p 581, of Trusses. **The limit of perpetual snow** at the equator is at the height of about 16000 feet, or say 3 miles above sea-level; in lat 45° north or south, it is about half that height; while near the poles it is about at sea-level.

EVAPORATION, FILTRATION, AND LEAKAGE.

The amount of evaporation from surfaces of water exposed to the natural effects of the open air, is of course greater in summer than in winter; although it is quite perceptible in even the coldest weather. It is greater in shallow water than in deep, inasmuch as the bottom also becomes heated by the sun. It is greater in running, than in standing water; on much the same principle that it is greater during winds than calms. It is probable that the average daily loss from a reservoir of moderate depth, from evaporation alone, throughout the 3 warmer months of the year, (June, July, August.) rarely exceeds about $\frac{3}{10}$ inch, in any part of the United States. $\frac{1}{10}$ inch during the 9 colder months; except in the Southern States. These two averages would a daily one of .15 inch; or a total annual loss of 55 ins, or 4 ft 7 ins. It probably is 3.5 to 4 ft.

By some trials by the writer, in the tropics, ponds of pure 8 ft deep, in a stiff retentive clay, and fully exposed to a very hot sun all day, lost during the dry son, precisely 2 ins in 16 days; or ⅛ inch per day; while the evaporation from a glass tumbler was ¼ inch per day. The air in that region is highly charged with moisture; and the dews are heavy. Every day during the trial the thermometer reached from 115° to 125° in the sun.

The total annual evaporation in several parts of England and Scotland is stated to average from 22 to 38 ins; at Paris, 34; Boston, Mass, 32; many places in the U. S., 30 to 36 ins. This last would give a daily average of $\frac{1}{10}$ inch for the whole year. Such statements, however, are of very little value, unless accompanied by memoranda of the circumstances of the case; such as the depth, exposure, size and nature of the vessel, pond, &c. which contains the water, &c. Sometimes the total annual evaporation from a district of country exceeds the rain fall; and vice versa.

On canals, reservoirs, &c, it is usual to combine the loss by evaporation, with that by filtration. The last is that which soaks into the earth; and of which some portion passes entirely through the banks, (when in embankt;) and if in very small quantity, may be dried up by the sun and air as fast as it reaches the outside; so as not to exhibit itself as water; but if in greater quantity, it becomes apparent, as *leakage*.

E. H. Gill, C E, states the average evaporation and filtration on the Sandy and Beaver canal, Ohio, (38 ft wide at water surface; 26 ft at bottom: and 4 ft deep,) to be but 13 cub ft per mile per minute, *in a dry season*. Here the exposed water surf in one mile is 200640 sq ft; and in order, with this surf, to lose 13 cub ft per min, or 18720 cub ft per day of 24 hours, the quantity lost must be $\frac{18720}{200640} = .0933$ ft, $= 1\frac{1}{8}$ inch in depth per day. Moreover, one mile of the canal contains 675840 cub ft; therefore, the number of da reqd for the combined evaporation and filtration to amount to as much as all the water in the canal $\frac{675840}{18720} = 36$ days. Observations in warm weather on a 22-mile reach of the Chenango canal, York, (40; 28; and 4 ft.) gave 65½ cub ft per mile per min; or 5 times as much as in the preceding case. This rate would empty the canal in about 8 days. Besides this there was an excessive leakage at the gates of a lock, (of only 5½ ft lift.) of 479 cub ft per min, 22 cub ft per mile per min; and at aqueducts, and waste-weirs, others amounting to 19 cub ft per mile per min. The leakage at other locks with lifts of 8 ft, or less, did not *exceed* about 350 cub ft per min. at each. On other canals, it has been found to be from 50, to 500 ft per min. On the Chesapeake and Ohio canal, (where 50, 32, and 6 ft,) Mr. Fisk, C E, estimated the loss by evap and filtration in 2 weeks of warm weather, to be equal to all the water in the canal. **Professor Rankine assumes 2 ins per day, for leakage of canal bed, and evaporation, on English canals.** J. B. Jervis, C E, estimated the loss from evap, filtration, and leakage through lock-gates, on the original Erie canal, (40, 28, and 4 ft.; at 100 cub ft per mile per min: or 144000 cub ft per day. The water surf in a mile is 211200 sq ft; therefore, the daily loss would be equal to a depth of $\frac{144000}{211200} = .682$ ft, $=$ say 8⅛ ins. See end of Rain, p 221.

On the Delaware division of the Pennsylvania canals, when the supply is temporarily shut off from any long reach, the water falls from 4 to 8 ins per day. The filtration will of course be much greater on embankts, than in cuts. In some of our canals, the depth at high embankts becomes quite considerable; the earth, from motives of economy, not being filled in level under the bottom of the canal; but merely left to form its own natural slopes. At one spot at least, on the Ches and Ohio canal, where one side is a natural face of vertical rock, this depth is 40 ft. Such depths increase the leakage very greatly; especially when, as is frequently the case the embankts are not puddled; and the practice is not to be commended, for other reasons also.

The total average loss from reservoirs of moderate depths, in case the earthen dams be constructed with proper care, and well settled by time, will not exceed about from ½ to 1 inch per day; but in new ones, it will usually be considerably greater.

The loss from ditches, or channels of small area, is much greater than that from navigable canals; so that long canal feeders usually deliver but a small proportion of the water which enters them at their heads.

HYDROSTATICS.

Art. 1. Hydrostatics treats of the pressure of quiet water: and other liquids. The pres of liquids against any point of any surface which they act, whether said surf be curved or plane, is always perp to the surf at that point.

point. At any given depth, the pres of water is equal in every direction; and is in direct proportion to the *vert* depth below the surf. In all cases whatever, the total pres of quiet water against, and perp to any surf, is equal to the wt of a uniform column of water, the area of whose cross-section parallel to its base, is everywhere equal to the area of the surf pressed; and whose height is equal to the *vert* depth of the *cen of grav* of the surf pressed, below the hor surf of the water. This fact is one of those important ones of frequent application, which the young student should impress firmly upon his memory. The wt of a cub ft of fresh water is usually assumed to be 62½ lbs avoir; which is sufficiently correct for ordinary engineering purposes; though 62¼ is nearer the truth for ordinary temperatures of about 70° Fah. Hence,

1 **To find the total pres of quiet water against, and perp to any surf whatever,** *as a dam, embkt, lock-gate, &c; or the bottom, side, or top of any containing vessel, water-pipe, &c, whether said surf be vert, hor, or inclined at any angle whatever; or whether it be flat, or curved; or whether it reach to the surf of the water, or be entirely below it:*

Rule. Mult together the area, in sq ft, of the surf pressed; the *vert* depth in ft of its cen of grav below the surf of the water; and the constant number 62.5. The prod will be the reqd pres in pounds.*

Ex. 1. The wall A, Fig 1, is 50 ft long; and the depth, *n o*, of water pressing against its *vert* back is uniformly 10 ft. What pres does the wall sustain?

The area of surf pressed is 50 × 10 = 500 sq ft. And the vert depth of its cen of grav below the surf of the water is 5 ft; hence,

500 × 5 × 62.5 = 156250 pounds, or about 70 tons, the pres reqd.

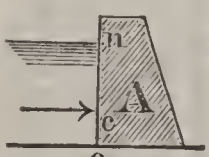


Fig 1

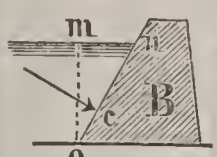


Fig 1½

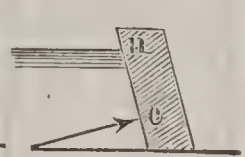


Fig 2

The pres in this case being perp to a *vert* surf, is *horizontal;* tending either to overturn the wall; or to make it slide on its base. The center of pressure is at c; or one-third of the vert depth from the bottom.

Ex. 2. As in the foregoing case, the wall B, Fig 1½, is 50 ft long; and the vert depth of water is 10 ft; but it presses against the *sloping* side of the wall; *n o* being 15 ft. What is the total pres, or the pres perp to *n o*; or in the direction of the arrow?

Here the area of surf pressed is 50 × 15 = 750 sq ft. And the vert depth of its cen of grav below the surf of the water is 5 ft, as before; hence,

750 × 5 × 62.5 = 234375 pounds, or about 105 tons, the total pres reqd.

The cen of pres as before, is at c, one-third of the depth from the bottom.

In such cases, the total pres perp to *n o*, may be considered as resolved into two pressures; one of them acting *hor*, either to overthrow the wall, or to make it slide; and the other acting *vert* to hold it in its place. And if the sloped line *n o* be taken at any scale to represent the total pres, then will the vert line *m o*, measured by the same scale, represent the hor pres; and the hor line *m n*, the vert one. See Art 34, Force in Rigid Bodies. Therefore, so long as the vert depth of water remains the same, the hor pres remains the same, no matter what may be the slope of *n o*; but the vert, as well as the total pres, will increase with *n o*. See Art 4. In Fig 2, the pres tends to *lift* the wall.

Rem. 1. This total pres of the water is of course distributed over the entire depth of the wetted part of the back of the wall; being least at top, and gradually increasing toward the bottom; but so far as regards the united action of every portion of it, in tending to overthrow the wall, considered as a single mass of masonry, incapable of being bent or broken, it may all be *assumed* to be applied at c; dist from the bottom of the water one-third of its vert depth; or, which is the same thing, at one-third of the sloping dist *o n*, Figs 1½ and 2. See Art 57, of Force in Rigid Bodies.

Rem. 2. **It follows, from the foregoing rule, that the amount of pres of water against any surf is entirely independent of the quantity of the water,** so long as the area pressed, and the vert depth of its cen of grav below the level surf remain unchanged. The wall A or B would sustain as great a pres from a layer of water only an inch thick behind it, as if the water had extended back for miles. From this cause, retaining-walls of mortar masonry carelessly backed, have been bulged, and cracked, by the infiltration of rain behind them; while walls of dry masonry would have permitted the water to escape through the open joints; and would therefore have stood safely.

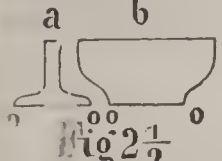


Fig 2½

Also in vessels *a*, *b*, Fig 2½, of any size or shape whatever, if they contain the same vert depth of water; and have equal bases *o o*, pressed by said depths of water, the pressures on the bases will all be equal, without any regard to the quantity of water. Or, if we have two water-pipes of the same diam, both full of water, one standing vertically, 10 feet long; and the other 20 miles long, and laid at an inclination of ½ ft per mile, so as to make its *vert* depth of water also 10 ft, the pressures at the lower ends of the two pipes will be equal. This fact, that the pres on a given surf at a given depth is independent of the quantity of water, is called the *hydrostatic paradox*. In the vessel *a*, the pres on the base is much greater than the wt of the water; but in *b*, it is less.

Rem. 3. Since the pres of water against any point is at right angles to the surf at that point, it fol-

* This is strictly true as regards the pres of the *water alone;* and this is usually all that is required. It should be borne in mind that the surf of the water is itself pressed by *the air;* to the average (at the level of the sea) of about 14.7 lbs per sq inch; or 2117 lbs, or nearly 1 ton per sq foot. Therefore, to find the true total pres, we should mult the area in sq ft of the surf pressed by the water, by 2117 lbs; and add the prod to the water-pres given by the rule. But in ordinary engineering cases,

lows that props *p p*, for strengthening such structures as the sloping dam D, Fig 3, should be placed at right angles to them in order to oppose the greatest possible resistance to the pres. Other considerations may at times prevent our doing so; thus the outer prop, *p*, if so placed, would be in danger of being broken by ice, or logs tumbling over the dam; and therefore, had better be more nearly vertical.

Rem. 4. It follows, from the foregoing rule, that in a cubical vessel, filled with water, the pres on the base is equal to the weight of the water; that on each of the four sides, to half the weight of the water; and that on the bottom and the 4 sides together, to 3 times the wt of the water. In a conical vessel, forming an entire cone, the pres on its hor base is equal to 3 times the wt of the water; and so likewise in a pyramidal vessel; for in both cases the wt of the water is but ⅓ that of a uniform column of water of the same height. In a full spherical vessel, the total pres against its entire interior surf, is also equal to 3 times the wt of the water, as in a cubical one.

Since the pres increases with the depth, the props in the dam, Fig 3, should be closest together near the bottom; also the hoops of a tank.

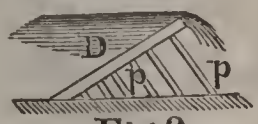


Fig 3

The following Table gives the pres to the nearest lb per sq ft at diff vert depths; and also the total pres against a plane one foot wide extending vert from the surface to those depths. The first increases as the depths; the last as the squares of the depths.

For the pres in lbs per sq inch at any given depth, mult the depth in ft by .434. For lbs per sq ft, mult by 62.5. For tons per sq ft, mult by .0279. **For the depth in ft** at which any given pres exists, divide the lbs per sq inch by .434; or the lbs per sq ft by 62.5; or the tons per sq ft by .0279.

D in Ft.	Per sq Ft.	Tot P.	D in Ft.	Per sq Ft.	Tot P.	D in Ft.	Per sq Ft.	Tot P.	D in Ft.	Per sq Ft.	Tot P.	D in Ft.	Per sq Ft.	Tot P.
1	62.	31	11	687.	3781	21	1312.	13781	31	1937.	30031	41	2562.	52531
2	125.	125	12	750.	4500	22	1375.	15125	32	2000.	32000	42	2625.	55125
3	187.	281	13	812.	5281	23	1437.	16531	33	2062.	34031	43	2687.	57781
4	250.	500	14	875.	6125	24	1500.	18000	34	2125.	36125	44	2750.	60500
5	312.	781	15	937.	7031	25	1562.	19531	35	2187.	38281	45	2812.	63281
6	375.	1125	16	1000.	8000	26	1625.	21125	36	2250.	40500	46	2875.	66125
7	437.	1531	17	1062.	9031	27	1687.	22781	37	2312.	42781	47	2937.	69031
8	500	2000	18	1125.	10125	28	1750.	24500	38	2375.	45125	48	3000.	72000
9	562.	2531	19	1187.	11281	29	1812.	26281	39	2437.	47531	49	3062.	75031
10	625.	3125	20	1250.	12500	30	1875.	28125	40	2500.	50000	50	3125	78125

Thus we see that at the depth of 36 ft, the pres of water against a single sq ft of surf, whether hor, vert, or oblique, is fully 1 ton; requiring great precaution to prevent leakage, or breaking. At 72 ft, it would be 2 tons, &c. A pres of 62½ lbs per sq ft gives a pres of .434 lbs per sq inch.

Further; let *a b*, Fig 3½, be a tube of 36 ft vert height; full of water; with a bore so small that the tube would contain say only one pound of water; and let this tube open at its lower end into a vessel also full of water; the top and bottom of which are 8 ft apart. Then the 1 lb of water in the tube, will cause each sq ft of the top of the vessel, (which is 36 ft below the surf of the water in the tube) to be pressed upward with a force of 2250 lbs, as per table. Each sq ft of the bottom of the vessel (which is 44 ft below the surf of the water in the tube) will be pressed downward with a force of 2750 lbs; and any particular sq ft of the sides of the vessel, will be pressed hor outward, with the force given in the table, opposite to the depth of the cen of grav of said sq ft below the same water surf of the top of the tube, whatever said depth may happen to be. Or, suppose, first only the lower vessel to be filled with water, and its inner surf to be sustaining the pres arising therefrom; if we then fill the 36 ft tube with its 1 lb of water, this 1 lb will create an *additional* pres of 2250 lbs against every sq ft of said inner surf; so that if the 6 sides of the vessel be each 8 ft square; or contain *in all* 384 sq ft of inner surf, this 1 lb of water will produce *additional* pres of 864000 lbs, or full 385 tons, against them. If we then press upon the top of the water with our thumb to the extent of 1 lb, we shall thereby redouble this enormous pres. This fact, however, belongs to Art. 7,

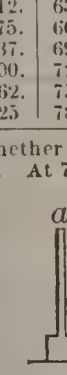


Fig 3½

Art. 2. Surfaces pressed on both sides; and immersed.

When two bodies of water of diff depths, press against two opposite sides of a plane which is completely immersed, whether vert or sloping; as, for instance, against the two sides *a b*, *n o*, Fig 4; or the two sides *d e*, *e r*, then, the total pres against *i b*, *i e*, *a b*, *n o*, or *e r*, &c, may still be found by the foregoing rule, in Art 1; but the excess *of pres against the part* a b, or d e, *of the immersed plane, beyond the counter-pres against the opposite part* n o, or e r, will be equal to the wt of a column of water whose section is equal to the area of the part *a b*, or *d e*, (as the case may be;) and whose vert height is equal to *m n*, or *x p*, the vert diff of level of the two bodies of water. Consequently, this *excess* of outward pres is found by mult together, the area of *a b* or *d e*, in sq ft; the vert height *m n* or *x p*, in ft; and the constant 62.5 lbs wt of a cub ft of water. Thus, if *a b* is 10 ft high, and 20 ft long: and the vert height *m n*, 12 ft; then the *excess* of pres against *a b*, over that against *n o*, will be $10 \times 20 \times 12 \times 62.5 = 150000$ lbs. The excess will be greater on *d e*, than on *a b*, although both are exposed to the same vert depths *m n*, *x p*; because the area of *d e* is greater than that of *a b*. Moreover, this *excess* of outward pres is *equally* distributed over the entire area of *a b* or *d e*; being no greater at *b* and *e*, than at *a* or *d*; in other words, every sq ft of area of *a b* or *d e* is pressed outward at right angles to its surf, by an *excess* of force equal to the wt of a column of water 1 ft sq; and of a height equal to *m n*, or *x p*.

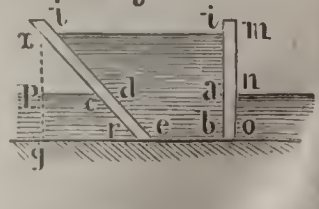


Fig 4

this pres of the air may, and *should be* omitted; because it is counterbalanced by an equal pres of air against the opposite side, face, or surf of the pressed body. It becomes necessary, therefore, to take it into consideration only when the opposite face of the body is not exposed to a counterbalancing atmospheric pressure; as when there is a vacuum on that side.

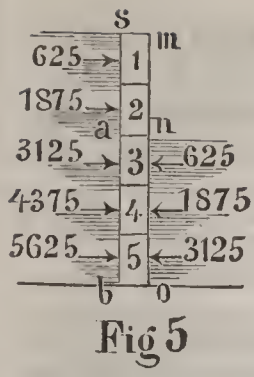


Fig 5

This will be understood by means of Fig 5, which may represent five planks, 1, 2, 3, 4, and 5, forming a dam, and seen endwise; each one 1 ft in depth, and say 20 ft long hor; making the area of each surf pressed, equal to 20 sq ft. The pres in lbs against each separate 20 sq ft of area, calculated by the rule in Art 1, is shown in the fig. Now, the outward pres against the upper *immersed* 20 ft area, or that of plank 3, is 3125 lbs; while the counter-pres against it from the other side is 625 lbs; making the *excess* of outward pres equal to $3125 - 625 = 2500$ lbs. Again, at the lowest plank, number 5, the outward pres exceeds the inward one by $5625 - 3125 = 2500$ lbs, the same as in the upper one. And so of any other equal area of surf, at any depth whatever; the excess depending upon the vert height of $m\ n$, will be equally distributed over $a\ b$. It only remains to show that the total excess of outward pres against $a\ b$, is equal in amount to the wt of a uniform column of water with a base equal in area to $a\ b$, and with a height equal to $m\ n$. Thus, we have seen that in the instance before us, the excess amounts to 3 times 2500 lbs, or to 7500 lbs. Now, the wt of the column of water will be 60 (or area of $a\ b$) $\times$ $m\ n$ (or 2 ft) $\times$ 62.5 lbs $=$ 7500 lbs; or the same as the *excess* pres on $a\ b$.

The excess of pres against the entire side $s\ b$, over that against n o, is evidently the diff between those two pressures calculated respectively by the rule in Art 1.

Art. 3. Surfaces, vert, as $b\ m\ c\ s$, $a\ n\ o\ t$, Fig 6, **or otherwise, of equal widths, $b\ m$, $a\ n$; commencing at the level, $b\ a\ n\ m$, of the water, but extending to diff depths, $m\ c$, $n\ o$, measured vert; and having the same inclination to the surf of the water; sustain total pressures proportional to the squares of those depths.**

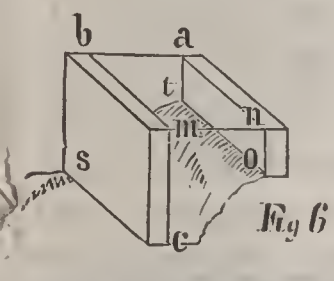


Fig 6

In Fig 6, let the two vert sides, $a\ n\ o\ t$, and $b\ m\ c\ s$, of a vessel, have the same width $a\ n$, and $b\ m$; then if the depth m c, be 2, 3, 4, 5, &c, times greater than the depth $n\ o$, the pres against the surf $b\ m\ c\ s$, will be 4, 9, 16, 25, &c, times greater than that against $a\ n\ o\ t$. This will be seen by referring to the pressures figured on the left side of Fig 5, where, as stated in Art 2, the surf of plank 1, exposed to the pres on the left side, is 20 sq ft; that of planks 1 and 2, 40 sq ft; that of planks 1, 2, and 3, 60 sq ft, &c. All these surfs commence at the level of the water; and all of them being vert, are of course at the same inclination with the water surf; but their depths are respectively 1, 2, and 3 ft. The pres against the surf of 1, is 625 lbs; that against the surf of 1, 2, is $625 + 1875 = 2500$; and that against the surf of 1, 2, 3, is $625 + 1875 + 3125 = 5625$. But 2500 is *four* times 625; and 5625 is *nine* times 625. And the pres against the entire surf $s\ b$, (which is 5 times as deep as plank 1,) is 25 times as great as that against plank 1; or $625 \times 25 = 15625$ lbs $=$ the sum of all the pressures marked on the left side of Fig 5.

This follows, from the Rule in Art 1; for twice the area of surf, mult by twice the vert depth of the cen of grav below the surf, must give 4 times the pres: three times the area, by three times the depth, must give 9 times the pres, &c. See third columns of table, p 224.

It follows, also, that at any particular *point*, or against any *given* area placed at various depths, the pres will increase simply as the vert depth: thus, if there be three areas, each one sq ft, placed in the same positions, but with their centers of grav respectively 8, 16, and 24 ft below the surf, the pres against them will be respectively *as* 8, 16, and 24; or *as* 1, 2, and 3. See second columns table, p 224.

Art. 4. The pressure of quiet water, in any one given direction, against any given surf, whether vert, hor, inclined, flat, or curved, is equal to the wt of a uniform column of water, the area of whose section, parallel to its base, is everywhere equal to the area of the *projection of the pressed surf taken perp to the given direction*; and the height of the column equal to the vert depth of the cen of grav of the pressed surf below the upper surf of the water. Hence the

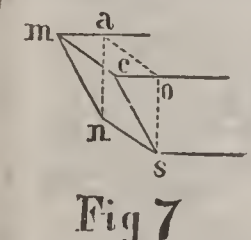


Fig 7

RULE. **To find the pres in lbs,** mult together the area in sq ft of the projection taken at right angles to the given direction; the vert depth in ft of the cen of grav of the pressed surf below the upper surf of the water; and the constant 62.5 lbs wt of a cub ft of water.

EX. Let $m\ c\ s\ n$, Fig 7, be an inclined surf, sustaining the pres of water which is level with its top m c. Then the *total* pres against $m\ c\ s\ n$, and at right angles to it, as found by the rule in Art 1, is an illustration of the present rule; because the projection of $m\ c\ s\ n$, taken at right angles to the given direction, or parallel to $m\ c\ s\ n$, is in fact $m\ c\ s\ n$ itself, or equal to it. Hence the rule in Art 1 is merely a simple modification of the present one, applicable to the case of *total* pres against any surf.

But if it be reqd to find only the vert or downward pres against $m\ c\ s\ n$, in pounds, mult together the area of the hor projection $a\ o\ c\ m$ sq ft; the vert depth in ft of the cen of grav of $m\ c\ s\ n$ below the surf; and 62.5. Or if only the *hor* pres against $m\ c\ s\ n$ be sought, mult together the area of the vert projection $a\ o\ s\ n$; the vert depth of the cen of grav of $m\ c\ s\ n$; and 62.5.

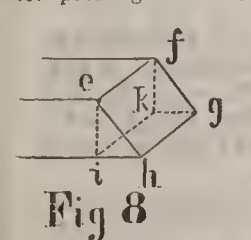


Fig 8

In Fig 8 also, the *total* pres against $e\ f\ g\ h$ is found by rule in Art 1; while the *hor* and vert pressures against it are found as in Fig 7, by using the projections $e\ f\ k\ i$, and $k\ i\ g\ h$. In Fig 7 the vert pres is downward: while in Fig 8 it is upward; but this circumstance in no respect affects the rule.

REM. 1. At any given depth, the pres, perp to any given surf, is the same in all directions; but Figs 7 and 8 show that the total pres *oblique* to a given surf will be less than the perp one at the same depth; because an oblique projection of a surf must be less than the surf itself, which last is the projection when the pres is perp to it. Thus, in a reservoir, the total pres perp to a sloping side, as $m\ n\ s$ c, Fig 7, is greater than either the vert or the hor pres upon it.

Again, let Fig 9 represent a conical vessel full of water; its base b c, 2 ft diam; its vert height a n, 3 ft; then the circumf of the base will be 6.2832 ft; the area of the base 3.1416 sq ft; the length of its slant side a b or a c, 3.16 ft; the area of its curved slanting sides will be $\frac{6.2832 \times 3.16}{2} = 9.93$ sq ft; and the vert depth of the cen of grav of the slanting sides will be at two-thirds of the vert height a n from the apex a, or 2 ft.

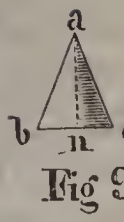


Fig 9

Here, to find the total pres against the base, we have by rule in Art 1, $3.1416 \times 3 \times 62.5 = 589.05$ lbs. For the total pres against the slant sides, by the same rule, $9.93 \times 2 \times 62.5 = 1241.25$ lbs. For the vert pres upward against the entire area of the slant sides, we have given the area of the base (which is here the hor projection of the slant sides) $= 3.1416$; and the vert depth of the cen of grav of the slant sides, 2 ft. Therefore, $3.1416 \times 2 \times 62.5 = 392.7$ lbs, the upward vert pres.

Finally, for the hor pres in any given direction against the slant sides of *one half* of the cone, we have the vert projection of that half, represented by the triangle a b c, with its base 2 ft, and its perp height 3 ft; and consequently, with an area of 3 sq ft. The depth of its cen of grav is 2 ft; therefore, $3 \times 2 \times 62.5 = 375$ lbs, the reqd hor pres.*

In Fig 10, which represents a vessel full of water, the *total* pres against the semi-cylindrical surf a v e m d k, and perp to it, must be also hor, because the surf is vert; but inasmuch as the surf is *curved*, this total pres, as found by rule in Art 1, acts against it in many directions, which might be represented by an infinite number of radii drawn from o as a center. But let it be reqd to find the hor pres in lbs, in one direction only, say parallel to o e, or perp to a d; which would be the force tending to tear the curved surf away from the flat sides a b n v, and d c s k, by producing fractures along the lines a v and d k; or which would tend to burst a pipe or other cylinder. In this case, mult together the area of the vert projection a d k v in sq ft; the depth of the cen of grav of the curved surf in ft; (which, in the semi-cylinder would be half of e m, or of o i;) and 62.5. Since the resulting pres is resisted equally by the strength of the vessel along the two lines a v and d k, it is plain that each single thickness along those lines need only be sufficient to resist safely *one half* of it; and so in the case of pipes, or other cylinders, such as hooped cisterns or tanks. See Art 17.

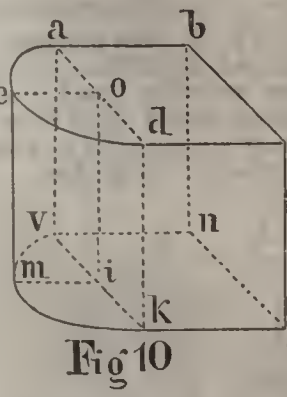


Fig 10

Should the pres against only one half of the curved surf, as e d m k be sought, and in a direction parallel to o d, tending to produce fractures along the lines e m, and d k, then use the vert projection o e m i; with the same depth; and 62.5, as before.

It follows, that if the face of a metallic piston be made concave or convex, no more pres will be reqd to force the piston through any dist, than if it were flat; for the pres against the face of the piston, in the direction in which it moves, must be measured by the area of a projection of that face, taken at right angles to said direction; and the area of said projection will be the same in all three cases.

Rem. 2. **If a bridge pier, or other construction, Fig 10½, be founded on sand or gravel,** or on any kind of foundation through which water may find its way underneath, even in a very thin sheet, then the upward pres of the water will take effect upon the pier; and will tend to lift it, with a force equal to the wt of the water displaced by the pier; (Arts 18, 19. In other words, the effective wt of the *submerged* portion of the pier, will be reduced 62½ lbs per cub ft; or nearly the half of the ordinary wt of masonry.

Fig 10½

But if the foundation be on rock, covered with a layer of cement to prevent the infiltration of water beneath the masonry, no such effect will be produced; but on the contrary, the vert pres downward, afforded by the battering sides of the pier, and by its offsets, will tend to hold it down, and thus increase its stability; which, in *quiet* water, will then actually be greater than on land.

Art. 5. To divide a rectangular surf, whether vert as a b c d, or inclined as m n o p, Fig 11, whose top a b or m n is level with the surf of the water, by a hor line x 2, such that the total pres against the part above said hor line, shall equal that against the part below it.

Rule. Mult one half of the length of b c, or m p, as the case may be, by the constant number 1.4142; the prod will be b 2, or m x.

Ex. Let b c $= 12$ ft. Then $6 \times 1.4142 = 8.4852$ ft; or b 2. Let m $p = 16$ ft. Then $8 \times 1.4142 = 11.3136$ ft, or m x.

Rem. The line x 2, thus found, must not be confounded with the *cen of pres.* which is entirely diff. See Art 8.

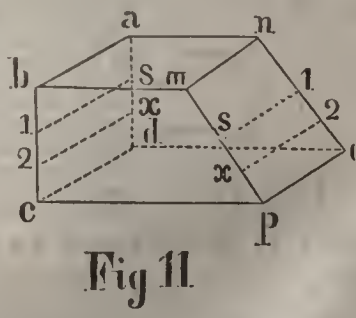


Fig 11

Art. 6. In a rectangular surf, whether vert as a b c d, or inclined as m n o p, Fig 11, whose top a b or m n coincides with the surf of the water, to find any number of points, as 1, 2, &c. through which if hor lines, as 1 s, 2 x, &c. be drawn, they will divide the given surf into smaller rectangles, all of which shall sustain equal pressures.

Rule. First fix on the number of small rectangles reqd. Then for point 1 from the top, mult the number 1, by this number of rectangles. Take the sq rt of the prod. Mult this sq rt by the entire length

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* **In a sphere** filled with a fluid the total inside pres $= 3$ times wt of fluid.

$b\ c$ or $m\ p$, as the case may be. Div the prod by the number of rectangles. The quot will be the dist b 1, or n 1, as the case may be.

For the dist b 2, or n 2, proceed in precisely the same way; only instead of the number 1, use the number 2 to be mult by the number of rectangles: and so use successively the numbers 3, 4, 5, &c, if it be reqd to find that number of points.

Ex. Let $b\ c = 10$ ft; and let it be reqd to find 2 points, 1 and 2, for dividing the rectangular surf $a\ b\ c\ d$ into 3 rectangular parts, which shall sustain equal pressures. Here we have for point 1,

$1 \times 3 = 3$. The sq rt of $3 = 1.732$. And 1.732×10 (or $b\ c$) $= 17.32$. And $\frac{17.32}{3 \text{ rectangles}} = 5.773$ ft $= b\ 1$.

For point 2, we have

$2 \times 3 = 6$. The sq rt of $6 = 2.449$. And 2.449×10 (or $b\ c$) $= 24.49$. And $\frac{24.49}{3 \text{ rectangles}} = 8.163$ ft $= b\ 2$.

And so for any number of points.

REM. 1. **This rule will be found useful in spacing the cross-bars of lock-gates; the hoops around cylindrical cisterns; and the props to a structure, like Fig 3.**

REM. 2. **For dividing any surf, as $o\ b\ c\ d$, Fig 12, which is not rectangular, in the same manner,** with an accuracy sufficient for most practical purposes, perhaps the following method is as convenient as any.

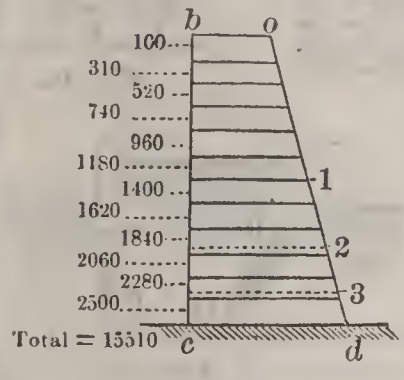

Fig. 12.

RULE. First div the surf, as in Fig 12, into several small hor parts, equal or not, at pleasure. Then by Rule in Art 1, find the pres on each part separately, as is supposed to be done in the numbers on the left-hand of the fig. The sum of these (in this case 15510) is the total pres against the entire surf $o\ b\ c\ d$. Now suppose we wish to div this surf in 4 parts bearing equal pres; first div 15510 by $4 = 3878$. Then beginning at the top, add together a number of the separate pressures sufficient to amount to 3878; by this means find point 1. Then proceed with the addition until the sum amounts to twice 3878, or 7756, which will indicate point 2; and in the same manner find point 3, by adding up to three times 3878, or 11634. Then the hor dotted lines ruled through points 1, 2, and 3, will give the reqd divisions approximately. In this manner the hoops of conical, and other shaped vessels, may be spaced nearly enough for practical purposes.

Art. 7. The transmission of pressure through water. Water, in common with other fluids, possesses the important property of transmitting pres equally in all directions. Thus,

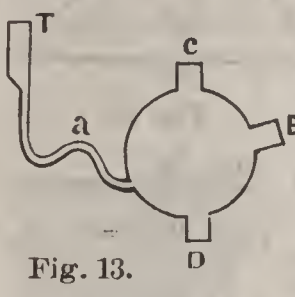

Fig. 13.

suppose the vessel, Fig 13, to be entirely closed, and filled with water; and suppose the transverse area of T, C, D, and E, to be each equal to one sq inch. Then, if by means of a piston, or otherwise, a pres of 1 lb, 1 ton, or any other amount, be applied to the one sq inch of area of T, C, D, or E, every sq inch of the inner surf of the vessel, and of the pipe a, will instantly receive, at right angles to itself, an equal pres of 1 lb, or 1 ton, &c; in addition to the pres which it before sustained from the water itself; and this will occur if the vessel consist of parts even miles asunder; as, for instance, if T were miles distant from E; and united to it by a long series of tubes. If the vessel were a strong steam boiler full of water, a single pres of a few hundred pounds at T, C, &c, would burst it. See also fig 3½.

The hydrostatic press acts on this principle. Any body, within the vessel, would also receive an equal additional pres on each sq inch of its surf.

If the top of T be open, the air will press upon the sq inch of the exposed surf of water to the extent of nearly 15 lbs; and the same degree of pres will also be transmitted to every sq inch of the interior surf of the vessel, and its connecting tubes; but no danger of bursting will result from this atmospheric pres, because the air also presses every sq inch of the outside of the vessel to the same extent.

Air, and other gaseous fluids, transmit pres equally in all directions, like liquids; but not as rapidly.

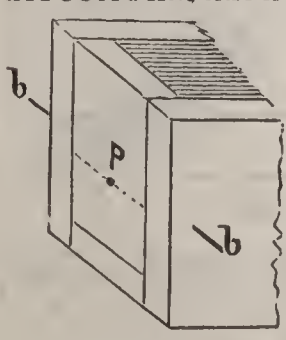

Fig. 14.

Art. 8. The center of pressure. Let Fig 14 represent a vessel full of water, and suppose the side P to be perfectly loose, so as to be thrown outward by the slightest pres of the water from within. Now, there is but one single point, P, in every surf so pressed, no matter what its shape may be, to which if we apply a force equal to the pres of the water, and in a direction opposite to said pres, the side P will be thereby prevented from yielding. Such point is called the *center of pressure.* It must not be understood by this that the actual amount of pres of the water against that part of the surface which is above the hor dotted line passing through P, is equal to that of the water below said line; but that the sum of the products of the several pressures above it, mult by their several leverages, or vert dists from P, is equal to the sum of the products of the pressures below, mult by their leverages; or, in other words, that the sum of the *moments* around the point P, of the pressures above the line, is equal to the sum of the *moments* of those below it; so that if a hor iron rod $b\ b$ were passed entirely through the side P, at the same level as the dotted line, as shown in the fig, so as to serve as a hinge for the side P to turn on, the side would have no tendency to turn.

Art. 9. To find the cen of pres of a quiet fluid, against a plane surface. Fig 15.

1. The center of pressure of a quiet fluid against any plane surface whose width is uniform throughout its depth, whether said surface be vertical, as *e o*, or inclined, as *c a*, (or inclined in the opposite direction:) and whose top *c*, or *e*, coincides with the hor water surf; is distant *vert* below the water surf, two-thirds of the vert depth, *s x*, from said water surf to the bottom of the plane; as at *n* and *i*. Inasmuch as a hor line at ⅔ of the depth of *s x*, interseets both *c a* and *e o* at ⅔ of their lengths respectively, we might say at once that the center of pres against a plane of uniform width is at two-thirds of its length below the water surface.

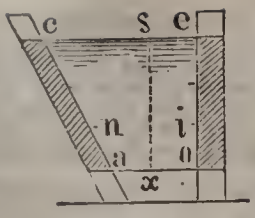

Fig. 15

Throughout Art 9 **any measure,** as yard, foot, or inch &c, may be used.

2. But if the hor top *a*, or *o*, Fig 16, of the rectangular plane *a g*, or *o h*, be covered to some depth with water, then the *vert* depth *s m*, of the cen of pres *d*, or *e*, below the surf of the water, will be equal to

$$\frac{2}{3} \text{ of } \frac{\text{cube of } s\,c - \text{cube of } s\,w}{\text{square of } s\,c - \text{square of } s\,w}$$

where *s c* is the vert depth of the bottom, and *s w* the vert depth of the top, of the pressed surf, below the water surf. Or, in words: From the cube of *s c*, take the cube of *s w*; and call the rem *a*. Then, from the square of *s c*, take the square of *s w*; and call the rem *b*. Div *a* by *b*, and take two-thirds of the quot for *s m*.

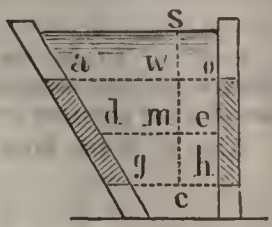

Fig. 16.

3. When a plane surf of any shape whatever, whether rectangular, triangular, or circular, &c; whether vert as *o p*, Fig 17, or inclined as *m n*, is *entirely immersed*, so as to be pressed over the entire area of *both* sides but by *diff depths* of water on its two sides; then the cen of pres coincides with the *cen of grav* of the pressed surf.

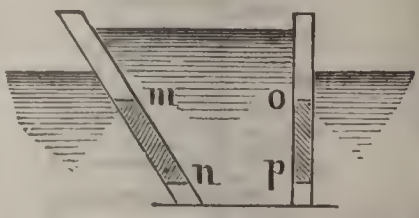

Fig. 17.

In the 3 foregoing figures the supposed surfaces are shown edgewise, so that their widths do not appear.

4. In any triangular plane surf, whether right-angled, or otherwise, as *a b c*, Fig 18; whether vert, or inclined; the *base a b* of which coincides with the hor surf of the water; the cen of pres *o*, will be in the center of the line *e r*, which bisects the base *a b*.

5. But if the triangle, as *a s c*, vert, or inclined, have its *apex*, *a*, at the surf of the water; and its base *s c*, *hor;* then the cen of pres *x*, will also be in the line *a m* which bisects the base; but *a x* will be ¾ of *a m*.

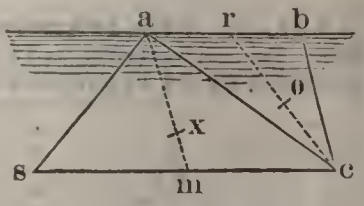

Fig. 18.

6. If any plane triangle *a b c*, Fig 19, base up, and hor; have its base *a b* covered to some depth *n d*, with water; then the cen of pres *o*, will be in the line *c s* which bisects the base; and *n o* will be equal to

$$\frac{m\,x^2 + (2\,m\,x \times m\,a) + 3\,m\,a^2}{(m\,x + 2\,m\,a) \times 2.}$$

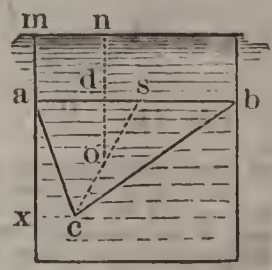

Fig. 19.

7. The center of pres against any plane *rectangular* surface, Fig 20, whether vert as *m n*, or inclined as *p o*, or *w x*; having its top coinciding with the surf of the water; and pressed by diff depths of water on its opposite sides, as shown in the fig; will be *vert* below the *upper* water surf, a dist equal to

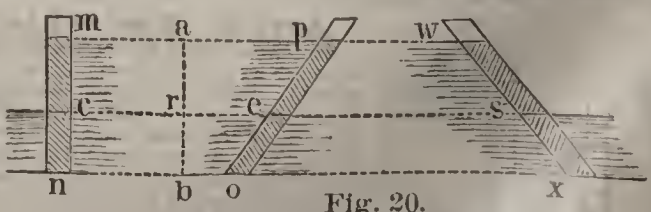

Fig. 20.

$$\frac{\left(\begin{matrix}\text{area of surf}\\ \text{m n, or p o, or w x}\end{matrix} \times \frac{\text{sq of vert depth a b}}{3}\right) - \left(\begin{matrix}\text{area of surf}\\ \text{c n, or e o, or s x}\end{matrix} \times \frac{\text{sq of vert depth r b}}{3}\right) - \left(\begin{matrix}\text{vert depth}\\ \text{a r}\end{matrix} \times \begin{matrix}\text{area of surf}\\ \text{or e o, or s x}\end{matrix} \times \frac{\text{vert depth r b}}{2}\right)}{\left(\begin{matrix}\text{area of surf}\\ \text{m n, or p o, or w x}\end{matrix} \times \text{half of a b}\right) - \left(\begin{matrix}\text{area of surf}\\ \text{c n, or e o, or s x}\end{matrix} \times \text{half of r b}\right).}$$

8. To find the cen of pres against either a circular, or an elliptic surf, pressed on one side only; whether vert, or inclined; and having its top either coinciding with the surf of the water, or below it.

Call the vert depth of the cen of pres below the water surf, h.

The vert (or inclined, as the case may be) *semi* diam of the surf, r.

The *vert* dist of the *cen* of the pressed surf, below the water surf, d.

Then, $h = \frac{r^2}{4d} + d$. In a **vert circle** with top at surf, $h = 1\frac{1}{4}$ rad.

Art. 10. Walls for resisting the pres of quiet water. A study of what we have said on retaining-walls for earth, will be of service in this connection. It is of course assumed that the water does not find its way under the wall; and that the wall cannot slide. In making calculations for walls to resist the pres of either earth, or water, it is convenient to assume the wall to be but one foot in *length;* (not height, or thickness;) for then the number of cub ft contained in it, is equal to that of the sq ft of area of its cross-section, or profile; so that these sq ft, when mult by the wt of a cub ft of the masonry, give the wt of the wall. In ordinary cases, it is well for safety to assume that the water extends down to the very bottom line of the wall.

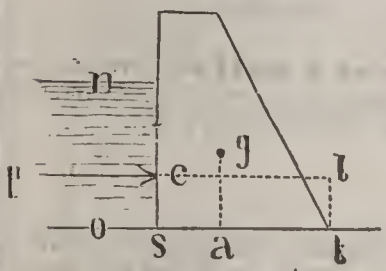

Fig 20 1/2

Now, by Art 1, the total pres of quiet water, against the rectilineal back of a wall, whether vert or sloping, is found in lbs, by mult together the area in sq ft of the part actually pressed, (or in contact with the water;) half the *vert* depth of the water, in ft, (being the *vert* depth of the cen of grav of a rectilineal back, below the surf;) and the constant 62.5 lbs; and this total pres is always *perp to the pressed area.*

When the back of the wall is vert, as in Fig 20½, this pres p is of course less than when it is battered; and is also *hor;* and it tends to overthrow the wall, by making it revolve around its outer toe, or edge t. The cen of pres is at c; $e\,s$ being ⅓ the *vert* depth $o\,n$; in other words, the entire pres of the water, so far as regards overthrowing the wall as one mass, (see Art 1, of Force in Rigid Bodies,) may be considered as concentrated at the point c; where it acts with an overthrowing leverage $t\,l$, (see Arts 46, 47, 49 of Force). The pres in lbs, mult by this leverage in feet, gives the *moment* in ft-lbs of the overturning force; (see Art 49, Force in Rigid Bodies.) The wall, on the other hand, resists in a vert direction $g\,a$, with a moment equal to its wt (supposed to be concentrated at its cen of grav g,) mult by the hor dist $a\,t$, which constitutes the leverage of the wt with respect to the point t as a fulcrum. If the moment of the water is greater than that of the wall, the latter will be overthrown; but if less, it will stand.

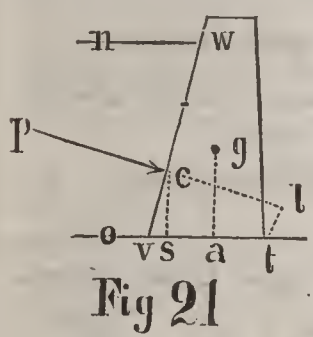

Fig 21

Rem. 1. Art 49 of Force in Rigid Bodies, will sufficiently explain the subjects of moments and leverage; and make it evident that the same principle applies also to sloping backs, as in Fig 21. Here the overturning moment of the water is equal to its calculated pres $p \times$ its leverage $t\,l$; while the moment of stability of the wall is equal to its wt $\times$ its leverage $a\,t$. By aid of a drawing to a scale, we may on this principle ascertain whether any proposed wall will stand. For we have only to calculate the pres p; then apply it at c, and at right angles to the back; prolong it to l; measure $t\,l$ by the same scale. Then calculate the wt of wall; find its cen of grav g; draw $g\,a$ vert, and measure the leverage $a\,t$. We then have the data for calculating the two moments. For finding the cen of grav, see Cen of Grav, Trapezoid, p 348.

Rem. 2. If the water, instead of being quiet, is liable to waves, the wall should be made thicker.

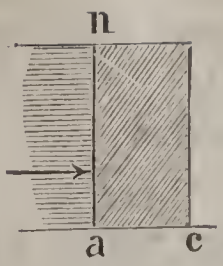

Fig. 22.

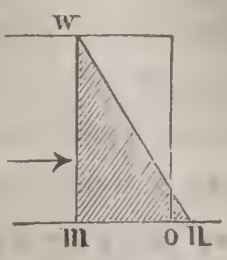

Fig. 23.

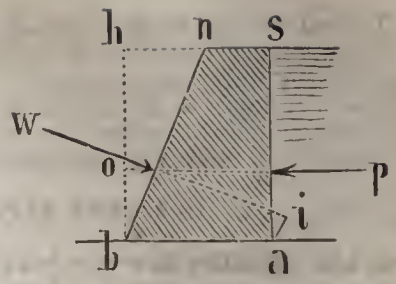

Fig. 24.

Art. 11. To find the thickness at base of a wall required to be safe against *overturning* under the pres of quiet water level with its top, and pressing against its entire vert back. **Caution.** See Art. 13, p 231.

(1st) Vertical wall, Fig 22.

$$\text{Thickness in feet} = \text{Height in feet} \times \sqrt{\frac{\text{Factor of safety *}}{3 \times \text{sp grav of wall}}} = \text{Height in feet} \times \text{the proper decimal in following table}$$

To change a vert wall into a battered one, see Art. 8, p 691.

(2d) Right angled triangular wall, Fig 23.

$$\text{Thickness at base in feet} = \text{Height in feet} \times \sqrt{\frac{\text{Factor of safety *}}{2 \times \text{sp grav of wall}}} = \text{Height in feet} \times \text{the proper decimal in following table}$$

$$= \text{thickness, } m\,o\text{, of vertical wall} \times 1.225.$$

Notwithstanding their greater thickness at base, such triangular walls contain, as seen by the fig, not much more than half the quantity of masonry reqd for vert ones of equal stability. This is owing to the fact that their cent of grav is thrown farther back; thus increasing the leverage by which the wt of the wall resists overthrow.

(3d) Wall with vertical back and sloping face, Fig 24.

$$\text{Thickness at base in feet} = \sqrt{\frac{(\text{Ht}^2, \text{ft} \times \text{factor of safety *}) + (\text{batter } h\,n^2, \text{ft} \times \text{sp grav of wall})}{3 \times \text{specific gravity of wall}}}$$

$$= \text{Height in feet} \times \text{the proper decimal in the following table.}$$

Fig. 22.	Sp. Gr.	Lbs per Cub Ft.	Resist = 1.5 pres.	Resist = 2 pres.	Resist = 3 pres.
Dressed Granite...	2.5	156	.447	.516	.673
Dressed Sandstone	2.2	137	.477	.550	.674
Mortar Rubble.....	2.	125	.500	.578	.707
Brickwork	1.8	112	.527	.609	.746
Fig. 23.					
Dressed Granite...	2.5	156	.548	.633	.775
Dressed Sandstone	2.2	137	.584	.675	.826
Mortar Rubble.....	2.	125	.613	.707	.866
Brickwork	1.8	112	.646	.746	.913

Fig. 24.	Sp. Gr.	Lbs per Cub Ft.	Resist = 1.5 pres.				Resist = 2 pres.			
			Batter 1 in. to a foot.	Batter 2 ins. to a foot.	Batter 4 ins. to a foot.	Batter 6 ins. to a foot.	Batter 1 in. to a foot.	Batter 2 ins. to a foot.	Batter 4 ins. to a foot.	Batter 6 ins. to a foot.
Dressed Granite...	2.5	156	.449	.458	.487	.532	.519	.526	.551	.593
Dressed Sandstone	2.2	137	.480	.488	.515	.558	.552	.560	.583	.622
Mortar Rubble.....	2.	125	.502	.510	.536	.578	.571	.586	.609	.646
Brickwork	1.8	112	.530	.539	.562	.602	.610	.618	.640	.674

* Factor of safety $= \frac{\text{Required moment of stability of wall}}{\text{overturning moment of water}}$. See p 229.

Art. 12. Table showing how the stability of a wall sustaining water is affected by a change in the form of the wall; the quantity of masonry remaining the same. REM. When the base of a triangular wall, of sp grav 2, is less than ½ the ht, the stability is greatest when the water presses the vert side; but if the base exceeds ½ the ht, the stability is greatest with the water on the battered side. **Caution.** See Art. 13.

	All these walls contain precisely the same quantity of masonry. The masonry is supposed to be mortar rubble, weighing 125 lbs per cubic foot; or twice as much as water; or about the same as ordinary rough mortar rubble. If the sp gr of the masonry is actually greater or less than this, the safety also will be greater or less, in precisely the same proportion.	Base in parts of height.	Approx resist of wall.
1	Vertical wall	.5	1.5
2	Face vertical; back batters one-tenth height	.55	1.8
3	" " " " one-fifth "	.6	2.2
4	" " " " one-fourth "	.625	2.6
5	" " " " one-third "	.667	3.5
6	" " " " four-tenths "	.7	4.9
7	" " " " one-half "	.75	14.0
8	Back vertical; face batters one-tenth height	.55	1.8
9	" " " " one-fifth "	.6	2.1
10	" " " " one-fourth "	.625	2.2
11	" " " " one-third "	.667	2.4
12	" " " " four-tenths "	.7	2.6
13	" " " " one-half "	.75	2.9
14	Back and face, each batter one-tenth height	.6	2.2
15	" " " " one-fifth "	.7	3.4
16	" " " " one-fourth "	.75	4.6
17	" " " " one-third "	.833	9.0
18	" " " " four-tenths "	.9	36.0

Art. 13. Liability of wall or foundation to crush under unequal distribution of pressure. Arts 11 and 12 apply only *to* the *stability* of a *rigid* wall resting upon a *rigid* base, and therefore incapable of failure *except by overturning as a whole.* They show that the *stability* is greatest when the water presses against the *sloping* side. But in practice the point where the resultant of all the pressures on the base of the wall cuts the base, must not be so near to either toe as to endanger a *crushing* of wall or of foundation. This consideration often makes it best to let the water press against the *vert* back, notwithstanding the consequent loss in stability.

Thus, Fig 25 represents, to scale, a dam wall at Poona, India, designed by Mr. Fife, C. E., of England. It is of mortar rubble, of 150 lbs per cub ft. Its total vert height is 100 ft; thickness *uv* at base, 60 ft 9 ins; at top, *rx*, 13 ft 9 ins. The front *ru* slopes 42 ft in 100 ft; and the back *xv*, 5 ft in 100 ft. Its foundation is 7 feet deep; but we here assume that the water presses against its *entire* back *xv*. Through the cen of grav G draw G*s* vert. From *c*, where the direction of the pres P of the water strikes G*s*, lay off *cn* by scale = 139.6 tons (of 2240 lbs) water pres against 1 ft in length of *xv;* and *ct* = 249.4 tons wt of 1 ft length of wall. Complete the parallelogram *cnmt* of forces. Its diag *cm* represents the resultant of all the pressures upon the base *uv*, and cuts the base at *a*, 20 ft back from the toe *u*. Doing the same with the 151.4 tons pres *p* against *ru*, we get the resultant *oy*, which is greater than *cm*, and cuts the base (at *i*) only 12.7 ft from the toe *v*, or 7.3 ft less than *a* is from *u*.

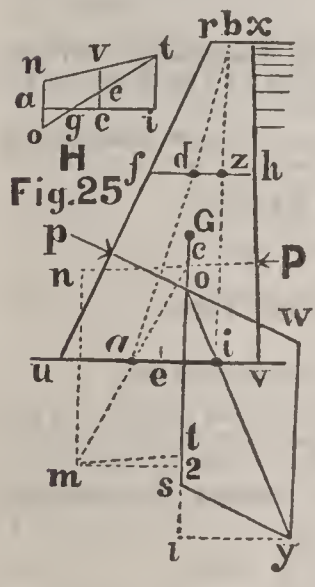

Fig. 25

Hence, when the water presses against *xv* the wall is less liable to fracture or crushing, and the earth foundation *uv* is more evenly loaded, and hence less liable to yield unequally so as to cause cracks in the wall. On this account *xv* is made the back of the wall, although the moment of stability of the wall is then only 2.2 (calling the overturning moment of the water 1), while if the water pressed against *ru* it would be 3, or 36 per cent greater.

For rules governing distribution of pressure, see Art 14, p 231*a*.

Art. 14. Distribution of pressure over the base uv, Figs 25, A, B, C and D. Let

uv = the length of the rectangular base, or of any rectangular surface common to two bodies which are pressed against each other; or (since the *width* of the surf is taken as 1) = the *area* of that surf.

P = the resultant of all the extraneous forces pressing one of the two surfs against the other. The *amount* of its pres is represented by the trapezoid $uosv$, Fig A; triangle uov, Fig B; *diff* of triangles $uod - dvs$, Fig C; triangle uod, Fig D; or by the parallelogram $untv$ in all four Figs. We confine ourselves to cases where P cuts the base in the *center* of its *width* measured at right angles to uv.

R = the resultant of all the resistances of the several points of the other surf. It is necessarily equal and opposite to P.

$un = \dfrac{untv}{uv} = \dfrac{P}{uv}$ = the *mean* pressure
$uo =$ (Figs A, B, C) the *maximum* pressure
$vs =$ (Figs A, B, C) the *minimum* pressure
} per unit of area uv.

This Art. applies equally whether the surf is hor, vert or inclined, and whether the forces are oblique to it (Figs A, B, C, D); or perp to it (Fig 71, p 357). If oblique, a portion of the resistance R is that of friction. See Art. 19, p 314.

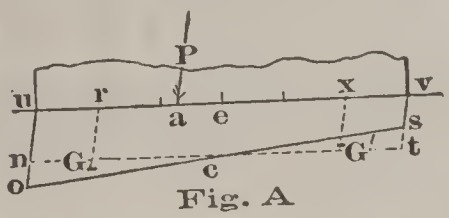

Fig. A

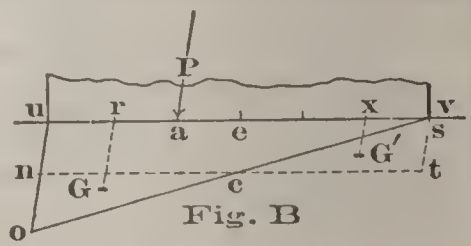

Fig. B

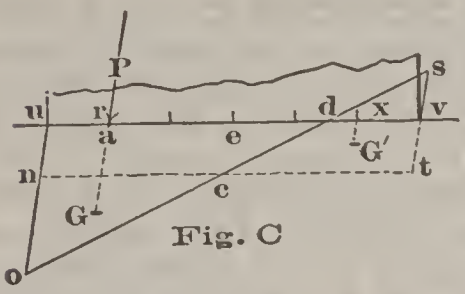

Fig. C

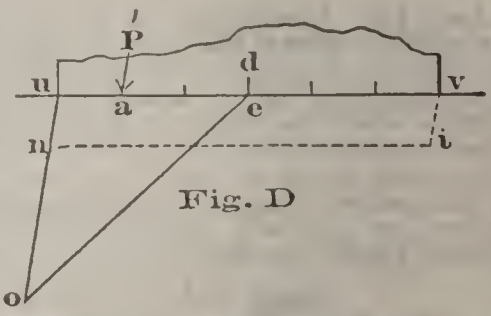

Fig. D

The parallelogram $untv$ represents the pres uniformly distributed over uv, as it would be if P cut uv at its center, e. The *intensity* of the pres, or its amount *per unit of area* of uv, would then be everywhere $= un$.

But when, as in our Figs, P cuts uv at any other point, a, its pres is *unequally* distributed; the nearer toe, u, receiving the max pres, uo.

If (Figs A, B) ea **does not exceed one-sixth** uv, then

$$\text{maximum pressure } uo = un\left(1 + \frac{6\,ea}{uv}\right)^{*}; \text{ and}$$

$$\text{minimum pressure } vs = (2\,un) - uo \quad \text{(See * and Figs.)}$$

If $ea =$ **one-sixth** uv (Fig B), this becomes

maximum pressure $uo = 2\,un$; and minimum pressure $vs = 0$.

If ea **exceeds one-sixth** uv (Fig C), vs is *less* than 0, or *minus; i e*, the toe v has a tendency to *rise*, and actually does so unless prevented either by the rigidity of the pressed part, $u\,d$, of the base, or by a *tensile* resistance (as by the adhesion of cement) in the remainder, dv. If it is thus resisted by tension in dv (Fig C) we still have

$$\text{maximum pressure } uo = un\left(1 + \frac{6\,ea}{uv}\right)^{*}; \text{ and}$$

$$\text{minimum pressure } vs = (2\,un) - uo,$$

---

* Demonstration. See Figs A, B, C. Suppose for a moment that R remains at e (so as to be uniformly distributed over uv, as represented by $untv$) while P is at a; then P and R form a *couple* (= one of the forces, as P, × their perpendicular distance from each other). This couple tends

vs being the tension, (or *minus* pres) per unit of area, at v. This rarely happens; for no ordinary mortar or cement can be depended upon to resist such tensions as might thus occur.

To find the neutral axis, d, or point of no pres; lay off uo and vs, and draw os. The *total* pressure, uod, on ud, is $= \frac{1}{2}\,uo.ud = P$ *plus* the tension, dvs, in $dv = P + \frac{1}{2}\,vs.dv$.

But when (as usual) uv is practically incapable of resisting tension, P is simply concentrated upon a portion, ud Fig D ($= 3\,ua$) of uv: $3\,ua$ is then practically the base; the remainder, dv, being idle. We then have

$$\text{mean pres on } 3\,ua = \frac{P}{3\,ua}$$

$$\text{maximum pressure } uo = 2 \times \text{mean pres on } 3\,ua = 2\,\frac{P}{3\,ua} = un \times \frac{2}{3\left(.5 - \frac{ea}{uv}\right)} \dagger$$

$$\text{pres at } d, \text{ and from } d \text{ to } v, = 0.$$

In any case, the maximum pressure uo should evidently not exceed the safe strength of the masonry or soil. Therefore (if, as usual, no part of the base is to be relied upon for *tension*) ea **in feet must not exceed**

$$uv, \text{ in ft} \times \left(.5 - \frac{2\,P}{3\,uv \times \text{safe load per sq ft}}\right); \ddagger$$

P and the safe load being in the same unit; as both in lbs, or both in tons, etc.

First class rubble in cement mortar, or good cement concrete, should be safe with 8 tons per sq ft, which limit will rarely be reached. Sound earth or gravel foundations, sunk to a depth sufficient to protect them from frost, rain, sliding etc, should be safe with from 2 to 4 tons per sq ft.

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to press u downward and raise v. This tendency causes (and is resisted by) a second couple, consisting of an increase, nco. of the pres on ue, and an equal decrease, tcs, of the pres on ev; *i e*, tcs, instead of pressing on ev as before, would be called upon to help resist the first couple. The points, r and x, at which the resultants of the resisting forces nco and tcs act, are opposite to the cens of grav, G and G', of those triangles. Each is therefore distant $\frac{2}{3}$ of half uv from e; and their dist rx from each other, measd along uv, is twice this, or $\frac{2}{3}\,uv$. The *resisting* couple must (p 351) be equal and opposite to the first one. Hence each of its forces, nco and tcs, must be $= \frac{\text{first couple}}{\text{leverage of } nco \text{ and } tcs} = \frac{P.ea}{\frac{2}{3}\,uv} =$ the *additional* pressure, nco, on ue. (If the dist apart of the 2 forces is thus measd along uv in *both* couples, said dists will be *in the same proportion to each other* as the *leverages*.) The *mean* additional pres on ue, or the middle ordinate of the triangle nco, is $= \frac{nco}{ue} = \frac{P.ea}{\frac{2}{3}\,uv} \div \frac{uv}{2}$; and the *max* additional pres, no, is = twice this, $= \frac{P.ea}{\frac{2}{3}\,uv} \div \frac{uv}{4} = \frac{P}{uv} \times ea \times \frac{3}{2} \times \frac{4}{uv} = un\,\frac{6\,ea}{uv}$ $\left(\text{because } \frac{P}{uv} \text{ is} = un\right)$. And

$$uo = un + no = un + \left(un\,\frac{6\,ea}{uv}\right) = un\left(1 + \frac{6\,ea}{uv}\right).$$

† Fig. D. $ua = \frac{uv}{2} - ea$; $3\,ua = 3\left(\frac{uv}{2} - ea\right)$.

$$uo = 2\,\frac{P}{3\,ua} = 2\,\frac{un.uv}{3\,ua} = un\,\frac{2\,uv}{3\left(\frac{uv}{2} - ea\right)} = un\,\frac{2}{3\left(.5 - \frac{ea}{uv}\right)}.$$

‡ Fig D. Safe load $= uo = un \times \frac{2}{3\left(.5 - \frac{ea}{uv}\right)}$; (see †) or $3\left(.5 - \frac{ea}{uv}\right) = \frac{2\,\frac{P}{uv}}{\text{safe load}}$:

$$\text{or } 3\,\frac{ea}{uv} = (3 \times .5) - \frac{2\,P}{uv \times \text{safe load}}; \quad \text{or } \frac{ea}{uv} = .5 - \frac{2\,P}{3\,uv \times \text{safe load}};$$

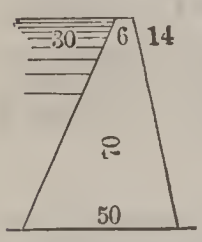
$$\text{or } ea = uv \times \left(.5 - \frac{2\,P}{3\,uv \times \text{safe load}}\right).$$

Art. 15. The points *a* and *i*, Fig 25, are called **centers of pressure** upon the base, or **centers of resistance** of the base. If similar points, as *d* and *z*, be found in the same way for other lines, as *f h*, by treating a part (as *r x h f*) of the wall as if it were an entire wall; a slightly curved line joining these points is called the **line of pressure.** Thus, *b a* is the line of pressure when the water presses against *x v*. Each point, as *d*, in *b a*, shows where any joint, as *f h*, drawn through that point, is cut by the resultant of all the forces acting upon said joint. *b i* is the line of pres when the water presses against *r u*. These lines do not show the *direction* of the resultants. Thus, at *a*, the latter is *c m*, not *b a*. The angle between the direction of the resultant and a line at right angles to the bed or joint, must be less than the angle of friction (p 355) of the materials forming the joint.

If from the end *m* or *y* of the resultant of the pressures upon any joint, we draw *m* 2 or *y l* hor, then *c* 2 or *o l* (as the case may be) measures the entire *vert* pres on that joint: and *m* 2 or *y l* measures the *hor* pres against the back of the wall, which tends to cause sliding at the same joint. If the direction of the resultant comes within the limit stated in the preceding paragraph, *m* 2 or *y l* will be less than the frictional resistance to sliding, which last is = *c* 2 (or *o l*) × the coeff of friction for the surfaces forming the joint. Hence sliding cannot take place.

Such sliding never occurs in the *masonry* of walls of ordinary forms. Good mortar, well set, aids to prevent sliding, but it is better not to rely upon it. See Rem 2, p 683. But entire walls have slidden on slippery foundations. See Art. 9, p 692.

Art. 16. In California is this dam of a mining reservoir, built of rough stone without mortar, founded on rock. Height, 70 feet; base, 50; top, 6; water-slope, 30 feet; outer-slope, 14. To prevent leaking the water-slope is only covered with 3-inch plank bolted horizontally to 12 by 12 inch strings, built into the stone-work. All laid with some care by hand, except a core of about one-fifth of the mass, which was roughly thrown in. Cost about $3 per cubic yard. It has been in use since 1860.



Rem. If a dam is compactly backed with earth at its natural slope, and in sufficient quantity to prevent the water from reaching the dam, the pressure against the dam will not be increased.

Art. 17. To find the thickness of a cylinder to resist safely the pressure of water, steam, &c, against its interior. If riveted, see next page.

Where the thickness is less than one-thirtieth of the radius, as it is in most cases, the usual formula

$$(1)\qquad \text{Thickness in inches} = \frac{\text{pressure}}{\text{safe strength}} \times \text{radius}^*$$

is employed. It regards the material as being subjected only to a direct tensile strain, which is sufficiently correct in such thin shells.

For somewhat greater pressures and thicknesses, Professor F. Reuleaux (Der Konstrukteur, p 52) gives

$$(2)\qquad \text{Thickness in inches} = \frac{\text{pressure}}{\text{safe strength}} \left(1 + \frac{\text{pressure}}{2 \times \text{safe strength}}\right) \times \text{radius}.^*$$

For very great pressures and thicknesses, as in hydraulic presses, cannons, &c, Professor Reuleaux (Konstrukteur, p 53) gives Lamé's formula:

$$(3)\qquad \text{Thickness in inches} = \left(\sqrt{\frac{\text{safe strength} + \text{pressure}}{\text{safe strength} - \text{pressure}}} - 1\right) \times \text{radius}.^*$$

The three formulæ give results as follows, pressures and strengths in lbs per square inch:

Diameter.	Radius.	Pressure.	Safe tensile strength.	Thickness, inches.		
				Formula (1).	Formula (2).	Formula (3).
20 inches.	10 inches.	50	10000	**.05**	.050125	.05
"	"	500	"	.50	**.5125**	.513
"	"	5000	"	5.00	6.25	**7.32**

The thicknesses given by the formulæ appropriate to the several pressures are printed in **heavy type.** It will be seen that in these cases the results differ but slightly, except for *very* great pressures.

* In all three formulæ take the *radius in inches*, and the pressure and strength in *pounds per square inch*.

Rem. 2. Want of uniformity in the cooling of thick castings makes them proportionally weaker than thin ones, so that in order to reduce thickness in important cases we should use only best iron remelted 3 or 4 times, by which means an ult cohesion of about 30000 lbs per sq inch may be secured. But even with this precaution **no rule will apply safely** in practice to cast cylinders whose thickness exceeds either about 8 to 10 ins, or the inner rad however small.

Under a pres of 8000 lbs per sq inch, **water will ooze through cast iron 8 or 10 ins** thick; and under but 250 lbs per sq inch, through .5 inch.

Table of thicknesses of single-riveted wrought iron pipes, tanks, standpipes, &c, by the above rule, to bear with a safety of 6 a quiet pressure of 1000 ft head of water, or 434 lbs pr sq inch; the ult coh of fair quality plate iron being taken at 48000 lbs per sq inch, or at 8000 lbs for a safety of 6; which is farther reduced to 8000 × .56 = 4480 lbs, to allow for weakening by rivet holes; for **single-riveted** cyls have but about .56 of the strength of the solid sheet; and **double-riveted** ones about .7. With the above pres and other data, the rule here leads to thickness = .1016 × inner rad in ins.

For a similar table for tanks, see p 803; and for **cast iron** and **lead** pipes, foot of this, and top of next page. (Original.)

Di. Ins.	Ths. Ins.	Di. Ins.	Ths. Ins.	Di. Ins.	Ths. Ins.	Di. Ins.	Ths. Ins.	Di. Ins.	Ths. Ins.	Di. Ins.	Di. Ft.	Ths. Ins.
.5	.025	5	.254	16	.813	30	1.52	60	3.05	120	10	6.09
1.0	.051	6	.305	18	.914	33	1 68	66	3.35	132	11	6.70
1.5	.076	8	.406	20	1.016	36	1.83	72	3.66	144	12	7.31
2.0	.102	10	.508	22	1.117	42	2.13	84	4.27	192	16	9.75
3.0	.152	12	.609	24	1.219	48	2.44	96	4.88	240	20	12.19
4.0	.203	14	.711	27	1.371	54	2.74	108	5.49	288	24	14.63

For a less head or pressure, or for any safety less than 6, it is safe and near enough in practice, to reduce the thickness of wrought iron cyls in the same proportion as said head, pres, or safety is less than the tabular one.

Double-riveted cylinders, Fairbairn says, are about 1.25 times as strong as single-riveted. Hence they may be one-fifth part thinner. **Lap-welded ones** are nearly 1.8 times as strong as single-riveted; and hence may be only .56 as thick.

Many continuous miles of **double-riveted pipes in California** have been in use for years with safetys of but 2 to 2.6. In one case the head is 1720 ft, with a pres of 746 lbs per sq inch; diam 11.5 ins; thickness, .34 inch; safety, 2.6 by rule p 232 for such iron as in our table.

Cast iron city water pipes must be thicker than required by formula (1), p 232, in order to endure rough handling and the effects of "water-ram" (due to sudden stoppage of flow, see second Rem, p 234), and to provide against irregularity of casting and the air bubbles or voids to which all castings are more or less liable. In the following table the ultimate tensile strength of cast iron is taken at 18,000 lbs per square inch. Column A gives thicknesses by Mr. J. T. Fanning's formula (Hydraulic Engineering, p 454).

$$\left.\begin{matrix}\text{Thickness}\\ \text{in inches}\end{matrix}\right\} = \frac{(\text{pres, lbs per sq in} + 100) \times \text{bore, ins}}{.4 \times \text{ultimate tensile strength}} + .333\left(1 - \frac{\text{bore, ins}}{100}\right).$$

These correspond with average practice. The addition of 100 lbs to the pres is made in order to allow for water-ram. Column B gives thicknesses by formula (1), p 232, taking coefficient of safety = 8 (thus making safe tensile strain = 2250 lbs per square inch) and adding three-tenths of an inch to each thickness given by the formula:

Head in feet	50		100		200		300		500		1000	
Pressure, lbs per sq in.	21.7		43.4		86.8		130		217		434	
Bore, ins.	**Thickness of pipe, in Inches.**											
	A	**B**	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**
2	.36	.31	.37	.32	.38	.34	.39	.36	.42	.40	.48	.51
3	.37	.31	.38	.33	.40	.35	.42	.40	.45	.45	.54	.60
4	.39	.32	.40	.34	.42	.38	.45	.42	.50	.50	.61	.72
6	.41	.33	.43	.36	.47	.42	.50	.48	.57	.60	.75	.94
8	.45	.34	.47	.38	.52	.47	.57	.55	.66	.70	.90	1.15
10	.47	.35	.50	.40	.56	.50	.62	.60	.74	.81	1.04	1.35
12	.49	.36	.53	.42	.60	.54	.67	.66	.82	.91	1.18	1.57
16	.55	.38	.60	.46	.70	.62	.79	.77	.98	1.10	1.46	2.00
18	.57	.39	.63	.48	.74	.65	.85	.84	1.06	1.21	1.60	2.20
20	.61	.40	.67	.50	.79	.68	.91	.90	1.15	1.31	1.75	2.50
24	.66	.42	.73	.53	.87	.77	1.02	1.01	1.30	1.51	2.03	2.84
30	.74	.45	.83	.59	1.01	.89	1.19	1.19	1.55	1.82	2.46	3.47
36	.82	.47	.93	.65	1.15	1.01	1.36	1.37	1.80	2.12	2.88	4.11
48	.98	.53	1.13	.77	1.42	1.24	1.70	1.73	2.28	2.73	3.73	5.38

Table of thickness of lead pipe to bear internal pressures with a safety of 6; taking the ultimate cohesion of lead at 1400 lbs per sq inch. By rule on p 232.

Rem. Although these thicknesses are safe against quiet pressures, they might not resist shocks caused by too sudden closing of stop-cocks against running water. See Service pipes, p 299.

Bore in Inches.	Heads in Feet. 100	200	300	400	500	Bore in Inches.	Heads in Feet. 100	200	300	400	500
	Pres in lbs per sq inch. 43.4	86.8	130	174	217		Pres in lbs per sq inch. 43.4	86.8	130	174	217
	Thickness in Inches.						Thickness in Inches.				
¼	.026	.055	.089	.128	.171	1	.102	.221	.357	.511	.682
⅜	.038	.083	.134	.192	.256	1¼	.127	.276	.447	.639	.853
½	.051	.111	.179	.256	.341	1½	.153	.332	.536	.767	1.02
⅝	.064	.138	.223	.320	.427	1¾	.178	.387	.626	.895	1 20
¾	.076	.166	.268	.383	.512	2	.204	.442	.714	1.02	1.36
⅞	.089	.193	.313	.447	.597						

Rem. The valves of water-pipes must be closed slowly, and the necessity for this precaution increases with their diams. Otherwise the sudden arresting of the momentum of the running water will create a great pressure against the pipes in all directions, and throughout their entire length behind the gate, even if it be many miles: thus endangering their bursting at any point. Hence stop-gates are shut by screws, which prevent any very sudden closing; but in large diams even the screws must be worked very slowly to avoid bursting.

Art. 18. The buoyancy of liquids. When a body is placed in a liquid, whether it float or sink, it evidently displaces a bulk of the liquid equal to the bulk of the immersed portion of the body; and the body in both cases, and at any depth, and in any position whatever, is buoyed up by the liquid with a force equal to the wt of the liquid so displaced. Thus, if we immerse entirely in water a piece of cork c, c, Fig 26, or any body of less sp gr than water, the cork will by its wt, or force of gravity, tend to descend still deeper; but the upward buoyant force of the water, being greater than the downward force of gravity of the cork, will compel the latter to rise with a force equal to the diff between the two. In this case, the cork receives a total downward pres equal to the wt of the vert column of water above it, shown by the vert lines in vessel 1; and a total upward pres equal to the wt of the column shown in vessel 2. The diff between these two columns is evidently (from the figs) equal to the bulk of the cork itself; therefore the diff between their wts or pressures, (or, in other words, the buoyancy of the water,) is equal to the wt or pres of the water which would have occupied the place of the cork: or, in other words, of the water which is displaced by the cork. This *diff*, or buoyancy, will plainly be the same at any depth whatever of *entire* immersion. Now the cork, if left to itself, will continue to rise until a portion of it reaches above the surf, as in vessel 3; so that the downward pressing column ceases to exist; and the cork is then pressed downward only by its own wt. But as it now remains stationary, we know (from the fact that when two opposite forces keep a body at rest, they must be equal to one another) that the upward pres of the water must be equal to the wt of the cork. But the upward pres of the water arises only from the shaded column shown in vessel 3; and this column is (as in the case of total immersion) equal to the bulk of water displaced. Therefore, *in all cases*, the buoyancy is equal to the wt of water displaced; and when the body *floats* on the surf, the buoyancy, or the wt of water displaced, is also equal to the wt of the body itself.

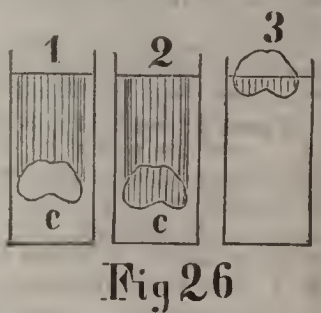


Fig 26

If the immersed body c, c, be of iron, or any other substance specifically heavier than water, the diff between the upward and downward pres will of course remain the same; or equal to the wt of water displaced. But the wt of the body is now greater than that of the water which it displaces; or, in other words, the downward force of gravity of the body is greater than the upward buoyant force of the displaced water; and therefore the body descends, or sinks, with a force equal to the diff between the two. Thus, if the body be a cub ft of cast iron, weighing 450 lbs, while a cub ft of fresh water weighs 62½ lbs, the iron will descend with an effective force of only 450 − 62½ = 387.5 lbs.

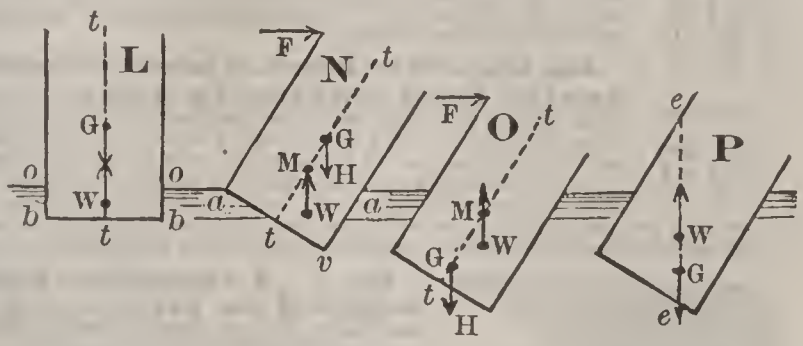
If the immersed body has the same sp gr as the fluid, it will neither rise nor sink; but will remain wherever it is placed; because then the wt of the body, and the buoyancy of the water, are equal.

The air also buoys bodies upward to an extent equal to the wt of air displaced: therefore, although a pound of iron, and a pound of feathers, weighed in the air, will balance each other yet in the exhausted bell-glass of an air-pump the feathers will outweigh the iron, by as much as the bulk of air which they displaced outweighs the bulk of air displaced by the iron.

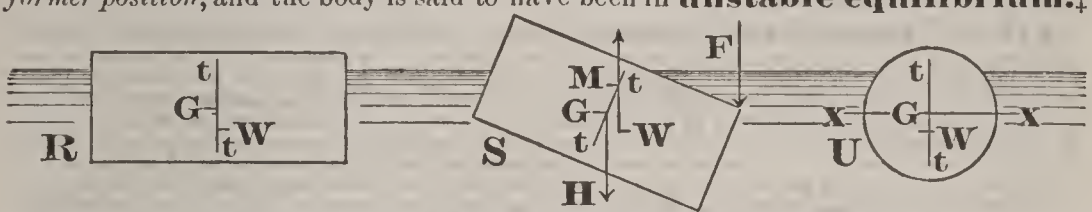
A balloon rises in the air on the same principle that cork rises in water. Its ascending force is equal to the diff between its wt when full of gas, and the wt of the bulk of air which it displaces. The balloon does not actually tend to rise, but to descend; but the air being, bulk for bulk, heavier than the balloon, pushes the latter upward with more force than the gravity, or the wt of the balloon, exerts to bring it down. So also warm smoke has no tendency in itself to rise. It is *pushed up* by the heavier cold air. No substance tends to rise; but all tend downward toward the center of the earth.

The downwd force of grav may be regarded (p 347) as concentrated at the cen of grav G of a floating body. The upwd pres, or buoyancy,† of the water may similarly be regarded as acting at the cen of gr W of the displaced water.* W is also called the **center of pressure,** or **of buoyancy,** of the water; and a vert line drawn through it is called the **axis,** or **vertical, of buoyancy,** or **of flotation.** Ordinarily,‡ W shifts its position with every change in that of the body. Thus in L it is at the cen of gr of the rectangle *o o b b*; and in N at that of the triangle *a a v*.

When a floating body, L, P or R, is at rest, and undisturbed by any third force, as F, it is said to be in **equilibrium,** and G and W are then in the same vert line *t t* Figs L and R, or *e e* Fig P; which line is called the **axis,** or **vertical, of equilibrium.**§



When a third force, as F, causes the axis of equilib to lean, as in Figs N, O and S, then if a vert line be drawn upwd from the cen W of buoy, the point M where said line cuts said axis, is called the **metacenter** of the body.‖ G and W are then no longer in the same vert line;‡ and the two opp and vert forces, grav and buoy, acting upon those points respectively, form a "couple" (Case 3, p 351); and, when the third force F is removed, they no longer hold the body in equilib, but cause it to rotate. If (as in Figs O and S) the positions of G and W are then such that the metacenter M is *above* the cen of gr G, this rotation will tend to *restore the body to its former position*, and the body is said to have been (before the application of the third force F) in **stable equilibrium.**¶ But if (as in N) M is *below* G the direction of rotation is such as to *upset* the body, by causing it to *depart further from its former position*, and the body is said to have been in **unstable equilibrium.**‡



The tendency or moment in ft-lbs of a floating body either to upset or to right itself, is,(Case 3, p 351,)

= the wt of the body (or the equal upwd pres of the water) in lbs × the hor dist between W M and G H, Figs N, O and S, in ft.

The third force F may of course be so great as to overpower the tendency of the body to right itself. Thus, a ship may upset in a hurricane, although judiciously loaded and ballasted for ordinary winds. A hor section of a body at water-line is called its **plane of flotation.**

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* **The body is in fact acted upon by other forces,** such as the hor pressures of the water against its immersed portions; but as all of these in any one given direction are balanced by equal ones in the opposite direction, they have no effect upon the forces G and W. It is also acted upon by the air, which presses it downwards with a force of 14.75 lbs per sq inch: but this is balanced by an equal pres of the surrounding air upon the surface of the water, and which is transmitted (art 7, vert upwards against the immersed bottom of the floating body.

† **This buoyancy is made up** of the parallel upward pressures of the innumerable vert filaments of the displaced water as shown by Fig 26, and the **axis of flotation** is their resultant, as in the case of parallel forces.

‡ The shape of a body (as that of a sphere or cylinder U) may be such that the position of its cen of buoy W, relatively to that of its cen of gr G, is not changed by the rotation of the body about a given axis (as any axis of the sphere or the *longitudinal* axis of the cyl), but remains constantly in the same vert line with G, so that the body, in rotating, remains in equilib. Such a body is said to be in **indifferent equilibrium** about said axis. But if a cyl U be made to rotate about its *transverse* axis *x x*, it plainly comes under the remarks on Figs R and S, and may (before rotating) be in either stable or unstable equilib about that axis according to the way in which its wt is distributed.

‖ This metacenter shifts its position on the line *t t* according to the inclination of the latter.

§ **Uneven loading,** instead of a third force, may cause a vessel at rest to lean as at P; and yet the vessel so leaning may be in equilib: for its axis *e e* of equilib may be vert, although not coinciding with the **axis of symmetry** of the vessel, as it does at *t t* in L.

¶ In *floating* bodies, this may sometimes (as in Figs R and S) be the case even when the *cen of buoy* W (*not* the *metacenter*) is *below* the cen of gr G; because, when the body is forced to lean, W moves to another point in it, and this point may be such as to bring M above G. W is always below G in bodies of uniform density, floating at rest, if any part of the body is above water. When such bodies are entirely submerged, W and G coincide.

Fig 27

Art. 19. A body lighter than water, if placed at the bottom of a vessel containing water, will not rise unless the water can get under it, to buoy it, or press it upward, as the air presses a balloon or smoke upward. Thus, if one side of a block of light wood, *perfectly* flat and smooth, be placed upon the similarly flat and smooth bottom of a vessel, and held there until the vessel is filled with water, the downward pres will keep it in its place, until water insinuates itself beneath through the pores of the wood. But if the wood be smoothly varnished, to exclude water from its pores, it will remain at the bottom.

On the other hand, a piece of metal may be prevented from sinking in water, by subjecting it to a sufficient *upward* pres only, while the downward pres is excluded. Thus, if the bottom of an open glass tube, *t*, Fig 27, and a plate of iron *m*, be made smooth enough to be water-tight when placed as in the fig; and if in this position they be placed in a vessel of water to a depth greater than about 8 times the thickness of the iron, the upward pres of the water will hold the iron in its place, and prevent its sinking; because it is pressed upward by a column of water heavier than both the column of air, and its own weight, which press it downward. On this principle iron ships float.

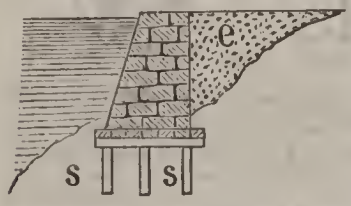


Fig 28

Rem. 1. **A retaining-wall, as in Fig 28, founded on piles,** may be strong enough to resist the pres of the earth *e* behind it, in case water does not find its way underneath; and yet may be overthrown if it does; or even if the earth *s s* around the heads of the piles becomes saturated with water so as to form a fluid mud. In either case, the upward pres of the water against the bottom of the wall will virtually reduce the wt of all such parts as are below the water surf, to the extent of 62½ lbs per cub ft; or nearly one-half of the ordinary wt of rubble masonry in mortar.

Rem. 2. Although the piles under a wall, as in Fig 28, may be abundantly sufficient to sustain the wt of the wall; and the wall equally strong *in itself* to resist the pres of the backing *e*; yet if the soil *s s* around the piles be soft, both they and the wall may be pushed outward, and the latter overthrown by the pres of the backing *e*. From this cause the wing-walls of bridges, when built on piles in very soft soil, are frequently bulged outward and disfigured. In such cases, the piling, and the wooden platform on top of it, should extend over the whole space between the walls; or else some other remedy be applied.

Art. 20. Draught of vessels. Since a *floating* body displaces a wt of liquid equal to the wt of the body, we may determine the wt of a vessel and its cargo, by ascertaining how many cub ft of water they displace. The cub ft, mult by 62½, will give the reqd wt in lbs. Suppose, for instance, a flat-boat, with vert sides, 60 ft long, 15 ft wide, and drawing unloaded 6 ins, or .5 of a ft. In this case it displaces $60 \times 15 \times .5 = 450$ cub ft of water; which weighs $450 \times 62\frac{1}{2} = 28125$ lbs; which consequently is the wt of the boat also. If the cargo then be put in, and found to sink the boat 2 ft *more*, we have for the wt of water displaced by the cargo alone, $60 \times 15 \times 2 \times 62\frac{1}{2} = 112500$ lbs; which is also the wt of the cargo. So also, knowing beforehand the wt of the boat and cargo, and the dimensions of the boat, we can find what the draught will be. Thus, if the wt as before be 140625 lbs, and the boat 60×15, we have $60 \times 15 \times 62\frac{1}{2} = 56250$; and $\frac{140625}{56250} = 2.5$ ft the required draught. In vessels of more complex shapes, as in ordinary sailing vessels, the calculation of the amount of displacement becomes more tedious; but the principle remains the same.

Art. 21. Compressibility of liquids. Liquids are not entirely incompressible; but for most engineering purposes they may be so considered. The bulk of water is diminished but about one-thousandth part by a pres of 324 lbs per sq inch, or 22 atmospheres; varying very slightly with its temperature. It is perfectly elastic; regaining its original bulk when the pres is removed.

HYDRAULICS.

Art. 1. Hydraulics treats of the flow or motion of water through pipes, aqueducts, rivers, and other channels; also through orifices or openings of various kinds; of machinery for raising water; as well as that in which water furnishes the moving power. The science of hydraulics, in many of its departments, is but imperfectly understood; therefore, some of the rules given on the subject are to be regarded merely as furnishing close approximations to the truth.

On the flow of water through pipes.

Inasmuch as the experiments on which the following rules are based, were made with pipes carefully laid in straight lines; and perfectly free from all obstructions to the flow of the water, some allowance must in practice be made for this circumstance. Workmen do not lay long lines of pipes in perfectly straight lines; it is almost impossible to avoid very numerous, although slight deviations, both vert and hor; the soil itself, in which the pipes are imbedded, especially when in embkt, will settle unequally; especially in streets liable to heavy traffic, which not only frequently deranges, but occasionally breaks water pipes whose tops are 3 or 4 ft below the surf. The material used for caiking the joints, may be carelessly left projecting into the interior, and thus cause obstructions; the water is frequently muddy, or is impregnated with certain salts, or gases, which form deposits, or incrustations, which materially impede the flow. Moreover, the pipes themselves are not cast perfectly straight, or smooth, or of uniform diam; and irregular *swellings*, by producing eddies.

retard the flow as well as *contractions;* and accumulations of air do the same. Under the most favorable circumstances, therefore, it is expedient to make the diams of pipes, even for temporary purposes, sufficiently large to discharge *at least* 20 per ct more than the quantity actually needed; and if there is occasion to anticipate deposit, or incrustation, a still larger allowance should be made in permanent pipes, especially in those of small diam; because in them the same thickness of incrustation occupies a greater *comparative* portion of the area. Perhaps it would be best to allow an equal increase, of say from ½ to 1½ inch, to each diam, whether great or small; inasmuch as the thickness of incrustation will be the same for all diams, or nearly so. The cost of pipes does not increase as rapidly as their discharging capacities; thus, if the diam be increased only $\frac{1}{10}$ part, the disch will be increased about 25 per cent; if ⅙ part, nearly 50 per cent; if ⅓ part, the disch will be doubled.

Within these limits, the increase of thickness for the larger diams, and the increased expense of laying, will add but little to the cost; which will therefore augment only a little more rapidly than the diams.

The increased diam involves no waste of water; since the disch may be regulated by stopcocks.

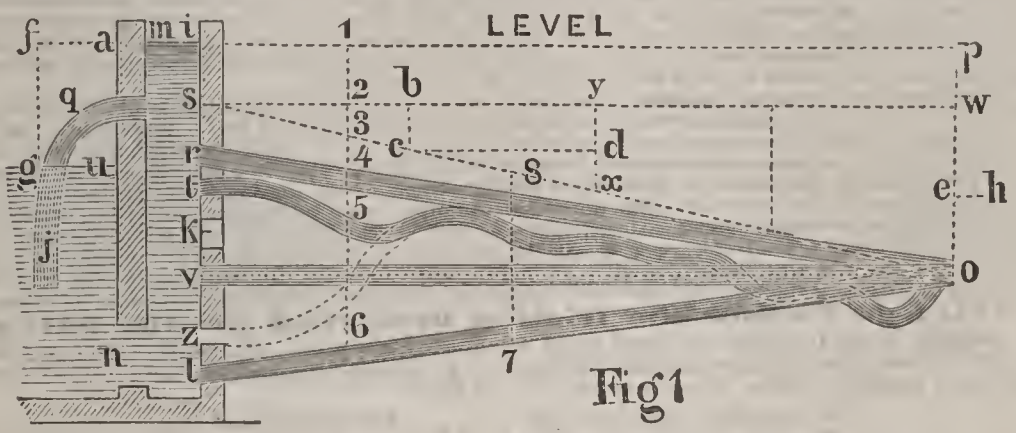


Fig 1

The term **HEAD** or **TOTAL HEAD** of water, as applied to the flowage of water through canals, pipes, or openings in reservoirs, &c, means the *vert* dist *i v* or *p o*, Fig 1, from the level surf, *m i*, of the water in the reservoir, or source of supply, to the center (or more properly to the cen of grav) *o*, of the orifice (whether the end of a pipe, *r o*, *t o*, *v o*, *z o*. *l o*; or any other kind of opening) through which the disch takes place freely, *into the air;* or the vert dist *a u*, or *f g*, from the same surf, *m i*, to the level *surf*, *g u*, of the water in the lower reservoir; when the disch takes place *under water*. Thus, in the case of disch *into the air*, the vert dist *i v* or *p o*, is the total head for either of the pipes *r o*, *t o*, *v o*, *z o*, or *l o*; and *i k* is the head for the orifice, *k*, in the side of the reservoir. And for disch *under water*, *a u*, or *f g*, is the head for either the pipe *j*, or the opening *n*; without any regard whatever to their depths below the surf of the lower water; which, according to the older authorities, do not at all affect their disch.

A portion of a pipe may have a head greater than the total head of the entire pipe. Thus the point 6 in the pipe *l o*, has a head 6 1; while the entire pipe has only the head *p o*.

Both in theory and in practice it is immaterial as regards the vel, and the quantity of water discharged, whether the pipe is inclined downward, as *r o*, Fig 1; or hor, as *v o*; or inclined upward, as *l o*; provided the total head *p o*, and also the length of the pipe, remain unchanged. If one pipe is longer than another, its sides will evidently present more friction against the water, and thus diminish the vel and the quantity of disch. The inclined pipes, *r o*, *l o*, being of course a little longer than the hor one *v o*, will therefore each disch a trifle less water; but if the hor one were extended slightly beyond *o*, so as to give it the same length as the others, then each of the three would disch the same quantity in the same time.

Art. 1 *a*. Divisions of the Total Head. In any pipe, as *s o*. *r o*, *t o*. *v o*, *z o*, or *l o*, Fig 1, the total head has three distinct duties to perform; 1st, to overcome the resistance to *entry* at *s*, *r*, *t*, *v*, *z*, or *l*; 2d, to overcome the resistances *within* the pipe; and, 3d, to give to the water, entering the pipe, the uniform *velocity* with which it actually flows.

For convenience, we regard the total head as divided into three portions, corresponding to these duties; namely, 1st, the *entry* head; 2d, the *resistance*, or *friction*, head; and, 3d. the *velocity* head.

Art. 1 *b*. The **velocity head** is the height through which a body must fall, in vacuo, to acquire the vel with which the water actually flows into the pipe. It is therefore $= \frac{v^2}{2g}$, in which v is the vel in ft per sec; and g is the acceleration of gravity, or 32.2

This head will be found in the table, p 258, opposite to the actual vel.

Art. 1 *c*. Experiment shows that, with the usual sharp-edged entry, the **entry head** is, near enough for practice, = half the vel head. If the entry is shaped like Fig 7, scarcely any entry head will be required. But, in pipes longer than about 1000 diameters, the entry head bears so slight a proportion to the total head, that this advantage is of but little importance. It becomes more apparent in shorter pipes.

Art. 1 *d*. In Fig 1 we will assume that for any of the pipes, *i s* represents the sum of the vel and entry heads. Then the remainder *s v*, or *w o*, of the total head, is the **friction head;** or the head which is just sufficient to *balance* the friction and other resistances *within* the pipe, and thus leave for the vel head nothing to do but to impart vel to

the unresisting water. If, by shortening the pipe, or by smoothing its inner surf, we diminish the total friction, then a less friction head will be required; but the vel will, at the same time, be increased, and this will require a greater vel head, and entry head, so that the three together make up the total head, as before. Since the *friction* is equal to the force or head reqd to overcome it, it also is represented by *w o*.

Art. 1 *e*. The friction head may as in *v o*, *z o*, and *l o*, Fig 1, be all *above* the entrance to the pipe, and therefore *outside* of the pipe; or, as in a pipe laid from *s* to o, it may be all *below* the entrance, and *within* the pipe; or, as in *r* o and *t* o, it may be partly above, and partly below, the entrance; and therefore partly within, and partly without, the pipe. The vel and disch, *after the pipe is filled*, are not affected by this difference in position of the entry end; but the *pressures* in the pipe, and the vels *while the water is filling an empty pipe*, are affected by it, as explained in Arts 1 *l* and 1 *o*.

Art. 1 *f*. But it is necessary that the entry end of the pipe should be placed so far below the surf *m i*, that there shall be left, above the cen of grav of the entry end, at least a head, *i s*, sufficient to perform the duties of the entry and vel heads. If the entry end of any of the pipes be raised above *s*, a portion of the *vel* head will be *in* the pipe. In other words, the head *in* the pipe will be more than sufficient to overcome the resistances in the pipe: and the surplus will act as vel head, and will give greater vel to the water *in* the pipe. The *reduced* head thus left above the entry end will plainly be insufficient to maintain the supply for the *greater* vel, and the pipe will run only partly full.

In ordinary cases of pipes of considerable length, the sum of the entry and vel heads theoretically required, is but a small portion of the total head, and rarely exceeds a foot. Indeed, in a pipe of considerable diameter, the upper half of its cross section at the entry end may often be more than enough to provide sufficient entry and vel heads above the cen of grav of said cross section: so that the top of the entry end might, so far as these considerations alone are concerned, project above the surf of the water in the reservoir. But the end of the pipe should in practice always be entirely below the surf; otherwise air and floating impurities will be drawn into it, and cause obstructions. Moreover, the water surf of reservoirs is always liable to considerable changes of height; and the entry end of the pipe must be placed at such a depth that the water can flow into it with sufficient vel when at its *lowest* stages. As before stated, this will cause no diminution or increase of disch.

Art. 1 *g*. To find the friction head reqd for any part of a pipe: knowing the fric head reqd for the whole pipe. Since the friction, in a pipe of uniform diam, is (other things being equal) in proportion to its length; and since *w o*, Fig 1, represents the total friction, or reqd friction head, we have

Total length of the pipe	:	Length of the given portion	: :	*w o*	:	The friction head reqd for that portion.

Or, having drawn *w* o by scale, *s w* hor, and *s* o;

Total length of the pipe	:	Length of the given portion	: :	*s o*	:	A dist, as *s c*, to be laid off from *s* on *s o*.

Or : : *s w* : A dist, as *s b*, to be laid off from *s* on *s w*.

Then a vert line, as *b c*, drawn from *b* or c, and joining *s w* and *s* o, gives by scale the friction head reqd.

Art. 1 *h*. If the pipe is straight, as *r o*, *v o*, *l o*, the friction in any part *beginning at the reservoir*, as *l* 6 in the pipe *l* o, may be found at once by drawing a line 6 1 vert upward from the axis of the pipe at 6. The line 2 3 will then give the friction in *l* 6. It also gives the friction in *r* 4, or in that part of *v o* which lies between *v* and the dotted line 1 6. It must be remembered that all the pipes in Fig 1 are supposed to be of the same actual length. They would thus end at different points o, and strictly, a separate diagram must be drawn for each pipe. In a part of the pipe *not* beginning at the reservoir, as in *r* o, *v* o, or *l* o, between points vertically under c and *x*, the amount of friction is given by the line *d x*, for it is plainly = *y x* − *b* e.

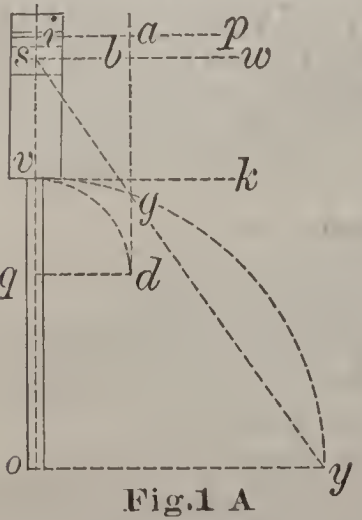


Fig. 1 A

Art. 1 *j*. If the pipe is vert, as *v o*, Fig 1 A; let *i s* (on its axis *i o*) represent, as before, the sum of the vel and entry heads. From *s*, *v*, and *o*, respectively, draw hor lines *s w*, *v k*, and *o y*, making *o y* = *v o*. Draw the oblique line *s y*. Then, to find the friction in any part, as *v q*, beginning at the reservoir: from *q* lay off *q d* hor, and equal to *v q*, and draw the vert line *a d* crossing *s y* at *g*. Then *b g* will give the friction in *v q*.

Art. 1 *k*. If the pipe is curved, and if the curvature is uniformly distributed along its length, or so slight that it may be neglected; the friction heads reqd for the several portions of the pipe, may be found in the same way as for straight pipes, as in Art 1 H. Otherwise they must be found by proportion, as in Art 1 G.

Art. 1 *l*. While water is filling an empty pipe, the excess of the total head above the requirements of friction, &c, gives to the water a **greater vel** than it has after the pipe is filled; but this gradually decreases as the advancing water encounters the friction along the increased lengths of pipe filled; and finally becomes least when the water fills the whole length, and begins to flow from the disch end, *o*. But if only the vel and entry heads are left above the entry end, as in a pipe laid from *s* to *o*, there will plainly be no such excess of total head, and, consequently, no such change of vel during the filling of the pipe.

When a pipe of uniform diameter is flowing full, and is entirely open at its discharge end, **the vel throughout the pipe is equal to that at the outflow.**

But if the opening *o* be contracted, but so shaped (see Fig 7, p 260) that the resistance of its edges may be neglected; then

Area of cross-section of pipe . Area of cross-section of outflow . . vel of outflow . vel throughout the pipe.

Art. 1 *m*. Of the outward, or bursting, pressure of water in pipes. When any pipe is full of water *at rest*, the *entire* head acts as *pressure* head, and the pres is greater than when the water is flowiug through it. Thus, at the point 4 iu the pipe *r o*, Fig 1, it is that due to the head 4 1; at the point 6 in the pipe *l o*, it is that duc to the head 6 1; at the point *o*, in any of the pipes, that due to o *p*. Therefore its amount in lbs per sq inch is = Total head in ft × .434, as per rule p 224.

But if an *opening* be made *in any part* of the pipe, or of the reservoir, the water will of course be set in motion, and although the level in the reservoir be maintained, the pres *in all parts* of the pipe *and of the reservoir* will be reduced. The greater the area, and number, of such openings, the greater will be the vel, and the greater will be the reduction of pres. A part of this reduction is, in all cases, due to the vel of the water in the pipe, which requires the consumption of a part of the head (the vel head) for its maintenance. Another part is due to resistance to entry, and the remainder is due to friction within the pipe.

Art. 1 *n*. The foregoing is true of all other vessels, as well as of pipes. Thus, if an opening o be made anywhere in a vessel V, Fig 1 B, the pres throughout will be reduced, and the water in the pipes *p* and *g*, will no longer stand at the same level *i* as that in the vessel, although the vessel be kept full; but will fall to some lower one, *l*, which will depend for its height upon the relative areas of cross section of the orifice o and of the vessel V. For this reason, two equal openings in a reservoir, at equal heights, will always give less than double the disch that one of them alone would give: but where the area of the reservoir is very large in comparison with that of the openings, the discrepancy is inappreciable. It follows, also, from the above, that a vessel V, Fig 1 C, although kept constantly full of water, will *weigh less* when the orifice *o* is open, than when it is closed; because the pres on the *bottom* of the vessel, as well as that on the sides, is reduced by the opening of the orifice.

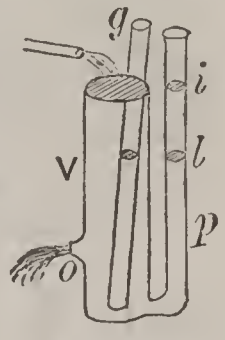


Fig.1 B

Art. 1 *o*. The pressure head of running water upon any point in a pipe between the orifice and the reservoir, is

$$= \left\{\begin{array}{l}\text{the } \textit{total}\\ \text{head on}\\ \text{that point}\end{array}\right\} \textit{minus} \left(\begin{array}{l}\text{the head}\\ \text{due to the}\\ \textit{vel} \text{ at}\\ \text{that point}\end{array} + \begin{array}{l}\text{the}\\ \text{entry}\\ \text{head}\end{array} + \begin{array}{l}\text{the head consumed}\\ \text{in overcoming } \textit{re-}\\ \textit{sistances} \text{ in the pipe}\\ \textit{between the reservoir}\\ \textit{and the point.}\end{array} \right)$$

Thus, at the point 6 in the pipe *l o* Fig 1, the pres head is = 3 6 = 1 6 *minus* (1 2 + 2 3); 1 2 being = *i s*, or = the sum of the vel and entry heads. At 4 in the pipe *r o*, the pres head is only 3 4 = 1 4 *minus* (1 2 + 2 3.) **In a straight inclined or hor pipe,** the pres head at any point is thus given by the length of a vert line drawn from the point to the line s *o*.

Art. 1 *p*. If the pipe has gentle curves, or if the curvature is uniformly distributed along the length of the pipe, the pres head may be found in the same way as for a straight pipe in Art 1 *o*. But if the curves are of considerable extent, and unevenly distributed along the pipe, first find, by Art 1 *g*, the friction head reqd for that part of the pipe between the reservoir and the point in question. Then find the pres head by the above formula.

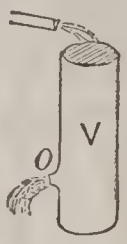


Fig.1 C

Art. 1 *q*. For a vert pipe *v o* Fig 1 A, draw the diagram as directed in Art I *j*. Then *g d* (= *i q* or *a d* − (*a b* + *b g*)) gives the pres head at *q*. At the point *o* in any of the pipes, Fig 1 or Fig 1 A, the pres head is zero, supposing the pipe to be entirely open at that point.

In a pipe laid along the line *s o*, Fig 1, the pres head will be zero at all points.

Art. 1 *r*. In a pipe *r m* or *r m'*, Fig 1 D, closed at its end, *m* or *m'*, but **having an orifice *o* between its end and the reservoir,** the pres at any point, *x*, between the orifice and the closed end, is equal to the pres in the tube opposite the orifice, *plus* that arising from the head *o' x* between the orifice and the point. If the point, as *x'*, is *higher* than *o*, this head is *negative*, and the pres at *x'* will be *less* than that opposite *o*. If the area of the orifice *o* is such as to pass the entire flow of the pipe without obstruction, the pres opposite o is zero. Otherwise there will be a pres at *o* varying with the amount of obstruction to outflow at that point.

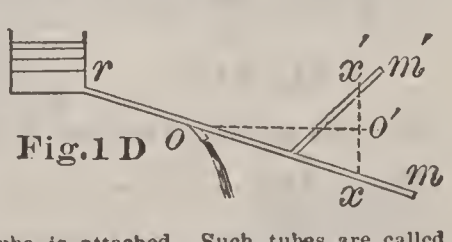


Fig.1 D

Art. 1 *s*. If a vert or oblique pipe be inserted into one containing water under pres, the water will rise in the first, and the *vert* ht to which it rises is the head producing the pres at the point where the tube is attached. Such tubes are called **piezometers,** or pressure-measurers. In order that the ht of the water in them may be known, they are made of glass, at least in that part of their length where the surface of the column is likely to be, or else they are provided with a floating index.

The piezometer is used for detecting the positions of obstructions in a line of pipes. If the water in the piezometer is found at any time to *fall below* its proper level, it shows that the pressure in the main pipe at that point has become *diminished* by some obstruction in the interval *between it and the reservoir*; but if the water *rises above* its proper level, it indicates that the pressure there has been *increased* by an obstruction *beyond* the piezometer. **By having several piezometers,** the point at which an obstruction has taken place can be approximately ascertained, and thus much of the labor of searching for it, avoided.

Art. 1 *t*. If we imagine any pipe, full of water, to be supplied with a number of piezometers, then a line, joining the tops of the columns of water in the several piezometers, is called the **hydraulic grade line.**

Art. 1 *u*. In a straight tube of uniform diam throughout, as *r o*, *v o*, or *l o*, Fig 1, running full and discharging freely into the air, the hyd grade line is a straight line drawn from its disch end o to a point *s* immediately over the entry end of the pipe, and at a depth below the surf equal to the sum of the vel and entry heads.

If the orifice at *o* be contracted, the hyd grade line must be drawn from *s* to some point, as *e*, immediately over o, and depending, for its height, upon the amount of contraction at o. But in this case the point *s* will also be higher than before, because the vel in the pipe is reduced by the contraction; and the sum *i s* of the vel and entry heads will be less.

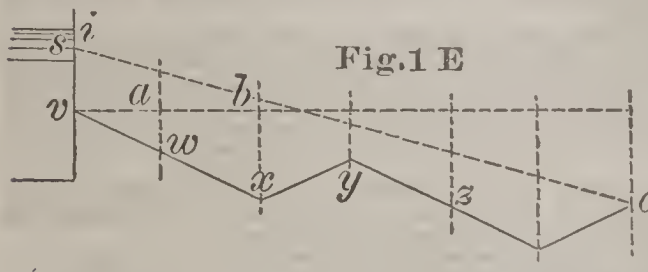


Fig. 1 E

If the disch at *o* is under water, the effect upon the position of the grade line will be the same as that of a contraction of the orifice at *o*. The point *e* will be on the surf of the lower water, and immediately over o.

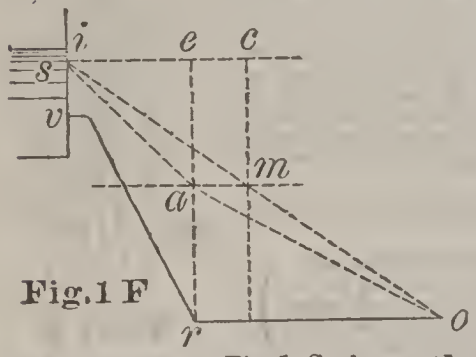


Fig. 1 F

Art. 1 *v*. If the pipe, of uniform diam, (whether discharging freely or through a contracted opening at *o*, whether into the air or under water), **is bent or curved,** the hyd grade line will still be straight, **provided** the resistances are equal in each equal division of the *hor* length of the pipe, as in Fig 1 E, where equal divisions *v w*, *w x*, &c, of the *total* length, correspond with equal divisions *v a*, *a b*, &c, of the *hor* length.

But in Fig 1 F, the hyd grade line will take the shape *s a* o. For if, in accordance with Art. 1 G, we divide *s* o into two equal parts, *s m*, *m* o, corresponding with the two equal parts *v r*, *r* o, of the length of the pipe, we obtain *m c* = *a e* for the head consumed in the resistances in *v r*, leaving only *r a* for the pres head at *r*.

Art. 1 *w*. Fig 1 G shows **the effect upon the hyd grade line, caused by a contraction** or other partial obstruction, as at *o*. Here, a vert line *a* 6 drawn from the axis of the wider pipe *l* o, shows 1st, the sum *a b* (= *i s*) of the vel and entry heads for *l* o; 2d, the total friction head *b m* (= *w e*) for *l* o; of which *b* e has been used in overcoming the resists between *l* and 6, leaving *c m* as a part of the pres head at 6 (which part decreases from *s v* at *l* to zero at o); and, 3d, a pres head *m* 6 (= *e* o) uniform throughout *l* o, and caused by the contraction at *o*. This last is the *total* head for o o′. Thus, the line 6′ *a*′ shows, 1st, the sum *a*′ *b*′ of the vel and entry heads for o o′; and 2d, its total friction head *b*′ 6′, of which *c*′ 6′ is the pres head at 6′.

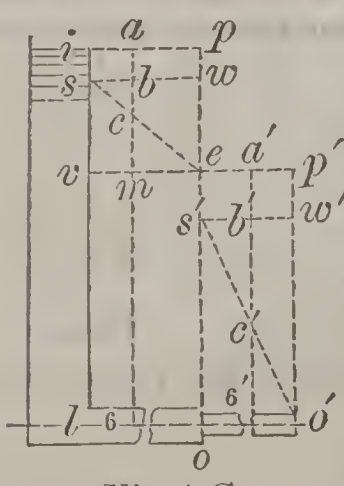


Fig. 1 G

In practice, owing to partial obstructions, such as curves, reductions of diameter, etc, the hyd grade line is rarely straight. **To find the vel** in a pipe like *l o*′, Fig 1 G, see p 254.

Art. 1 *x*. If the pipe *a g n y o*, Fig 1½, of uniform diam, is *first filled* with water, which is *then* allowed to flow out at *o*; **the pipe becomes a siphon.**

The pres at any point, *g* or *y*, in the pipe *below* the hyd grade line *s o*, is given by vert lines *g u* or *y v* drawn from the point to *s* o. But at every point, as *n*, *above s o*, there is a *negative* pres, or *tendency to vacuum*, the amount of which is given by a vert line, as *n r*, drawn from the point in question to *s* o.

This tendency to vacuum at *n* causes an accumulation there of particles of air that have been carried along by the water or which find their way into the pipe through imperfect joints, etc; and these retard the flow. If this could be prevented, the vel and disch through *a g n y o* would be the same as they would be if the pipe were revolved about a straight line joining its two ends, so as to bring each part of it below *s* o.

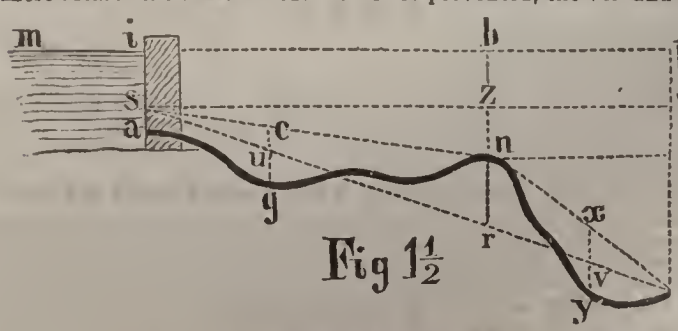


Fig 1½

But if the water be admitted to the empty pipe at *a*, *while the orifice at o is open*, the part *a g n* will run full, and will have *s n* for its hyd grade line, and *z n* for its friction head. But the *shorter* part *n* o has a *greater* friction head *t o*. Consequently, the water in it will move more rapidly than that in *a g n*, which will thus be unable to maintain a sufficient supply at *n*; and *n o* will be only part full, and will carry off the water as in an open gutter.

The syphon, or siphon. If one leg *a b* of a bent tube or pipe *a b c*, Fig M, of any diam, filled with water, and with both its ends stopped, be placed in a reservoir of water, as in the fig; and if the stoppers be then removed, the water in the reservoir will begin to flow out at c, and will continue to do so until its level is reduced to *t*, which is the same as that of the *highest* end c of the pipe or syphon. The flow will then stop. The parts *a b* and *b c* are called the legs of the syphon, *b* being its highest point; and this is correct so far as relates to it merely as a piece of tube; but considering it purely with regard to its character as a hydraulic machine, the part *t a* below the level of the highest end c, may be entirely neglected; for the water in the reservoir will not be drawn down below the level of the highest end, whether that be the inner or the outer one. Therefore, if the disch end be above the water in the reservoir, as, for instance, at *w*, no flow will take place. The *vert* height *b o*, from the highest part of the syphon, to the lowest level *t*, to which the reservoir is to be drawn down, must not, theoretically, exceed about 33 or 34 ft; or that at which the pres of the air will sustain a column of water. Practically it must be less, to allow for the friction of the flowing water, and for air which forces its way in. And still less at places far above sea level; for at such the reduced weight of the atmospheric column will not balance so great a height of water. In order readily to understand, or at any time to recall the principle on which the syphon acts, bear in mind that we may theoretically consider the end of the inner leg to be not actually immersed *below* the water surf, but only to be kept precisely *at* it, as the surf descends while the water is flowing out: but may regard the vert dist *b o* as the length of the outer leg; and a varying dist, which at first is *b s*, and finally *b o* (as the surf of the reservoir descends) as the length of the inner leg; and that the flow continues only while *this* outer leg is longer than *this* inner one. The books are wrong in saying that the outer leg *b c must* be longer than the inner one *b a*, in order that the water may run at all. The principle then is simply this: that both these legs *b c*, and *b i*, being first filled with water, (the part *i a* being considered at first as a portion of the *reservoir*, and not of the syphon,) it follows that when the stoppers are removed from the ends c and *a*, the air presses equally against these ends; but the great vert head of water *b o* in the outer leg *b c*, presses against the air at c, with more force than the small head of water *b s* in the inner leg *b i*, does against the air at *a* or *i*.* Consequently, the water in *b c* will tend to fall out more rapidly than that in *b i*; and as it commences to fall, would produce a vacuum at *b*, were it not that the pres of the air against the other end *a* or *i*, forces the water up *i b*, to supply the place of that which flows out at c. In this manner the flow continues until the surf of the water in the reservoir descends to *t*, on the same level as c. The pressures of the vert heads *b o*, *b o*, in the two legs *b c*, *b t*, being then equal, it ceases.

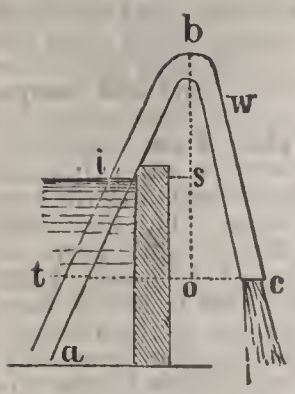


Fig. M.

The syphon principle may be employed for draining ponds into lower ground at a considerable dist, even though an elevation of several feet (in practice perhaps not *exceeding* about 28 ft above the level to which the pond is to be reduced) may intervene. In such a case an **escape** must be provided at the summit (or summits, if there are more than one) of the bends, for the disch of free air, which will inevitably enter, and soon stop the flow, unless this precaution be taken. The air-valve p.297, will not answer for this, because as soon as the valve *v* opens, the syphon becomes in effect two separate tubes open at top; and the water will fall in both. An orifice at the escape will be needed for filling the syphon at the start; and to prevent the water thus introduced, from running out, stopcocks must be provided at the ends, and kept closed until the filling is completed.

The greatest pains must be taken to make all the joints perfectly air-tight.

The motive power or head which causes the flow in a syphon, is the vert dist *s o*, from the surf of the reservoir, to the disch end *c;* or in other words, it is the diff, *s o*, between the theoretical lengths *b s* and *b o*, of the two legs. Consequently, the farther *c* is below *s* the more rapid will be the flow; and it is plain that as the surf gradually sinks below *s*, the less rapid will the flow become. Having this head, the entire length *a b c* of the syphon, and its diam, all in ft, the disch may be found approximately by either of the rules given in Art 2 for straight pipes. These rules give 55½ galls per min, instead of the 43½ galls actually dischd by Col Crozet's syphon, with a head of 20 ft, **as stated on p 242, which see.**

* Said pressure of the air is of course not exerted directly at *a* or *i*; but is transmitted to *a* through the water in the vessel; and thence upward to *i* through the water in the siphon.

At Blue Ridge Tunnel, Virginia, Col. C. Crozet constructed a drainage syphon 1792 ft long of cast iron faucet pipes 3 ins bore, 9 ft long. Its summit was 9 ft above the surface of the water to be drained; and its discharge end was 20 ft below said surface, thus giving it a head of 20 ft. At the summit 570 ft from the inlet, was an ordinary cast iron **air-vessel** with a chamber 3 ft high and 15 ins inner diam. In the stem connecting it with the syphon was a **cut-off stop-cock;** and at its top was an opening 6 ins diam, closed by an air tight screw lid. At each end of the syphon was a stopcock. **To start the flow** these end cocks are closed, and the entire syphon and air-vessel are filled with water through the opening at top of air-vessel. This opening is then closed airtight, and the two end cocks afterwards opened; the cut-off cock remaining open. The flow then begins, and **theoretically** it should continue without diminution, except so far as the head diminishes by the lowering of the surface level of the pond. **But in practice** with very long syphons this is not the case, for air begins at once to disengage itself from the water, and to travel up the syphon to the summit, where it enters the air-vessel, and rising to the top of the chamber gradually drives out the water. If this is allowed to continue the air would first fill the entire chamber, and then the summit of the syphon itself, where it would act as a wad completely stopping the flow. **The water-level in the air chamber** can be detected by the sound made by tapping against the outside with a hammer.

To prevent this stoppage, the cut-off at the foot of the chamber is closed before the water is all driven out; and the lid on top being removed the chamber is refilled with water, the lid replaced, and the cut-off again opened. The flow in the meantime continues uninterrupted, but still gradually diminishing notwithstanding the refilling of the chamber; and after a number of refillings it will cease altogether, and the whole operation must then be repeated by filling the whole syphon and air chamber with water as at the start.

At Col. Crozet's syphon at first owing to the porosity of the joint-caulking, which was nothing but oakum and pitch, air entered the pipes so rapidly as to drive all the water from the chamber and thus require it to be refilled every 5 or 10 minutes; but still in two hours the syphon would run dry. The joints were then thoroughly recaulked with lead, and protected by a covering of white and red lead made into a putty with Japan varnish and boiled linseed oil. But even then the chamber had to be refilled with water about every two hours; and after six hours the syphon ran dry, and the whole had to be refilled. In this way it continued to work.

In the writer's opinion an inside, and probably an outside coating of the pipes and air-vessel with the coal pitch varnish, Art 33, p 291, would effect a great improvement.

Art. 2. Approximate formulæ for the velocity of water in straight, smooth, cylindrical iron pipes, as *r o*, *v o*, *l o*, Fig 1, p 237. Having the total head *p o*, and the length and diameter of the pipe.

$$\left.\begin{matrix}\textbf{Approx}\\ \textbf{mean vel}\\ \text{in ft per sec}\end{matrix}\right\} = \begin{matrix}\text{coefficient}\\ \textbf{m}\\ \text{as below}\end{matrix} \times \sqrt{\frac{\text{diam in ft} \times \text{total head in ft}}{\text{total length in ft} + 54 \text{ diams in ft}}}$$

Table of coefficients "m".

$\sqrt{\frac{\text{diam} \times \text{head}}{\text{length} + 54 \text{ diams}}}$*	diameter* of pipe, in feet							
	.05	.10	.50	1	1.5	2	3	4
	m	**m**	**m**	**m**	**m**	**m**	**m**	**m**
.005	29	31	33	35	37	40	44	47
.010	34	35	37	39	42	45	49	53
.020	39	40	42	45	49	52	56	59
.030	41	43	47	50	54	57	60	63
.050	44	47	52	54	56	60	64	67
.100	47	50	54	56	58	62	66	70
.200 and over	48	51	55	58	60	64	67	70

The above coefficients are approximate averages deduced from a large number of experiments. In most cases of pipes in fair condition, carefully laid, and straight or nearly so, they should give results within say from 5 to 10 per cent of the truth. But slight differences as to roughness etc, may cause much greater variations, especially in small pipes, for in such a given roughness of surface bears a greater proportion to the whole area of cross section than in a pipe of large diameter. Extreme accuracy is not to be expected in such matters.

As in a river the velocity half way across it, and at the surface, is usually greater than at the bottom and sides, so in a pipe the velocity is greater at the center of its cross section than at its circumf. The **mean velocity** referred to in our rules is an assumed uniform one which would give the *same discharge* that the actual ununiform one does.

Hence

$$\begin{matrix}\textbf{Discharge}\\ \text{in cub ft per sec}\end{matrix} = \begin{matrix}\textbf{Mean velocity}\\ \text{in ft per sec}\end{matrix} \times \begin{matrix}\textbf{Area of cross section}\\ \text{of pipe in sq ft.}\end{matrix}$$

See tables pp 125 to 140, 157, 247.

1 cubic foot = 7.48052 U.S. gallons
1 U. S. gallon = .13368 cubic foot = 231 cubic inches.

For Kutter's formula, as applied to pipes, see p 244.

* For intermediate diameters, etc, take intermediate coefficients from the table by simple proportion.

In the case of long pipes with low heads, the sum of the velocity and entry heads (see pp 237, 238) is frequently so small that it may be neglected. Where this is the case, or where their amount can be approximately ascertained, **Kutter's formula,** although designed for open channels, may be used. This formula is the joint production of two eminent Swiss engineers, E. Ganguillet and W. R. Kutter, but for convenience it is usually called by the name of the latter.

It is, properly speaking, a formula for finding the coefficient c in the well known formula,

$$\textbf{Mean velocity} = c\sqrt{\text{mean radius} \times \text{slope}}$$

$$= c\sqrt{\frac{\text{diameter}}{4} \times \text{slope}}$$

According to Kutter,

For English measure.		**For metric measure.**
$\mathbf{c} = \dfrac{41.6 + \dfrac{.00281}{\text{slope}} + \dfrac{1.811}{n}}{1 + \dfrac{\left(41.6 + \dfrac{.00281}{\text{slope}}\right)n}{\sqrt{\text{mean rad in } \textit{feet}}}}$	$=$	$\dfrac{23 + \dfrac{.00155}{\text{slope}} + \dfrac{1}{n}}{1 + \dfrac{\left(23 + \dfrac{.00155}{\text{slope}}\right)n}{\sqrt{\text{mean rad in } \textit{metres}}}}$

See also **tables of c,** pp 275 to 278.

The mean radius is the quotient, in feet or in metres, obtained by dividing the area of wet cross section, in square feet or in square metres, by the wet perimeter (see below) in feet or in metres. In pipes running full, or exactly half full, and in semicircular open channels running full, it is equal to one fourth of the inner diameter.

The wet perimeter is the sum, *a b c o* Figs 28, 29, 30, p 271, of the lengths, *a b*, *b c*, *c o*, in feet or in metres, found by measuring (at right angles to the length of the channel) such parts of its sides and bottom as are in contact with the water. In pipes running full, it is of course equal to the inner circumference.

$$\textbf{The slope is} = \frac{\textit{friction} \text{ head } w\,o \text{ Fig 1, p 237}}{\text{length of pipe measured in a straight line from end to end.}}$$

$$= \text{sine of angle } w\,s\,o, \text{ Fig 1.}$$

In open channels, this becomes

$$\textbf{slope} = \frac{\text{fall of water surface in any portion of the length of the channel}}{\text{length of that portion}}$$

$$= \text{fall of water surface per unit of length of channel}$$

$$= \text{sine of the angle formed between the sloping surface and the horizon.}$$

The number indicating the slope in any given case is plainly the same for English, metric and all other measures.

"**n**" is a "**coefficient of roughness**" of wet perimeter, and of course depends chiefly upon the character of the inner surface of the pipe. For iron pipes in good order and from 1 inch to 4 feet diameter, n may be taken at from .010 to .012; the lower figures being used where the pipe is in exceptionally good condition.

If the diameter, or the mean radius, is in feet, metres etc, the velocity will be in feet, metres etc, per second.

The diameter or the slope, required for a given velocity, may be found by trial as follows: assume a diameter, or a slope, as the case may be; take the corresponding c from tables, pp 275, etc. Then say

$$\text{Approx Diam required for the given vel} = \text{mean radius} \times 4 = \left(\frac{\text{velocity}}{c\sqrt{\text{slope}}}\right)^2 \times 4$$

$$\text{Approx Slope required for the given vel} = \left(\frac{\text{velocity}}{c\sqrt{\text{mean radius}}}\right)^2 = \left(\frac{\text{velocity}}{c\sqrt{\frac{1}{4}\text{ diam}}}\right)^2.$$

With the approximate diameter (or slope) and c, thus obtained, say $v' = c\sqrt{\text{mean radius} \times \text{slope}}$. If v' is near enough to the given velocity, the assumed diameter (or slope) is the proper one. If not, try again, assuming a *greater* diameter or slope than before if v' is *less* than the required velocity, and *vice versa.*

To reduce cub ft to U. S. gallons, mult by 7.48. Since, therefore, 8 cub ft are equal to 60 gals, (very nearly,) if we divide the cub ft per 24 hours, by 8, we get the number of persons that may be daily supplied with 60 gals each, by a pipe *constantly running full,* and at the vel given in the third col. This condition does not exist in city water-pipes; the water in them being comparatively stagnant. Therefore, the results of the rule and table do not at all apply to them.

Rem. **If the pipe, instead of being straight, has easy curves,** (say with radii not less than 5 diams of the pipe,) either hor or vert, the disch will not be materially diminished, so long as the total heads, and total actual lengths of pipes remain the same; provided the tops of all the curves are kept below the hydraulic grade-line; and provision be made for the escape of air accumulating at the tops of the curves. See Fig 44 A, p 297.

Notwithstanding what is said about bends on pages 255, 256, we advise to make the radius as much *more* than 5 diams as can conveniently be done.

To find either the area of pipe, opening, or channel-way; or the mean vel; or the quantity discharged, when the other two are given. This applies to openings in the sides of vessels, to rivers, and to all other channels as well as to pipes.

$$\text{Area in sq feet.} = \frac{\text{Disch in cub ft per second}}{\text{mean vel in feet per sec.}} \qquad \text{Mean vel in ft per sec} = \frac{\text{Disch in cub ft per second}}{\text{area in sq feet.}}$$

$$\text{Disch in cub ft per second} = \text{area in sq feet} \times \text{mean vel in ft per second.}$$

Or all the terms may be in inches instead of feet; and minutes or hours instead of seconds.

TABLE 2. Weight of water (at 62¼ lbs per cub foot) contained in one foot length of pipes of different bores. (Original.)

Bore. Ins.	Water. Lbs.	Bore. Ins.	Water. Lbs.	Bore. Ins.	Water. Lbs.	Bore. Ins.	Water. Lbs.	Bore. Ins.	Water. Lbs.	Bore. Ins.	Water. Lbs.
⅛	.00531	2.	1.3581	3⅛	5.0980	7½	19.098	13½	61.877	22	164.33
¼	.02122	⅛	1.5331	4.	5.4323	¾	20.392	14.	66.545	23	179.60
⅜	.04775	¼	1.7188	¼	6.1325	8.	21.729	½	71.384	24	195.56
½	.08488	⅜	1.9150	½	6.8750	¼	23.109	15.	76.392	25	212.20
⅝	.13263	½	2.1220	¾	7.6601	½	24.530	½	81.568	26	229.51
¾	.19098	⅝	2.3395	5.	8.4880	¾	25.993	16.	86.916	27	247.51
⅞	.25994	¾	2.5676	¼	9.3580	9.	27.501	½	92.434	28	266.18
1.	.33952	⅞	2.8063	½	10.270	½	30.641	17.	98.121	29	285.53
⅛	.42969	3.	3.0557	¾	11.225	10.	33.952	½	103.97	30	305.57
¼	.53050	⅛	3.3156	6.	12.223	½	37.432	18.	110.00	31	326.27
⅜	.64190	¼	3.5862	¼	13.262	11.	41.082	½	116.20	32	347.66
½	.76392	⅜	3.8673	½	14.345	½	44.901	19.	122.56	33	369.74
⅝	.89654	½	4.1591	¾	15.469	12.	48.891	½	129.10	34	392.48
¾	1.0398	⅝	4.4615	7.	16.636	½	53.049	20.	135.81	35	415.90
⅞	1.1936	¾	4.7745	¼	17.846	13.	57.379	21.	149.73	36	440.00

And in larger pipes, as the squares of their bores. Thus a pipe of 40 or 60 ins bore, will contain 4 times as much as one of 20 or 30 ins bore; and one of $\frac{3}{16}$, ¼ as much as one of ⅜ inch. At 62¼ lbs per cub ft, a sq inch of water 1 ft high weighs .432292 of a lb.

TABLE 3. Areas and Contents of Pipes; and square roots of Diams. (Original) Correct.

Diam. in Ins.	Diam. in Feet.	Area in sq ft, also cub ft, in 1 foot length of Pipe.	Sq. rt. of Diam. in Ft.	Diam. in Ins.	Diam. in Feet.	Area in sq ft, also cub ft, in 1 foot length of Pipe.	Sq. rt. of Diam. in Ft.	Diam. in Ins.	Diam. in Feet.	Area in sq ft, also cub ft, in 1 foot length of Pipe.	Sq. rt. of Diam. in Ft.
¼	.0208	.0003	.145	4.	.3333	.0873	.579	15.	1.250	1.227	1.118
5-16	.0260	.0005	.161	⅛	.3438	.0928	.588	¼	1.271	1.268	1.127
⅜	.0313	.0008	.177	¼	.3542	.0985	.596	½	1.292	1.310	1.136
7-16	.0365	.0010	.191	⅜	.3646	.1040	.604	¾	1.313	1 353	1.146
½	.0417	.0014	.204	½	.3750	.1104	.612	16.	1.333	1.396	1.155
9-16	.0469	.0017	.217	⅝	.3854	.1167	.621	¼	1.354	1.440	1.163
⅝	.0521	.0021	.228	¾	.3958	.1231	.629	½	1.375	1.485	1.172
11-16	.0573	.0026	.239	⅞	.4063	.1296	.637	¾	1.396	1.530	1.181
¾	.0625	.0031	.250	5.	.4167	.1363	.645	17.	1.417	1.576	1.190
13-16	.0677	.0036	.260	⅛	.4271	.1433	.653	¼	1.437	1.623	1.199
⅞	.0729	.0042	.270	¼	.4375	.1503	.660	½	1.458	1.670	1.207
15-16	.0781	.0048	.280	⅜	.4479	.1576	.669	¾	1.479	1.718	1.216
1.	.0833	.0055	.289	½	.4583	.1650	.677	18.	1.5	1.767	1.224
1-16	.0885	.0062	.297	⅝	.4688	.1725	.685	½	1.542	1.867	1.241
⅛	.0938	.0069	.305	¾	.4792	.1803	.693	19.	1.583	1.969	1.258
3-16	.0990	.0077	.314	⅞	.4896	.1878	.700	½	1.625	2.074	1.274
¼	.1042	.0085	.322	6.	.5	.1964	.707	20.	1.667	2.182	1.291
5-16	.1094	.0094	.330	¼	.5208	.2131	.722	½	1.708	2.292	1.307
⅜	.1146	.0103	.338	½	.5417	.2304	.736	21.	1.750	2.405	1.323
7-16	.1198	.0113	.346	¾	.5625	.2485	.750	½	1.791	2.521	1.339
½	.1250	.0123	.354	7.	.5833	.2673	.764	22.	1.833	2.640	1.354
9-16	.1302	.0133	.361	¼	.6042	.2867	.777	½	1.875	2.761	1.369
⅝	.1354	.0144	.368	½	.6250	.3068	.791	23.	1.917	2.885	1.384
11-16	.1406	.0155	.375	¾	.6458	.3276	.803	½	1.958	3.012	1.399
¾	.1458	.0167	.382	8.	.6667	.3491	.817	24.	2.000	3.142	1.414
13-16	.1510	.0179	.389	¼	.6875	.3712	.829	25.	2.083	3.409	1.443
⅞	.1563	.0192	.395	½	.7083	.3941	.841	26.	2.166	3.687	1.472
15-16	.1615	.0205	.402	¾	.7292	.4176	.854	27.	2.250	3.976	1.500
2.	.1667	.0218	.408	9.	.75	.4418	.866	28.	2 333	4.276	1.528
1-16	.1719	.0232	.414	¼	.7708	.4667	.879	29.	2 416	4.587	1.555
⅛	.1771	.0246	.420	½	.7917	.4922	.890	30.	2.500	4.909	1.581
3-16	.1823	.0260	.427	¾	.8125	.5185	.902	31.	2.584	5.241	1.607
¼	.1875	.0276	.433	10.	.8333	.5454	.913	32.	2.666	5.585	1.633
5-16	.1927	.0291	.440	¼	.8542	.5730	.924	33.	2.750	5.940	1.658
⅜	.1979	.0308	.445	½	.8750	.6013	.935	34.	2.834	6.305	1.683
7-16	.2031	.0324	.451	¾	.8958	.6303	.946	35.	2.916	6.681	1.708
½	.2083	.0341	.457	11.	.9167	.6600	.957	36.	3.000	7.069	1.732
9-16	.2135	.0358	.462	¼	.9375	.6903	.968	38.	3.166	7.876	1.779
⅝	.2188	.0375	.467	½	.9583	.7213	.979	40.	3.333	8.727	1.825
11-16	.2240	.0394	.473	¾	.9792	.7530	.990	42.	3.500	9.621	1.871
¾	.2292	.0412	.478	12.	1.	.7854	1.000	44.	3.666	10.56	1.914
13-16	.2344	.0432	.484	¼	1.021	.8184	1.010	48.	4.000	12.57	2.000
⅞	.2396	.0451	.489	½	1.042	.8522	1.020	54.	4.500	15.90	2.121
15-16	.2448	.0471	.495	¾	1.063	.8866	1.031	60.	5.000	19 63	2.236
3.	.2500	.0491	.500	13.	1.083	.9218	1.041	66.	5.500	23.76	2.345
⅛	.2604	.0532	.510	¼	1.104	.9576	1.051	72.	6.000	28.27	2.449
¼	.2708	.0576	.520	½	1.125	.9940	1.060	78.	6.500	33.18	2.550
⅜	.2813	.0621	.530	¾	1.146	1.031	1.070	84.	7.000	38.48	2.646
½	.2917	.0668	.540	14.	1.167	1.069	1.080	90.	7.500	44.18	2.739
⅝	.3021	.0716	.550	¼	1.187	1.108	1.090	96.	8.000	50.27	2.828
¾	.3125	.0767	.560	½	1.208	1.147	1.099				
⅞	.3229	.0819	.570	¾	1.229	1.187	1.110				

For contents in gallons, see p 157.

Art. 3. To find the total head required for a **given velocity,** or **given discharge,** through a straight, smooth, cylindrical iron pipe of known diam and length.

If the pipe is curved, see Rem p 245.

If the *discharge* is given, first find

$$\text{mean velocity in feet per second} = \frac{\text{discharge in cubic feet per second}}{\text{area of cross section of pipe in square feet}}$$

Then

$$\text{approx } \sqrt{\frac{\text{diam} \times \text{head}}{\text{length} + 54 \text{ diams}}} = \frac{\text{mean velocity in feet per second}}{\text{the proper divisor as follows}}$$

diam of pipe in ft	.05	.10	.50	1	1.5	2	3	4
divisor	40	43	46	48	51	54	58	61

(for intermediate diams, take intermediate divisors by guess.)

From table Art 2, p 243, take the coefficient m corresponding to this value of $\sqrt{\frac{\text{diam} \times \text{head}}{\text{length} + 54 \text{ diams}}}$, and to the given diam. Then

$$\textbf{Total head in feet} = \frac{\text{mean velocity}^2 \text{ in feet per sec} \times (\text{length in ft} + 54 \text{ diams in ft})}{m^2 \times \text{diam in feet}}$$

To find the Friction head. Weisbach's formula.

$$\text{Friction head in feet} = \left(.0144 + \frac{.01716}{\sqrt{\text{vel in ft per sec}}} \right) \times \frac{\text{Length in feet}}{\text{Diam in feet}} \times \frac{\text{Vel}^2 \text{ in ft per sec}}{64.4}$$

For the total head, we have only to add together, the **friction head** so found, the **velocity head,** taken from the next table, or from Table 10, p 258, opposite the given velocity, and the **entry head** (= say half the velocity head). The sum of the velocity head and entry head rarely amounts to a foot.

TABLE 4½. Of the vel, and discharge of water through straight, smooth, cylindrical cast-iron pipes; with the friction head required for each 100 feet in length; and also the velocity head. Calculated by means of Weisbach's formula, by James Thompson, A M; and George Fuller, C E, Belfast, Ireland. The vel head remains the same for any *length* of pipe; being dependent only on the *velocity* of the water in the pipe.

The entry head is equal to about half the vel head.

TABLE 4½.

Vel. in Feet per Sec.	Vel-head in Feet.	Diam. in Inches. 3		3½		4		4½		5	
		Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min
2.0	.062	.659	5.89	.565	8.02	.494	10.4	.439	13.2	.395	16 3
2.2	.075	.780	6.48	.669	8.82	.585	11.5	.520	14.6	.463	18.0
2.4	.090	.911	7.07	.781	9.62	.683	12.5	.607	15.9	.547	19.6
2.6	.105	1.05	7.65	.901	10.4	.788	13.6	.701	17.2	.631	21.3
2.8	.122	1.20	8.24	1.03	11.2	.900	14.6	.800	18.5	.720	22.9
3.0	.140	1.35	8.83	1.16	12.0	1.02	15.7	.905	19.8	.815	24.5
3.2	.160	1.52	9.42	1.31	12.8	1.14	16.7	1.02	21.2	.915	26.2
3.4	.180	1.70	10.0	1.46	13.6	1.27	17.8	1.13	22.5	1.02	27.8
3.6	.202	1.89	10.6	1.62	14.4	1.41	18.8	1.26	23.8	1.13	29.4
3.8	.225	2.08	11.2	1.78	15.2	1.56	19.9	1.39	25.2	1.25	31.0
4.0	.250	2.28	11.8	1.96	16.0	1.71	20.9	1.52	26.5	1.37	32.7
4.2	.275	2.49	12 3	2.14	16.8	1.87	22.0	1.66	27.8	1.50	34.3
4.4	.302	2.71	12.9	2.33	17 6	2.03	23.0	1.81	29.1	1.63	36.0
4.6	.330	2.94	13.5	2.52	18.4	2.21	24.0	1.96	30 4	1.76	37.6
4.8	.360	3.18	14.1	2.72	19.2	2 38	25.1	2.12	31.8	1.91	39.2
5.0	.390	3.43	14.7	2.94	20.0	2.57	26.2	2.28	33.1	2.05	40.9
5.2	.422	3 68	15.3	3.15	20.8	2.76	27.2	2.45	34.4	2.21	42.5
5.4	.455	3.94	15.9	3.38	21.6	2.96	28.2	2.63	35.8	2.37	44.2
5.6	.490	4.22	16.5	3.61	22.4	3.16	29.3	2.81	37.1	2.53	45.8
5 8	.525	4.50	17.1	3.85	23.2	3.37	30.3	3.00	38.4	2.70	47.4
6.0	.562	4 78	17.7	4.10	24.0	3.59	31.4	3.19	39.7	2.87	49.1
6.2	.600	5.08	18.2	4.36	24.8	3.81	32.4	3.39	41.0	3.05	50.7
6.4	.640	5.39	18.8	4.62	25.6	4.04	33.5	3.59	42.4	3.23	52.3
6.6	.680	5.70	19.4	4.89	26.4	4.28	34.5	3.80	43.7	3.42	54.0
6.8	.722	6.02	20.0	5.16	27.3	4.52	35.6	4.01	45.0	3 61	55.6
7.0	.765	6 35	20.6	5.45	28.0	4.77	36.6	4.24	46 4	3.81	57.2

Vel. in Feet per Sec.	Vel-head in Feet.	Diam. in Inches. 6		7		8		9		10	
		Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min
2 0	.062	.329	23 5	.282	32.0	.247	41.9	.220	53.0	.198	65.4
2.2	.075	.390	25.9	.334	35.3	.293	46.1	.260	58.3	.234	72.0
2.4	.090	.456	28.2	.390	38.5	.342	50.2	.304	63.6	.273	78.5
2.6	.105	.526	30.6	.450	41.7	.394	54.4	.350	68.9	.315	85.1
2.8	.122	.600	32.9	.514	44 9	.450	58.6	.400	74.2	.360	91.6
3.0	.140	.679	35.3	.582	48.1	.509	62.8	.453	79.5	.407	98.2
3.2	.160	.763	37.7	.654	51.3	.572	67.0	.508	84.8	.458	105
3.4	.180	.851	40 0	.729	54.5	.638	71.2	.567	90.1	.510	111
3.6	.202	.943	42.4	.808	57.7	.707	75.4	.629	95.4	.566	118
3.8	.225	1.04	44.7	.892	60.9	.780	79.6	.693	101	.624	124
4.0	.250	1.14	47.1	.979	64.1	.856	83.7	.761	106	.685	131
4.2	.275	1.25	49.5	1.07	67.3	.935	87.9	.832	111	.748	137
4.4	.302	1.35	51.8	1.16	70.5	1.02	92.1	.905	116	.814	144
4.6	.330	1.47	54.1	1.26	73.7	1.10	96.3	.981	122	.883	150
4.8	.360	1.59	56.5	1.36	76.9	1.19	100	1.06	127	.954	157
5.0	.390	1.71	58 9	1.47	80.2	1.28	105	1.14	132	1.03	163
5.2	.422	1 84	61.2	1.58	83.3	1.38	109	1.23	138	1.10	170
5.4	.455	1.97	63.6	1.69	86.6	1.48	113	1.31	143	1.18	177
5.6	.490	2.11	65.9	1.81	89.8	1.58	117	1.40	148	1.26	183
5.8	.525	2 25	68.3	1.93	93.0	1.68	121	1.50	154	1.35	190
6 0	.562	2.39	70.7	2 05	96.2	1.79	125	1.59	159	1.43	196
6.2	.600	2.54	73.0	2.18	99.4	1.90	130	1.69	164	1.52	203
6.4	.640	2 69	75.4	2.31	102	2.02	134	1.79	169	1.61	209
6.6	.680	2.85	77.7	2.44	106	2.14	138	1.90	175	1.71	216
6.8	.722	3.01	80.1	2 58	109	2.26	142	2.01	180	1.81	222
7.0	.765	3.18	82.4	2.72	112	2.38	146	2.12	185	1.90	229

TABLE 4½ — (Continued.)

Vel. in Feet per Sec.	Vel. head in Feet.	Diam. in Inches. 11		12		13		14		15	
		Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min
2.0	.062	.180	79.2	.165	94.2	.152	110	.141	128	.132	147
2.2	.075	.213	87.1	.195	103	.180	121	.167	141	.156	162
2.4	.090	.248	95.0	.228	113	.210	133	.195	154	.182	176
2.6	.105	.287	103	.263	122	.242	144	.225	167	.210	191
2.8	.122	.327	111	.300	132	.277	156	.257	179	.240	206
3.0	.140	.370	119	.339	141	.313	166	.291	192	.271	221
3.2	.160	.416	127	.381	151	.352	177	.327	205	.305	235
3.4	.180	.464	134	.425	160	.393	188	.365	218	.340	250
3.6	.202	.514	142	.472	169	.435	199	.404	231	.377	265
3.8	.225	.567	150	.520	179	.480	210	.446	243	.416	280
4.0	.250	.623	158	.571	188	.527	221	.489	256	.457	294
4.2	.275	.680	166	.624	198	.576	232	.534	269	.499	309
4.4	.302	.740	174	.679	207	.626	243	.582	282	.543	324
4.6	.330	.803	182	.736	217	.679	254	.631	295	.589	339
4.8	.360	.867	190	.795	226	.734	265	.682	308	.636	353
5.0	.390	.935	198	.857	235	.791	276	.734	321	.685	368
5.2	.422	1.00	206	.920	245	.850	287	.789	333	.736	383
5.4	.455	1.07	214	.986	254	.910	298	.845	346	.789	397
5.6	.490	1.15	222	1.05	264	.973	309	.903	359	.843	412
5.8	.525	1.22	229	1.12	273	1.04	321	.964	372	.899	427
6.0	.562	1.30	237	1.19	283	1.10	332	1.02	385	.957	442
6.2	.600	1.38	245	1.27	292	1.17	343	1.09	397	1.01	456
6.4	.640	1.47	253	1.35	301	1.24	354	1.15	410	1.08	471
6.6	.680	1.55	261	1.42	311	1.31	365	1.22	423	1.14	486
6.8	.722	1.64	269	1.50	320	1.39	376	1.29	436	1.20	500
7.0	.765	1.73	277	1.59	330	1.46	387	1.36	449	1.27	515

Vel. in Feet per Sec.	Vel. head in Feet.	Diam. in Inches. 16		17		18		19		20	
		Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min
2.0	.062	.123	167	.116	189	.110	212	.104	236	.099	262
2.2	.075	.146	184	.138	208	.130	233	.123	260	.117	288
2.4	.090	.171	201	.161	227	.152	254	.144	283	.137	314
2.6	.105	.197	218	.185	246	.175	275	.166	307	.158	340
2.8	.122	.225	234	.212	265	.200	297	.189	331	.180	366
3.0	.140	.255	251	.240	284	.226	318	.214	354	.204	393
3.2	.160	.286	268	.269	302	.254	339	.241	378	.229	419
3.4	.180	.319	284	.300	321	.283	360	.269	401	.255	445
3.6	.202	.354	301	.333	340	.314	382	.298	425	.283	471
3.8	.225	.390	318	.367	359	.347	403	.328	449	.312	497
4.0	.250	.428	335	.403	378	.380	424	.360	472	.342	523
4.2	.275	.468	352	.440	397	.416	445	.394	496	.374	550
4.4	.302	.509	368	.479	416	.452	466	.429	519	.407	576
4.6	.330	.552	385	.519	435	.490	488	.465	543	.441	602
4.8	.360	.596	402	.561	454	.530	509	.502	567	.477	628
5.0	.390	.642	419	.605	473	.571	530	.541	590	.514	654
5.2	.422	.690	435	.650	492	.614	551	.581	614	.552	680
5.4	.455	.740	452	.696	511	.657	572	.623	638	.592	707
5.6	.490	.791	469	.744	529	.703	594	.666	661	.632	733
5.8	.525	.843	486	.793	548	.749	615	.710	685	.674	759
6.0	.562	.897	502	.844	567	.798	636	.755	709	.718	785
6.2	.600	.953	519	.897	586	.847	657	.802	732	.762	811
6.4	.640	1.01	536	.951	605	.898	678	.851	756	.808	838
6.6	.680	1.07	553	1.01	624	.950	700	.900	780	.855	864
6.8	.722	1.13	569	1.06	643	1.00	721	.951	803	.904	896
7.0	.765	1.19	586	1.12	662	1.06	742	1.00	827	.953	916

TABLE 4½. —(Continued.)

Vel. in Feet per Sec.	Vel-head in Feet.	Diam. in Inches. 22		24		26		28		30	
		Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min	Fr head Ft per 100 ft.	Cub ft per Min
2.0	.062	.090	316	.082	377	.076	442	.070	513	.066	589
2.2	.075	.106	348	.097	414	.090	486	.083	564	.078	648
2.4	.090	.124	380	.114	452	.105	531	.097	616	.091	707
2.6	.105	.143	412	.131	490	.121	575	.112	667	.105	766
2.8	.122	.164	443	.150	528	.138	619	.128	718	.120	824
3.0	.140	.185	475	.170	565	.157	663	.145	770	.136	883
3.2	.160	.208	507	.191	603	.176	708	.163	821	.152	942
3.4	.180	.232	538	.213	641	.196	752	.182	872	.170	1001
3.6	.202	.257	570	.236	678	.218	796	.202	923	.189	1060
3.8	.225	.284	601	.260	716	.240	840	.223	974	.208	1119
4.0	.250	.311	633	.285	754	.263	885	.244	1026	.228	1178
4.2	.275	.340	665	.312	791	.288	929	.267	1077	.249	1237
4.4	.302	.370	697	.339	829	.313	973	.290	1129	.271	1296
4.6	.330	.401	728	.368	867	.339	1017	.315	1180	.294	1355
4.8	.360	.434	760	.397	905	.367	1062	.341	1231	.318	1414
5.0	.390	.467	792	.428	942	.395	1106	.367	1283	.343	1472
5.2	.422	.502	823	.460	980	.425	1150	.394	1334	.368	1531
5.4	.455	.538	855	.493	1018	.455	1194	.423	1385	.394	1590
5.6	.490	.575	887	.527	1055	.486	1239	.452	1437	.422	1649
5.8	.525	.613	918	.562	1093	.519	1283	.482	1488	.450	1708
6.0	.562	.652	950	.598	1131	.552	1327	.513	1539	.478	1767
6.2	.600	.693	982	.635	1168	.586	1371	.544	1590	.508	1826
6.4	.640	.735	1013	.673	1206	.622	1416	.577	1642	.539	1885
6.6	.680	.778	1045	.713	1244	.658	1460	.611	1693	.570	1943
6.8	.722	.821	1077	.753	1282	.695	1504	.645	1744	.602	2003
7.0	.765	.867	1109	.794	1319	.733	1548	.681	1796	.635	2061

TABLE 5. Of fifth roots and fifth powers.

Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.
.0000100	.1	.000142	.170	.004219	.335	.077760	.60	.695688	.93	8.11368	1.52
		.000164	.175	.004544	.340	.084460	.61	.733904	.94	8.66171	1.54
.0000110	.102	.000189	.180	.004888	.345	.091613	.62	.773781	.95	9.23896	1.56
		.000217	.185	.005252	.350	.099244	.63	.815373	.96	9.84658	1.58
.0000122	.104	.000248	.190	.005638	.355	.107374	.64	.858734	.97	10.4858	1.60
		.000282	.195	.006047	.360	.116029	.65	.903921	.98	11.1577	1.62
.0000134	.106	.000320	.200	.006478	.365	.125233	.66	.950990	.99	11.8637	1.64
		.000362	.205	.006934	.370	.135012	.67	1.	1.	12.6049	1.66
.0000147	.108	000408	.210	.007416	.375	.145393	.68	1.10408	1.02	13.3828	1.68
.0000161	.110	.000459	.215	.007924	.380	,156403	.69	1.21665	1.04	14.1986	1.70
.0000176	.112	.000515	.220	.008459	.385	.168070	.70	1.33823	1.06	15.0537	1.72
.0000193	.114	.000577	.225	.009022	.390	.180423	.71	1.46933	1.08	15.9495	1.74
.0000210	.116	.000644	.230	.009616	.395	.193492	.72	1.61051	1.10	16.8874	1.76
.0000229	.118	.000717	.235	.010240	.400	.207307	.73	1.76234	1.12	17.8690	1.78
.0000249	.120	.000796	.240	.011586	.41	.221901	.74	1.92541	1.14	18.8957	1.80
.0000270	.122	.000883	.245	.013069	.42	.237305	.75	2.10034	1.16	19.9690	1.82
.0000293	.124	.000977	.250	.014701	.43	.253553	.76	2.28775	1.18	21.0906	1.84
.0000318	.126	.001078	.255	.016492	.44	.270678	.77	2.48832	1.20	22.2620	1.86
.0000344	.128	.001188	.260	.018453	.45	.288717	.78	2.70271	1.22	23.4849	1.88
.0000371	.130	.001307	.265	.020596	.46	.307706	.79	2.93163	1.24	24.7610	1.90
.0000401	.132	.001435	.270	.022935	.47	.327680	.80	3.17580	1.26	26.0919	1.92
.0000432	.134	.001573	.275	.025480	.48	.348678	.81	3.43597	1.28	27.4795	1.94
.0000465	.136	.001721	.280	.028248	.49	.370740	.82	3.71293	1.30	28.9255	1.96
.0000500	.138	.001880	.285	.031250	.50	.393904	.83	4.00746	1.32	30.4317	1.98
.0000538	.140	.002051	.290	.034503	.51	.418212	.84	4.32040	1.34	32.0000	2.00
.0000577	.142	.002234	.295	.038020	.52	.443705	.85	4.65259	1.36	36.2051	2.05
.0000619	.144	.002430	.300	.041820	.53	.470427	.86	5.00490	1.38	40.8410	2.10
.0000663	.146	.002639	.305	.045917	.54	.498421	.87	5.37824	1.40	45.9401	2.15
.0000710	.148	.002863	.310	.050328	.55	.527732	.88	5.77353	1.42	51.5363	2.20
.0000754	.150	.003101	.315	.055073	.56	.558406	.89	6.19174	1.44	57.6650	2.25
.0000895	.155	.003355	.320	.060169	.57	.590490	.90	6.63383	1.46	64.3634	2.30
.000105	.160	.003626	.325	.065636	.58	.624032	.91	7.10082	1.48	71.6703	2.35
.000122	.165	.003914	.330	.071492	.59	.659082	.92	7.59375	1.50	79.6262	2.40

TABLE 5. Of fifth roots and fifth powers — (Continued.)

Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.
88.2735	2.45	2824.75	4.90	85873	9.70	2609193	19.2	20511149	29.0	459165024	54.
97.6562	2.50	2971.84	4.95	90392	9.80	2747949	19.4	21228253	29.2	503284375	55.
107.820	2.55	3125.00	5.00	95099	9.90	2892547	19.6	21965275	29.4	550731776	56.
118 814	2.60	3450.25	5.10	100000	10.0	3043168	19.8	22722628	29.6	601692057	57.
130.686	2.65	3802.04	5.20	110408	10.2	3200000	20.0	23500728	29.8	656356768	58.
143.489	2.70	4181.95	5.30	121665	10.4	3363232	20.2	24300000	30.0	714924299	59.
157.276	2.75	4591.65	5.40	133823	10.6	3533059	20.4	26393634	30.5	777600000	60.
172.104	2.80	5032.84	5.50	146933	10.8	3709677	20.6	28629151	31.0	844596301	61.
188.029	2.85	5507.32	5.60	161051	11.0	3893289	20.8	31013642	31.5	916132832	62
205.111	2.90	6016.92	5.70	176234	11.2	4084101	21.0	33554432	32.0	992436543	63.
223.414	2.95	6563.57	5.80	192541	11.4	4282322	21.2	36259082	32.5	1073741824	64.
243.000	3.00	7149.24	5.90	210034	11.6	4488166	21.4	39135393	33.0	1160290625	65.
263.936	3.05	7776.00	6.00	228776	11.8	4701850	21.6	42191410	33.5	1252332576	66.
286.292	3.10	8445.96	6.10	248832	12.0	4923597	21.8	45435424	34.0	1350125107	67.
310.136	3.15	9161.33	6.20	270271	12.2	5153632	22.0	48875980	34.5	1453933568	68.
335.544	3.20	9924.37	6.30	293163	12.4	5392186	22.2	52521875	35.0	1564031349	69.
362.591	3.25	10737	6.40	317580	12.6	5639493	22.4	56382167	35.5	1680700000	70.
391.354	3.30	11603	6.50	343597	12.8	5895793	22.6	60466176	36 0	1804229351	71.
421.419	3.35	12523	6.60	371293	13.0	6161327	22.8	64783487	36 5	1934917632	72.
454.354	3.40	13501	6.70	400746	13.2	6436343	23.0	69343957	37.0	2073071593	73.
488.760	3.45	14539	6.80	432040	13.4	6721093	23.2	74157715	37.5	2219006624	74.
525.219	3.50	15640	6.90	465259	13.6	7015834	23.4	79235168	38.0	2373046875	75.
563.822	3.55	16807	7.00	500490	13.8	7320825	23.6	84587005	38.5	2535525376	76.
604.662	3.60	18042	7.10	537824	14.0	7636332	23.8	90224199	39.0	2706784157	77.
647.835	3.65	19349	7.20	577353	14.2	7962624	24.0	96158012	39.5	2887174368	78.
693.440	3.70	20731	7.30	619174	14.4	8299976	24.2	102400000	40.0	3077056399	79.
741.577	3.75	22190	7.40	663383	14.6	8648666	24.4	108962013	40.5	3276800000	80
792.352	3.80	23730	7.50	710082	14.8	9008978	24.6	115856201	41.0	3486784401	81.
845.870	3.85	25355	7.60	759375	15.0	9381200	24.8	123095020	41.5	3707398432	82.
902.242	3.90	27068	7.70	811368	15.2	9765625	25.0	130691232	42.0	3939040643	83.
961.580	3.95	28872	7.80	866171	15.4	10162550	25.2	138657910	42.5	4182119424	84.
1024.00	4.00	30771	7.90	923896	15.6	10572278	25.4	147008443	43.0	4437053125	85.
1089.62	4.05	32768	8.00	984658	15.8	10995116	25.6	155756538	43.5	4704270176	86.
1158.56	4.10	34868	8.10	1048576	16.0	11431377	25.8	164916224	44 0	4984209207	87.
1230.95	4.15	37074	8.20	1115771	16.2	11881376	26.0	174501858	44.5	5277319168	88.
1306.91	4.20	39390	8.30	1186367	16.4	12345437	26.2	184528125	45.0	5584059449	89.
1386.58	4.25	41821	8.40	1260493	16.6	12823886	26.4	195010045	45.5	5904900000	90.
1470.08	4.30	44371	8.50	1338278	16.8	13317055	26.6	205962976	46.0	6240321451	91.
1557.57	4.35	47043	8.60	1419857	17.0	13825281	26.8	217402615	46.5	6590815232	92.
1649.16	4.40	49842	8.70	1505366	17.2	14348907	27.0	229345007	47 0	6956883693	93.
1745.02	4.45	52773	8.80	1594947	17.4	14888280	27.2	241806543	47.5	7339040224	94.
1845.28	4.50	55841	8.90	1688742	17.6	15443752	27.4	254803968	48.0	7737809375	95.
1950.10	4.55	59049	9.00	1786899	17.8	16015681	27.6	268354383	48.5	8153726976	96.
2059.63	4.60	62403	9.10	1889568	18.0	16604430	27.8	282475249	49.0	8587340257	97.
2174.03	4.65	65908	9.20	1996903	18.2	17210368	28.0	297184391	49.5	9039207968	98.
2293.45	4.70	69569	9.30	2109061	18.4	17833868	28.2	312500000	50.0	9509900499	99.
2418.07	4.75	73390	9.40	2226203	18.6	18475309	28.4	345025251	51.		
2548.04	4.80	77378	9.50	2348493	18.8	19135075	28.6	380204032	52.		
2683.54	4.85	81537	9.60	2476099	19.0	19813557	28.8	418195493	53.		

TABLE 6. Of the square roots of the fifth powers of numbers. In this table the numbers and the roots are supposed to be in the same dimensions; that is, both in inches, or both in feet, &c. See the next table.

No.	Sq. Rt. of 5th Power.	No.	Sq. Rt. of 5th Power.	No.	Sq. Rt. of 5th Power.	No.	Sq. Rt. of 5th Power.	No.	Sq. Rt. of 5th Power.	No.	Sq. Rt. of 5th Power.
.25	.031	7.	129.64	17.5	1281.1	31.	5351	49	16807	76	50354
.5	.177	7.25	141.53	18.	1374.6	31.5	5569	50	17678	77	52027
.75	.485	7.5	154.05	18.5	1472.1	32.	5793	51	18575	78	53732
1.	1.	7.75	167.21	19.	1573.6	32.5	6022	52	19499	79	55471
1.25	1.747	8.	181.02	19.5	1679.1	33.	6256	53	20450	80	57243
1.5	2.756	8.25	195.50	20.	1788.9	33.5	6496	54	21428	81	59049
1.75	4.051	8.5	210.64	20.5	1902.8	34.	6741	55	22434	82	60888
2.	5.657	8.75	226.48	21.	2020.9	34.5	6991	56	23468	83	62762
2.25	7.594	9.	243.	21.5	2143.4	35.	7247	57	24529	84	64669
2.5	9.882	9.25	260.23	22.	2270.2	35.5	7509	58	25620	85	66611
2.75	12.541	9.5	278.17	22.5	2401.4	36.	7776	59	26738	86	68588
3.	15.588	9.75	296.83	23.	2537.	36.5	8049	60	27886	87	70599
3.25	19.042	10.	316.23	23.5	2677.1	37.	8327	61	29062	88	72646
3.5	22 918	10.5	357.2	24.	2821.8	37.5	8611	62	30268	89	74727
3.75	27 232	11.	401.3	24.5	2971.1	38.	8901	63	31503	90	76843
4.	32.	11.5	448.5	25.	3125.	38.5	9197	64	32768	91	78996
4.25	37.24	12.	498.8	25.5	3283.6	39.	9198	65	34063	92	81184
4.5	42.96	12.5	552.4	26.	3446.9	39.5	9806	66	35388	93	83408
4.75	49 17	13.	609.3	26.5	3615.1	40.	10119	67	36744	94.	85668
5.	55.90	13.5	669.6	27.	3788.	41.	10764	68	38131	95	87965
5.25	63.15	14.	733.4	27.5	3965.8	42.	11432	69	39548	96	90298
5.5	70.94	14.5	800.6	28.	4148.5	43.	12125	70	40996	97	92668
5.75	79.28	15.	871.4	28.5	4336.2	44.	12842	71	42476	98	95075
6.	88.18	15.5	945.9	29.	4528.9	45.	13584	72	43988	99	97519
6.25	97.66	16.	1024.	29.5	4726.7	46.	14351	73	45531	100	100000
6.5	107.72	16.5	1105.9	30.	4929.5	47.	15144	74	47106		
6.75	118.38	17.	1191.6	30.5	5138.	48.	15963	75	48714		

TABLE 6½. Numbers, in inches. Square roots of fifth powers, in feet.

	Sq. Rt. of 5th Pow.		Sq. Rt. of 5th Pow.		Sq. Rt. of 5th Pow.		Sq. Rt. of 5th Pow.		Sq. Rt. of 5th Pow.
Ins.	Feet.	Ins.	Feet.	Ins.	Feet.	Ins.	Feet.	Ins.	Feet.
¼	.00006	3¾	.0547	12.	1.000	22½	4.813	42	22.92
⅜	.00017	4.	.0341	½	1.108	23	5.086	43	24.31
½	.00035	¼	.0731	13.	1.221	½	5.365	44	25.74
⅝	.00062	½	.0827	½	1.342	24	5.657	45	27.23
¾	.00098	¾	.0971	14.	1.470	25	6.264	46	28.77
⅞	.00144	5.	.1120	½	1.605	26	6.909	47	30.36
1.	.0020	¼	.1271	15.	1.747	27	7.593	48	32.00
⅛	.0027	½	.1428	½	1.896	28	8.316	49	33.69
¼	.0035	¾	.1590	16.	2.053	29	9.079	50	35.44
⅜	.0044	6	.1768	½	2.217	30	9.882	51	37.25
½	.0055	½	.2160	17.	2.389	31	10.73	52	39.13
⅝	.0067	7.	.2599	½	2.567	32	11.61	53	41.02
¾	.0081	½	.3088	18.	2.756	33	12.54	54	42.96
⅞	.0096	8.	.3628	½	2.950	34	13.51	55	44.97
2.	.0113	½	.4228	19.	3.155	35	14.53	56	47.05
¼	.0152	9.	.4871	½	3.365	36	15.59	57	49.17
½	.0198	½	.5577	20.	3.586	37	16.69	58	51.35
¾	.0252	10.	.6339	½	3.813	38	17.84	59	53.60
3.	.0312	½	.7162	21.	4.051	39	19.04	60	55.90
¼	.0383	11.	.8043	½	4.297	40	20.29	61	58.27
½	.0459	½	.8990	22.	4.551	41	21.58		

Art. 4 *a*. To find the discharge through a compound pipe *v o*, Fig 1 H, composed of any number of pipes, *a*, *b*, *c*, *z*, of **different diameters,** which decrease from the reservoir toward the outflow *o*.

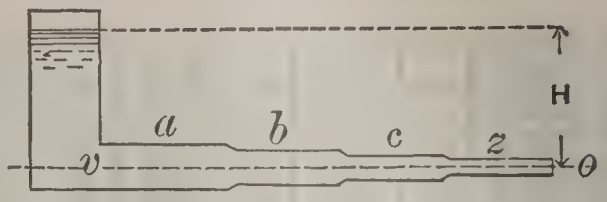


Fig. 1 H

First find what part of the total head H is employed in forcing the water *through the last pipe z alone*, thus:

Let L *a*, L *b*, L *c*, and L *z*, be the lengths in ft of the pipes *a*, *b*, *c*, and *z*, respectively; D *a*, D *b*; D *c*, and D *z* their diameters in ft; and A *a*, A *b*, A *c*, and A *z* the areas of their cross sections in sq ft. Then

$$\left.\begin{array}{l}\text{The head in ft employed in forcing the water through the last pipe } z \text{ alone}\end{array}\right\} = \frac{\text{The } \textit{total} \text{ head H in feet}}{1+\left[\frac{\text{A } z^2 \times \text{D } z}{\text{L } z+(54\times \text{D } z)} \times \left(\frac{\text{L}a+(54\times \text{D}a)}{\text{A } a^2 \times \text{D } a}+\frac{\text{L } b+(54\times \text{D } b)}{\text{A } b^2 \times \text{D } b}+\frac{\text{L } c+(54\times \text{D } c)}{\text{A } c^2 \times \text{D}c}\right)\right]}$$

However many divisions the pipe may have, proceed in the same way as above, using

$\frac{\text{Area}^2 \times \text{Diam}}{\text{Length} + 54 \text{ Diams}}$ for the *last* or narrowest division; and $\frac{\text{Length} + 54 \text{ Diams}}{\text{Area}^2 \times \text{Diam}}$

for each of the *others*.

Then, by the formulæ, Art. 2, find the velocity in ft per second, and discharge in cub ft per second, of the last pipe *z*, using its actual diameter, length, and cross sectional area, and the head just found. Said *discharge* is evidently the discharge for the compound pipe.

For the *velocity in any portion*, as *b*, say

area of cross section of the given portion : area of cross section of *z* :: velocity in *z* : velocity in the given portion.

For the above rule and formula, we are indebted to Mr. Howard Murphy, C E, of Phila; and for the opportunity of testing it experimentally, to Messrs Morris, Tasker & Co, Limited, Pascal Iron Works, Philadelphia.

Art. 5. On the resistance which curved bends oppose to the flow of water through round pipes. Well-rounded bends of large rad, whether vert or hor, produce but little resistance; except so far as the first may cause accumulations of sediment, or of air. According to Weisbach, the rules of DuBuat, Navier, and other authorities, are erroneous; and he gives the following one for ascertaining the *additional* head reqd to overcome the resistance produced in a circular pipe, by a bend formed by an arc of a circle: Knowing the rad *r e* of the pipe, (or, in other words, half its diam,) in feet; the rad *r s*, of the axis *r n o* of the bend, in feet; the central angle *r s o* in degrees; (which is equal to the angle *d b x*, or *c b a*,) and the reqd vel of the water in the pipe, in ft per sec.

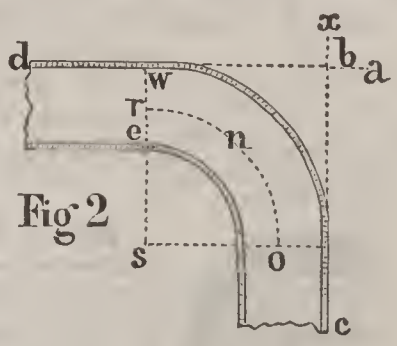


Fig 2

Rule. Div the central angle *r s o* in deg, by 180. Call the quot *a*. Next, square the reqd vel. Div this sq by the constant number 64.4. Call the quot *b*. Div the inner rad *r e* of the pipe, in ft, by the rad *r s* of the axis *r n o* of the bend, in ft. Call the quot c. Take from the following Table 7, the number in column *d*, which corresponds to c; (unless c be *less* than .1, in which case always take .13 as *d*.) Finally, mult together this number *d*, the quot *a*, and the quot *b*. The prod will be the reqd head in feet; which must either be added to the head previously calculated for the straight pipe, if the original vel is required to be maintained; or must be subtracted from it, in case the head does not admit of increase, and a new calculation made to ascertain the diminished vel under the head thus reduced. If there is more than one bend of the same dimensions, an equal alteration of head must be made for each; or, if they are of diff radii, and with diff central angles *r s o*, a separate calculation must be made for each. Rennie's experiments, at the end of this Art, seem to prove that this is by no means the case. So far as the writer is aware, we have no reliable data for calculating the effects of a succession of bends.

In shape of a formula, Weisbach's rule stands thus: R being the rad of the axis of the bend, and *r* the rad of the pipe:

$$\text{Additional head in feet} = .131 + 1.847\left(\frac{r \text{ in ft}}{R \text{ in ft}}\right)^{\frac{7}{2}} \times \frac{\text{square of vel in ft per sec}}{64.4} \times \frac{\text{central angle in degrees}}{180.}$$

The expression $\frac{7}{2}$ means the sq root of the 7th power. When the rad of the bend exceeds 5 diams of the pipe, then $1.847\left(\frac{r}{R}\right)^{\frac{7}{2}}$ becomes inappreciable in practice, and may be omitted from the formula. When the pipe is square, instead of circular, the formula becomes

$$\text{Additional head} = .124 + 3.104\left(\frac{r}{R}\right)^{\frac{7}{2}} \times \frac{\text{vel}^2}{64.4} \times \frac{\text{central angle}}{180.}$$

TABLE 7.

c.	d.	c.	d.	c.	d.	c.	d.	c.	d.
.1	.131	.325	.17	.5	.29	.675	.60	.85	1.18
.15	.135	.35	.18	.525	.32	.7	.66	.875	1.29
.2	.138	.375	.195	.55	.35	.725	.73	.9	1.41
.225	.145	.4	.206	.575	.39	.75	.80	.925	1.54
.25	.15	.425	.225	.6	.44	.775	.88	.95	1.68
.275	.155	.45	.24	.625	.49	.8	.98	.975	1.83
.3	.16	.475	.264	.65	.54	.825	1.08	1.	2.

Ex. A straight pipe 1 mile long, and 18 ins diam, with a total head of 20 ft, will disch water with a vel of 4 ft per sec; but it has been found necessary to introduce a circular bend of 90°, with a rad *s r*, Fig 2, of 5 feet. What addition must be made to the 20 ft head, to compensate for the additional resistance caused by the bend; so that the reqd vel of 4 ft per sec may still be maintained?

Here, $90° \div 180 = .5 = a$. Next, the square of the reqd vel in ft per sec, is $4 \times 4 = 16$. And $16 \div 64.4 = .2484 = b$. The rad *r e* of the pipe (.75 ft), div by the rad *r s* of the bend (5 ft), $= .75 \div 5 = .15 = c$; and opposite this .15 in the column c of the foregoing table, we find $d = .135$. Finally, $a \times b \times d = 5 \times .2484 \times .135 = .0168$ ft, or about one-fifth inch only, the additional head reqd. See next table, No. 8.

Du Buat's rule for the additional head required to overcome the resistance of circular bends in water pipes. Having diam of pipe, in ft; rad of bend, in ft, central angle *w s o*, Fig 2; and vel in ft per sec. Div *e r*, Fig 2, or *half* the diam of the pipe, by the rad *s w* of the outer side of the bend. The quot will be the nat versed sine of Du Buat's **angle of reflexion.** Take this versed sine from unity, or 1. The Rem will be the nat cosine of the same angle. From the Table of Nat Sin and Tang, take both the angle and the nat sine corresponding to this nat cosine. Call the angle R. Also, square the nat sine, and call this square S. Take the angle *w s o* from 180°. Div the rem by *twice* the angle R of reflexion just found. Call the quot T. Finally, mult together the constant dec .00375, the square of the vel in ft per sec, the quot T, and the square S. The prod will be the reqd extra head in feet,

Ex. The same as the foregoing one for Weisbach's rule; that is, a pipe of 18 ins, or 1.5 ft diam; rad *s w* of *outer* side of bend, 5.75 ft; vel 4 ft per sec. What extra head will the bend require, in order that this vel may not be diminished?

Here, $\frac{.75}{5.75} = .13043$ = nat versed sine of angle of reflexion. And $1 - .13043 = .86957$ = nat cos of same angle. In the Table of Nat Sin. &c, we find, opposite the nat cos .86957, the angle R = 29° 35'; and its nat sine .4937. The square of .4937 = .2437 = S. Again, 180° — 90° = 90°. And $\frac{90°}{59°\ 10'} = \frac{5400 \text{ min}}{3550 \text{ min}} = 1.52$, or T. Finally, the square of the vel is 16; hence, we have .00375 × 16 × 1.52 × .2437 = .0222 ft. the reqd extra head; or about ¼ of an inch. Weisbach's rule gave .0168 ft, or about one-fifth of an inch. Hence we see that the resistance produced by well-rounded bends is not great.

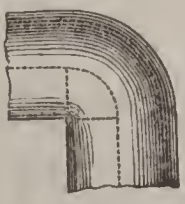


Fig. 3.

When the rad *r s*, Fig 2, of the bend, is less than about two diams of the pipe, which will rarely happen, the resistance to the flow of the water increases very rapidly; while, on the other hand, by Weisbach's rule, as we understand it, no advantage appears to be gained by using a rad greater than 5 diams of the pipe.* Employing Weisbach's formula, the writer has drawn up the following table of heads reqd to overcome the resistance of one bend of 90°, for diff vels in ft per sec; and for any diam whatever. This table extends from a rad of 5 diams down to one of ½ diam; which is the smallest possible, inasmuch as it leads to a bend like Fig 3.

A vel of 12 ft per sec is equal to 8.18 miles per hour; one which will rarely occur, inasmuch as it requires a head of about 330 feet per mile.

TABLE 8. Heads required to overcome the resistance in circular bends of 90°. Original.

	Velocity in feet per Second.										
	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.	6 ft.	7 ft.	8 ft.	9 ft.	10 ft.	12 ft.
	HEADS IN FEET.										
Rad = 5 diams of the pipe.	.001	.004	.009	.016	.025	.036	.050	.065	.082	.101	.145
Rad = 3 diams...	.001	.004	.010	.017	.027	.038	.052	.069	.086	.106	.153
Rad = 2 diams...	.001	.005	.011	.019	.029	.042	.057	.074	.094	.116	.167
Rad = 1½ diams...	.001	.005	.012	.021	.033	.048	.066	.086	.108	.134	.192
Rad = 1¼ diam...	.002	.007	.015	.026	.041	.059	.080	.104	.132	.163	.235
Rad = 1 diam...	.002	.009	.020	.036	.056	.081	.110	.144	.182	.225	.324
Rad = ¾ diam...	.005	.018	.041	.072	.113	.162	.221	.288	.365	.450	.649
Rad = ½ diam...	.016	.062	.140	.248	.388	.559	.761	.994	1.26	1.55	2.24

If the central angle *r s o*, Fig 2, should be either greater, or less than 90°, then the heads given in the table, must be increased, or diminished directly in the same proportion.

Experiments by Rennie, with a pipe 15 ft long; and ½ inch bore; with 4 ft head, gave the following disch in cub ft per sec:

Straight..... .00699 cub ft.	One bend at right angles near end .... .00556
15 semicircular bends.....00617 "	24 bends at right angles..... .00253

The mean of many careful experiments tried at Liverpool, England, with a leaden pipe, 75 ft long, ⅜ inch bore, under 8 ft head, gave the following number of secs to discharge one gallon of water:

75 ft pipe, straight and horizontal.. 81.56 sec.	4 vertical bends near discharge end 85.00 sec.
2 hor bends near discharge end ... 83.33 "	4 vertical bends near supply end... 84.00 "
2 hor bends near supply end..... 81.80 "	

The rad of the bends is not stated. See Minutes Trans Inst Civ Eng, vol 12, page 501.

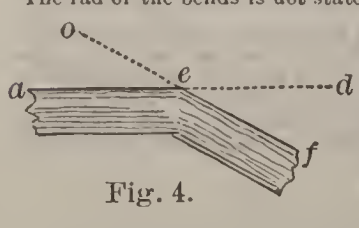


Fig. 4.

On knees, or angular bends in water pipes, Fig 4. The bends in lines of water pipes should always form circular arcs; because knees create a much greater resistance. According to Weisbach, the head in ft reqd to overcome the additional resistance caused by a knee in a round pipe is as follows:

$$\text{Additional head in ft} = \frac{\text{Sq of vel in ft per sec}}{64.4} \times \text{constant in following}$$

table opposite the angle of deflection, *a e o*, or *d e f*,† Fig 4.

* Notwithstanding this, we advise to use as many more than 5 as can conveniently be done.

† The constant for any angle of def, is equal to .946 times the square of the nat sine of half the angle of def; × 2.05 times the 4th power of the same sine.

TABLE 9.

Ang. of Def. in Degs.	Constant	Ang. of Def. in Deg.	Constant.	Ang. of Def. in Degs.	Constant.
140°	2.431	70°	.533	25°	.049
130	2.158	60	.364	20	.030
120	1.861	50	.234	15	.016
110	1.556	40	.139	10	.007
100	1.260	35	.102	5	.002
90	.984	30	.073		
80	.740				

Constants intermediate of those in the table may be obtained near enough by simple proportion.

The disch is diminished by swellings, or enlargements in pipes, as well as by contractions, bends, and knees; on account of the eddies which they produce, &c.

Art. 6. Inasmuch as the pres of quiet water against, and perp to, any given surf. is (other things being equal) in proportion to the *vert* height of the water above the cen of grav of the pressed surf, (see Art 1, Hydrostatics,) it follows that in two pipes of the same diams, as *a b*, and *c h*, Fig 5, the pres against, and at right angles to, the equal bases, *m n* of the vert pipe, and *o p* of the inclined one, are equal; because the vert heights, *a b* and *h g*, of the water above the cen of grav *a* and e, of the equal bases, are equal in the two pipes. If the base of the inclined pipe be cut so that *t y* becomes the base, then the base is no longer a circle, but an ellipse; the area of which will always be greater than that of the circular one; and since the vert height *h g* remains unchanged, the pres against the base *t y*, and perp to it, will be greater than that against *o p*, in the same proportion as the two areas.

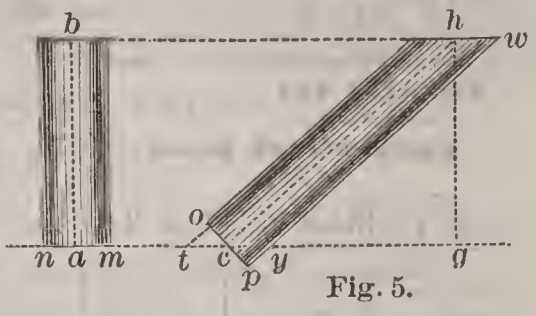


Fig. 5.

The upright pipe may be but 1 ft long; and the inclined one 1 mile, or 10 miles long, still the pres at the base *m n* will be the same as that at the base *o p*, so long as the vert height *a b* is equal to the vert height *g h*. The greater *weight* of the water in *c h*, does not increase the *pres* at its lower end *o p*; said weight being sustained by the under part *p w* of the inclined pipe.

If, therefore, two steam pumps, with plungers of equal diameter, were employed; one to force the water up the one foot long vertical pipe, and the other to force it up the ten miles of inclined pipe; both engines would have to exert the same force to *balance* their respective columns of water; *i e*, to *uphold* them. In other words, the *static* pressure of the water is the same in both cases; being equal to the weight of a cylindrical column of water of the same diameter as the *plunger*,* and as long as the *vertical* stretch, *a b* or *g h*, of the pipes.

But in order to *move* the water in either pipe at a given velocity, an *additional* force is required, equal to the weight of a cylindrical column of water, the diameter of which is equal to that of the *plunger*, and the length of which is equal to the total head (calculated by Art 3, p 248) required to force water at the given velocity through a pipe of the given dimensions. The weight of this second column is the pressure, or *motive force*, necessary to give the required velocity to the water, to overcome its friction in the pipe, and to put water into the pipe at its lower end as fast as it passes out at the upper end. See Art 1 a, p 237.

The *total* force, or total steam pressure required in the cylinder, is the *sum* of these two pressures; namely, of the static pressure and the motive force.

In the foregoing we have assumed that the resistance to entry is only such as would be encountered by water when forced by any means from a reservoir into the open end of an ordinary pipe; so that the "entry head" may be taken as equal to about ½ the velocity head. In practice, a much greater resistance to entry is offered by valves, by sudden bends in pipes leading to and from air chambers, etc. Still, in most cases the entry head, even as thus increased, is but trifling in comparison with the total head.

Art. 7. The flow of water through openings, or apertures, in the sides or bottom of the containing vessel, or reservoir. *Theoretically*, the vel with which water should flow through such an opening, is equal to that which would be acquired by a heavy body falling freely through a height equal to the *head*, or depth of water, measured *vert* from the level surf of the water in the reservoir, to the *center* of the opening; or, more correctly, to its *cen of grav*. This theoretical vel is found in ft per sec, by mult the sq rt of said head, or vert depth in ft, by the constant number 8.03; or, mult the head itself in ft, by

* Because the plunger now forms practically the bottom, *n m* or *o p*, of the pipe. See Rem 2 p 223.

64.4, and take the sq rt of the prod. In practice, we may use 8 and 64, as near enough. The theoretical, as well as the actual *disch*, or the *quantity* in cub ft, which flows out per sec, is evidently equal in all cases to the prod of the theoretical, or of the actual vel, (as the case may be,) in ft per sec, mult by the area of the opening in sq ft.

These theoretical laws apply equally to all fluids, whatever may be their sp grav; thus, theoretically, mercury, water, air, &c. will all flow with equal vels from openings of equal sizes, under equal heads.

Practically, however, only the mean vel, and the disch through the **vena contracta,** or **contracted vein,** (see Fig 11,) which forms itself just outside of certain kinds of openings, (and which is smaller than the openings themselves,) are actually *very nearly* equal to the theoretical ones; but through the *very opening itself* they are *usually* less. The discrepancy is greater in some cases than in others; depending *chiefly* on the shape of the opening.

On this account, the theoretical vel and disch found by the foregoing rule, must usually be diminished by mult them by certain decimal numbers corresponding to the various kinds of openings; and called *coefficients of discharge.* These coeffs have in many cases been determined by experiment *very approximately;* and will be found in the following articles. It will be seen in Remark 5, p 260, that by the use of a peculiarly formed adjutage, or attachment, to small openings, the *actual* disch may even be increased *beyond* the theoretical one.

The following table will save the trouble of calculating the *theoretical* vel, previously to mult it by the corresponding coeff of disch, for obtaining the actual vel. The coeffs for diff kinds of openings will be found further on.

TABLE 10. Of the theoretical velocities in feet per sec,

with which water should flow out into *the air*, under diff heads, through openings in the bottom or sides of the containing reservoir; the surf level of which remains constantly at the same height.

Weisbach says (see third footnote to Art 9) that when water flows out of an opening *under water*, as at n, Fig 1, the vel and disch are about $\frac{1}{75}$ part less than when it flows into the open air, under equal heads. When the disch is made under water, the vert dist $a\,n$, Fig 1, between the surf levels of the two reservoirs, must be taken as the head. These theoretical vels are *very* nearly the actual mean ones at the *contracted vein;* see Art 9. Calling the head, H, then

$$\underset{\text{in ft per sec}}{\textbf{Theoret vel}} = \sqrt{2\,g\,H} = \sqrt{64.4\,H} = 8.03 \text{ times the sq rt of the head in ft.}$$

$$\underset{\text{in feet}}{\textbf{Theoretical head}} = \frac{vel^2}{2\,g} = \frac{vel^2}{64.4} = \left(\begin{matrix}\text{square of theoret vel}\\ \text{in ft per sec}\end{matrix}\right) \times .0155.$$

Head	Vel.	Head	Vel.	Head	Vel.	Head	Vel.	Head	Vel.	Head	Vel	Head	Vel.
Feet.	Ft per sec.	Feet.	Ft per sec.	Feet.	Ft per sec.	Feet.	Ft per sec.	Feet.	Ft per sec.	Feet.	Ft per sec.	Fect.	Ft per sec.
.005	.57	.29	4.32	.77	7.04	1.50	9.83	7.	21.2	28	42.5	76	69.9
.010	.80	.30	4.39	.78	7.09	1.52	9.90	.2	21.5	29	43.2	77	70.4
.015	.98	.31	4.47	.79	7.13	1.54	9.96	.4	21.8	30	43.9	78	70.9
.020	1.13	.32	4.54	.80	7.18	1.56	10.0	.6	22.1	31	44.7	79	71.3
.025	1.27	.33	4.61	.81	7.22	1.58	10.1	.8	22.4	32	45.4	80	71.8
.030	1.39	.34	4.68	.82	7 26	1.60	10.2	8.	22.7	33	46.1	81	72.2
.035	1.50	.35	4.75	.83	7.31	1.65	10.3	.2	23.0	34	46.7	82	72.6
.040	1.60	.36	4.81	.84	7.35	1.70	10 5	.4	23.3	35	47.4	83	73.1
.045	1.70	.37	4.87	.85	7.40	1.75	10.6	.6	23.5	36	48.1	84	73.5
.050	1.79	.38	4.94	.86	7.44	1.80	10.8	.8	23.8	37	48.8	85	74.0
.055	1.88	.39	5.01	.87	7.48	1.85	10.9	9.	24.1	38	49.5	86	74.4
.060	1.97	.40	5.07	.88	7.53	1.90	11.1	.2	24.3	39	50.1	87	74.8
.065	2.04	.41	5.14	.89	7.57	1.95	11.2	.4	24.6	40	50.7	88	75.3
.070	2.12	.42	5.20	.90	7.61	2.	11.4	.6	24 8	41	51 3	89	75.7
.075	2.20	.43	5.26	.91	7.65	2.1	11.7	.8	25.1	42	52.0	90	76.1
.080	2.27	.44	5.32	.92	7.70	2.2	11.9	10.	25.4	43	52.6	91	76.5
.085	2.34	.45	5 38	.93	7.74	2.3	12.2	.5	26.0	44	53.2	92	76.9
.090	2.41	.46	5.44	.94	7.78	2.4	12.4	11.	26.6	45	53.8	93	77.4
.095	2.47	.47	5 50	.95	7.82	2.5	12.6	.5	27.2	46	54.4	94	77.8
.100	2.54	.48	5.56	.96	7.86	2.6	12.9	12.	27.8	47	55.0	95	78.2
.105	2.60	.49	5.62	.97	7.90	2.7	13.2	.5	28.4	48	55.6	96	78.6
.110	2.66	.50	5.67	.98	7.94	2.8	13.4	13.	28.9	49	56.2	97	79.0
.115	2.72	.51	5.73	.99	7.98	2.9	13.7	.5	29.5	50	56.7	98	79.4
.120	2.78	.52	5.79	1 Ft.	8.03	3.	13.9	14.	30.0	51	57.3	99	79 8
.125	2.84	.53	5.85	1.02	8.10	3.1	14.1	.5	30.5	52	57.8	100	80.3
.130	2.89	.54	5.90	1 04	8.18	3.2	14.3	15.	31.1	53	58.4	125	89.7
.135	2.95	.55	5.95	1 06	8.26	3.3	14.5	.5	31.6	54	59.0	150	98.3
.140	3.00	.56	6.00	1.08	8.34	3.4	14.8	16.	32.1	55	59.5	175	106
.145	3.05	.57	6.06	1.10	8.41	3.5	15.	.5	32.6	56	60.0	200	114
.150	3.11	.58	6.11	1.12	8.49	3.6	15.2	17.	33.1	57	60.6	225	120
.155	3.16	.59	6.17	1 14	8.57	3.7	15.4	.5	33.6	58	61.1	250	126
.160	3.21	.60	6.22	1.16	8.64	3.8	15.6	18.	34.0	59	61.6	275	133
.165	3.26	.61	6.28	1.18	8.72	3.9	15.8	.5	34.5	60	62.1	300	139
.170	3.31	62	6.32	1 20	8.79	4.	16.0	19.	35.0	61	62.7	350	150
.175	3.36	.63	6 37	1.22	8.87	.2	16.4	.5	35.4	62	63.2	400	160
.180	3.40	.64	6.42	1.24	8.94	.4	16.8	20.	35.9	63	63.7	450	170
.185	3.45	.65	6.47	1.26	9.01	.6	17.2	.5	36.3	64	64.2	500	179
.190	3.50	.66	6.52	1.28	9.08	.8	17.6	21.	36.8	65	64.7	550	188
.195	3.55	.67	6 57	1.30	9.15	5.	17.9	.5	37.2	66	65.2	600	197
.200	3.59	.68	6.61	1.32	9.21	.2	18.3	22.	37.6	67	65.7	700	212
.21	3.68	.69	6.66	1.34	9.29	.4	18.7	.5	38.1	68	66.2	800	227
.22	3.76	.70	6.71	1.36	9.36	.6	19.	23.	38.5	69	66.7	900	241
.23	3.85	.71	6.76	1.38	9.43	.8	19 3	.5	38.9	70	67.1	1000	254
.24	3.93	.72	6.81	1.40	9.49	6.	19.7	24.	39.3	71	67.6		
.25	4.01	.73	6.86	1.42	9.57	.2	20.0	.5	39.7	72	68.1		
.26	4.09	.74	6.91	1.44	9.63	.4	20.3	25	40.1	73	68.5		
.27	4.17	.75	6 95	1.46	9.70	.6	20.6	26	40.9	74	69.0		
.28	4.25	.76	6.99	1.48	9 77	.8	20.9	27	41.7	75	69.5		

Art. 8. On the flow of water through vertical openings furnished with short tubes. When water flows from a reservoir, Fig 6, through a vert partition *m m a a*, the thickness *a m* of which is about 2½ or 3 times the least transverse dimension of the opening, (whether that dimension be its breadth, or its height;) or when, if the partition be very thin, as *n n*, the water flows through a tube, as at *t*, the length of which is about 2 or 3 times its least transverse dimension, then the effluent stream will entirely fill the opening, or the tube, as shown in Fig 6; or, in technical language, will *run with a full flow; or a full bore;* and will disch more water in a given time, than if the tube were either materially longer or shorter. For if longer than 3 times the least transverse dimension, the flow will be impeded by the increased friction against the sides of the tube; and if shorter than about twice the least transverse dimension, the water will not flow in a full stream, but in a contracted one, as shown by Fig 11. This will be the case whether the tube be circular, or rectilinear, in its cross-section.

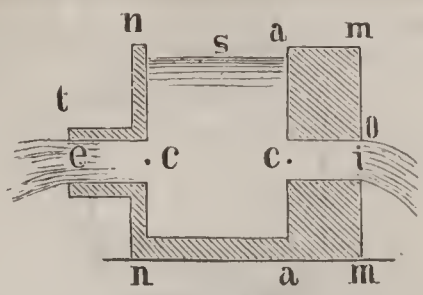


Fig. 6.

To find approximately the actual vel. and disch into the air, through a tube, or opening, either circular or rectilinear in its outline, or cross-section; and whose length *c i*, or *c e*, in the direction of the flow, is about 2½ or 3 times its least transverse dimension; when the surface-level, *s*, Fig 6, remains constantly at the same height; and which height must not be below the upper edge of the tube, or opening.

Rule 1. Take out the theoretical vel from Table 10, corresponding to the head measured *vert* from the center (or more properly, the cen of grav) *c*, of the opening, to the level water surf *s*. Mult it by the coeff of disch .81. The prod will be the reqd vel, in ft per sec. Mult this actual vel by the transverse area of the opening, in sq ft. If circular, knowing its diam. this area will be found in Table 3. The prod will be the quantity of water dischd, in cub ft per sec; within, probably, 3 or 4 per cent.

Rule 2. Find the sq rt of the head in ft. Mult this sq rt by 6.5. The prod will be the actual vel in ft per sec.

Ex. An opening *c o*; or box-shaped tube *c t*, Fig 6, is 3 feet wide, by .25 of a ft high; and its length in the direction *c t* or *c e* in which the water flows is about .62 of a ft, or about 2½ times its least transverse dimension, or its height. The head from the cen of grav *c*, of the opening, to the constant surf-level *s*, is 4 feet. What will be the vel of the water: and how much will be dischd per sec?

By Rule 1. The theoretical vel (Table 10.) corresponding to a head of 4 ft is 16 ft per sec. And $16 \times .81 = 12.96$ ft per sec, the actual vel reqd. Again the transverse area of the opening, or of the tube, is 3 ft $\times$ .25 ft $= .75$ sq ft. And $.75 \times 12.96 = 9.72$ cub ft; the quantity dischd per sec.

By Rule 2. The sq rt of 4 is 2. And $2 \times 6.5 = 13$ ft per sec. the reqd vel, as before; the very slight diff being owing to the omission of small decimals in the coeffs.

Rem. 1. **If the short tube *t* projects partly inside of the vert partition *n n*,** the disch will be diminished about ⅛ part. In that case, use .71 or .7 instead of the .81 of Rule 1; or 5.7 instead of the 6.5 of Rule 2.

Rem. 2. When the thickness *a m* of the vert partition *m m a a*; or the length *c e* of the tube *t*, Fig 6, is increased to about 4 times the least transverse dimension of the opening; or of the diam, when circular; then the additional friction against its sides begins appreciably to lessen the vel and disch. In that case, or for still greater lengths, up to 100 diams, they may be found approximately, by using instead of the coeff of disch .81 in Rule 1, the following coeffs, by which to mult the theoretical vels of Table 10. Or use Rule, p 243.

TABLE 11.

Length of Pipe in Diams.	Coeff.	Length of Pipe in Diams.	Coeff.
4	.80	40	.62
6...	... .76	50...	... .60
10	.74	60	.57
15...	... .71	70...	... .55
20	.69	80	.52
25...	... .67	90...	... .50
30	.65	100	.48

Rem. 3. When the length of the opening or tube, in the direction in which the water flows, becomes *less* than about twice its least transverse dimension, the disch is diminished; so that for lengths from 1½ times, down to openings in a very thin plate, we may use .61, instead of the .81 of Rule 1. For such openings, however, see Arts 9 and 10.

Rem. 4. But on the other hand, the disch through such short openings and tubes as are shown in Fig 6, may be increased to nearly the theoretical ones of Table 10, by merely rounding off neatly the edges of the entrance end or mouth, as in Fig 7; which is the shape, and half actual size of one with which Weisbach obtained .975 of the theoretical vel and discharge, when the head was 10 ft; and .958

with a head of one foot; so that in similar cases, .975, and .958 may be used instead of the coeff .81 in Rule 1.

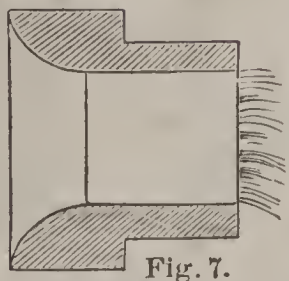


Fig. 7.

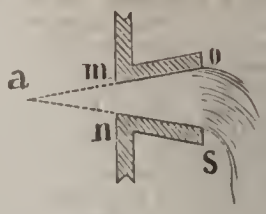


Fig. 8.

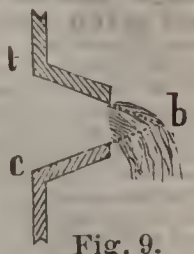


Fig. 9.

As much as .92 to .94 may be obtained by widening the opening, *m n*, toward its outer mouth, o *s*, Fig, 8, making the divergence, or angle *a*, about 5°: or by widening it toward its inner mouth, as at *t c*, Fig 9; but increasing the angle of divergence, at *b*, to from 11° to 16°. In all cases, we consider the small end as being the opening whose area must be multiplied by the vel to get the discharge.

In some experiments made with large pyramidal wooden troughs 9.5 ft long, with an inner mouth of 3.2 × 2.4 ft, and a discharging one of .62 × .44 ft; and under a head of 9½ feet, the discharge was .98 of the theoretical one, due to the smaller end. Therefore, .98 may be used in such cases, instead of the .81 of Rule 1.

Rem. 5. **The discharge through a short opening of small transverse section may even be made 50 per cent greater than the theoretical one,** by adopting the shape, Fig 10; where *m n* is supposed to be the diameter of the opening. The best proportions appear to be about as follows: o *y* = 9 inches; *m n* = 1 inch; *b c* = 1.8 inch; o *s* = ½ inch; *a d* = 2 ins; the curves, *a m*, and *d n*, being quadrants; the angle, *x*, of divergence, about 5° 6'; and the tube of polished metal. In this case use 1.55, or more safely, 1.5, instead of the .81 of Rule 1. The only experiments with this form have been on a very small scale. To what extent it may be applicable is unknown.

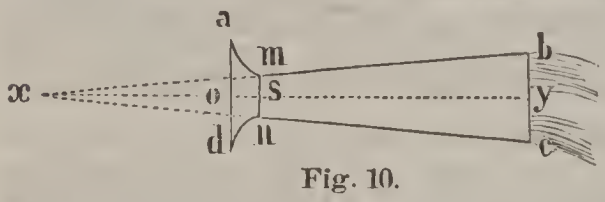


Fig. 10.

So far as regards the ordinary operations of the engineer, this subject is perhaps more curious than useful; for he will rarely have any difficulty in making his openings large enough, without resorting to such aids; except, perhaps, that of rounding off the inner edges, as in Fig 7; which is usually done.

Art. 9. On the disch of water through openings in thin vert partitions, with plane or flat faces, *e e*, or *n n*, Fig 11.* If the face *e e*, or *n n*, instead of being plane, and vert, should be curved, or inclining in diff directions toward the opening, then the disch will be altered. When water flows from a reservoir, Fig 11, through a vert plane plate or partition *n n*, which is not thicker than about the least transverse dimension of the opening, whether that dimension be its breadth, or its height *o o*;† or when, if the partition *e e* itself is much thicker, we give the opening the shape shown at *b*, (which evidently amounts to the same thing,) then the effluent stream will not pass out with a *full flow*, as in Fig 6, but will assume the shape shown in Fig 11; forming, just outside of the opening, what is called the *vena contracta*, or *contracted vein*. In order that this contraction may take place to its fullest extent, or become *complete*, the inner sharp edges of the opening must not approach either the surf of the water, or the bottom or sides of the reservoir, nearer than about 1½ times the least transverse dimension of the opening. The contracted vein occurs at a dist of about half the smallest dimension of the orifice, from the orifice itself. In a circular orifice, at about half the diam dist; and ordinarily its area is about .62, or nearly ⅝ that of the orifice itself. At this point the actual mean *vel* of the stream is very nearly (about .97) the theoretical vel given by Table 10, and hence the actual *discha* are but .62, or nearly ⅝ of the theoretical ones.

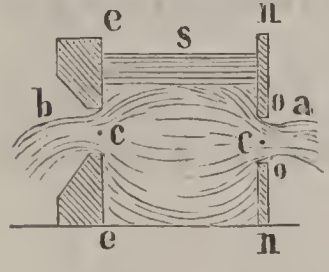


Fig. 11.

Case 1. **To find the actual disch into air,‡ through either a circular or rectilinear§ opening in a thin vert plane parti-**

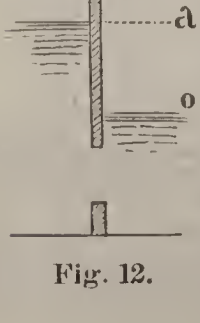


Fig. 12.

* We believe that these rules for thin plate are also sufficiently approximate for most practical purposes, if the opening be in the *bottom* of the reservoir; or in an *inclined*, instead of a *vert* side.

† When the side of a reservoir, or the edge of a plank, &c, over which water flows, has no greater thickness than this, the water is said to flow through, or over, **thin plate**, or **thin partition.**

‡ Should the disch take place *under water*, as in Fig 12, *both surf-levels remaining constant*, then the head to be used is the *vert* diff *a o*, of the two levels. After making the calculation with this head, we should, according to Weisbach, deduct the $\frac{1}{75}$ part; inasmuch as he states that the disch is that much less when under water, than when it takes place freely into the air. Other experimenters, however, assert that it is precisely the same in both cases.

§ If the shape of the opening is oval, triangular, or irregular, the head must be measured vert from its cen of grav.

tion, when the contraction is complete; and when the surf-level, *s*, remains constantly at the same height; water being supplied to the reservoir as fast as it runs out at the opening.*

Rule 1. When the head, measured vert from the center (or rather from the cen of grav) *c*, of the opening, to the surf-level *s* of the reservoir, is not less than 1 ft, nor more than 10 ft; and when the least transverse dimension of the opening is not less than an inch, mult the theoretical vel in ft per sec due to the head, (Table 10,) by the coefficient of disch .62. The prod will be the actual mean vel of the water through the opening. Mult this vel by the area of the opening in sq ft; the prod will be the disch in cub ft per sec, approximately.

When the head is greater than 10 ft, use .6, instead of .62.

Rule 2. Find the sq rt of the head in ft. Mult this sq rt by 5; the prod will be the vel in ft per sec; which mult by the area as before for the disch.

Ex. What will be the disch through an opening in complete contraction, whose dimensions are 6 ins, or .5 ft vert; and 4 ft hor; the vert head above the cen of grav of the opening being constantly 6 feet?

By Rule 1. The theoretical vel (Table 10,) corresponding to 6 ft head, is 19.7 ft per sec. And $19.7 \times .62 = 12.214$ ft, the reqd vel. Again, the area of the opening $= .5 \times 4 = 2$ sq ft; and $12.214 \times 2 = 24.428$ cub ft per sec; the disch.

By Rule 2. The sq rt of $6 = 2.45$; and $2.45 \times 5 = 12.25$ ft per sec, the reqd vel; and $12.25 \times 2 = 24.5$ cub ft per sec, the disch.

Both very approx even if the orifice reaches to the surface of the issuing water.

Rem. 1. The coef .62 is a mean of results of many old experimenters. In 1874 Genl. T. G. Ellis of Massachusetts conducted an elaborate series (Trans Am Soc C E, Feb 1876) on a large scale, the general results of which, within less than 1 per ct, are given in the following table. See also Rem 3. The sharp edged orifices were in iron plates .25 to .5 inch thick.

Orifice.	Head above Center.	Coef.
2 ft sq.	2. to 3.5 ft.	.60 to .61
2 " long, 1 ft high	1.8 to 11.3 "	.60 to .61
2 " long. .5 high	1.4 to 17.0 "	.61 to .60
2 " diam.	1.8 to 9.6 "	.59 to .61

Rem. 2. Extreme care is reqd to obtain correct results; but for many purposes of the engineer an error of 5 to 10 per ct is unimportant.

It will rarely happen that greater accuracy is required than may be obtained by the foregoing rules; but when such does occur, aid may be derived from the following **table deduced from the experiments of Lesbros and Poncelet.** on openings 8 ins wide, of diff heights, and with diff heads. Use that coeff in the table which applies to the case, instead of the .62 of Rule 1. In some of the cases in this table, the upper edge of the opening is nearer the surf-level of the reservoir than 1½ times its least transverse dimension.

TABLE 12. Coefficients for rectangular openings in thin vertical partitions in full contraction.*

Head above cen. of grav. of opening in Feet.	Head above cen. of grav. of opening in Inches.	The breadth in all the openings = 8 inches. HEIGHT OF OPENING. Ins. 8	Ins. 6	Ins. 4	Ins. 3	Ins. 2	Ins. 1	Ins. .4
.033	.4	.....	.....	.....	.....	.....	.....	.70
.0666	.8	.....	.....	.....	.....	.....	.65	.69
.0833	1	.....	.....	.....	.....	.....	.64	.68
.125	1½	.....	.....	.....	.....	.61	.64	.68
.1666	2	.....	.....	.....	.60	.62	.64	.68
.2083	2½	.....	.....	.59	.61	.62	.64	.67
.250	3	.....	.....	.60	.61	.62	.64	.67
.2917	3½	.....	.57	.60	.61	.62	.64	.66
.3333	4	.....	.58	.60	.61	.63	.64	.66
.3750	4½	.56	.59	.60	.61	.63	.64	.66
.4167	5	.57	.59	.61	.62	.63	.64	.66
.6666	8	.59	.60	.61	.62	.63	.64	.65
1	12	.60	.60	.61	.62	.63	.63	.64
3	36	.60	.60	.61	.62	.62	.63	.63
5	60	.60	.60	.61	.61	.62	.62	.62
10	120	.60	.60	.60	.60	.60	.61	.61

Rem. 3. **Careful experiments on openings 4¼ ft wide, and 18 ins high,** under heads of from 6 to 15 ft, show that the coeff .62 will give results correct within $\frac{1}{30}$ part, for openings of that size also, under large heads; although the thickness of the partition varied on its diff sides, from 12 to 20 ins. It must be recollected, however, that nothing more than close *approximations* are to be attained in such matters.

Rem. 4. It has been asserted by some writers, that **when two or more *contiguous* openings** are discharging at the same time from the same reservoir, they disch less in proportion than when only one of them is open. Other experiments, however, seem to show that this is not the case; it is therefore probable, at least, that the diff, if any, is but trifling. See Art 1 N, p. 239.

* See first footnote on preceding page.

Case 2. **The discharge through thin vert partitions in complete contraction, when the surface-level, *m*, Fig 13, descends as the water flows out into the air.** In this case, if the reservoir is prismatic, that is, if its hor sections are everywhere equal; and if no water is flowing into the reservoir, to supply the place of that which flows out, then, to find the time reqd to disch the reservoir.

RULE. Inasmuch as the time in which such a reservoir entirely discharges itself, is twice that in which the same quantity would flow out under a constant head, as in Case 1, therefore, calculate the disch in cub ft per sec by Rule 1, Art 9; div the number of cub ft contained in the reservoir, above the level *g* of the bottom of the opening, Fig 13, by this disch; the quot will be the number of sec in which a volume equal to that in the reservoir, to the depth *g*, would run out in Case 1, of a *constant* head. And *twice* this number will be the seconds reqd to empty the reservoir in Case 2, of a *varying* head.

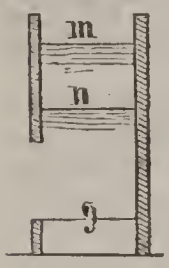


Fig. 13.

REM. If it should be reqd to find the time in which such a prismatic reservoir would *partly* empty itself, as, for instance, from *m* to *n*, Fig 13, first calculate, by the above rule, the secs necessary to empty it if it had only been filled to *n*: and afterward calculate as if it had been filled to *m*. The diff between the two times will evidently be the time reqd to empty it from *m* to *n*. If the opening is not in complete contraction, see Arts 11, &c.

If the disch is into a lower reservoir, whose surf-level remains constant, proceed in the same manner; only use the diff of level of the two surfs as the head, and afterward (according to Weisbach) increase the time $\frac{1}{75}$ part.

Art. 10. Disch from a reservoir R, Fig 14, the surf-level, *s*, of which remains constantly at the same height; through an opening, *o*, in thin vert partition; and in complete contraction; but entirely under water; and into a prismatic reservoir, *m*.

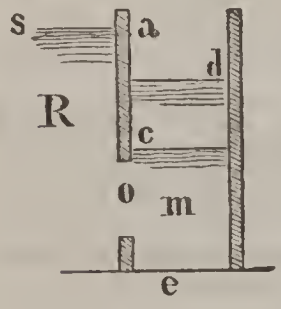


Fig. 14.

Seconds required to discharge a quantity = *e d a, the level* c *remaining constant.*

$$= \frac{\sqrt{\text{height } a\,c \text{ in ft}} \times \text{hor area of } m \text{ in sq ft}}{\text{area of opening } o \text{ in sq ft} \times .62 \times 8.03}$$

Seconds required to *raise level* in *m* from e to *a*

$$= \frac{\sqrt{\text{height } a\,e \text{ in ft}} \times \text{hor area of } m \text{ in sq ft} \times 2}{\text{area of opening } o \text{ in sq ft} \times .62 \times 8.03}$$

Seconds required to *raise level* in *m* from e to *any other level, d.*

$$= \frac{\left(\sqrt{a\,c} - \sqrt{a\,d} \text{ both in ft}\right) \times \text{hor area of } m \text{ in sq ft} \times 2}{\text{Area of opening } o \text{ in sq ft} \times .62 \times 8.03}$$

REM. 1. **If it should be reqd to find the time of filling *m*, from its bottom *e*, up to *d*,** we may do so very approximately by calculating by the first rule in Art 9, the time reqd from *e* to the center of the opening *o*, as if all that portion of the disch took place into air; and afterward, from the center of the opening to *d*, by the rule just given. This case is similar to that of filling a lock from the canal reach above, in which the surf-level may be considered constant.

REM. 2. **If the bottom of the opening *o*, should coincide with the bottom of the reservoir,** then the coeff will become greater than .62. See Art 11, for obtaining coeffs for imperfect contraction.

REM. 3. **If the opening, instead of being in complete contraction,** is of any of the shapes Figs 6 to 9, then a reference to Art 8 will show what coeff must be substituted for .62.

Case 3. **Disch from one prismatic reservoir, Fig 15, W, into another, X, of any comparative sizes whatever, through an opening *o*, in a plane thin vert partition, and in complete contraction; when the water rises in X, while it falls in W.**

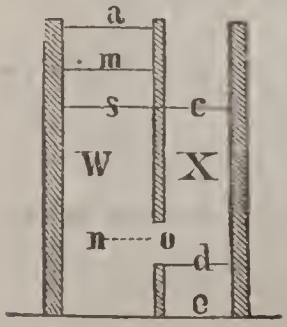


Fig. 15.

To find the time in which the water, flowing from W *into* X, *through* o, *will fall through the dist* a s, *so as to stand at the same level* s e, *in both reservoirs.*

In this case, the water reqd to fill X from *e* to *d*, (*d* being the bottom of the opening o,) flows out into the air; and the time necessary for it to do so, must be calculated separately from that reqd above *d*, which flows into water.

RULE. First from *e* to *d*. Find the hor area of each reservoir, in sq ft. Mult the hor area of X, by the vert depth *d e* in ft, for the cub ft contained in that portion. Div these cub ft by the hor area of W. The quot will be the dist *a m*, in feet, through which the water in W must descend, in order to fill X to *d*.

Seconds required to lower from *a* to *m*, and raise from *e* to *d*.

$$= \frac{\text{Twice the hor area of W in sq ft} \times \left(\sqrt{\text{head } a\,n \text{ in ft}} - \sqrt{\text{head } m\,n \text{ in ft}}\right)}{\text{Area of opening } o \text{ in sq ft} \times .62 \times 8.03}$$

Seconds required to lower from m to s, and raise from d to c. (Very approx)

$$= \frac{\text{Hor area of X in sq ft} \times \text{twice the hor area of W in sq ft} \times \sqrt{\text{head } m\,n \text{ in ft}}}{\text{Area of opening o in sq ft} \times \left(\frac{\text{hor area of W}}{\text{in sq ft}} + \frac{\text{hor area of X}}{\text{in sq ft}}\right) \times .62 \times 8.03}$$

Ex. Let the hor area of W be 100 sq ft: and that of X, 60 sq ft. Let $a\,n$ be 20 ft; and $m\,n$ 16 ft; and the area of the opening o, 3 sq ft. In what time will the water descend from a to s, and rise from e to c?

Inasmuch as the method of finding the time for filling from e to d, by the water falling from a to m, requires no further exemplification, we will confine ourselves to the *additional* time necessary for filling from d to c, by the water falling from m to s. To find this, we have, the sq rt of the head $m\,n = 4$ ft; and the sum of the 2 areas $= 100 + 60 = 160$. Hence, $\frac{4 \times 100 \times 60 \times 2}{160 \times 8.03 \times 3 \times .62} = \frac{48000}{2389.73} =$ 20.1 sec; the additional time reqd, very approximately.

Note 1. **If the opening, as d, Fig 16, reaches to the very bottom of the reservoirs,** we may consider all the water flowing from R into T, as flowing into water. Therefore, using the head $a\,m$, we at once calculate the time necessary for the water in the two reservoirs to arrive at the same level s c, by the last process of the preceding rule; or, in other words, by the process given in the preceding example. But in this case it must be borne in mind that the opening o is no longer in *complete* contraction, inasmuch as the contraction along its lower edge is suppressed.

The disch will consequently be somewhat increased; and a coeff greater than .62 becomes necessary. The method of finding this, is given in the following Case 4. A reference to Art 8 will give the coeff in case the opening is shaped as Figs 6 to 9.

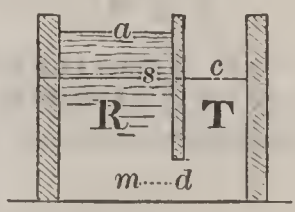


Fig. 16.

Art. 11. *Case* 4. The discharge through openings in plane thin vert partitions; but in incomplete contraction.

The opening may be such that contraction will take place along one portion of its perimeter, or at the top of the opening a, Fig 17; while it is suppressed on another portion; as at the bottom and two ends of the opening a; where suppression is caused by the addition of *short* side and bottom pieces c, c, c. Or it may be caused by the bottom, or ends, or both, coinciding with the bottom and sides of the reservoir. In such cases the disch will be greater than in those of complete contraction; but less than in those of full flow; inasmuch as the opening now partakes somewhat of the character of the *short* tubes of Art 8; and the coeff will rise from .62, or that which *usually* pertains to openings in full contraction; and will approach .8, or that of full flow, in proportion to the extent of perimeter along which contraction is suppressed; or even to .9 or .98 by the use of such openings as are shown by Figs 7, 8, 9.

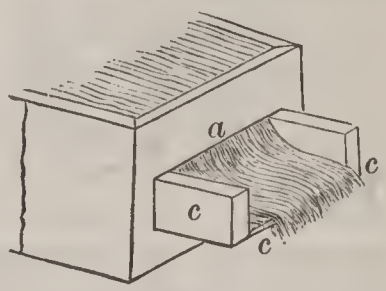


Fig. 17.

To find approximately a new coeff of disch; and the disch itself, in cases of incomplete contraction.

Rule. First find by the foregoing rules, what would be the disch in the particular case that may be under consideration, supposing the contraction to be complete. Then div that portion of the perimeter of the opening on which contraction is suppressed, by the entire perimeter. Mult the quot by the dec .152 if the opening is rectangular, or by .128 if circular. To the prod add unity, or 1. Call the sum, g. Then say, as unity, or 1, is to g, so is the coeff for complete contraction in ordinary cases (usually .62) to the reqd new coeff. Finally, repeat the original calculation, only substituting this new coeff in the place of .62.

According to this rule, we have the following coeff of discharge for rectangular openings within probably 3 or 4 per cent when contraction is not suppressed on more than ⅛ of the perimeter. The theoretical discharge multiplied by the corresponding coeff will give the actual discharge. When the contraction is carried farther, the coeff becomes extremely irregular, and is probably indeterminable.

For complete contraction (ordinarily)	.62
When contraction is suppressed on ¼ the perimeter	.64
" " " " " ½ " "	.67
" " " " " ¾ " "	.69
" " " " *entirely around the orifice*	.80

Intermediate ones can be estimated nearly enough, mentally.

Rem. 1. **When, instead of a short spout, as in Fig 17, the opening is provided with an indefinitely long hor trough,** similarly attached, and open at top, there will be no practically appreciable diminution of disch below that through the simple opening as at a, Fig 11; provided the head measured above the cen of grav of the opening be at least as great as 2 or 2½ times the height of the opening itself. Therefore, under such circumstances the disch may be calculated by the rules in Art 9. But with smaller heads the disch diminishes considerably; so when the head above the center becomes but as great as the height of the opening, it will be but about $\frac{4}{5}$ of the calculated one. With still smaller heads, the flow becomes less much more rapidly; but has not been reduced to any rule.

Rem. 2. **If, instead of being hor, the trough is INCLINED**

as much as 1 in 10, the disch will be *increased* very slightly, (some 3 or 4 per cent) over that calculated by the rules in Art 9, for the plain opening. These results were obtained by experiments on a very small scale; and should be considered as mere approximations.

Art. 12. In a case like Fig 18, where contraction is supposed to be suppressed at the bottom, and at both vert sides of the opening *o*, in consequence of their coinciding with the bottom and sides of the reservoir: but where the front of the reservoir, instead of being vert, is sloped as at *f*; and when the water, after leaving the opening, flows away over a slightly sloping apron, *g*, then the disch in cub ft per sec may be approximately found by Rule 1, Case 1, Art 9, only substituting .8 in place of .62, when *f* slopes back 45°, or 1 to 1; or .74 when *f* slopes back 63°, or with a base of 1 to a rise of 2. In such cases of inclined fronts, the height of the opening must be measured *vert*, or rather *at right angles to the floor of the reservoir;* and not in a line with the sloping front.

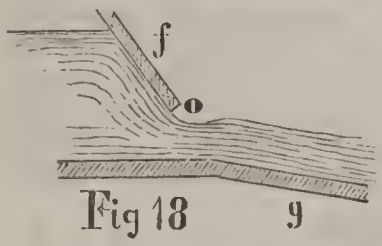


Fig 18

REM. **When the front, *f*, of the reservoir is vert, and a sloping apron or trough, *g*, is used,** having its upper edge level with the bottom of the opening, the disch is not appreciably diminished below that which takes place freely into the air, provided the head above the cen of grav of the opening is not less than from

18 to 24 ins, for an opening 6 to 9 ins high.
12 to 16 " " " " 4 ins high.
9 " " " 2 ins or less, high.

Art. 13. To find, approximately, the time reqd for the emptying of a pond, or any other reservoir, as Fig 19, which is not of a prismatic shape; through an opening, *n*, near the bottom.

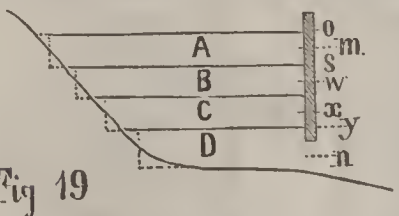


Fig 19

RULE. First ascertain the exact shape and dimensions of the reservoir. If large, and irregular, it must be carefully surveyed; and soundings taken, and figured upon a correct plan and cross-sections. Next, consider the entire body of water to be divided into a series of thin hor strata, A, B, C, D; the top line of the lower one being at least a few ins above the top of the opening *n*. It is not necessary that these strata should be of equal thickness; although the thinner they are, the more correct will the result be. The depth of the lower one, D, will vary to some extent with the height of the opening; those next above it should not exceed about a foot in thickness, until a depth of 6 or 8 feet is reached; then they may conveniently, and with sufficient accuracy, be increased to about 2 ft, for 6 or 8 ft more; and so on; becoming thicker as they approach the surf. By aid of the drawings, calculate the content of each stratum in cub ft. Now, since the strata are thin, we may, without serious error, assume each of them to be prismatic, as shown by the dotted lines; and may assume that the head under which each stratum (except the lowest) empties itself through *n*, is equal to the vert height from the center of the opening to the center of the stratum. Thus, *m n* will be the head of A; *w n*, the head of B; *x n*, the head of C. Then, for the stratum A, by Rule 1, Art 9, (only using *m n* as the head instead of *o n*,) and instead of the coeff .62 of that rule (which can only be used if *n* is in complete contraction) using .64, or whatever other coeff near the end of Art 11 applies to the case, calculate the disch in cub ft per sec. Div the content of the stratum A by this disch, and the quot will be the number of sec reqd for discharging A. Using the head *w n*, proceed in precisely the same way with the stratum B; and using the head *x n*, do the same with C. Finally, for the lower stratum D, find by Rule 1, Art 9, (with the same caution as before respecting the proper coeff.) in what time it would empty itself under a *constant* head equal to *y n*, measured from its *surf* to the center of the opening. *Double* this time will be that reqd to empty itself in the case before us, under its *varying* head. Finally, add together all these separate times; and their sum will be the entire time reqd to empty the pond, or reservoir, approximately enough for practical purposes.

Art. 14. On the discharge of water over weirs, or overfalls. Experiments and observations on a grand scale, were made on this subject at Lowell, Mass, by **Mr James B. Francis,** C E, one of the most accomplished hydraulicians of the age. (See his "Lowell Hydraulic Experiments.") To apply the rule arrived at by Mr Francis, the following conditions must exist.

The crest *a*, Fig 20, or top of the weir over which the water discharges, must be a hor, sharp corner, in thin plate, or thin partition. (See p 260; first footnote.) Its inner side must form a vertical straight line, *a h*, with the inner face of the dam, to a depth *a h*, *not less than* twice the depth, or head *a m*, measured vert from *a* to the level *o m* of the *hor* portion of the water surf; and not to c, the *curved* surf of the falling sheet of water. The head *a m* may vary from 6 to 24 ins in height. The ends *a h*, *a h*, of the weir, Figs 21 and 22, must be vert; and its length, *a*, *a*, not less than 3 times the head *a m*.

These conditions being observed, we may distinguish 2 cases; namely, Case 1st, that in which, as in Fig 21, the weir extends entirely across the reservoir: so that its ends *a h*, *a h*, coincide with, or form portions of, the sides *s*, *s*, of the reservoir: in which case contraction takes place only along the upper edge *a*, *a*, of the weir, Fig 21, as shown at *a* in Fig 20; but is suppressed entirely at the ends, so that the water flows out as shown in plan by Fig 23. And Case 2d, in which, as in Fig 22, the vert ends *a h*, *a h*, as well as the crest *a*, *a*, are formed with a sharp corner in thin plate; and are, moreover, removed from the sides *v*, *v*, of the reservoir, a dist equal *at least* to the head *a m*; so that contraction takes place at the ends of the weir, as well as along its crest; and less water flows out, as shown in plan at *a*, *a*, Fig 24.

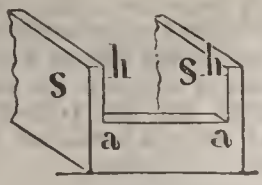


Fig 21

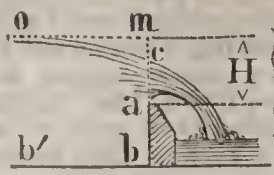


Fig 20

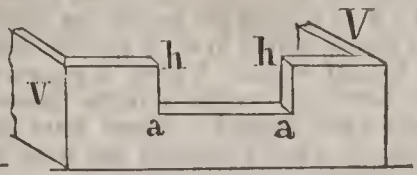


Fig 22

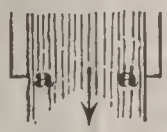


Fig 23

Fig 24

To find the discharge over a weir in thin plate.

Case 1. When there is no contraction at either end of the weir. Figs 21 and 23:

$$\text{Discharge in cub ft per sec} = 3.33 \times \text{the length } a\,a \text{ of the overfall, in ft} \times \sqrt{\text{cube of the head } a\,m \text{ in ft.}}$$

Ex. How many cub ft per sec will flow over such a weir in thin plate, 200 ft long; having a head, $a\,m$, of 1.5 ft, measured to the *level* surf $o\,m$, of the reservoir; and with no contraction at either end?

Here, the cube of $1.5 = 3.375$. And the sq rt of $3.375 = 1.837$. And $3.33 \times 200 \times 1.837 = 1223.4$ cub ft per sec, the reqd disch.

This rule will also be very approximate even when there is contraction at both ends, provided the length of the weir is at least 10 times as great as the head $a\,m$; and provided the head is not less than 2 or 3 ins in depth. Indeed, it will be within about 6 per cent of the truth, for weirs with contraction at the ends, and whose lengths are but 4 times the head; and for the many cases in which no closer approximation is reqd, the disch may be taken at once from table 13.

Case 2. When contraction takes place at both ends, as in Figs 22 and 24, **or at one end only,** use the same formula as in Case 1; except that when there is contraction at *both* ends, one-*fifth* of the head $a\,m$ is to be taken from the length $a\,a$ before using it as a multiplier; and when there is contraction at *one* end only, one-*tenth* of $a\,m$ must first be taken from the length $a\,a$.

Ex. How many cub ft per sec will flow over such a weir in thin plate, 200 ft long; having a head $a\,m$, of 1.5 ft; with contraction at both ends?

Here, the cube of $1.5 = 3.375$. And the sq rt of $3.375 = 1.837$. Again, one-fifth of $a\,m = .3$ of a ft. And $200 - .3 = 199.7$. Therefore we have $3.33 \times 199.7 \times 1.837 = 1221.6$ cub ft per sec, the reqd disch; or (in this case) *practically* the same as when there is no contraction.

Rem. If, instead of 3.33, we use 3.41, the two foregoing rules will apply to heads $a\,m$, as small as ½ an inch; and coeffs between 3.33 and 3.41 may be used for heads between about 5 inches, and ½ an inch, where more than common accuracy is aimed at. We may also use 3.3 instead of 3.33, for heads greater than 2 feet.

Eytelwein's formula for weirs, over a thin edge, and having no contraction at either end, is identical with the above. His coefficient is 3.4.

From the Lowell experiments, by Mr Francis, it appears that when the depth, $a\,m$, is 1 foot, and the entire sheet of water, after passing over the weir, strikes a hor solid floor placed only about 6 ins below the crest a of the weir, the disch is thereby diminished but about the one-thousandth part; and that when the head $a\,m$ is about 10 ins, and falls into water of considerable depth, no diff whatever is perceptible in the disch, whether the surf of that water be about 4, or about 13 ins below the crest a; and that a fall below the crest a, equal to one-half of the head $a\,m$, is quite sufficient.

Art. 14 A. If the water in the reservoir, or in the feeding canal, instead of being stagnant, has a slight current toward the weir, the disch will be but very little increased thereby when the head $a\,m$ is several ins. Mr Francis observed that a current of 1 foot per sec, or nearly .7 of a mile per hour, increased the disch but about 2 per cent, when the head was 13 ins; and one of 6 ins per sec, about 1 per ct, when the head was 8 ins. Whenever the effect of the current, however, is so great as to require notice, proceed as follows: First find the approx disch as directed in Art 14, under Case 1 or Case 2, according to circumstances. Then, **in Case 2,** find the area in sq ft of cross section of channel of approach at $o\,b'$, Fig 20, measured between its two sides, and from bottom to surf of water. Divide this area by the length of the weir in ft, corrected for contraction as directed in Case 2 above. Call the quot D. **In Case 1,** D is = the depth $o\,b'$. In either case, divide the head $a\,m$ or H, in ft, by D. Find this last quot in the column $\frac{H}{D}$ in table 13 A, p 266. Mult the approx disch by the corresponding coeff in column K.

For this method of correction for vel of approach, and for table 13 A, we are indebted to Messrs A. W. Hunking and Frank S. Hart, Civ Engs, of Lowell, Mass. They are, in fact, simply a convenient method of applying **Mr. Francis' rule** for similar cases, which is as follows: Find in Table 10, the theoretical head h corresponding to the observed vel of approach. Add this head to the head $a\,m$, or H, Fig 20. Then, in the formula, Art 14, instead of

$$\sqrt{\text{cube of head } a\,m}, \text{ use } \sqrt{\text{cube of } (H + h)} - \sqrt{\text{cube of } h.}$$

TABLE 13. Of actual discharges in cub ft per sec, for each foot in length of weir in thin plate; and without contraction at either end; *a b*, Fig 20, being vert, and not less than twice the head *a m*. Very approximate also, when there is contraction at both ends, provided the length be at least 10 times the head. And but about 6 per cent in excess of the truth, if the length be but 4 times the head. (Original.)

The decimal .01 of a foot, is precisely .12 of an inch; or scant $\frac{1}{8}$ inch.

Head, *a m*, in Feet.	Cub. ft. per Second.	Head, *a m*, in Feet.	Cub. ft. per Second.	Head, *a m*, in Feet.	Cub. ft. per Second.	Head, *a m*, in Feet.	Cub. ft. per Second.	Head, *a m*, in Feet.	Cub. ft per Second.
.03	.017	.22	.351	.58	1.47	.94	3.04	2.4	12.2
.04	.027	.24	.401	.60	1.54	.96	3.14	2.5	13.0
.05	.038	.26	.452	.62	1.62	.98	3.24	2.6	13.8
.06	.050	.28	.505	.64	1.70	1.	3.33	2.7	14.6
.07	.063	.30	.560	.66	1.78	1.1	3.85	2.8	15.4
.08	.077	.32	.603	.68	1.86	1.2	4.38	2.9	16.2
.09	.092	.34	.659	.70	1.95	1.3	4.94	3.	17.1
.10	.108	.36	.719	.72	2.03	1.4	5.51	3.1	18.0
.11	.124	.38	.780	.74	2.12	1.5	6.11	3.2	18.9
.12	.142	.40	.842	.76	2.21	1.6	6.73	3.3	19.8
.13	.160	.42	.907	.78	2.30	1.7	7.37	3.4	20.7
.14	.178	.44	.972	.80	2 38	1.8	8.04	3.5	21.6
.15	.198	.46	1.04	.82	2.47	1.9	8.72	3.6	22.5
.16	.218	.48	1.11	.84	2.56	2.	9.42	3.7	23 5
.17	.239	.50	1.18	.86	2.65	2.1	10.0	3.8	24.4
.18	.260	.52	1.25	.88	2.74	2.2	10.8	3.9	25.4
.19	.282	.54	1.32	.90	2.84	2.3	11.5	4.	26.4
.2	.305	.56	1.40	.92	2.94				

In calculating this table, the coeff 3.41 was used for heads from .03 ft to .3 ft; then 3.33 to 2 ft; then 3.3 to the end.

TABLE 13 A. Coefficients K for velocity of approach. For the use of this table, see Art 14 A, p 265.

$\frac{H}{D}$	K	$\frac{H}{D}$	K	$\frac{H}{D}$	K	$\frac{H}{D}$	K	$\frac{H}{D}$	K
.01	1.0000	.09	1.0020	.17	1.0072	.24	1.0143	.31	1.0239
.02	1.0001	.10	1.0025	.18	1.0081	.25	1.0155	.32	1.0254
.03	1.0002	.11	1.0030	.19	1.0090	.26	1.0168	.33	1 0271
.04	1.0004	.12	1.0036	.20	1.0100	.27	1.0181	.34	1.0287
.05	1 0006	.13	1.0042	.21	1.0110	.28	1.0195	.35	1.0305
.06	1.0009	.14	1.0049	.22	1.0121	.29	1.0209	.36	1.0322
.07	1.0012	.15	1.0056	.23	1.0132	.30	1.0224	.37	1.0341
.08	1.0016	.16	1.0064						

The following rule for finding the discharge over an overfall, or weir, approximately, has been prepared by the author from various data.

Mult together the length, *a*, *a*, of the weir, Figs 21 and 22, in feet; the head, *a m*, Fig 20, measured to the *level* surf of the reservoir, in ft; the theoretical vel in ft per sec, corresponding to the head *a m*, (Table 10, p 258;) and that coeff from the following table, which agrees most nearly with the case. The prod will be the reqd disch in cub ft per sec, near enough for ordinary purposes; and probably quite as close as can be arrived at without actual measurement in each case that presents itself.

Ex. How much water will be dischd over a weir 60 ft long; the crest of which is level, smooth, and 3 ft wide, or thick; and over which the head *a m*, Fig 20, is 8 ins, or .6666 ft thick? Here the theoretical vel for a head of 8 ins, (Table 10.) is 6.55 ft per sec. The coeff for a weir whose crest is level, and 3 ft wide, with a head of 8 ins, is by the following table .31. Consequently, $60 \times .6666 \times 6.55 \times .31 = 81.21$ cub ft per sec; the reqd disch approximately.

TABLE 14. Of coefficients of approximate discharge over weirs of different thicknesses, varying from a sharp edge to 3 feet. —(Original.)

Head $a\,m$ in Feet.	Head $a\,m$ in Inches.	Sharp Edge.	2 Inches Thick.	3 Ft Thick; smooth; sloping outward: and downward, from 1 in 12 to 1 in 18.	3 Ft Thick; smooth, and level.
.0833	1	.41	.37	.32	.27
.1666	2	.40	.38	.34	.30
.25	3	.40	.39	.34	.31
.3333	4	.40	.41	.35	.31
.4166	5	.40	.41	.35	.32
.5	6	.39	.41	.35	.33
.5833	7	.39	.41	.35	.32
.6666	8	.39	.41	.34	.31
.8333	10	.38	.40	.34	.31
1.	12	.38	.40	.33	.31
2.	24	.37	.39	.32	.30
3.	36	.37	.39	.32	.30

Art. 15. If the inner face of the weir and dam, instead of being vert, as $a\,b$, Fig 20, is sloped, as a or b, Fig 25; the contraction on the crest will be diminished; and consequently the disch will be increased. This will also be the case if the inner corner or edge of the crest be rounded off, instead of being left sharp; or if the sides of the reservoir converge more or less as they approach the weir; so as to form wings for guiding the water more directly to it; or if $a\,b$, Fig 20, be less than twice $a\,m$. Indeed, so many modifying circumstances exist to embarrass experiments on this, and similar subjects, that some of those which have been made with great care, are rendered inapplicable as other than tolerable approximations, in consequence of the neglect to take into consideration some local peculiarity, which was not at the time regarded as exerting an appreciable effect. Unless, therefore, circumstances admit of our combining all the conditions mentioned in the first part of Art 14, thereby securing *very* approximate results, we must either resort to an actual measurement of the disch in a vessel of known capacity; or else be contented with rules which may lead to errors of 5, 10, or more per cent, in proportion as we deviate from these conditions. Frequently even 10 per ct of error may be of little real importance.

Fig. 25.

Rem. 1. **When the water, after passing over a weir, Fig 26, instead of falling freely into the air, is carried away by a slightly inclined apron or trough,** T, the floor of which coincides with the crest, a, of the weir, then the disch is not appreciably diminished thereby when the head $a\,m$ is 15 ins or more. But if the head $a\,m$ is but 1 ft, then the calculated disch must be reduced about $\frac{1}{10}$ part; if 6 inches, $\frac{2}{10}$; if 2½ inches, $\frac{3}{10}$; and if 1 inch, $\frac{5}{10}$, or one-half, as approximations.

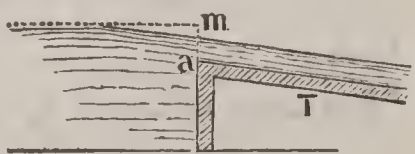


Fig 26

Rem. 2. **Professor Thomson, of Dublin, proposed the use of triangular notches, or weirs,** for measuring the disch; inasmuch as then the periphery always bears the same ratio to the area of the stream flowing over it; which is not the case with any other form. Experimenting with a right-angled triangular notch in thin sheet iron, Fig 26¼; with heads of from 2 to 7 ins, measured vert from the bottom of the notch, *to the level surf of the quiet water*, he found the disch in cub ft per sec to be as follows:

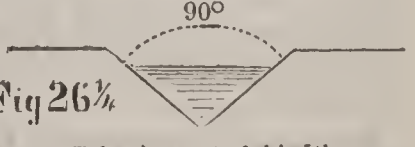


Fig 26¼

Find the fifth power of the head in *inches*, (Table 5, pages 251-2.) Take the sq rt of this 5th power, (Table 6,) Mult this sq rt by .0051. Or by formula,

$$\frac{\textit{Disch in}}{\textit{cub ft per sec}} = \textit{Head in inches}^{\frac{5}{2}} \times .0051.$$

ON THE FLOW OF WATER IN OPEN CHANNELS.

Art. 16. The mean velocity of flow is an imaginary uniform one, which, if given to the water at every point in the cross section, would give *the same discharge* that the actual ununiform one does. Or

$$\text{mean velocity} = \frac{\text{volume of discharge}}{\text{area of cross section}}$$

In channels of uniform cross section, **the maximum velocity** is found about midway between the two banks, and generally at some dist below the surface. This dist varies in diff streams; but, as an average, it seems to be about one third of the total depth. Where the total depth is great in proportion to the width, (say ½ the width or more), the max vel has been found as deep as midway between surf and bottom; while in small shallow streams it appears to approach the surf to within from .1 to .2 of the total depth. Many experiments upon shallow streams have indeed indicated that the max vel was *at* the surf.

The ratio between the velocities in different parts of the cross section varies greatly in diff streams; so that but little dependence can be placed upon rules for obtaining one from the other. With the same surf vel, wide and deep streams have greater mean and bottom vels than small shallow ones. In order **to approximate roughly to the mean vel** when the greatest surf vel is given, it is frequently assumed that the former is = ⅘ (or .8) of the latter. But Mr. Francis found, in his experiments at Lowell, that *surface* floats of wax, 2 ins diam, floating down the center of a rectangular flume 10 ft wide, and 8 ft deep, actually moved about 6 per cent *slower* than a tin tube 2 ins diam, reaching from a few ins above the surf, down to within 1½ ins of the bottom of the flume; and loaded at bottom with lead, to insure its maintaining a nearly vert position. While the wax *surf* float moved at the rate of 3.73 ft per sec, the rate of the tube (which was evidently very nearly the same as that of the center *vert* thread of water) was 3.98 ft per sec. Also, that in the same flume, with vels of the center tube varying from 1.55 to 4 ft per sec, the vel of the tube was *less* than that of the mean vel of the entire cross section of water in the flume, about as .96 to 1, for the lesser vel; and .93 to 1 for the greater vel. While, in another rectangular flume 20 ft wide and 8 ft deep, with vels varying from 1.16 to 1.84 ft per sec, that of the tubes was *greater* than that of the entire mass of water, about as 1.04 to 1. In a flume 29 ft wide, by 8.1 ft deep, with vels of about 3 ft per sec, it was as 1 to .9; and in a flume 36½ ft wide, by 8.4 ft deep, with vels of about 3½ ft per sec, as 1 to .97.

Charles Ellet, Jr, C E, found in the Mississippi "at diff points on the river, in depths varying from 54 to 100 ft; and in currents varying from 3 to 7 miles an hour that the speed of a float supporting a line 50 ft long, is almost always greater than that of the surf float alone." The same results were obtained with lines 25 and 75 ft long; the excess of the speed of the line floats being about 2 per cent over that of the simple floats: and Mr. Ellet concludes, therefore, that the mean vel of the entire cross section of the Mississippi, instead of being less, is absolutely greater by about 2 per cent, than the MEAN *surf* vel. He, however, employed .8 of the *greatest* surf vel as representing approximately, in his opinion, the mean vel of the *entire* cross section of water. In *shallow* streams, he always found the surf float to travel more rapidly than a line float.

European trials of the *mean* vel of *separate single verticals*, in tolerably deep rivers, have resulted in from .85 to .96 of the surf vel at each vertical. The mean of all may be taken at .9.

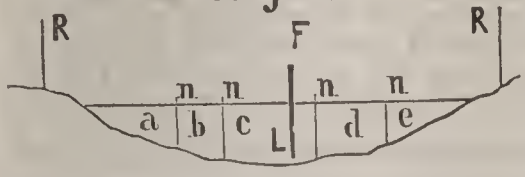
Bottom velocity. In streams of nearly uniform slope and cross section, there is a great reduction of vel near the bottom. As a very rough approximation, the deepest measurable vel, in streams of uniform slope etc, appears to be from ½ to ¾ of the mean vel. But see rems on "scour", p 279*f*.

Art. 17. To measure the surface velocity, select a place where the stream is for some dist (the longer the better) of tolerably uniform cross section; and free from counter-currents, slackwater, eddies, rapids, etc. Observe, by a seconds-watch, or pendulum, how long a time a float (such as a small block of wood) placed in the *swiftest* part of the current, occupies in passing through some previously measured dist. From 50 feet for slow streams, to 150 ft for rapid ones, will answer very well. This dist in ft, or ins, div by the entire number of seconds reqd by the float to traverse it, will give the greatest surf vel in ft or ins per sec.

The surf vel should be measd in perfectly calm weather, so that the float may not be disturbed by wind; and, for the same reason, the float should not project much above the water. The measurement should be

repeated several times to insure accuracy. In very small streams, the banks and bed may be trimmed for a short dist, so as to present a uniform channel-way. The float should be placed in the water a little dist above the point for commencing the observation; so that it may acquire the full vel of the water, before reaching that point.

Fig 27



Art. 18. To gauge a stream by means of its velocity. Select a place where the cross-section remains for a short dist, tolerably uniform, and free from counter-currents, eddies, still water, or other irregularities. Prepare a careful cross-section, as Fig 27. By means of poles, or buoys, *n*, *n*, divide the stream into sections, *a*, *b*, c, &c. Plant two range-poles, R, R, at the upper end; and two others at the lower end of the dist through which the floats are to pass; for observing the time, by a seconds watch, or a pendulum, which they occupy in the passage. Then measure the *mean* vel of each section *a*, *b*, c, &c, separately, and directly, by means of long floats, as F L, reaching to near the bottom: and projecting a little above the surf. The floats may be tin tubes, or wooden rods; weighted in either case, at the lower end, until they will float nearly vert. They must be of diff lengths, to suit the depths of the diff sections. For this purpose the float may be made in pieces, with screw-joints. The area of each separate section of the stream in sq ft, being mult by the observed mean vel of its water in ft per sec, will give the disch of that section in cub ft per sec. And the discharges of all the separate sections thus obtained, when added together, will give the total disch of the stream. And this total disch, div by the entire area of cross-section of the stream in sq ft, gives the mean vel of *all* the water of the stream, in ft per sec.

Rem. If the channel is in common earth, especially if sandy, the loss by soakage into the soil, and by evaporation, will frequently abstract so much water that the disch will gradually become less and less, the farther down stream it is measured. Long canal feeders thus generally deliver into the canal but a small proportion of the water that enters their upper ends.

The double float is used for ascertaining vels at diff depths. It consists of a float resting upon the surface of the water, and of a heavier body, or "lower float", which is suspended from the upper float by means of a cord. The depth of the lower float of course depends upon the length of the suspending cord (which may be increased or diminished at pleasure until the lower float is believed to be at that depth for which the vel is wanted), and upon its straightness, which is more or less affected by the current. Owing to this latter circumstance, it is difficult to know whether the lower float is really at the proper depth. Moreover it is uncertain to what extent the two floats and the string interfere with one another's motions. In deep water the string may oppose a greater area to the current than the lower float itself does. It thus becomes doubtful to what extent the vel of the upper float can be relied upon as indicating that of the water at the depth of the lower one.

Art. 19. Castelli's quadrant, or hydrometric pendulum, consisted of a metallic ball suspended by a thread from the center of a graduated arc. The instrument was placed in the current, with the arc parallel to the direction of flow; and the vel was then calculated from the angle formed between the thread and a vert line.

Gauthey's pressure plate was a sheet of metal suspended by one of its ends, about which it was left free to swing. The plate was immersed in the stream, with its face at right angles to the current. The vel was estimated by means of the weight required to make the plate hang vert in opposition to the force of the current.

Pitot's tube is a tube bent at right angles like the letter L. One leg is held hor under the water, with its open end facing the current. The vel is measd by the height to which the water rises in the vert leg above the general surf.

These instruments could not well be used except for points near the surface; and they gave only the vel at the time of observation.

Art. 20. Wheel meters. For accurate current measurements, especially in streams of irregular cross section, where long floats cannot well be used; wheel meters, similar in principle to Woltmann's tachometer, are now largely employed.

Such a meter consists of a wheel which is turned by the current, and which communicates its motion, by means of its axle and gearing, to indices which record the number of revolutions. The inst may be clamped to any part of a long pole reaching to the bottom of the stream, and thus may be used at any depth. The observer, by means of a wire, rod or string, reaching down to the inst, throws the registering apparatus first into, and then out of, gear with the wheel (applying a brake to the former at the instant it is thrown out of gear), and carefully noting the times when he does so. The inst is then raised, the number of revolutions in the measd time is read off from the indices, and from it the vel is calculated.

Various plans have been tried for so arranging such wheel meters that their revolutions may be registered above the surface at the time. This is now generally accomplished by electricity; the wheel, at each rev, automatically breaking and re-establishing a galvanic current generated by a battery. The wire carrying this current is thus made to operate Morse telegraphic registering apparatus placed in a boat or on shore.

A number of meters, so arranged, can be attached at diff points on the same pole at the same time, and thus **simultaneous observations of velocities at different depths** may be made and registered.

Wheel meters are made by Messrs. Buff & Berger, No 9 Province Court, Boston. The prices range approximately from $125 to $225 each. Most of their meters are so arranged that they can freely swing hor about the long vert pole to which they are clamped, and are provided each with a vane or tail similar to that of a wind-mill, for keeping the wheel in the proper position as regards the current. The wheels are generally made like those of a wind-mill; *i e* with blades set at such an angle as to present a sloping surface to the current; and with the axis of the wheel parallel to the direction of flow. The axis runs in agate bearings. When desired, the rim of the wheel is furnished with an air-chamber, which just counter-balances the weight of the wheel, and thus removes journal friction due to it. Meters are made both with and without electrical registering apparatus. In the latter case the gearing and indices etc are sometimes enclosed in a glass case, to prevent them from becoming clogged by weeds, sediment etc.

In all of the above methods, except those with floats which move along with the current, it is necessary, in order to calculate the vel from an observation, to first **rate the meter;** *i e*, to ascertain what effects different vels produce upon the inst. This is best done by moving the inst at a known vel through still water, and noting the effect produced. In this way a coefficient is obtained for each meter, which, when multiplied by the number of revs etc recorded in any given case, gives the vel for that case.

Art. 21. Kutter's formula for the mean vel of water flowing in open channels of uniform cross section and slope throughout.

Caution. The use of all such formulæ is liable to error arising from the difficulty of ascertaining the exact condition of the stream as regards roughness of bed, surface slope,* etc.

Rem. 1. **Care must be taken that the bottom vel is not so great as to wear away the soil.** If there is any such danger artificial means must be applied to protect the channel-way; or it may be advisable to reduce the rate of fall, and increase the cross section of the channel; so as to secure the same disch, but with less vel. A liberal increase should also be made in the dimensions of such channels, to compensate for obstructions to the flow, arising from the growth of **aquatic plants**, or deposits of mud from rain-washes, etc; or even from very strong winds blowing against the current. See also Rem, p 269.

Rem. 2. **Water running in a channel with a horizontal bed, or bottom, cannot have a uniform vel, or depth, throughout its course;** because the action of gravity due to the inclined plane of a sloping bottom, is wanting in this case; and the water can flow only by forming its *surface* into an inclined plane; which evidently involves a diminution of depth at every successive dist from the reservoir.

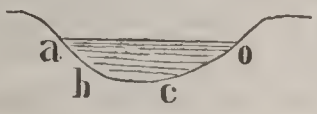


Fig. 28.

Fig. 29.

Fig. 30.

Theory of flow. It is generally held that the resistances to the flow of water in a pipe or channel are directly proportional to the area of the bed surface with which the water comes in contact (*i e*, to the product of the "wetted perimeter" as *a b c o* Figs 28, 29, 30 mult by the length of the channel, or of the portion of it under consideration); and to the square of the vel of the flowing water; and, inasmuch as the resistance at any given point in the cross section appears to be inversely as the dist of that point from the bottom or sides, we conclude that the total resistances are inversely as the area of the cross section; because the greater that area, the greater would be the mean dist of all the particles from the bottom and sides. (The resistance is *independent of the pressure.* See p 374*c*.)

In short, the resistances are assumed to be in proportion to

$$\frac{\text{vel}^2 \times \text{wet perimeter} \times \text{length}}{\text{area of cross section}} \quad \text{or} \quad \frac{v^2 p l}{a}$$

and the head h'' in feet or in metres etc, required to overcome those resistances, is

$$\text{resistance head} = \text{a coefficient } C \times \frac{\text{vel}^2 \times \text{wet perimeter} \times \text{length}}{\text{area of wet cross section}} \quad \text{or} \quad h'' = C\frac{v^2 p l}{a}$$

from which we have

$$v^2 = \frac{a h''}{C p l} \quad \text{and} \quad v = \sqrt{\frac{1}{C}\frac{a}{p}\frac{h''}{l}} \quad \text{or} \quad \text{vel} = \sqrt{\frac{1}{C}} \times \sqrt{\frac{\text{area of wet cross section}}{\text{wet perimeter}} \times \frac{\text{resistce head}}{\text{length}}}$$

---

* "In measuring the slope of a large river, the ordinary errors of the most careful leveling are a large proportion of the whole fall; the variation of level in the cross section of the surface is often as great as the slope for ten miles or more; the exact point where the level should be taken is often uncertain: the rise and fall of the water makes it extremely difficult to decide when the levels should be taken at the upper and lower points; waves of translation may affect the inclination to a great and uncertain degree, and may even make the surface slope the reverse way." Genl T. G. Ellis, Trans Am Soc Civ Engrs, Aug 1877.

But $\frac{\text{area of wet cross section}}{\text{wet perimeter}}$ or $\frac{a}{p}$ is the "hydraulic radius" or "mean depth" or "**mean radius**," R, of the cross section;

and $\frac{\text{resistance head}}{\text{length}}$ or $\frac{h''}{l}$ is the inclination or slope, S, (frequently denoted by "I") of the hydraulic grade line, p 240, or the sine of the angle *w s o* Fig 1½, p 240. In open channels, it is = the fall of the surface per unit of length.

We therefore have $$\text{velocity} = \sqrt{\frac{1}{C}} \times \sqrt{\text{mean radius} \times \text{slope}}$$

or, by using a coeff $(c) = \sqrt{\frac{1}{C}}$,

$$\text{velocity} = \text{coefficient } c \times \sqrt{\text{mean radius} \times \text{slope}}$$

or $$v = c\sqrt{RS}$$

The earlier hydraulicians gave (each according to the results of his investigations) *fixed* **values for the coeff c**, (generally about 95 to 100 for channels in earth or gravel, as in our early editions), making it, in other words, a *constant*, and independent of the shape, size, slope and roughness of the channel. But according to Messrs. E. Ganguillet and W. R. Kutter, eminent Swiss engineers, the coeff is affected by diffs in any of these particulars.

According to their formula (generally called, for convenience, "Kutter's formula")

For English measure. **For metric measure.**

$$\mathbf{c} = \frac{41.6 + \frac{.00281}{\text{slope}} + \frac{1.811}{n}}{1 + \frac{\left(41.6 + \frac{.00281}{\text{slope}}\right) \times n}{\sqrt{\text{mean rad in } \textit{feet}}}} = \frac{23 + \frac{.00155}{\text{slope}} + \frac{1}{n}}{1 + \frac{\left(23 + \frac{.00155}{\text{slope}}\right) \times n}{\sqrt{\text{mean rad in } \textit{metres}}}}$$

Tables giving values of c for diff grades, mean radii and degrees of roughness, and for English and metric measures, are given on pp 275 etc.

Here **n** is a "**coefficient of roughness**" of sides of channel as given below. These values of *n* were obtained from experience, by averaging a large number of experiments made under very different circumstances. They therefore embrace all the disturbing effects arising from obstructions existing upon the bottom and sides of the channel in the cases experimented upon. In small artificial channels of uniform cross section and slope, these obstructions may be said to consist entirely of the comparatively minute roughnesses of the *material* of which the bed of the channel consists. But in rivers and earth canals, even where the *general* direction, slope and cross section are tolerably uniform, (as they were in the cases upon which our list is based), there are still many considerable irregularities in the sides and bottom; and these exert a much greater retarding effect upon the mean vel than the mere *roughness* of the material of the banks. We therefore find larger values given for *n* in such cases than for small regular artificial channels, although the material of the sides etc was in many cases smooth mud; and we must not apply to such comparatively irregular channels the small values of *n* obtained by experiments with small and carefully made straight flumes of uniform section and slope, even if we suppose the bottom and sides of the former to be made as smooth as those of the latter.

No general formula is applicable to cases of **decided bends** in the course of a natural stream, or of **marked irregularities in the cross section.** Such cases would require still higher coefficients *n* than those here given for rivers and canals; but they would have to be ascertained by experiment for each case, and would be useless for other cases. For such streams we must therefore depend upon actual measurements of the velocity, either direct or by means of the disch.

There is much room for the exercise of judgment in the **selection of the proper coefficient n** for any given case, even where the condition of the channel is well known. It may frequently be necessary to use values of n intermediate between those given; for careless brickwork may be rougher than well finished rubble; side slopes in "very firm gravel" may have very diff degrees of roughness; etc etc. The engineer should make lists of values of n from his own experience, fully noting the peculiarities of each case, and calculating n from the tables, pp 275 etc, as directed.

A given diff in the deg n of roughness exerts a much greater effect upon the coefficient c, and thus upon the velocity, in small channels than in larger ones. It is therefore especially necessary in small channels that care be exercised in finding (by experiment if necessary) the proper value of n; and, where a large disch is desired, the sides of small channels should be made particularly smooth.

Table of n, or coefficient of roughness.

In any given case the value of **n is the same whether the mean radius is given in English, metric or any other measure.**

Artificial channels of uniform cross section.

Sides and bottom of channel lined with	n =
well planed timber	.009
neat cement* (applies also to glazed pipes and very smooth iron pipes).	.010
plaster of 1 measure of sand to 3 of cement;* (or smooth iron pipes).	.011
unplaned timber (applies also to ordinary iron pipes)	.012
ashlar or brickwork	.013
rubble	.017

Channels subject to irregularity of cross section.

Canals in very firm gravel	.020
Canals and rivers of tolerably uniform cross section, slope and direction, in moderately good order and regimen, and free from stones and weeds	.025
having stones and weeds occasionally	.030
in bad order and regimen, overgrown with vegetation, and strewn with stones and detritus	.035

Art. 22. The following tables give values of the coefficient c as obtained by Kutter's formula for diff slopes (S) mean radii (R) and degrees of roughness (n).†

Caution. Different values of c must be used with English and with metric measures. We give tables for both measures.

1st. Having the slope S, the mean rad R and the deg n of roughness; **to find the coeff c.** Turn to the division of the table corresponding to the given slope S. In the first column find the given mean rad, R. In the same line with this R, and under the given n, is the proper value of c.†

2d. Having the slope S, the mean rad R and either c or the actual or reqd vel v; **to find the actual, or the greatest permissible, deg n of roughness** of channel. If the vel is given, and not c, first find

$$c = \frac{\text{velocity}}{\sqrt{\text{slope} \times \text{mean radius}}}.$$

Turn to the division of the table corresponding to the given S, and in the first col find the given R. In the same line find the value given, or just obtained, for c; over which will be found the reqd n.†

3d. Having the slope S, the deg n of roughness, and the actual or required vel v; **to find the actual or necessary mean rad, R.** Assume a mean rad; and from the division of the table corresponding to the given S take out the value of c corresponding to the given n and the assumed R. Then say

$$v' = c \text{ so found} \times \sqrt{\text{assumed mean radius} \times \text{slope}}$$

---

* For experiments on abrasion of cements, see p 678.

† It is often necessary to interpolate values of S, R, n and c intermediate of those in the tables. This may be done mentally by simple proportion.

If this v' is the same as the given vel, or near enough to it, take the assumed R as the proper one. Otherwise, repeat the whole process, assuming a new R, *greater* than the former one if v' is *less* than the given vel, and *vice versa*.*

4th. Having the dimensions of the wetted portion (*a b c o* Figs 28, 29, 30.) of the channel, the deg *n* of roughness, and the actual or reqd vel; **to find the actual or necessary slope,** S:

$$\text{Find the mean rad, R} = \frac{\text{area of wet cross section}}{\text{length, } a\,b\,c\,o, \text{ of wet perimeter}}$$

Assume one of the four slopes of the tables to be the proper one. From the corresponding division of the table take out the value of *c* corresponding to the given R and *n*.

If R is 3.28 feet, or 1 metre, the value of *c* thus found is the proper one (because then *c*, for any given *n*, remains the same for all slopes); and the slope, S, may be found at once, thus:

$$\text{Slope, S} = \left(\frac{\text{given velocity}}{c \times \sqrt{\text{mean radius}}}\right)^2$$

But if R is greater or less than 3.28 feet, or 1 metre, say

$$v' = c \text{ thus found} \times \sqrt{\text{mean radius} \times \text{assumed slope}}$$

If this v' is near enough to the given vel, take the assumed S as the proper one. Otherwise, assume a new S, *greater* than the former one if v' is *less* than the given vel, and *vice versa;* and repeat the whole process.*

* It is often necessary to interpolate values of S, R, *n* and *c* intermediate of those in the tables. This may be done mentally by simple proportion.

Table of coefficient c, for mean radii in *feet.*

Slope S = .00025 per unit of length, = 1 in 4000, = .132 foot per mile.

Mean rad R feet	Coefficients n of roughness												Mean rad R feet
	.009	.010	.011	.012	.013	.015	.017	.020	.025	.030	.035	.040	
	c	c	c	c	c	c	c	c	c	c	c	c	
.1	65	57	50	44	40	33	28	23	17	14	12	10	.1
.2	87	75	67	59	53	45	38	31	24	19	16	14	.2
.4	111	97	87	78	70	59	51	42	32	26	22	19	.4
.6	127	112	100	90	81	69	60	49	38	31	26	22	.6
.8	138	122	109	99	90	77	66	55	43	35	30	25	.8
1	148	131	118	106	97	83	72	60	47	38	32	28	1
1.5	166	148	133	121	111	95	83	69	55	45	38	33	1.5
2	179	160	144	131	121	104	91	77	61	50	43	37	2
3	197	177	160	147	135	117	103	88	70	59	50	44	3
3.28	201	181	164	151	139	121	106	91	72	60	52	46	3.28
4	209	188	172	158	146	127	113	96	78	65	56	49	4
6	226	206	188	174	161	142	126	108	88	74	64	57	6
8	238	216	199	184	171	151	135	117	96	82	71	63	8
10	246	225	207	192	179	159	142	124	102	87	76	68	10
12	253	231	214	198	186	165	149	129	107	92	81	72	12
16	263	242	223	208	195	174	157	138	115	100	88	79	16
20	271	249	231	215	202	181	164	144	121	106	94	84	20
30	283	261	243	228	215	193	176	157	133	117	104	95	30
50	297	274	257	241	228	207	190	170	147	130	117	107	50
75	306	284	267	251	238	217	200	180	157	140	127	117	75
100	312	290	273	257	244	223	207	187	163	147	134	124	100

Table of coefficient c, for mean radii in *feet*.—CONTINUED.

Slope S = .00005 per unit of length, = 1 in 20000, = .264 foot per mile.

Mean rad R feet	Coefficients n of roughness .009	.010	.011	.012	.013	.015	.017	.020	.025	.030	.035	.040	Mean rad R feet
	c	c	c	c	c	c	c	c	c	c	c	c	
.1	78	67	59	52	47	39	33	26	20	16	13	11	.1
.15	91	79	69	62	56	46	39	31	23	19	16	13	.15
.2	100	87	77	68	62	51	44	35	26	21	18	15	.2
.3	114	99	88	79	71	59	50	41	31	25	21	18	.3
.4	124	109	97	88	79	66	57	46	35	28	24	20	.4
.6	139	122	109	98	90	76	65	53	41	33	28	24	.6
.8	150	133	119	107	98	83	71	59	46	37	31	27	.8
1	158	140	126	114	104	89	77	64	49	40	34	29	1
1.5	173	154	139	126	116	99	87	72	57	47	40	34	1.5
2	184	164	148	135	124	107	94	79	62	51	44	38	2
3	198	178	161	148	136	118	104	88	71	59	50	44	3
3.28	201	181	164	151	139	121	106	91	72	60	52	46	3.28
4	207	187	170	156	145	126	111	95	77	64	56	49	4
6	220	199	182	168	156	137	122	105	85	72	63	56	6
8	228	206	189	175	163	144	129	111	91	78	68	61	8
10	234	212	195	181	169	149	134	116	96	82	72	64	10
12	238	217	200	185	173	153	138	120	99	86	75	68	12
16	245	223	206	191	180	160	144	126	106	91	81	73	16
20	250	228	211	196	184	165	149	131	110	96	85	77	20
30	257	236	219	204	192	172	157	139	118	103	92	84	30
50	266	245	228	213	201	181	165	148	127	112	101	93	50
75	272	250	233	218	207	187	171	153	133	119	108	99	75
100	275	254	237	222	210	190	175	158	137	123	112	104	100

Slope S = .0001 per unit of length, = 1 in 10000, = .528 foot per mile.

.1	90	78	68	60	54	44	37	30	22	17	14	12	.1
.2	112	98	86	76	69	57	48	39	29	23	19	16	.2
.3	125	109	97	87	78	65	56	45	34	27	22	19	.3
.4	136	119	106	95	86	72	62	50	38	31	25	22	.4
.6	149	131	118	105	96	81	70	57	44	35	30	25	.6
.8	158	140	126	114	103	88	76	63	48	39	33	28	.8
1	166	147	132	120	109	93	81	67	52	42	35	31	1
1.5	178	159	144	130	120	103	89	75	59	48	41	35	1.5
2	187	168	151	138	127	109	96	81	64	53	45	39	2
3	198	178	162	149	137	119	104	89	71	59	51	45	3
3.28	201	181	164	151	139	121	106	91	72	60	52	46	3.28
4	206	186	169	155	143	125	111	94	76	64	55	49	4
6	215	195	178	164	152	134	119	102	84	71	61	54	6
8	221	201	184	170	158	139	124	107	88	75	66	59	8
10	226	205	188	174	162	143	128	111	92	78	69	62	10
15	233	212	195	181	169	150	135	118	98	85	75	68	15
20	237	216	200	185	173	154	139	122	102	89	79	71	20
30	243	222	206	191	179	160	145	128	108	95	84	77	30
50	249	227	211	197	185	166	151	134	114	100	91	83	50
100	255	234	218	204	191	172	158	140	121	108	98	91	100

Slope S = .0002 per unit of length, = 1 in 5000, = 1.056 feet per mile.

.1	99	85	74	65	59	48	41	32	24	18	15	12	.1
.2	121	105	93	83	74	61	52	42	31	25	21	17	.2
.3	133	116	103	92	83	69	59	48	36	29	24	20	.3
.4	143	125	112	100	91	76	65	53	40	32	27	23	.4
.6	155	138	122	111	100	85	73	60	46	37	31	26	.6
.8	164	145	131	118	107	91	79	65	50	41	34	29	.8
1	170	151	136	123	113	96	83	69	54	44	37	32	1
1.5	181	162	146	133	122	105	91	77	60	49	42	36	1.5
2	188	170	154	140	129	111	97	82	64	54	45	40	2
3	200	179	163	149	137	119	105	89	72	59	51	45	3
4	205	185	168	155	143	125	111	94	76	63	55	48	4
6	213	193	176	162	150	132	117	100	82	69	60	53	6
8	218	198	181	167	155	137	122	105	87	73	64	57	8
10	222	201	185	170	158	140	125	108	89	76	67	60	10
15	228	207	190	176	164	145	131	113	95	82	72	65	15
20	231	210	194	180	168	149	134	117	98	85	76	68	20
30	235	215	198	184	172	154	139	122	103	89	80	73	30
50	240	220	203	189	177	158	143	126	108	94	85	78	50
100	245	224	208	194	182	163	148	131	113	99	90	83	100

Table of coefficient c, for mean radii in *feet*.—CONTINUED.

	Mean rad R feet	Coefficients n of roughness.												Mean rad R feet
		.009	.010	.011	.012	.013	.015	.017	.020	.025	.030	.035	.040	
		c	c	c	c	c	c	c	c	c	c	c	c	
Slope S = .0004 per unit of length, = 1 in 2500, = 2.112 feet per mile.	.1	104	89	78	69	62	50	43	34	25	19	16	13	.1
	.15	116	101	90	80	71	59	50	40	29	23	19	16	.15
	.2	126	110	97	87	78	65	54	44	32	25	21	18	.2
	.3	138	120	107	96	87	73	62	50	37	30	24	21	.3
	.4	148	129	115	104	94	79	68	55	42	33	27	23	.4
	.6	157	140	126	113	103	87	75	62	47	38	31	27	.6
	.8	166	148	133	121	110	93	81	67	51	42	35	30	.8
	1	172	154	138	125	115	98	85	70	55	45	37	32	1
	1.5	183	164	148	135	124	106	93	78	61	50	42	37	1.5
	2	190	170	154	141	130	112	98	83	65	54	45	40	2
	3	199	179	162	149	138	119	105	89	71	59	51	45	3
	4	204	184	168	154	142	124	110	94	76	63	55	48	4
	6	211	191	175	161	149	130	116	99	81	69	60	53	6
	10	219	199	183	168	157	138	123	107	88	75	66	59	10
	20	227	207	190	176	164	146	131	115	96	83	73	66	20
	50	235	215	198	184	173	154	139	123	104	91	82	75	50
	100	239	219	203	189	177	158	143	127	108	96	87	80	100
Slope S = .0010 per unit of length, = 1 in 1000, = 5.28 ft. per m.	.1	110	94	83	73	65	54	45	36	27	21	17	14	.1
	.2	129	113	99	89	81	66	57	45	34	27	22	18	.2
	.3	141	124	109	98	89	74	63	51	39	30	25	21	.3
	.4	150	131	117	105	96	80	69	56	43	34	28	24	.4
	.6	161	142	127	115	104	88	76	63	48	39	32	27	.6
	.8	169	150	134	122	111	94	82	68	52	42	35	30	.8
	1	175	155	139	127	116	99	86	71	56	45	38	33	1
	1.5	184	165	149	136	124	108	93	78	62	50	43	37	1.5
	2	191	171	155	142	130	112	98	83	66	54	46	40	2
	3	199	179	163	149	138	119	105	89	71	59	51	45	3
	4	204	184	168	154	142	124	110	93	75	63	54	48	4
	6	211	190	174	160	149	130	116	99	81	68	59	52	6
	10	218	197	181	167	155	136	122	105	87	74	65	58	10
	20	225	205	188	175	163	144	129	113	94	81	72	65	20
	50	232	212	196	182	170	151	137	120	101	89	79	72	50
	100	236	216	200	186	174	155	141	124	105	94	85	77	100
Slope S = .01 per unit of length, = 1 in 100, = 52.8 feet per mile.	.1	110	95	83	74	66	54	46	36	27	21	17	14	.1
	.15	122	105	93	83	75	62	52	42	31	24	20	17	.15
	.2	130	114	100	90	81	67	57	46	34	27	22	19	.2
	.3	143	125	111	100	90	76	64	52	39	31	25	22	.3
	.4	151	133	119	107	98	82	70	57	44	35	29	24	.4
	.6	162	143	129	116	106	90	77	64	49	39	33	28	.6
	.8	170	151	135	123	112	95	82	68	53	43	35	31	.8
	1	175	156	141	128	117	99	87	72	56	45	38	33	1
	1.5	185	165	149	136	125	107	94	79	62	51	43	37	1.5
	2	191	171	155	142	130	112	99	83	66	55	46	40	2
	3	199	179	162	149	138	119	105	89	71	59	51	45	3
	3.28	201	181	164	151	139	121	106	91	72	60	52	46	3.28
	4	204	184	167	154	142	123	109	93	76	63	55	48	4
	6	210	190	173	160	148	129	115	99	81	68	59	52	6
	10	217	196	180	166	154	136	121	105	86	74	65	58	10
	20	225	204	187	173	161	143	128	112	93	80	71	64	20
	50	231	210	194	181	168	150	135	119	100	87	78	71	50
	100	235	214	197	184	172	153	139	122	104	91	82	75	100

For slopes steeper than .01 per unit of length, = 1 in 100 = 52.8 feet per mile, *c* remains practically the same as at that slope. But the *velocity* (being $= c \times \sqrt{\text{mean radius} \times \textit{slope}}$) of course continues to increase as the slope becomes steeper.

Table of coefficient c, for mean radii in *metres*.

Mean rad R metres	Coefficients n of roughness. .009	.010	.011	.012	.013	.015	.017	.020	.025	.030	.035	.040	Mean rad R metres
	c	c	c	c	c	c	c	c	c	c	c	c	
Slope = .00025 per unit of length, = 1 in 40000.													
.025	34	29	25	22	20	17	14	11	9	7	6	5	.025
.05	44	38	33	30	27	22	19	16	12	9	8	7	.05
.1	58	50	44	40	36	30	26	21	16	13	11	9	.1
.2	72	63	56	51	46	39	34	28	21	18	15	13	.2
.3	82	72	64	58	53	45	39	33	25	21	17	15	.3
.4	89	79	71	64	59	50	44	37	29	23	20	17	.4
.6	99	88	80	72	67	57	50	42	33	28	23	20	.6
1.	111	100	90	83	77	67	59	50	40	33	28	25	1.
1.50	121	109	100	92	85	74	66	57	46	38	33	29	1.50
2	127	115	106	98	91	80	71	61	50	42	37	32	2
3	136	124	114	106	99	87	78	68	56	48	42	37	3
4	142	130	120	111	104	93	83	73	61	52	46	41	4
6	149	137	127	119	111	100	90	80	67	58	51	46	6
10	158	145	135	127	120	108	98	88	75	66	59	53	10
15	164	151	141	133	126	114	104	94	81	72	64	59	15
20	167	155	145	137	130	118	108	98	85	75	68	62	20
30	172	160	150	142	135	123	113	103	90	81	74	68	30
Slope = .00005 per unit of length, = 1 in 20000.													
.025	40	35	30	26	24	20	17	13	10	8	7	5	.025
.05	52	44	39	34	31	26	22	18	13	11	9	7	.05
.1	65	57	50	44	40	34	29	24	18	14	12	10	.1
.2	79	69	62	55	51	43	37	30	23	19	16	13	.2
.3	87	77	69	62	57	48	42	35	27	22	18	16	.3
.4	93	83	74	67	62	53	46	38	30	25	21	18	.4
.6	102	90	82	74	69	59	52	43	34	28	24	21	.6
1.	111	100	90	83	77	67	59	50	40	33	28	25	1.
1.5	118	107	97	90	83	73	65	55	45	38	33	28	1.5
2	123	111	102	94	87	77	68	59	48	41	35	31	2
3	129	117	108	100	93	83	74	64	53	45	40	35	3
4	133	121	112	104	97	86	77	68	56	49	43	38	4
6	138	126	117	109	102	91	82	72	61	53	47	42	6
10	143	131	122	114	107	96	87	78	66	58	52	47	10
15	147	135	126	118	111	100	91	82	70	62	56	51	15
20	150	137	128	120	113	103	94	84	72	64	58	53	20
30	152	140	131	123	116	105	97	87	76	68	62	57	30
Slope = .0001 per unit of length, = 1 in 10000.													
.025	47	40	35	31	28	22	19	15	11	9	7	6	.025
.05	59	50	44	40	35	29	25	20	15	12	10	8	.05
.1	72	62	55	50	45	37	32	26	19	16	13	11	.1
.2	84	74	66	60	54	46	39	32	25	20	17	14	.2
.3	91	81	73	66	60	51	44	37	28	23	19	17	.3
.4	97	86	77	70	64	55	48	40	31	25	21	18	.4
.6	104	92	83	76	70	60	53	45	35	29	25	21	.6
1.	111	100	90	83	77	67	59	50	40	33	28	25	1.
1.5	117	105	96	88	82	72	64	54	44	37	32	28	1.5
2	120	109	100	92	85	75	67	57	47	40	34	30	2
4	128	116	107	99	92	82	73	64	53	46	40	36	4
6	131	119	110	102	96	85	77	67	56	49	43	39	6
10	135	123	114	106	100	89	81	71	60	53	47	43	10
15	137	126	116	109	102	92	83	74	63	55	50	46	15
30	141	129	120	112	106	95	87	78	67	59	54	50	30
Slope = .0002 unit of length, = 1 in 5000.													
.025	52	45	40	35	31	25	21	17	12	9	8	6	.025
.050	63	55	48	43	39	32	27	21	16	12	10	8	.050
.1	75	66	59	53	48	40	34	27	21	16	13	11	.1
.2	87	77	69	62	57	48	41	34	26	21	17	15	.2
.4	99	88	80	72	66	57	49	41	32	26	22	19	.4
.6	104	93	84	77	71	61	53	45	36	29	25	22	.6
1	111	100	90	83	77	67	59	50	40	33	28	25	1
2	118	107	98	90	84	74	65	56	46	39	34	30	2
4	124	113	104	97	90	79	71	62	51	44	39	35	4
10	130	119	110	102	96	85	77	67	57	50	45	40	10
30	135	124	114	107	100	90	82	73	62	55	50	46	30

Table of coefficient c, for mean radii in *metres*.—CONTINUED.

	Mean rad R meters	Coefficients n of roughness. .009	.010	.011	.012	.013	.015	.017	.020	.025	.030	.035	.040	Mean rad R metres
		c	c	c	c	c	c	c	c	c	c	c	c	
Slope = .0004 per unit of length, = 1 in 2500.	.025	55	47	41	37	33	27	22	17	13	10	8	7	.025
	.050	66	58	51	45	40	33	28	23	17	13	11	9	.050
	.1	78	68	61	55	50	42	35	28	21	17	14	12	.1
	.2	90	80	70	64	59	49	42	35	27	22	18	15	.2
	.3	95	85	76	70	63	54	47	39	30	24	21	17	.3
	.4	99	89	80	73	67	57	50	42	32	27	22	20	.4
	.6	105	94	85	78	72	62	54	45	36	30	25	22	.6
	1	111	100	90	83	77	67	59	50	40	33	28	25	1
	2	117	106	97	89	83	73	65	56	45	38	34	30	2
	4	123	111	102	95	88	78	70	61	50	43	38	34	4
	6	125	114	105	97	91	81	72	63	53	46	40	36	6
	10	128	117	108	100	93	83	75	66	55	48	43	39	10
	30	132	121	112	104	98	87	79	70	60	52	48	43	30
Slope = .0010 per unit of length, = 1 in 1000.	.025	57	50	43	38	34	28	23	18	13	11	9	7	.025
	.050	69	59	52	47	42	34	29	23	17	13	11	9	.050
	.1	80	70	63	56	50	42	36	30	22	17	14	12	.1
	.2	90	80	72	65	60	50	43	35	27	22	18	16	.2
	.3	96	86	77	70	64	54	47	39	30	25	21	18	.3
	.4	100	89	81	74	67	58	50	42	33	27	23	19	.4
	.6	104	94	85	78	72	62	54	46	36	30	25	22	.6
	1	111	100	90	83	77	67	59	50	40	33	28	25	1
	2	116	106	97	90	83	72	64	55	45	38	33	29	2
	4	121	111	102	94	87	77	69	60	50	42	37	33	4
	6	124	113	104	97	90	80	71	62	52	45	40	36	6
	10	127	115	106	99	92	82	73	64	54	47	42	38	10
	30	130	119	110	102	96	86	77	68	58	51	46	42	30
Slope = .01 per unit of length, = 1 in 100.	.025	59	50	44	39	35	28	24	19	14	10	9	7	.025
	.05	69	60	53	48	43	35	29	24	18	14	11	9	.05
	.1	81	71	63	57	51	43	36	30	22	18	15	12	.1
	.2	91	81	72	65	60	50	44	36	27	22	18	16	.2
	.3	97	86	77	71	65	55	48	40	31	25	21	18	.3
	.4	101	90	81	74	68	58	50	42	33	27	23	20	.4
	.6	106	95	86	78	72	62	54	46	36	30	25	22	.6
	1.	111	100	90	83	77	67	59	50	40	33	28	25	1.
	1.5	115	104	94	87	80	70	62	53	43	36	31	27	1.5
	2	117	105	96	89	83	72	64	55	45	38	33	29	2
	4	121	110	101	93	87	76	68	59	49	42	37	33	4
	10	126	114	105	98	91	81	73	64	53	46	41	37	10
	30	129	118	108	101	95	84	77	67	57	50	45	41	30

For slopes steeper than .01 per unit of length, = 1 in 100, the *coefficient* c remains practically the same as at that slope. The *velocity*, however, being $= c \times \sqrt{\text{mean radius} \times \textit{slope}}$, continues to increase as the slope becomes steeper.

To construct a diagram, fig 30 A, from which the **values given by Kutter's formula** may be taken by inspection.

Draw xz hor, and say from 2 to 4 ft long; and oy vert at any point o within say the middle third of xz. On oy lay off, as shown on the left, the values of c for which the diagram will probably be used. If a scale of .05 inch, or .002 metre, per unit of c be used, and be made to include $c = 250$ for English measure, or 150 for metric measure, oy will be about 1 ft long. For the sake of clearness we show only the larger divisions in this and in what follows.

On oz lay off, as shown on its upper side, the *square roots* of all the values of the mean rad R for which the diagram is to be used. One inch per ft, or .06 metre per metre, of sq rt, is a convenient scale. *Mark* the dividing points with the respective values of the *mean radii themselves.*

Having decided upon the *flattest* slope to be embraced in the diagram, say

$$w = 41.6 + \frac{.0028}{\text{flattest slope per unit of length}}$$ for English measure.

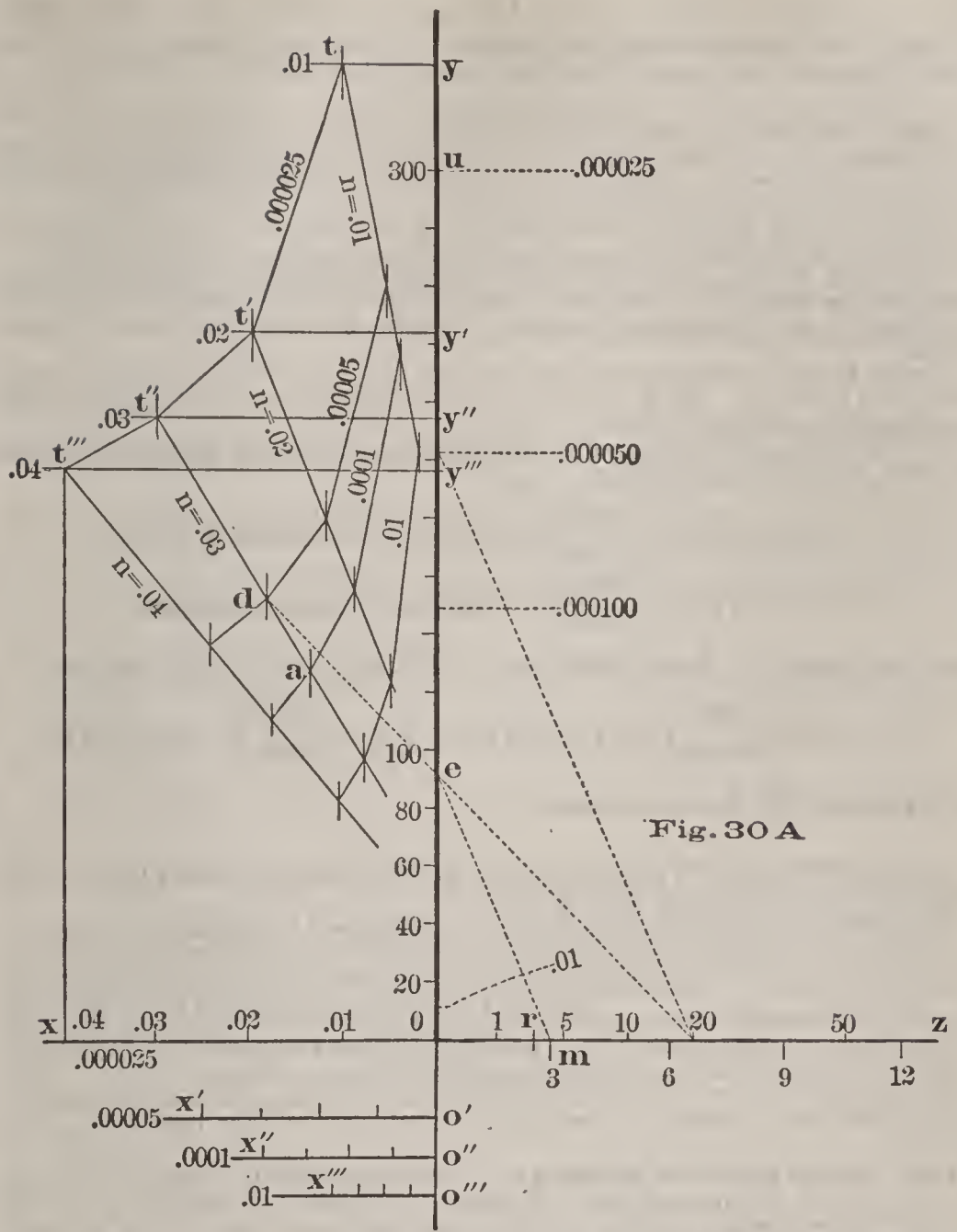


Fig. 30 A

$$w = 23 + \frac{.00155}{\text{flattest slope per unit of length}}$$ for metric measure.

For each value of n to be embraced in the diagram, say

$y - w = \frac{1.811}{n}$ for English measure; or $y - w = \frac{1}{n}$ for metric measure.

To each value of $y - w$, add w, thus obtaining values of y. We take .000025 per unit of length as the flattest slope,* and .01, .02, .03 and .04 for n.† Hence (using English measure) $w = 41.6 + \frac{.0028}{.000025} = 41.6 + 112 = 153.6$.

$y - w = \frac{1.811}{.01}, \frac{1.811}{.02}, \frac{1.811}{.03}, \frac{1.811}{.04}$; = 181.1, 90.5, 60.4, 45.3 respectively;

* This is about as flat as is likely to occur in practice.
† In most cases many intermediate ones would be used.

and $y = 181.1 + 153.6$, $90.5 + 153.6$, $60.4 + 153.6$ and $45.3 + 153.6$; or 334.7, 244.1, 214.0 and 198.9 respectively. Lay off these values of y on oy in pencil, as at y, y', y'', and y''', using the scale already laid off for c on oy.

From each point, y, y' etc, draw a hor pencil line yt, $y't'$ etc, and mark on it, in pencil, the value of n used in determining its height oy etc.

Next say $x = w \times$ *greatest* value of n. Make $ox = x$ by the scale of *sq rts* of R on oz. In our case $ox = 153.6 \times .04 = 6.144$ by the scale of *sq rts* of R, or $= 6.144^2 = 37.75$ by the scale of R.

Divide ox into as many equal spaces (4 in our case) as .01 is contained in greatest n. Mark the dividing points with the values of n, as in our Fig.

From each dividing mark on ox erect a perpendicular, (xt''' etc) in pencil, to cut that hor line ($y'''t'''$ etc) which corresponds to the same value of n. The intersections are points in a hyperbola. Join them by straight lines $t'''t''$, $t''t'$, $t't$ etc.

From r in oz (corresponding to a mean rad of 3.28 ft, or 1 metre) draw radial lines, rt, rt', rt'' etc. Mark them "$n = .01$", "$n = .02$" etc, the same as their corresponding lines yt, $y't'$ etc.

For each slope (S) to be used in the diagram (except the flattest, for which this has already been done) say

$$x', x'' \text{ etc} = \left(41.6 + \frac{.0028}{\text{slope}}\right) \times \textit{greatest } n, \quad \text{for English measure.}$$

$$x', x'' \text{ etc} = \left(23 + \frac{.00155}{\text{slope}}\right) \times \textit{greatest } n, \quad \text{for metric measure.}$$

Thus, our slopes are $= .000025$, $.00005$, $.0001$ and $.01$ per unit of length. Hence,

$$x' = \left(41.6 + \frac{.0028}{.00005}\right) \times .04 = 3.904; \quad x'' = \left(41.6 + \frac{.0028}{.0001}\right) \times .04 = 2.784;$$

$$x''' = \left(41.6 + \frac{.0028}{.01}\right) + .04 = 1.675.$$

Lay off each value of x', x'' etc from oy on a separate hor pencil line $o'x'$ etc, using the scale of sq rts of R as on oz.

Mark each line $o'x'$ etc in pencil with the slope used in fixing its length.

Divide each dist $o'x'$ etc into the same number of equal parts as ox. From the dividing points (which, like those of ox, represent the values of n) erect perps to cut the radial lines rt''', rt'' etc, each perp cutting that radial line which corresponds to the value of n represented by the point at the foot of the perp. The intersections corresponding to each line $o'x'$ etc form a hyperbolic curve. Mark each curve with the slope of its corresponding line, ox, $o'x'$ etc.

The drawing is now in the shape proposed by Mess Ganguillet and Kutter, and is ready for use in finding either c, n, R or S when the other three are given. Thus:

1st. Having R, S and n, **to find c.** For example let R = 20 ft, S = .00005, $n = .03$. From the intersection d of slope curve .00005 and radial line $n = .03$, draw* d–20 to the point (20) in oz corresponding to the given R. At e, where d–20 cuts oy, is the reqd c, $= 96$ in this case.

2d. Having R, S and c, **to find n.** For example let R = 20 ft, S = .00005, $c = 96$. Through the points R = 20 in oz, and $c = 96$ in oy, draw* d–20 to cut curve .00005. n (= .03) is found by means of the radial lines nearest to the intersection, d.

3d. Having S, n and c, **to find R.** For example let S = .00005, $n = .03$, $c = 96$. Find curve .00005 and radial line $n = .03$. From their intersection d draw d–20 through the point e showing $c = 96$. Its intersection with oz shows the reqd R, 20 in this case.

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* Instead of *drawing* these lines, we may use a fine black thread with a loop at one end. Drive a needle either into one of the points R or into one of the intersections, d etc. Slip the loop over the needle. The other end of the thread is held between the fingers, and the thread is made to cut the other points as reqd. The diagram should lie perfectly flat, and the string be drawn tight at each observation, in order that friction between string and paper may not prevent the string from forming a straight line. Or the free end of the string may rest on a pamphlet or other object about ⅛ inch thick, to keep the string clear of the diagram. Special care must then be taken to have the eye perp over the point observed.

4th. Having R, c and n, **to find S.** For example let R = 20 ft, c = 96, n = .03. Through R = 20 and c = 96 draw d–20. S (.00005) is found by means of the curves nearest to the point d of intersection of d–20 with radial line n = .03.

The following addition to Kutter's diagram, proposed by Mr Rudolph Hering, Civil and Sanitary Engineer, Philadelphia,* enables us **to read the velocity from the diagram.**

Find the sq rt of the reciprocal of each slope to be embraced in the diagram $= \sqrt{\frac{1}{\text{slope per unit of length}}}$. Lay off these sq rts on the right of oy, using the scale of c already laid off on its left. In our fig we have so proportioned the two scales that $\frac{c}{\sqrt{\text{recip of S}}} = \frac{1.5}{1}$. *Mark* the dividing points with the *slopes per unit of length.*

On oz lay off the vels to be embraced in the diagram, using the scale of sq rts of R already laid off on oz, and making $\frac{\text{vel}}{\sqrt{\text{R}}} = \frac{c}{\sqrt{\text{recip of S}}}$

1st. Having R, S and n; **to find v.** For example let R = 20 ft, S = .00005, n = .03. From R = 20 draw d–20 to the intersection d of curve .00005 with radial line n = .03. d–20 cuts oy at e, where c = 96. With a parallel ruler join R = 20 with S = .00005 on oy. Draw a parallel line through c = 96. It cuts oz at m, giving the reqd vel, 3.03 ft per sec.

2d. Having R, S and v; **to find n.** For example let R = 20 ft, S = .00005, v = 3.03 ft per sec. With a parallel ruler join R = 20 and slope .00005 on oy. Draw a parallel line through v = 3.03. It cuts oy at e, where c = 96. Through R. = 20 and c = 96, draw d–20 to cut curve .00005. The point d of intersection, being on radial line n = .03, shows .03 to be the proper value of n.

Any line drawn to the curves from R = 3.28 ft or 1 metre, is one of the radial lines used in making the diagram. It therefore necessarily cuts all the slope curves at points showing the same value of n.

3d. Having S, n and v; **to find R.** For example, let S = .00005, n = .03, v = 3.03 ft per sec. *Assume* a value of R, say 10 ft. Find curve .00005 and radial line n = .03. Join their intersection d with R = 10 ft. The connecting line cuts oy at c = 82. with a parallel ruler join c = 82 with v = 3.03. Draw a parallel line through slope = .00005 on oy. It cuts oz at R = 27.3. showing that a new trial is necessary, and with an assumed R *greater* than 10 ft.

If R thus found is the same as the assumed one, the latter is correct. If they are nearly equal, their mean may be taken.

4th. Having R, n and v; **to find S.** For example, let R = 20 ft, n = .03, v = 3.03 ft per sec. *Assume* a slope (say .0001). Find its curve, and radial line n = .03. Join their intersection with R = 20, and note the value (89) of c where the connecting line cuts oy. With a parallel ruler join c = 89 with v = 3.03. Draw a parallel line through R = 20. It cuts oy at slope .000058, showing that a new trial is necessary, and with an assumed S *flatter* than .0001. If R is 3.28 ft, or 1 metre, the diagram gives the correct S at the first trial, no matter what S was assumed at starting. With any other R, if the diagram gives the same S as that assumed, the latter is correct. If the two differ but slightly, we may take their mean.

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* Transactions of the American Society of Civil Engineers, January 1879.

Table of vels in Circular Brick Sewers when running full, by Kutter's formula, p 272, but taking **n** at .015 instead of his .013, in consideration of the rough character of sewer brickwork generally.

When running only half full the vel will be the same as when full, but this is not the case at any other depth whether greater or less. At greater ones it increases until the depth equals very nearly .9 of the diam, when it is about 10 per cent greater than when either full or half full. From depth of .9 of the diam the vel decreases whether the depth becomes greater or less. At depth of .25 diam the vel is about .78 of that when full; and then diminishes much more rapidly for less depths. All this applies also to pipes.

The vel for any fall or diam intermediate of those in the table can be found by simple proportion. Original.

Fall in ft per mile.	Diameters in feet.								Fall in ft per 100 ft.
	2	3	4	6	8	12	16	20	
	Velocities in feet per second.								
.1	.19	.27	.35	.50	.64	.89	1.10	1.34	.0019
.2	.30	.42	.53	.74	.93	1.26	1.56	1.84	.0038
.4	.46	.65	.80	1.08	1.39	1.81	2.20	2.60	.0076
.6	.59	.81	1.00	1.35	1.70	2.22	2.70	3.18	.0114
.8	.69	.95	1.17	1.57	1.94	2.56	3.08	3.60	.0151
1.0	.79	1.07	1.32	1.77	2.16	2.84	3.43	3.96	.0189
1.25	.89	1.21	1.49	1.98	2.42	3.17	3.8	4.5	.0237
1.50	.98	1.33	1.64	2.18	2.64	3.5	4.2	4.9	.0284
1.75	1.06	1.44	1.78	2.34	2.85	3.8	4.5	5.3	.0331
2.0	1.15	1.55	1.91	2.53	3.1	4.0	4.8	5.6	.0379
2.5	1.32	1.78	2.18	2.85	3.5	4.5	5.4	6.3	.0473
3.0	1.44	1.94	2.38	3.2	3.8	5.0	6.0	6 9	.0568
3.5	1.58	2.10	2.58	3.4	4.1	5.3	6.5	7 4	.0662
4.	1.68	2.2	2.7	3.6	4.4	5.7	6.9	7.9	.0758
5.	1.90	2.5	3.1	4.1	4.9	6.3	7.6	8.7	.0947
6.	2.06	2.7	3.3	4.4	5.4	6.9	8.3	9.6	.1136
7.	2.2	3.0	3.6	4.8	5.8	7.5	9.0	10.4	.1325
8.	2.4	3.2	3.8	5.1	6.2	8.0	9.7	11.1	.1514
9.	2.5	3.4	4.1	5.4	6.6	8.5	10.3	11.8	.1703
10.	2.7	3.5	4.3	5.7	6.9	9.0	10.8	12.5	.1894
12.	2.9	3.9	4.8	6.3	7.6	9.9	11.9	13.6	.2273
15.	3.3	4.4	5.4	7.1	8.5	11.0	13.3	15.3	.2841
18.	3.6	4.8	5.9	7.7	9.3	12.1	14.5	16.7	.3409
21.	3.9	5.1	6.3	8.4	10.0	13.0	15.7	17.9	.3975
24.	4.2	5.5	6.8	8.9	10.8	13.9	16.8	19.2	.4546
27.	4.5	5.9	7.2	9.5	11.4	14.8	17.9	20.4	.5109
30.	4.7	6.2	7.5	9.9	12.0	15.6	18.8	21.5	.5682
35.	5.0	6.7	8.2	10.8	13.0	16.8	20 4	23.2	.6629
40.	5.4	7.1	8.7	11.5	13.9	18.0	21.7	24.8	.7576
45.	5.6	7.5	9.2	12.2	14.8	19.1	23.0	26.3	.8523
50.	5.9	8.0	9.7	12.8	15.5	20.1	24.2	27.7	.9470
60.	6.5	8.7	10.7	14.1	17.0	22.1	26.5	30.3	1.136
70.	7.0	9.4	11.5	15.2	18.4	23.9	28.5	32.8	1.326
80.	7.4	10.1	12.3	16.2	19.7	25.5	31.0	35.0	1.515
90.	7.9	10.7	13.1	17.2	20.9	27.0	32.3	37.1	1.705
100.	8.4	11.3	13.8	18.2	22.0	28.5	34.1	39.1	1.894

A vel of 10 ft per sec = 600 ft per minute = 36000 ft, or 6.818 miles per hour. About 5 ft per sec is as great as can be adopted in practice to prevent the lower parts of the sewers from wearing away too rapidly by the debris carried along by the water.

Art. 23. The rate at which rain water reaches a sewer or culvert, etc, may, according to the admirable "Report on European Sewerage Systems" by Mr. Rudolph Hering, Civ. and San. Eng. of Philada, be found approximately by the following formula by Mr Burkli-Ziegler. See Trans. Am. Soc. C. E, Nov 1881.

$$\text{Cub. ft. per second per acre, reaching sewer} = \text{A coef according to judgment} \times \text{Av. cub. ft. of rainfall per } \textbf{second} \text{ per } \textbf{acre}, \text{ during heaviest fall.} \times \sqrt[4]{\frac{\text{Av. slope of ground in feet per 1000 ft}}{\text{No. of acres drained}}}$$

His coefficient for paved streets is .75; for ordinary cases .625; and for suburbs with gardens, lawns, and macadamized streets .31. **His average heaviest fall** is from $1\frac{3}{4}$ to $2\frac{3}{4}$ ins per hour. To this the writer will add that each **inch** of rainfall per **hour**, corresponds closely enough to 1 **cub ft** per **sec** per **acre**; so that if we liberally allow for 3 or 4, etc, ins per hour of average heaviest rainfall, the third term of the above equation also becomes simply 3 or 4, etc.

Example. If an area of 3100 acres (nearly 5 sq miles), with an average slope of 5 ft per 1000 ft, receives a rainfall averaging 3 ins per hour when heaviest, then, assuming a coefficient of .5, the rate at which the water would reach the mouth of a sewer at the lower end of the 3100 acres would be

$$.5 \times 3 \times \sqrt[4]{\frac{5}{3100}} = .5 \times 3 \times .203 = .305 \text{ cub ft per sec per acre;}$$

or $.305 \times 3100 = 945.5$ cub ft per sec, total.

Now suppose the fall of the intended sewer to be say 4 ft per mile; and that for fear of the too rapid wearing away of its brickwork by debris swept along by the water, we limit its vel to 6.3 ft per sec, which may be permitted on occasions as rare as rains of 3 ins per hour, although for tolerably constant flow, where liable to debris, it should not exceed about 5 ft per sec. **To find the diameter,** look in the Table of Vels in Sewers, p 279c, for a diam corresponding as near as may be, to a vel of 6.3, and to a fall of 4 ft per mile. We find this diam to be 14 ft, the area of which is 154 sq ft. Hence, $154 \times 6.3 = 970$ cub ft per sec = capacity of sewer. This is a trifle more than our 945.5 cub ft per sec of rainfall; nevertheless, to allow for deposits in the sewer, it would be advisable to increase the diam say to 14.5 or 15 ft.

REM. **Mr Wicksteed, an experienced English hydraulician, gives the following table of the least vels and grades or falls, to be given to drain-pipes and sewers in cities,** in order that they may under ordinary circumstances keep themselves clean, or free from deposits. He recommends that no drain pipe, even for a single common dwelling, shall be *less* than 6 ins diam.

Diam. in Inches.	Vel. in ft. per Min.	Grade, 1 in	Grade. Feet per Mile.	Diam. in Inches.	Vel. in ft. per Min.	Grade, 1 in	Grade. Feet per Mile.
4	240	36	146.7	18	180	294	18 0
6	220	65	81.2	21	180	343	15.4
7	220	76	69 5	24	180	392	13 5
8	220	87	60.7	30	180	490	10 8
9	220	98	53.9	36	180	588	9.0
10	210	119	44.4	42	180	686	7.7
11	200	145	36.7	48	180	784	6.8
12	190	175	30.2	54	180	882	6.0
15	180	244	21.6	60	180	980	5.4

Weight per foot run, and price per foot run, of glazed terra cotta pipes for drains etc; made by Moorhead Clay Works, Spring Mill (office No 11 South 7th St) Philadelphia.

Drain pipe, with socket joint						Sewer pipe, with sleeve joint					
Bore	Wt	Price	Bore	Wt	Price	Bore	Wt	Price	Bore	Wt	Price
ins	lbs	$	ins	lbs	$	ins	lbs	$	ins	lbs	$
2	4	0.13	6	18	0.30	15	45	1.25	30	150	4.50
3	7	0.16	8	22	0.45	18	65	1.60	36	195	Special contract.
4	10	0.20	10	30	0.70	21	89	2.30	42	203	
5	12	0.25	12	33	0.80	24	100	3.00	48	230	

The joints are filled with cement mortar; or, when used for drainage only, with clay. Drain pipes (3 to 12 ins bore) are about $\frac{7}{8}$ inch thick. A bend or branch costs about as much as from 3 to 5 feet of pipe. The 48-inch pipes are about 2 ins thick.

Art. 24. When the area of cross section of channel is reduced at any point, as by a dam (Fig 33, p 279 *e*), or by narrowing it, either at its sides (Fig 32) or by placing in it a pier etc, Fig 34; a portion at least of the force of grav (which would otherwise be giving *vel* to the water up-stream from the point where the obstruction takes place), causes *pressure* against the dam etc. This pres maintains the up-stream water at a higher level than it

would otherwise have. Said water is then practically in a reservoir; *i e*, it has less vel and greater pres than before. If the reservoir has no outlet, there is *no* vel; and *all* of the head, or force of grav, acting on the water is expended in *pres*

But if there is an outlet, as over the dam, or between the piers etc, a portion *c o*, Figs 31, 33, 34, of this pres or head, is expended in giving *vel* (or an acceleration of vel) to the water escaping by that outlet; after which only so much head (in the shape of surface slope) is needed as will overcome the resistances of the channel down-stream from the obstruction, and so *maintain uniform* the vel given to the water by the head *c o*.

Where a large canal, such as those intended for navigation, is fed from a reservoir, the fall *c o* in feet is approximately

$$= \text{mean velocity}^2 \text{ in canal, in feet per second, } \times .017;$$

and in smaller canals, such as mill courses,

$$= \text{mean velocity}^2 \text{ in canal, in feet per second, } \times .02.$$

The abruptness of the fall may be diminished by rounding off or sloping the edges of the piers, or the corners at the sides of the channel (Fig 32) or the approach to the dam (Figs 1 to 4, pp 283, 284).

Fig 33 is a cross section of **Clegg's dam**, across Cape Fear River, N. C. It is from measurements made by Ellwood Morris, C E; by whom they were communicated to the writer. The dam is of wooden cribwork; and its level crest, 8 ft 5 ins wide, is covered with plank; along which the water glides in a smooth sheet, 6 ins deep, (at the time of measurement). At the upper end of this sheet, and in a dist of about 2 ft, a head *c o* of 9 ins forms itself, as in the fig.

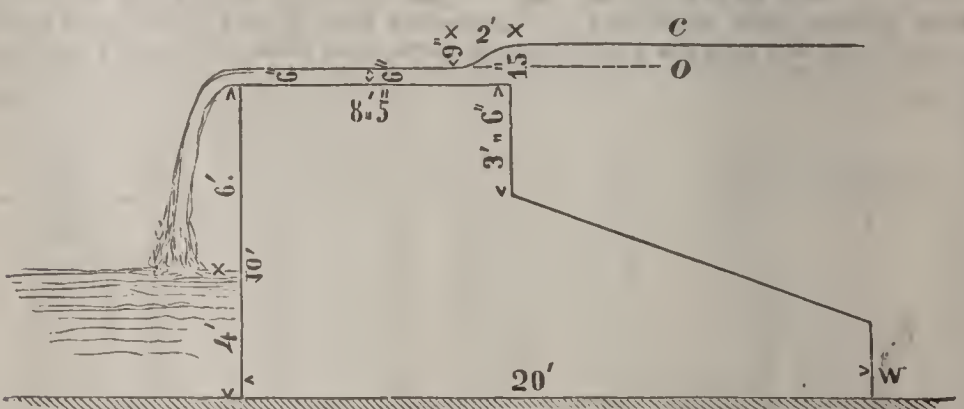

Fig 33

For **Construction of Dams**, see p 282, etc.

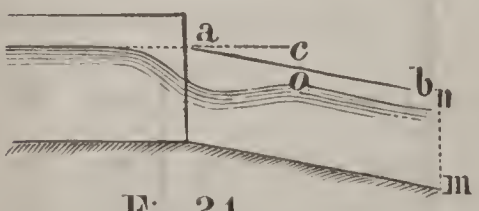

Fig 31

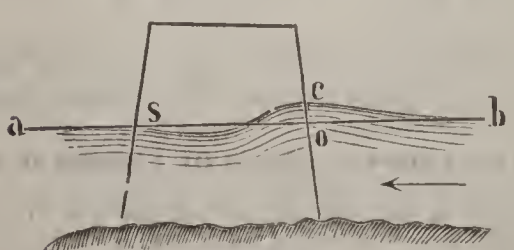

Fig 34

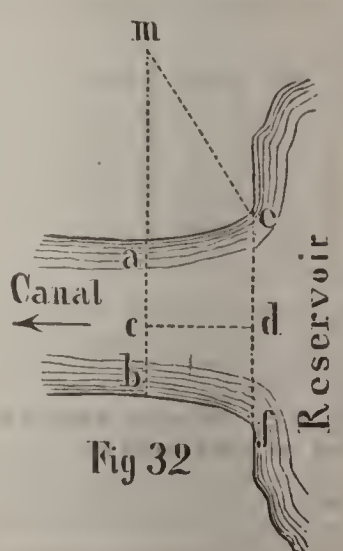

Fig 32

Art. 25. Scour. In a channel of uniform and constant slope and cross section, the vel of the particles of water immediately adjoining the bottom and sides is very slight; and but little scouring takes place. But when irregularities in the slope or cross section occur, as in the last article, the scour is greatly increased in their immediate neighborhood.

The erection of one or two piers in a quite large stream, will frequently produce an almost incredible amount of scour, if the bottom is at all of a yielding nature. The greatest scour of course takes place during freshets; and near the obstruction.

Scouring action is supposed to be as square of vel.

According to Smeaton, a vel of 8 miles an hour will not derange quarry rubble stones, not exceeding half a cub ft. deposited around piers, &c; except by washing the soil from under them.

1 inch per sec. = 5 ft per min. = .056818 of a mile, or 300 ft per hour.
1 foot per sec. = 60 ft per min. = .681816 of a mile, or 3600 ft per hour.

To reduce inches per sec, to feet per minute, multiply by 5.
" " " " " " " " " hour, " " 300.
" " " " " to miles per hour, divide by 17.6.
One mile per hour = 88 ft per min = 1.4667 ft, or 17.6 ins per sec.

The two following tables are (with many corrections) from Nicholson's Architecture; and must be looked upon merely as probable approximations. They suppose the piers, &c, to be properly rounded or pointed at their upstream ends, so as to give as free a passage as possible to the water. He says that if they are square-ended, the head will be increased about 50 per ct. The subject is an extremely intricate one, and admits of no precise solution. If the increased vel scours away the bottom until the area of water-way becomes as great as it originally was, the head disappears; and the vel also becomes reduced to its original rate. This is common in soft bottoms.

TABLE Of heads produced by obstructions to streams.

Original Vel. of Stream.* Per Sec. Ins.	Ft.	Per Hour. Miles.	Kind of Bottom which begins to wear away under Bottom Vel. equal to those in the first three cols.	Proportion of Area of original Water-way, occupied by the Obstructions. $\frac{1}{12}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$
				Head of Water produced at the Obstructions; in Feet.								
3	¼	.170	Ooze, and Mud...	.0003	.0004	.0004	.0006	.001	.0014	.0033	.0067	.0162
6	½	.341	Clay	.0011	.0014	.0017	.0023	.004	.0058	.0133	.0267	.0646
12	1	681	Saud.....	.0045	.0056	.0069	.0091	.015	.0231	.0532	.1069	.2584
24	2	1.36	Gravel	.0182	.0225	.0276	.0364	.060	.0924	.2128	.4276	1.036
36	3	2.04	Small Shingle....	.0409	.0507	.0621	.0819	.135	.2079	.4788	.9621	2.326
48	4	2.72	Large "	.0728	.0902	.1104	.1456	.240	.3696	.8412	1.710	4.144
	5	3.41	Soft Shistus.....	.1137	.1410	.1725	.2275	.375	.5775	1.320	2.672	6 475
	6	4.09	Stratified Rocks..	.1638	.2030	.2484	.3276	.540	.8316	1.915	3.848	9.304
20	10	6.81	Hard Rocks.....	.4550	.5640	.6901	.9100	1.50	2.310	5.280	10.69	25.9

TABLE Increased velocities produced at and by rounded, or pointed obstructions. If square, these vels must, according to Nicholson, be increased ⅛ part.

Original Vel. of Stream.* Per Sec. Ins.	Ft.	Per Hour. Miles.	Proportion of Area of Water-way, occupied by the Obstructions. $\frac{1}{12}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$
			Velocity produced at the Obstruction in Feet per Second.								
3	¼	.170	.28	.29	.30	.32	.35	.394	.52	.7	1.05
6	½	.341	.56	.58	.60	.64	.70	.788	1.05	1.4	2 1
12	1	.681	1.13	1.16	1.20	1.26	1.40	1.58	2.1	2.8	4.2
24	2	1.36	2.27	2.33	2.40	2.52	2.80	3.16	4.2	5.6	8.4
36	3	2.04	3.39	3.48	3.60	3.78	4.20	4.74	6.3	8.4	12.6
48	4	2.72	4.54	4.66	4.80	5.04	5.60	6.32	8.4	11.2	16.8
60	5	3.41	5.60	5.80	6.00	6.40	7.00	7.88	10.5	14.0	21.0
72	6	4.09	6.78	6.96	7.20	7.56	8.40	9.48	12.6	16.8	25.2
20	10	6.81	11.3	11.6	12.0	12.6	14.0	15.8	21.0	28.0	42.0

* A very vague expression. Does it refer to the greatest surface vel at mid-channel; or to the mean vel of the entire cross-section?

Art. 26. The resistance of water against a flat surface moving through it at right angles, is nearly as the squares of the vel; and, according to Hutton, its amount in lbs per sq ft approx = Square of vel in ft per sec. **Or like the pres of a running stream** against a perp fixed flat surface, it is = wt of a col of water whose base = pressed surf, and whose ht = head due to the vel as per table p 258.

The resist of a sphere is to that of its great circle about as 1 to 2.9.

When the moving surf, instead of being at right angles to the direction in which it moves forms another angle with it, the resistance becomes less in about the following proportions. Therefore, when the surf is inclined, first calculate the resistance as if at right angles: and then mult by the following decimals opposite the angle of inclination:

90°....1.00	60°.... .88	40°.... .58	20°.... .16
80 .... .98	55 .... .83	35 .... .46	15 .... .10
70 .... .95	50 .... .76	30 .... .34	10 .... .06
65 .... .92	45 .... .68	25 .... .24	5 .... .02

The scour, or abrading power of moving water is considered to be as the square of its vel.

Art. 27. To calculate the horse-power of falling water, on the ordinary assumption that a horse-power is equal to 33000 lbs lifted 1 foot vert per min. That of average horses is really but about ⅔ as much, or 22000 lbs, 1 foot high per min. Mult together the number of cub ft of water which fall per min; the vert height or head in feet, through which it falls; and the number 62.3, (the wt of a cub ft of water in lbs;) and div the prod by 33000. Or, by formula,

$$\textit{The number of horse-powers} = \frac{\textit{cub ft per min} \times \textit{vert height in ft} \times \textit{lbs } 62.3}{33000.}$$

Ex. Over a fall 16 ft in vert height, 800 cub ft of water are disehd per min. How many horse-powers does the fall afford?

$$\text{Here, } \frac{^{\text{cub ft}}\ 800 \times 16 \times 62.3}{33000} = \frac{797440}{33000} = 24.17 \text{ h-pow.}$$

Water-wheels do not realize all the power inherent in the water, as found by ou rule. Thus, undershots realize but from ¼ to ⅓; breast-wheels, ½; overshots, from ⅔ to ¾; turbines, ¾ to .85 of it; according to the skill of design, and the perfection of workmanship. Even when the wheel revolves in a close-fitting casing, or breast, elbow buckets give considerably more power than plain radial or center-buckets. Of the power actually received by a wheel, part is expended in friction, &c; while the remainder does the *useful* or *paying* net work of raising water, grinding grain, sawing, &c.

Observations by Genl Haupt, in 1866, gave the following results for a small **hydraulic ram.** Head of water to ram = 8.812 ft; diam of drive-pipe = 1½ ins; length 15 ft. Diam of delivery-pipe = ¾ inch; length 200 ft. Vert height to which the water was raised by the delivery-pipe, 63.4 feet. Strokes of ram per min, 170. Quantity of water which worked the ram = 768 cub ins. = 3.31 galls, = 27.73 lbs per min. Quantity raised 63.4 ft high per min, = 48 cub ins, = 1.736 lbs. Hence the power expended per min, was $^{\text{lbs water}}\ 27.73 \times 8.812 = {}^{\text{ft-lbs}}\ 244.35$. And the useful effect, was $^{\text{lbs}}\ 1.736 \times {}^{\text{ft}}\ 63.4 = {}^{\text{ft-lbs}}\ 110.06$. Hence the ratio which the *useful effect* bears to the power in this instance, is $\frac{110.06}{244.35}$, or .45. The *actual power* of the ram is, however, greater than this, inasmuch as it has to overcome the friction of the water along the delivery-pipe.*

To find the horse-power of a running stream. Water-wheels with simple float-boards,† instead of buckets, are sometimes driven by the mere force of the ordinary natural current of a stream, without any appreciable fall like that in the foregoing case. In such cases, we must substitute the *virtual* or *theoretic* head; which is that which would impart to it the same vel which it actually has. This virtual head may be taken at once from Table, p 258. Thus, a stream has a vel of 2.386 miles per hour; or 210 ft per min; or 3½ ft per sec; and in the column of heads in Table 10, opposite to 3.5 vel per sec, we find the reqd head .190 of a ft. Having thus found the head, we must now find the quantity of water which passes any given area of the stream in a min. Thus, suppose that the *immersed* part of a float when vert is 5 ft long, and 1 ft wide or deep; then the area of this part which receives the force of the current, is 5 × 1 = 5 square feet. Hence, $^{\text{area}}\ 5 \text{ sq ft} \times {}^{\text{vel}}\ 210 = 1050$ cub ft per min. Having now the cub ft per min, and the vert height or head, the number of horse-powers *of the stream* of the given area, is found by the foregoing rule, or formula.

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* **A committee of the Franklin Institute,** in 1850, gave .71 as th coefficient for a ram at the Girard College, in which the diam of drive-pipe was 2½ ins; its length 160 ft; fall, 14 ft. Delivery-pipe, 1 inch diam; 2260 ft long; vert rise, or height to which the water was raised, 93 ft. No details of the experiment are given. Some large rams in France give a *useful effect* of from .6 to .65 of the whole power expended. It is an excellent machine for many purposes; and is sometimes used for filling railway tanks at water stations.

† **Such wheels, for floating mills, in Europe,** rarely exceed 15 ft diam. Whatever the diam, they may have about 18 to 20 floats. The floats are from 8 to 16 ft long; and about $\frac{1}{8}$ to $\frac{1}{10}$ as deep as the diam of the wheel. They should not dip their entire depth into the water, but nearly so. They should not be in the same straight line with the radii; but should incline from them 30° up stream, to produce their full effect. All these remarks apply to wheels moving freely in a wide or indefinite channel; as in the case of a floating mill, built on a scow, and anchored out in a stream: but not to wheels for which the water is dammed up, and acts with a practical fall. No great exactness is to be expected in rules on this subject. The best vel for the wheel is about .4 that of the stream.

$$\text{Thus, } \frac{\text{No of}}{\text{H. Pow.}} = \frac{\underset{1050}{\text{cub ft per min}} \times \underset{.190}{\text{vert ht in ft}} \times \underset{62.3}{\text{lbs}}}{33000} = \frac{12429}{33000} = .377 \text{ of a H. Pow.}$$

But in practice the *wheels* actually realize but about $\frac{4}{10}$ of this power *of the stream*, when working in an open channel; and still less when the water flows *with the same vel* through a narrow artificial channel, but little wider than the wheel. Therefore, the actual power of our wheel will be but $.377 \times .4 = .1508$; or about $\frac{1}{7}$ of a horse power; or $33000 \times .1508 = 4976$ ft-lbs per min. Making a rough allowance for the friction of the machine at its journals, &c, we should have say about 4400 ft-lbs of *useful* power; that is, the wheel would actually raise about 440 lbs 10 ft high; or 44 lbs 100 ft high, &c, per min. The vel of the stream must not be measured at the surface; but at about $\frac{1}{6}$ of the depth to which the floats are to dip, or be immersed. This, however, is chiefly necessary in shallow streams, in which the depth of the float bears a considerable ratio to that of the water.

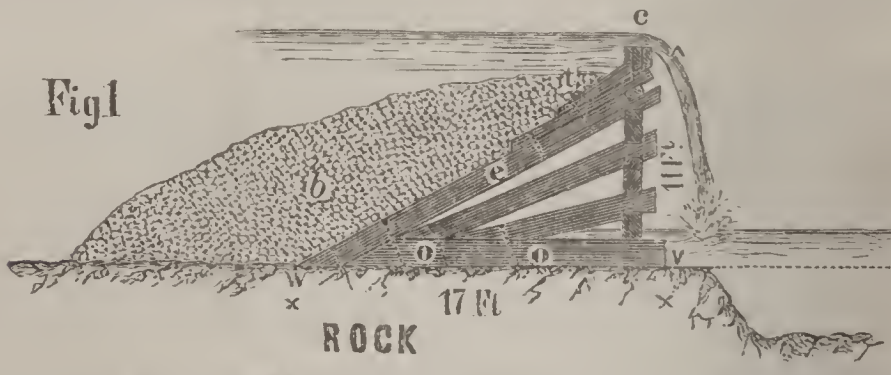
This power of a running stream, (for any given area of transverse section,) increases as the cubes of the vels; for, as we have seen, the power in ft-lbs per min is found by mult together the weight of water which passes through the section in a min, and the virtual head in ft; and since this weight increases as the vel, and this head as the square of the vel, the prod of the two (or the power) must be as the cube of the vel. Therefore, if the vel in the foregoing case had been 10.5 ft per sec, or 3 times 3.5 ft, the power of the wheel would have been 27 times as great, or $.1508 \times 27 = 4.07$ horse-powers.

DAMS.

We can devote but little space to this subject, in addition to what is said on earthen dams for reservoirs, p 287; and on stone ones, p 229, &c. Those we shall now describe will also answer for such reservoirs, when the perishable nature of timber is not an objection.

Primary requisites, in the erection of dams, are, a foundation sufficiently firm to prevent them from settling, and thus leaking; the prevention of leaks through their backs, or under their bases; and the prevention of wear of the bottom of the stream in front of the dam, by the action of the falling water. For the first purpose, hard level rock bottom is of course the best; and should be chosen, if possible. In that case, thick planks, *t t*, Fig 6, (single or double, as the case may be,) closely jointed, and reaching from the crest, *c*, to the back lower edge *w*, (where they should be scribed down to the rock;) with a good backing, *b*, of gravel, will suffice to prevent leaks. Gravel, or rather very gravelly soil, is far better than earth for this purpose; for if the water should chance to form a void in it, the gravel falls and stops it. To prevent this backing from being disturbed near the crest of the dam, by floating bodies swept along by freshets, a rough pavement of stones, about 15 to 18 inches deep, as shown in Fig 7, should be added for a width of about 10 to 20 feet; or until its top becomes 3 to 5 feet below the crest *c* of the dam, according to circumstances.

Fig 1

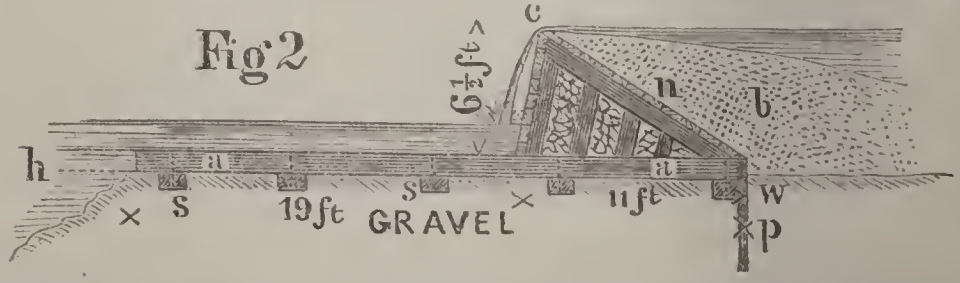


In Fig 1, (a dam on the Schuylkill navigation,) the upper timbers, *e*, are all close jointed, and laid touching, so as not to require planking in addition.

But if the bottom of the stream is gravel or earth, there must in addition to these be used two thicknesses of sheet piles, *p*, Fig 2, &c, close driven, breaking joint, to a depth of several feet, to prevent leaking through the soil beneath the base of the dam. Frequently but one thickness is used. If the bottom is soft or open for a depth of only a few feet, it is at times better to remove it, and base the dam on the firmer stratum below; still, however, using the sheet piles. Old decayed timber and other rubbish should be removed from the base. In very bad soils of greater depth, it may be necessary to support the dam entirely upon a platform resting on bearing piles. Here great precautions are necessary against leaks; but the case occurs so rarely, that we shall not stop to consider it.

As to the wearing away of the bottom of the stream by the water falling over the front of the dam, precautions should be used in all cases except that of very

Fig 2



hard rock, or of medium rock protected by a considerable depth of water. The dam, Fig 1, was built upon a tolerably firm micaceous gneiss in nearly vertical strata, covered by about 2 feet of water in ordinary stages. In 39 years the rock was

worn away in front of the dam, as shown in the fig, to the average depth of 3 feet; or very nearly 1 inch per year. The depth of water on the crest c, was usually from 6 to 18 ins; rarely 5 or 6 ft during freshets; and but a few times during the whole period, 8 or 9 ft.

At Jones's dam, on Cape Fear River; height of dam, 16 ft; front vert; fall, usually 10 ft, into 6 ft depth of water; the soft shale rock, in vert strata, was, in the course of a few years, worn away 16 ft; and the dam was undermined to such an extent as to fall into the cavity.

In another case, dam 36 ft high; front vert; the water falling upon nearly vert strata of hard shale rock, usually covered by but about 2 ft of water: in about 20 years wore it to an irregular depth of from 10 to 20 ft; and extending from the very face of the dam, to 70 or 80 ft in front of it.

In Fig 2, upon a stream subject to very violent freshets, the gravel was washed away for a considerable width and depth beyond the apron, as at *h*. To prevent a repetition, the cavity was filled with cribwork full of stone, clear across the river.

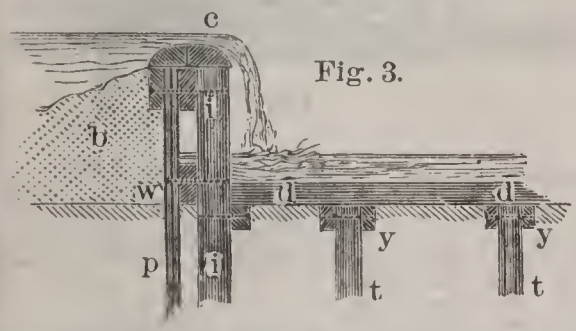


Fig. 3.

A deposit of blocks of loose stone, of even a ton weight or more, will not serve as a protection in front of a dam exposed to high freshets; but will soon be swept away. A common precaution against this wear, in low dams, is an *apron*, *a a*, Fig 2; or *d d*, Fig 3; of either rough round tree trunks, or of hewn timber, laid close together; extending under the entire base of the dam, and from 15 to 30 ft in front of its face. These are sometimes bolted to pieces, *s s*, Fig 2; or *y y*, Fig 3; laid under them across the stream. In Fig 3, with very soft bottom, these pieces *y y* are supposed to be bolted to short piles *t t*, driven for that purpose.

At times a distinct wide low timber crib, filled with stone, and covered on top with stout plank, has been placed in front of the dam, to receive the fall of the water; and is effective in protecting the bottom. Also, in some cases, a dam of less height, and of cheap character, has been built at a short distance down stream from the main one, in order to secure at all times a deep pool in front of the latter for breaking the force.

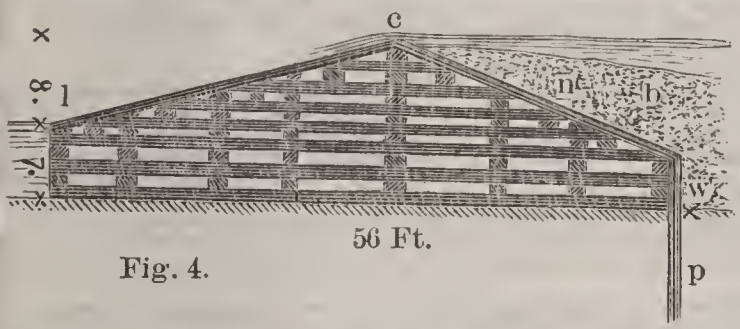


Fig. 4.

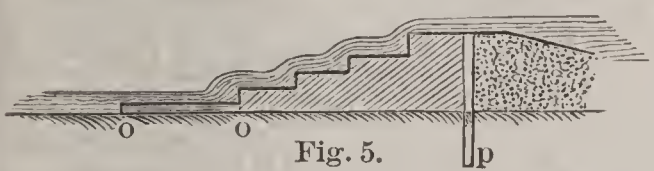


Fig. 5.

Another precaution is to substitute a sloping front like *c l*, Fig 4, or such as Figs 1 and 2 would form if reversed, for the nearly vert one of the other figs; thus to some extent reducing the force of the water. This, however, is but a partial remedy, especially for soft bottoms in shallow water; for the sliding sheet still descends with great force. The best form of dam, perhaps, in such cases, is that shown in Fig 5, in which the front consists of a series of steps of about 1 vert, to 3 or 4 hor. These effectually break the force of the water; and, with the addition of an apron *o o*, secure a satisfactory result. It is objected against this form, as also against Figs 4 and 6, that their fronts are liable to be torn by descending trees, ice, and other bodies swept along during freshets; but experience shows that this objection has but little weight; for when such bodies pass, the sheet of water is thicker than usual; and protects the front timbers. On the Sch Nav, the timbers *c l*, Fig 6, scarcely wear thin at the rate of an inch in 10 to 15 years.

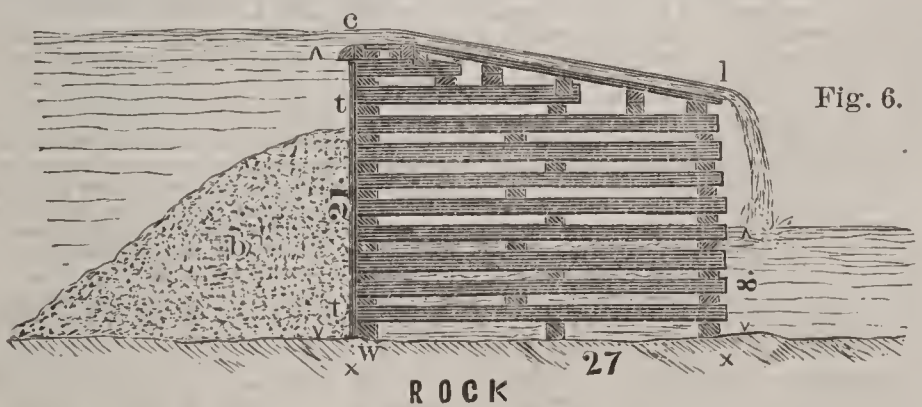


Fig. 6.

The forms of wooden dams are many; (see the figs, which show those most used:) varying with the circumstances of the case, and with the fancy of the designer. In the United States they are usually of cribwork, of either rough round logs with the bark on, or of hewn timber; in either case about a foot through. These timbers are merely laid on top of each

other, forming in plan a series of rectangles with sides of about 7 to 12 ft. They are not notched together, but simply bolted by 1 inch square bolts (often ragged or jagged) about 2 to 2½ feet long, through two timbers at every intersection. These are not found to rust or wear seriously, even when exposed to a current. Square bolts hold best. Round logs are flattened where they lie upon each other. Experience shows that firmer but more expensive connections are entirely unnecessary. The cribs are usually, but not always, filled with rough stone. In triangular dams, disposed as in Figs 1, 2, and 7, this stone filling is not so essential as in other forms; because the weight of the water, and of the gravel backing, tends to hold the dam down on its base. Still, even in these, when the lower timbers are not bolted to a rock bottom, or otherwise secured in place, some stone may be necessary to prevent the timbers from floating away while the work is unfinished, and the gravel not yet deposited behind it. On rock, the lowest timbers are often bolted to it, to prevent them from floating away *during construction;* and when the water is some feet deep, this requires coffer-dams.

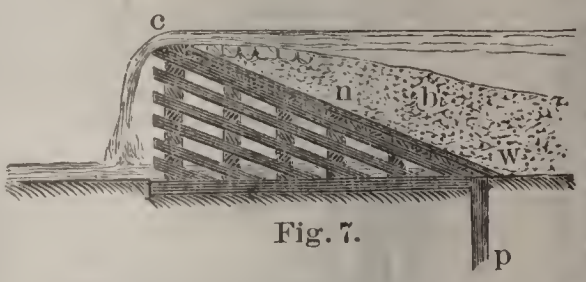


Fig. 7.

Or, the cribs may be built at first only a few feet high; then floated into place, and sunk by loading them with stone; for the reception of which a rough platform or flooring will be reqd in the cribs, a little above their lowest timbers. The bolting to the rock may then be dispensed with. The water may flow through the open cribwork as the building higher goes on; attention being paid to adding stone enough to prevent it floating away if a freshet should happen. Or, cribs shown in plan at c c, Fig 8, loaded with stone, may be sunk, leaving one or more intervals, like that at o o o o, between them, for the free escape of the water. These openings to be finally closed by floating into them closing-cribs shaped like *n*.

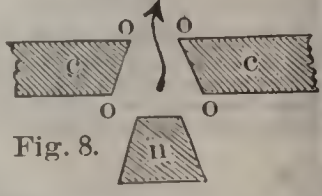


Fig. 8.

The workmanship of a dam in deep water can of course be much better executed in coffer-dams, than by merely sinking cribs. The joints can be made tighter: the stone filling better packed; the sheet piling more closely fitted, &c.

When a very uneven rock bottom in deep water, or the introduction of sluices in the dam, or any other considerations, make it expedient to build dams within coffer-dams, both should be carried on *in sections;* so as to leave part of the channel-way open for the escape of the water. Commencing at one or both shores, the first section of the coffer-dam may reach say quarter way or more across the stream. In the section of the dam itself built within this enclosing coffer-dam, ample sluices should be left for the water to flow through when we come to build the *closing* section of the coffer-dam. When the dam has been finished, these sluices may be closed by **drop-timbers*.** Before removing one section of coffer-dam, the outer end of the enclosed section of dam itself must be firmly finished in such a manner as to constitute a part of the inner end of the next section of coffer-dam. It is impossible to give details for every contingency; the engineer must rely upon his own ingenuity to meet the peculiarities of the case before him. In some cases of shallow water, mere mounds of earth may answer for coffer-dams; or rough stone mounds, backed with earth or gravel.

After the water has passed beyond the crest, c in the figs, there is no necessity for preventing its leaking down among the crib timbers: on the contrary, the thick sheeting planks, (or squared timbers, as occasion may require,) *c l*, Figs 4 and 6, which form the slopes along which the water then flows in some dams, are usually not laid close together, but with open joints of about ½ inch wide between them, for the express purpose of allowing part of the water to fall through them, so as to keep the timbers beneath them partially wet; which, to some extent, renders them more durable. In Figs 1, 4, 6, and 7, the water of the lower pool flows freely back among the crib timbers, and rough quarry stones with which the cribs are filled either partly or entirely. In Figs 4 and 6, these stones are not shown. In the dam, Fig 1, none were used. In Fig 2, they were as shown.

A substantial, and not very expensive dam of the form of Fig 7, may be built of rough stone in cement. Some hewn timbers should be firmly built horizontally into the masonry of the sloping back *c n w*, at a few feet apart, with their tops level with the surf of the masonry. To these must be well spiked close-jointed sheeting-plank *c n w*, for protecting the masonry from the action of the water, and of floating bodies. The gravel backing *b*, may be omitted; but the sheet piles *p*, and an apron in front of the dam, will be as indispensable in yielding soils, as if the dam were of timber.

Figs 1, 2, 4, 6, and 7, are sections drawn to a scale, of existing dams in Pennsylvania, that have stood successfully the force of heavy freshets for a long series of years.† These freshets at times carry along large bodies of ice, trees, houses, bridges, &c.; and have risen to 11 ft above the crests. Fig 1, on the Sch Nav, was built in 1819, and served perfectly for 39 years, until in 1858 the decay of much of its timber, especially of the close-laid top ones, *e*, rendered it necessary to build a new one just in front of it. It was of extremely simple construction; and was never filled with stone. The bottom timbers, o o, 10 ft apart, were bolted to the rock; and immediately over each of them, was such a series of inclined timbers as is shown in the fig. The top ones, *e*, however, were close-jointed, and laid touching, so as to form the top sheeting, instead of thinner planks. The short pieces at *t* were laid in the same way. No coffer-dam was used: but the bottom pieces were first bolted to the rock; 10 ft apart; then the stringers and the sloping pieces were added. The close covering (*e*) was carried forward from each end of the dam, until at last a space of only about 60 ft was left in the center, for the water to pass. The close covering for this space being then all got ready, a strong force of men was set to work, and the space was covered so rapidly that the river had not time to rise sufficiently high to impede the operation.

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* Timbers ready prepared for closing an opening through which water is flowing; and suddenly *dropped* into place by means of grooves or guides of some kind for retaining them in position. Several such timbers may at times be firmly framed together, and then be all dropped at once; closing the opening or sluice at one operation: especially when it is of small size. In some cases, a crib may be sunk on the up-stream side of such an opening, for closing it.

† Those on the Schuylkill Navigation were obligingly furnished by James F. Smith, Esq. chief engineer and superintendent of that work. Other valuable information from the same source will be found in different parts of this volume.

Fig 2 is a canal feeder dam on the Juniata. Here *s s* are timbers stretching clear across the stream, (about 300 ft,) and sustaining the apron *a a*, of stout hewn timbers laid touching. This dam was filled with stone, for the retention of which the front sheeting planks were added.

Fig 6 is on the Sch Nav; was built in 1855. It is a form much approved of on that work, for such situations; namely, firm rock foundation, with a considerable depth of water in front. The highest dam (32 ft) on the Sch Nav. is very similar to it; built in 1851. All the dams on this work are of hewn timber, chiefly white and yellow pine. The water occasionally runs from 8 to 12 feet deep over their crests; and then overflows and surrounds many of the abuts. The vertical back allows the overflowing water to leak down among *all* the lower timbers of the dam, and thus tend to their preservation.

Fig 4 shows the dams on the Monongahela slackwater navigation; W. Milnor Roberts, eng. They are of round logs, with the bark on; flattened at crossings. The longest ones in the fig are 10 feet apart along the length of the dam. Experience shows that such dams possess all the strength necessary for violent streams. On rock, the lowest timbers are bolted to it.

Fig 7 has been successfully used to heights of 40 ft.*

Fig 3 is intended merely as a hint for a very low dam on yielding bottom. Its main supports are piles *i i*, from 4 to 8 ft apart, according to the height of the dam; and other circumstances; and *t t* are short piles for sustaining the apron *d d*. It may be extended to greater heights by adding braces in front; which may be covered by stout planks, to form an inclined slide for the overfalling water. Many effective arrangements of piles, and sloping timbers for dams on soft ground, will suggest themselves to the engineer. Thus, at intervals of several feet, rows of 3 or more piles may be driven transversely of the dam; the top of the outer pile of each row being left at the intended height of the crest, while those behind are successively driven lower and lower; so that when all are afterward connected by transverse and longitudinal timbers, and covered by stout planking, and gravel, they will form a dam somewhat of the triangular form of Fig 7. It would be well to drive the piles with an inclination of their tops up stream.

There is much scope for ingenuity both in designing, and in constructing dams under various circumstances; and in turning the course of the water from one channel to another, by means of ditches, pipes, or troughs, &c., at diff heights; aided at times by low temporary dams or mounds of earth; or of sheet piles, &c; or by coffer-dams; so as to keep it away from the part being built. Each locality will have its peculiar features; and the engineer must depend on his judgment to make the most of them.

Abutments of dams as a general rule should not contract the natural width of the stream; or, if they must do so, as little as possible; for contractions increase the height, and violence of the overflowing water in time of freshets; during which a great length of overfall is especially desirable. They should be very firmly connected with the ends of the dams; and should, if the section of the valley admits of it, be so high, and carried so far inland, that the high water of freshets will not sweep either over them, or around their extremities; and thus endanger undermining, and destruction. In wide, flat valleys they cannot be so extended without too much expense; and the only alternative is to found them so deeply and securely as to withstand such action; making their height such that they will, at least, be overflowed but seldom. Their ends adjacent to the dam, should be rounded off, so as to facilitate the flow of the water over the crest.

They are best built of large stone in cement; for although sufficient strength may be secured by timber, that material decays rapidly in such exposures. If of earth only, they are very apt to be carried away if a freshet should overtop them.

Sluices should be placed in every important dam, in order that all the water may be drawn off, if necessary, for the purpose of repairs; or of removing mud deposits; or finding lost articles of importance, &c. They may be merely strong boxings, with floor, sides, and top of squared timbers; and passing through the breadth of the dam, just above the bottom. To prevent trees, &c, from entering and sticking fast in them, some kind of strong screen is expedient. In common cases a sluice should not *exceed* about 3½ ft by 5 ft in cross-section; otherwise it becomes hard to work. Two or more such openings may be used when much water is to be voided. They should be near the abutments. The gates or valves for opening and shutting them, should be at the up-stream end; for if at the lower one, accumulations of mud, &c, will fill the sluices, and prevent them from working. They are usually of timber; and slide vertically in rebates; being raised and lowered by rack and pinion; but in very important dams they may be of cast iron. Two sets of sluices are desirable; that one may be always ready for use if the other is stopped for repairs.

The part of the apron in front of the sluice should be particularly firm, so as not to be deranged by the water rushing out under a high head.

Dams are sometimes, but rarely, built in the form of an arch; convex up stream. This form is strong; and when the shores are of rock it may be expedient to use it; but if the banks are soft, they will be exposed to wear by the current thrown against them at the abuts of the arch.

At times dams are built obliquely across the stream, with the object of increasing the length, and consequently reducing the depth of water over the crest in times of freshets. The argument, however, appears to the writer to be of but little weight, inasmuch as the reduction of depth would extend but a trifling distance up stream from the dam; and would therefore scarcely have an appreciable effect in diminishing the injury to the overflowed district above. Moreover, the increased expense is probably always more than commensurate with any advantage gained.

Some dams are subject to "tremblings," which have not been satisfactorily accounted for. They exhibit themselves chiefly as undulations of the air, produced by the falling water; and which occasionally cause a rattling of windows within a distance of ½ a mile or more. We have known this to be stopped unintentionally in one case, by building a well-covered

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* **Cost of crib dams.** With common labor at $1.50 per day; $20 per 1000 ft board measure, delivered; stone for filling, $1 per cub yard; gravel 50 cts per cub yd; iron for bolts, &c, 4 cts per lb; such dams in shallow water usually cost complete, from 9 to 12 cts per cub ft; or $2.43 to $3.24 per cub yd of crib.

wide crib apron, a few feet high, against the front of the dam, for preventing the abrasion of the bottom. In other cases a series of oblique timbers placed against the front of the dam, and part way up it, at a slope of about 1½ to 1, and covered with plank, has been perfectly effective in stopping it.

The proper time for building dams is of course at the longest period of low stage of water.

To ascertain in advance approximately, the height to which the water will rise above the crest of a dam; or rather, a little back from it; the crest being above the level of the original water. This will vary with the shape of the crest, as may be seen by reference to Fig 26½, page 268, which, however, is a very peculiar case. Still, until we have more experiments, appreciable deviations from the results of such rules must be expected in practice. Square the disch of the stream in cub ft per sec. Call this square, *s*. Square the length of the overfall in ft. Mult this square by 7. Call the prod *p*. Divide *s* by *p*. Take the cube rt of the quot.* This cube rt will be the reqd approximate height *of rise* in ft.

When, in times of freshets, the water rises above the crest to a height equal to that of the dam itself, there is no perceptible fall at the dam; and boats may pass in safety over the crest.

For measuring the disch over dams, see pp 264, 267. See also Art 1 of Hydrostatics. In shape of a formula, the foregoing rule will be,

$$\text{Rise in ft} = \text{cube rt of } \left(\frac{\text{discharge in cub ft per sec}^2}{7\ (\text{length of overfall in ft}^2).}\right)$$

When the dam is originally a submerged, or drowned one, as D, Fig 9; the following is a rough approximation, *probably somewhat in excess; o g* being the natural level of the water previous to building the dam; and *o c* the natural depth in ft, of the water above the intended crest. Then the required depth *a c*, of the up-stream water, above the crest when built, will be, approximately,

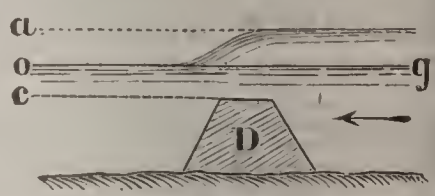

Fig 9

$$a\,c = o\,c + \text{cube root of } \left(\frac{\text{discharge}^2}{7 \times \text{length}^2}\right)$$

Having *a c*, deduct *o c*; and the rem will be *a o*, or the required rise produced by the dam.*

The rules given for the varying rise of surface for considerable distances up stream from dams: as well as for some allied subjects in hydraulics; are extremely complicated; and require much greater knowledge of mathematics than is usually found among civil engineers; and so far as regards their application to the actualities of common occurrence, they are probably no less useless than complicated.

Table of thickness of white pine plank required not to bend more than $\frac{1}{480}$ part of its clear horizontal stretch, under different heads of water. (Original.)

Stretch in Ft.	Heads in feet.				
	40	30	20	10	5
	Thickness in Inches.				
3	3½	3	2¾	2¼	1¾
4	4½	4	3½	2¾	2¼
6	6¾	6	5¼	4¼	3¼
8	9	8	7	5½	4½
10	11¼	10	8¾	7	5½
12	13½	12¼	10¾	8½	6¾
15	16¾	15	13	10½	8½
20	22¼	20	17½	14	11

* These two rules are from Rankine, who says they apply to "crests either flat or slightly rounded." But that, in itself, is very vague.

WATER SUPPLY.

The quantity of water required in cities, has been found by experience to increase faster than the population. About 60 gallons, or 8 cubic feet, per day, to each inhabitant, is usually considered a fair ample allowance. Many European cities have not half as much; while New York, and some others, use and waste half as much more. With efficient means for preventing waste, 60 gallons would probably suffice for any commercial city; but inasmuch as cleanliness and health are promoted by its free use, as few restrictions as possible should be introduced.

Water for city use should not be drawn from the very bottom of the reservoir, because it will then be apt to carry along the sediment; which not only injures the water, but creates deposits within the pipes; thus obstructing the flow. In fixing upon the necessary capacity of a reservoir, this must be taken into consideration; inasmuch as all the water below the level for drawing off, must be regarded as lost. When circumstances justify the expense, it is well to curve up the reservoir end of the service main, so as to provide it with valves at different heights; for drawing off only the purest stratum that may be in the reservoir. With this view, the valve-tower (page 289) generally has such valves communicating with the water in the reservoir; and by this means only the purest is admitted into the tower: and from it, into the city pipes. This refinement, however, is rarely practicable. Such valves must of course be worked by watchmen. **For rainfall,** see p 220.

Art. 1. Reservoirs. In important reservoirs of earth, for storing water to moderate depths for cities, experience appears not to sanction **dimensions** *bolder* than 10 feet thick at top; inner slope 2 to 1; outer slope 1½ to 1.* A top width of 15 feet to 20 feet, and inside slopes of 3 to 1, are adopted in some important cases; with outer slopes of 2 to 1. Both slopes, however, are at times made only 1½ to 1. The level water surface should be kept at least 3 or 4 feet below the top of the embankment; or more, if liable to **waves.** In a large reservoir, a quite moderate breeze will raise waves that will run 3 feet (measured vertically) up the inner slope. A low wall, or close fence, *w*, Fig. 37, is sometimes used as a defence against them. The top and the outer slopes should be protected at least by sod or by grass. To assist in keeping the top dry, it should be either a little rounding, or else sloped toward the outside.† The soft soil and vegetable matter should be carefully removed from under the entire base of the embankments; which should be carried down to soil itself impervious to water, in order that leakage may not take place *under* them. To aid in this, a double row of sheet piles, or a sunk wall of cement masonry, carried to a suitable depth below the bottom, may be placed along the inner toe in bad cases. If there are springs beneath the base, they must either be stopped, or led away by pipes. The embankment should be carried up in layers, slightly hollowing toward the center, and not exceeding a foot in thickness; and all stones, stumps, and other foreign material, such as *clean* gravel, sand, and decomposed mica schists, &c, that may produce leakage, carefully excluded. These layers should be well consolidated by the carts; and the easier the slopes are, the more effectively can this be done. The layers, however, should not be distinct, and separated by actual plane surfaces; but each succeeding one should be well incorporated with the one below. This has sometimes been done by driving a drove of oxen, or even sheep, repeatedly over each layer; in addition to the carting. Rollers are not to be recommended, as they tend to produce seams between the layers. This might possibly be obviated by projections on the circumference of the roller.

Gravelly earth is an excellent **material,** perhaps the best. The choicest material should be placed in the slope next to the water; and should be deposited and compacted with special care in that portion, so as to prevent the water from leaking into the main body of the dam, and thus weakening it. It is not amiss to introduce a bench, *b*, Fig 37, in the outer slope, to diminish danger from rainwash by breaking the rapidity of its descent.

If the bottom of the reservoir itself is on a leaky soil, or on fissured rock, through the seams of which water may escape, it must be carefully covered with from 1½ to 3 feet of good puddle; which, in turn, should be protected from abrasion and disturbance, by a layer of gravel; or of concrete, either paved or not, according to circumstances.

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* The writer suggests that a top width equal to 2 feet + twice the square root of the height in feet, will be safe for any height whatever of reservoir properly constructed in other respects.

† Some engineers slope the top toward the *inside.*

Reservoirs constructed with the foregoing dimensions, and with care, *may* remain safe for an indefinite period; but where serious damage would result from failure, the following **additional precautions** should be taken. The inner slopes should be carefully faced up to the very top, with at least a close dry rubble-stone pitching, not less than 15 to 18 inches thick; as a protection against wash, and against muskrats. These animals, we believe, always commence to burrow under water. If the slopes are much steeper than 2 to 1, this dry pitching will be apt to be overthrown by the sliding down of the softened earth behind it, if the water in the reservoir should for any cause be drawn down rather suddenly. It will be much more effective, but of course more costly, if laid in hydraulic cement; and still more so if laid upon a layer a few inches thick of cement-and-gravel concrete; especially if this last be underlaid by a layer about 1½ to 3 feet thick of good puddle, spread over the face of the slope; the great object being to protect the inner slope from actual contact with the water. If this can be effectually accomplished, slopes as steep as 1½ to 1 will be perfectly secure; for the danger does not arise from any want of weight of the earth for resisting overthrow. **Special care should be bestowed upon the inner toe of the slope,** to prevent water from finding its way beneath it, and softening the earth so as to undermine the stone pitching. Near the top, reference should be had to danger of derangement by ice frost, rain, and waves. Flat inner slopes tend not only to prevent the displacement of the pitching; but increase the stability of the embankment, by causing the pressure of the water (which is always at right angles to the slope) to become more nearly vertical; and thus to hold the embankment more firmly to its base than if there were no water behind it. Sometimes the toes of both the inner and outer slopes abut against low retaining-walls in cement. This gives a neat finish, and tends to preservation from injury.

Many engineers, in order to prevent leaking, either through or beneath the embankment, construct a **puddle-wall,** *p*, Fig. 37, of well-rammed impervious soil, (gravelly clay is the best,) reaching from the top to several feet below the base. This wall should not be less than 6 or 8 feet thick on top, for a deep reservoir; and should increase downward by *offsets* (and not by slopes or batters) at the rate of about 1 in total thickness, to 3 or 4 in depth. Other engineers object to these puddle-walls; and contend that leakage should be prevented by making both the inner slopes and the bottom of the reservoir, water-tight, by means of puddle, concrete, and stone facing in cement, as just alluded to. They argue that if the embankment is well constructed, it is itself a puddle-wall throughout.

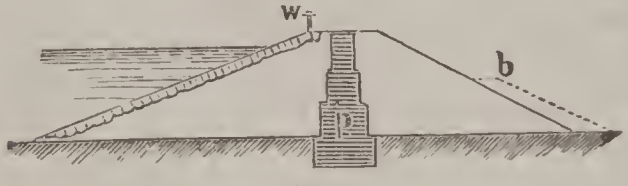


Fig. 37.

Near San Francisco, Cal, are two earthen reservoir dams built about 1864, one 95 feet high, 26 on top, inner slope 2.75 to 1, outer 2.5 to 1. The other 93 high, 25 on top, inner slope 3.5 to 1, outer 3 to 1. In each the puddle-wall is carried 47 feet deeper than the base. No stone facing.

It is difficult to prevent water under high pressure from finding its way through considerable distances along seams where earth is in contact with smooth rock, wood, or metal; as, for instance, along the surfaces of iron pipes laid under reservoir embankments; or along the tie-rods sometimes used through the puddle of coffer-dams; and the same is apt to occur under the bases of embankments which rest on smooth rock. Special care should be taken that the earth used in such positions is not of a porous nature; and that it is thoroughly compacted all along the seam; and the straight continuity of the seam should be interrupted or broken as frequently as possible by projections. Faucets or flanges do this to a limited extent in the case of iron pipes; and something similar, but on a larger scale, should at short intervals be constructed in the shape of collars or yokes of cement stonework, in the case of rock or masonry. See also Dams, p 282, also p 229, &c.

It is usually advisable to **divide reservoirs into two parts,** so that while the water in one part is being drawn off for use, that in the other may purify itself by settling its sediment. Also, one part may remain in use, while the other is being cleaned or repaired. Many days, or even two or three weeks, sometimes, are required for the complete settlement of the very fine clayey particles in muddy water; depending on the depth of the reservoir. One or more flights of steps to the bottom of the reservoir should be provided.

Mud in Reservoirs. The reservoirs of the New River Water Co, London, England, were uncleaned for 100 years, during which mud 8 feet deep was

deposited, or about an inch annually. At Philadelphia it is about .25 inch per annum from the Schuylkill, and 1 inch from the Delaware River. At St. Louis, Missouri, about 3 to 4 feet per year! Vegetation is apt to take place in sha low reservoirs and near the edges of deep ones. especially in very warm weather; and the plants, on decaying, injure the water.

Water flowing through marsh lands is sometimes unfit for drinking purposes. That, for instance, in some sections of the Concord River, Massachusetts, was reported by the eminent hydraulic engineer, Loammi Baldwin, of Boston, to be absolutely *poisonous* from this cause.

The construction of a large deep reservoir is not only a very costly, but a very hazardous undertaking. With every watchfulness and care, it is almost impossible entirely to prevent leaking; although this may not manifest itself for months, or even years. Should a break occur, especially near a city, it would probably be attended by great loss of life and property. If the water once finds its way in a stream, either across the unpaved top, or through the body of the embankment, the rapid destruction of the whole becomes almost certain.

Art. 1a. Storing Reservoirs. The entire annual yield of a stream may be much more than sufficient for supplying a certain population with water; and yet in its natural condition the stream may not be available for this purpose, because it becomes nearly dry in summer, when water is most needed; while, at other seasons, the rains and melted snows produce floods which supply vastly more than is required; and which must be allowed to run to waste. A storing reservoir is intended to collect and store up this excess of water, so that it may be drawn off as required during the droughts of summer, and thus equalize the supply throughout the entire year. This, when the locality permits, is effected by building a dam across the stream, to form one side of the reservoir; while the hill-slopes of the valley of the stream form the other sides. The stream itself flows into this reservoir at its up-stream end. When the stream is liable to become nearly dry during long summer droughts experience shows that the **capacity** of the reservoir should be equal to from 4 to 6 months' supply, according to circumstances. During the construction of the dam, a free channel must be provided, to pass the stream without allowing it to do injury to the work. If the dam were built precisely like Fig 37, entirely of earth, it would plainly be liable to destruction by being washed away in case the reservoir should become so full that the water would begin to flow over its top. To provide against this we may, by means of masonry, or of cribs filled with broken stone, or otherwise, construct either the whole, or part of the dam, to serve as an **overfall,** or a **waste-weir.** Or a side channel (an open cut, pipes, or a culvert, &c) may be provided at one or both ends of the dam, and in the natural soil, at such a level as to carry away the surplus flood water before it can rise high enough to overtop the earthen dam. Besides these, and the pipes for carrying the water to the town, there should be an outlet, with a valve or gate, at the level of the bottom of the reservoir; in order that, if necessary for repairs, or for cleaning by scouring, all the water may be drawn off. The entrances to the city pipes should be protected by gratings, to exclude fish, &c.

To facilitate repairs or renewals of all valves, &c, which are under water, the reservoir ends of the pipes or culverts to which they are attached, may be surrounded by a water-tight box or chamber, which will usually be left open to the reservoir; but may be closed when repairs are required. Access may then be had to them by entering at the outer end, after the water has flowed away from inside. In case the outlet is through a long line of pipes which cannot thus be entered, a special entry for this purpose may be cast in the pipe itself, near the outer toe of the embankment; to be kept closed except in case of repairs. Sometimes a better, but more expensive means of access to such valves, is secured by enclosing them in a **valve-tower** of masonry. This is a hollow vertical water-tight chamber, like a well; but near the toe of the inner slope; having its foundation at the bottom of the reservoir; whence the tower rises through the water to above its surface. This chamber is provided with valves or gates usually left open to the reservoir; but which may be closed when repairs are needed; and the water in the tower allowed to escape from it through the open valves of the outlets. This done, workmen can descend through the tower by ladders from the aperture at its top.

At times the outlets for the discharge of surplus flood water are, like those for scouring, placed at, or just above, the level of the bottom of the reservoir. In order that these may work in case of a sudden flood at night, &c, they must be furnished with self-acting valves, which will open of their own accord when the flood is about to rise too high. This may be effected by attaching them to floats, the rising of which, when the water is high, will pull them open. All such outlets should be large enough to let men enter them for repairs. They should by

no means be laid through the artificial earthen body of the dam itself, without being supported upon masonry reaching down to a firm natural foundation; otherwise they are very apt to be broken by the subsidence of the embankment. It is usually safer to carry them through the firm natural soil near one end of the dam. Their valves, if only single, should be at their inner or reservoir end, so as to leave the outlets themselves usually empty, for inspection; but it is better to have two valves, so that one may be used when the other needs repair; and in this case one may be placed at each end. Reservoirs which are supplied by pumps, need no precautions against overflow; because the pumping is stopped when they are filled to the proper height. Large storing reservoirs necessarily submerge more or less land, which has therefore to be purchased. By intercepting the descending water, they frequently prevent spring floods from injuring low lands farther down stream. If there are mills down stream from the reservoir, they would evidently be deprived of water for driving them, unless a portion of that stored in the reservoir be devoted to that purpose. Water thus applied to *compensate* for the loss of the natural stream, is called **compensation water**; and the reservoir, a *compensating* one.

Art. 1b. Distributing reservoirs. Frequently a valley fit for a storing reservoir can be found only at a long dist (sometimes many miles) from the town; and it then becomes expedient to construct also an additional one of smaller size than the storing one, near the town; and at as great an elevation above it as circumstances will permit; but lower than the storing one. This is called, by way of distinction, a *distributing* reservoir, because from it the water, after having flowed into it from the storing reservoir, through the long *supply pipe* which connects them, is distributed in various directions through the town, by means of the street mains, or pipes. This small reservoir should hold a supply sufficient at least for a few days; a few weeks would be better; and the end of the supply pipe which terminates in it, should be provided with a valve for shutting off the supply from the storing reservoir. These precautions permit repairs to be made along the line of supply pipe without depriving the town of water in the mean time. With a view to such repairs; as well as to scouring out sediment from the supply pipe, this last should be provided with **outlet valves** at various low points along the entire interval between the two reservoirs; especially at those at which the valves may disch into natural watercourses. On opening these valves, the outrush of the water carries away sediment; and leaves the pipe empty for inspection.

In fixing upon the diams of pipes for supplying cities, it is necessary to bear in mind, that by far the greater portion of the 24 hours' yield is actually drawn from them during only 8 to 12 hours of daylight; and therefore the capacity of the pipes must be sufficient to furnish the daily supply in much less than 24 hours. Again, during the hot summer months, much more water is used than during the winter ones; and this consideration necessitates a still larger diam.

Art. 2. Systems of street pipes for supplying cities. The writer knows of no practical rules for proportioning the diams for such systems. The various complications involved, render a purely scientific investigation of little or no service. With much hesitation, he ventures the following purely empirical rules of his own; based on such limited observations as have casually fallen under his notice.

Rule 1. *When, at no point in a system of city pipes, is the head, or vert dist below the surface of the reservoir, compared with the hor dist from the reservoir, less than at the rate of 50 ft per mile, then the population in the last column of the following Table A, may be abundantly supplied, for all city purposes, by either one pipe of the inner diam or bore in the 1st col; or by 2, 3, &c. pipes of the diams in the other cols.* These diams are given to the nearest safe ⅛ inch. The supply is assumed to be about 60 gallons per day to each inhabitant.

TABLE A. (Original.)

NUMBER OF PIPES.								
1	2	3	4	6	8	12	24	Population.
Diam. Ins.	Diam. Ins.	Diam. Ins.	Diam. Ins.	Diam. Ins.	Diam. Ins.	Diam. Ins.	Diam. Ins.	
6	4⅝	3⅛	3½	3	2⅝	2⅜	1⅞	1647
8	6⅛	5¼	4⅝	4	3½	3⅛	2⅜	3465
10	7⅝	6½	5⅞	5	4⅜	3⅞	3	5908
12	9⅛	7¾	7	5⅞	5¼	4½	3½	9324
14	10⅝	9⅛	8⅛	7	6⅛	5⅜	4⅛	13706
16	12⅛	10⅜	9¼	7⅞	6⅞	6	4⅝	19141
18	13¾	11⅝	10½	8⅞	7⅞	6¾	5⅛	25677
20	15¼	13	11⅝	9⅞	8¾	7½	5¾	33426
22	16¾	14¼	12¾	10⅞	9½	8¼	6¼	42433
24	18¼	15½	13⅞	11¾	10½	9	6⅞	52671
26	19¾	16¾	15	12¾	11¼	9¾	7½	64447
28	21¼	18⅛	16¼	13¾	12¼	10½	8	77565
30	22⅞	19⅜	17⅜	14¾	13⅛	11¼	8⅝	91580
32	24⅜	20⅝	18½	15⅝	14	11⅞	9	108160
34	25⅞	22	19⅝	16¾	15	12¾	9¾	125840
36	27⅜	23¼	20⅞	17¾	15¾	13½	10¼	144480
40	30½	25⅞	23⅛	19⅝	17⅝	15	11½	188320
44	33½	28⅜	25⅜	21⅝	19½	16½	12⅝	239600
48	36½	31	27¾	23⅝	21⅛	18	13¾	297600
54	41	34⅞	31¼	26½	23⅞	20⅛	15⅜	391200
60	45⅝	38¾	34⅝	29½	26⅜	22⅜	17⅛	511900
66	50⅛	42⅝	38⅛	32⅜	29⅛	24⅝	18¾	650400
72	54¾	46½	41⅝	35⅜	31⅝	26⅞	20½	800000
80	60⅞	51⅝	46¼	39¼	35¼	29⅞	22¾	1064000

It is well to allow in addition from ½ inch to 1 inch, or more, (depending on the character of the water,) to each diameter; for deposits and concretions.

The water, after reaching the city through one or more large main pipes from the reservoir, must be distributed through the streets by means of smaller mains branching from the larger ones. The diameters of these smaller ones also may be found by Table A. Thus, if a street, with its alleys, &c, contains about 6000 persons, (the rate of head being, as before, not less than 50 feet to a mile at any point of the system,) then we see by the table that a 10-inch pipe will answer. It would be well to lay no city street pipes of less than 6 inches diameter.

Mains which cross each other should be connected at some of their intersections, to allow the water a more free circulation throughout the entire system; so that if the supply at any point is temporarily cut off from one direction by closing the valves for repairs, or is diminished by excessive demand, it may be maintained by the flow from other directions.

Avoid dead ends when possible, as the water in them becomes foul and unwholesome.

RULE 2. *With the same diameters, different rates of head will supply the proportionate populations in column 3 of Table B. Or, to find the diameters which at different rates of head will supply the same populations given in the last column of Table A,* multiply the diameter given in Table A, by the corresponding number in column 4 of Table B; or (approximately) do as directed in column 5.

TABLE B. (Original.)

Col. 1.	Col. 2.	Col. 3.	Col. 4.	Col. 5.
Rate of Head, in Feet per Mile.	Rate of Head, compared with that in Table A.	Proportionate Populations.	Proportionate Diam. to supply the Populations in Table A.	Remarks.
5	.1	.32	1.58	
10	.2	.45	1.37	
12½	.25	.50	1.32	Add one-third.
15	.3	.55	1.27	Add full one-fourth.
20	.4	.64	1.20	Add one-fifth.
25	.5	.71	1.14	Add one-seventh.
30	.6	.78	1.11	Add one-ninth.
35	.7	.84	1.07	Add one-fourteenth.
37½	.75	.87	1.06	Add one-sixteenth.
40	.8	.90	1.05	Add one-twentieth.
45	.9	.95	1.02	Add one-fiftieth.
50	1.0	1.00	1.00	
75	1.5	1.23	.92	Deduct one-thirteenth.
100	2.0	1.41	.88	Deduct one-eighth.
125	2.5	1.59	.83	Deduct full one-sixth.
150	3.0	1.73	.80	Deduct one-fifth.
200	4.0	2.00	.76	Deduct nearly one-fourth.
250	5.0	2 25	.73	Deduct nearly two-sevenths.
300	6 0	2 46	.69	Deduct three-tenths.
400	8.0	2.83	.66	Deduct full one-third.
500	10.0	3.18	.63	

Example. By Table A we see that with the rate of head of 50 feet per mile, a 30-inch pipe will supply a population of 91580; but with three times that rate of head, or 150 feet per mile, we see by column 3, Table B, that the same pipe will supply 1.73 times as many persons, or $91580 \times 1.73 = 158433$ persons. But if, at this greater rate of head, we still wish to supply only 91580 persons, then we find in column 4, Table B, that we may diminish the diameter of the pipe from 30, down to $30 \times .80 = 24$ inches; or, by column 5, we have $30 - 6 = 24$ inches.

Again, after the water has reached the city by the 30-inch pipe of Table A, if we wish to distribute it through the city by say eight branches or smaller mains, we see by column 6, Table A, that each of them must have at least 13⅛ inches diameter. From these eight, other smaller ones may branch off into the cross streets, alleys, &c; and in estimating the supply required for any particular street main, we must evidently add what is required also for such cross streets, &c, &c, as are to be fed from said main.

If certain limited parts of a city pipe system have considerably less rates of head than most of the remainder, it may become expedient to supply the former by a special separate main of larger diameter; which may start either directly

from the reservoir; or as a branch from the grand leading main which feeds the lower parts, according to circumstances.

It must be remembered, that although by increasing the diameters, an abundant supply may be obtained under a small rate of head, as well as under a great one, yet the water will not rise to as great a height in the service pipes for supplying the different stories of dwellings, &c. Even with the diameters in Table A, the water, under ordinary use, will not rise in these pipes to the full height of the surface of the reservoir; and if an unusual drawing-off is going on at the same time at many parts of the system, as in case of an extensive fire, or frequently during the hot summer months, it may not rise to even one-half of that height.

Art. 3. The following has been found very effective for preventing concretions in water pipes. Formerly in Boston, cast-iron city pipes, 4 inches diameter, became closed up in 7 years; and those of larger diameter became seriously reduced in the same time. But later, during 8 years, in which this varnish was used, no concretions formed.

Coal-pitch varnish to be applied to pipes and castings, made for the Water Department of Philadelphia, under the following conditions:

First. Every pipe must be thoroughly dressed and made clean, free from the earth or sand which clings to the iron in the moulds: hard brushes to be used in finishing the process to remove the loose dust.

Second. Every pipe must be entirely free from rust when the varnish is applied. If the pipe cannot be dipped immediately after being cleansed, the surface must be oiled with linseed oil to preserve it until it is ready to be dipped: no pipe to be dipped after rust has set in.

Third. The coal-tar pitch is made from coal tar, distilled until the naphtha is entirely removed, and the material deodorized. It should be distilled until it has about the consistency of wax. The mixture of five or six per cent of linseed oil is recommended. Pitch which becomes hard and brittle when cold, will not answer for this use.

Fourth. Pitch of the proper quality having been obtained, it must be carefully heated in a suitable vessel to a temperature of 300 degrees Fahrenheit, and must be maintained at not less than this temperature during the time of dipping. The material will thicken and deteriorate after a number of pipes have been dipped; fresh pitch must therefore be frequently added; and occasionally the vessel must be entirely emptied of its old contents, and refilled with fresh pitch; the refuse will be hard and brittle like common pitch.

Fifth. Every pipe must attain a temperature of 300 degrees Fahrenheit, before it is removed from the vessel of hot pitch. It may then be slowly removed and laid upon skids to drip.

All pipes of 20 inches diameter and upward, will require to remain at least thirty minutes in the hot fluid, to attain this temperature; probably more in cold weather.

Sixth. The application must be made to the satisfaction of the Chief Engineer of the Water Department; and the material be subject at all times to his examination, inspection, and rejection.

Seventh. Payment for coating the pipes will only be made on such pipes as are sound and sufficient according to the specifications, and are acceptable independent of the coating.

Eighth. No pipe to be dipped until the authorized inspector has examined it as to cleaning and rust; and subjected it thoroughly to the hammer proof. It may then be dipped, after which, it will be passed to the hydraulic press to meet the required water proof.

Ninth. The proper coating will be tough and tenacious when cold on the pipes, and not brittle or with any tendency to scale off. When the coating of any pipe has not been properly applied, and does not give satisfaction, whether from defect in material, tools, or manipulations, it shall not be paid for; if it scales off or shows a tendency that way, the pipe shall be cleansed inside before it can be recoated or be receivable as an ordinary pipe.

Art. 4. The pipes are laid to conform to the vert undulations of the street surfaces. The tops of the pipes are laid not less than 3½ feet below the surface of the street; but in 3-inch pipes the water has at times been frozen at that depth.

In Philada, in 1885, there are about 784 miles of street pipes; or about 1 mile to every 1100 inhabitants. The population is about 860,000; residing in about 150,000 dwellings.

No galvanic action has been observed where lead pipes or brass unite with cast-iron ones. **No pipe** less than 6 inches diam should be laid in cities: and even they only for lengths of a few hundred feet. Their insufficiency is chiefly felt in case of fire. 8 ins would be a better minimum. No more leakage occurs in winter than in summer; except from the bursting of *private service-pipes* by freezing.

To compact the earth thoroughly against the pipes excludes air, and greatly impedes rust. Pipes may be corroded by the leakage of gas through the body as well as through the joints of **adjacent gas-pipes.**

For thickness of metal pipes to resist safely the pressure of various heads, see p 233 of Hydrostatics.

WEIGHT OF CAST-IRON WATER-PIPES,

As used in Phila, and tested by hydraulic press before delivery to an internal pres of 300 lbs per sq inch. This table includes spigots, and faucets or bells. The pipes are required to be made of remelted strong tough gray pig iron. such as may be readily drilled and chipped; and all of more than 3 ins diam to be cast vertically, with the bell end down. Deviations of 5 per cent above or below the theoretical weights, are allowed for irregularities in casting, which it seems impossible to avoid.

The pipes are in lengths from 3 to 3½ ins longer than 12 ft; so that when laid they measure 12 ft from the mouth, *f*, Fig 38, of one bell to that of the next.

Diam.	Thickness.	Wt per length.	Diam.	Thickness.	Wt per length.	Diam.	Thickness.	Wt per length.
Ins.	Ins.	Lbs.	Ins.	Ins.	Lbs.	Ins.	Ins.	Lbs.
3	5/16	158	16	5/8	1322	36	15/16	4334
4	3/8	211	20	"	1654	36	1 1/16	4862
6	7/16	385	20	11/16	1798	36	1 3/16	5366
8	"	460	30	13/16	3313	48	1 1/8	7282
10	½	667	30	.9	3610	48	1 3/8	8667
12	9/16	899	30	1	3964	48	1½	9378

Price, Phila, 1886, about $35 to $45 per ton; or say 1½ to 2 cents per lb; depending on size and quantity ordered. Elbows, connections, &c, about 75 to 100 per cent more. In ordering anything by the ton, be careful to specify the number of lbs (2240). This prevents *misunderstandings.*

The following sizes **of lap-welded wrought-iron water-pipe** are made by the National Tube Works Co, McKeesport, Pa, and fitted with their **"Converse lock-joint."** One end of each length of pipe has the lock-joint permanently attached (leaded) to it at the works before shipping. The "weights per foot" include these joints. The weight of "lead per joint" given is that required to be poured in *laying* the pipe, or that for one side only of the joint.

Outer **diam,** ins.....	2	3	4	5	6	8	10	12	16
Weight per ft, lbs.....	1.86	3.48	5.26	7.33	8.76	13.20	17.08	25.12	47.70
Lead per joint, lbs.....	5/8	1 9/16	2 1/8	3¼	3 9/16	5¾	6	8¾	16
Average **car load:**									
Number of lengths.....	800	380	275	145	126	128	80	56	40
" " feet.....	11500	5600	4500	2600	2000	2000	1200	800	630

The pipes are tested for a bursting pressure of 500 lbs per square inch, or higher if desired. They are furnished either coated with asphaltum, or **"kalameined;"** or, if desired, first kalameined and then coated with asphaltum. Kalameining consists in "incorporating upon and into the body of the iron a non-corrosive metal alloy, largely composed of tin." The surface thus formed is not cracked by blows, or by bending the pipe, either hot or cold.

The joint, or coupling, is of cast-iron, and has internal recesses which receive and hold lugs on the outside of each length of pipe, near each of its ends. The joint is then poured with lead in the usual way (see next page), either with clay collars, or with a special pouring clamp furnished by the Co. This clamp resembles the "jointer," Figs 39 &c, except that it is in two rigid semi-circular pieces, connected together by a hinge-joint, and furnished with handles like those of a lemon-squeezer, and has a hole in one side for pouring. The coupling forms a flush inner surface with the pipe at the joint, thus avoiding much of the resistance of cast-iron pipes to flow. For cases where it may be necessary to make frequent changes, the couplings are made in two pieces, which are bolted together by flanges.

Wrought-iron, for pipes, has the great **advantages** over cast-iron of lightness, toughness, and pliability. The lightness of wrought-iron pipes renders them easier to handle, and cheaper *per foot* notwithstanding that their cost *per ton* is about 25 per cent greater. They are not liable to breakage in transportation or from rough handling, and they may be bent through angles up to about 25°. They therefore require no special bend castings for such angles. The National Co supply bending machines, to be worked by two men. One machine can, by changing the dies, be used in bending all sizes of pipe. The pipes are in lengths of from 15 to 18 feet, instead of 12 feet, as in the case of cast-iron, so that **fewer joints are required per mile.**

The Co furnish special "service clamps" and tapping machines for **attaching service pipes to mains.** This may be done (as in the case of the Payne machine, page 299) while the main is under pressure. The service clamp is a cast-iron saddle, which, before the main is tapped, is attached to it by means of a U bolt, and which remains permanently so attached after the tapping. A sheet-lead gasket is placed between clamp and main. The clamp has a tapped cylindrical opening through it, into which the corporation stop (see page 299) is screwed before the pipe is tapped. The drill of the tapping machine passes through the stop, and through the cylindrical opening in the clamp, and drills through the lead gasket and through the side of the main.

The Co furnish also pipe-cutting machines, and special castings (reducers, crosses, &c, &c) fitted with the Converse joint.

Art. 5. Wrought-iron pipes corrode much more rapidly than cast.

A gutta-percha pipe, ⅛ inch thick, and ¾ inch bore, has sustained safely an internal pres of more than 250 lbs per sq inch; equal to nearly 600 feet head. It merely swelled slightly at 337 lbs. In 1851 a tube of that material, 2½ ins bore, about ½ inch thick, and 1350 ft long, was sunk in the East River, New York, to carry the Croton water to Blackwell's Island. It was held down by weights. It proved unsatisfactory owing to abrasion caused by tidal currents, and injury from the anchors of dragging vessels. A wrapping of canvas, confined by spun yarn, was useful in preventing the former, but not the latter. This pipe was replaced in 1870 by wrought-iron pipes.

Ball's patent iron and cement pipe, is made by The Patent Water and Gas Pipe Co, of Jersey City, N. J. It is formed of riveted sheet-iron, and each length is dipped into, and coated with, a hot mixture of coal tar and asphalt. The lining of hydraulic cement is then applied. This ranges, in thickness, from ⅝ inch for 12-inch pipes to 1 inch for 20-inch pipe. This pipe is made up to diams of 36 ins. It is laid in a bed of cement mortar, and completely covered with the same. Suitable means are provided for making all the attachments, &c, required in city pipes for water and gas. More than 1300 miles of it are in use in various towns, some of it for 35 years; and it appears to give general satisfaction. Tubercles do not form in these pipes, as they are apt to do in cast-iron ones. There is every reason to suppose that they are durable. The trenches being dug, the Jersey City Co furnish pipes and lay them (including the cement).

The Wyckoff Pipe Co, Williamsport, Pa, make **wooden water pipes.** For pressures of 15 to 20 lbs per sq inch, they furnish either plain pipes, 3½ to 7 ins square externally, and from 1½ to 4 ins internal diam; or round pipes, 1 inch to 16 ins bore, coated externally with asphaltum cement. At their ends, both the square and the round pipes are banded with iron. For pressures from 40 to 160 lbs per sq inch, the round wooden pipes, before being coated with cement, are spirally wrapped, by steam power, with hoop iron, which is first passed through a preparation of coal-tar. The iron is wound so tightly as to be imbedded in the pipe, leaving its outer surface flush with that of the wood. The ends of each length of pipe receive extra banding. The asphaltum cement coating is then applied. These pipes have been extensively and successfully used for both water and gas. Suitable arrangements are provided for joints and connections.

Water pipes of bored oak and pine logs, laid in Philada 50 to 60 years ago, are frequently quite sound, and still fit for use, except where outer sap wood is decayed. When this is removed, many of these old pipes have been relaid in factories, &c. Clay *well packed* around wooden pipes, excludes the contact of air, and thus contributes greatly to their durability. Loose porous soils, such as gravel, &c, on the contrary, are unfavorable.

Pipes made of bituminized paper, prepared under great pressure, have been used for both water and gas. They are much less liable to break than cast-iron, and do not weigh or cost more than about half as much. Pipes of 5 ins bore and ½ inch thick, have resisted test strains of 220 lbs per sq inch; equal to a water head of 507 ft.

Art. 6. Fig 38 is a **standard form of pipe-joint,** Phila, 1886. The clear distance, *d*, between the spigot and the faucet, is nearly uniform for all sizes of pipe, varying only from $\frac{5}{16}$ inch for 4-inch pipe, to $\frac{7}{16}$ inch for 30-inch pipe. The depth, *m n*, of the faucet varies from 3 ins in 4-inch pipe, to 4 ins in 30-inch pipe.

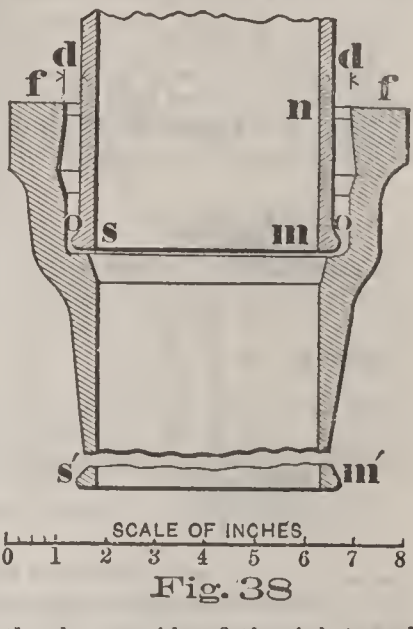


Fig. 38

The small beads at *s* and *m*, *s'* and *m'* on the spigot end of the pipe, project about $\frac{1}{4}$ inch; and are to prevent the calking material from entering the pipe. The calking consists of about 1 to 2 ins in depth of well-rammed, untarred gasket, or rope yarn; above which is poured melted lead, confined from spreading by means of clay plastered around the joint. The lead is afterwards compacted by a calking hammer.

The lead is poured through a hole left in the clay on the upper side of the pipe. In large pipes two additional holes are left in the clay, one at each side of the pipe, and lead is first poured into the side holes by two men at once, one man pouring into each side hole until the joint is half full. The side holes are then stopped, and, after the lead already poured has hardened, the two men finish the pouring by means of the top hole. This course is necessary, because the great weight of melted lead in the entire large joint would press away the clay at the lower side of the joint, and thus escape.

The moisture in the clay is liable to freeze in cold weather, and to render it too hard to be used. It is also liable, at all times, as is also any dampness in the pipe, to be converted into steam by the heat of the melted lead. The steam sometimes breaks out, or "blows" through the clay, allowing the lead to escape.

Art. 7. The Watkins patent "**Pipe Jointer**" avoids these difficulties by dispensing with the ring of clay. It consists of a ring R, Figs 39 and 40, of square

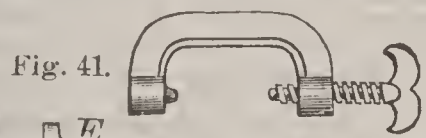


Fig. 41.

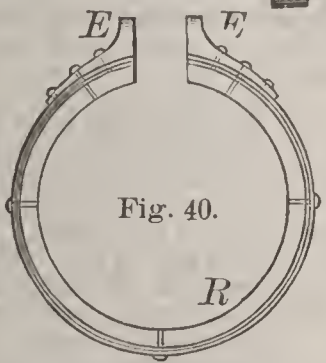


Fig. 40.

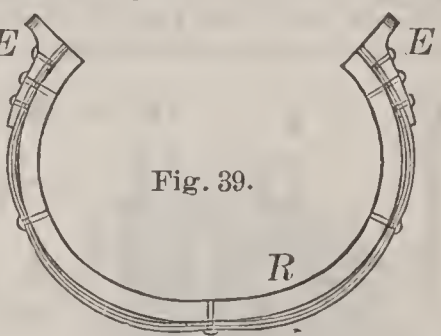


Fig. 39.

cross-section, and made of packing composed of alternate layers of hemp cloth and India rubber. This ring is encircled by one or more thin strips of spring steel, which are riveted to it at intervals, as shown. E E are iron-elbows riveted outside of the steel bands. After the gasket has been rammed into its place, the ring is placed around the spigot near the faucet, in the position shown in Fig 40, and is held loosely by the clamp, Fig 41, one point of which enters a small pit in each of the elbows, E E. The ring is then, by means of a hammer, driven close up against the end, *f*, of the faucet, Fig 38; the screw of the clamp is tightened somewhat, so as to bring the ring close to the spigot; a small dam of clay is placed in front of the aperture between the two elbows, E E; and the joint is ready for pouring. After the lead has hardened, the "jointer" is removed, and is ready for use at another joint. One can be used for several hundred joints. They of course dispense with the services of the men who prepare the clay collars, and supply them to the pourers. Upon the removal of the "jointer" the lead is found smooth, requiring no chipping, as it is apt to do when poured in the ordinary way. One of these jointers, for 4-inch pipe, costs, 1886, $4; for 8-inch pipe, $8, and so on, at $1 for each inch of diam. of pipe. Thos. Watkins, Patentee and Sole Manufacturer, Johnstown, Pa.

Art. 8. As a further preventive against the **escape of any of the gasket into the pipe,** Mr Chas. G. Darrach, Hydraulic Engineer, Phila, places a ring of lead pipe in the joint before the gasket is inserted. This lead pipe is of such diameter that it can just be pushed through the space, *d*, Fig 38, between the spigot and the faucet; and of such length as just to encircle the water-pipe. It is driven as closely as possible into the narrow annular space at *o o*, Fig 38. The gasket is then rammed in, and the lead poured, as usual.

Art. 9. John F. Ward's flexible joint for pipes laid across the irregular bottoms of streams, is shown at Fig 42. A portion, *a o*, of the inside of the faucet F, is bored out truly to form the middle zone of a sphere; and the spigot end, *e o*, of the other pipe is cast with two raised collars, *o* and *e*. The inner collar, *o*, is of such a height as barely to allow it to pass into the faucet. The outer one, *e*, is a little lower, so as to allow melted lead (shown black) to be poured in at *a*. The outer edge or diam of the spigot end at *o* is carefully turned so as to fit the turned spherical zone; so that the joint will admit of considerable play without danger of leaking. In laying the pipes under water, the joints are filled with melted lead, as usual, on board of suitable vessels or floats. As fast as they are thus filled, the floats are moved forward, and the pipes, if small, and the water shallow, are passed into the water without further care. But for large pipes in deep water, suitable apparatus is used for lowering them without undue strain on the joints. Mr Ward has been perfectly successful in laying this pipe under water, in one case 40 feet deep. One of the mains of the Philada water-works was thus laid across the Schuylkill River.

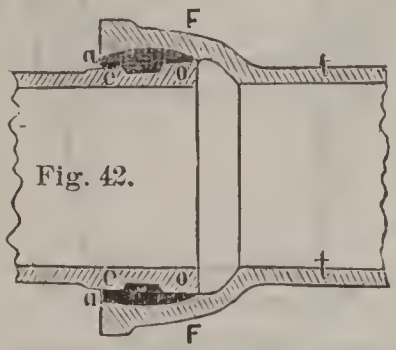
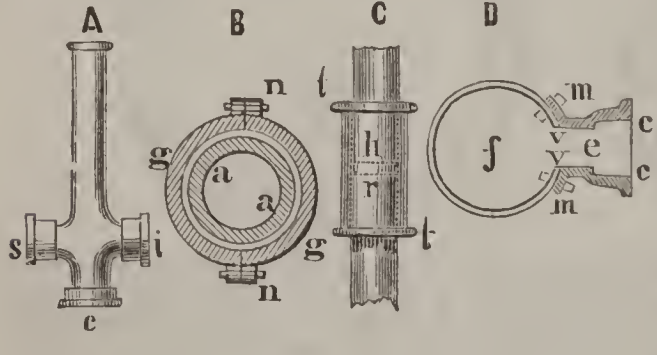


Fig. 42.

In some cases preliminary dredging may be expedient, to diminish abrupt irregularities of the bottom.

Art. 10. In Figs 43, A is a **double branch;** which is a pipe having, in addition to the faucet, *c*, at one end, two others, *s* and *i*, to which pipes leading in opposite directions (as at cross-streets) may be attached. If either *s* or *i* be omitted, the pipe is a **single branch.** The pipe is stronger when these extra faucets are near its end, than if they were at its middle. In a long line of pipes, for the sake of expedition, different gangs of men are frequently laying detached portions some distance apart; and when two ends of different portions are brought near enough together to be united, as *h* and *r*, Fig C, their junction cannot be effected by the usual spigot-and-faucet joint. In this case a cast-iron **sleeve,** *t t*, is used, which is first slid upon one of the pieces of pipe; and (after the other piece also is laid) is slid back into the position in the fig, so as to cover the joint. Sleeves are usually about a foot long; as thick as the pipe; and their diam is sufficient to allow the usual joint of gasket and lead. There is of course such a joint at each end of the sleeve.



Figs. 43.

Art. 11. When a crack occurs in a pipe, *a a*, Fig B, already in use, it is repaired by means of a cast-iron sleeve, *g g*, made in two parts, bolted together by means of flanges as at *n n*. In other respects it is like the preceding sleeve. The intermediate white ring is the lead joint. If the crack is too long, or otherwise too bad to be remedied by a sleeve, the pipe is broken to pieces; and the lead joints at its ends melted out, so as to allow of its removal. Then, since an entire new pipe cannot now be inserted, owing to the overlapping of the spigot-and-faucet ends, two short pieces must be substituted for it. One end of each of these is lead-jointed to the pipes already laid; while the other two ends, which will probably be a few inches apart, are covered by a sleeve, *t t*, Fig C.

Cracks may at times be temporarily repaired in an emergency, by a wrapping of folds of canvas thoroughly saturated with white-lead paint; and tightly confined to the pipe by a spiral banding of thin hoop-iron or wire. Or, by an iron band, made in two parts, B B, Fig 44, and clamped together by screw-bolts, S S. Such bands are useful, also, for strengthening pipes that are considered to be in danger of bursting.

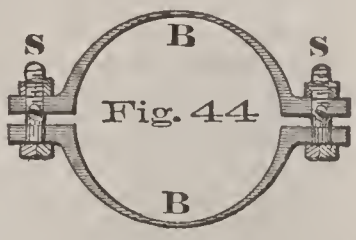


Fig. 44

To attach a pipe, *c*, Fig 43, to one, *f*, already in use, but in which no provision has been made for such attachment, a piece may be cut out of *f*, as at *v v*, and a casting, *e*, furnished with flanges, *m m*, bolted over the opening, by screw-bolts passing through female screws tapped in the thickness of the pipe. If the new pipe is so large that the opening, *v v*, if circular, would be inconveniently wide, it may be made *oval*, with the longest diameter in the direction of the *length* of the pipe, *f*. In that case the casting *e* will be oval at its flanges; and circular at *c c*.

Art. 12. The following table, arranged from data kindly furnished by the late Isaac Newton, C E, Ch Eng, Dept of Pub Wks, New York, and his Asst, Mr. James Duane, gives the **average prices, &c, of pipes and laying,** in that city, for three years prior to 1885. The pipes are in lengths of 12 ft, and cost $35 per 2000 pounds. Unpaving, digging trenches, re-filling and re-paving not included.

Diam of pipe in ins	6	12	20	36	48
Weight in lbs of pipe alone, per length	430	1000	2000	4860	8250
" " " " " " ft	36	83	167	405	688
Cost in $ of pipe alone, per length	7.52	17.50	35.00	85.05	144.38
" " " " " " ft	.63	1.46	2.92	7.09	12.03
Lead* per joint, lbs, at 5 cts per lb	13	22	41	80	146
" " " $	.65	1.10	2.05	4.00	7.30
Yarn per joint, lbs, at 9 cts per lb	1/8	1/4	1	2 1/2	5
" " " $	.01	.02	.09	.23	.45
Hemlock lumber per length, $	...	...	.40	.75	1.04
Coke, clay, &c, per joint, $	.02	.04	.08	.15	.25
Hauling per length, $	.40	.80	2.00	3.40	5.00
Labor per length, $	.60	.86	1.27	1.95	3.25
Cost, in $, of laying, exclusive of pipe, per length	1.68	2.82	5.89	10.48	17.65
" " " " " " " " " ft	.14	.24	.49	.87	1.47
Total cost, in $, of pipe and laying, per length	9.20	20.32	40.89	95.53	162.03
" " " " " " " " " ft	.77	1.69	3.41	7.96	13.50

Art. 13. Air valves. Air is apt to collect gradually at the high points of vert curves along the supply pipes; and, unless removed, produces more or less obstruction to the flow. This may be prevented by air valves, see Fig 44A, which is 1/8 of the full size of those once used in Philada. This simple device consists of a cast-iron box, *e e d d*, confined to the main pipe *m m*, by screw-bolts passing through its flange *d d*. It has a cover *g n g*, confined to it by screws *t t*; and at the top of which is an opening *n*, for the escape of air from within. In this box is a float *f*, which may be a close tin or copper vessel, or of layers of cork, as supposed in the fig; or &c. This float has a spindle or stem *s s*, fast to it; which passes through openings in the bridge-bars *a a*, and *o*; thereby allowing the float to rise and fall freely, but preventing it from moving sideways. When the pipe *m m* is empty, the float is down; its base *y* resting on the cross-bar *a a*. The stem *s s* has fixed to it a valve *v*, which rises and falls with it and the float. Suppose the pipe *m m* to be empty, and consequently the float, and the valve *v*, down. Then, if water be admitted into the pipe, it will rise and fill also the box as far up as *e*; and in doing so will lift the float *f*, and the valve *v*, to the position in the fig; thus preventing

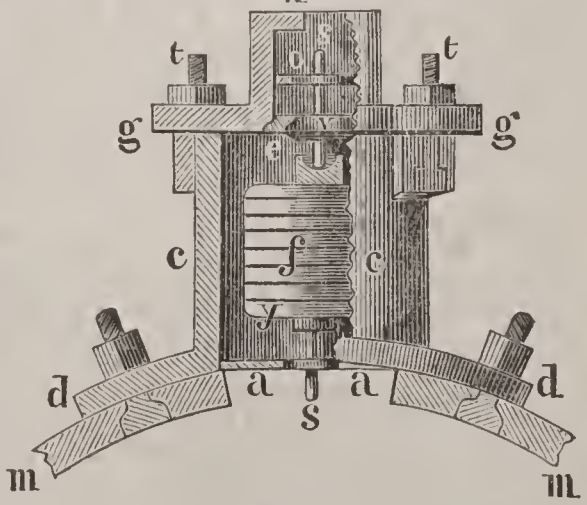


Fig. 44A.

* Mr Thomas Wicksteed, engineer of the East London water-works, England, says that more than 50 years' experience proves that slightly tapering wedges of pine, about 4 ins long, 2 ins wide, and 3/4 ins thick at the butt, carefully shaped to suit the curve of the pipe, and well driven, answer all the purposes of lead joints, at considerably less cost.

egress to the outer air, by closing the opening at *v*. Now, air carried along by the water, will, on account of its lightness, ascend to the highest points it meets with. Hence, when such air arrives under the opening *a a*, it will rise through it, and ascend to *e*; the closed valve preventing it from going farther. Thus successive portions of air ascend, and in time accumulate to such an extent as gradually to force much of the water downward out of the box. When this takes place, the float, which is held up only by the water, of course descends also; and in doing so, pulls down with it the valve *v*. The accumulated air then instantly escapes through the openings at *v* and *n*, into the atmosphere; and the water in the pipe *m m*, immediately ascends again into the box, carrying with it the float; and thus again closing the valve *v*. The valve, and the valve-seat *e*, are faced with brass, to avoid rust, and consequent bad fit. The whole is protected by an iron or wooden cover, reaching to the level of the street.

Air valves are no longer used in city pipes; their place being supplied by the fireplugs at average distances of about 150 yards apart. These, being placed as much as possible at the summits of undulations in the lines of pipes, for convenience of washing the streets, and being frequently opened for that purpose, permit also the escape of accumulated air.

The escape of compressed air through an air valve, or other opening, has been known to produce bursting of the main pipes; for the escape is instantaneous, and permits the columns of water in the pipes on both sides of the valve, to rush together with great forces, which arrest each other, and react against the pipes.

Air-Vessels. Motion is imparted to the water in a line of pipes, by the *forward* stroke of the piston of a single-acting pump; but during the *backward* stroke, this motion is stopped; and the water in the pipes comes to rest. Therefore, at the next forward stroke, all the water has to be again set in motion; and the force that must be exerted by the pump to do this is much greater than would be required if the motion previously imparted had been maintained during the time of the backstroke. The addition of an air-vessel secures this maintenance of motion, and thus effects a great saving of power; besides diminishing the danger of bursting the pipes at each forward stroke. It is merely a tall and strong air-tight iron box, usually cylindrical, strongly bolted on top of the pipes just beyond the pump, and communicating freely with them through an opening in its base. It is full of air. The forward stroke of the piston then forces water not only along the pipes, but also into the lower part of the air-vessel, through the opening in its base; thus compressing its contained air. But during the backstroke, this compressed air, being relieved from the pressure of the pump, expands; and in so doing presses upon the water in the pipes, and thus keeps it in motion until the next forward stroke; and so on. An air-vessel also acts as an *air-cushion;* permitting the piston to apply its force to the water in the pipes gradually: thus preserving both the pipes and the pump from violent shocks. The air in the vessel, however, becomes by degrees absorbed and taken away by the water; and its action as a regulator then ceases. To prevent this, fresh air must be forced into the vessel from time to time by a condenser, or forcing air-pump. A *double-acting* pump does not *so much* need an air-vessel. There is no particular rule for the size or capacity of air-vessels. In practice it appears to vary from about 5 to 50 times that of the pump; with a height equal to two or more times the diameter. A stand-pipe (see below) is sometimes used instead of an air-vessel.

A stand-pipe is sometimes used for the same purpose as an air-vessel (see above). It is a tall pipe, open to the air at top; and communicating freely at its foot with the water-pipe, in the same manner as in an air-vessel. Its top must be somewhat higher than that to which the pump has to force the water through the system of pipes; otherwise the water would be wasted by flowing over its top. The area of its transverse section should be *at least* equal to that of the pipe or pipes which conduct the water *from* it; but it is at times better to have it much larger, as a stand-pipe may then answer, especially in a small town, *as a reservoir*, if the pumping should cease for a few hours. A stand-pipe should be cylindrical, not conical; for if thick ice should form on top of the water in a conical one, a sudden forcing of it upward by the pump might strain the stand-pipe seriously. The stand-pipes connected with the Philadelphia Water-Works are from 125 to 170 feet high; 5 feet diameter; and made of riveted boiler-iron about $\frac{3}{8}$ inch thick near the base, and about $\frac{1}{4}$ inch near the top. They have no protection from the weather; nor are they braced in any manner; but retain their positions by their own inherent strength, although exposed at times to violent winds.

Art. 14. The service-pipes for supplying single dwellings, are of lead; and of ½ to ⅝ inch bore. They are connected with the street mains, *n n*, Fig 45, by a **brass ferrule,** *f*, here shown at ⅛ real size. The dotted lines show its ½ inch bore. The tapering ferrule is merely hard driven into a cylindrical hole reamed out of the main, as at *s*. The lead pipe, *o*, is attached to the other end of the ferrule; overlapping it about 1½ ins; and the joint soldered, *t*. The extra thickness near *f*, is for giving proper shape and strength for hammering the ferrule into the main. The pipe and solder are shown in section. Besides the stopcocks attached to each service-pipe, and to its branches through the house, there is an underground one by which the city authorities can stop off the water in case of delinquency in payment of dues; and another by which the plumber can stop it off when so required during indoor repairs. **Galvanized iron tubes** are being much used for service-pipes, especially for hot water; being less subject to contraction and expansion, which produce leaks. See near bottom of page 218, for such water pipes. Brass service-pipes are now much used in Boston. See bottom of page 218; also page 417.

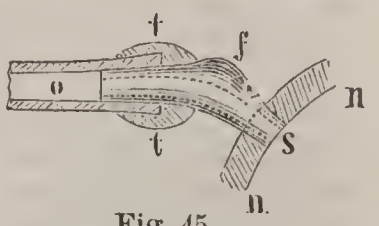


Fig. 45.

Art. 15. The so-called **"corporation stops"** or "corporation cocks" are inserted into the pipe by a special machine, Fig 46. Their great advantage over the ferrule, Fig 45, is that **they can be inserted into a pipe when the latter is full of water under pressure.** Besides, inasmuch as they are *screwed* into the pipe, they are in no danger of being forced out of it by any pressure within it. As their name implies, they are furnished with a stop-valve, which is kept closed while the valve is being inserted into the pipe, and is then opened, and generally remains open permanently.

Pipe-tapping machines, for drilling and tapping the pipe, and for attaching these stops, are made in a variety of forms. Fig 46 shows one made by **Walter S. Payne & Co.,** Fostoria, Ohio. Each of these machines is furnished with a number of malleable cast-iron saddles, which fit the various diams of pipe with which it is to be used. The saddle is not shown in the fig. It is fastened to the pipe by a chain slung around the latter The chain is tightened by a bolt and nut at each end.

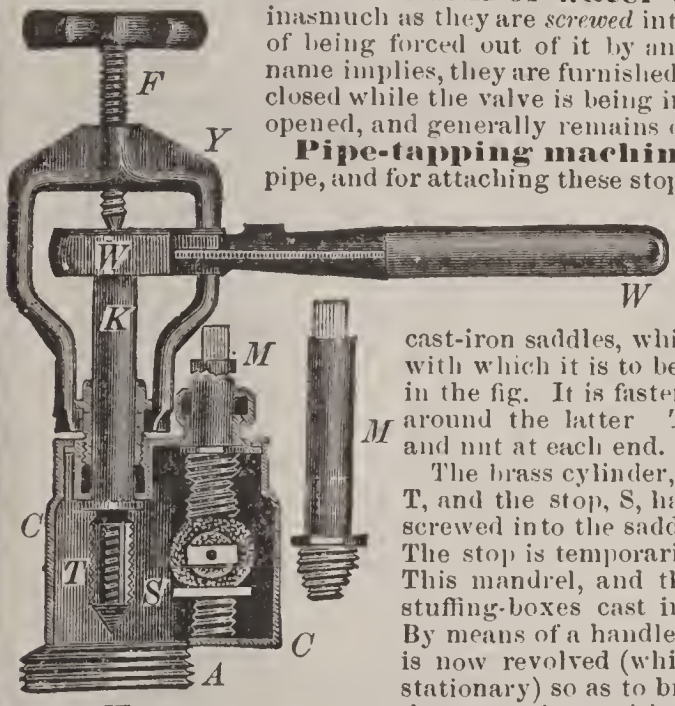


Fig. 46.

The brass cylinder, C C (into which a tap-and-drill, T, and the stop, S, have first been inserted), is then screwed into the saddle by means of the thread at A. The stop is temporarily screwed on to a mandrel, M. This mandrel, and the drill-shank, K, pass through stuffing-boxes cast in one with the head of the cyl. By means of a handle, not shown in the fig, this head is now revolved (while the body of the cyl remains stationary) so as to bring the drill, T, and stop, S, into the respective positions shown in the fig. When the cyl head comes to the proper position, it is stopped by a lug inside of the cyl. The drill is then immediately over the center of a large circular opening in the base of the cyl, C C, and over a similar opening, through the saddle, to the surface of the pipe to be tapped. It is then pushed down until it touches said pipe. The ratchet-wrench, W W, is then set on the square head of the drill-shank, K; the feeder-yoke, Y, with feed screw, F, is put in position as shown; and the pipe is drilled and tapped by working the wrench; whereupon the water in the pipe, if under pressure, rushes out through the hole thus made, and fills the cylinder.

By reversing the position of the switch on the ratchet in the wrench, W, and by working the latter, the tap is now withdrawn from the hole, but remains in the cyl. The cyl head is now revolved so as to reverse the positions of S and T; the lug inside of the cyl stopping the head when the *stop* is immediately over the hole. By means of the ratchet-wrench, applied to the square head of the mandrel, M, the stop, S (*the valve of which must be closed*), is now screwed into the hole, but only far enough to hold securely, and thus prevent the further escape of the water from the pipe when the machine is now removed. The stop is now screwed firmly into place by means of a wrench applied to a square on the stop itself. When the pres in the pipe exceeds about 200 lbs per sq inch, the feeding apparatus, as used with the *drill* (see fig), may be also used in aiding the insertion of the stop.

The mandrel, M, is made in two lengths (one of which screws into the other) in order that the upper part may be out of the way of the wrench-handle while drilling. It has three or more diff threads at its foot, to suit diff sizes of stop. Stops, made to suit the machine, are furnished as wanted.

The machine can work in any direction radial to the pipe, and can therefore be used for tapping a pipe in any part of its circumference.

After the stop is inserted, the service-pipe is attached to its outer end by a coupling nut passing over the thread there shown.

The machines are guaranteed to tap under a pressure of 600 lbs per square inch. They are made in four sizes. No 0 taps holes from ¼ to ½ inch diam, and weighs 15 lbs; No 1 from ¼ to 1 inch, 22 lbs; No 2, from ¼ to 1½ ins, 30 lbs; No 3, from ¼ to 2 ins, 35 lbs. Card prices, 1886, No 0, $75; No 1, $100; No 2, $125; No 3, $150. These prices are for the machines complete, including 3 taps and drills and 3 saddles, but exclusive of stops.

Other forms of pipe-tapping machines are the Boston, made by Whittier Machine Co., office, Granite and First Sts, Boston, Mass; the Lennox, by Lennox Foundry and Machine Works, Marshalltown, Iowa; Young's by the Easton (Pa) Brass Works; Letzkus', James H. Harlow, Eng'r, agent, Pittsburgh, Pa; Sperring's, Mueller's, and Hadesty's. Any of these can be had through the larger manufacturers and dealers in plumbers' supplies; as Haines, Jones & Cadbury, 1136 Ridge Ave, Phila; McCambridge & Co., 527 Cherry St, Phila; Chas Perkes, 627 Arch St, Phila.

Art. 16. Stop-valves, or **gates,** opening vertically in grooves, are placed across the street pipes at intervals of from 100 to 300 yds. Their use is to shut off the water from any section during repairs; the water of such sections being allowed to run to waste, and to soak into the ground.

The details are much varied by diff makers.

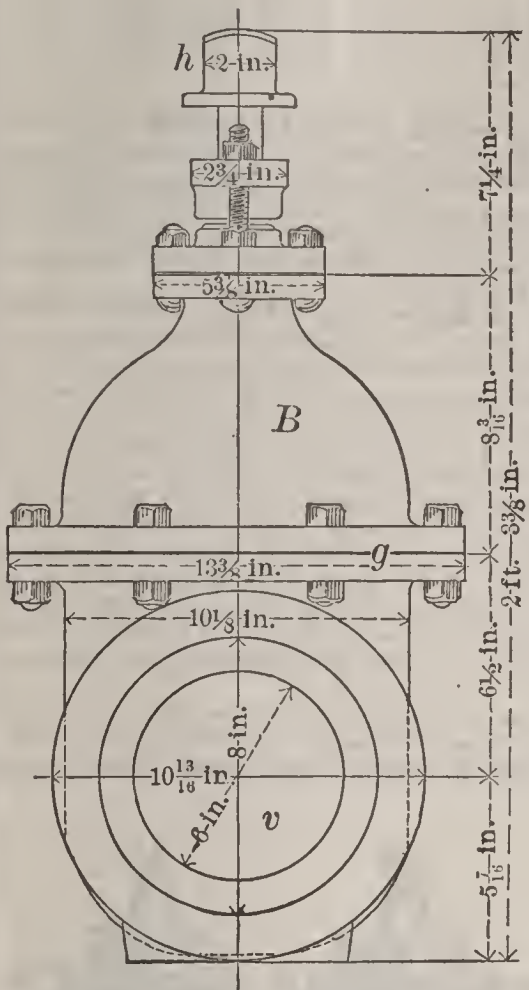


Fig. 47.

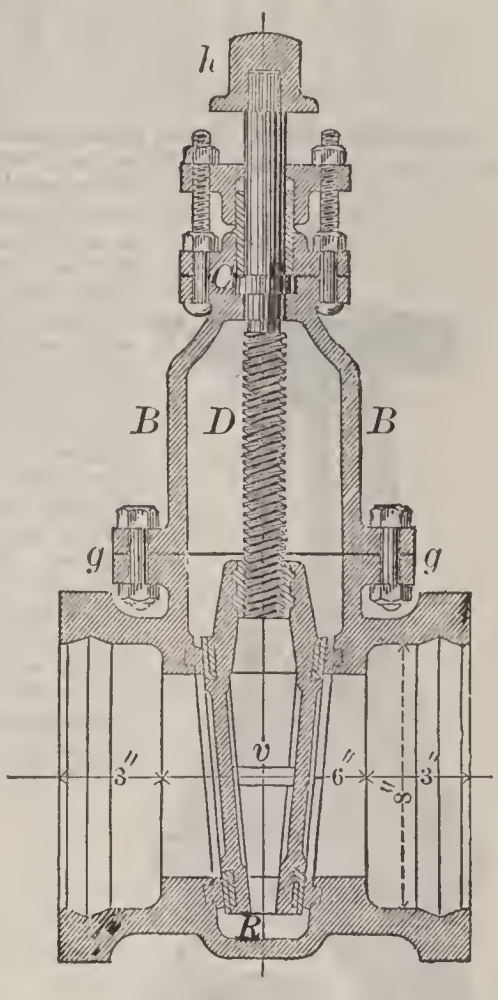


Fig. 48.

Figs 47 and 48 show such a gate made by Chapman Valve Mfg Co, Indian Orchard, Mass. The valve, *v*, is cast in one piece. When down, as in the figs, it closes the pipe. As in other styles, it opens vert by means of a screw, D, the valve rising into the cast-iron case or box, B B, and leaving, when all the way up, an opening of the full diam of the pipe. The screw is turned by a wrench fitting on its square head, *h*. The screw, D, itself, is prevented from moving vert by the collar, C.

The two principal castings which compose the box or cover are bolted together by means of flanges, *g*. The joint faces of the castings are carefully smoothed; and a thin strip of lead is inserted between them, as a precaution against leaks. The recess, R, admits small particles of foreign matter which might otherwise prevent the gate from closing perfectly. The valve seats are faced with Babbitt metal. At the top of the cover, the screw stem passes through a stuffing-box, which prevents leaking at that point. Very careful workmanship is required throughout.

The following are the **weights** and **approx prices** (1886) of these valves. They give a tolerable average of the wts and prices of similar gates by other first-class makers, among whom are Isaac S. Cassin, 2d St and Germantown Ave, Phila; Ludlow Valve Mfg Co, 938 River St, Troy, N Y; and Whittier Mach Co, office, Granite and First Sts, Boston. The latter make Coffin's patent double-disk valve, besides other patterns, fire hydrants, locomotive and stationary engines, dredging machines, iron bridges, and heavy tools and machinery in general.

Chapman Bell-end Water-gates.

Bore. ins.	Wt. lbs.	List price.* $	Bore. ins.	Wt. lbs.	List price.* $	Bore. ins.	Wt. lbs.	List price.* $	Bore. ins	Wt. lbs.	List price.* $
2	32	10	6	195	30½	12	600	82	20	1700	230
3	55	15	7	245	36	14	843	112	24	2750	333
4	116	19	8	290	45	16	1080	150	30†	6400	709
5	135	25	10	439	62	18	1475	194	36†	8300	1200

Art. 17. Fig 49 shows an arrangement **with outside screw** for raising and lowering the valve. Here the screw, D, does not revolve, but is attached to the valve, and rises and falls with it, being raised or lowered by turning the wheel, W, at the center of which is a nut through which the screw passes. The nut is fixed in the wheel, and is so confined that it, and the wheel, cannot move vertically.

Fig. 49

Art. 18. A four-way stop, or **four-way valve.** Figs 50 and 51, is placed at the intersection of two mains; the four ends of which are attached, respectively, to the four openings, M M M M. At the bottom is an additional opening, connecting, by means of an elbow, H, with pipes running to a fire-hydrant at the street curb. See Arts 20 and 21. Two or more of such bottom openings may be made, if desired, for the supply of as many fire-hydrants. All of the openings are opened or closed at one time by raising or lowering the valve or plug, P, by means of a wrench or key applied to the square head, S, of the screw stem. As in Figs 47 and 48, the screw turns, but is prevented from rising and falling, and the plug moves up and down on the screw.

Inasmuch as all sediment escapes into the bottom opening which leads to the fire-hydrant, the valve is not liable to clogging through this cause. The fire-hydrant, being fed from both of the mains, obtains a fuller supply than would be possible if it were fed, as usual, through only one main.

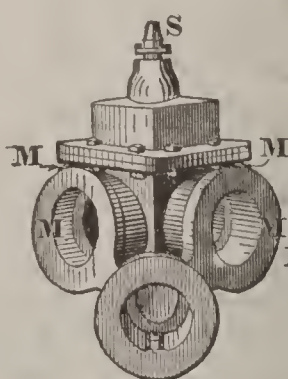


Fig. 50

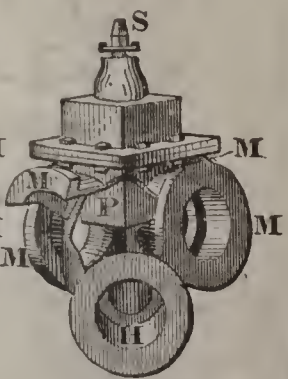


Fig. 51

Figs 50 and 51 represent Viney's four-way valve, made by Keystone Valve Co, office 1510 Brown St Philadelphia. The plug, P, is a hollow iron casting, in the shape of a truncated four-sided pyramid. Each of its sloping sides is faced with brass; and the seats are of white metal. For prices and discounts, address as above. The weights are as follows: for 4 inch pipes, 170 lbs; 6 inch, 395 lbs; 8 inch, 495 lbs; 10 inch, 700 lbs; 12 inch, 1000 lbs. The Co. also makes *three*-way valves.

* **Discount,** 1886, about 35 per cent.

† Furnished with gearing to aid in opening and closing the valve. The 20 inch and 24 inch gates can also be furnished with such gearing when desired. This increases the weight about one eighth, and the cost about one fourth.

Art. 19. Whatever the style of the gate may be, it is, when attached to the pipe, **protected by a surrounding box,** generally of plank or cast-iron, with four sides, which taper so that the box is of smaller hor section at top than at bottom. It is open at bottom, but has a movable iron top, level with the street. This top is taken off when the valve is to be opened or closed, or inspected. Two of the opposite sides of the box of course have openings for the passage of the pipes to or from the valve.

The gates, especially of large mains, must be closed very slowly. Otherwise, the too sudden arresting of the momentum of the flowing water would be apt to break either them or the covers; or burst the pipes. As a precaution against this, the covers for very large valves are cast with outside strengthening ribs.

No self-acting air-valves (Fig 44 A) are now placed at street summits, to allow confined air to escape. The fire-plugs answer instead. The rad for hor bends in mains is if possible not less than about 12 times their diams; they are made as large as the widths of the streets will admit; usually about 50 ft. **Fire-plugs,** Figs 52, &c, are placed as much as possible at summits, so as to serve also for washing the streets; and for the escape of accumulated air. They average about 8 in number to each mile of pipe; or 1 to each block of buildings.

Art. 20. Fig 52 represents a common street **fire-plug, or fire-hydrant,** as made by Gloucester Iron Works, office, No 6 N 7th St, Phila, and used in that city.

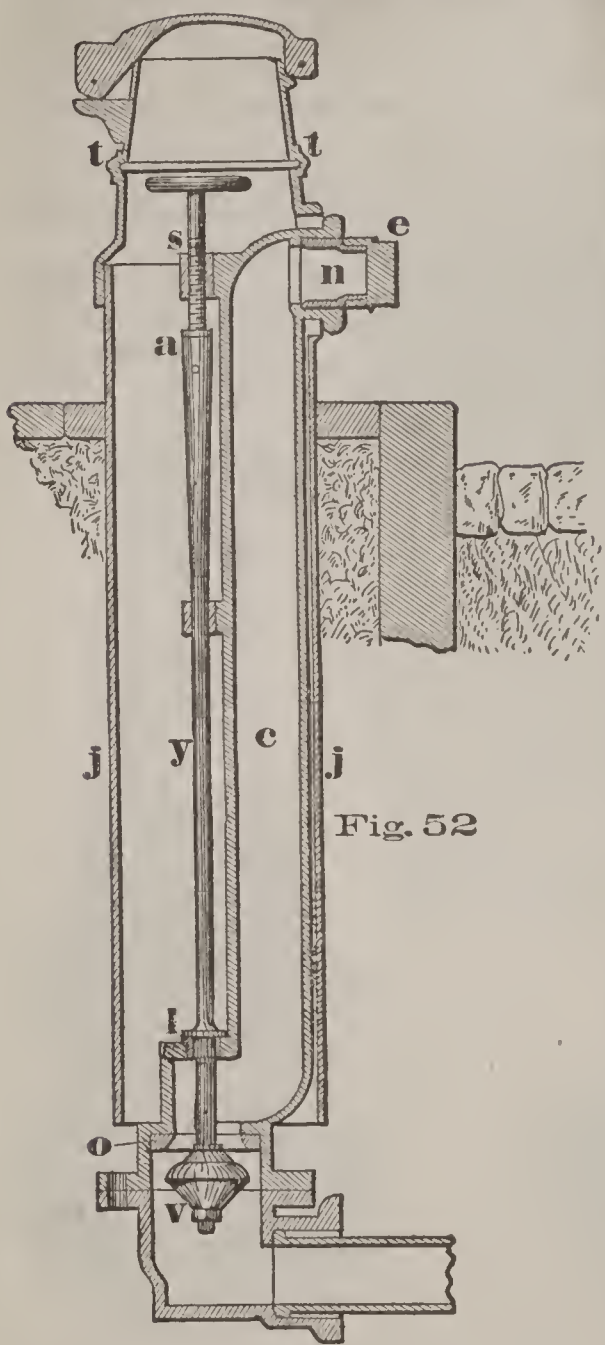


Fig. 52

The valve, *v*, is of layers of well-hammered sole-leather; and, when closed, shuts against a brass ring seat, *o*, which is confined to its place by a lead joint. The valve is opened by working down the screw, *s*, which, by means of a swivel joint at *a*, can revolve without turning the valve-rod, *y*. When the valve, *v*, is closed, after the plug has been in use, the chamber, *c*, is full of water, which, if allowed to remain, would be in danger of freezing, and of bursting the plug. But, in closing the valve, we raise the flange, *l*, on the rod, and thus allow the water to escape through the opening at *l*, whence it runs to waste into the ground, through the open lower end of the **frost-jacket,** *j j*, which is a hollow cast-iron cylinder surrounding the working parts of the hydrant. Being free to slide vert it rises and falls when the level of the ground is disturbed by frost, and the hydrant is thus protected against injury from this cause.

The top, *t*, of the hydrant case, is cast in one piece with the chamber, *c*.

The stopper, *e*, screws on over the nozzle, *n*.

Art. 21. In the **Chapman fire-hydrant,** Figs 53, 54, and 55, made by the Chapman Valve Mfg Co, Indian Orchard, Mass, **the valve, v v, is a *sliding* one.** The stem, *y*, Fig 53, to which the screw, *s*, is attached, is, like that in Figs 47 and 48, prevented from moving *vert* by a collar, fast to it near its top, and confined in a circular groove. When the rod and screw are made to revolve, by means of a wrench applied to the square head of the former, the valve slides up or down on the screw, admitting the water to, or shutting it off from, the hydrant. The valve slides on the two guides, *g g*, which are cast in one piece, respectively, with the two *vert* sides of the lower part of the hydrant. Its circular face, where it comes into contact with the hydrant case, *h*, is faced with a gun-metal ring, *e e*, which bears against a similar ring, made of Babbitt metal, let into the hydrant case.

The water, left in the hydrant case after closing the valve, **escapes** through a cylindrical hole, *d*, about ⅜ inch diam, bored through the guide, *g*, and case, *h*. This hole is at such a ht as to be just above the top of the loose plate, *p*, when the valve is closed, as in Fig 53. This plate lies in a vert groove in the side of the valve-casting, and is pressed against the side of the case, *h*, by two spiral springs confined in cavities, *c c*, in the valve casting. When the valve begins to rise, the hole, *d*, is closed by the plate, *p*, and remains so until the valve is again entirely closed.

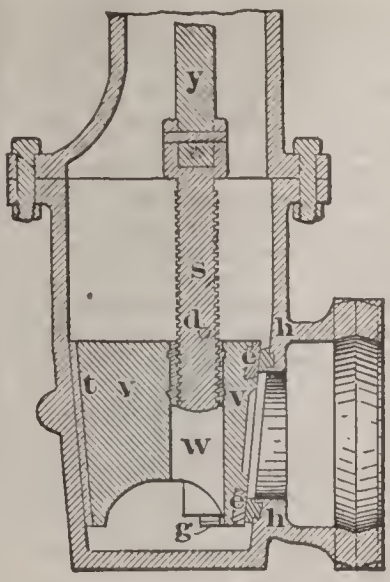


Fig.53

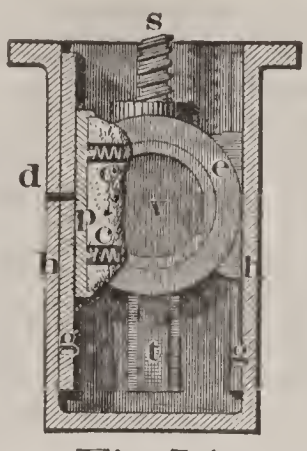


Fig. 54

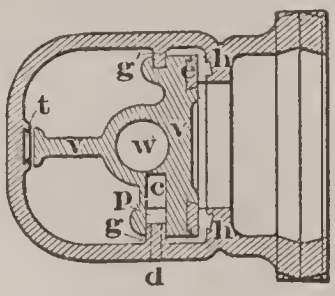


Fig.55

MECHANICS. FORCE IN RIGID BODIES.

Art. 1. Mechanics (the very foundation of civil engineering) is that branch of science which treats of the effects of force upon matter. This broad application of the term necessarily includes hydrostatics, pneumatics, &c; but ordinarily it is restricted to the effects produced upon matter by the application of *outward, extraneous*, or *mechanical* force, from whatever source it may be derived; whether from steam, water, air, wind, gravity, animals, &c. Some of these effects consist in either deranging, or separating the particles of matter which compose a body, by pushing them close together; or by pulling them farther apart; and in all such cases the extraneous force is to be considered only as acting against the natural inherent forces, or *strength* of the materials of which the bodies consist. This class of effects

is comprised under the distinct head of Strength of Materials. Another and very important class refers only to the action of force upon *entire* bodies; either to *move* them; or *to keep them stationary*. Force, when it moves bodies, is called **Dynamic** force; and when it keeps them stationary, **Static.** Hence, we have the sciences of **Statics** and **Dynamics**; which imply merely the effects of force as to giving motion to bodies; or to keeping them at rest.

In examining the abstract static and dynamic effects of force thus applied to *entire* bodies, we of course have to assume that it does not break, bend, penetrate, or alter *in any way*, the shape of the body. The body is to be regarded merely as something that *receives the force*; but is in no way affected thereby, further than that it *moves*, or *stands (or rests) when the force does so.* That is, the forces are assumed to act upon *each other* only, when there are more than one; the body being only the field of their action. This assumption is of course not strictly true in any case, because no body is so perfectly rigid as not to have its shape somewhat changed by the application of force; still, it will be seen, as we proceed, that it is not liable to produce error. Thus, it will be evident that the illustrations of force in the following pages would not apply to walls, arches, beams, &c, of snow, cotton, loose sand, or soft clay; and in the same manner, although to a less extent, would they be influenced by the assumption of any yielding whatever of the particles composing bodies of metal, stone, or wood. Such yielding has of course to be taken into consideration in nearly all cases *in practice*; but it must be done by a totally distinct process, under the head of Strength of Materials. The two processes do not interfere with each other.

It is so absolutely essential in the study of Statics and Dynamics, and in reading the following articles, to keep this assumption constantly in mind, that we repeat it; namely, that the force is to be considered only as acting upon *entire* bodies; or upon bodies *as a whole*; incapable of being broken, bent, or affected by it in any manner whatever, other than in merely being moved; or kept stationary by it. For instance, if we wish to ascertain the effects of a given force *f h* to overturn or upset a stone, S, p 337, around one of its edges, *n*, we suppose the stone to remain whole. But if we wish to know whether the edge *n* will be in danger of being fractured when the whole weight of the stone comes upon it during the process of upsetting, we resort to the crushing strength of stone. It is plain, that a body when pushed, or pulled in several directions at once, may not *as a whole*, have the slightest tendency to move in one direction rather than another; yet some of its *particles* must tend to move in one direction; and others in others. Statics and Dynamics regard the body only in the first point of view.

Rem 1. When not otherwise stated, or apparent, the force in many of the following examples, is supposed to be imparted to the body in a direction passing through its cen of grav: so as to move, pull, or press it, without tending to make it revolve, or upset. By examining first the action of forces imparted in that manner, certain elementary principles become much more easy of comprehension.

Rem. 2. If the direction of the imparted force does not pass through the cen of grav of a body free to move, the body will still move forward in that direction just as far as before; but while so doing, it will also revolve around its cen of grav to the same extent as if it did not move forward at all, and as it would around a fixed axis, and under the full action of the force.

Rem 3. Lastly, when not otherwise stated, the force is supposed to be applied to the rigid body in a direction at right angles to its surf at the point where it is applied; otherwise (except as per Art 19) only a part of the force will be *imparted* to the body; that is, will *be put into* it, or *enter* it; and *produce an effect* upon it.

Art. 2. Matter is any substance whatever, as metal, stone, wood, water, air, steam, gas, &c. **A body** is any piece of matter, as a stone, a house, a mountain, &c. **The weight** of a body is the amount of *vert* pres, or pull, which the force of gravity in that body exerts upon it. In ordinary practical mechanics, the **quantity** of matter in a body is measured by its wt; for a method which is the only theoretically correct one, but which is not adapted to popular use, see note to p. 317. **Motion** is change of place, or of position. Since all bodies are constantly in motion in consequence of the revolution of the earth, the ordinary use of the word here refers to the motion produced by the extraneous force under consideration at the time.

The base of a body, as ordinarily understood, is that part of it which is underneath, and upon which it stands when acted upon by its own wt, or by the wt of other bodies also, which may be placed upon it. But in fact, a base may be at any part whatever of a body; at its top, on one side, &c; as when we press a block of wood upward against the ceiling of a room; or horizontally against a wall; the part which touches the ceiling, or wall, is the base.

Strain is the effect produced by the action of *equal* forces or of equal parts of unequal forces, *against each other*; or in *opposite* directions. It may consequently be applied to the effect which these equal parts or wholes produce among the *particles* of the body upon which they act: as tending either to push them closer together; or to pull them farther apart; in other words, to crush, or to rend the body. But as before stated, this view of strain belongs, not to pure Statics and Dynamics, but to Strength of Materials. Again, it may refer to the action of the forces *upon each other*; or, as usually stated, *upon the body as a whole.*

Thus, if two men pull or push, with equal forces, a body that is between them, their forces merely react against, counterbalance, or destroy *each other;* but they produce no effect whatever upon the body *as a whole; it* remains at rest; and has no more tendency to move in any direction, than if both forces were absent. The forces then merely strain (pull or push against, without moving) each other, and keep each other at rest; but no practical error will arise from adopting the common phraseology, and saying they act upon the *body* to keep *it* at rest.

Strain is measured by weight: as by pounds, tons, &c. Its amount or quantity is equal to that of only one of the two equal opposing forces. Thus, if two men pull against each other at two ends of a rope, each with a force of 30 lbs, the strain on the rope is but 30 lbs; and it is equal throughout the length of the rope. The two 30 lb forces strain each other, 30 lbs; as is made manifest if a spring-balance is inserted at any part of the rope. If a rope passes over a pulley, and equal wts be suspended at each end of it, then the two equal forces of gravity of the two wts strain against each other; and also strain the rope; to an amount equal to one of them.

For more on Strain, see Art 13.

Two equal opposing forces produce strain and a tendency to motion, among the *particles* which compose a body to which they are imparted; but exert no tendency to motion upon the body *as a whole;* because the particles are held together by their internal cohesive force; and this cohesive force reacts against our extraneous imparted forces. If it is not sufficient to do so completely, the body is broken.

Art. 3. Force is that principle of which, considered simply as a mechanical agent, we know but little more than that when it is *imparted*, that is, *put into*, a body, it produces either motion alone; or strain, with or without motion. This is all that force, mechanically considered, can do under any circumstances whatever. When it produces motion alone, it is called *moving* force. When it produces strain alone, or without motion, *straining* force; or simply strain, pull, push, tension, or pres, as the case may be. As, for instance, when a body rests quietly on a table, or on a post, &c; or is suspended by a chain, rope, &c; its effect is simply strain; a push or pres on the post or table, and a pull or tension, on the rope. There can be no strain except where there are at least two forces to strain against each other. When it produces strain and motion at the same time; or in other words, when a part of it produces strain; and a part of it, motion; it is called *working* force; and its effect is *work*. Thus, when a man is lifting a wt, a part of his force is straining against the equal force of gravity of the wt; while the other part of his force is giving motion to the wt; so that all his force combined, is at work; or is working force; or performs work. See Art 11.

These are not different *kinds* of force; but diff *effects* of the same force; for there are no diff kinds of mechanical force; it is all the same: no matter from what source it may be derived. Indeed, we might generally, and without fear of being misunderstood, call it simply motion, work, or strain, according to which of the three effects it is producing at the time; and we shall frequently do so in the following pages. The only means we have of measuring it, is by measuring the quantity of these its three effects.

Nothing but force can resist force. When all the force imparted, or put into, a body, is unresisted by opposing force; it produces motion only; when it is all resisted, it produces strain only; when a part of it is unresisted, and a part resisted, the first part produces motion; and the last part strain; which two combined constitute *work*. A single force can produce motion; but at least two are required to produce strain. See Art. 13.

Force is sometimes defined to be that principle which either *does* produce, or *tends* to produce motion: or which either *does* prevent, or *tends* to prevent it.

Rem 1. We infer from both reason and observation, that force, when once imparted, or put into a body, would, if not resisted by other force, remain in it, and move it forever, in a straight line: in the direction in which the force was imparted; and at a uniform velocity, or rate of speed.

For we observe in practice, that motion continues longer in proportion as we remove resisting forces; hence we infer, that if all resistances could be removed, it would continue forever. But in practice we cannot remove all resisting forces, such as those of gravity, wt, friction, the air, &c. These strain against, or resist the forces which we may impart; so that with the exception of a body falling by the force of gravity, in a vacuum, it is difficult to mention a case of motion alone, without strain.

Rem 2. **Matter, in itself, cannot resist** force. See Arts 5 and 11. When force produces motion alone, without strain, it is often called *Dynamic* force; when

continuous strain without motion. *Static:* when a sudden strain for an instant only, as a blow of a hammer, *Impulsive;* and its action, *an impulse.* See Art 10, p 310.

Rem 3. The **Living Force,** or **vis viva** of scientific writers, is nothing more than an expression referring to the quantity of work (motion and strain combined) which the force in a body at any given instant, could perform, if left to itself, without afterward receiving any additional force. See Art. 24.

Art. 4. When force is imparted in any direction which passes through the cen of grav of a homogeneous rigid body, *perfectly free to move,* it quickly diffuses itself equally through every part; and gives to every atom composing the body, the same tendency to move onward in the same direction.

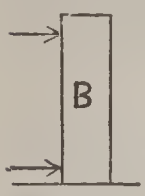


Fig 1

But although this diffusion takes place *rapidly,* especially in compact bodies, still it requires *some* time; generally so short as not to be appreciable without close observation. If we put the body B, Fig 1, into motion by striking it a blow near the top or bottom, the struck end will for an instant move faster than the other; after which both will move with equal velocity; because the great force which for an infinitely short time acted upon the top only, becomes equally diffused throughout the body. We are not, however, now treating of cases of this kind, in which the direction of the force does not pass through the cen of grav of the body; for the body may then rotate, or whirl arouud as it moves forward.

This equal diffusion of force is owing to the inherent cohesive force of bodies, which holds their atoms together, somewhat as lime holds together the grains of sand in a piece of mortar; and will not permit one grain to move unless the others move with it. In producing this diffusion, the cohesive forces, or the lime, do not *diminish the quantity* of the imparted force; but merely cause that instead of continuing to give a great velocity to that part only at which it was imparted, it shall give a slower, but equal one, to every part.

But if the rigid body to which we impart force in any direction, *is not free to move,* being prevented from so doing by opposing force, then our imparted force strains against the opposing one; and thus produces *strain,* instead of *motion,* in the body.

Art. 5. We must clearly distinguish between merely moving a body, and lifting it. The smallest imaginable force would, if unresisted by other force, *move* the greatest imaginable body; but if we apply force to lift a body, then its wt, or force of gravity, resists. Force applied merely to move things without lifting them, is often called traction: as in the case of horses or men towing a canal boat, or hauling a vehicle along *level* ground. The same word, however, is generally used for hauling, even when the thing is partially lifted; as when a horse is hauling up hill; in which case he is lifting both himself and the vehicle with its load. See Traction.

Matter always *appears* to resist force; but this deception is caused by its gravity, or by friction, or some other resisting force acting upon it at the same instant.

A man cannot *lift* a wt of 20 tons; but if it be placed hor upon proper friction rollers, he can *move* it, as we see in some drawbridges, turntables, &c; and if friction and the air could be entirely removed, he could move it by a single breath; and it would continue to move forever. It would, however, move very slowly, because the force of the single breath would have to diffuse itself among 20 tons of matter. He can move it, if it be placed in a suitable vessel in water, or if suspended from a long rope. A powerful locomotive that may move 2000 tons, cannot lift 10 tons vertically.

If we imagine two bodies, each as large and heavy as the earth, to be precisely balanced in a pair of scales without friction, it is plain that since all of their two opposing gravities strain against each other, and thus neutralize each other's motions, a single grain of sand added to either scale-pan, would give motion to both bodies.

The inherent cohesive force of matter, by which its particles are held in close union, frequently causes the matter itself to appear to resist straining force; thus, a cake of ice may sustain a great pres; but if we destroy its cohesive force by converting it into water, it will yield readily. So with the metals and stones if reduced to dust. It is not the *material* that resists being broken; but the inherent *cohesive force* which holds the particles of the material firmly together.

The fact that matter in itself opposes no resistance to force, is called its inertia, or inertness; but the term "inertia" is often used to denote a supposed *resistance* exerted by matter itself against force. We see no reason for assuming the existence of such a resisting property in matter, and we prefer to apply to "inertia" its literal definition of inertness or the *absence* of all tendency either to move or to resist moving force.

In most kinds of machinery, nearly all the parts are supported by either pivots

or gudgeons, which resist the action of gravity of the parts; so that the working force, or power, has only to overcome the *friction;* and not to lift the *weight* of the parts. The expression, "**overcoming the inertia of a body.**" is founded upon radical error; for there is no such resistance as inertia to be overcome. Inertia is the name of a *fact;* not of a *force;* and that fact is simply, that matter does not resist force. What is called overcoming inertia, is simply *putting in force;* as when we pour water into a glass, we do not overcome any resistance of the glass against being filled; we simply apply the water to the glass, which receives it, and retains it; until, by applying force, we take it out again. "Overcoming the inertia" of a railroad train, &c, therefore, means nothing more than that if we wish the train to move, we must put moving force into it. It is inert, and cannot move of itself.

Art. 6. Care must be taken not to confound motion with vel. **Velocity is the rate of speed** of motion; or, in other words, it is the *dist* moved over *in a given time*, without any regard to the wt of the body. Thus we say that it moves with a vel, or at the rate, of 10 ft per sec; 20 miles per hour, &c; whether it weighs an ounce or a ton.

Art. 7. We impart force to a body, by **applying** to it (that is, by bringing into close contact with it) a second body in which the force already is. We must discriminate between *applied*, and *imparted* force; and the precaution is the more necessary because writers generally appear not to recognize the distinction.

Applied force is merely carried *to* a body; imparted force is put *into* it. A heavy body containing great force may be applied to a small body containing none; and yet the former may impart but little of its force to the latter. If we place a body on, that is, apply it to, a steep inclined plane, we apply also the wt of the body, or its force of gravity: yet only a portion of this force is imparted, or put into the plane; while another portion remains in the body; and being unresisted by the plane, produces the motion of sliding down it. Force may be applied to a body in any direction whatever; but can be imparted to it only at right angles to the surface, except as explained Art 19.

That point of a body to which force is applied, is called the *point of application;* but in fact we cannot in practice apply force to a *point*, according to the scientific meaning of that word; but have to apply it distributed over an appreciable area (sometimes very large) of the surface of the body; still, as explained in Art 57, it may, as regards its action upon the *entire mass* of a *rigid* body, be considered as applied at one point. The expression, *direction of a force*, refers to the direction in which it is *applied*.

Art. 8. When diff forces act upon a body, it is absolutely essential, in considering their effects upon it, to know whether they all act **in the same plane;** for if they do not, their effects become totally diff.

A flat piece of paper is a plane, and if on it we draw any number of straight lines, in any direction whatever, they will represent so many forces all acting in that same plane; that is, the same flat surface coincides with the directions of all of them. It will evidently do the same in whatever position this plane surf may be placed, whether hor, vert, or inclined. Straight lines drawn on the floor of a room, will represent forces in *that* same plane: lines on the ceiling, forces in *that* same plane; which of course is not the same plane of those on the floor; so with lines on the sides of the room. All the lines *o t*, *i t*, *a t*, *c t*, Fig 2, are in the same plane *t o i c*. Although *t* is in the plane *t o i c*; and *t e* at the same time in the plane *t c g e*, and in the plane *t o n e*; and *s e* in the plane *s n e g*; and *t s* in the plane *i t e s*, still all of these, namely, *i t*, *t e*, *s e*, and *t s*, are evidently in the same plane *i t e s*. Any two lines which meet, or would meet or intersect each other if sufficiently extended in either direction, are in the same plane; as *o t*, *i t*; or *s t*, *i t*; or *t s*, *e s*. Still two lines may be in the same plane, and yet not meet if extended; as, for instance, the *parallel* lines *c t*, *g e*, in the plane *t c g e*. The lines *a t* and *g e*, being in parallel planes, *o i c t* and *n s e g*, cannot meet if extended.

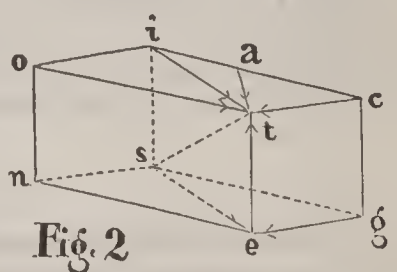


Fig. 2

Rem. We must not confound acting *in*, with acting *on*, *upon*, or *against* the same plane. The floor of a room is a plane; and *upon* or *against* that same plane, forces *in* a thousand diff planes may act. The distinction is so self-evident, that a bare allusion to it will prevent mistake. See Fig 53½, Art 55.

Art. 9. The **quantity.** or **amount. of any motion.** of any body, is measd by mult the wt of the body by the dist through which it is moved; thus, if a body of 4 lbs is moved 3 ft; or one of 6 lbs, 2 ft; or one of 120 lbs, $\frac{1}{10}$ of a ft; there is in each case an entire motion of 12 foot-pounds, without reference to the length of the time during which the motion takes place. Most authors apply the term "quantity of motion" to *momentum* (see below); but we see no necessity for thus employing two terms to express one idea; and we therefore use the above definition of "quantity of motion" as being at once more rational and more convenient.

Momentum. scientifically speaking, is the product of the *mass* of a body (see foot-note p 317) multiplied by its velocity. But for practical purposes the term is applied to the product of the *weight* and the velocity; the weight being expressed in pounds, and the velocity in feet per second; and we so employ it. Thus, if three bodies weigh respectively 4, 6 and 12 pounds, and move with velocities of (respectively) 3, 2 and 1 foot per second, the momentum of each is 12 foot-pounds per second. Momentum, as thus defined, is of course "g" times (or about 32.2 times) the true scientific momentum, and is plainly the same as the "quantity of motion" *per second.*

Forces are estimated or measured by. or are in proportion to, the momentums which they can respectively produce in a given time. Thus a 1 pound weight and a 2 pound weight, after falling for one second from a state of rest, have each a velocity of say 32.2 feet per second. Their momentums at the end of the second are therefore respectively 32.2 and 64.4 foot-pounds per second, or in proportion to the gravity forces (1 pound and 2 pounds respectively) acting upon the two weights.

It follows from the above that forces are proportional to the *velocities* which they can in a given time impart *to a given body.*

In thus studying the relation between force and velocity, it is important to bear in mind the difference between the *total* force acting upon a body in a given direction, and the *unresisted* or net or resultant force acting in that direction. Thus, suppose, for convenience, that the resistance of a certain railroad train remains = 1 ton at all speeds (see Art. 20, p 374 *e*); that is, that a pull of 1 ton on the draw-bar would in all cases balance the resistance and maintain the speed uniform. Suppose the train to be pulled first by an engine exerting a constant pull of 2 tons on the draw-bar; and then by one pulling 3 tons. Strictly speaking these engines impart to the train, in a given time, velocities which are to each other as 2 to 3, or as the respective pulling forces of the two engines; but in both cases the supposed uniform resistance of 1 ton would, in the same time, cause a *retardation* (or *counter*-acceleration) of 1. Hence the *net* or *resultant* velocities (*i e*, the observed velocities) would be = 2 — 1 and 3 — 1, or = 1 and 2; or in proportion to the *net* or resultant *forces.*

Art. 10. An impact. blow, stroke or collision takes place when a moving body encounters another. If a body of 100 pounds has a velocity of 3 feet per second; and another body of 5 pounds has a velocity of 60 feet per second, the momentum of each is 300 foot-pounds per second; and if they meet each other when moving in opposite directions, they will bring each other suddenly to rest.

"In some careful experiments made at Portsmouth dock-yard, England, a man of medium strength, and striking with a maul weighing 18 lbs, the handle of which was 44 ins long, barely *started* a bolt about $\frac{1}{8}$ of an inch at each blow; and it required a quiet pres of 107 tons to press the bolt down the same quantity; but a small additional weight pressed it completely home."

Art. 11. Working force. When the force imparted to a body, not only moves it, but does so in spite of resisting forces; or in other words, when one part of it strains against, and balances the resistances equal to itself; while the other part of it moves the body, it is called working force. The quantity of working force expended; as well as the quantity of work thereby accomplished, is estimated by mult the resistance overcome, (which frequently consists of wt *lifted*,) by the dist through which it is overcome. Thus, if the resistance be a friction of 4 lbs, overcome at every point along a dist of 3 ft; or if it be a wt of 4 lbs, lifted 3 feet high, then either the working force, or the work done by it, amounts to $4 \times 3 = 12$ ft-lbs. The *lifting* of wts; and the friction encountered in merely *moving* them by sliding, or rolling, constitute the principal sources of resistance, and work, in practice. The quantity of any work may evidently be considered by itself, without regard to the time reqd to perform it; but (as in motion) we generally require to know the *rate* at which work can be done; that is, how much can be done within a certain time.

Thus, in selecting a steam-engine, it is important to know how much it can do per minute, hour, or day. We therefore stipulate that it shall be of so many horse-powers; which means nothing more than it shall be capable of overcoming resisting forces, at the rate of so many times 33000 ft-lbs per min. See Arts 22 to 25.

Working force is often called **power**; horse, steam, water power, &c. Since work, in a *strictly scientific* sense, involves motion and strain combined, a man who is standing still is not considered to be working, any more than a post or a rope sustaining a heavy load; although he may be supporting an oppressive burden; or holding a car-brake with all his strength; for in both cases his force is *strain* only. The work done by a horse drawing a heavy load on a *level* road, consists entirely in overcoming the *friction* at the axles and rims of the wheels; but in drawing it *up hill*, he partly *lifts* the *load*, and *himself* also. In the first case his work (scientifically) is not so much wt of load moved a certain dist; but so many pounds of rolling and axle-friction overcome through said dist. Up hill, his work consists of friction, overcome through one dist; namely, the *length* of the hill; and of the wt of the load, vehicle, and himself, lifted to another dist; namely, the *vert height* of the hill. On a perfect level road, and without friction, the horse would encounter no resistance in hauling his load, however great it might be. He would have nothing to do but impart motion to unresisting matter. Therefore, in such a case his hauling would not be *work*, in the scientific sense.

The ordinary unit of work. or standard for measuring it, is one lb of wt *lifted* one foot high; or one lb of any kind of *resistance*, overcome in any direction whatever, for the dist of one foot; and is called a *foot-pound*. Or, when more convenient, we may use foot-tons, &c, &c; as we use a two-ft rule, a yardstick, or a tape-line, as may best suit our purpose. **The unit** of **RATE of work,** or the quantity done in a *given time*, is *one ft-lb per sec.*

The same quantity of force which will overcome a given resistance through a given dist, in a given time, will also overcome any other resistance through any other dist, in that same time, provided the resistance and dist when mult together give the same amount as in the first case. Thus, the force that will lift 50 lbs through 10 ft in a sec, will lift 500 lbs, 1 ft; or 25 lbs, 20 ft; or 5000 lbs $\frac{1}{10}$ of a ft in a sec; and in all these cases the amount of force expended, as well as of work done, is precisely the same. In practice, the adjustment of the speed to suit diff amounts of resistance, is usually effected by the medium of **cog-wheels, belts, or levers.** By means of these, the engine, water-wheel, horse, or other motive power, may be made to overcome small resistances rapidly; or great ones slowly, by the same working force.

Art. 12. When vel undergoes no change, it is said to be **uniform**; so with force, motion, strain, and work. When any of them becomes gradually greater, it is said to be *accelerated*; when gradually less, *retarded*. If the acceleration, or retardation is in exact proportion to the time; that is, when during any and every equal interval of time, the same degree of change takes place, it is *uniformly* accelerated, or retarded. When otherwise, the words *variable*, and *variably* are used.

Gravity is a uniformly accelerating force when it acts upon a body falling freely; for it then increases the vel at the uniform rate of .322 of a foot during every hundredth part of a sec; or 32.2 ft in every sec. Also when it acts upon a body moving down an inclined plane; although in this case the increase is not so rapid, because it is caused by only a part of the gravity; while another part presses the body to the plane; and a third part overcomes the friction. It is a uniformly retarding force, upon a body thrown vert upward; for no matter what may be the vel of the body when projected upward, it will be diminished .322 of a foot in each hundredth part of a sec during its rise; or 32.2 ft during each entire sec. At least, such would be the case were it not for the varying resistance of the air at diff vels. It is a uniformly straining force when it causes a body at rest, to press upon another body; or to pull upon a string by which it is suspended. The foregoing expressions, like those of momentum, strain, push, pull, lift, work, &c, do not indicate diff *kinds* of force; but merely diff kinds of *effects* produced by the one grand principle, force.

The above 32.2 ft per sec is called the **acceleration of gravity**; and by scientific writers is conventionally denoted by a small **g**: or, more correctly speaking, since the acceleration is not precisely the same at all parts of the earth, **g** denotes the acceleration per sec, whatever it may be, at any particular place. See note to Art 25.

Art. 13. Reaction. Strain. Strain, as before said, in Art 2, is either a pull or a push. The term may be applied equally to the *act*, or to its *effects*; that is, it may be said to be either the action of opposing forces against each other; or the effects which that action produces upon the particles of the body in which they act. A single force cannot produce strain; for since nothing but force can oppose, resist, pull, or push, against force, there must be at least two, to strain by pulling or pushing against each other. This mutual opposition or straining is called also the *reaction* of the forces. It can take place only between equal forces, or equal portions of unequal ones. When the forces are equal, and meet from diametrically opposite directions; that is, in the same straight line, but in opposite directions along it, they become *entirely* converted into reaction, or strain. If they are unequal; or are not

in the same straight line; but meet obliquely; then only equal portions of them will react against each other; while the remainder will continue as motion; unless some third force is present to prevent it; as when friction is generated. The reaction of the equal wholes or parts consists in their mutually resisting, opposing, arresting, balancing, equilibrating, straining, pulling, pushing, counteracting, or destroying each other.

All these words are equally applicable. As a matter of *convenience* only, we often say that the *bodies themselves* react. If a cannon-ball in its flight cuts a leaf from a tree, we say that the *leaf* has reacted against the *ball* with precisely the same degree of force that the ball acted against the leaf. That degree of force was sufficient to cut off a leaf, but not to arrest the ball; for, after a small portion of the moving force of the ball had been converted into straining force to react against the resistance of the leaf, the remainder was sufficient to carry it onward in its course. It, has, however, lost precisely as much force as that which the leaf opposed to it. A ship of war, in running against a canoe, receives as violent a blow as it gives; but the same blow that will upset or sink a canoe, will not *appreciably* affect the motion of a ship. The fist of a pugilist striking his opponent in the face receives as severe a blow as it gives; but the blow which may seriously damage a nose, mouth, or eyes, may have no such effect upon hard knuckles. The pain received in the one case, and not in the other, is of course no measure for force.

Rem. We have just said that strain is the mutual expenditure of two equal amounts of force. This may readily be conceived in cases where two bodies come into sudden contact, and arrest each other's progress by a mutual blow; for then their forces no longer produce either motion, strain, or work. But in *continuous* strains (pulls, or pushes) the principle is not so evident, at first. For instance, when a wt rests upon a table, and continues to strain it day after day, it may be asked where is the loss of gravity force in the weight; inasmuch as it weighs as much after pressing for a long time, as it did at the beginning; or where is the loss of inherent cohesive force in the material of the table, which is as strong as at first?

The reply is that gravity, and the natural strengths, or inherent forces, of matter, are being incessantly maintained or renewed, by unceasing streams, as it were, of those forces. The gravity of a wt which presses a table, or pulls on a rope, at one moment, is not identically the same that pressed or pulled it the moment before; and so with the cohesive force of the material of the table. If it were not so, a post or a rope which would be broken by a single force of 10 tons, would also be broken by sustaining one ton 10 consecutive times; for the one ton would each time counteract or overcome one ton of the inherent resisting force of the post or rope, and in ten applications would overcome it all. We must therefore conclude that these natural forces are being unceasingly supplied from that inexhaustible source of power "by which all things are upheld." When we lean forward against a strong wind, we are continuously exerting new force against the continuous stream of force which assails us; and in the same way does a post or a chain sustain its load. Continuous strains produced by the force of water, steam, animals, &c, we well know can only be maintained by a continuous supply of said force; to be as continuously counteracted or overcome by whatever opposing force of grinding, pumping, &c, it is directed against. The strain of an impulse, blow, or stroke, lasts only for an instant, because new force is not supplied to make it continuous. Two equal forces, straining against each other, do not even keep a body at rest; but the body rests merely because the two forces balance each other, and therefore cannot prevent it from resting. As a matter of convenience only, we may, however, say they keep it at rest.

Art. 14. While a horse is hauling a boat on a canal, it is not the boat and its load which react against his force; because matter cannot react against force; it is the force of friction of the boat against the water; and the resistance of the water in front of the boat, produced by its cohesive force. So with an engine and train on a *level* railroad; the only resistance to the steam force of the engine, is the force of friction at the axles and tires of the wheels, and the pres force of the air in front. But on an up grade, the engine has also to partly *lift* the train; or, in other words, to react against its forces of friction and gravity combined.

Neither the horse nor the engine exerts any more force upon the opposing forces, than these last exert upon *them;* but both the horse and the engine possess a surplus of power beyond what is necessary to strain against or destroy the forces opposed to them: and this surplus, as moving force, enables them in addition to move forward, as in the case of the cannon-ball just alluded to; and since the *unresisting matter* of the boat, load, and train, is attached to them by the tow-rope and coupling-links, they of course must follow.

The resistance which an abutment opposes to the pres of an arch; or a retaining-wall to the pres of the earth behind it, is no greater than those pres themselves; but the abut and wall are, for the sake of safety, made *capable* of sustaining much greater pressures, in case accidental circumstances should produce such.

Art. 15. The mere fact that a body is subjected to great strains from equal forces reacting upon it in opposite directions, does not **of itself** render the body more difficult to move than if it were free from strains; but in the cases which usually present themselves in practice the straining forces generate **friction,** which does oppose motion.

Thus let B, Fig 3, be a block resting on a hor support, and acted upon by a downward force *d* of say 100 tons, produced say by an immense block of granite resting upon B. Now it is plain that this 100 tons downward force will be met and balanced by a 100 tons upward force *u*, being the resistance of the hor support. Hence these two equal reacting forces produce in the body B a strain of 100 tons; but evidently do not impart to it as a whole any tendency to move in any direction whatever; nor do they tend to prevent it from being moved in any direction. The body therefore remains as before a mere inert mass incapable of resisting the slightest moving force.

Now suppose no friction to exist at either the base or the top of B. Then the slightest hor force *h*, a mere breath, would slide B along the hor support, moving it from

under the 100 ton block on its top. No matter how heavy B might be, the same smallest force would slide it, the only difference being that the heavier it was the less would be its velocity.

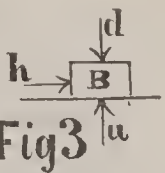


Fig 3

The heaviest bodies resting upon the surface of the earth, as well as ourselves, would be swept along by the slightest breeze if it were not for friction.

If the screw of a vise be worked until it produces a great strain in the jaws of the vise, the vise is not thereby rendered more difficult to move.

Again, if a strain of thousands of tons were produced by the jaws of a vise, in a body weighing an ounce, this immense strain would not prevent, nor even tend in the smallest degree to prevent, the ounce body from falling down from the jaws of the vise. It is prevented by the third force, friction, which compels the one ounce of gravity force to become vert strain, instead of motion. The two forces of thousands of tons each, which produce the strain of the vise, are thereby *entirely destroyed*, as regards their action upon the body *as a whole*. Hence *they* could not prevent the one ounce from producing motion in it; nor could they affect it as a whole in any way; for all their action is actually *against each other*. It is on this principle alone that strains do not interfere with motions.

If a body H, Fig 4, of 10 tons weight, is suspended from a long rope, its reaction against the equal opposing force at the other end of the rope, produces a continuous strain among all the particles which compose the rope; but which does not in the least affect the rope considered as a whole; inasmuch as it does not tend to move it in any direction. Now, in this case, there is no friction to be overcome; and we know from daily experience that it is therefore easy to move the unresisting body a little dist, by applying a very small hor force *f*. We cannot move it far, as, for instance, to *m*, because we then have not only to *move*, but to *lift* it, or overcome its gravity force, through the vert height *v c*. In doing this, it is true our force does not have to sustain the *entire* wt of the body; because most of it is sustained by the rope. Still, if we move it *at all*, we have to overcome *some* of its weight; otherwise, a mere breath would move it, although very slowly. If we attempt to move it by an upward force *u*, we shall have still more of its wt to resist us; and if by a downward one *d*, we shall be resisted by the cohesive force of the rope. Therefore, in this case, we can move it more readily by the hor force *f*.

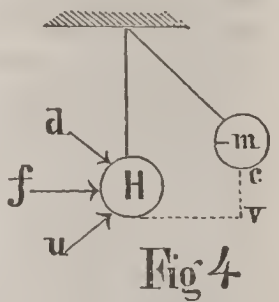


Fig 4

Art. 16. If two unequal forces, which for illustration we will call 10 and 12, are imparted, either as pulls or as pushes, in precisely opposite directions, to a rigid body on which no other force is acting, then the small force 10, and 10 parts of the large one, will destroy each other as strain; after which, of course, they can produce no effect of any kind; but the remaining two parts of the large one, meeting with no opposing force to react against, will continue onward as motion, in the same direction as before; taking the unresisting body along with them. In such a case, the large force is said to *overcome* the small one; and such an expression is very convenient, in reference to the entire original forces. But in a *strictly scientific* sense, one force cannot overcome another.

Thus, in the foregoing case, so far as the strain is concerned, the large force must be considered as separated into 10 straining, and 2 moving parts. Neither of the two 10 forces which strained against each other overcame, or gained any advantage whatever over the other; for the two reacted equally on each other, and mutually arrested each other. And this is plainly the most that one force can do to another.

Since two equal opposing forces, or equal portions of unequal ones, thus bring each other to a *stand-still*, or *equilibrate* each other, they are called *Static;* from the Latin "*Sto*, I stand;" and that branch of the science of force which treats only of cases in which all the applied forces keep each other at rest, is called "Statics," or "Equilibrium."

Art. 17. When motion has once been given to a body, it can only be taken out again by the reaction of some opposing force. The same identical portion of any force cannot produce both motion and strain at the same time. When continuous force is applied, as in mills, &c, to do both, (or, in other words, to work,) it must be considered to divide itself into two parts for those separate purposes. If, while at work, the resistances to be overcome become less, the strain also becomes less, and the motion becomes greater; and vice versa. Motion is diminished by converting part of it into strain; and strain, by converting it into motion.

Art. 18. If force f be imparted to any rigid body, as N, Fig 5, at any point c; and if $f o$ represent the direction in which it was imparted, whether as a pull or a push, then the force would produce the same effect upon the body considered as an entire mass, as if it had been imparted as either a pull, or a push, in the same direction, at any other point of the body in said line; as at i, t, s, o, &c.

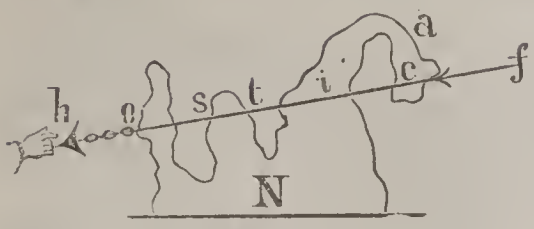


Fig 5

Under Composition and Resolution of Forces, it will be seen to be sometimes necessary to consider a push $f c$, to be changed to a pull $o h$; and vice versa; when we wish to ascertain the joint effect or resultant of a pull and push imparted to a body at the same time. See Remark 1, Art 29.

The foregoing important principle holds good, no matter how many diff forces may be acting upon the body at the same time, in diff directions; or how much the direction of their joint effect, or resultant, may differ from that of any one of them; the action of each force, considered *separately*, may be regarded as just stated. The tendencies of several forces, acting at the same moment, may therefore frequently be first investigated one by one; and these tendencies then combined into one; or the forces themselves may first be combined into one or more resultants, as directed under Composition and Resolution of Forces, and the effect of these resultants considered. The engineer has generally to divide all the forces acting upon his structures into two classes; namely, those whose tendency is to secure the stability he requires; and those which tend to impair that stability. He therefore first finds the resultant, or joint effort of each class separately; and then compares these two resultants with each other. The mode of doing this will be shown further on. See Arts 35, 72, &c.

It is plain that if, instead of regarding the body as rigid, we considered it as elastic, or as breakable, an entirely diff course would be necessary, as the question would then become one on *the strength of materials*; for the force f, applied at c or t, as a push, might break off the pieces c and t; and so with the same force as a pull at s or o. Although masonry, iron, timber, and other building materials are by no means absolutely rigid, yet generally they may be assumed to be so when we are investigating the effect of force to overthrow, or derange the structure as a whole.

Art. 19. The full amount of a given force cannot (theoretically) **be put into a resisting body B, Fig 6, except when applied in a direction at right angles to the surf of the body at the point of application.** If it be applied in a direction at all oblique to the surf, then only a portion of it will (according to theory) enter the body, and produce any effect on it. The remainder will continue to produce motion in that other body in which the applied force was carried to the body B; unless some third force be present to receive and react at right angles to that remainder also. Thus, the forces $m o$ and $n e$, being at right angles to the surf at the points of application o and e, will all enter B, *if the resistance of* B *is as great or greater than they*; if not, they will move B. But the force F g is not at right angles to the surf at its point of application g; therefore, only a portion of it will (by the theory) be imparted to the body B, however great may be the resistance of B.

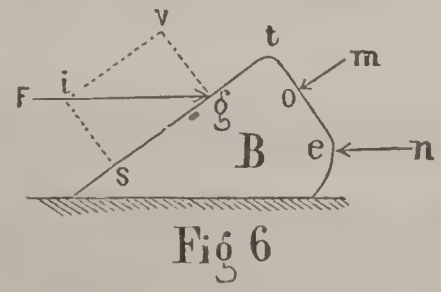


Fig 6

Under the head Composition and Resolution of Forces, it will be seen that when a force is thus applied obliquely to a surf, its action at the point g is precisely the same as that of two entirely separate forces; one of which, $v g$, is at right angles to the surf at g; and the other, $s g$, parallel to the same surf. Only $v g$ will enter the body B; while $s g$ will remain in the body F g, which carried the entire force to B; and (if F g also is rigid) will (by theory) cause it to move in the direction $g t$, unless some third force be opposed at right angles to it also. The quantity of each of the forces $v g$ and $s g$, is very readily found thus: On the line F g measure off by any convenient scale a dist $g i$, to represent the amount in lbs. tons, &c, of the force F g; then on $g i$ as a diag draw a parallelogram $i v g s$, having two of its sides perp to the surf at g, and the other two parallel to it. Then $i s$, or $v g$, measd by the same scale as $g i$, will give the force imparted to the body B; and $i v$ or $s g$ will give that which remains in the body F g, and will move it from g toward t, unless prevented by some other force. If F g is not thus prevented from moving toward t, it is plain that the force $v g$, which is transferred to the body B, cannot remain stationary at g; but must move along with F g, and thus press upon the surf at every point between g and t.

This will appear in a clearer light on referring to a body placed upon an inclined plane so steep that the body will slide down. We know that a part of the force of gravity, or wt of the body, presses, as strain, upon the plane, and at right angles to it; while a part remains in the body, as motion; and causes it to slide down the

plane; see Art 60. As it slides, it is evident that the pressure part of the force also moves along with it; and is thus imparted at every point of the plane along the dist slid over. The case is identically the same with the pres force $v\,g$ in Fig 6; this will slide *up* the plane from g toward t; being carried along by the sliding force $s\,g$.

But in practice this theory does not hold good because it ignores the existence of friction, which pressures always generate at points of application. This friction always acts in the direction of the pressed surfaces, and consequently always opposes more or less resistance to that component, $s\,g$, which tends to produce sliding along the surface $s\,t$; and so much of $s\,g$ as is thus resisted is thereby changed from motion to strain, and enters the body B. If it were not for friction a body, Fig 64, would slide down an inclined plane $w\,x$, no matter how slight its inclination might be; but we know that friction often prevents such sliding even when the plane forms a considerable angle, $y\,x\,w$, with a hor line $y\,x$. When this angle becomes so great that the body is just on the point of starting to slide down it, it is called the **angle of friction;** and in Fig 6, if the force F g does not form with the perp $v\,g$ an angle $v\,g$ F, greater than this angle of friction, then friction will oppose **all** the sliding force $s\,g$, no matter how great it may be; so that the entire force F g will then enter B at g, and in its original direction F g. Study Art 63.

This remark is particularly applicable to the case of the masonry joints in the abuts of stone arches; especially those of large span, with small rise. The pres which such an arch exerts upon its abuts is very great; and its line of direction changes at each joint, as at $o\,n\,m$, Fig 7. It therefore becomes necessary first to find the position of this line, (see Art 72.) so as to know how to draw the varying inclinations of the joints nearly at right angles to it; otherwise, the upper courses, if hor, may slide outward upon the lower ones, as shown by the arrow. In small arches of considerable rise, the sliding portion of this force may be safely resisted by good mortar or cement, if sufficient time be first allowed for it to harden properly; but in large ones, the direction of the joints must be relied on, unless we increase the expense by making the abuts unduly thick.

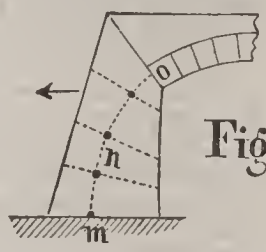


Fig 7

The angle of friction of masonry on masonry (see table, p 373) is about 32°. Therefore if at any bed-joint of masonry, as in the abut of Fig 7, the *resultant* that cuts said joint at the line of pressures, $o\,n\,m$, does not differ more than 32° from a perp to said joint, there will be no unresisted tendency to slide at that joint.

Fig 8 is added merely to illustrate more strikingly the necessity for clearly distinguishing between *applied*, and *imparted* forces. Here the great force $a\,o$ is applied to the body B B at the point o; but all of it that is theoretically put into, or enters the body; or produces any kind of effect upon it, is the very small amount represented by $c\,o$, at right angles to the surf of B B at o, but we have seen that *in practice* friction makes it more.

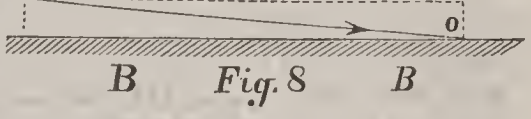


Fig. 8

All this will be better understood after studying Comp and Rcs of Forces, Art 28.

Art. 20. It rarely happens in practice that we impart very great force, or velocity, to a heavy body instantaneously; or by a single effort. In the machinery of mills, in railroad trains, in steamships, &c, it is done very gradually: and indeed, few water or steam powers possess sufficient force to do it otherwise. The principle explained in the following example, applies to the others. Thus, no locomotive, or marine engine, has sufficient power to start a heavy train, or a steamship, at once at a rapid speed. The first stroke of the piston imparts, or puts into the train, a quantity of force sufficient not only to react or strain against, balance, or destroy (and be destroyed by) all the resisting forces of friction, the air, grade, curvature, &c, which present themselves along the short portion of the road passed over during the first stroke: but a small excess besides, which, being unresisted, is not destroyed; but continues in the train, as motion; giving a slow movement to the *unresisting matter* of which it is composed. This last portion would remain in the train, and move it at the same very slow vel forever, if only we could remove all resisting forces from before it The next stroke in the same manner furnishes a new instalment of force; which, like the first, divides itself into two parts; one of them as strain to destroy, and be destroyed by the new resistances met with in the next short dist along the track; and the other as motion, to remain stored up in the train, united with the motion put there by the first stroke: thereby imparting increased vel to the unresisting matter of the train; and this process is repeated during a great number of strokes: the train moving faster and faster. But after a while it is found that no matter how powerful the engine may be, or how light its train, the speed no longer increases.

The reason of this is that the resistances of the air, friction, &c, increase with the speed, but at a more rapid rate; so that after a certain vel has been attained, they require *all* the power the engine is capable of, in order as strain to react against, balance, or destroy them alone; without leaving any more surplus to be stored away, as motion, in the train. After this point is attained, the power of the engine no longer draws the train; but barely suffices to remove all the resistances which would otherwise impede it; and thus permits the moving force (which has *previously* been stored, and accumulated in the train, by small instalments) to continue to move the unresisting matter of the train at an undiminished vel. The train might now be said to "move itself;" and it would do so forever, without requiring any additional moving force, if the engine would only continue to destroy the resistances in its way. Should a stiff up-grade, sharp curvature, or other resistance present itself, so that the opposing forces actually *exceed* the *entire* power of the engine, for a short time, the train will actually come to its assistance, and by converting part of its own *motion* into *strain*, will, as it were, lend pressure-force to the engine, to help it push through its increased work; performing in fact the part of a fly-wheel in machinery. Frequently, when an engine appears to be pulling a heavy train up a short sharp grade, it is actually, so to speak, the train that is pushing itself up; the engine probably could not do it. This state of affairs can, however, continue for but a short time; otherwise all the motion of the train would be converted into strain; and being thereby destroyed, the whole would come to rest.

When a train is going at speed, and it becomes necessary to stop at a station some dist ahead, steam is shut off, so that the steam force of the engine shall no longer counterbalance, or destroy the resisting forces in front of it; and the number of the resistances themselves is increased by adding to them the friction of the brakes. Against all these combined, the train has now to "work" its way unaided; and it does so by the gradual conversion of its previously accumulated motion, or moving force, into straining force. Work, as before stated, consists in motion and strain combined. When the conversion has been completely effected, the train stops. Thus, we see, that up to the time of its stopping, the force which had gradually accumulated in it as *motion* before it had reached its greatest vel is gradually taken out of it as *work*, after steam is shut off. We may therefore speak of it as well under the head of *accumulated*, or *stored-up work*; as of accumulated *motion*, when we intend, in any kind of machinery, to convert such motion into work. Thus, the *motion* gradually accumulated in a fly-wheel, is also accumulated *work*, held in reserve until some extra strain on the machinery calls for its aid.

Art. 21. Up to the time that the vel ceased to increase because *all* the power of the engine became required as strain to react against resisting forces, and consequently could no longer spare any as motion to the train, the *work* of the *engine* was what is termed *variable*; being *gradually accelerated*. Also, when steam was shut off, the work of the train was variable; being gradually *retarded*. Such is the case in almost every kind of machinery, during the interval between starting, and ultimately attaining its maximum speed. When the latter point is reached, supposing the resistances afterward to remain the same, the work is termed *uniform*, or *steady*. All these remarks, as well as those of Art 20, apply alike to all kinds of heavy machinery; no matter by what kind of force or power it is driven; the machinery takes the place of the train just spoken of; and the friction of the cog-wheels, gudgeons, pivots, the grinding, sawing, or whatever the work may be, takes the place of the grades, curves, and friction of the train. All alike are simply cases of force in its various shapes of motion, strain, and work.

Art. 22. The quantity of any work, considered by itself, without reference to the time reqd to perform it, is plainly to be measd by mult together the resistance in lbs, by the dist through which it is overcome in ft, as stated in Art 11.

After work becomes *uniform*, that is, when neither its strain nor its motion undergoes any change; its **rate** is measured by mult the resistance, or strain, in lbs, tons, &c, by the vel, or dist, in feet, &c, through which the resistance is overcome *in a given time*, as a sec, min, hour, &c.

Thus, if the resistance is 3300 lbs, and is overcome through a dist of 10 ft in every min; or if the resistance is 33 lbs, and is overcome through a dist of 1000 ft per min, the rate of the work is in each

lbs vel lbs vel

case the same, namely, 33000 ft-lbs per min, or one horse-power; for $3300 \times 10 = 33 \times 1000 = 33000$ ft-lbs per min. The quantity of motion of a body (Art 9) is also estimated in ft-lbs; and under the head Levers, it will be seen that the tendencies (called *moments*) which the power and the weight respectively have to commence motion about the fulcrum as a center, are measured in the same term. Work, mere motion, and moments, are, however, effects of force so diff from each other, that confusion is no more likely to occur, than in applying the same measure, one foot, to materials as diff as cloth, bar iron, boards, &c.

In scientific phraseology, **work is either useful or prejudicial,** the latter being the quantity of force lost by friction, by the resistance of the air, &c. Thus, in pumping water, part of the applied force or power is lost in the friction of the diff parts of the pump; so that a steam or water power of 100 lbs, moving 6 ft per sec, cannot raise 100 lbs of water to a height of 6 ft per sec. Therefore machines, so far from *gaining power*, according to the popular idea, actually lose it, in one sense of the word. In the practical application of all machinery, the object is twofold; namely, to enable us conveniently to apply straining force, to balance, react against, or destroy, the resisting forces of friction, and the cohesive forces of the material which is to be operated on; and moving force, to give motion to the unresisting matter of the machine, and of the material operated on, after the resisting forces which had acted upon them have thus been rendered ineffective.

Art. 23. The total quantity of work that will be performed by the moving force that is in a body at any given moment, provided that after changing from mere moving force into working force, it is left to expend itself in uniformly retarding work, without receiving any additional force to aid it, (as in the case of the *moving* force in the locomotive in Art 20, after steam is shut off; and when said force begins to *work* against the resistances of the road,) is as, or in proportion to,

(not equal to,) the wt of the body, mult by the **square** of its vel at the moment it begins to work. For example, if a train at the time steam is shut off, has in it an accumulated moving force due to a vel of 10 miles an hour; and if that force will by itself work the train against the resistances of the road for a dist of one-quarter of a mile, before coming to a stop; then, if steam is shut off while the train is moving at 2, 3, or 4 times that vel, and consequently with 2, 3, or 4 times the moving force, it will work through 4, 9, or 16 times the dist of the first case, before coming to rest. If bullets of equal wt be fired with vels proportioned to each other as 1, 2, 3, they will respectively penetrate a plank to depths as 1, 4, 9. If an engine, water-wheel, &c, works *steadily* in a mill, grinding at 2, 3, or 4 revolutions per min, it performs only 2, 3, or 4 times as much *uniform* work per min. as when at but 1 rev per min. But if steam or the water be suddenly shut off at 2, 3, or 4 revs per min, then the 2, 3, or 4 times quantity of *moving* force accumulated in the machinery at that moment, will, as *working* force, run the mill through 4, 9, or 16 times as many revs before stopping, as if shut off at 1 rev. If a rolling ball, started against a row of bricks, will overcome their resistances, and knock them down for a distance of 4 ft; then, if it be started at a vel 3, 4, or 5 times as great, it will overcome and knock them down for dists of 9, 16, and 25 times 4 ft; and in but 3, 4, or 5 times the time.

But in all these cases the *rate* of the work done, that is, the quantity done *in any given time*, as one sec, is *directly* as the vels. Thus, the locomotive whose steam is shut off at 20, 30, or 40 miles an hour, will require but 2, 3, or 4 times as many seconds for running its 4, 9, or 16 dist before it comes to a stop; in other words, when its moving force is 2, 3, or 4 times as great, it will overcome but 2, 3, or 4 times the resistances *in the same time*, although the *total* amount of resistances overcome will be as 4, 9, and 16. And so with the other examples.

Rem. We know that the dist through which a body must fall by the unif accel force of gravity in order to acquire any given vel and moving force, is as the square of said vel; but directly as the time of falling. Also if a body is thrown vert upwards with any given vel or force, grav will retard it unif, and the height to which it will rise by the time that grav destroys all the force with which it was thrown, will be as the square of said vel: but the time will be directly as said vel. And so with anybody moving in any direction, and acted upon by any unif accel or retard force whatever. It will either acquire or part with its moving force within dists proportionate to the square of its vel, and in times proportionate to its simple vel. See **Caution,** Gravity, p 362.

Art. 24. Vis viva, or living force. The preceding article serves as an introduction to this subject; of which we shall endeavor to give some idea in plain language. The term itself is merely one of those absurdities to which savants resort, in order to impart an air of mystery to their writings. We might with the same propriety speak of a brickbat viva, or a living hod of mortar.

We have seen in Art 23, that if that portion of force in a body which is occupied in giving motion *alone* to the body, be suddenly converted into *working* force, the quantity of work which it would perform against *uniformly retarding* resistances, before being entirely destroyed, or coming to rest, would be in proportion to the *square* of its vel at the time of beginning to work. Now, if "vis viva," or "living force," were merely the name given to this force; or to the quantity of work done by it, (as measured in ft-lbs, by mult the resistances in lbs, by the dist in ft through which they were overcome,) the expressions, although silly, would still convey an idea readily understood by practical men. We could then say, for instance, of a moving body, that its vis viva was 100 foot-lbs; meaning thereby that it would overcome a uniform resistance of 1 lb through a dist of 100 ft; or a resistance of 100 lbs through a dist of 1 ft, &c. But scientific writers apply the terms to a purely *imaginary* quantity, equal to *twice* this: and which does not exist in any body, under any circumstances. The reason they do so is that it facilitates some of their calculations. But the practical engineer need not concern himself with either this reason, or vis viva itself; the simple statement of facts contained in the preceding and following articles, probably contains all that it is essential for him to know on the subject of moving force, converted into uniformly retarded working force.

Art. 25. The amount of work, in foot-pounds, that can be accomplished by a given moving body, by virtue of its momentum alone, is

$$\textbf{Work} = \text{Weight of moving body, in lbs} \times \frac{\text{square of its velocity in feet per second}}{2g}$$

$$= \text{Weight of moving body, in lbs} \times \text{fall in feet required to give the velocity}$$

($\mathbf{g}$ = about 32.2. $\mathbf{2\,g}$ = about 64. 4). See "Falling bodies", p 362.

An *imaginary* force equal to double this, will be the vis viva, or living force of the savants: now seldom used:

$$\textit{Vis viva} = \frac{\textit{weight of body in lbs}}{} \times \frac{\textit{sq of its vel, in ft per sec}^*}{g}$$

* For the purposes of abstract science, it is not sufficiently exact to measure the quantity of matter by its wt; because the wt or gravity of a body, as shown by a spring balance, varies somewhat in diff latitudes, and at diff heights above the level of the sea. But the vel with which it will fall by that grav is a proper measure of the grav; for it varies in the *same proportion;* all force being in proportion to the velocity it imparts to a given body in a given time. Therefore, if the wt of a body at any place, be div by the vel imparted by gravity in one sec at the same place, (and called the *acceleration of gravity* of that place,) the quot will be the same at all places. And since the *quantity* of matter undergoes no change at diff places, the *measure* of that quantity should likewise

Art. 26. To perform a given amount of work, we must expend an equal amount of **energy.** A moving body is said to have a **kinetic energy** equal to the amount of work which the body can perform by means of its momentum alone; i e,

$$\begin{matrix}\textbf{kinetic energy}\\ \text{in foot-pounds}\end{matrix} = \begin{matrix}\text{weight of moving}\\ \text{body, in lbs}\end{matrix} \times \frac{\text{vel}^2 \text{ in ft per sec}}{2g}$$

$$= \begin{matrix}\text{weight of moving}\\ \text{body, in lbs}\end{matrix} \times \begin{matrix}\text{fall in ft required}\\ \text{to give the velocity}\end{matrix}$$

($\mathbf{g}$ = about 32.2. $\mathbf{2\,g}$ = about 64.4.)

For instance, a train weighing 1,120,000 lbs, and moving 22 ft per second, has a kinetic energy of

$$1{,}120{,}000 \text{ lbs} \times \frac{22^2}{64.4} = 8{,}400{,}000 \text{ ft-lbs.}$$

That is, if steam be shut off, the train will perform a work of 8,400,000 ft-lbs in coming to rest. Thus, if the sum of all the resistances (of friction, air, grades, curves etc) remained constantly = 5000 lbs, the train would travel

$$\frac{8{,}400{,}000 \text{ ft-lbs}}{5000 \text{ lbs}} = 1680 \text{ ft.}$$

Energy, thus expended in work, is not destroyed. It is either transferred to other bodies, or else stored up in the body itself; or part may be thus transferred, and the rest thus stored. But, although energy cannot be destroyed, it may be rendered useless to us. Thus, a moving train, in coming to rest on a level track, transfers its kinetic energy into other kinetic energy; namely, the useless heat due to friction at the rails, brakes and journals; and this heat, although none of it is *destroyed*, is dissipated in the earth and air so as to be practically beyond our recovery.

Potential energy, or possible energy, may be defined as stored-up energy. We lift a one-pound body one foot by expending upon it one foot-pound of energy. But this foot-pound is stored up in the body as an addition to its stock of potential energy. For, in falling through one foot, it will acquire a kinetic energy of one foot-pound, and will part with one foot-pound of its potential energy.

Familiar instances of potential energy are—the weights or springs of a clock when fully or partly wound up, and whether moving or not; the pent-up water in a reservoir; the steam pressure in a boiler; and the explosive force of powder. We have mechanical energy in the case of the weights or spring or water; heat energy in the case of the steam, and chemical energy in the case of the powder.

In many cases we may conveniently estimate the *total* potential energy of a body. Thus (neglecting the resistance of the air) the potential energy of a pound of powder is = the weight of any given cannon ball × the height to which the force of that powder could throw it, = the weight of the ball × (the square of the initial velocity given to it by the explosion) ÷ $2g$. But in other cases we care to find only a certain definite *portion* of the total potential energy. Thus, the *total* potential energy of a clock-weight would not be exhausted until the weight reached the center of the earth: but we generally deal only with that portion which was stored in it by winding-up, and which it will give out again as kinetic energy in running down. This portion is = the weight × the height which it has to run down = the weight × (the square of the velocity which it would acquire in falling *freely* through that height) ÷ $2g$.

We thus see that kinetic and potential energy are measured in the same way.

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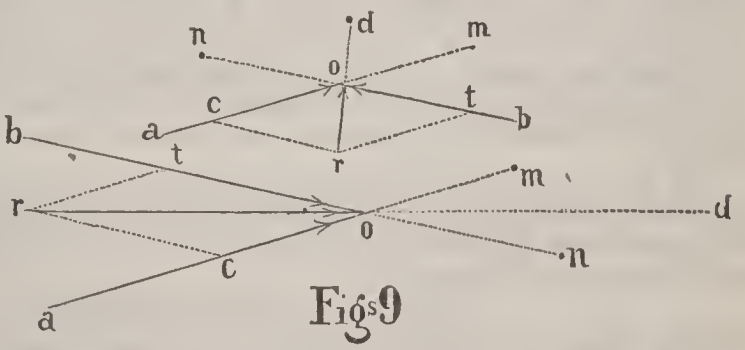
undergo none. Therefore scientific men adopt $\frac{\text{Weight}}{\text{acceleration}}$ to denote the *quantity* of matter in a body; and they call the resulting quot the "*mass*" of the body, to distinguish it from mere *wt*. Thus at any place where the acceleration of gravity is 32.2 feet per sec, and where a body weighs 20 lbs by a spring balance, the body's **mass,** or scientific quantity, is equal to $\frac{20}{32.2} = .621$. To the practical man, *this* mass or quantity conveys no idea whatever; but it is plain that the ordinary measure by weight cannot be perfectly correct, because the weight changes at diff places, while the quantity remains the same; and the measure by size would be equally incorrect, because the size varies with the temperature.

Since, therefore, the only absolutely correct measure of quantity in a body is the scientific mass; and since the imaginary vis viva is the quantity, mult by the square of the vel, we have vis viva represented strictly by, Mass × Vel^2; or the M.V^2 of scientific writers. In science, the mass of 100 lbs of iron is equal to that of 100 lbs of cotton. The greatest discrepancy that can occur at various heights and latitudes, by adopting wt as the measure of quantity, would not be likely to *exceed* 1 in 300; or, under ordinary circumstances, 1 in 1000.

Art. 27. As explained in Art. 9, p 310, forces are proportional to the velocities which they can produce in a given body in a given time. From this it follows that a greater force is required to impart a given velocity to a given body in a short time than in a longer time. For instance, the forward coupling links of a long train of cars would snap instantly under a pull sufficient to give to the train in five seconds a velocity of twenty miles per hour, supposing a locomotive of sufficient power to exist. In many such cases, therefore, we have to be contented with a slow, instead of a rapid, acceleration.

A string may safely sustain a weight of one pound suspended from our hand. If we wish to impart a great upward velocity to the weight *in a very short time*, we evidently can do so only by exerting upon it a great force; in other words, by jerking the string violently upward. But if the string has not tensile strength sufficient to transmit this force from our hand to the weight, it will break. We might safely give to the weight the desired velocity by applying a *less force* during a *longer time.*

Art. 28. Composition and resolution of forces. We have already said that when diff forces are *imparted* * (whether so *applied*, or not) in the same direction to a rigid body *free to move unresistedly*, they all act as motion alone, in that same direction. If two equal forces are imparted in diametrically opposite directions, they mutually destroy each other entirely, as strain (pull or push) against each other; thereby producing strain among the *particles* of the body: but having no tendency to move the body, *as a whole*, in either direction. If unequal, and in diametrically opposite directions, the whole of the small one, and a part of the large one, equal to the small one, destroy each other as strain; while their diff remains as motion, (or a moving of the whole body,) in its original direction. But if two forces, *a o* and *b o*, Figs 9, whether equal or unequal, are imparted at the same time to an unresisting rigid body *o*, *in directions either converging toward*; or *diverging from, the same point* o, *at any angle whatever*: then the body *o* cannot possibly be kept at rest by them; or in other words, equilibrium cannot exist between them; or they cannot balance, or completely react against each other; the body must move. *Equal parts* of each of the two forces will mutually destroy each other as strain among the *particles* of the body; while the remaining portions will unite to constitute a single force *r o*, which will move the *whole body* in a direction *o d*, in the line *r o* extended; and which direction *o d* will always be somewhere *between* those in which the separate forces would have moved it.



Figs 9

If we lay off *c o* and *t o* by any convenient scale, to represent respectively the amount of the forces *a o* and *b o*, and then complete the parallelogram *o c r t*; the diag *r o*, measured by the same scale, will represent both the direction and the amount, of the single remaining force.

* It is *absolutely necessary* to keep distinctly in mind the diff between *applied* and *imparted* force. Writers carelessly confound the two very frequently. See Art 19.

The same process will answer also for forces which instead of motion, produce strain, not only in the particles of the body, but in the body itself considered as a whole; or, in other words, a tendency to press or pull the entire body in a certain direction. Thus, suppose that two men were either pulling or pushing with the forces c o and t o; trying in vain to detach a piece o of rock, from a cliff of which it forms a portion; and which, by its inherent *force of cohesion* to the cliff, defies their efforts. Here we have a case of *extraneous forces*, resisted, or reacted against, or balanced, by *strength of material*.

As in the case of motion, the two forces partly destroy each other as strain among the particles of the body; and the remainders combine to form the single force r o, which tends to move the whole body toward d. The rock resists this single force, by a cohesive force precisely equal, and diametrically opposite to it; and so long as it does so, there is strain but no motion. The piece of rock may have strength enough to oppose a much greater resistance; but cannot actually *exert* it unless the men also exert more force.

In the matter of comp and res of forces, it must be remembered that when force is applied to a body in order to produce *motion*, care must be taken that there is no other force to prevent it: but when the force is intended to produce strain, it is equally necessary that other force should be present to oppose it; for strain is the *opposition* of forces.

The fig o c r t, Figs 9, is called the **parallelogram of forces.** The two original forces c o, t o, are called the *components* of the force r o; which *results* from their joint action; and the force r o is called the *resultant* of the original ones which *compose* it. The principle of the parallelogram of forces, than which there is none more important in the whole range of mechanical science, may be expressed thus: If any two forces, (both motions, or both strains,) whose directions either converge toward, or diverge from, the same point, be represented both in quantity and in direction by two adjacent sides of a parallelogram: then will their resultant be similarly represented by the diag of the parallelogram.*

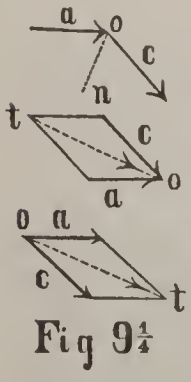
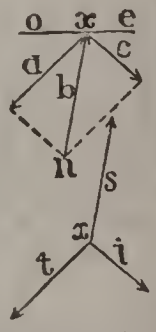


Fig 9¼

Rem. 1. **If one of the forces, as c,** upper Fig 9¼, **is a pull, and the other a push,** then to find their resultant o t we must, before drawing the parallelogram of forces, move (or imagine to be moved) one of the forces to the opposite side of the point o, so as to change it from a pull to a push, or *vice versa*, so that both shall be pulls, or both pushes, as shown by the two lower figs. Otherwise we should obtain a wrong resultant n o of the top fig. Either a push or a pull equal to o t, if applied at o, would be equal in effect to the push a and the pull c. The remark is of frequent use when finding strains in bowstring and crescent trusses; as in many other cases.

Rem. 2. When any three forces as a, b, c, forming only two angles a x b and b x c, balance each other at any point x, then a straight line as o e can be drawn through that point so that all three forces shall be on one side from it; then also a parallelogram x n can be drawn on the three lines a, b c, having the middle line b for its diagonal; and this diagonal will be of a different character from the two outer forces a and c; that is, if **they** are pulls, **it** will be a push, and vice versa. But if as in the three balancing forces t, i, s, three angles as s x t, t x i, s x i, are formed, neither such a line, nor such a parallelogram can be drawn; and the three forces will all be alike, all pulls or all pushes. All this is evident from the two figures.



Rem. 3. We have alluded to equal parts of each component as being *lost*, or destroyed, by reacting against each other; thus producing within the body a straining of its particles; and therefore having no tendency to move, push, or pull, the body *as a whole*, in any direction.

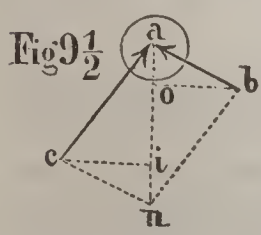


Fig 9½

Let b a and c a be any two components, and n a their resultant. From the two angles b and c, opposite to the diagonal, draw b o and c i at right angles to the diagonal; or to the diagonal extended, if necessary, as in Fig 9¾. These two lines, b o, i c, will always be equal to one another; whatever may be the lengths and directions of the components b a, c a. When two forces, as b a, c a, are imparted at a, there occurs a loss of force equal to what would result from the reaction of two forces equal to b o and c i. It is lost by becoming strain against the cohesive forces of the particles which compose the body a. In anticipation of what is said in Art 31, we will state that the force b a may be regarded as made up of the forces b o, o a; and the force c a, of c i, i a; which act also in those directions, when b a and c a *converge* toward a, as in Fig 9½; or in the directions a o, o b, and a i, i c, when the forces *diverge* from a, as in Fig 9¾. In either case, however, these forces, b o, a o, c i, i a, &c, must be considered as being imparted at a. This being supposed, it becomes plain that when b a and c a meet at a, inasmuch as b o and c i destroy each other as strain against the internal cohesive forces of the body, there remains nothing to act upon the body considered as a whole, except o a and i a; which, being together equal to n a, (as seen in the fig,) are, in other words, equal to, or actually compose, the resultant n a of the two components b a, c a. See Rem 5.

* **Components and Resultants may be calculated** by the formulas in Art 45, when a diagram is not considered sufficiently accurate.

We conceive that each of the original forces endeavors as it were to compel the other to leave its own course, and follow that of its antagonist; and the struggle continues until they have succeeded in forcing each other into *the same* direction. This is of course effected by their reactions against each other; and, as occurs in all cases of reaction, they expend equal parts of their forces on each other. When the two forces act in diametrically opposite directions, where there is no neutral diag direction that can be adopted, there is no alternative but for the larger force to react against or destroy the smaller one entirely; thereby losing an equal amount of its own force. Its remains totter on slowly in their former unchanged direction. The writer can see no difference of principle between the reaction of opposite forces; that of oblique ones; and that of those at right angles to each other.

Rem. 5. When the direction $a\,b$, Fig 9¾, of one of the forces, forms an angle $b\,a\,n$, greater than 90°, with the diagonal, the shape of the parallelogram of forces becomes such that the two equal lines $b\,o$ and $c\,i$, cannot be drawn at right angles to the diag $a\,n$ itself; or within the parallelogram; in which case the diag must be extended each way, as to o and i; and the lines $b\,o$, $c\,i$, must be drawn at right angles to the extensions.

When this occurs, the componcnt forces $a\,o$, $a\,i$, cannot as in Fig 9½ be measured on the diag $a\,n$ of the parallelogram; because they will be greater than it; hut must, like $b\,o$, $c\,i$, be measured outside of the fig. And here it must be remembered that $a\,o$ and $a\,i$ no longer measure forces *acting* (like those in Fig 9½) *in the same direction*. Thus the strain along $a\,b$ may be considered (see Comp and Res of Forces, Art 31) to be made up of two forces imparted at a; namely, a hor force equal to $o\,b$, and a vert one equal to $a\,o$, actiug *upward*. And the strain along $a\,c$, as made up of one hor force equal to $i\,c$, and a vert one $a\,i$. (greater than the whole diag,) acting *downward;* both of them imparted at a. Hence, the resultant $a\,n$ we find is equal to the diff between the two vert components $a\,o$ and $a\,i$. Thus it is seen that this shape of the parallelogram in no way affects the principle laid down in Remark 3.

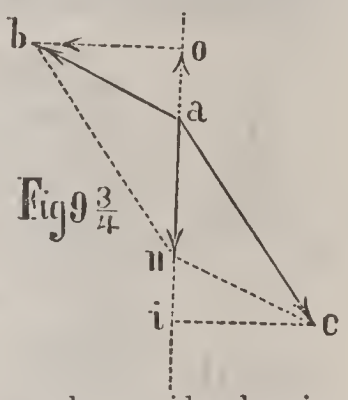


Fig 9 ¾

Art. 29. According to Art 18, the force $w\,e$, Fig 10, may be considered as imparted to the rigid body B at any point whatever in its line of direction $w\,c$; also, the force $x\,i$, at any point in its direction $x\,d$; consequently, both of them may be considered as imparted at the same point a; inasmuch as it is situated in both these lines. Hence, it is immaterial, so far as regards the effect of those two converging forces upon the body considered as one entire rigid mass, whether they are actually imparted like $z\,o$ and $y\,o$, at the same point o; or like $w\,e$ and $x\,i$, at diff points i and e. For in either case their resultant, or joint effect upon the body as a whole, is precisely the same; namely, a tendency to move the body in the same line of direction $o\,a\,t$. This tendency will actually produce motion if no opposing force prevents; otherwise it will produce strain in the body.

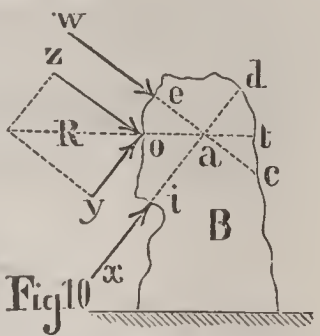


Fig 10

Rem. 1. Hence the resultant R, of two converging forces F f, Fig 10¼; or of two diverging ones F f, Fig 10½, acting in the same plane, but imparted at diff points

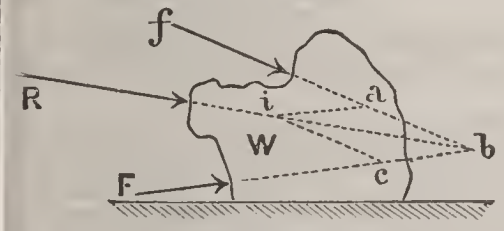


Fig 10 ¼

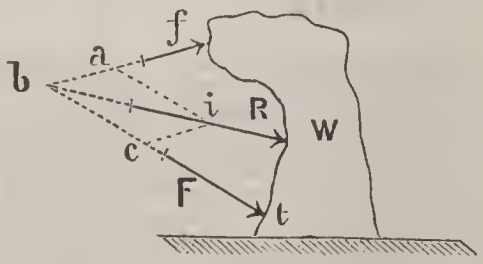


Fig 10 ½

of a rigid body W, may be found as readily as when imparted at the same point; as at *o*, Figs 9, or Fig 10.

Thus, produce their lines of direction, either forward as in Fig 10¼: or backward as in Fig 10½ as the case may require, until they meet, as at *b*. Make *b a* by any scale, equal to the force *f*; and *b c* equal to the force F. From *a* and *c*, draw lines respectively parallel to *b c* and *b a*; thus completing the parallelogram of forces, *b a i c*. The diag *b i* of this parallelogram, measured by the same scale, will represent the reqd resultant R, both in quantity, and in direction. It is thus seen that it is not necessary that the point *b* shall be in the body itself.

Rem. 2. It is perhaps almost useless to again remind the young student that the bodies are all along assumed to be rigid; or inelastic, and incapable of being broken or bent by the imparted forces. For otherwise the force *f*, in Fig 10½, might split off the top of the body; or F might crush to dust its toe *t*; or both might penetrate it. But, assuming that the material is sufficiently strong to resist such splitting, crushing, and penetration, we at present confine ourselves to the effect of the forces, whether as motion, push, or pull, upon the body *as a whole*. The splitting, crushing, &c, is a matter that must be considered under the head of *Strength of Materials*. It is of course quite as necessary in practice to pay attention to these effects as to the others, but it must be done by a separate process.

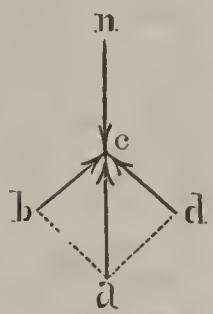


Fig 11

Art. 30. Since the effect produced upon a rigid body (considered as a whole) by the resultant (*a c*, Fig 11) of any two forces (*b c*, *d c*) tending to or from the same point, is the same as the joint effect of those two forces themselves, it follows that if we oppose to those two forces a third one (*n c*) equal to the resultant (*a c*), and diametrically opposite to it, that this third force will completely react against, balance, or destroy said two forces; or rather their remains. It is frequently necessary to consider such a third force, (*n c*,) equal and opposite to a resultant (*a c*); and inasmuch as we do not know that any specific name has been applied to it, although one is needed, we suggest **anti-resultant.** Resultant (*a c*) may be defined to be a single force which will produce upon a body considered as a whole, the same result that its components (*b c*, *d c*) produce. Or as a force which, if its direction were reversed, (thus making an anti-resultant,) would balance its components.

In the preceding Figs, the arrows represent *pressures;* if all the arrows be reversed, thus indicating *pulls*, the principle and processes remain precisely the same; for force is still only force; and its effect upon a rigid body, considered as a whole, is the same whether it act as a pull, or as a push. See Art 18.

When the forces diverge from the same point, their strain is a pull, or a tension; when they converge toward it, a push, or pres, or compression.

Art. 31. By a process the reverse of that in Art 28, any single force, *o d*, Fig 12, may be resolved into two component ones, *n d*, *m d*, one on each side of it, and in the same plane with it; which would produce the same effect as it upon a rigid body, *d*, (considered as a whole,) by merely drawing from *d*, 2 lines *d g*, *d t*, showing the *directions* of the two forces; and then, drawing from *o* two other lines *o n*, *o m*, respectively parallel to *d g*, *d t*; thus completing the parallelogram (*d n o m*) of forces, upon *o d* as its diag. Then measure *d n*, and *d m*, by the same scale as *o d*; and they will give the *amount* of each of those forces.

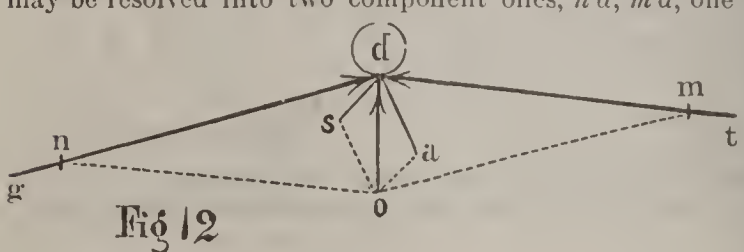


Fig 12

It is plain that an infinite number of differently proportioned parallelograms, such as *d n o m*, *d s o a*, &c, may be drawn upon any line o *d* as a diag; and in any one of them, two adjacent sides will represent components equal in effect to the single force o *d*, represented by the diag. Thus the forces *n d*, *m d*, are equal to o *d*, as regards their effect upon a rigid body *d*, as a whole. So are also the forces *s d* and *a d*; consequently the effect of *s d*, and *a d*, is equal to that of *n d*, and *m d*. It will be observed that the longer any two components on the same diag are, (as *n d*, *m d*, longer than *s d*, *a d*,) the more nearly in a straight line, and more directly opposed to each other, do they become: and consequently the more nearly do they mutually destroy each other; leaving smaller portions of each to act upon the body. Thus the portion of the great forces *n d*, *m d*, left to act upon the body *d*, is no greater than that of the small forces *s d*, *a d*; this remainder being in both cases represented by the resultant *o d*.

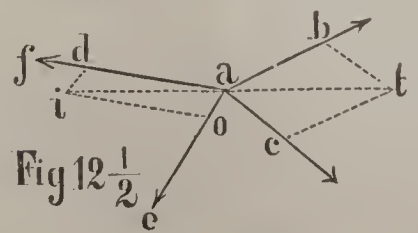


Fig 12½

Rem. Hence, if we have two forces, as the two pulls *a b*, *a e*, Fig 12½, whose amounts and directions both are given; and which are counteracted, or held in equilibrium, by two other forces such as the two pulls *a f*, *a e*, whose *directions* alone are known, it becomes easy to find the *amounts* *a d* and *a o* of these last, thus: Complete the parallelogram *b a c t*; and draw its diag *a t*. Make *a i* equal to *a t*, and in a line with it. Complete the parallelogram *a d i o*; then plainly *a d* will be the amount of the force in the direction *a f*; and *a o* that in the direction *a e*.

Art. 32. It follows from the foregoing articles, that a single force cannot be resolved into *two* components, one of which only is in the same direction as that force itself; for if a line representing that force be taken as a diag, it is self-evident that no parallelogram can be drawn upon it which shall have any of its sides parallel to said diag.

Therefore a rope, as $a\,b$. Fig 15, sustaining a wt w, so long as it remains perfectly vert, that is, precisely in the direction of the force of gravity of the wt, will receive no assistance in upholding the wt by having added to it a single rope as $o\,b$, or $b\,y$; or one extending from the wt itself in any inclined direction. In other words, a perfectly vert rope cannot sustain one part of a load, and one inclined rope another part. All this, indeed, is a result of the fact stated in Art 15; that any force, however great, (as the vert force of an immense suspended weight w, Fig 15.) will be turned out of its direction by any other force, however small. (as a slight pull from a rope $o\,b$, or $b\,y$,) unless there be some third force to prevent it. In the present instance, this third force might be a third rope; for the rope $a\,b$ will be relieved, and still remain vert, if we employ *two* oblique ones to assist it, *provided they be exactly opposite each other;* or, in other words, that all three ropes, or forces, be in one plane.

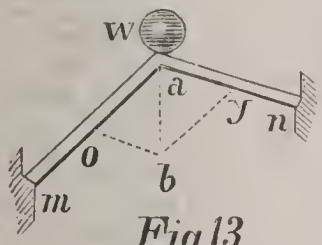


Fig 13

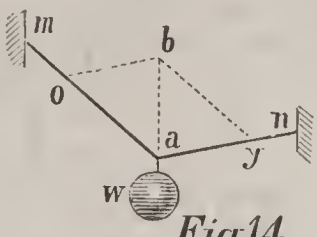


Fig 14

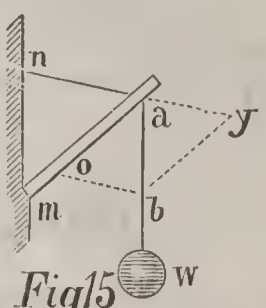


Fig 15

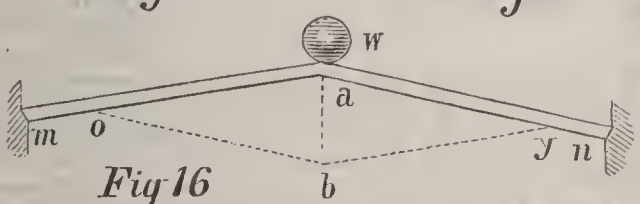


Fig 16

So also in the case of a vert post sustaining a load; the pres from the load cannot pass vert through the axis of the post, if the load at the same time is partly sustained by a single oblique brace pressing against the post. Indeed, such a brace, by turning away the direction of the strain from the axis of the post, may very materially diminish the power of the latter to sustain the load; for it will be found under Strength of Materials, that if the strain along a post or column does not pass directly through its axis, the column may in some cases lose two-thirds of its strength. The principle of course applies to force in any other direction, as well as vert.

A resultant may be greater or less than either one of its two oblique components: but it can never be greater, or even quite equal, to both of them: on the plain principle that any two sides of a triangle are greater than the third side. If the components are equal, and inclined to each other at an angle of 120°, the resultant will be equal to one of them; therefore, the same weight that would break a single vert rope, or post, would break two ropes each of the same strength as the single one, or two posts, inclined 120° to each other. If the angle $o\,a\,b$, or $y\,a\,b$, which either of the forces form with the diag $a\,b$, exceeds 90°, see Rems 5, of pp 321, 327.

Art. 33. The principle of the parallelogram of forces is of constant application in constructions of every kind; for instance, bridges, centers, roofs, retaining-walls, &c. Figs 13, 14, 15, 16, show a few of the most simple cases of force (the load w) applied to produce strain; by reacting against opposing forces $y\,a$, $o\,a$, presented by the walls. In all these, the load w, applied at a, is a single force of *gravity;* and consequently acts in a *vert* direction downward. It is to be resolved into two component forces in the direction $a\,m$, $a\,n$, in order that we may find the strains which it produces (according to the ordinary phraseology) along the pieces $a\,m$, $a\,n$, so that we may proportion their dimensions to resist those strains; which strains are in fact produced by the reactions of the *three* forces, of the load, and the two walls. To do this, in all the figs, from a draw a vert line $a\,b$, to represent the *direction* of grav, or of the force in the load w. On this line, lay off by any convenient scale, the dist $a\,b$ to represent the *amount* in lbs, tons, &c, of the load w. Also, from a draw the two lines $a\,m$, $a\,n$, in the directions of the reqd component forces. Then complete the parallelogram of forces, by drawing lines $b\,o$, $b\,y$, from b, respectively parallel to $a\,m$, $a\,n$. Then will $a\,o$, measd by the same scale as $a\,b$, give the amount of strain, whether push or pull, which the load w produces along the piece $a\,m$; and in like manner will $a\,y$ give the amount which it produces along the piece $a\,n$.

It must be especially borne in mind, that we here speak only of the amounts and directions of the strains produced by the extraneous load w alone; without reference to those produced by the weight of the pieces themselves. If the force acting at a is not vert, but oblique, then the direction of $a\,b$ must of course be drawn oblique; but if the force at a is gravity or wt, it *must* be vert.

Caution. See foot-note, p 556.

Fig 16¼ shows that the strains *e t*, *e s*, are really due to the action and *reaction* of the wt *and the walls;* although we often speak of them as due to the load *l alone* which is represented by the diag *e i*. We have said that a force cannot produce strain unless there is opposing force to strain against. Now, when we place the force of the load *l* at the point *e*, it is evident that it is upheld by the walls at A and B; or in other words, that it reacts against these walls; and the walls against it. The wall A furnishes the force indicated by the arrow A; and which may be considered as the resultant of the hor force *c*; and of the vert one *o*. So also the force B; as the resultant of *m* and *n*.

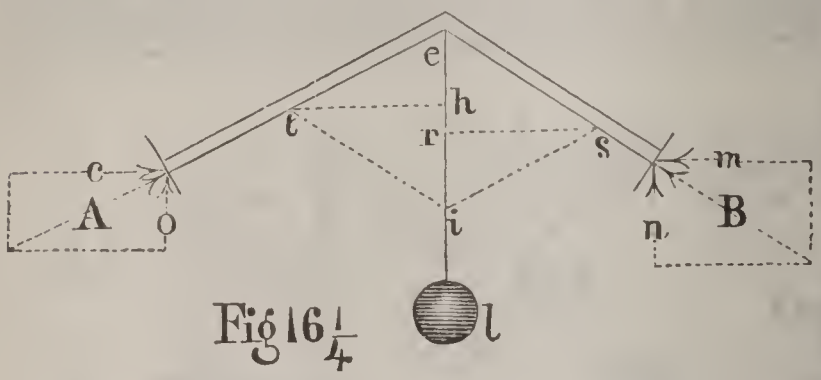


Fig 16¼

Now these forces A and B are applied at the point *e*, just as well as the load *l* is; for they pass up as pushes, along the rafters; as the force of *l* passes up as a pull, along the rope. The rafters and rope are merely the mediums through which the three forces reach *e*; and the forces in passing through them from end to end, of course produce in them strains respectively proportionate to the forces. Now, the forces *e t* and *e s*, which are usually said to be produced by the load, are nothing more or less than the two forces A and B, produced by reaction of the walls; and which, for convenience of drawing the parallelogram of forces in practice, are laid off each way from *e*. We have then three forces *t e*, *s e*, and *e i*, all acting at *e*, to produce *strain* alone; and this they must do by straining against each other.

The following is the manuer in which they do so. The two hor componcnts *m* and c, (which will always bc equal to each other; uo matter how different the slopes of the two rafters may be,) being diametrically opposite in directiou, react or strain against, or balance, each other; thereby produciug a hor strain, equal to one of theiu, throughout every part of each rafter. The two vert compoueuts *o* aud *n*, (however unequal they may be,) will *together* be equal to the load *l*; or to its representative *e i*; and having a direction cxactly opposed to it, they react against, or balance it; thereby producing in every part of the rafter *e s*, a vert strain cqual to *n*; aud iu the rafter *e t*, ouc cqual to o. Therefore, since *n*, is here greater than o, the rafter *e s* bears more of the load *l*, than the rafter *e t* does; and in the same proportion.

Thus, we scc that every part of each of the threc forces *e i*, *e t*, *e s*, produccs strain, by balancing an equal part of oue of the others. The walls really oppose to the load no force greater than its own; namely, o aud *n*, against *e i*. With thc hor componeuts *m* and c, the walls react only against each other.

As it is difficult, however, to introduce a new phraseology, in place of one which, although erroneous, is in universal use, we also shall speak of component *strains* like *e t*, *e s*, as if they were really produced by the resultant, or load, *e i*. And in alluding to resultant *motion*, we shall prohably often say they arc the effects of *components*, instead of effects of their *remainders*, after the components have partially destroyed each other's moving forces by straining against each other to produce change of direction.

Rem 2. The truth of such examples as Fig 14, with a rope or string, may easily be shown by means of two spring balances, to which the ends *m* and *n* of the string may be fastened. Suspend a weight *w* from the string, and the balances will show the strains along *a m* and *a n*. The balances must be held in inclined positions.

The student should try all such experiments. This one will show that in proportion as the two parts *a m*, *a n*, of the rope, approach nearer to one straight line, the greater will be the strain produced upon them by any given load, or force *w*; and so great will this bc, that if the weight *w* bc only one pound, two of the strongest men cannot strain the rope perfectly straight between them. Or if they stretch the rope alonc to as nearly a straight line as possible, and if then a weight of a few lbs be suspended from it, this small weight will pull the men closer together. Or if the rope bc stretched nearly straight between the two spikes so firmly driven as to require a great force to draw them, it will be found that a much smaller force applied as at *w*, will draw them readily. In other words, a rope so situatcd, and with force, or power *w*, applied to it in this manner, *between its ends, and oblique to its direction*, bccomes a *machine;* for by it *power may*, (to use the

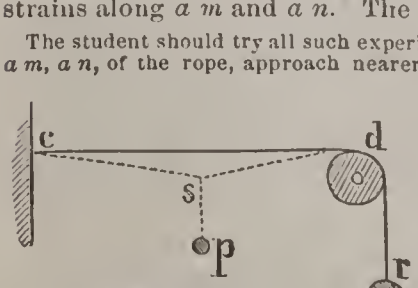


Fig 16½

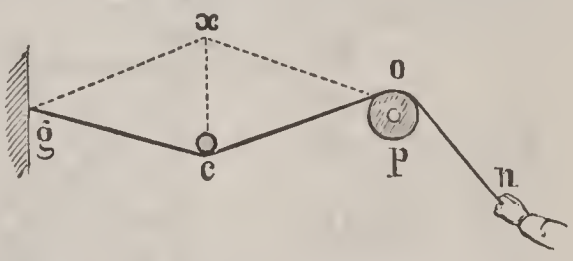
ordinary incorrect expression,) *be gained.* It is called the **funicular machine;** or sometimes simply the **cord.** Fig 16½ shows the principle on which this machine is frequently employed for overcoming a great resistance, *r*, through a short distance, by a small power *p*. One end c, of a rope *c d r*, is firmly fixed. The rope passes over a pulley *d*; and its other end is tied to the resistance, or load *r*. By applying a small downward force *p*, at the center of the rope, drawing it down to *s*, the load *r* is thereby raised a short dist; for the same great strain which the small force *p* produces from *s* to *d*, extends also down the rope, from *d* to *r*; except a slight loss produced by the friction of the pulley. Thus, the strain along the back-stays of a suspension bridge, is equal to that on the main chains just inside of the suspension piers; supposing the cables to rest upon rollers. In the *theoretical* consideration of ropes and chains, they are in most cases assumed not to stretch; to be perfectly flexible; without weight; and infinitely thin.

In such a machine the two parts *s c*, *s d*, Fig 16½, are to be considered as two entirely distinct ties, in the same manner as *a m* and *a n*, Figs 13 and 16, are two distinct struts. Each of these ties may have to sustain a different amount of strain, depending on their respective inclinations to *s p*. Thus if the load *p*, Fig 16½, be suspended from a perfectly frictionless pulley or slip knot resting on the perfectly flexible cord *c s d r*, and if this pulley or knot be at first placed near c or *d*, it, with its load *p* will descend by gravity along the cord until it comes to rest at *s*, which is the lowest point that the cord admits of its attaining and at which alone **the angles of inclination of s c and s d to s p become equal;** and the strains on the two parts will then be equal. But if as in Fig 14 the short string which sustains W is **tied fast** to the cord (so as not to move as the pulley did) at any point *a*, such that the angles of inclination of *a m* and *a n* to the diagonal *a b* shall be different, then the strains or pulls along *a m* and *a n* will also be different.

It is immaterial whether m and n, Fig 14, or c and d, Fig 16½, are at the same height or not.

For more on the funicular machine see p 344.

Let the end *g* of the rope *g c o n* be fixed; a power of 9 tons at *n*; the rope passing over a pulley at P; and bent out of line at c by a fixed pin. Make *c g* and *c o* by scale each equal to the power 9 at *n*; and complete the parallelogram; the diagonal *c x* of which is then found to be, say 6; or a resultant of 6 tons. Now, in this case, theoretically the strain lengthwise of the rope is everywhere equal to the power *n*, or 9 tons; and we have found that it produces also a strain *c x*, against the pin at c, of 6 tons. It also produces a pushing strain on the pulley P. Its amount may be found in the same way, by measuring 9 tons by scale each way from o toward c and *n*; completing the parallelogram; and measuring its diagonal resultant. But now let us use this rope as a funicular machine; and apply a power *x* c of 6 tons at c. We find that this 6 tons produces a strain *c g* or c o, of 9 tons along the rope; and this strain along *c* o will pass along to *n*; and thus the power of 6 balances a resistance of 9 tons acting at *n* in the direction *n o*.



The diagonal *c x* or any other will plainly be vertical only when the angles of inclination of *c g* and *c o*, with the horizon are equal. If they differ, both the direction and the length of the diagonal will change.

All will remain the same if the end *g* instead of being fixed, is passed over a pulley as at P, and a load or a pull equal to that at the other end is applied to it.

Rem. 3. The surfaces of contact of pieces used in construction, are called **joints.** When a piece is intended to resist compression, or push, it is called a **strut;** or if inclined, it is often called a **brace;** or if vertical, a **post, pillar,** or **column.** When to resist tension or pull, a **tie.** When to resist both tension and pull alternately, a **tie-strut,** or a **strut-tie.** A strut should be stiff or inflexible; but a rope, chain, or thin rod, may answer for a tie.

Rem 4. **To distinguish a tie from a strut** at a glance is sometimes difficult; but it may be done thus. From the point *a*, Figs 16¾, at which the force acts, draw a line *a c*, in the direction in which the force, if at liberty, would move away from that point. On any part, *a o*, of that line as a diag, draw a parallelogram of forces. Through the point *a* draw a line *i i*, parallel to the *other* diag *t t*. Then all the pieces which are on the same side of that line, that *a c* is, are struts; while those on the opposite side, are ties. We may also frequently determine, by imagining the piece to be a rope or chain, instead of a beam; and seeing whether it would then bear the strain. If it would it is a tie; if not, a strut.

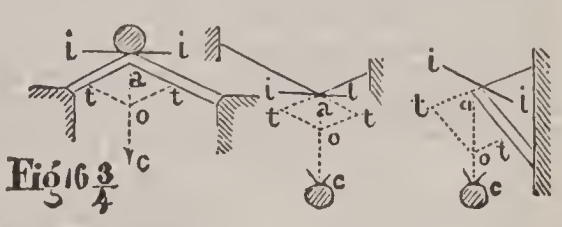


Fig 16 3/4

When a piece of material is used to resist forces which tend to bend or break it crosswise, or transversely of its length, as in Figs 47, 48, 49, 50, it is called *a beam;* such as joists, girders, &c. The same piece, however, frequently acts at once, as a beam, and as a tie, &c. Its own weight strains a beam transversely; but in our present illustrations of comp and res of forces, this strain, although frequently the most important one, could not be well considered at the same time.

Art. 34. Since any single force may be resolved into two oblique ones in the same plane with it, and which shall produce the same effect upon a rigid body considered as a whole, it follows that the single strain along any piece $a\,m$ or $a\,n$, of the four figs on p 323, may be thus resolved. In practice, it is frequently necessary to do this; and especially so for finding components at right angles to each other, in hor and vert directions.

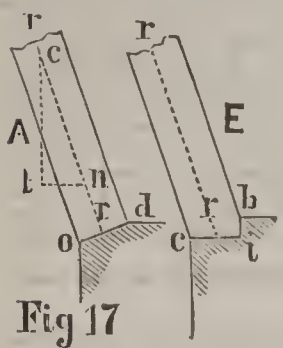


Fig 17

For instance, the joint o d, Fig 17, at the foot of the beam A, if made at right angles to the resultant $r\,r$ of all the pressures along the beam, of course receives the whole of these pressures; which consequently are all imparted to the abutment; leaving no portion unresisted, so as to produce sliding; or even a tendency to slide along the joint o d. Consequently, this joint is perfectly adapted to its duty. See Art 19. But a joint of the form of $b\,i\,c$, which is equally effective, is sometimes reqd for receiving a single strain (like that along A) along a piece E; and in order to properly proportion the vert and hor faces $b\,i$, and $c\,i$, of the joint, we must find the proportion existing between the vert, and the hor components equal to the single strain $r\,r$ along E. To do this is very easy; for we have only to lay off by scale, any length $e\,n$ along $r\,r$, to represent the single strain in that direction; and on it as a diag, from e and n draw vert and hor lines $e\,t$, $n\,t$, meeting in t. Then $e\,t$ measured by the same scale, will give the vert strain; while $n\,t$ will give the hor one. The parts $b\,i$, $i\,c$ of the joint, must consequently have the same proportion as these two components have to each other; bearing in mind, however, that since joints should be at right angles to the forces they have to sustain, the vert part $b\,i$ must bear the hor strain; and the hor part $i\,c$, the vert one.

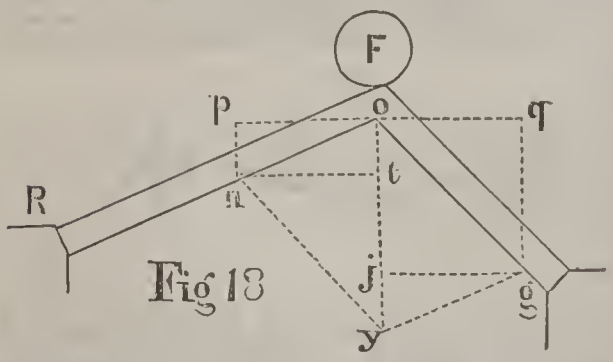


Fig 18

When, by Art 33, we are finding, by means of the parallelogram of forces $o\,n\,y\,g$, Fig 18, the total strains $o\,n$, $o\,g$, which an extraneous load F produces along two beams FR, Fg, it is easy at the same time to find the vert and hor components also; by drawing the two hor lines $n\,t$, $g\,j$, and measuring them by the same scale used for the diag $o\,y$. Likewise measure $o\,t$, and $o\,j$, for the corresponding vert forces at the joints; because when $n\,t$ and $g\,j$ may be drawn *inside* of the parallelogram, (which is not always the case; as see Fig 18¼.) the component forces in the direction of any diag, whether vert or not, are measured respectively from the point o, where the extraneous force F is imparted to the beams; to those points t and j, where the diag is met by the equal lines $n\,t$, $g\,j$.

Rem. 1. It is an important fact that however diff may be either the inclinations, or the lengths of the two beams; or how diff the total strains in the directions of their respective lengths; the *hor* strains, caused both by the extraneous load and by the weights of the beams themselves, will always be equal on both of them. Thus, in Fig 18, $n\,t$ is equal to $g\,j$; and in Figs 13 to 16, if hor lines be drawn from o and y, to $a\,b$, those in any one fig will be equal to each other.

Rem. 2. It is plain that each beam may be considered as receiving from the load F, **either one force or its two components.**

The vert component $o\,j$, of the triangle $o\,g\,j$, being longer than $o\,t$, of the triangle $o\,t\,n$, shows that the beam $o\,g$ bears more of the vert force or weight of the load F, than oR does; and in the same proportion as $o\,j$ is to $o\,t$. The two components on the diagonal, (when inside of the parallelogram,) will always together equal the length of the diag, or the weight F. But as $n\,t$ and $g\,j$ are of the same lengths, they indicate that both beams are pressed *sideways*, or *hor*, to the same extent.

When we come to treat on trusses, we shall find this method of obtaining vert components, by means of $o\,t$ and $o\,j$, very useful. The lengths of the beams o R, $o\,g$, do not affect the amount of strains produced upon them by the load F at their summits; but as their own wts must increase with their lengths, the strains arising from them must increase also; but we have not yet taken their own wts into consideration; neither are we yet prepared to do so.

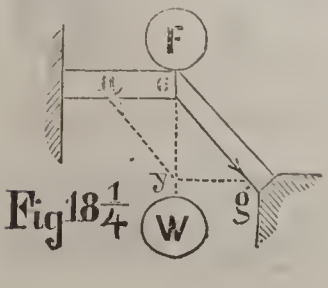


Fig 18¼

Rem. 3. If, as in Fig 18¼, one of the beams, as $n\,o$, is hor, it is plain that all of the diag $o\,y$, that is, all of the weight F or W, is borne by the other beam $o\,g$; and $n\,o$ sustains hor strain only. The beam $o\,g$ of course bears an equal hor strain also, as shown by $y\,g$, equal to $n\,o$.

Rem. 4. It is immaterial (Art 18) whether the load rests on top, as F; or is suspended below, as W; for in either case it is simply vert force imparted at o.

Rem. 5. When one of the forces, as $n\,o$, makes an angle $n\,o\,y$, greater than 90°, with the diag $o\,y$, the positions of the beams $o\,n$, $o\,g$, become as in Fig 18½, and we have a case like Fig 9¾; that is, the hor lines $n\,b$, $g\,a$, from the angles n and g, and at right angles to the diag, cannot be drawn *inside* of the parallelogram. Therefore we must extend the diag both ways, to a and b. If we wish to consider each of the forces $o\,n$ and $o\,g$ as made up of two components; then for those of $o\,n$, we have $b\,n$, and $o\,b$; and for those on $o\,g$, we have $a\,g$ and $o\,a$. Hence when the angle $n\,o\,a$ exceeds 90, **the vert strain on $o\ g$ is greater than the load w,** which (according to the ordinary phraseology) produces it. But the part $a\,y$ of the vert $o\,a$, has no reference to the load; but represents an upward vert force produced by the wall M, to balance a downward vert one equal to $b\,o$ from the wall P. This excess $a\,y$ over the diag, occurs only when one of the beams forms an angle greater than 90° with the diag. We call the attention of the student to this case, because we do not remember to have met with it in any book. It will perhaps be a new idea to many, that the vert pres on the wall M, can be greater than the entire load.

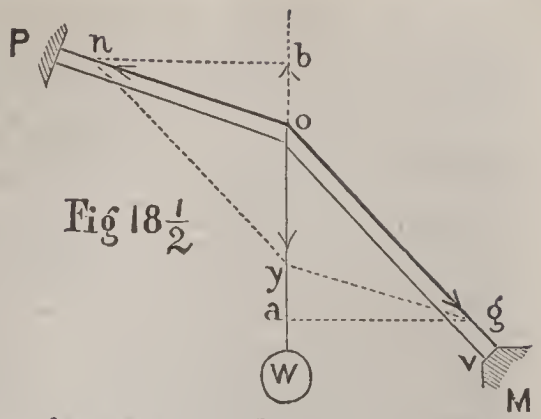


Fig 18½

Art. 35. As a simple practical example of very common occurrence, of the application of the foregoing principle of finding the resultant of two forces in the same plane, and tending to one point; let S, Fig 19, represent a block of stone weighing 3 tons; and standing on a *hor* base $m\,n$; but not attached to it in any way by cement, &c., but with a stop at n, merely to prevent sliding toward b.

In this case there will be no force acting upon the body in such a way as to prevent its being overturned around its toe n as a turning point, except its wt, or force of gravity, which always acts vert downward. By Arts 56, 57, all this force may be considered to be concentrated at the cen of grav i of the stone; and as acting at any point whatever in its vert line of direction $l\,g$.

Now suppose a pres $f\,h$, of 2 tons, (which may be either one simple force, or the resultant of many forces,) to be imparted to the stone, in the same plane with the force of gravity; which it will evidently be only if its direction $f\,c$ meets the direction of gravity $l\,g$, at some point; as, for instance, at a; because then a plane surface would coincide with both directions. Art. 8. The question is, which of these two forces will prevail; the two tons of $f\,h$, to overturn the stone; or the 3 tons of gravity to prevent its being overturned? By Art 29, both forces may be considered to be imparted to the rigid stone, at the point a, where their lines of direction meet; and we may make $a\,c$ equal to 2 inches, ft, &c, to represent the amount and the direction of the 2 tons of pres of the force $f\,h$: and $a\,v$, by the same scale, for the direction, and 3 tons of pres of the weight of the stone. From c draw a line $c\,d$ parallel to $a\,v$; and from v draw $v\,d$ parallel to $a\,c$; these will meet at d: thus completing the parallelogram of forces $a\,c\,d\,v$. The diag $a\,d$ of this parallelogram, measd by the same scale, will give about 2¾ tons for the single resultant force, which would by itself produce upon the rigid stone the same effect as gravity and $f\,h$ combined. This force by Art 18, may be considered as imparted to the stone at any point in the line of its direction *(w o)* through the stone; as a *push* at w, as shown by the arrow $z\,w$; or as a *pull* at o, by means of a rope $b\,o$, fastened to the stone at o. Since this resultant is supposed *to take the place* both of gravity and of $f\,h$, the two last must of course be considered as annihilated; so that the stone becomes as it were an *unresisting body of matter without weight;* and acted upon by the force $a\,d$, or $z\,w$, which must of course move it, and thus compel it to overturn around n as a pivot.

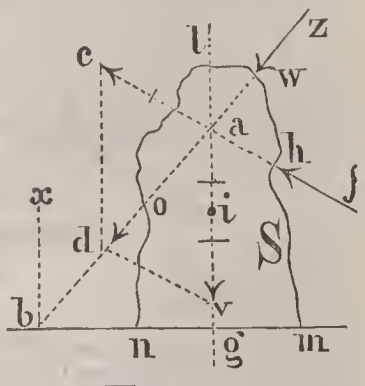


Fig 19

Rem. 1. Had the direction of the resultant $a\,d$ struck the base of the stone *at* n, instead of striking *outside* of the base as at b, the stone would barely have stood; because then the resultant, on leaving the body at n, would have encountered the resisting force of the ground on which the stone stood, acting upon the body at that point. Had the direction struck *within* n, that is, between n and m, the stone would stand still more firmly; and the more firmly in proportion as it strikes nearer g, where the direction $l\,g$ of the gravity of the stone meets the base. The direction of a resultant may strike within the base; and the body remain firm, so far as regards *overturning;* but yet may *slide.* See Art 63; *very important.*

This example shows also the necessity for assuming at times that bodies are rigid, or unbreakable. For in the case of stones of but little strength, the application of the great force of the resultant so near to n, would break the body at that point; and might, besides, mash n into the yielding earth on

which it stood. A knowledge of the direction of the resultants of forces acting on bridge abutments, retaining-walls, &c, is therefore of use also by enabling us to guard against such accidents, by selecting the strongest stones for the most strained parts of the structure; as well as by adopting extra precautions in preparing those portions of the earth foundations, upon which those parts rest.

It must be remembered, however, in such cases as the foregoing, that with the exception of the point o, at which the resultant *leaves* the body, $a\,d$ is not the direction which the resultant *actually follows in* the body; but is one which we may assume it to have, so long only as we assume the body to be *practically* rigid: that is, that it cannot be in any way broken, bent, or have its form changed, by the forces *actually imparted*. Frequently we cannot safely assume a mass of masonry to be thus rigid, for it may be composed of many separate pieces merely placed in contact with each other without mortar, as in dry masonry; or even if mortar or cement be used to unite these pieces, it may not have time to set or harden properly, before the deranging forces are brought to bear upon it. In that case, although the resultant might fall entirely within the body, and within the base, thus denoting perfect security to a *rigid* body; yet the structure might be completely destroyed by the sliding or other derangement of *its parts among one another*, under a force much less than would be required to overturn it. On this account, if we wish to obtain security at the least expense, we must frequently trace the actual curved direction of the resultant through its entire course; so that we may at every point of it place the joints of our masonry at right angles to it, as in Fig 7; or adopt other precautions to prevent the parts of the structure from separating. See Art 72.

Rem. 2. If in Fig 19 we suppose strong mortar or cement to exist between the base of the stone and a rigid foundation of masonry or rock, upon which we may assume it to stand, then it may not be overthrown, although the direction of the resultant $a\,d$ falls outside of the base $m\,n$. For then a third force, namely, the cohesive strength of the mortar, is brought to act upon the stone; and the resultant of *all three forces* may fall within the base. In all cases where a body remains at rest, notwithstanding that the resultant of the forces falls outside of its base, (whether the base be hor, vert, or inclined,) we may be certain that it is because some other force, which we have neglected, is acting upon it at the same time; for when the direction of *all* the forces passes beyond its base, and is consequently *force unresisted*, the body *must move*. See Rem, Art 65; also see Art 72. When one body is thus cemented to another, the two become in fact *one* body, so long as the cement does not give way under the imparted forces: so that a problem which is one in *Statics*, if there is no cement, may become one in *Strength of Materials*, when there is cement. The cement takes the place of natural cohesive force between the bodies which it unites.

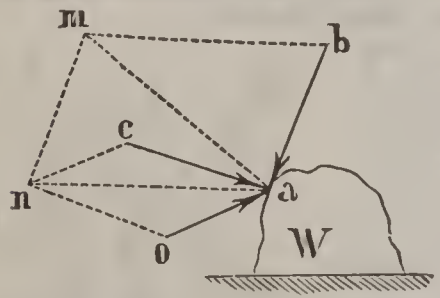


Fig 19¼

Art. 36. When the number of forces in the same plane, whether tending to or from the same point or not, is greater than two, **their resultant may be found in the manner already given for two.** Thus, with the three forces $b\,a$, $c\,a$, $o\,a$, Fig 19¼, first find the resultant of any two of them: as, for instance, the resultant $n\,a$, of $o\,a$, and $c\,a$. Then consider $o\,a$ and $c\,a$ as removed and $n\,a$ as taking their place; and then find the resultant $m\,a$, of $n\,a$ and $a\,b$; then is $m\,a$ the single resultant that will produce upon the rigid body W, the same effect as the three forces $o\,a$, $c\,a$, $b\,a$. If the three forces are imparted at diff parts of the body, proceed as in Figs 10¼ and 10½. If the number of forces be greater than 3, the process is precisely the same; find 1st the resultant of two of them; then the resultant of the 1st resultant and 3d force; then the resultant of the 2d resultant and 4th force; and so on to the end.

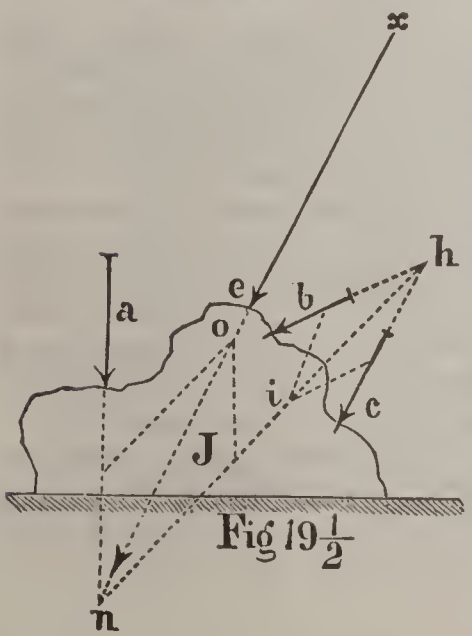


Fig 19½

Fig 19½ illustrates a case in which three forces a, b, and c, in the same plane, do NOT tend towards the same point. We may begin with any two of the forces at pleasure. We will take b and c; and, as at Fig 10½, prolong them backwards to h, and find their resultant $h\,i$. Then prolonging $h\,i$, and the third force a to meet at n, we lay off from n two sides of the parallelogram equal respectively to a and to $h\,i$, and complete the parallelogram $n\,o$. Then the diagonal $o\,n$ is the required resultant, to be applied to the body J at e, as shown by $x\,e$. This resultant would then by itself produce upon the rigid body considered as a whole, the same effect as would the three forces a, b, c. Or if its direction were inverted, so as to *pull*, instead of *push* at e, it would become an antiresultant to the three forces, and thus hold them in equilibrium.

The same process applies to any number of forces.

Art. 37. It sometimes happens, after having found the resultant of all the forces except the last one, that said resultant and remaining force are in the same straight line. Thus, with the forces u, v, w, Fig 20; the resultant r of u and w, is in the same straight line with the last remaining force v; and of course no parallelogram of forces can be drawn which shall have v and r for two of its sides. When this happens, if v and r are of diff lengths, we have the case given in Art 16, of two unequal forces meeting in the same straight line; but in opposite directions along it. Consequently, the small one, and an equal part of the large one, mutually destroy each other; and the remainder of the large one is the resultant of the two.

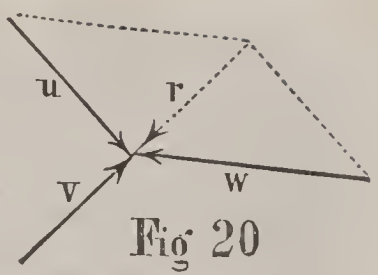


Fig 20

But if v and r are of the same length, then we have the case of two equal opposing forces, which mutually destroy each other entirely; and the body remains at rest. Consequently, there is no resultant in this case; for no single force can have the effect of keeping a body at rest; but will always move it. In other words u, v, and w, are then in equilibrium.

Art 38. The polygon of forces. The resultant of any number of forces in the same plane; may be found by means of the polygon of forces, thus: Let a, b, and c, be three such forces; whose resultant R is to be found. Begin with any one of them, as a, and draw a', parallel, and equal to it; and place an arrow-head at the proper end of it, to show its direction. From this arrow-head, draw b', equal and parallel to b; placing an arrow-head at its end. From this second arrow-head, draw c', equal and parallel to c; and so on with any number of forces; taken in any order. Finally, from the arrow-head d of the last of the forces, draw a line A to the butt-end, n, of the first one; thus closing the figure; and place an arrow-head as on the others. Now, this closing line A, or $d\,n$, with its arrow, represents both in quantity and direction the antiresultant of the three given forces; or if its arrow be reversed, it will represent their resultant. Consequently, we have only to draw from o a line, o R, parallel to A; and to make R equal to A; but pointing in the opposite direction. Then is R the resultant. **This process will give different figures,** according to which force we begin with; or whether we take the forces in right, or left-hand order; still, A will always come out the same in all of them. If the three given forces (or any greater number, as the case may be) had been in equilibrium with each other, that is had mutually *destroyed* each other's tendency to cause motion, they of course could have no resultant, or single force that would produce an equal effect; because a single force, if the only one acting on a body, must produce motion. When this is the case the forces will of themselves form a closed polygon. In either case some of the lines may cross each other as do a′ and c′ at A, or not, as at N below. **If the forces do not all act through the same point,** see Art. 40, p 332.

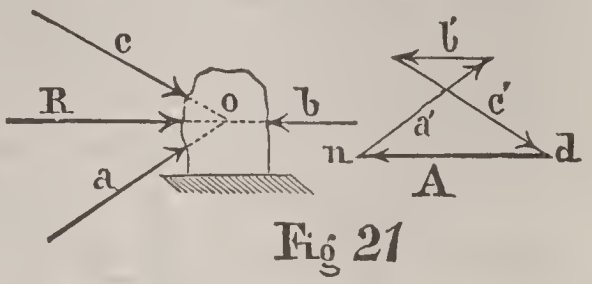


Fig 21

When any number of forces are in equilibrium, any one of them is the anti-resultant of all the rest, because it keeps them all in equilibrium; Also, any number of the forces balance all the rest.

If any number of forces, as $a'\,b'\,c'\,d'$ Fig 22, (whether they act through one point, as s, or not) are in equilibrium; and if we know the *directions* of *all* of them, and the *amounts* of all but *two;* then the polygon N will give us the *amounts* of those two. Suppose we know the amounts of a' and b' and require those of c' and d'. First draw by scale the known ones a and b (see polygon N) parallel to their actual directions, and then complete the polygon with lines c and d, respectively parallel to c' and d'. Then the *lengths* of c and d will give the *amounts* of c' and d' respectively.

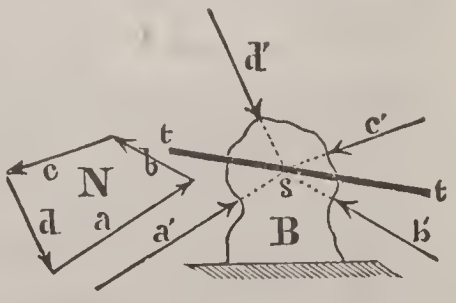


Fig 22

Any diagonal across a polygon of forces, represents the resultant of all the forces on either side of it.

If the forces are not all in one plane. the polygon is not in one plane, and cannot be drawn on a flat surface, but is twisted, or "**gauche,**" as is *o c t y*, Fig 36, p 333.

Rem. 1. It must not be inferred because forces balance *each other*, that therefore they balance *the body* to which they are imparted; for the body might move under the influence of other forces. Thus, the forces a', b', c', d', may be supposed to be the balancing forces of several persons holding a body at rest in a railroad car moving with great speed. Their forces prevent *each other* from giving motion to the body; but do not prevent the steam force of the engine from doing so. It is only when *all* the forces imparted to a body, including its own weight, are in equilibrium, that the *body itself* is also at rest. Or, if we hold a book, ruler, &c, vert between our thumb and forefinger; the opposite and equal pressures of the thumb and finger, hold each other in equilibrium, so that they cannot move the book either to the right or left, and the friction between them and the book, holds the gravity or weight of the book in equilibrium, so that it does not fall. So that so far as these forces are concerned, they produce no motion in the book; but we can move it vertically up and down, by introducing the third force of our wrist; or hor by stretching out our arm; or by walking; none of which will interfere with the equilibrium of the other forces; they only prevent *each other* from producing any motion.

Rem. 2. A triangle being a polygon of 3 sides, if any 3 forces which form a triangle, be applied in one plane, to a body, and in directions parallel to the sides of the triangle; and tending either to or from one point; they will hold each other in equilibrium. And, vice versa, when we see a body kept at rest solely by the action of 3 forces which are not parallel to each other, we may be sure that those forces are proportional to the sides of a triangle drawn parallel to them; that they are in one plane; that they all tend either to or from one point; and that any one of them acts in the direction of an antiresultant to the other two. Moreover, each of the forces is proportionate to the sine of the angle included between the other two; so that if we know one of the forces, we can readily find the others if we have the angles. This is very often of use in practice; as in finding by calculation alone, the line of pressures through an arch; the pres of earth against retaining-walls, &c. It must be remembered that the wt of a body usually constitutes one of the forces to be considered as acting upon it. **This rem is very important.**

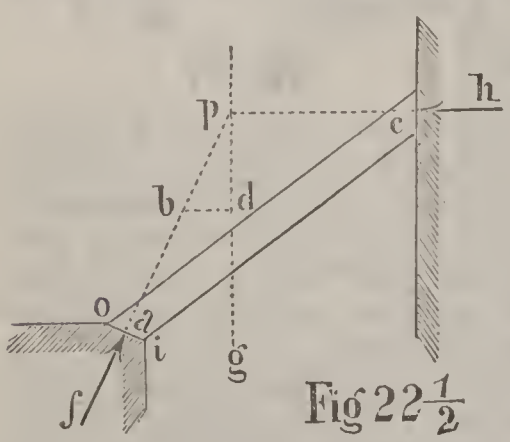


Fig 22½

Ex. 1. Let a c, Fig 22½, be a beam; its foot resting on o i; and its head c merely *leaning against* a smooth *vert* wall; and whether a c be unloaded; or whether it supports a load placed in any manner upon it, or suspended from it; let the vert line which passes through the cen of grav of the beam and its load (both of which are supposed to be known) be represented by $p\,g$. The beam and its load may be regarded as a single body, acted upon, and kept at rest, by three forces; namely, its own gravity or wt; the force h, at c; and the force f, at a. No other forces act on it. Now, gravity acts vert only; and in the case before us it may all be regarded as acting in the line $p\,g$. The force at c can act only at right angles to the surf or joint at that place, (see Art 19;) and since the joint is vert, the force h must be hor, or along $h\,p$. The question now is, how to find the direction of the third force f. To do this we must avail ourselves of the principle that when three forces, not parallel to each other, hold a body at rest, or in equilibrium, as these three forces hold the beam a c, their directions all tend to or from one point; which is either at the cen of grav of the body; or in a vert line passing through said cen. Hence, since the vert direction $p\,g$ of the force of gravity of the body; and the direction $h\,p$ of the force h, meet at p, therefore, the direction $f\,p$, of the force f, must also meet there. Hence we have only to draw a line $f\,p$, in order to find the reqd direction. A post intended to support the end a of the beam, should have the position $f\,a$; and the joint o i should be at right angles to $f\,a$; and not to a c, as might at first be supposed from Figs 13 and 16, in which the wt of the beams is not considered, and in which the extraneous weight w is applied only at the *joint*, a.

Having found the directions of the three forces in Fig 22½, it only remains to find their amounts. To do this, we already have one of them given, namely, gravity, or the wt of the beam and its load; and we know that they must be in proportion to the sides of the triangle drawn parallel to their directions. Consequently, if on the vert direction $p\,g$, we lay off by scale any portion whatever, as $p\,d$, to represent the force of gravity, then will the hor side of the triangle, $p\,d\,b$, represent by the same scale the hor pres at c; and the side $b\,p$, the oblique pres at a. The hor pres at the foot is equal to that at the head of the beam. It is of course included in the oblique pres f; which is compounded of said hor force, and of the vert force at a. The vert force is equal to the weight of the beam and its load; none of which is sustained at c; nor can be, so long as the joint at the head and wall is vert. See Fig 5½, p 552.

Ex. 2. This is very similar to the preceding. Let $a\,b\,i\,j$, Fig 22¾, be the half of any arch bridge, loaded or unloaded equally throughout; and of which we know in either case the total wt; and that the cen of grav of said wt is somewhere in the vert line $g\,g$. Now this half bridge is, like the preceding beam, kept in equilibrium, or at rest, by three forces only; namely, the wt; a hor pres h, at the crown, arising from the other half of the arch; and an oblique pres o, at the springing, or skewback $j\,i$. To find the directions and amounts of these forces, from a draw a hor line, meeting the vert $g\,g$ at c. From c draw a line to the center of $j\,i$; this is the direction of the oblique force o. From c measure down by scale any dist $c\,s$, on the vert direction $g\,g$, to represent the weight; and from s, draw $s\,t$ hor. Then $s\,t$, measd by the same scale, will be the hor pres h; and $c\,t$, the oblique one o. The joint $j\,i$, at the spring of the arch, bears all of $c\,s$; that is, all the wt of the half arch, and half

load. No vert pres or wt is sustained at the center *l n* of the arch; nothing but the hor pres. But *j i* also sustains this hor pres, for *c t* is composed of *c s* and *s t*.

The oblique force *c t* constitutes the total thrust exerted by the *entire* arch, against *each* of its two abuts; and the line *c t* shows the direction in which this thrust *enters* the abut at the skewback *j i*. After entering at that point, it begins to curve downward, on the principle explained in Art 72. Since *c t* is the hypothenuse of a right-angled triangle, of which the leg *c s* represents the half wt; and *s t* the hor pres of the arch, it follows that the total thrust *c t* of an arch may be found thus: *add together the square of half its wt; and the square of the hor pres*; *and take the sq rt of the sum.* This applies also to arches of iron or wood.

The joints of any arch which is a portion of a circle, are usually drawn toward the center of the circle; and, practically, this answers every purpose; but it is plain that strict theory would require the joint *i j* to be at right angles to c o. So also the other joints of the archstones would be reqd to be perp to the pres which they have to sustain.

Moreover, since the pres *c t*, upon the joint *j i*, is much greater than the pres *s t*, upon the joint *l n*, theory would require the joint *j i* to be proportionally deeper than *l n*; whereas in practice they are usually made the same, except in very large arches. See Stone Bridges.

The last few lines of Ex 1, respecting the hor pres at the foot, and at the head, apply equally here. The young student should familiarize himself thoroughly with the principle illustrated by these two examples, as it is one of very frequent application in practice; as in retaining-walls, abutments, &c.

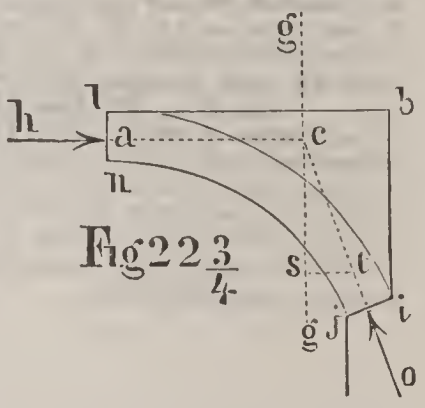


Fig 22 3/4

Here, as in the preceding example of the beam, we do not consider the strains produced along the length of the arch *l n j i* itself; but merely the two forces which, acting at its center *l n*, and at its foot *j i*, keep each other, and the wt of the half arch and its load, in equilibrium. For the others, see Stone Bridges.

Art. 39. A third mode of finding the resultant R, Figs 23, of any number of forces E, F, G, in one plane; **and acting through one point, *x*.**

Draw two lines, H H, and V V, at right angles to each other. From their point of intersection *o*, draw lines by any convenient scale, to represent the directions and amounts of the forces. By Art 34, resolve each of these forces into two component ones parallel to H H and V V. Thus F o is resolved into *u o* and *e o*; G *o*, into *m o* and *i o*; E o, into *t o* and *n o*. Then measure by the scale, and add together, those components *i o* and *t o*, parallel to H H, and which tend to move the point *o*

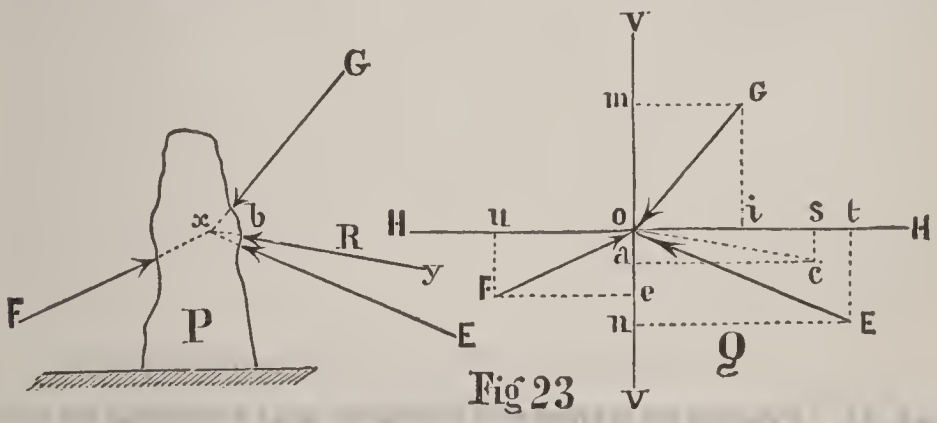


Fig 23

toward the *left* hand. Also add together those (in this case only *one*) *u o*, which tend to move *o* toward the *right* hand. Subtract the least sum from the greatest; their diff, equal to *s o*, will be the resultant of the two sets of forces which respectively tend to move o to the right, and to the left. In this instance, this *s o* must evidently be placed to the right of o, because the components on that side give the greatest sum.

Next, add together those components *e o*, *n o*, parallel to V V, which tend to move the point *o upward*. In like manner add together those (in this case only *one*) components *m o*, parallel to V V, which tend to move *o downward*. Subtract, as before, the least sum from the greatest; their diff, equal to *a o*, will be the resultant of the two sets of forces which respectively tend to move *o* upward and downward. In this instance, *a o* must be measd off *below o*, because the upward tendency is the greatest. By this process, then, we first reduce *all* the original forces to two components, *s o* and *a o*. This being done, we have only to complete the parallelogram of forces *o s c a*, and draw its diag *e o*; which will be the final single resultant of all the original forces. From *x* draw *x y* parallel to *c o*, and make *b y* equal to *c o*; then is *b y*, or R, the reqd resultant; and *b* the point for imparting it to the body P, so that its effect may be equal to that of the three original forces combined.

Art. 40. Even when any number of forces in the same plane do NOT tend to or from the same point, the principle of the polygon of forces, or of Art 39, may be used in precisely the same manner as at Figs 21 and 23, for finding the length and direction of their resultant. Or if they are in equilibrium, and hence can have no resultant, they will still form a closed figure as A Fig 21, or N Fig 22, as well as if they acted through one point. **There will however be this difference,** that when all the forces, as *a*, *b*, and *c*, Fig 21, tend to or from one point *o*, we know that their resultant, as R, must be applied parallel to A, and must tend to or from that same point *o*. In other words, **we know where its point of application must be.** And so with the resultant *co*, Art 39. But when the forces do not tend to or from one point, and we find their resultant by Art 38 or 39, we know only its amount and direction; **but do not know where to apply it.** In such cases we may use Art 36, Fig 19½. **If the lines,** representing any number of forces in one plane, **form a polygon, then we know** that those forces, *if they all act through one point*, are in equilibrium; and we know that if they do *not* all act through one point they are either in equilibrium, and hence have no resultant; or else they have *two* resultants, *equal* and *parallel* to each other, and acting in opposite directions; forming a "couple" (case 3, p 351) and causing the body to revolve around a point midway between them; while said point, and the body *as a*

CAUTION.

Page 333, lines 30 to 41. **Model representing four forces.** When there are but *three* forces, as in Fig. 30, the model, being a *triangular* pyramid, does not admit of change of form, so long as its sides remain flat. Hence, *such* a model, if correctly made, *necessarily* represents the forces in their true relative positions. But where there are *four* forces, as in Figs. 34 and 35, the completed model is a *four*-sided pyramid, and can be made to assume different shapes by changing the inclinations of its sides to each other. Each such change in the shape of the model gives a different representation of the relative positions of the original forces, and thus makes a difference in the position of the resultant, relative to the positions of the original forces; and, in most cases, a difference in its *amount* also. Hence, when there are four forces, we must know, not only the angle between each *two* adjacent *forces*, but also the angle between two *planes* formed by *three* of the forces; and must take care to so hold the model that its corresponding planes shall form that same angle.

Art. 41. Forces in different planes; but tending to or from the same point. Such forces cannot, like those in one plane, be correctly represented together on one flat surf, such as a sheet of paper. Thus, let Fig 27 be a cube; and tx, cx, ix, three forces acting in the directions of its edges; and all tending to the same point x. It is plain that the relative positions of these forces are not correctly represented; for txc, txi, and cxi, are in reality right angles; whereas, in the fig, txc appears to be an acute one; cxi, a right angle; and txi an obtuse one.

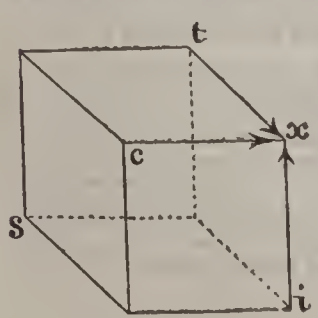


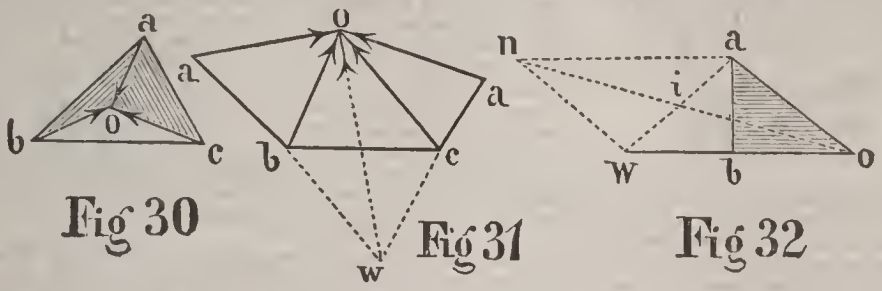
Fig 27

On this account the resultant of such forces cannot be had by measurement from a drawing. Recourse must therefore be had to calculation; which, however, will be facilitated by a drawing. The theoretical principle is very simple; being, in fact, the same as when the forces are all in one plane; namely, first find by Art 28 the resultant of any two of them, (for any two are really in one plane;) then find the resultant of this resultant and the third force; and so on to the end. It is easy to find the first resultant; but the others are more troublesome. Instances are comparatively rare, in which the resultants of such forces are reqd to be found: the attention of the engineer being generally confined to those in one plane; as when proportioning bridges, roofs, retaining-walls, &c.

Art. 42. To find the resultant of forces in different planes, but all tending through one point.

In cases where mathematical accuracy is not necessary, and the number of forces only three, or four, the writer will venture to propose a method by *models;* which, if open to the objection of empiricism, has the advantage of requiring less time than other processes; is sufficiently correct for most practical purposes; and shows the resultant *in its actual position*, which is done by no method of calculation.

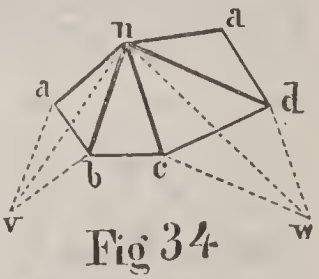
Let $a\,o$, $b\,o$, $c\,o$, Fig 30, be the three forces, meeting at o; their angles with each other, $a\,o\,b$, $b\,o\,c$, $c\,o\,a$, (which alone are necessary in this method,) being of course known. Draw on pasteboard the



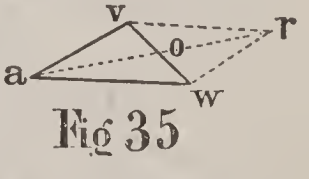
three forces $a\,o$, $b\,o$, $c\,o$, as in Fig 31, with their *actual* angles $a\,o\,b$, $b\,o\,c$, $c\,o\,a$. By Art 28, draw the parallelogram of forces for the middle pair $b\,o$, $c\,o$; and draw its diag $w\,o$, which will be the resultant of those two; leaving the resultant of it, and $a\,o$, yet to be found. Cut away neatly the whole fig, $a\,o\,a\,c\,w\,b\,a$. Make deep knife-scratches along $o\,b$, $o\,c$, so that the two outer triangles may be more readily turned at angles to the middle one. Turn them until the two edges $o\,a$, $o\,a$, meet; and then paste a piece of thin paper along the meeting joint, to keep them in place. Stand the model upon its side $o\,b\,w\,c$ as a base; and we shall have the *slipper* shape $a\,o\,b\,w$, Fig 32; $o\,w$ being the sole, and $a\,o\,b$ the hollow foot.

We now have the first resultant $w\,o$, and the third remaining force $a\,o$, in their *actual relative positions*. Now, to find *their* resultant, also in its actual position, cut a separate triangular piece of pasteboard of the size and shape of $w\,a\,o$. Find the center i, of the edge $w\,a$, and draw a line $i\,o$ on each side of it. Finally, by means of the edges $a\,o$, $w\,o$, paste this piece to the inside of the model, along its center-line $w\,o$. This done, $i\,o$ represents *one half* of the reqd resultant, in its actual position.

The reason why it represents but one-half of it is plain; for, as before stated, we now have $a\,o$ and $w\,o$ in their actual positions in the model; consequently, if we complete the parallelogram of forces $w\,o\,a\,n$, and draw its diagonal $n\,o$, this last will be their resultant. But since the two diags of every parallelogram divide each other into two equal parts, the diag $a\,w$ thus divides the resultant: consequently $i\,o$ is one-half the resultant.



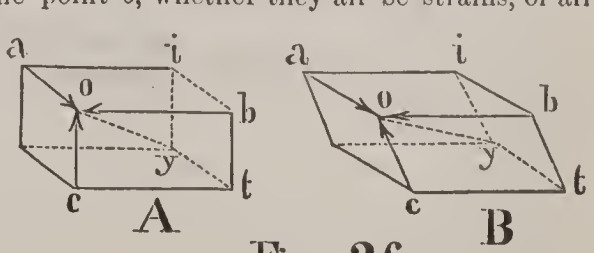
If there be four forces, as $a\,n$, $b\,n$, $c\,n$, $d\,n$, Fig 34, draw them as in the fig, with their actual angles $a\,n\,b$, $b\,n\,c$, &c. Draw also the resultants $n\,v$, of $a\,n$ and $b\,n$; and $n\,w$, of $n\,c$ and $n\,d$. Then cut out the entire fig, as before; and paste together the two edges $a\,n$, $a\,n$. Then we have the two resultants $a\,v$, $a\,w$, Fig 35, forming two simple forces, in their *actual relative positions;* and we have only to measure their dist apart from v to w; and thence find their resultant $a\,r$, which will evidently be that of the four original forces.



Or, as in the preceding case, cut out a separate piece of pasteboard, $a\,v\,w$, Fig 35, and having drawn on each side of it a line from a to the center o of $v\,w$, paste it inside of the model. Then will $a\,o$ represent *one-half* of the resultant of the four forces, in its actual position.

Should the model be exposed to hard usage by workmen, it should be made of wood; the triangles $a\,n\,b$, $b\,n\,c$, &c, being cut out separately; the joining edges bevelled; and then glued together. See also Art 43.

Art. 43. The parallelopiped of forces. If any three forces, $a\,o$, $b\,o$, $c\,o$, Figs 36, in diff planes, meet at one point o, whether they all be strains, or all motions, their resultant or joint effect will be represented, both in quantity and in direction, by the diag $y\,o$, of a parallelopiped $a\,c\,t\,b\,i$, of which three converging edges may be assumed to represent the three converging forces.



This suggests another mode of showing the resultant of three such forces by a model; for it is only necessary to prepare a box A or B, as the case may be; and $y\,o$ will represent the reqd resultant.

No three forces in different planes can be in equilibrium.

Art. 44. Forces in different planes; and *not* tending to or from one point. It is but rarely that such forces have a resultant, or anti-resultant; that is, no single force can usually be found either to produce an equal effect, or to balance them. It is so seldom that they present themselves to the engineer's attention, and their solution is so tedious, except in very simple cases, that we shall confine ourselves to one of that kind. As in Art 41, the resultants cannot be had by measurement from a drawing.

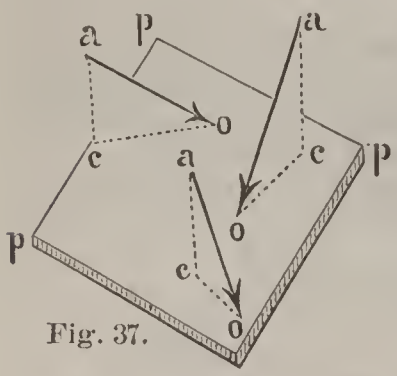


Fig. 37.

Let *a* o, *a* o, *a* o, Fig 37, be three such forces; and suppose them all to act *against* (see Remark, Art 8) the same plane *p p p*; and against the same side, or surf of it; that is, none of them pointing upward against the under side of the plane in the fig. Having the points o of application, and the relative positions of the forces themselves, as well as the angles *a* o c which they form with the plane *p p p*, resolve each of the forces into two components; one of which, c o, coincides with the plane; while the other (parallel and equal to *a* c, but meeting c o at o) is at right angles to c o, or to the plane. We then, have two sets of forces; one set in the plane, and the other at right angles to it. Since those in the plane do not tend to or from one point, their resultant must be found by Fig 19½,

while that of the several parallel components (equal to *a* c, but applied at o) may be obtained by Arts 56 and 59. These two resultants will rarely be in the same plane with each other, and consequently can have no joint resultant. If they should chance, however, to fall in the same plane, use Art 28 for finding their resultant. In simple cases, where the forces act against one plane, as in our fig, pieces of wire, cut to lengths to represent the forces; and stuck into a piece of smooth board, in their proper relative positions, will greatly facilitate the finding of the resultant approximately enough for most practical cases.

The same general process must be used, no matter how great may be the number and directions of the forces. A plane must be assumed to pass somewhere through the system: and the directions of all the forces must be conceived to be so extended as to terminate at points of application in said plane. Each force must then be resolved into two, as in the foregoing example; and the resultants of the two sets of forces, as well as their joint resultant, if they have one, must be found as before. If any of the forces should be *parallel* to the assumed plane, but not *in* it, it evidently cannot be resolved into two, one of which shall be *in* the plane; for (Art 32) no force can have one of its components parallel to itself. Hence, in such a case, the resultant cannot be found by this process.

Art. 45. It is comparatively seldom that strict mathematical accuracy is reqd in finding the resultants of forces in engineering practice; therefore, the foregoing easy methods by measurement from a drawing, or model, will usually answer every purpose. Moreover, they appeal to the eye; and are therefore much less liable to serious errors than methods involving numerous calculations. But when more correct results are needed, they may be had by means of a table of nat sines, tangents, &c. Thus, in the case of two components and their resultant, calling the components, Fig 38, C and *c*: and the resultant R, *then*

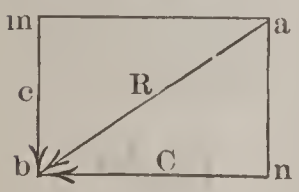


Fig. 38.

If the angle m b n between the components C, c is 90°, R will $= \sqrt{C^2 + c^2}$; and $C = \sqrt{R^2 - c^2}$; and $c = \sqrt{R^2 - C^2}$. Or R will = C ÷ cosine of a b n = c ÷ cosine of a b m. And C will = R × cosine of a b n; and c = R × cosine of a b m.

Or whether the angle between the components C and c be 90°, or more, or less, as in Fig 39,

$$C \text{ will} = \frac{R \times \text{sine of v}}{\text{sine of C b c}}; \text{ and } c = \frac{R \times \text{sine of x}}{\text{sine of C b c}}.$$

Observe that *v* is used for finding *C*; and *x* for finding *c*.

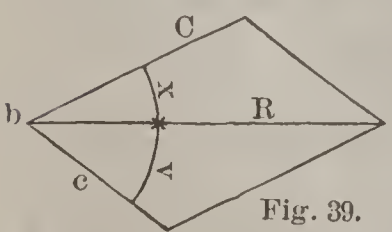


Fig. 39.

$$\text{And R will} = \frac{c \times \text{sine of Cbc}}{\text{sine of } x}; \text{ or } \frac{C \times \text{sine of Cbc}}{\text{sine of } v}.$$

If the angle Cbc, or either of the others exceeds 90°, subtract it from 180°, and use the sine of the remainder.

Art. 46. Moments. Leverage. If *a b*, Fig 40, represent any force acting in any direction whatever; and if *o*, or *i*, be any point whatever, whether in or out of the body on which the force is acting; and if from said point a line *o s*, or *i c*, be drawn *at right angles* to said direction of the force, then said line *o s*, or *i c*, is called the **arm, or leverage** of the force *a b*, *about said point.* And if the amount of the force in lbs, &c, be mult by the length of the arm or leverage in feet, &c, the prod in ft-lbs, &c, is called the **moment** of the force *about that point.* Thus, if the force *a b* be 8 lbs, or tons; and the line *o s*, 6 feet, then the moment of *a b* about *o* is $8 \times 6 = 48$ ft-lbs; or ft-tons. A force has of course no moment about any point *through which it passes.*

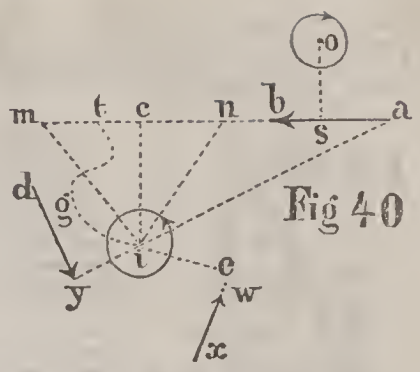


Fig 40

This moment represents the total tendency of the force to produce motion about the given point. We cannot hold hor. between the ends of a thumb and forefinger, a piece of stick a foot long, which has a 3 lb wt at the other end of it; because the tendency of the wt to produce motion is too great for the force of our fingers to resist; but we can in that manner hold a stick two feet long, with a 3 lb wt at each end, if we take it at the center. For although in this case there is twice as much moment as before exerted *at* our fingers; yet it is not now exerted *against them;* because we now have two equal moments in opposite directions, reacting *against each other;* and leaving nothing for the fingers to react against, except the mere vert *wt* of 6 lbs.

Since the moment about *o* tends to produce motion at that point in the direction in which the hands of a watch move, or from the left hand, toward the right, it is called a *right-hand* moment. But the moment of the same force, about the point *i*, tends to produce motion at that point, from right to left, as shown by the arrow-head on the small circle; hence it is called a *left-hand* moment. The moment of the force *d y*, with its leverage *y i*, about the point *i*, is a left-hand one; as is also that of *x w* with its leverage *e i*.

When the arm *o s*, or *i c*, instead of being merely an imagined *dist*, is a rigid *bar*, at one end of which, as *s* or c, the force is imparted; thus giving the bar a tendency to move around the point *o* or *i* as a fixed center, it is frequently called a LEVER; and the point *o* or *i*, the FULCRUM of the lever.

If the lever, instead of being like *c i*, at right angles to the direction *a m* of the force, should be oblique to it, as in *i m*, or *i a*; or should be curved, or bent in any way, as *i g t*; this in no way affects the leverage, or moment of the force; for the **leverage is always the perp dist,** *or in other words,* **the shortest dist from the fulcrum. to the direction of the force;** and is entirely independent of the length of the lever itself. **This is a grand fundamental principle of all levers. and leverages;** and the young student should carefully impress it upon his memory, inasmuch as it is of constant application in practice.

The fulcrum is not always at one end of the lever, but may be between the two ends; so that there are two arms. Cog-wheels are merely continuous circular levers, with the fulcrum at the center.

Art. 47. As a further illustration, let *a f b*, Fig 41, be a bent lever, turning on its fulcrum *f*; and *m* and *n* two wts suspended from its ends, constituting two forces acting in the vert directions *a n*, *b c*. Now, *f c*, at right angles to the direction *b c*; and *f a*, at right angles to the direction *a n*, are the arms, or leverages of the forces *m*, and *n*, about the point *f*.

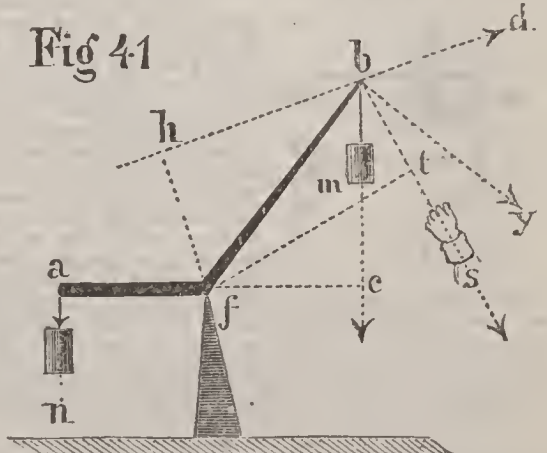


Fig 41

In the fig these leverages are equal, say each is 6 ft; and let each wt, *m* and *n*, be 100 lbs; then the right-hand moment of *m*, and the left-hand one of *n*, about *f*, are each $6 \times 100 = 600$ ft-lbs; and since both the forces, and *f*, *are all in the same plane*, it is evident that the two opposite moments, balance each other; although the *lever f b*, is much longer than *f a*.

Now suppose the wt *m* to be removed: and that instead of it, a person pulls in the direction *b s*, by means of a string fastened at *b*. With what force must he pull in order to balance the wt *n*? First measure the leverage *f t*, from the fulcrum, and at right angles to the direction *b s* of the new force; and suppose it is found to be 9 ft. Now we have already found the moment of *n* about *f* to be 600 ft-lbs; and we require the same moment on the opposite side; so that all that is reqd is to find what number the 9 feet leverage must be mult by, in order to make 600. This is plainly $\frac{600}{9} = 66.66$ lbs, for the pull which the person must exert; because $9 \times 66.66 = 600$.

So it is seen that with the same length of *lever, f b*, we can have diff *powers*, (so called,) or *lever-*

ages, according to the *direction* in which we apply our force to the lever. This, however, evidently has its limit; for the greatest power is gained (to use the popular expression) when we apply our force in the direction $b\,y$, at right angles to a line $f\,b$, drawn from the fulcrum to the outer end of the lever. If we apply it in the direction $b\,d$, we get only the leverage $f\,h$.

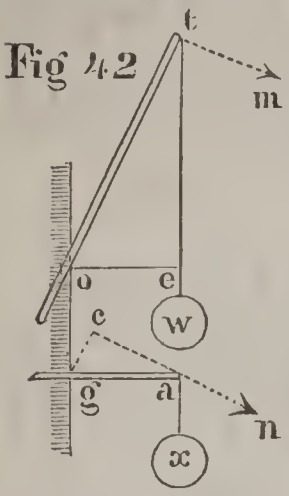


Fig 42

On the same principle as in the foregoing example, if $o\,t$ and $g\,a$, Fig 42, be two beams of equal scantling, but of diff lengths; with one end of each firmly fixed in a vert wall, and both sustaining equal suspended wts w, x; the moments of the wts about the points o and g will be equal, because the arms or leverages $o\,e$, $g\,a$, are equal. Therefore the wt w will have no more tendency to break off the long beam at o, than x has to break the short one at g. The wts of the beams themselves are not here taken into consideration; and this is always the case in speaking of levers, unless otherwise expressed. In very many cases, the wt of levers of two arms does not affect the result aimed at, *provided the arms are so proportioned as to balance each other when unloaded;* no matter what their comparative lengths or wts may be.

If, in Fig 42, we apply pulls to the beams, in the parallel directions $t\,m$, $c\,n$, at right angles to $o\,t$, then the leverages become changed from $o\,e$ and $g\,a$, to $o\,t$ and $g\,c$; and since $o\,t$ measures 6 times the length of $g\,c$, it follows that the beam $o\,t$ would be broken off at o, by $\frac{1}{6}$ part as much force in this new direction, as $g\,a$ would; for the leverage being 6 times as great, must be mult by only $\frac{1}{6}$ as much wt, in order to have an equal breaking moment.

Art. 48. In ordinary phraseology, the load, or resistance of any kind, which we wish to move, overcome, or balance, by means of a lever, is called *the weight;* while the force of whatever kind which we apply to accomplish this, is called *the* **power.*** Usually, but not always, the power is applied to the longer arm. Equilibrium, or balance, or equal moments in opposite directions, will then plainly take place, when the long leverage (not lever) has the same proportion to the short one, that the wt has to the power; because then only can the long leverage mult by the small power, have the same moment as the short leverage mult by the great wt.

This is seen in the common steelyard, Fig 43; which is merely an iron lever, turning on a fulcrum f, and having the wts of its two arms $f\,a$, $f\,b$, so proportioned as to balance each other when unloaded. Here the power, P 1, at the dist of two divisions from the fulcrum; balances the wt, W 2, at the dist of one division. If the wt W 2 were suspended at y, only half a division from f, it would balance the power P 1, suspended where W 2 is in the fig, at a whole division from f. If the power is reqd to *move*, or *overcome* the wt, it is plain that either the power itself, or the length of its arm, must be greater than when mere equilibrium is to be effected; in other words, besides the two *straining* forces, which by their mutual action balance or equilibrate each other, we need some *unresisted* force to impart *motion* to the inert matter.

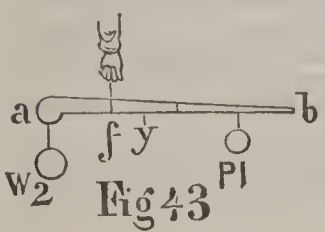


Fig 43

In the two levers of the same length, Fig 44, the leverage $f\,w$ of the wt w, is of the same length in both; namely, one division; but the leverage $f\,p$, of the power p, is but two divisions long in the upper one, and three divisions in the lower. Therefore a power of 1 lb will balance only a wt of 2 lbs in the upper one, and of three lbs in the lower. In the upper one, the power will move twice as fast as the wt; in the lower one, three times as fast. When the fulcrum is between the wt and the power, as in the upper one, the lever is said to be of the *first class;* when the fulcrum is at one end, and the power at the other, *second* class; fulcrum at one end, and wt at the other, *third* class. In all cases it is assumed that the fulcrum is in the same plane (Art 8) with the directions of both the wt and the power; otherwise the principles do not apply. When two weights balance each other on two arms of a lever, as a steelyard, or common weighing scales, &c, their directions are the vert lines passing through their centers of grav; and the same imaginary vert plane which coincides with those directions, coincides with, or passes through, the fulcrum also. When this is not the case, no equilibrium can exist. This may be readily proved by experiment; for we cannot balance a bow-shaped piece of stick or wire, so long as the bow is hor; for it will turn on the fulcrum, of its own accord, until the bow becomes vert; so that the same vert plane that passes through the fulcrum shall pass also through the cen of grav of each half of the bow. If all the forces acting on the lever are hor, or oblique, the imaginary plane must be so too.

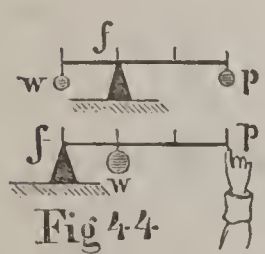


Fig 44

Rem. From what has been said respecting the lower Fig 44, it follows that when a load w is borne at any point of a beam $f\,p$ supported at both ends, then the portion of the load supported by each of these points f and p, is in the same proportion to the whole load that the respective portions $f\,w$ and $p\,w$ of the beam, are to the whole span $f\,p$ of the beam; *but inversely;* that is, the smallest portion of the load is borne by the support at p; and the largest portion by f. In the fig, $f\,w$ is one third, and $p\,w$ is two thirds of the clear length or span of the beam; hence, p supports $\frac{2}{3}$ of the load w; and f supports $\frac{1}{3}$. **Or as** $f\,p : w :: f\,w$ **: load at** p. **And as** $f\,p : w :: p\,w$ **: load at** f.

* The fact that by means of leverage a small power can be made to move a great wt, is in common parlance styled a **gain of power.** In a scientific sense the expression is absurd, yet in practice it has by its universal use become very convenient, and we shall therefore employ it. When the lever, instead of merely *balancing* the power and the weight, has to be *put into motion*, it is plain that there must be *some* excess of force applied at the power end, to produce the motion.

Art. 49. This example is the same as in Fig 19, where the question is solved independently of leverage and moments.

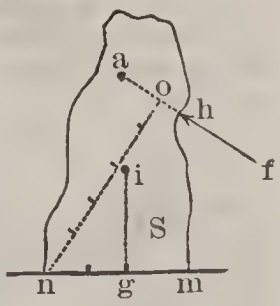


Fig. 25.

Let S be a stone of 3 tons; standing on a hard hor base *n m;* let *i* be its cen of grav; and let the dist *n g* from the toe *n*, and at right angles to the vert direction, *i g*, of the gravity of the stone, be 2 feet. Also let *f h* be a force of 2 tons, imparted to the stone at *h;* and let the dist *n o* from the toe *n*, and at right angles to the direction *f a* of the force *f h* be 5 ft. Will the force *f h* upset the stone around the toe *n*, as a turning point?

Here *n g*, or 2 ft. is the leverage of the force (3 tons) of gravity; consequently, the moment of the stone about the poiut *n*, is equal to 2 ft × 3 tons = 6 foot-tons; and this moment (which is called the *moment of stability* of the stone, about *n*) aloue tends to prevent the stone from overturning about *n*. Again, *n* o, or 5 ft, is the leverage of the force, (2 tons,) of *f h*; consequently, the momeut of *f h* about the poiut *n*, is equal to 5 ft × 2 tons = 10 foot-tons; and this moment alone tends to overturn the stone about *n*. Since the overturning momeut is the greatest, the stone will of course upset. The foregoing case resembles that of an abutment resisting the thrust of an arch; or that of a retaining-wall, sustaining the thrust of earth against its back; said thrust being supposed to be concentrated at its *center of pressure*. (Art 57.) It is analogous to a simple bent lever *e f o*, Fig 25¼, supported at its fulcrum *f*; around which it may revolve. The short arm *f e*, of 2 ft, is acted upon by the 3 tons wt of the stone; moment = 2 × 3 = 6 ft-tons. The long arm *f o*, of 5 ft, is acted upon by the 2 ton force *h*; moment 5 × 2 = 10 ft-tons; which, being greater than the 6 ft-tons of the stone, equilibrium cannot exist; and motion must ensue. The *wt* of the body S, in Fig 25, constitutes one of the forces acting upon it; while its inert *matter* constitutes the two lever arms at Fig 25¼. It frequently thus happens that a body is at the same time the resistance to be overcome; and the lever with which to overcome it. It is plain, that in the same manner as above, we may find separately the moments of any number of forces acting in the same plane, upon a body; and may afterward ascertain their united effects, by adding into one sum those which tend to overturn it; and into another sum, those which tend to prevent its overturning; the diff between these sums will be their joint effect.

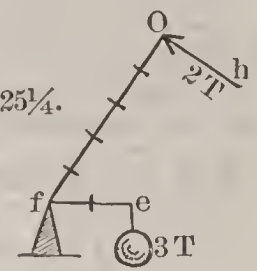


Fig. 25¼.

Rem. In Fig 25 we have supposed the body S to turn about a single *point*, *n*; but in practical cases they turn about *edges*, as *d y*, Fig 25½, the assumed turning edge of the body B; which may be regarded as a retaining-wall, or abutment, &c. In making calculations for the strength of such structures, it is usual to restrict ourselves, for convenience, to a supposed vert slice, one foot thick, like the shaded end of the fig; in which case the turning edge is one foot long instead of extending along the toe, *d y*, of the entire structure; (see Art 70.) This, however, causes no change in the calculations, which remain the same as for Fig 25; for we suppose all the wt of the slice to be concentrated at its cen of grav; and the forces to be imparted in the same vert plane with the direction of the gravity; so that it amounts virtually to the same thing as if we assumed our one-foot slice to be infinitely thin; but still to have the same wt as if it were one foot thick. We of course restrict ourselves, also, to the *forces* acting upon such 1 ft slice.

Fig. 25½.

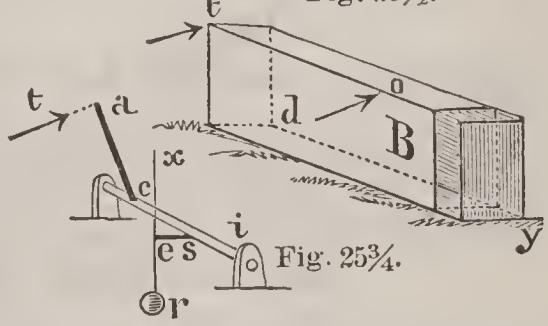


Fig. 25¾.

But in fact, it is not absolutely necessary in such cases, to suppose our applied forces to be in the same vert plane with the gravity of the wall, *provided that both our structure, and the base against which the edge* d y *bears, and revolves, may be regarded as practically rigid, or unyielding, under the action of those forces.* For, if there be no yielding whatever along the edge *d y*, then it is immaterial, so far as regards overturning, whether the force be applied at o, or at *t*; for *d y* then becomes analogous to the rigid axle *c i*, Fig 25¾, with two lever-arms *s e*, and *c a*. The wt *r*, may be supposed to be that of the structure; and from that common machine, the wheel and axle, we know that it is immaterial whether the applied force, and other lever-arm, are attached to the axle at *c*; or at any other point along its length; so long as the axle is equally unyielding at every point. Art 53 will perhaps make this more clear.

Caution. Although it is immaterial, as just explained, as regards *overturning*, whether the force, Fig 25½, be applied at *t* or at *o*; it is plainly *not* immaterial as regards a tendency to swing the body around horizontally, or as regards pressnres (and consequent friction) at the ends of the stick, Fig 25¾, or in the bearing *f f*, Fig. 45.

Art. 50. Equilibrium of forces, and of moments. The student must distinguish clearly between those cases in which **forces** hold each other in equilibrium, and those in which the **moments** of forces do so. Thus, if two equal forces n and R, Fig Y, act against each other in the same straight line n R, but in opposite directions, we correctly say that these two **forces** are in equilibrium, or balance each other, or prevent each other from giving motion to the body. But also in the case of a lever, one arm of which is say 10 ft long, and the other only 2 ft, we usually say that 2 lbs of force at the end of the long arm, will balance or hold in equilibrium 10 lbs at the end of the short one. But this is not scientifically correct; for a force of 2 lbs cannot possibly balance one of 10 lbs. It is actually the **moment** of the 2 lbs that balances the **moment** of the 10 lbs. That is, 2 lbs $\times$ 10 ft leverage = 20 ft-lbs, the **moment** of the 2 lbs, balances the 10 lbs $\times$ 2 ft leverage = 20 ft-lbs, the **moment** of the 10 lbs. As to the two *forces* 2 and 10, they are balanced by the upward 12 lb reaction of the fulcrum.

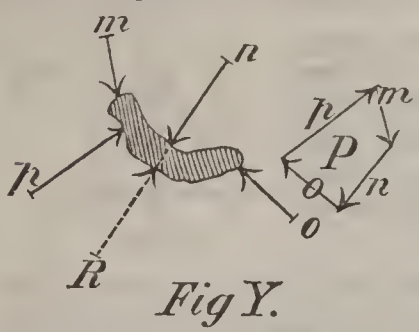


Fig Y.

Rem. When any number of **forces** as m, n, o, p, Fig Y, in the same plane, whether acting through one point or not, hold each other in equilibrium, then any one of them, as n, is equal to the resultant R of all the rest; and will be in the same straight line with it, but will act in the opposite direction. And this is the most ready **method** that suggests itself to the writer **for determining whether several given forces are in equilibrium or not.** Art 36 may be used for finding the required resultant R. See Art 40.

Art. 51. Equality of moments. This principle consists in the following: If any number of **forces** as a, b, c, d, Fig 26, all in the same plane, and acting upon a body N, in any directions whatever in that plane, hold each other in equilibrium, then if any point i be taken in that same plane, whether within the body, or out of it, the **moments** of the forces will hold each other in equilibrium around that point; that is, the moments of all those forces (b and d) which tend to turn the body N in a right hand direction, (or like the hands of a watch,) around the point i, will together be equal to the moments of all those (a and c) which tend to turn it around i in a left hand direction.

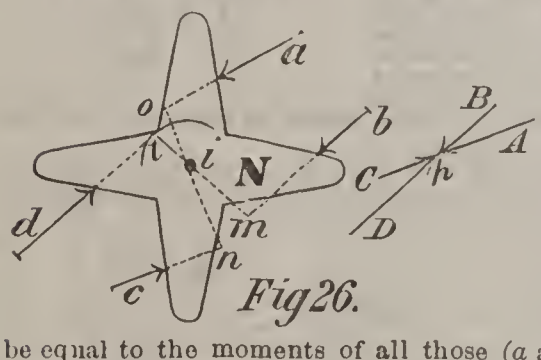


Fig26.

If the four **forces** a, b, c, d, mutually prevent each other from imparting to the body N, any tendency to move, as a whole, in any straight direction whatever, they are themselves in equilibrium: and this being the case, their right and left hand **moments** around the point i will also be found to be in equilibrium, as shown below. And so also with any other point in the same plane. But in that case the arms or leverages would be changed.

Forces.	Arms.	Right-hand Moments.	Left-hand Moments.
a=6	3.9	....	23.4
c=3	5.8	....	17.4
b=4	4.6	18.4	....
d=7.2	3.111	22.4	....
		40.8 Total.	40.8 Total.

But moments around a point may hold each other in equilibrium even when the forces themselves do not balance each other; as in the case of the lever in Art 50.

Rem. **If any number of forces as a, b, c, d, Fig 26, in the same plane, whether acting through the same point, or not, hold each other in equilibrium, they will do the same if they all be made to act in the same directions through one point;** as A, B, C, D, through p.

But it does not follow, that because forces may balance each other when applied at one point p, they will do so if applied to a body N *at any points whatever*, in the same plane.

Art. 52. Virtual velocities. Whenever the power and the wt balance each other, either in a single lever, or in a connected system of levers, or leverages, of any kind whatever, then if we suppose them to be put into motion about the fulcrum, their respective vels will be in the same proportion or ratio as their leverages; that is, if the leverage of the power is 2. 5, or 50 times as long as that of the wt, the power will move 2, 5, or 50 times as fast as the wt. Therefore, by observing these vels, we may determine the ratio of the leverages. The wt and the power are to each other, therefore, *inversely* as their vels, as well as inversely as their leverages; and this is based upon *the principle of virtual velocities;* and is very important.

Art. 53. Neither the amount, nor the effect of leverage is changed, if the arms of the lever, (whether straight or crooked, or in whatever relative positions they may be,) instead of being in one piece, and supported by a single point or edge as a fulcrum, as in Fig 44, p 336; should consist of two separate pieces m, n, Fig 45, firmly united to a straight rigid axle $a\ x$, of any length; (usually placed at right angles to the levers m, n,) and supported at two or more points f, f. The moments are then about the axis, or longitudinal center-line of the axle; any point of which may be regarded as the fulcrum. It is immaterial at what points along the axle, the lever-arms m, n, may be attached to it; both may be between f, f; or both outside; or one in each position. But see **Caution,** p 337. This is illustrated by the common **wheel and axle.** The rad of the wheel, and that of the axle, measd from their common axis, constitute two continuous levers. Also by series of cog-wheels, which we see placed indifferently at any points along extended shafts, or axles, whether vert, hor, or inclined.

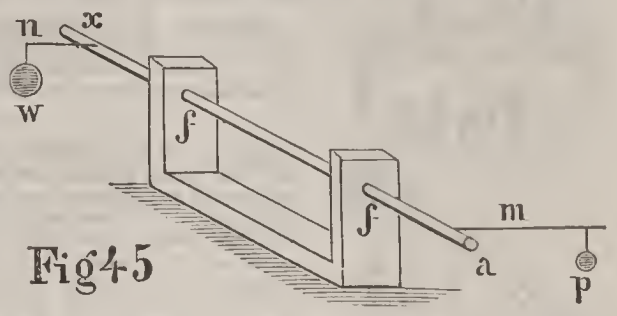


Fig 45

If the levers m, n, are not at right angles to the length of the axle, then their leverages, and not their actual *lengths*, (measd from the center line of the axle,) must evidently be used in calculating the moments of forces acting upon them. See Remark, Art 49.

Rem. The assumption of an imaginary axle, or axis, as in Rem to Art 49, enables to solve such cases as this. The entire load being known which is sustained by three vert supports; also the position of its cen of grav; to find how much of it is sustained by each of the supports: Draw by scale, a triangle $a\ b\ c$, Fig 46, showing in plan the correct position of the supports; and of the cen of grav g, of the sustained load. Assume any side, as $a\ c$, to be an axis; and from it draw the two perps: $i\ b$, to the opposite angle b; and $e\ g$. Now, it is plain that $e\ g$, and $i\ b$, may be considered as two leverages from the supposed axle $a\ c$. At g is placed the entire load, whose moment about the axis of the axle, is equal to the load, (say 5 tons,) mult by the measured (by scale) length of $e\ g$, say 8 ft. And this moment ($5 \times 8 = 40$ ft-tons) is balanced by that of an upward force at b: which upward force must be equal to that share of the load which rests upon b, inasmuch as the two are in equilibrium. To find the amount of the force at b, we have only to div the moment (40 ft-tons) of g, by the leverage $i\ b$, of the reqd force b. Suppose $i\ b$ is found by the scale to be 25 feet; then the upward force at b, is $\frac{40}{25} = 1.6$ tons; and we have its moment about the axis $= 1.6 \times 25 = 40$ ft-tons, the same as that of g. Therefore b supports 1.6 tons of the load. In precisely the same manner we may assume each side in turn to be an axis; and find how much is sustained at the opposite angle. The pressure on each of four legs cannot be calculated.

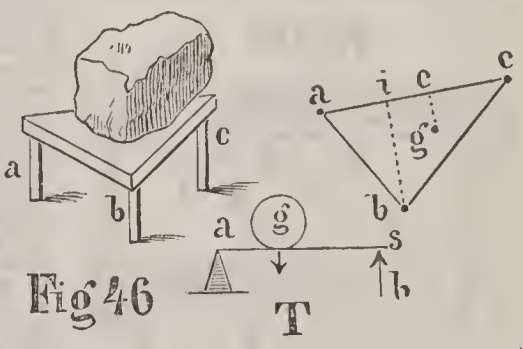


Fig 46

The Fig T, in which a represents one end of the axis; $a\ g$ the leverage $e\ g$ of the load g; and $a\ s$ that ($i\ b$) of the force b, will make the principle more apparent.

Art. 54. Ex. 1. The condition of a beam $a\ b$, Fig 47, may often be examined on the principle of a lever. Suppose it to be of uniform depth and thickness; its length $a\ b$, in the clear between its supports, 20 ft; its wt 600 lbs; and its position hor. In this case we know that one-half its wt, or 300 lbs, is borne by each support. To prove this, we may consider its entire weight to be concentrated at its cen of grav, which in this case will be at its center t. See Arts 56, 57. Then we suppose one of the supports, as o, to be removed; and an upward force f to be acting upon a lever $a\ b$, 20 ft long, without wt; and sustaining a load of 600 lbs at 10 ft from the fulcrum a. Since the force and the load both act vert, and the beam is hor, or at right angles to their directions, therefore the dist $a\ t$ and $a\ b$ from the fulcrum, are the true leverages of said force and load. Now, the moment of the load, about the fulcrum, is 600 lbs mult by 10 ft $= 6000$ ft-lbs; and to

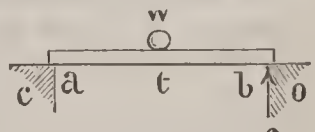


Fig 47

balance this the upward force f must be equal to $\frac{6000}{20} = 300$ lbs, or one-half of the wt of the beam, the proof reqd.

If, in addition to its own wt, the beam had actually sustained a load *w at its center*, we must add this load to the wt of the beam; and then proceed as before. So also if it sustains a load *uniformly* distributed over its length.

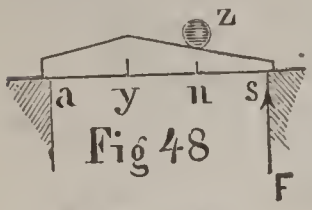


Fig 48

Ex. 2. If the cen of grav of the beam be at any point *y*, Fig 48, not at its center, we use the leverages *a y* and *a s*, instead of *a t* and *a b*, and if in addition it sustains a load *z* at any point *n* whatever, we first find as before the force reqd at F for the beam alone; and afterward, by using *a n*, and *a s* as leverages, we find the force reqd by the load: and add the two forces together for the total F.

Rem. To find the portions of *z* borne by *a* and by *s* say, as the whole span *s a* is to the whole load *z*, so is *n a* to the load on *s*; and as *s a* is to *z* so is *n s* to the load on *a*.

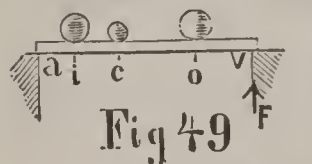


Fig 49

Ex. 3. If the beam sustains several loads at diff points, as in Fig 49, calculate for each of them separately, using the leverages *a i*, *a c*, *a o*, &c; and add all together for total F. For portions of each of these loads borne by *a* and *v* see above Rem.

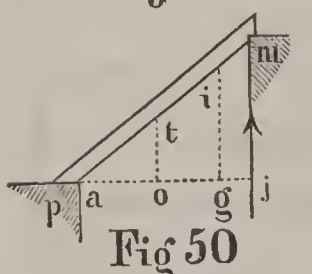


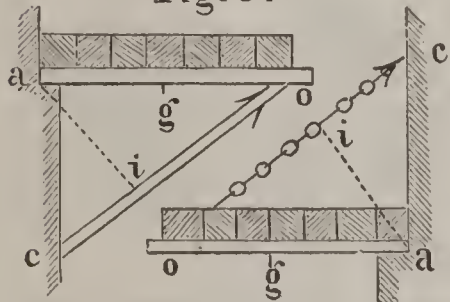
Fig 50

Ex. 4. If the beam in any such case, is inclined, as in Fig 50, the hor dist *a o*, *a g*, &c, must be taken as measured from the fulcrum *a*, instead of *a t*, *a i*, &c; because, since all the forces are vert in direction, only a hor line can be at right angles to them, and serve to measure their leverages from the fulcrum *a*. If the beam be *rigid*, and its ends cut hor, as shown in this fig, it will have no tendency to slide; because all the forces which through it are *applied* to the bodies *m* and *p* are vert; and since the joints are at right angles to those bodies at those points, the entire forces will be *imparted* also; no portion of them remaining *unresisted*, to act as *motion*, *so long as the beam remains rigid, and consequently straight*. But if it bends under either its own wt, or that of its load, new forces come into action, which will tend to push the supports outward from each other; so also in the foregoing cases.

It is only where we may practically regard a beam as rigid, or unchangeable under the forces, that the foregoing concentration of *entire* weights or forces at the cen of grav, can be safely assumed. It will not apply when we are investigating the *strength*, and *deflections* of beams; see Art 58.

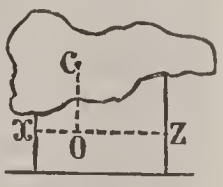
After having thus obtained F, in any of these cases, or in other words, having found how much of the entire wt of beam and load bears upon one support, we have only to subtract it from the entire wt, to obtain that on the other support. It is plainly immaterial which end of the beam is assumed to be the fulcrum in any of these cases.

Ex. 5. Let *a o*, Figs 51, be a hor beam 10 ft long, projecting from a vert wall *a c*; and resting at one end on a step *a*; the other end being sustained by either a strut, or a tie o c, 12½ ft long. The beam, and its uniform load, weighing together 3 tons, what will be the pushing strain along the direction of the strut; or the pulling strain along the tie? Draw the Fig to scale; and measure *a i* (which will be found to be 6 ft) at right angles to c o. Now, the weight of a rigid body, when considered only with regard to its effect in moving the entire unaltered body, or in straining it bodily against another body, acts the same as if it were all concentrated at its cen of grav; and since we are now about to consider it in that light, and not as tending to *bend* or *break* the beam *a o* (in which case only *half* its uniform load, and wt must be assumed to be concentrated at its cen of grav;) we consider the 3 tons wt to act at *g*, 5 ft, or half the length of the beam from *a*. Now, the 3 tons, being a force of grav, will act in a vert direction; and since the beam is hor, *a g* is at right angles to this direction of the force exerted by the beam and its load. Consequently, if we assume the beam to be a lever, movable about *a*, as a fulcrum, *a g* is the leverage of that force of grav; and the moment of that force about *a*, consequently, is $3 \times 5 = 15$ foot-tons. But this moment is reacted against by that of another force in the direction c o; which acts at the point o of the lever *a o*, to uphold the beam and its load. The leverage *a i* of this force, that is, the dist from the fulcrum *a*, and at right angles to the direction c o of the force, has already been found to be 6 ft; consequently, the force itself, in order to have a moment of 15 ft-tons about *a* (as the beam and its load have) must evidently be $\frac{15}{6} = 2.5$ tons, the reqd strain along the strut, or along the tie, o c; for $2.5 \times 6 = 15$.



Figs 51

A load resting on two props either at its ends or otherwise. When a load c of any shape whatever, rests in any position upon two props x and z, the portions of its wt borne by the respective props will be to each other **inversely** as the horizontal distances $o\,x$, $o\,z$, from the cen of grav c of the load, to the props. Thus if $o\,z$ is two, three or four times as great as $o\,x$, then will x bear two, three, or four times as much of the entire wt of the load c as z does. **Therefore to find how much each prop bears,** first find the cen of grav c of the entire load; and its hor dist (say $o\,x$) from **either** one of the props, (say x.)



Then as *The entire hor dist x z between the two props* : *Entire wt of load* :: *Dist o x* : *Wt borne by the* **other** *prop z.*

And this wt taken from the entire load leaves the wt borne by x.

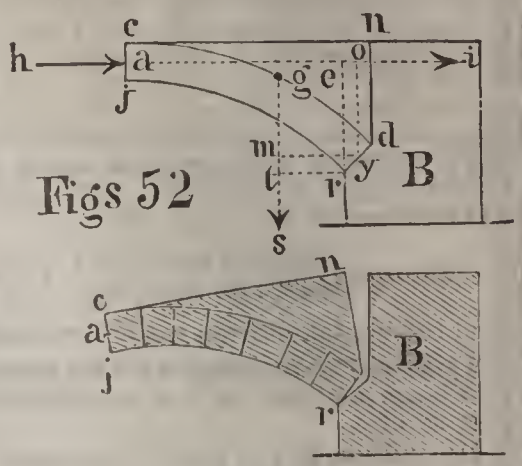
This all amounts to the same as if we consider $x\,z$ to be the clear span of a beam without wt, and supporting a load equal to c, concentrated at the cen of grav of c.

Conversely, to place two props x and z so that each may bear a **given** portion of the entire load c, take any two hor dists $o\,x$ and $o\,z$ from the cen of grav c, **inversely** as the two portions of the load to be borne by each. Thus if x is to bear two-thirds of the wt, make $o\,z$ equal to two-thirds of $x\,z$.

Ex. 6. The following is very important in its application to arches of any material.

Let $c\,n\,d\,r\,j$, Fig 52, represent one half of a bridge arch. If this half were not prevented by the hor pres, $h\,a$, of the opposite half, it would evidently fall, as in the shaded fig. by turning about the point r as a fulcrum. (See Rem 1.) Let us find what this hor pres amounts to in any case. To do this, we may consider the half bridge $c\,n\,d\,r\,j$ to be a lever. Suppose its wt to be 80 tons; and to be concentrated at its cen of grav g; the vert line $g\,s$ being of course the direction in which it would act. And let its leverage $r\,t$, about r, be 6 ft; $r\,t$ of course being at right angles to the direction

$g\ s$. Then is its moment about r equal to $80 \times 6 = 480$ ft-tons. (Art 46.) Now, whatever may be the amount of the hor force $h\ a$, which acts at the end a of this lever, to counteract this moment of 480 ft-tons, its *leverage* (Art 46) is plainly equal to $r c$, measured from the fulcrum r, and at right angles to the direction $h\ i$ of said force. Suppose we find by measurement from the drawing that $r\ e$ is 8 feet. Then the force itself must necessarily be $\frac{480}{8} = 60$ tons; which is the hor pres which the opposite half of the bridge exerts against the keystone a, of the arch; for $60 \times 8 = 480$ ft-tons of moment.

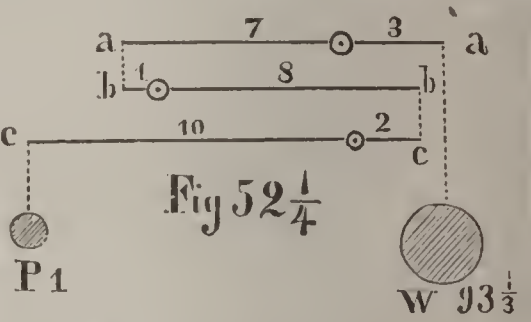


Rem. 1. But so long as an arch is not deranged, but remains firmly in position, the half arch, instead of tending to revolve about the point r, presses equally over the entire surf $r\ d$ of its skewback. Therefore, the leverage with which the hor force h acts upon the skewback, is actually $y\ o$, measured from the center of $r\ d$, and in practice it must be used instead of $r\ e$. In the same manner, $y\ m$ becomes the leverage for the wt, instead of $r\ t$.

Rem. 2. **The cen of grav of the half arch, can be found** by making a drawing $e\ n\ d\ r\ j$, about 4 to 6 ins long, on pasteboard, or on a stiff drawing-paper, to a scale. Cut out the fig; and balance it flatways on a sharp straight-edge, or over the edge of a table, in two directions or positions. Where these two directions intersect each other is the cen of grav. It is not indeed this cen itself that is needed, but the line $g\ s$, of its direction; which may be found at once by taking care that the straight-edge is parallel to the back $n\ d$, while balancing the fig.

Rem. 3. Under the head leverage, may be classed the tread-wheel; windlass and lever; capstan and lever; and all axles turned by a winch or by a crank; such as the drum and winch with which a water-bucket is raised from a well, &c. They are all merely continuous simple levers, of which the axis is the fulcrum; the rad of the circle described by the power is one arm, and the rad of that described by the shaft, drum, &c, is the other.

Rem. 4. **Compound levers.** $a\ a$, $b\ b$, $c\ c$, Fig 52¼, may be used where there is not space for the arms of a single lever of sufficient power. They need not extend in one line; but may be placed one over the other; or in such other positions as may be convenient. Their effect is much greater than the combined effects of the three simple levers, and is found thus: As the product of the weight-arms, $2 \times 1 \times 3 = 6$; is to that of the power-arms, $10 \times 8 \times 7 = 560$; so is the power to the weight; or, as $6 : 560 ::$ P 1 : W 93⅓. These arms are measd in all cases from the *fulcrum*; which is sometimes at the *end* of one or more of the levers, when compounded; see Fig 44. The combined effects of the three simple levers would be but $5 + 8 + 2\frac{1}{3} = 15\frac{1}{3}$; or P 1, W 15⅓.



A series or train of toothed pinions and wheels, working into each other, is merely a series of continuous compound levers. These are generally set in motion by a winch-handle, the rad of which is the first leverage of the series; while at the other end of the train the wt is usually suspended from a drum, the rad of which is the last leverage. To find the effect, mult into one prod the radii of the winch and of the wheels, and into another prod the radii of the pinions and of the drum; then, as the last of these prods is to the first, so is the power to the wt, as in the preceding case.

In both the foregoing cases of compound leverage, as in all other cases whatever of leverage, the vel of the wt is to that of the power, as the power itself is to the wt; thus, in Fig 52¼, the wt will move only $\frac{1}{93\frac{1}{3}}$ part as fast as the power; or the power must descend 93⅓ inches, in order to raise the weight 1 inch; on the principle of virtual vels. See Art 52.

Rem. 5. **The Screw** is a combination of leverage, with an inclined plane; a spiral inclined plane being formed by the threads of the screw. While the power applied to the lever which turns the screw moves around an entire circle, the body moves only the dist between the centers of two threads; and since in all mechanical contrivances, the wt is to the power as the vel of the power is to that of the wt, so in this case, *theoretically*, the wt is to the power, as the entire circumf of the circle described by the power, is to the dist between the centers of two threads; but in practice, the friction of the screw (which under heavy loads becomes very great) has also to be overcome by the power; and this fact makes the calculations of but little use.

The Pulley, also, when a fixed one, is referable to leverage. In the *fixed* pulley A, Fig 52½, there is no gain of power; for here the diam $a\ b$ is a lever of two equal arms, revolving around its fulcrum at the center of the pulley. Consequently, the wt and the power have equal leverages;

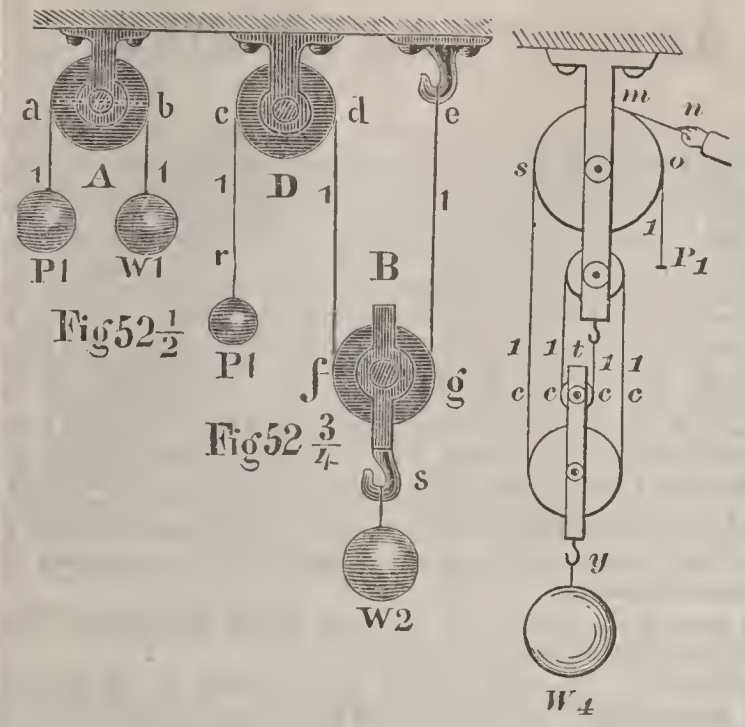
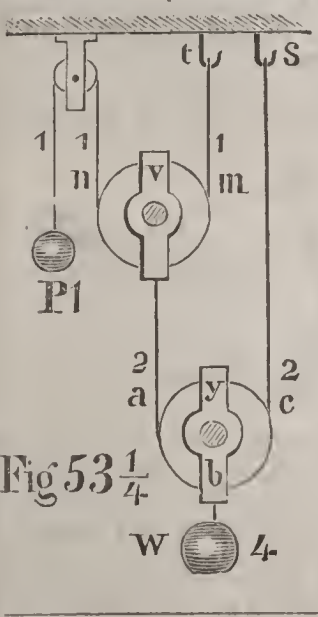


Fig.53

each equal to the rad of the circle; and in order to *balance* a wt W of say 1 ton, the power P must also be I ton; for if one of them moves, the other must plainly move with the same vel. To *raise* the wt, the power must *exceed* the wt; because some *unresisted* force is required to give *motion*, as well as to overcome the friction of the axle around which the pulley revolves, and the friction of the rope in the groove around its circumf. These frictions become so great when many pulleys are combined, that theoretical calculations of the power are of little value. Although a fixed pulley gives no *gain of power*, it is very *convenient* for allowing *change of direction in applying the power;* so that by pulling downward, or hor, &c, we can cause the wt to rise vert. It is plain that the rope in this pulley is equally strained at all points. Theoretically, this is the case with any one single rope, as *r c d f g e*, Fig 52¾, passing around any number of pulleys, whether fixed, as A or D, or movable, as B; and all the *theoretical* calculations of the power may be based upon this principle alone. They will, however, be incorrect in practice, on account of the friction just alluded to. In Fig 52¾, where only one rope is used, the lower pulley-block B *s*, to which the wt W is attached, is directly upheld by the *two* parts *d f* and *e g* of the rope. Consequently each of these parts is equally strained. Since both parts are vertical the strain on each is that due to a force equal to one *half* the wt*; and since the whole single rope is theoretically strained to the same extent, that part of it to which the power is applied must be strained equal to half the wt W; or, in other words, the power itself must be equal to half the wt, and will move twice as fast, and, of course, twice as far in the same time.

In Fig 53 the lower pulley-block *t y* is sustained directly by the 4 parts e c c c of the single rope; therefore, each part of the rope, and, consequently the whole of it, is equally strained by a force equal to ¼ of the wt W; and the power P must be = the same ¼, and will move 4 times as fast as the wt, or 4 times as far in the same time. It is immaterial whether the two pulleys in the lower block, Fig 53, be placed one above the other, as shown, or (as usual, and more convenient), side by side; so also with those in the upper block. If there were 3, 4, or 5, &c, pulleys in each block, then there would be 6, 8, or 10 sustaining parts c c, &c, of rope, each stretched equal to $\frac{1}{6}$, $\frac{1}{8}$, or $\frac{1}{10}$ of the wt W; and the power would also be in the same proportions to the wt; in other words, to find the theoretical proportion of the power to the wt, when there is but one rope throughout, as in Figs 52¾ and 53, divide I by the number of parts c c c c of rope which directly sustain the lower block. In fig 53 the number of parts is 4. When more than one rope is used in a system of pulleys, the principle is the same as above, but the rule must be differently worded. See Fig 53¼.



Here the lower block *y b*, with its attached wt of say 4 tons, is directly sustained by the two parts *a* and *c* of one rope. Consequently, each part has a strain of ½ the wt, or 2 tons; which is uniform throughout *that* rope. But all the 2 tons strain on the part *c* is sustained by the hook *s*; while that on the part *a* is sustained by the two parts *n* and *m* of the other rope; each of which plainly sustains one half of it, or 1 ton, which is uniform throughout this second rope to its very end. Therefore the power also is 1 ton, or ¼ of the wt W. The mode of proceeding is the same, whatever may be the number of movable pulleys. To find the theoretical proportion of the power to the wt, mult together continuously as many 2s as there are movable pulleys, and div 1 by the prod. Thus, here we have two movable pulleys, and $2 \times 2 = 4$; and ¼ = the answer. If there were 4 movable pulleys, we should have $2 \times 2 \times 2 \times 2 = 16$; and $\frac{1}{16}$ = answer.

In all our figs, that end of the rope to which the power P 1 is applied is represented as hanging vertically, and parallel to the other parts of the rope; but this was done merely because the power is supposed to be a *weight*, and of course acting vert. But if the power is muscular force, or any other kind that may act in any direction whatever, then the power end of the rope, as *m n*, Fig 53, may have that direction in which it is most convenient. The amount of power required will not be thereby changed; for it is plain that leverage from the center of the pulley *o s* to *m*, when the power is at *n*, and acting in the inclined direction of the rope, is equal to that from the same center to o, when the power is at P, and acting vert.

* If the two parts, *d f* and *e g*, of the rope were *inclined*, the strain on each would be *greater* than half W, as at *a c*, *a o*, Fig 5, p 345.

THE CORD OR FUNICULAR MACHINE.

Art. 1. Some allusion to this subject has already been made on page 325, which see. **Theory requires** that the cord, rope or string, &c, shall be **absolutely** flexible, inextensible, frictionless, without weight, and infinitely thin; and that such pulleys, posts, pins or pegs, loops or rings as may be used with the cord shall also be absolutely frictionless; and at times devoid of weight unless said wt is included in one of the acting forces. These assumptions cannot of course be realized in practice, which however will agree with theory in proportion as we approximate to them; and this we can frequently do so far as to render the theory of great use. We know that all cords have wt and thickness, and can be stretched; and that so far from being flexible, they may require very considerable force to bend them around pulleys, posts, pins, &c. They also possess friction. Also that all pulleys, pins, sliding rings or loops &c, have more or less friction; which when there are many of them, may entirely vitiate all calculations. **A pulley has, in itself, no advantage over a smooth cylindrical pin, or post** (as the case may be) except that its friction being a **rolling** one, is less than the **sliding** friction which takes place between a cord and a pin, &c. Therefore in what follows we may usually (so far as theory is concerned) substitute one for the other.

Rem. It will be seen that the principle of the cord serves for finding the strains on the ropes of the different systems of pulleys.

Art. 2. Assuming then such a theoretical cord, pulleys, pins, &c, all devoid of friction, **the broad principle of the cord** is that any force *f*, Figs 1, imparted to either end of a cord (whether the cord be straight as *f g* or *f c*; or bent out of its course as *f e* by any number of frictionless pulleys or pins as *n r m s*, &c, however placed) is all transmitted along the entire cord to its other end, straining it uniformly in every part. In *f g* or *f c* this may be regarded as self-evident; in *f e* it is proved by experiment. It is perhaps needless to add that a straining force cannot be imparted to one end if there is not an equal one at the other end to react or strain against it.

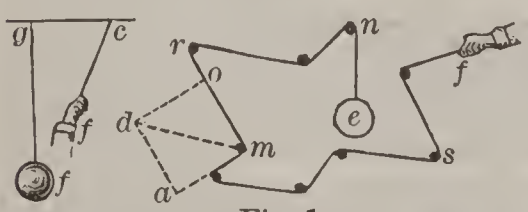


Fig. 1.

If at any one of the pulleys, pins, &c, as *m*, we make *m o*, *m a*, each equal to the force at either *f* or *e*, and complete the parallelogram *m o d a* of forces, then the diagonal or resultant *d m* will give both the direction and amount of strain which the cord produces **on that pin.** This strain will differ at the several pins according to the angles formed by the two components; and may thus be greater, or less, or equal to the force at *f* or *e*.

Rem. With theoretical cords, pulleys, pins, &c, the two components *m o*, *m a* at any pin will always form **equal angles with the resultant** *m d*; and upon this fact the **principle of the cord depends.**

Art. 3. The principle of the cord applies also when as in Figs 2, 3, 4, a load or third force *f* is imparted **between** the ends of the cord as at *a*, by means of

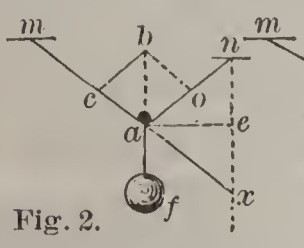


Fig. 2.

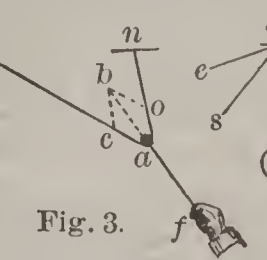


Fig. 3.

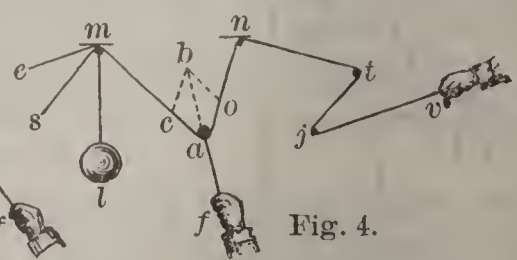


Fig. 4.

a **frictionless** pulley, ring or loop which can move along the cord with perfect ease. If such a pulley, ring or loop be first placed upon the cord near *m* or *n*, it will with its load or other force move down along the steepest part of the cord until it comes to rest at that point *a* at which *a m* and *a n* form **equal angles** *b a c*, *b a o* with the direction *a b* of the force *f*. If both ends *m n* of the cord are **at the same height,** and the force *f* a load and of course acting vertically, then *a* will be at the middle of the cord; but if the **ends are at different**

heights as in Fig 2, from either of them as n draw a vertical, as $n\,x$; and from the other one, as m lay off the whole length $m\,x$ of the cord; bisect $n\,x$ at e, and draw $e\,a$ horizontal; a will be the required point.

If we draw $a\,b$ to show both the direction and the amount of the force f, and complete the parallelogram $b\,c\,a\,o$ of forces, then either $a\,c$ or its equal $a\,o$ will give the uniform strain which f produces from end to end (m to n) of the cord. From which it is plain that a force equal to $a\,c$ or $a\,o$ may be considered to be imparted at each end m, n, of the cord, and there to react against $a\,c$ and $a\,o$, thereby straining the cord uniformly throughout.

Rem. 1. Precisely the same result will follow if one or both of the ends m, n, instead of being fixed or fastened at those points as is supposed in Figs 2 and 3, are continued as at m, Fig 4, over a frictionless pulley or pin, and prolonged either **vertically** as $m\,l$, in which case it will sustain a **load** or vertical force l equal to $a\,o$ or $a\,c$; or in any other direction as $m\,s$ or $m\,e$, in which case it will sustain at s or e a pull equal to $a\,o$ or $a\,c$ acting in said directions. And the same will take place if one or both ends as n, Fig 4, be extended over or under **any number** of such pulleys or pins, (no matter what angles they form) say to v. The strain at v will still be equal to $a\,c$ or $a\,o$, and will be uniform from v to l. **The strain on any pulley or pin** may plainly be found as in Art 2.

Rem. 2. If we know the angles $b\,a\,o$, $b\,a\,c$, and the force or resultant $a\,b$, **we can calculate the strains** or components $a\,o$, $a\,c$; or knowing the angles and components we can calculate the force $a\,b$, all by the formulas in Art 45, p 334, which are based upon plane trigonometry, which enables us to find unknown parts of a triangle when certain other parts are given.

Rem. 3. If the angle m, a, n, is 120°, each component $a\,o$, $a\,c$, will be equal to the resultant $a\,b$; if it is less than 120°, each component will be less than the resultant; and vice versa.

Art. 4. A little reflection will enable the student to see that this Art is merely a farther illustration of the preceding ones. In Fig 5 a load s, or any equal vertical pull will strain the strings $s\,a$ and $z\,v$ each to an amount equal the load, **because they both act in its own direction.** If the parts $m\,a$, $g\,a$ of the cord $a\,m\,n\,g\,a$ which passes over the **frictionless** pulley or pin z were also vertical, each of them would be strained equal to **one-half** the load s; but they are not vertical, and if we make $a\,b$ to represent s, and complete the parallelogram $b\,o\,a\,c$, we shall find that the resulting strains $a\,c$, $a\,o$ are each rather **more** than half of s; and they will increase if the angles at a increase; and may be **calculated** by the formulas in Art 45, p 334.

Fig. 5.

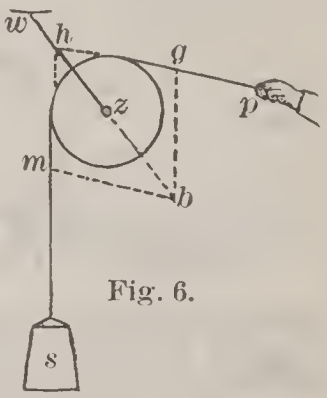


Fig. 6.

In Fig 5 we had the force s or $a\,b$ given as a resultant, and from it we found its two components or strains $a\,o$, $a\,c$. In Fig 6 we have the two components $h\,m$, $h\,g$ given, (each evidently equal to the load s) and from them we deduce the resultant $h\,b$, which represents the strain on the pulley, and on the string $z\,w$. It is plain that this string cannot be vertical as $z\,v$ is, but must (as $z\,v$ does) adjust itself to the direction of the force or resultant which pulls it. If the load s is the same in both Figs, we see that the pull on $z\,w$ will be about twice as great as on $z\,v$; and the one along the cord $s\,m\,g\,p$ about twice as great as that along $a\,m\,n\,g\,a$. This difference will of course vary with the angles.

Rem. 1. Let s, Fig 5, instead of the load there represented, be a suspended man weighing 200lbs, and holding $m\,a$, $g\,a$ together at a with both hands; or let $m\,a$ and $g\,a$ both hang **vertically** from the pulley, and let him hold their lower ends apart by one hand at each end. Now if each part of the rope will bear but a little more than 100 lbs, and $z\,v$ a little more than 200 lbs, he will be safe from falling. But if he lets go one end of the rope, putting it into the hands of an-

other man standing by to hold it; or if he hooks one end to a projecting spike in a wall close at hand, the rope will break, and he will fall, because he at once doubles the strain along both the ropes *a m n g a*, and *z v*.

Rem. 2. Fig 7 shows a device by which boatmen sometimes haul a boat out of the water, and up on to the beach, at a landing, when it is too heavy for their unaided efforts. The rope *e m x n b* is to be considered as horizontal in this case. One end *b* being fastened to the bow of the boat, the rope is carried past one smooth post *n*, to another at *m*, around which it makes a whole turn; and a man stands at that end *e* to take in the slack while the others, taking hold of the rope midway between *m* and *n*, pull it into a position *m a n* in which, if **the angle m a n exceeds 120°,** (see Rem 3, Art 3) each component *a n*, *a m* of their force exceeds said force itself; and a strain equal to one of these components (except so far as it may be reduced by the rigidity of the rope and by its friction against the post *n*) is transmitted uniformly to the boat *b*, drawing it a short distance up the beach. The rope is then straightened again from *m* to *n* by taking in the slack at *e*, and the operation is repeated as often as necessary.

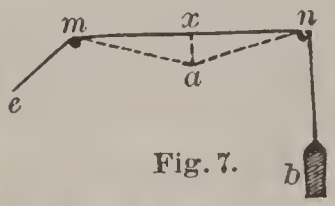


Fig. 7.

To find the strain *a m* or *a n*, divide the force by **twice** the nat cosine of the angle *x a n*, or *x a m*. Or to find how many times the strain exceeds the force divide the distance *a n* by **twice** the distance *a x*. Here *a x* represents only half the force, or half the diagonal or resultant of *a m* and *a* n.

The force of a man pulling by jerks at *x* or *a* will average between 30 and 80 lbs.

Art. 5. We will now dispense with the **frictionless** pulley required by the principle of the cord. and which would of itself roll along the cord until it came to rest at a point which would cause **equal angles** and consequently **uniform strain** throughout. We will substitute for it a **tight knot which cannot slide,** at a point *a*, Fig 8, which causes **unequal angles** *b a m*, *b a n*, between the direction *a b* of the force *f*, and the two parts *a m*, *a n*, of the cord. Here, completing the parallelogram *b c a o*, we find that the part *a n* of tho cord is strained to an amount denoted by *a c*; and that same strain would affect any extension of the cord **on that side,** like that to *v*, Fig 4. The part *a m* would be strained equal to *a o*; and that strain would continue to the end *m*, or to the end of any extension of **that part.** Hence **the strain is not uniform** from end to end of the cord, as it would have been if the knot had been **frictionless;** in which case it would have slid along the cord until the angles would be equal.

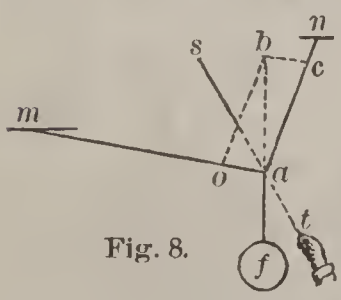


Fig. 8.

Rem. 1. But even in this case of a **tight knot** at *a*, there is always **one** direction, as *t a s*, in which the force can be imparted so as to cause **equal angles** *s a m*, *s a n*, with the direction *a s* of the force *t*, and then the strain will be **uniform** from end to end, as if a frictionless pulley had been used.

Rem. 2. With a tight knot at *a* it is plain that a force may be imparted there from any direction as *f a*, *t a* &c. **If the direction coincides with either part a m, or a n of the rope,** that part will bear **all** the strain; the other part remaining entirely free.

Rem. 3. From Rule 1, p 150, for drawing an **Ellipse,** it will be seen that at whatever point as *a*, Fig 8, we apply force to an inextensible cord *n a m* with fixed ends, that point will be in the circumference of an ellipse, the foci of which are **at** the ends *m*, *n*.

Art. 55. Parallel forces are those whose directions, as in Fig 53½, (whether opposite to one another, or not; or whether in the same plane, or not.) are parallel. In Fig 53½, the forces, although acting *upon* one plane, *o o o o*, are not *in* one plane, but in several.

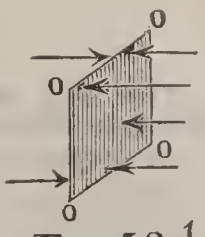


Fig 53 ½

It is a peculiarity of parallel forces *in one plane*, that all their arms, or leverages with respect to any given point in the same plane are in the same straight line. Thus, if *l*, *m*, *n*, *o*, Fig 54, be in the same plane, then their leverages *p q*, *p r*, *p a*, about the point *p*, are all in the same straight line *p a*. The point *p* is supposed to be in the same plane as the forces.

Two parallel forces are evidently always in the same plane; that is, the same flat surf could coincide with both of them; and their resultant or antiresultant will be in that same plane: which amounts to saying, in other words, that if 3 parallel forces hold each other in equilibrium, they are in the same plane.

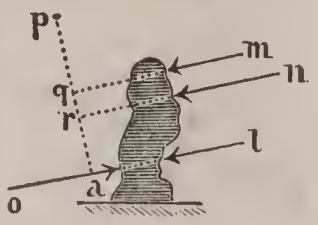


Fig 54

Art. 56. The resultant of any number of parallel forces, whether in the same plane, or in the same direction, or not, is always parallel to them. If they all act in the same direction, whether in the same plane or not, their resultant is equal in amount to their sum: or, in other words, an antiresultant force sufficient to balance them, must be equal to all the forces added together. But if they are in opposite directions, their resultant will be equal to the diff between those which act in one direction, and those which act in the opposite one; and its direction will be that of the greater sum. Thus, in Fig 53½, if the forces pointing to the left amount to 10 tons, and those to the right 4 tons; then the resultant will be 10 — 4 = 6 tons; and it will point to the left.

Fig 55

The parallel vert downward forces of gravity, upon the innumerable separate particles, situated in the infinite number of imaginary vert planes, in any body, as W, Fig 55, is an illustration of this. If any such body be suspended by a string from a spring-balance B, the vert upward pull of the string will balance or equilibrate all these innumerable forces. Consequently, the string represents their antiresultant, which is equal to their resultant. We know that the vert pull on the string, as shown by the spring-balance, is equal to the wt of the body; which wt is made up of the innumerable parallel vert forces alluded to. Thus we see that when any number of parallel forces, whether in the same plane or not, act in the same direction, their antiresultant is parallel to them, and equal to their sum; consequently their resultant must be so also. The same principle applies to parallel forces in any direction whatever.

When a body thus acted on by gravity is kept at rest, or balanced, as in the fig, then the direction of the resultant or antiresultant, or of the string in the fig, passes through a certain point, called the **center of gravity** of the body. This is a certain point, upon which when acted upon by gravity only, the body will balance itself, in whatever position it may be placed; and if the entire wt or grav of the body could be concentrated into that single point, its effect, whether regarded as moving the entire rigid body, or as producing strain (pull or push) between it and another rigid body, would remain precisely the same as it actually is with the grav diffused throughout the entire mass.*

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* In some bodies the cen of grav is also the center of the wt of the body; but very frequently this is not the case. Thus, in a body *a b c*, Fig 55½, with its cen of grav at e, there is more wt on the side *a* e, than on the side e *b*.

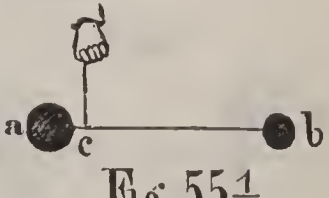


Fig 55½

If a body W, Fig 55, suspended freely from any point *n*, is at rest, its cen of grav is directly under said point. If the body W be pushed a little to one side, and then left to itself, it will plainly tend of itself to swing back to its first position; and when this is the case, it is said to be in *stable* equilibrium. But if the body, instead of being suspended, be balanced on top of a slim rod, and if we then push it a little to one side, it will not tend to return, but will fall over; and therefore the equilibrium of a body so balanced is said to be *unstable*. Also in such cases as that of a grindstone supported by its hor axis passing through its cen of grav, if we cause it to revolve a short dist, stop it, and then leave it to itself, it will have no tendency either to return, or to keep on revolving; and its equilibrium is called *indifferent*.

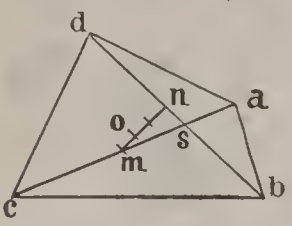
CENTER OF GRAVITY.

The cen of grav of a square, rectangle, rhombus, or **rhomboid,** is at the intersection of its two diagonals.

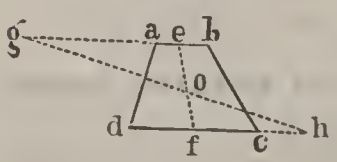
Of a circle, ellipse, or **regular polygon,** in the center of the figure.

Of a triangle, at the intersection of lines drawn from any two angles, to the middles of the sides respectively opposite said angles.

Or, draw a line from any one of the angles, to the middle of the side opposite said angle; the cen of grav is in this line at $\frac{1}{3}$ of its length from the side which it bisects.

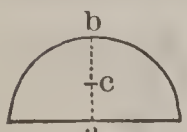


Of either a trapezium, or a trapezoid, draw the two diags $a\,c$ and $b\,d$. Div either of them, say $a\,c$, into two equal parts as at m. Take the longest part $d\,s$. of the other diag $d\,b$, and set it off from b to n. Join $n\,m$, and div it into 3 equal parts. The cen of grav will be at o, the first of these divisions from m.



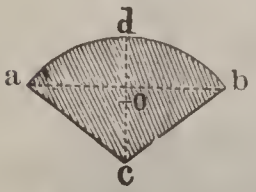
Of a trapezoid only. Prolong either parallel side, as $b\,a$, in either direction, say toward g; and make $a\,g$ equal to the opposite side $d\,c$. Then prolong the other parallel side $d\,c$, in the opposite direction; making $c\,h$ equal to the side $b\,a$. Join $g\,h$. Find the centre e of $a\,b$; and the center f of $d\,c$. Join $e\,f$. Then o is the cen of grav.

$$\textbf{Or } fo = \frac{d\,c + 2\,a\,b}{d\,c + a\,b} \times \frac{e\,f}{3}$$



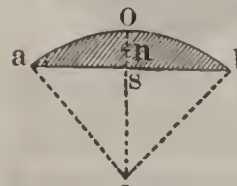
Of a semicircle. Mult the height or rad $a\,b$ by .4244; the prod will be $a\,c$; and c is the cen of grav.

Of a cycloid. See "The cycloid," p 154.



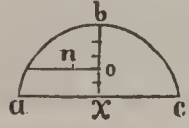
Of a sector of a circle, $a\,d\,b\,c$. Mult twice the chord $a\,b$, by the rad $a\,c$. Div the prod by 3 times the length $a\,d\,b$ of the arc of the sector; (see Lengths of Arcs, p 141, &c.) The quot is $c\,o$; and o is the cen of grav.

Of a quadrant, $c\,o = c\,d \times .6002$.



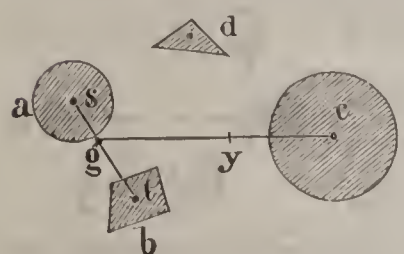
Of a segment of a circle, $a\,o\,b$. Cube the chord $a\,b$. Div this cube by 12 times the area of the segment; (see Areas of Segments, page 146.) The quot will be $c\,n$; and n is the cen of grav.

Of a circular arc $a\,o\,b$, (the line alone,) not exceeding a semicircle. As the length of the arc is to its chord, so is rad $c\,o$ to $c\,n$; n being the cen of gr. Or, quite approx, mult the rise $s\,o$ by .65 for $s\,n$. Or more correctly, if the rise $s\,o$ is .01 of the chord $a\,b$, or less, mult it by .666 for $s\,n$; if .1, mult by .665; if .15 by .663; if .2, by .660; if .25 by .657; if .3, by .653; if .35, by .649; if .4, by .645; if .45, by .641; if .5, by .637.



Of a parabola $a\,b\,c$, at o in the axis $x\,b$, $\frac{3}{8}$ths of its length from x.

Of a semiparabola $a\,b\,x$. At n; $o\,n$ being 3-eighths of the half base $a\,x$, and $o\,x$ being $\frac{3}{8}$ths of $x\,b$.



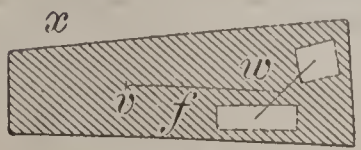
To find the common cen of grav, g, of two figs, a and b. First find their separate cens of grav, s and t, and areas. Then

Sum of the areas : $s\,t$: : area a : $g\,t$: : area b : $g\,s$.

To find the common cen of grav, y, of any number of figs, as a, b, and c. Having found the area and cen of grav of each fig separately, begin with any two of them, as a and b, as just explained. Then, taking any other one, as c,

Sum of areas of a, b, and c : $g\,c$: : sum of areas of a and b : $y\,c$: : area c : $y\,g$.

In the same manner proceed with Fig d, if required.



To find the cen of grav, v, of a hollow fig, x. Find the common cen of grav, w, of the openings, as directed above for two or more figs. Find the cen of grav, f, of the *entire* fig as though there were no openings. Then

NOTES ON "CENTER OF GRAVITY," pages 348 and 349.

Page 348.

After **"Of a quadrant,"** line 30, add **Of a sextant,** $co = cd \times .6366$.

Lines 35–37. The rule given for center of gravity of a **circular arc** (the line alone), viz.: $cn = \frac{\text{radius } co \times \text{chord } ab}{\text{arc } aob}$, applies to *all* circular arcs, whether less than, equal to, or greater than, a semi-circle.

Lines 42–45. These rules, for **parabola** and **semi-parabola,** refer to *surfaces*, as do all other rules on page 348, except those for circular arcs, lines 35 to 41, which refer to *lines* only.

Page 349.

Lines 9 to 47. Erase, and substitute the following, to the end of page iv.

The curved or slanting surface only, of a **right cone** (circular or elliptic) or of a **right pyramid.** The center of gravity is in the axis (the line joining the apex and the center of gravity of the base) and at one-third its length from the base.

The curved surface only, of a **hemisphere** or other **spherical segment,** or of a **spherical zone.** The center of gravity is in the center of the axis.

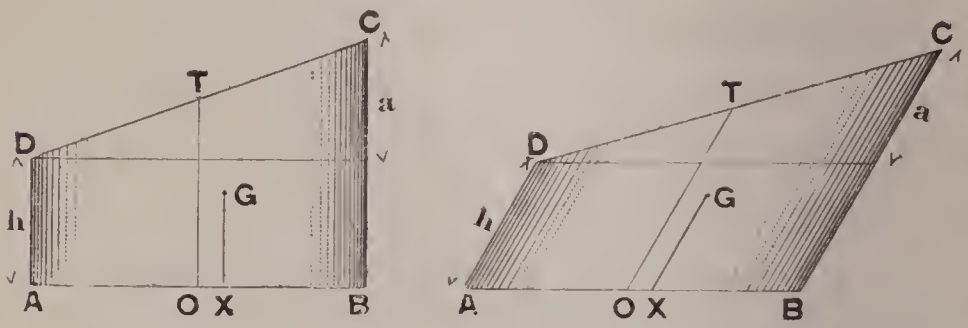
CENTERS OF GRAVITY OF SOLIDS.

The solids are of course supposed to be *homogeneous*, *i. e.*, of uniform density throughout.

Sphere or **spheroid, (ellipsoid).** The center of gravity is the center of the body.

Cube, parallelopiped, or other **prism,** regular or irregular, right or oblique; and **cylinder,** circular or elliptic, etc., right or oblique. The center of gravity is in the center of the axis joining the centers of gravity of the two parallel ends of the solid.

Ungula of a cylinder, circular, or elliptic (provided one of the axes of the ellipse coincides with the oblique cutting plane); right or oblique

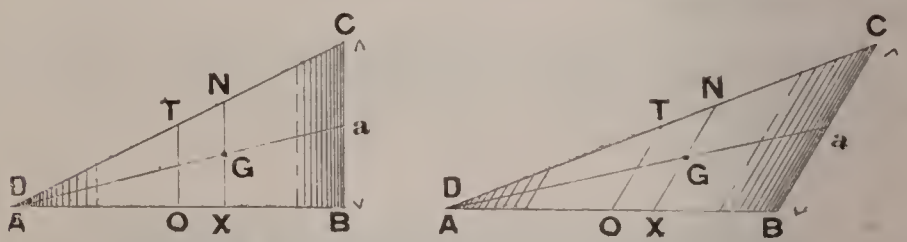


Let G be the center of gravity; O T the axis (joining the centers of gravity of the ends) and X G a line drawn parallel to the axis in the plane passing through the axis and through the uppermost and lowermost points C and D of the oblique cutting plane. Then the position of G in said plane is found thus:

$$O X = \frac{O B}{4} \times \frac{a}{2 h + a};$$

$$X G = \tfrac{1}{4}\left(2 h + a + \tfrac{1}{4} \frac{a^2}{2 h + a}\right).$$

If the oblique plane C D *meets* the base at A, so that $h = 0$, while C D remains

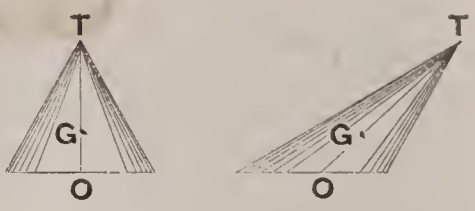


a complete ellipse or circle, this becomes

$$O X = \frac{O B}{4};$$

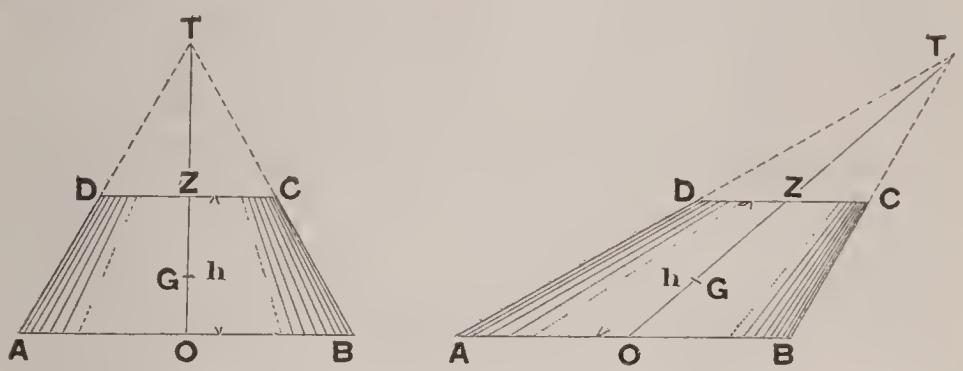
$$X G = \frac{X N}{2} = \tfrac{5}{16} a.$$

Cone, circular, elliptic, etc., right or oblique; or **pyramid,** regular or irregular, right or oblique. The center of gravity G is in the axis O T, drawn from the apex, or top, T, to the center of gravity O of the base; and



$$O G = \frac{O T}{4} .$$

Frustum of a cone, circular or elliptic, right or oblique; or **of a pyramid,** regular or irregular, right or oblique; in which the two ends A B and C D are parallel.



Call the area of the large end A, that of the small end a, and the height O Z of the frustum, *measured along its axis*, h.

The center of gravity, G, of the frustum is in the axis O Z, which joins the centers of gravity O and Z of the two ends; and its distance from the base, *measured along the axis*, is

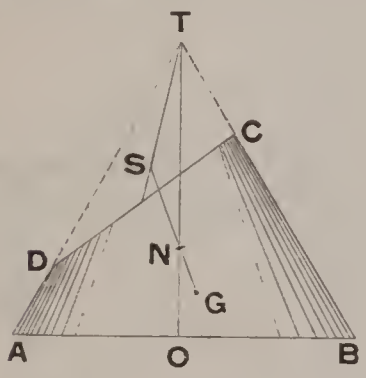
$$O G = \frac{h}{4} \times \frac{A + 2\sqrt{A a} + 3 a}{A + \sqrt{A a} + a} ,$$

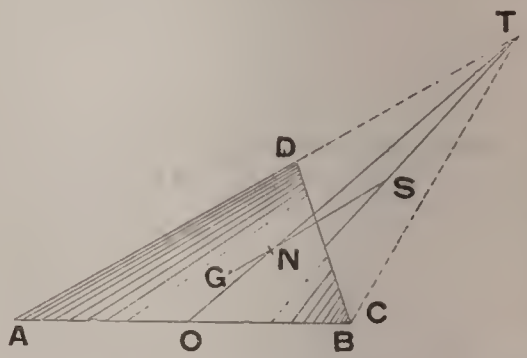
In a **frustum of a circular cone,** right or oblique, with parallel ends, this becomes

$$O G = \frac{h}{4} \times \frac{R^2 + 2 R r + 3 r^2}{R^2 + R r + r^2} ,$$

Sphere or pl... the radii of the large and small ends of the frustum the body.

Frustum, A B C D **of a cone,** circular, elliptic, etc., right or oblique; or **of a pyramid,** regular or irregular, right or oblique; whether the ends are parallel or not. By the rule given above, find the center of gravity N of the *entire* pyramid (or cone, as the case may be) A B T, of which the frustum forms the lower part; and the center of gravity S of the smaller pyramid or cone D C T, = entire pyramid or cone, minus the frustum. Also find the *volume* of each:





$$\text{Volume of pyramid or cone} = \frac{\text{area of base} \times \text{perpendicular height}}{3},$$

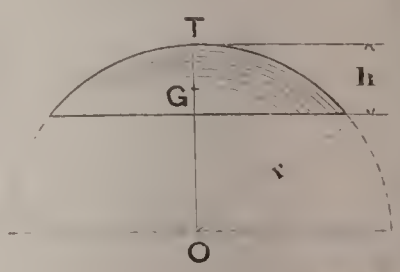
and the

$$\begin{matrix}\text{Volume of}\\ \text{the frustum}\\ \text{A B C D}\end{matrix} = \begin{matrix}\text{volume of}\\ \text{entire pyramid}\\ \text{or cone, A B T}\end{matrix} - \begin{matrix}\text{volume of}\\ \text{smaller}\\ \text{one, D C T}\end{matrix}.$$

Then the center of gravity G of the frustum A B C D is in the extension of the line S N; and

$$N G = S N \times \frac{\text{volume of smaller pyramid or cone}}{\text{volume of frustum.}}.$$

Segment of a sphere. Let O be the center of the sphere. The center of gravity G of the segment is in the axis O T. Let r = radius of the sphere, and h = the height or rise of the segment. Then



$$O G = \tfrac{3}{4} \frac{(2 r - h)^2}{3 r - h}.$$

In a **hemisphere,** this becomes

$$O G = \tfrac{3}{8} O T = \tfrac{3}{8} \text{ radius } r.$$

Area of fig *minus* the openings : $f w$ :: area of *entire* fig : $v w$ :: sum of areas of openings : $f v$.

In an irregular fig which may be divided into triangles, trapeziums, trapezoids, &c, the process is the same as with the figs a, b, c, d, and x.

The cen of grav of any plane fig may be found by drawing it to a scale on pasteboard; then cut out the figure; balance it in two or more positions over the edge of a table, or on a sharp knife-edge; and mark on it the directions of the edge. Where these directions intersect each other, will be the reqd point, near enough for most practical purposes. The paper on which the fig is prepared, must be so stiff that the fig will not bend when balanced.

Of a cube, parallelopiped, cylinder, prism, sphere, spheroid, ellipsoid; the cen of grav is in the cen of the body.

Of any frustum of either a right or oblique cylinder; *or of a right or oblique prism;* whether cut parallel to its base, or obliquely. In the center of the axis of the frustum.

Of a right pyramid, or cone; in its axis, at ¼ of its length from the base.

Of any pyramid, or cone; whether right, or oblique. In a line from its vertex, or top, to the cen of grav of its base; and at ¼ the length of this line from the base.

Of a frustum of a right cone, cut parallel to its base. Call the rad of the larger end R; and that of the smaller end r; and the height of the frustum measd on its axis, h. Then,

$$\frac{h}{4} \times \frac{R^2 + 2Rr + 3r^2}{R^2 + Rr + r^2} = \text{dist on axis from greater end, or base.}$$

Of a frustum of a right pyramid or of a right cone, cut parallel to its base. Call the area of the large end A; that of the small end a; the height measd on the axis, h;

$$\frac{h}{4} \times \frac{A + 2\sqrt{A}\,a + 3a}{A + \sqrt{A}\,a + a} = \text{dist on axis from greater end, or base.}$$

Of any frustum of any pyramid, or cone; whether right or oblique; or whether the ends of the frustum are parallel to the base or not; it is in a line t o, drawn between the centers of grav of the two ends of the frustum; but its dist y c above the larger end $a n$, and perp to it, must first be found upon the line $t y$, drawn from the cen of grav of the small end, to, and at right angles with, the large end $a n$. Having first found the line $t y$, whether for a pyramid or a cone, use it instead of the height h, in the preceding formula, for a frustum of a right *pyramid*. The result will be y c; and by drawing c s parallel to $a n$, we find the reqd point s, in the line t o.

Of the curved or slanting surface only, of a right cone; in the axis; and at ⅓ of its length from the base.

Of a hemisphere; in its axis, at ⅜ of its length from the base.

Of a segment of a sphere. From twice the rad of the sphere, take the height of the segment. Square the remainder. Then from 3 times the rad of the sphere, take the height of the segment. Div the square just found by this last rem; and take ¾ of the quot, for the dist from the cen of the sphere to the cen of grav of the segment.

Of the curved surface only of a hemisphere; of a spherical segment; or of a zone: at the middle of its axis, or height.

Of a paraboloid: in its axis, and at ⅓ of its length from the base.

Of a hollow body, or of any number of solid or hollow bodies, use the last three paragraphs on p 348, except that *weights* must be taken instead of areas.

Rem. We must not confound cen of *grav* with cen of *weight*. It does not follow when a body balances on a knife-edge, that there is equal wt on both sides of it; but merely that the wts of the several particles on one side, when mult by their respective leverages, or dists, at right angles from the knife-edge, have a united *moment* about the knife-edge, equal to that of the particles on the other side of it, when mult by *their* leverages. See Leverage, &c.

Art. 57. That point through which the direction of a single antiresultant force must pass, in order to balance several other forces acting at diff points; or, in other words, that point through which the direction of the resultant of those forces must pass, is called the **center of pressure,** or **of force,** or **of strain,** of those forces, as the case may be.

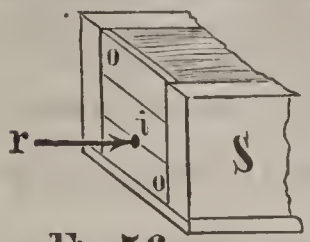


Fig 56

For instance, let S, Fig 56, be a common wooden box; but having one side, as o o, loosely fitted, so as barely to allow of pushing it backward and forward. *Fill* the box with dry sand, (clean small gravel will be better,) and it will be found that there is but one single point, *i*, at which we can, by holding to it a thin rod *r i*, balance the pres of the gravel against the opposite side of o o. If we apply the rod at any other point, o o will give way before the sand; if the rod is held above *i*, the bottom of o o will be pushed outward; if held below *i*, the top of *o* o will move outward. This point *i* is dist above the bottom of the sand one-third of the depth of the sand: in other words, the cen of pres of sand of any depth is, like that of water, at one-third of that depth from the bottom. In the case before us, the depth is supposed to be uniform, so that the cen of pres is at the same height above the bottom, clear across the box.

Now the balancing force applied through the rod at *i*, is the antiresultant of all the pressures resulting from the several particles of gravel against the opposite side of *o* o; and its effect upon the rigid body o o, (omitting of course any tendency to bend or break it, which comes under the head of Strength of Materials,) is precisely the same as that of all those forces combined; except that it is in the opposite direction. Its tendency to push o o bodily, or as an entire mass, toward the right hand, is precisely the same as that of the gravel to push it to the left hand; or it is the same as would result were we to heap up sand *in front* of o o, so as to balance the sand *behind* it.

Rem. It is this important principle of the cen of pres, that enables us to adopt the convenient practice of representing, by a single line, the effect of force actually distributed over a considerable surf. Thus, in Fig 52, the hor force *h a*, by which each half of the arch mutually prevents the other half from falling, is actually distributed over an area whose depth is the depth *e j* of the keystone; and its breadth, that of the whole bridge, as measd *across* the roadway. Yet the arrow *h a*, when drawn to a scale, perfectly represents the effects of this distributed force in upholding the half arch, considered as an *entire rigid mass*. So far as regards splitting or cracking the stone immediately at *a*, the effects would of course be diff; but as the whole force is only *supposed*, for convenience, to be applied at *a*, this diff is merely ideal in this instance. See Arts 5 and 8, p 226, 227.

It is evident from the foregoing that "cen of grav" means nothing more than "cen of force;" except only that the former is a convenient term for denoting that the *force* is that of *grav* alone.

Art. 58. When either grav or other forces are to be considered with regard to their effects in *bending*, or *breaking* bodies; instead of moving or straining them as entire masses, supposed to be rigid, or incapable of change of form, they cannot be assumed to be concentrated at either the cen of force, or the cen of grav.

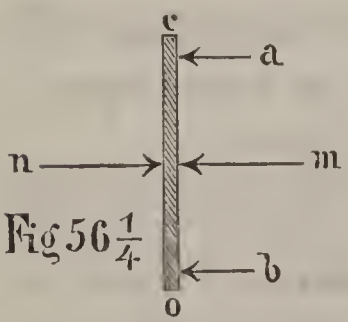


Fig 56 1/4

In the last case our applied extraneous forces are supposed to be brought to bear only upon other extraneous forces acting upon the bodies at the same time; the bodies themselves being regarded merely as unalterable mediums through which said forces are enabled to act upon *each other;* but in *bending, breaking, twisting, shearing,* &c, our extraneous or mechanical forces must be considered to act against the *inherent cohesive forces* of the bodies *themselves;* therefore these forces, which before were entirely neglected, now acquire a primary importance; the question of strength of materials comes in, and the assumption of perfect rigidity must be altogether discarded. Thus, in Fig 56¼, so far as regards either moving a rigid body c o, or straining it against the force *n*, it is immaterial whether we employ the two equal parallel forces *a, b*, or a single force *m*, equal to both of them, and acting at their center of force.

But since no bodies are absolutely rigid, but may all be bent or broken, it is plain that the two forces *a, b*, straining the body c o against the force *n*, would bend or break it much more readily than the force *m* would.

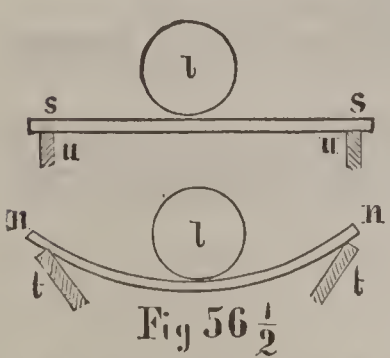


Fig 56 1/2

An absolutely rigid hor beam *s s*, would sustain any amount of load *l*, without bending; and consequently would always press vert upon its upright supports *u, u*, without any tendency to press them sideways. But an actual beam *n n*, if overloaded, will bend; thereby generating at its ends forces which are not vert, but which will tend to overthrow the supports *t t*.

Art. 59. To find the point of impartation of the resultant of parallel forces. *Case* 1. Two equal or unequal parallel forces, Fig 57, i

the same direction; and consequently in the same plane. Draw any straight line $a\,o$, uniting their directions $1\,m$, $3\,n$. Measure this line, and div it into two parts, $i\,o$, $i\,a$, proportioned like the forces; but placed *inversely* to the forces; that is, place the longest part near the smallest force, and vice versa. Through the point i draw the direction of the resultant R, parallel to the forces. Then is t the point of impartation. Make the resultant, $t\,4$, equal to the sum of the two forces. If the two forces are equal, R is evidently midway between them.

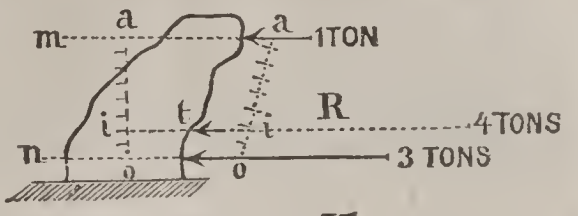


Fig 57

The foregoing, as well as some other facts connected with parallel forces, will be more easily recalled to mind, by associating them with the idea of the common steel-yard, Fig 58. Here the two forces in Fig 57 are represented by the wt 1 lb at c, and the wt 3 lbs at d; one of them 3 times as great as the other. We know that these weights, when suspended at dists $f\,a$, $f\,b$, one of them 3 times as great as the other, from the fulcrum f, are balanced by their antiresultant $f\,h$, which is equal to $3+1=4$ lbs, or the sum of the two forces. This antiresultant has precisely the same tendency to pull the steel-yard upward, that the two weights have to pull it downward. Also $f\,s$, equal to $f\,h$, but acting in the opposite direction, is the resultant of the two forces; its effect to pull the steel-yard bodily downward, when applied to it at f, is precisely equal to that of the two wts applied at a and b. The effect of the two wts at a and b, to *bend* or *break* the steel-yard, would plainly be very diff from that of their resultant $s\,f$, applied at f. See Art 58.

Fig 58

Case 2. Two *unequal* parallel forces, as $d\,a$, 3 tons, and $h\,f$, 4 tons, Fig 59, imparted to a rigid body in opposite directions, but not in the same straight line.

Starting from the line of direction $d\,m$, of the all force, draw any line $m\,n$, passing through of *beyond* the direction $h\,f$ of the large one. Now and the *amount* of the resultant R. This, by Art 56, is equal to the diff between $h\,f$ and $d\,a$, since they are in opposite directions; and has the same *direction* as the large one; therefore it is equal to $4-3=1$ ton; and its direction is the same as that of $h\,f$. Then say, as the resultant is to the small force $d\,a$, so is the dist $o\,m$, to the dist $o\,s$, along the line $m\,n$. Through s draw $s\,c$, parallel to the two forces; and c will be the reqd *point of impartation* of the resultant R; the tendency of which to move the entire rigid body, will he equal to the joint tendency of $d\,a$ and $h\,f$. An anti-resultant of 1 ton at t would counteract both the revolviug tendency of $h\,f$ and $d\,a$ (supposing R removed), and their tendeucy to move the body as a whole.

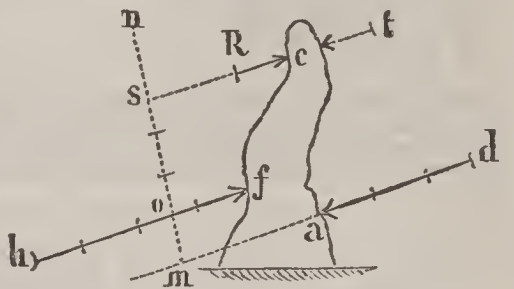


Fig 59

This case, like the preceding one, is illustrated by the steel-yard, Fig 58; where $a\,d$, and $f\,h$, represent the two forces in the last fig; while $b\,c$ represents their balancing antiresultant, correspouding to t.

Case 3. **Couples.** Two *equal* parallel forces, a, and b, Fig 59 a, imparted to a rigid body in opposite directions, but not in the same straight line, are called *a couple*. The *force* of a couple, means simply *one* of the forces; the perp dist c, between the directions of the two forces, is called the *arm*, or *leverage* of the couple. If one of the forces be mult by this arm, the prod is the *moment* of the couple, in foot-lbs, &c. A couple has no tendency to move the entire body forward in the direction of either force; but merely to make it rotate around a point o, half-way between the points at which the two forces are imparted. No single force can balance a couple. To do this requires a second couple whose moment is equal and opposite to that of the first one. Hence a couple cannot have a single force as a resultant.

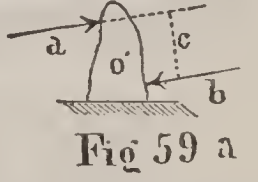


Fig 59 a

Case 4. Any number of parallel forces, whether in the same plane, or in the same direction, or not. The process in this case consists in a mere repetition of that in Case 1, Fig 57, as follows.

Namely, commencing with those forces which point in the same direction, as a, b, c, Fig 60, which all point downward; between the directions of any two of them, as a and b, draw any straight line i, and div it into two parts $j\,o$, $j\,i$, proportioned like the forces, but placed *inversely* to the forces. Through j draw $m\,s$, parallel and equal to the forces a and b. Then is $m\,s$ the resultant of those two forces. Next find the resultant of this resultant $m\,s$ and any third force, as c, in the same manner. That is, draw any line $t\,w$, uniting their directions; div it into two parts $l\,t$ and $l\,w$, proportioned to the forces, but placed inversely. Through l draw $n\,g$, and make it equal to $m\,s+c$. Then is

$n\,g$ the resultant of the three forces a, b, c; and g is the cen of force of those forces, and consequently is the point of application of their resultant. So do with any number in the same direction. Then proceed in the same manner with those which point in the opposite direction, as d, e, x, y; and having found *their* resultant, find by Case 2 or 3, or by Art 13 or Art 16, as the case may be, the resultant of the two resultant forces, now obtained, and acting in opposite directions.

It is not at all necessary that the forces be supposed to act upon a plane surf, as in Fig 60; the process is precisely the same when, as in Fig 61, they are imparted at points x, y, z, not in the same plane. This is a consequence of the principle laid down in Art 18, namely, that the effect produced upon a rigid body by an imparted force, remains the same, no matter at what point of the body it be imparted, so long as that point *is in the line of the direction of that force.*

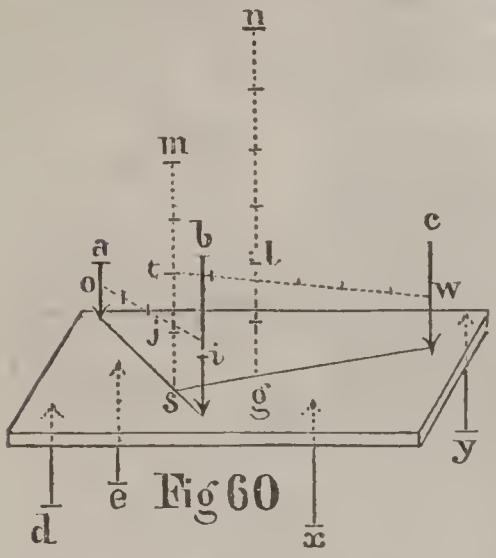


Fig 60

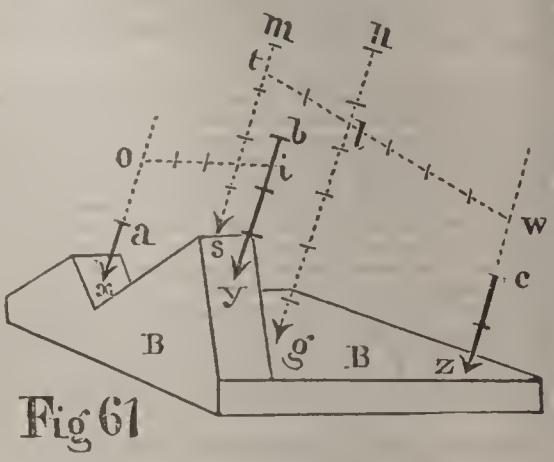


Fig 61

Although Figs 60 and 61 serve to illustrate the principle, they plainly do not give the actual positions of the forces and resultant; because they are necessarily drawn in a kind of perspective, so that all the parts cannot be measd by a scale. The *amounts* of the resultants are easily found by calculation; inasmuch as they are equal to the sums of the forces.

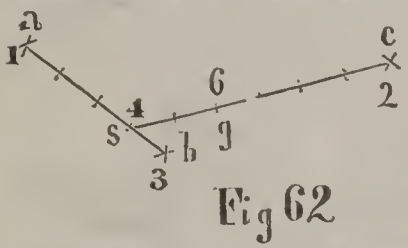


Fig 62

The points for imparting them can be found correctly from a drawing *in plan*, like Fig 62; where the stars represent by scale the *actual* dists apart of the directions of the forces. The *quantities* of the forces, instead of being shown by lines, are to be written in figures, as shown in the fig. This being done, it is easy to find the points s, g, &c, of the resultants.

Art. 60. The Inclined Plane is a rigid straight plane surf, as $a\,b$, Fig 63, not hor. If a vert line $b\,c$ be drawn from the top b of the plane, to meet a hor line $a\,c$, drawn from its bottom a, then $b\,c$ is called the *height* of the plane; $a\,c$ its *base;* and $a\,b$ its *length.* The angle $b\,a\,c$, which the plane forms with the hor line $a\,c$, is called its *inclination*, *slope*, or *steepness;* which, however, is frequently expressed also by the proportion which the base bears to the height; thus, if the length $a\,c$ of the base be 1, 1½, 2, &c, times that of the height $b\,c$, the inclination or slope is said to be 1 to 1, 1½ to 1, 2 to 1, &c. The angle $b\,a\,c$ is the *angle of inclination* of the plane.

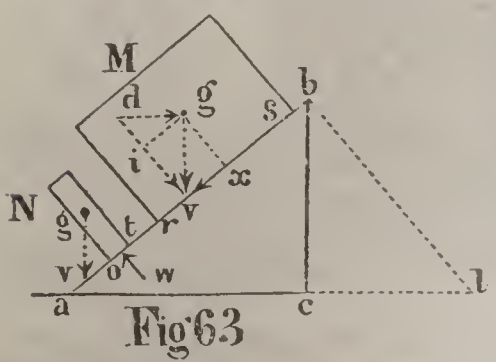


Fig 63

By Art 57, when one rigid body as N or M, Fig 63, is placed loosely upon another, as upon the rigid plane $a\,b$, the effect produced by its wt is the same as if all that wt were concentrated at its cen of grav g, and acted in the direction of a vert line $g\,v$ drawn through said center. When we assume the wt to be thus concentrated at the point g, we must remember that *all other parts of the body must be considered to be without weight;* although still retaining their inherent cohesive force, or strength.

If, as in N, this vert line $g\,v$, which now represents the direction of the *entire wt* of the body, passes beyond, or outside of the base, the body must fall; because this wt meets with no opposing force, in the direction $v\,g$, to react against it; and thus prevent it from producing motion.

But if, as in the body M, the line $g\,v$ falls within the base $r\,s$, the body will not upset: but we shall have (Art 19) a force $g\,v$ equal to the wt of the body, and applied obliquely to a rigid surface $a\,b$, at

the point v; and consequently resolvable into two components; namely, $i\,v$, perp to the surf $a\,b$, and therefore imparted to it as a pressure; and $x\,v$, parallel to the surf, and consequently not imparted to it. All these lines may be drawn by scale, to represent their respective forces. When we consider a single force as $g\,v$ to be thus resolved into two compouents, with a view to ascertaining their effects, it is plain that said single force must then be considered as no longer existing; but as being replaced by its components. Now the component *force* $x\,v$ being parallel to the plane, it follows (Art 15) that the *pressure* or *strain* $i\,v$, no matter how great it may be, cannot in the slightest degree oppose the cross action of the moving force $x\,v$, no matter how small it may be; and $x\,v$ must therefore produce motion in the body, causing it to slide down the plane; unless some third force, not yet spoken of, shall present itself, opposed to $x\,v$. But any forces which press bodies together, always produce a new force, friction, at the joint, or surfs of contact of the bodies; and this friction acts in direct opposition to any force, in any direction whatever that is in the plane of that joint or surfs. Therefore the force $i\,v$ produces fric between the surfs, $r\,s$, of the body M and of the inclined plane; and this fric acts in the direction $r\,s$, or diametrically opposite to that of the force $x\,v$. The amount of fric depends upon that of the pres; as also upon the nature of the bodies at whose surfs it is produced, upon the degree of smoothness of those surfs, and upon whether they are lubricated or not. If the fric is greater than the force $x\,v$ in the opposite direction, the body of course cannot move; but if less, it will move, under the action of a force equal to the excess of $x\,v$ over the fric. It must be remembered that the pres component $i\,v$, which produces fric on an inclined plane, is not equal to the wt of the body, but is less than it. It is equal only when the surf is hor, so that the vert force $g\,v$, representing the entire wt of the body, is at right angles to the joint, and when, consequently, it *all* acts as pres. Therefore, the steeper the plane becomes, the less is the fric; because then less of the wt of the body acts as pres, and more of it as moving force. Hence, a locomotive has less adhesion on an inclined grade, than on a level; for the *so called* adhesion is in reality nothing but fric. But although both the perp pres and the fric become less in amount as the plane becomes steeper, yet they retain practically the same *proportion* to each other; until pressures become so great that *abrasion* of the surfs of contact takes place; the proportion of the fric to the pres then increases.

Rem. It is evident that when we wish to push a body *up* an inclined plane, we must overcome both the fric, and the parallel force $x\,v$; but in pushing it *down*, we are opposed only by the fric; for the parallel force assists us.

Art. 61. Experiment has determined the amount of fric which takes place between the surfs of such materials as are employed in construction; that is, it has determined the proportion (or, more correctly, the ratio) between the pres and the fric. Any person may easily do this for himself, thus: A body as M, Fig 63, is placed upon the plane surf $a\,b$; of which one end, as b, is gradually raised until the body is barely about to begin to slide. When this takes place, we know that the *force* x v *has become barely equal to the fric;* and the angle $b\,a\,c$, which the plane then makes with the hor $a\,c$, is called the *angle of friction*, or *limiting angle of resistance*, or *angle of repose*, for the particular kind of surf experimented on.

Now, a little reflection will show that whatever may be this angle, $b\,a\,c$, of fric, the line $x\,v$, which measures not only the parallel force, but also the *fric* existing at that moment, (and at no other one,) is to the line $i\,v$, which measures the perp *pres*, (not the *wt* of the body;) as the vertical height $b\,c$ of the plane at the same moment, is to its hor base $a\,c$. That is, *at the point of sliding*, as Friction : Perp Pressure :: Height : Base; or, as Base : Ht :: Perp Pres : Friction. Therefore, when a body barely begins to slide, measure $a\,c$ hor, and $b\,c$ vert; div the last by the first, and the quot will be the proportion which the fric of the bodies experimented upon, bears to the pres which causes it. Or, measure the angle $b\,a\,c$ in degrees, &c; the nat tang of this angle will be that same proportion. This proportion is called the *coefficient of friction* for those bodies; a table of which will be found on page 373. A *hor* line $d\,g$, drawn from g, and terminating in $v\,i$ extended, will, when measd by the same scale as $g\,v$, $i\,v$, $x\,v$, give a hor force which, without the aid of friction, would react against the force $x\,v$, and prevent it from moving the body down the plane. Or if the length $a\,b$ of the plane be taken by scale to represent the wt of a body, then $b\,l$, perp to $a\,b$, to meet $a\,c$ produced at l, will give that same hor force.

Art. 62. If the length $m\,n$, Fig 63½, of an inclined plane, be taken by a scale, to represent the wt in lbs, tons, &c, of any body placed upon it; then the base $o\,n$ will, by the same scale, give the perp pres in lbs, tons, &c, which the body imparts to the surf of the plane; and the height $m\,o$ will give the amount of force parallel to $m\,n$, and which tends to move the body down the plane, either by sliding or rolling. If the pres $o\,n$ be mult by the proper coeff of fric, the prod will plainly be the actual amount of fric in lbs, &c. If the fric thus obtained proves to be greater than the sliding force $m\,o$, then the body will remain at rest on the plane; but if less, then sliding or rolling down the plane will be the result; and the amount of force which *starts* or *begins* the motion, will be equal to the excess of $m\,o$ over the fric.

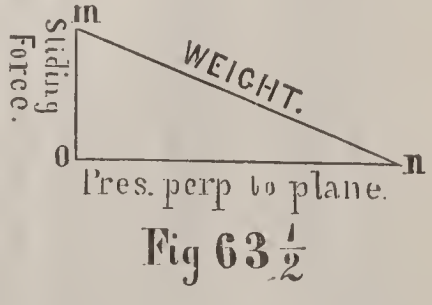


Fig 63 ½

As the motion continues, it will be accelerated by the accumulation of gravity.

When a body is placed upon an inclined plane, whether it slides or not, the pres which it produces at right angles to the surf of the plane, is equal to the wt × nat cosine of angle of slope; the sliding force parallel to the surface of the plane, = wt × nat sine of angle of slope; the actual amount of fric = wt × nat cosine angle of slope × coeff of fric. For the fric does not vary as the angle of slope of the plane, but as the *cosine* of that angle; in the same manner as the perp pres varies.

Ex. 1. Suppose we wish to slide a wooden box M, Fig 63, filled with stone, and weighing in all

1200 lbs, *up* the iron rails of an inclined plane, sloping 5°; what force must we use, parallel to the plane; assuming the coeff of wood on iron to be .4, or $\frac{4}{10}$ of the perp pres? Here we have to overcome the parallel force $x\ v$, and the fric. Now, as just stated, this parallel force $x\ v$ is equal to, wt × nat sine of slope, = 1200 × .087 = 104.4 lbs. The fric is equal to, wt × nat cos of slope × coeff of fric: = 1200 × .996 × .4 = 478. Consequently, 104.4 + 478 = 582.4 lbs, is the force reqd. In fact, however, this force merely *balances* the downward tendency of the box, together with its fric; thus rendering them incapable of resisting any *additional* upward force; but it is plain that we must apply *some* additional force, in order to impart *motion* to the now unresisting box.

Now, suppose we wish to slide the box *down* the plane, what force must we use? Here nothing resists us but the fric, just found to be 478 lbs. The parallel force helps us to the amount of 104.4 lbs; therefore we need only to add 478 — 104.4 = 373.6 lbs.

The box will then be upon the point of moving. Any additional force will move it.

For acceleration on inclined planes see p 363.

The following table will facilitate calculations respecting the draft required on grades, inclined planes, &c. **In practice, allowance for friction must be made in the last 2 cols.** Original.

Inclination or Slope of the Plane. The sloping length is = vertical height / sine, col 6.					Pres. on Plane, in parts of the wt. Or, nat. cos. of angle of Plane.	Pres. on Plane, in lbs per ton.	Tendency down the Plane, in parts of the wt. Or, nat. sine of angle of Plane.	Tendency down the Plane, in lbs per ton.
Vert.	Hor.	Ft. per mile.	Deg.	Min.				
1 in	3.	1760.00	18	26	.9487	2125	.3162	708.
1 in	4.	1320.00	14	2	.9702	2173	.2125	543.
1 in	5.	1056.00	11	19	.9806	2196	.1962	439.
1 in	6.	880.00	9	28	.9864	2210	.1645	368.
1 in	8.	660.00	7	8	.9923	2223	.1242	278.
1 in	9.	588.66	6	20	.9939	2226	.1103	247.
1 in	10.	528.00	5	43	.9950	2229	.0996	223.
1 in	11.4	461.94	5	00	.9962	2231	.0872	195.
1 in	12.	440.00	4	46	.9965	2232	.0831	186.
1 in	14.3	369.23	4	00	.9976	2232	.0698	156.
1 in	15.	352.00	3	49	.9978	2233	.0666	149.
1 in	19.1	276.73	3	00	.9986	2237	.0523	117.
1 in	20.	264.00	2	52	.9987	"	.0500	112.
1 in	23.1	229.04	2	30	.9990	"	.0436	97.7
1 in	25.	211.20	2	17	.9992	2238	.0398	89.2
1 in	28.6	184.36	2	00	.9994	"	.0349	78.2
1 in	30.	176.00	1	55	"	"	.0334	74.8
1 in	32.7	161.47	1	45	.9995	2239	.0305	68.4
1 in	35.	150.86	1	38	.9996	"	.0285	63.8
1 in	38.2	138.22	1	30	.9997	2240	.0262	58.6
1 in	40.	132.00	1	26	"	"	.0250	56.0
1 in	45.8	115.29	1	15	"	"	.0218	48 8
1 in	50.	105.60	1	9	.9998	"	.0201	45.0
1 in	57.3	92.16	1	0	"	"	.0175	39.1
1 in	60.	88.00	0	57½	.9999	"	.0167	37.4
1 in	70.	75.43	0	49	"	"	.0143	32.0
1 in	76.4	69.12	0	45	"	"	.0131	29.3
1 in	80.	66.00	0	43	"	"	.0125	28.0
1 in	90.	58.67	0	38	"	"	.0111	24.9
1 in	100.	52.80	0	34	1.0000*	"	.0100	22.4
1 in	114.6	46.07	0	30	"	"	.0087	19.6
1 in	125.	42.21	0	27⅔	"	"	.0080	17.9
1 in	150.	35.20	0	23	"	"	.0067	15.0
1 in	175.	30.17	0	19⅔	"	"	.0057	12.8
1 in	200.	26.40	0	17	"	"	.0050	11.2
1 in	229.2	23.04	0	15	"	"	.0044	9.77
1 in	250.	21.12	0	14	"	"	.0041	9.18
1 in	300.	17.60	0	11⅓	"	"	.0033	7.39
1 in	313.9	15.35	0	10	"	"	.0029	6.52
1 in	400.	13.20	0	8⅔	"	"	.0025	5.60
1 in	500.	10.56	0	7	"	"	.0020	4 48
1 in	600.	8.80	0	6	"	"	.0017	3.81
1 in	800.	6.60	0	4¾	"	"	.0013	2.91
1 in	1000.	5.28	0	3¾	"	"	.0010	2.24
1 in	3137.	1.54	0	1	"	"	.0003	0.65
Level.		0.00	0	0	"	"	.0000	0.00

For other tables of grades, see pp 176, 723, 724, 725.

Art. 63. The following principle is one of great practical importance. When

* Near enough for practice; actually .99995, or less by one part in 20000, or about 1 lb in 9 tons.

a plane $w\,x$, Fig 64, has just that inclination, $y\,x\,w$, at which the fric of any given body is balanced; and sliding is about to commence, from the action of any force $h\,o$, applied to the plane, through the body, in any direction $h\,o$, not perp to it; if from the point o of application, we draw a line $o\,p$, at right angles to the surf of the plane, then the angle $h\,o\,p$ will always be equal to the angle of fric $y\,x\,w$ of the body. If the plane is so steep that sliding must take place, then the angle formed between the force $h\,o$, and a perp $o\,p$ to the plane, becomes greater than the angle of fric; but if the steepness is so slight that the body rests firmly on the plane, then said angle is less than the angle of fric.

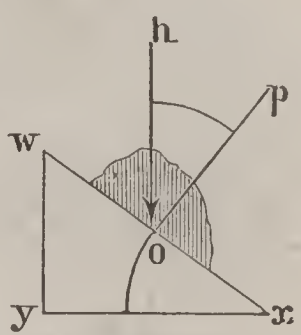


Fig 64

The practical applications of this principle are very numerous; they extend to pressures in any directions whatever; and apply to plane surfs in any positiou whatever, whether inclined, vert, or hor; for any given pres produces precisely the same amouut of fric, whether we impart it to the ceiling, the floor, or the wall of a room; provided they all be of the same material. The angle of fric of cut stone upon cut stone is about 32°; that is, one block of cut stone will not slide upon auother at a less slope than about 32°; the fric then being full $\frac{6}{10}$ of the pres. Therefore, if the floor f, Fig 65, ceiling c, and walls w, w, of a room be of cut stone; aud p, p, p, p, lines at right angles to them; we may press a piece s of cut stone against them *with any force whatever*, applied in the direction of the stone itself, without danger of its sliding; provided only that the direction of the pres along s does not form with the perp p an angle exceeding 32°. But sliding will take place, whether the pres be great or small, if, as at o, o, o, o, said angle exceeds 32°. The angle of fric is, by some writers, called, in such cases, **the limiting angle of resistance.**

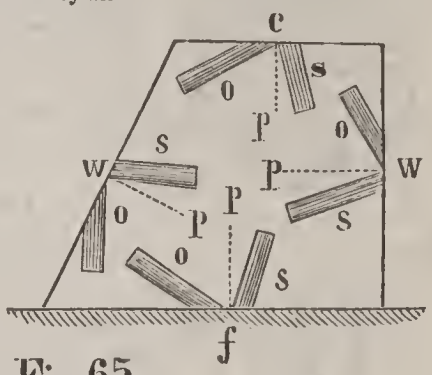


Fig 65

Rem. The friction at the feet of rafters when highly inclined diminishes very much their horizontal pressure and tendency to split off the ends of the tie-beams.

The angle of fric of oak endwise against hard limestone, is, according to Morin, 20¾°; therefore, if the walls, &c, of a room consisted of such limestone, we could not press a piece of oak endwise against it without sliding, if the angle with p exceeded 20¾°; and the legs of a wooden trestle, Fig 66, would not spread, on the level surf of such limestone, under any wt w, if the augle $a\,b\,c$ be less than 20¾°; but certaiuly would if it be greater, unless other preventives besides fric at the feet be depended on. In this case the fric amounts to very nearly $\frac{4}{10}$ of the pres; that being the proportion corresponding to 20¾°. These two illustrations show how wide is the application of this principle; for the announcement of which we are (the writer believes) indebted to Moseley.

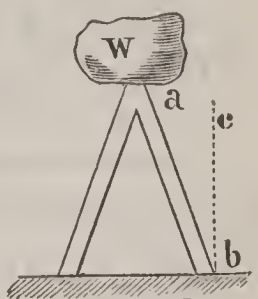


Fig66

Art. 64. To find the effect of an extraneous force ($f\,g$, **Fig 67.**) **imparted in any direction, to a rigid body (B) on an inclined plane,** $i\,p$; when we know the angle of fric, and the wt of the body. The principle laid down in the preceding Art enables us to do this.

Through the cen of grav c, of the body, draw a vert line $a\,w$; and extend the direction $f\,g$ of the force, to meet this line, as at o. Make $o\,a$ by scale, to represent the wt of the body; and $o\,z$ to represent the amount of the force $f\,g$. Then is o a point at which we may assume both these forces to be imparted to the body. (Art 29.) Complete the parallelogram of forces $a\,x\,z\,o$, by drawing $a\,x$, and $z\,x$, parallel and equal to $o\,z$, and $o\,a$. Draw the diag $x\,o$, and extend it to meet the plane, as at t. Make the line $t\,v$ perp to the surf of the plane. This done, we have a single force $x\,o$, equal in effect upon the rigid body, to its wt, and $f\,g$ combined.

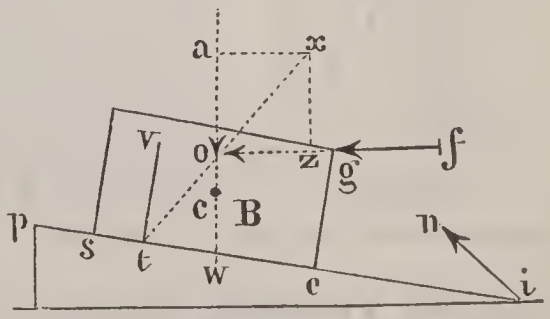


Fig 67

This single force may (Art 18) be considered as imparted to the body at any point that lies in its line of direction $x\,t$; therefore, we will assume it to be imported at t, where it encounters the force of fric acting in the direction $s\,e$, of the joint formed between the body, and the plane. Now, if i strikes within the base $s\,e$, $t\,v$ being at right angles to this joint, it follows from the last Art. that if the angle $x\,t\,v$ is less than the angle of fric corresponding to the nature of the materials which

compose the body and plane, then the body will remain at rest on the plane. But if the angle $x\,t\,v$ be greater than said angle of frie, the body will slide *up* or *down* the plane, (according to circumstances, stated in the next paragraph;) if the angles be equal, the body will be just on the point of beginning to slide either up or down.

When the angle $x\,t\,v$ is on the *down hill* side of $v\,t$, as in the fig, the tendency of the body will evidently be to move *up* the plane; but if, in consequence of a diff direction of the force $f\,g$, (and consequently of the resultant $x\,o$,) the angle $x\,t\,v$ is on the *up hill* side of $v\,t$, then the tendency will be *down* the plane.

Rem. 1. If the direction of the resultant $x\,o$, or the point t, falls *outside* of its base $s\,e$, the body, instead of sliding, will upset. It will fall up hill, if t strikes $p\,i$ up hill from the base; and down hill, if t strikes down hill from the base. See Art 65.

Rem. 2. In order to draw the parallelogram of forces $a\,x\,z\,o$, and its resultant diag $x\,o$, the line $a\,o$, which represents the wt, may sometimes have to be regarded as *pulling* instead of *pushing* downward at the point o, where the other force meets it. See Fig 9¼, and Fig 69, below.

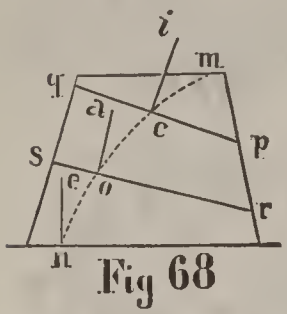


Fig 68

Rem. 3. It follows from the foregoing, that when at the joints $p\,q$, $r\,s$, Fig 68, of a mass of masonry; or at the joints of timbers in carpentry, iron work, &c. the frie alone is depended on to prevent sliding, the resultant as $m\,e$, $c\,o$, $o\,n$, &c, of all the forces acting at any joint, must not form an angle $m\,e\,i$, $c\,o\,a$, $o\,n\,e$, with a perp $c\,i$, $o\,a$, $n\,e$, to the joint, greater than the angle of frie corresponding to the nature of the materials whose surfaces constitute the joint.

Rem. 4. The extraneous force reqd to move a body up a plane, will be the least when its direction, $i\,n$, Fig 67, makes with surf $i\,p$, of the plane, an angle, $n\,i\,p$, equal to the angle of frie.

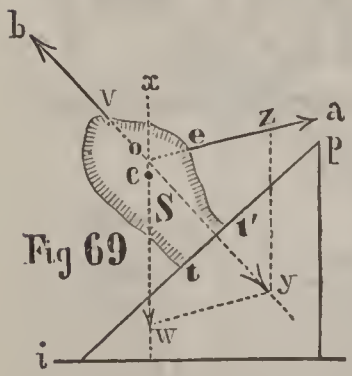


Fig 69

Art. 65. To find the force required to prevent a body S, Fig 69, from moving; when the direction, $o\,w$, of its wt, strikes outside of its base $t\,t'$. Thus, suppose we wish to impart a pulling force at e, and in the direction $e\,a$, to prevent the body from sliding either up or down the frictionless plane $i\,p$, and from upsetting down the plane. Through the cen of grav e, draw a vert line $x\,w$; and continue the line of direction of $a\,e$ to meet it at o. From o draw $o\,y$ at right angles to the plane $i\,p$. By scale make $o\,w$ equal to the wt of the body: and from w draw $w\,y$ parallel to $o\,a$. Make $e\,a$ equal to $w\,y$; then is $e\,a$ the reqd force, which will resist all tendency of the body to move. For in the parallelogram of forces $o\,w\,y\,z$, we have the force $o\,w$ tending to make the body fall; and the force $o\,z$ (equal to $e\,a$) tending to prevent it from falling; and the resultant $o\,y$, of these two forces, *equal to their joint effect*, is at right angles to the surf of the plane; and is consequently (Art 19) all imparted to it as pres; no part being left unresisted, to produce motion in any direction. For as before said, when two forces, as $o\,w$, $o\,z$, are compounded into one resultant force $o\,y$, those two forces must be considered as no longer existing; thus, in this case, so long as we regard the joint effect of $o\,w$ and $o\,z$ as being concentrated in their resultant $o\,y$, we cannot of course, consider them as acting in other directions at the same time; so that there is, as it were, no longer any wt, $o\,w$, tending to make the body fall; nor any force $e\,a$, tending to uphold it; but only the single force $o\,y$, which presses the inert body against, and at right angles to, the surf $i\,p$; imparting to it a tendency to move only in the direction $o\,y$; which tendency is reacted against by the inherent cohesive force, or strength, of the plane. If the body is prevented, by friction or by a stop at its lower toe t, from *sliding* down the plane, and we wish to know the least force in the direction $o\,z$ which will just prevent the body from overturning about t; the line $o\,y$, instead of being drawn perpendicular to the plane $i\,p$ as in Fig 69, must be drawn from o through the lower toe t as in Fig 69 a. The lines $o\,w$ and $z\,y$ of the parallelogram $o\,z\,y\,w$, representing the weight of the body, will of course remain the same as for the same body in Fig 69, but the lines $o\,z$ and $w\,y$, representing the extraneous force, although the same in *direction* as before, will evidently be much *shorter*.

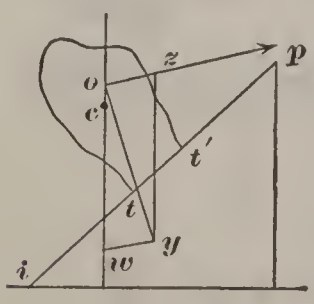


Fig.69 *a*

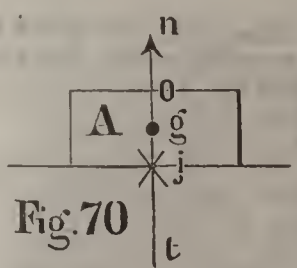


Fig.70

Art. 66. Stability. The stability of a structure, or of any body, is, strictly speaking, that resistance which its wt alone enables it to oppose against forces tending to change its position. Such resistance may be assisted by extraneous wts, or by other forces properly applied; but such must be distinguished from the stability inherent in the structure, or body itself. To insure the stability of a structure, the disposition of its parts, as well as that of the entire mass, must be such that neither of them shall move, either by *sliding*, or by *overturning*, under the action of the imparted forces. Stability is therefore a branch of Statics; or of forces at rest, or in equilibrium with each other; Art 16.

Stability must not be confounded with strength. A structure may be very strong, and yet very unstable. A block of stone is quite as strong while sliding down a smooth plane, or rolling down a steep bank, as when resting on a firm hor base; but it has stability only in the last case. A pyramid of weak chalk may have great stability; while a globe of granite or cast iron, has very little. We generally have to examine into the strength, as well as the stability of our structures: but it must be done by diff processes. The stability has reference to the structure considered as consisting of one or more *rigid* bodies, which may be *moved* as entire masses, but not *broken*, or *changed in form*, by the applied forces. See Remark 2, Art 29.

Those forces which tend to impair the stability of a structure, are called *acting* ones; and those which tend to maintain it, *resisting* ones. This distinction is merely a matter of convenience; for all the forces act, and resist.

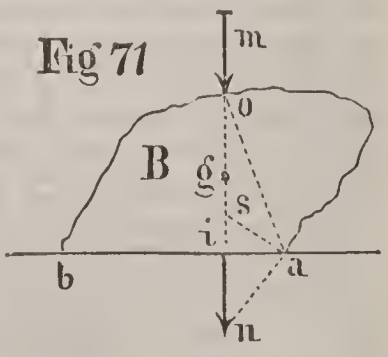
The forces which affect the stability of a rigid structure considered as one mass, are its wt; extraneous wts, or strains; and the foundation, or support; which last reacts as an antiresultant against the others. When these three balance each other, the structure is stable. When the structure is to be considered as composed of several rigid bodies, then the joints or surfaces of contact between these bodies must also be regarded as so many secondary foundations, and these also must respectively balance the forces acting upon them; otherwise these parts may slide, or overturn, while other parts may remain firm.

Art. 67. In order to guard against accidents, a structure must generally be so designed as to be capable of resisting much greater forces than those which it sustains under ordinary circumstances. The *proportion* which, with this object, we give to the resisting forces, in excess of the acting ones, is called the *coefficient of stability;* or simply the stability, or the safety, of the structure. Thus, if we make it capable of resisting 2, 3, or 6 times the amount of the ordinary acting forces, we say it has a stability, or a coeff of stability, or a safety, of 2, 3, or 6.

Art. 68. Since the stability of a structure, considered apart from its foundation, consists entirely in the resistance which its several parts, as well as the entire mass, can present against both sliding and overturning, it follows that two precautions, already adverted to in previous articles, must be resorted to. Namely, 1st, **against sliding,** take care that the resultant of all the forces acting upon any joint, (including that between the base and the foundation,) shall act either at right angles to said joint; so as to be entirely imparted to it as strain, (press or pull,) leaving no part unresisted to tend to produce motion; or else that it shall not deviate from a right angle, to a greater extent than the angle of fric corresponding to the materials which compose the joint; so that the portion of it which is not imparted at right angles, shall be resisted by friction; and thus be prevented from producing a motion of sliding.

Otherwise, instead of relying upon the *position* of the joints, resort must be had to the cohesive *strength* of *joint-fastenings*, such as bolts, spikes, cramps, joggles, mortises and tenons, mortar, cement, &c, to prevent sliding. As to mortar and cement, however, it is important to remember that frequently, and especially in very massive work, they have not time to harden, or acquire their full strength, before the acting forces are brought to bear upon them: therefore, great care is necessary, when we use them as substitutes for position. On this account we frequently cannot consider a mass of masonry to be a single rigid body, but must regard it as composed of several detached rigid bodies; the stability of each of which must be separately provided for, before we can secure that of the whole. Therefore, in large massive structures of importance, we should, as far as possible, omit all consideration of the strength of the mortar, and rely for stability chiefly upon placing the joints at or nearly at right angles to the forces acting upon them.

Art. 69. Moment of stability. We have already stated (see Arts 46 and 49) that the resistance which any rigid body as B, Fig 71, opposes against being overturned about any given point a, is equal to that which would be produced if the entire wt of the body were concentrated at its cen of grav g; and acted at the end i of a straight lever $a\,i$, of which a is the fulcrum; or at the end o, s, or n, of *any* straight lever (Art 49) $a\,o$, $a\,s$, $a\,n$; or of any bent lever $a\,i\,n$, $a\,i\,s$, $a\,i\,o$, $a\,s\,o$, provided that in every case there is the same leverage $a\,i$, measured from the fulcrum a, and at right angles to the direction $m\,n$ of the force of grav of the body. So far as regards tendency to resist overturning, it is immaterial (Art 18) at what point of the body, *in this line of direction* $m\,n$, we conceive the grav to act: or whether as a push at o, or a pull at i, as denoted by the arrows. We have also said that the tendency, or moment, of this force of grav, or wt, to produce or to resist motion about the fulcrum a, through the medium of any of these levers, is found by mult the force or wt in lbs, by the leverage $a\,i$ in feet. The prod in ft-lbs is generally called the moment of the *force* about the point a; but in cases like that before us, in which this moment becomes the measure of the stability of the body, it is called the *moment of stability* (or simply *the sta-*



bility) of the body, about that point. Therefore, if bodies of the same size and shape and with their centers of gravity in the same position, have diff wts, or sp gravities, their respective stabilities will be in proportion to their wts, or sp grs.

A body may have diff moments of stability, about diff points. Thus, it would be far more difficult to overturn B about the point *b*, than about *a*; because the leverage *b i* is 2½ times as great as *a i*; and since the wt and the point of the cen of grav remain unchanged, the moment about *b* is 2½ times as great as about *a*.

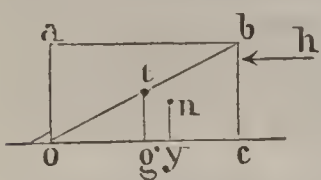


Fig 72

Rem. 1. Let *a b c* o, Fig 72, be a squared block of stone 6 feet long; on a hor base; and weighing 12 tons; and *h*, a force applied to overturn it about the toe o. Since its length o *c*, is 6 feet, its cen of grav *t*, will be dist o *g*, or 3 ft back from o. Consequently, the moment with which the block resists being overturned, is 12 (tons) × 3 (ft leverage) = 36 ft-tons. Now, suppose the upper half *a b* o, to be removed; the remainder o *b c* will weigh but 6 tons. But its cen of grav *n*, is farther from o, than that of the whole block was. Being now triangular in shape, the dist *o y* will be ⅔ of o *c*; or will be 4 ft. Consequently, the resisting moment will be 6 (tons) × 4 (ft leverage) = 24 foot-tons. So that although the block has but half the wt of the first one, it has ⅔ as great resisting power. It is on this principle, that in order to save masonry, the faces of retaining-walls, &c, are sloped, or *battered* back.

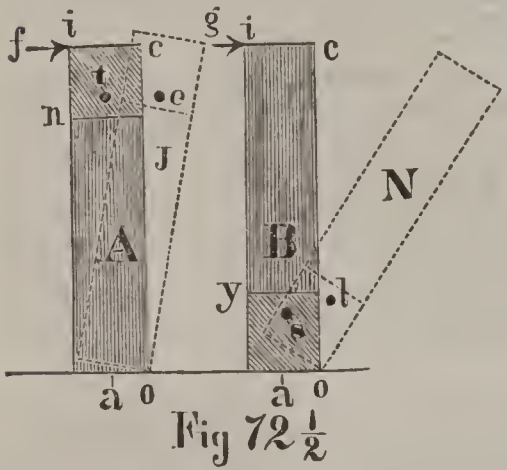


Fig 72½

Rem 2. In Fig 72½ let the upper part, *cn*, of A, and the lower part, *oy*, of B, be made of lead, and the remaining part of each, of cork. Then the center of gravity of the entire body A will be near the dot *t*; and that of B, near the dot *s*. Let the weights of A and B be equal, and their centers of gravity, *t* and *s*, at the same *horizontal* distance, *ao*, from the fulcrums, *o* and *o*, around which the bodies are to be overturned. Their moments of stability must then be equal. Consequently they will require, in order to *begin* to overturn them, equal forces, applied in any same given direction, and at any same given point; as, for example, the equal forces, *f* and *g*, applied horizontally at *i* and *i*. But, in order to *entirely overthrow* B, the overturning force must act *through a greater space*, *i e*, must do *more work*, than is required to overthrow A; for A will be overthrown when the force, *f*, has acted through only the small space necessary to move it into the position of the dotted lines J. Its center of gravity being then at *e*, which is beyond the base, A must necessarily fall. But the force, *g*, acting upon B, must move through the longer space necessary to move B into the position N; for not until then will its center of gravity, *s*, be in a position, *l*, beyond the base. As either A or B turns, about its fulcrum *o*, from its original position (A or B) to that (J or N) in which it is about to fall, its leverage, *ao*, of stability plainly diminishes, until the center of gravity is over *o*. The leverage is then = 0. Since the *weight* remains unchanged, the *moment* of stability decreases (and ceases) with the *leverage* of stability, as does also the *overturning* moment required to keep the body moving. The overturning leverage varies with the rising, and subsequent falling, of the corner, *i*, at which the force is applied; but in bodies shaped like A and B this variation is but slight and hence the horizontal *force*, *f* or *g*, required, decreases nearly as the leverage of stability decreases, ceasing when it ceases. The object of the civil engineer is generally to secure his structures against *beginning* to overturn. Hence he is chiefly anxious about the amount of the *force*, *f* or *g*, (in pounds, tons, etc) required to *start* motion; and seldom needs to concern himself about the amount of *work* (in foot-pounds, foot-tons, etc) required to *complete* the overthrow.

Rem. 3. It is not alone the wt of the body itself, which contributes to its stability in all cases; for this may be assisted by extraneous wts or loads. Thus, the wt of a pier P, Fig 73, gives it in itself a certain degree of stability; but when we add the wt of the two equal arches, its stability is thereby increased, supposing the foundation to be secure. And a passing load, when it is directly over the pier, increases it still more. It is true that the wt of the arches might crush the pier to fragments, if the stone be soft; but this is a matter of Strength of Materials; not of stability; and must be examined into by itself. If the two arches be of unequal sizes, or if there be but one arch, the stability of the pier may become either increased or diminished, according to circumstances; as will appear farther on.

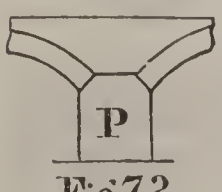


Fig 73

Whether the force acting for or against the stability of a structure, be gravity, or pushes, or pulls, produced from other sources, is, as in other cases, a matter of no importance: for force is simply force, no matter whence derived. We have, therefore, only to look at the diff forces acting upon our structures, as so many *tendencies to produce motions* in certain directions. If these tendencies are reacted against, or destroyed by others, they will not produce it; but if they meet no resistance, motion must take place. The only peculiarity we need assign to grav. is that the direction of its action is *always vert downward*; while other forces may be imparted in either that, or any other direction. The *resultant* of grav *combined with other force or forces*, may be in any direction.

Art. 70. Fig 73½. In the principal cases of stability that present themselves to the civil engineer, both the acting and resisting forces *w x*, *f*, *f*, *f*, &c, may all be considered to be imparted and acting in the same plane; which is a vertical one, *e l o c*, passing through the center of gravity, *v*, of the structure, *m n p q r s t u*: and of course, coinciding with the line of direction *w x*, of its weight, or force of gravity. The plane *e l o c*, and all the forces, therefore, may be considered as coinciding with a leaf of paper standing vertically on one edge. This renders the calculations much more simple than if the forces were in different planes; in which case diagrams alone would not suffice for determining the resultants, leverages, moments, &c. See Art 44. Whereas, when they are in the same plane, such diagrams, neatly drawn to a convenient scale, will usually possess all the accuracy required in practice.

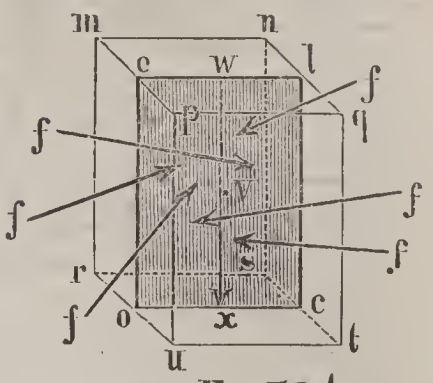


Fig 73 1/2

Thus, in examining the strains on the diff pieces composing the truss of a roof, or of a bridge, &c, not only their wts, but all the straiuing forces, may be assumed to act in a vert plane passing lengthwise of the truss from end to end: splitting it into two equal parts. In the case of a structure of masoury, such as a retaining-wall, or a stone bridge B, Fig 74, we base our calculations upon a thin vert slice of it, having a *length a o*, *i i*, or *y t*, of only one ft; no matter what may be the *height y s*; or the *thickness y c*, of the structure. Through the center of this one ft of length, we suppose a vert plane to pass; splitting, as it were, the body B into two precisely similar parts; and all the forces actually diffused equally throughout the whole one ft of length, are supposed to be concentrated, and to act upon one another, in this plane. See Remark, Arts 49 and 57,

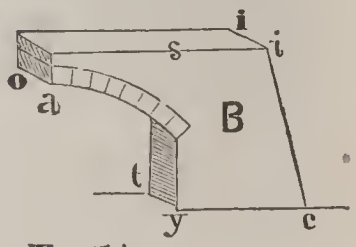


Fig 74

Art. 71. In order to ascertain the effect of diff forces to produce either sliding, or overturning, of either an entire rigid body, or of one composed of several rigid bodies placed together without joint-fastenings; we first find the direction of the resultant of those forces. If the body or structure is not composed of diff parts; or if, being so composed, these parts are so firmly united together by joint fastenings, as to constitute virtually but a single rigid mass, then we need do nothing more than (as in Art 35) find the resultant *a d*, Fig 19, of all the forces. If the direction of this resultant strikes *inside* of the base, the body will not *overturn* (Remark 2, Art 72,); and if, besides striking inside of the base, it forms with a line *b x*, at right angles to the base, and angle *a b x*, less than the angle of fric between the body and its support *n m*, then it cannot *slide*.

At Fig 67, the direction of the resultant *x o* strikes at *t*, inside of the base *s e*; therefore the body B cannot *upset*; whether it will *slide* or not, depends upon whether the angle *o t v* is greater or less than the angle of fric corresponding to the materials composing the body, and the plane. If both are of ordinary dressed granite, this angle must not *exceed* about 32°. See other angles, under head Friction.

Questions on overturning, may also be solved on the principle of leverage. See Art 49.

Art. 72. We will now consider a case in which the structure is assumed to be composed of several rigid bodies, merely placed together without joint-fastenings of any kind, such as cramps, bolts, mortar, &c; but depending entirely upon their wt and positions, for securing their stability. The process in this case is the same as in Fig 19, except that instead of assuming the body to have but the one joint *n m*; and finding the effect of the resultant *a d* with reference to this joint alone; we consider it to have several joints, as P Z, F L, W X, &c, Fig 75; and then examine the effect of the diff resultants which they must respectively sustain in consequence of the diff wts of the several parts N M P Z, N M F L, N M W X, resting on them.

Let H N J T be one half of a stone arch; or rather a vert slice of it, 1 ft thick; and let N M W X be a similar slice of a dressed stone abut which has been designed to sustain the thrust of the arch; and the fitness of which for the purpose, we wish to ascertain.

Suppose the thrust of the slice of the arch, (that is, the resultant of its wt, and of its hor pres,) to have been previously ascertained by Ex 2, p 330, to be 30 tons; that this is concentrated at o, (the center of the skewback;) and that its direction is *o b*. Also, suppose the wt of the part N M P Z be found to be 10 tons, and to be concentrated at its cen of grav G; and, consequently, to act in the vert direction G *a*. Now (Arts 18 and 35) we may suppose the 30 tons resultant of the arch, and the 10 tons grav of the part N M P Z, to act at the same point c, at which their directions G *a* and *o b* meet. Make *c d* by scale equal 30 tons, and *c a* 10 tons; complete the parallelogram of forces *c d y a*; and draw its diag *c y*, which by the same scale will give the resultant of all the forces acting upon the part N M P Z.

Now we see that the direction *c y* of this resultant strikes at *i*, or *within the base* P Z; consequently (Art 60, &c) N M P Z cannot *upset*, no matter how great may be the pres *c y*; see Remark 2. From *i*

draw *it*, at right angles to the line P Z; and measure the angle *cit*, which the resultant *cy* forms with it. This, we find, is greater than 32°; that is, it exceeds the angle of fric between surfaces of dressed stone. Therefore, the part N M P Z must *slide* along the joint P Z. This might possibly be prevented by good mortar, if time be allowed it to solidify properly, before the centers are eased so

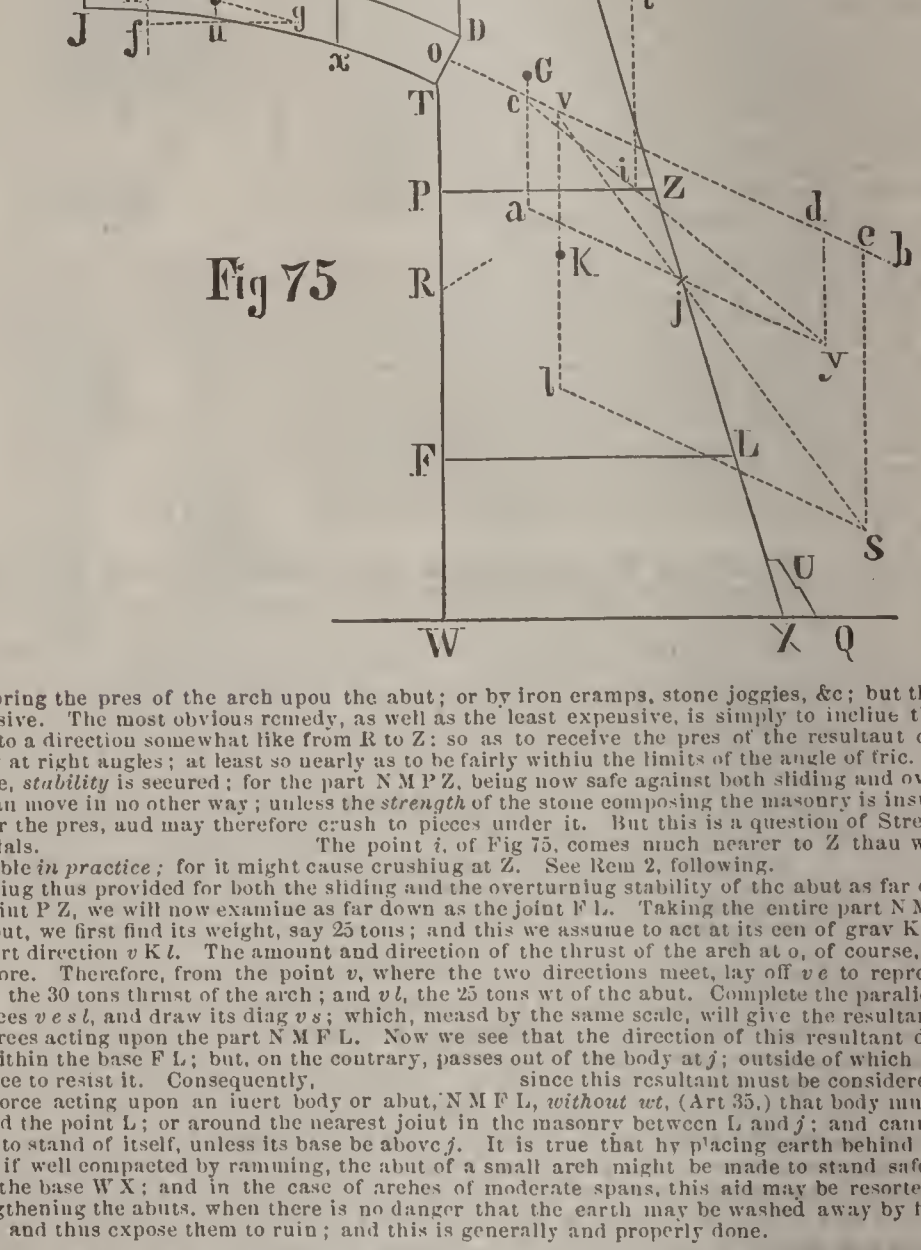


Fig 75

to bring the pres of the arch upon the abut; or by iron cramps, stone joggies, &c; but these are expensive. The most obvious remedy, as well as the least expensive, is simply to incline the joint P Z into a direction somewhat like from R to Z: so as to receive the pres of the resultant *cy* more nearly at right angles; at least so nearly as to be fairly within the limits of the angle of fric. If this is done, *stability* is secured; for the part N M P Z, being now safe against both sliding and overturning, can move in no other way; unless the *strength* of the stone composing the masonry is insufficient to bear the pres, and may therefore crush to pieces under it. But this is a question of Strength of Materials. The point *i*, of Fig 75, comes much nearer to Z than would be desirable *in practice;* for it might cause crushing at Z. See Rem 2, following.

Having thus provided for both the sliding and the overturning stability of the abut as far down as the joint P Z, we will now examine as far down as the joint F L. Taking the entire part N M F L of the abut, we first find its weight, say 25 tons; and this we assume to act at its cen of grav K, and in the vert direction *v* K *l*. The amount and direction of the thrust of the arch at o, of course, remain as before. Therefore, from the point *v*, where the two directions meet, lay off *ve* to represent as before the 30 tons thrust of the arch; and *vl*, the 25 tons wt of the abut. Complete the parallelogram of forces *v e s l*, and draw its diag *v s*; which, measd by the same scale, will give the resultant of all the forces acting upon the part N M F L. Now we see that the direction of this resultant does not fall within the base F L; but, on the contrary, passes out of the body at *j*; outside of which it meets no force to resist it. Consequently, since this resultant must be considered as an only force acting upon an inert body or abut, N M F L, *without wt*, (Art 35,) that body must upset around the point L; or around the nearest joint in the masonry between L and *j*; and cannot continue to stand of itself, unless its base be above *j*. It is true that by placing earth behind it, especially if well compacted by ramming, the abut of a small arch might be made to stand safely even upon the base W X; and in the case of arches of moderate spans, this aid may be resorted to for strengthening the abuts. when there is no danger that the earth may be washed away by floods or rains, and thus expose them to ruin; and this is generally and properly done.

Rem. 1. If in the same manner that the point *i* was found in the joint P Z, others between P Z and W X be determined also: then a curve, commencing at the skewback o, and drawn through them, will represent the *line of pressures or of resistance, or of thrust,* through the abut. At any point whatever in this line, say at *i*, the entire pres above said point may be supposed to be concentrated; while the entire length, as *cy*, of that resultant which cuts said point, gives *the amount* of said pres at that point; and the direction, as *ci*, of the same resultant is also *the direction in which said pres acts upon said point.* See Art 13 of Hydrostatics, p 231.

Rem. 2. The line of pres enables us to determine another very important point connected with the stability of a structure. It is not sufficient in practice that this line should strike *merely within* the base; it must strike at a *considerable dist* within. If the structure and its foundation were *absolutely rigid*, so that no conceivable force could bend or break them, this would not be necessary; but all materials are more or less weak, so that if great pressures come too near to their edges, there is danger of splitting or crushing at those points; or if near the edge of a base, an unequal settlement of the soil beneath may take place. Therefore, even in structures of but small size, the dist *i* Z, Fig 75, of the line of pres, from the outer point Z, should never be *less* at any joint than ¼ of the width of that joint; except, perhaps, in a case like that of a small arch in which the earth filling is depos-

ited behind the abuts before the centers are removed. In important works, it should not be less than about ⅛ of the width of the joint; and it is still better, when possible, at ½; or, in other words, at the center of each joint. When, as at Q, a footing U is added at a base, W X should be taken as the joint; not W Q.

Rem. 3. **The line of pressure in an arch itself,** as H J N T, Fig 75, may also be found much in the same way, thus: First divide the half arch H J N T, and the filling above it, by vert lines $r\,u$, $w\,x$, &c, which need not be at equal dists apart. Four such lines will suffice for a flat arch, and about six for a semicircular one. We then consider in turn, and separately, each part, as $r\,u$ H J, $w\,x$ H J, N D T H J, &c, which extends from these lines to the center H J of the half arch; the last of these being the entire half arch. The cen of grav, and the wt, of each of these parts, must be found; also, (Ex 6, p 341,) the hor pres at the keystone.

Now each of these parts, like the beam in Ex 1, or the half arch in Ex 2, p 330, is acted upon, and kept in equilibrium, by three forces; namely, the hor pres at the keystone, (see Ex 6, p 341;) its own wt, acting vert; and the reactiug force of the part next behiud it. We proceed with each part separately, as we did with the entire beam alluded to, thus: Beginning with the part $r\,u$ H J, from its ceu of grav, m, draw a vert line $m\,f$. From the center E of the keystone, draw a hor line E n, to meet $m\,f$. From n lay off $n\,f$ by scale, to represent the wt of the part $r\,u$ H J; and from f lay off $f\,g$, hor by scale, to represent the hor pres at the key. Draw the diag $n\,g$; which will give, by scale, the resultant of these two forces. The point b, at which the diag $n\,g$ intersects the vert $r\,u$, is a point in the liue of pres reqd. Next go to the part $w\,x$ H J; and in the same manner find another point in the line $w\,x$, using the cen of grav and the wt of that part. Finally, treat the entire half arch N D T H J in the same way. The resultant diag of this last will pass through o, the center of the skewback, if the archstones have the same depth throughout. A curve drawn by hand through the points thus found, will be the reqd line of pres. These points will not all fall equally well withiu the thickness of the archstones.

Art. 73. The stability of bodies on inclined planes, as regards overturning, is measd in the same way as when the base is hor; namely, by mult their wt, by the perp dist ($a\,o$, or $c\,t$, at A, B, and D, Fig 76,) from the fulcrum, or turning-point a or c, to the vert line of direction ($g\,o$) drawn from the cen of grav

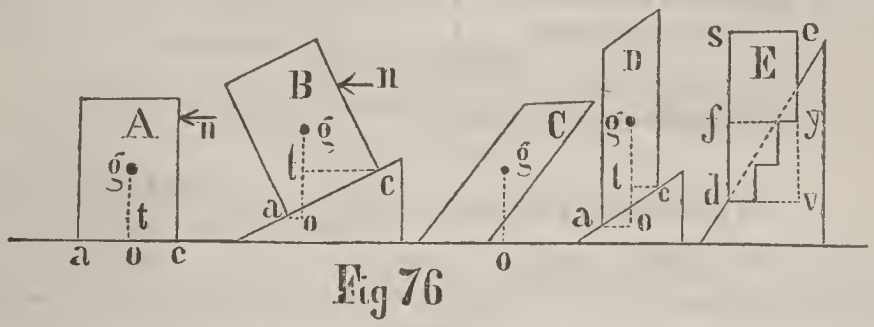


Fig 76

of the body. Hence, it is evident that the body B has less overturning stability about its toe a, than the similar body A has, when the force, n, tends to upset it down hill. But it has more than A, when the force tends to upset it up hill, or about the toe c; for the leverage $t\,c$ of B is greater than that, $o\,c$, of A.

The body C, which would overturn upon a level base, because the line $g\,o$ strikes *outside* of the base; would be stable against overturning, if placed as D upon an inclination, where the vert $g\,o$ strikes *within* the base. In our fig the longer sides of D are supposed to be vertical, so that the line of direction $g\,o$ cuts the *center* of the base $a\,c$. The two leverages, $a\,o$ and $t\,c$, are therefore equal, as are, consequently, the moments of stability of D about a and c respectively. Similarly, a given upward *vertical* force would have the same upsetting effect whether applied at a (to upset *up* hill) or at c (to upset *down* hill) because its leverage, $a\,o + t\,c$, is the same in both cases. But a *horizontal* force, applied at any given height, as at g, has a greater leverage ($= g\,o$) when pushing *down* hill than when pushiug *up* hill. For in order to upset down hill the body must rotate on the corner a, whereas, in order to upset up hill, it must rotate on the corner c, which gives to a horizontal force applied at g, only the shorter leverage $= g\,t$.

Structures built upon slopes are, however, liable to slide; that is, they are deficient in *frictional stability*. In practice this is remedied by cutting the slope into hor steps, as at E. But works so constructed are not as strong as if the base were a continuous hor line; because the vert faces of the steps *break the bond* of the masonry; and because the mortar in the higher portions $s\,d$, being in greater quantity than that in the lower portions $e\,y$, necessarily allows more settlement of the masonry in the former; and thus renders the work liable to crack, or split open vertically. The case is analogous to that of a foundation, firm in some parts, and compressible in others. Therefore, when circumstances permit, the foundation should be levelled off as at $d\,v$; or if the masonry has to sustain down-hillward pressures, v should be lower than d; and the courses of masonry be laid with a correspondiug inclination.

GRAVITY, FALLING BODIES.

Art. 1. Bodies falling vertically. A body, falling freely in vacuo from a state of rest, acquires, by the end of the first second, a velocity of about 32.2 feet per second; and, in each succeeding second, an *addition* of velocity, or acceleration, of about 32.2 ft per sec. This dist is called the **acceleration of gravity,** and is denoted by **g.** It increases from about 32.1 ft per sec at the equator, to about 32.5 at the poles. In the latitude of London it is 32.19. These are its values at sea-level; but at a height of 5 miles above that level it is diminished by only about 1 part in 400. For most practical purposes it may be taken at 32.2.

Caution. Owing to the resistance of the air none of the following rules give perfectly accurate results in practice, especially at great vels. The greater the sp gr of the body the better will be the result. The air **resists** both **rising** and **falling** bodies.

If a body be thrown vertically upwards with a given vel, it will rise to the same height from which it must have fallen in order to acquire said vel; and its vel will be retarded at the rate of 32.2 ft per sec. Its average ascending velocity will be half of that with which it started; as in all other cases of uniformly retarded vel. In falling it will acquire the same vel that it started up with, and in the same time. See above Caution.

In the following, the falls are in feet, the times in seconds, and the velocities and accelerations in feet per second.

Acceleration acquired

in a given time $= g \times$ time

in a given fall from rest $= \sqrt{2\,g \times \text{fall}}$. See table, p 258

in a given fall from rest and given time $= \dfrac{\text{twice the fall}}{\text{time}}$

Time required

for a given acceleration $= \dfrac{\text{acceleration}}{g}$

for a given fall from rest $= \sqrt{\dfrac{\text{fall}}{\frac{1}{2}\,g}} = \dfrac{\text{fall}}{\frac{1}{2}\text{ final velocity}}$

for a given fall from rest or otherwise $= \dfrac{\text{fall}}{\text{mean vel}} = \dfrac{\text{fall}}{\frac{1}{2}\,(\text{initial vel} + \text{final vel})}$

Fall

in a given time (starting from rest) $= \text{time} \times \frac{1}{2}$ the final vel $= \text{time}^2 \times \dfrac{g}{2}$

in a given time (starting from rest or otherwise) $= \text{time} \times \text{mean vel} = \text{time} \times \dfrac{\text{initial vel} + \text{final vel}}{2}$

reqd for a given acceleration (starting from rest) $= \dfrac{\text{acceleration}^2}{2\,g}$. See table, p 258.

during any one given second (counting from rest)

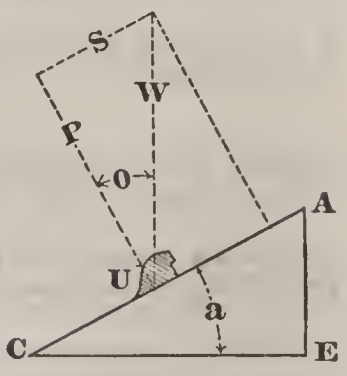
$= g \times \left(\text{number of the second (1st, 2d, \&c)} - \frac{1}{2}\right)$

during any equal consecutive times (starting from rest) $\propto$ 1, 3, 5, 7, 9, &c.

Calling $g = 32.2$, we have

	At the end of the seconds									
	1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.	9th.	10th.
Velocity; ft per sec.	32.2	64.4	96.6	128.8	161.0	193.2	225.4	257.6	289.8	322.0
Dist fallen since end of preceding sec; ft.	16.1	48.3	80.5	112.7	144.9	177.1	209.3	241.5	273.7	305.9
Total dist fallen; ft.	16.1	64.4	144.9	257.6	402.5	579.6	788.9	1030.4	1304.1	1610.0

Art. 2. Descent on inclined planes. When a body, U, is placed upon an inclined plane, A C, its whole weight W is not employed in giving it velocity (as in the case of bodies falling vertically) but a portion, P, of it (= W × cosine of o = W × cosine of a*) is expended in perpendicular pressure against the plane; while only S, (= W × sine of o = W × sine of a,*) acts upon U in a direction parallel to the surface A C of the plane, and tends to slide it down that surf. See Art. 60, p 352.



The acceleration, generated in a given body in a given time, is proportional to the force acting upon the body in the direction of the acceleration (Art. 9, p 310). Hence if we make W to represent by scale the acceleration g (say 32.2 ft per sec) which grav would give to U in a sec if falling freely, then S will give, by the same scale, the acceleration in ft per sec which the actual sliding force S would give to U in one sec if there were no friction between U and the plane. We have therefore

theoretical acceleration down the plane = g × sine of a.

Therefore we have only to substitute "g. sin a" in place of "g;" and the *sloping* distance or "slide" A C in place of the corresponding *vertical* distance or "fall" A E in the equations, p. 362, in order to obtain the accelerations etc as follows:

on an inclined plane without friction.

In the following, the slides A C are in feet, the times in seconds, and the velocities and accelerations in feet per second.

Acceleration of sliding velocity

$$\text{in a given time} = \left.\begin{matrix}\text{vert accel acquired in falling}\\ \text{vert during the same time}\end{matrix}\right\} \times \sin a$$

$$= g.\sin a \times \text{time}$$

$$\left.\begin{matrix}\text{in a given slide, as A C,}\\ \text{from rest}\end{matrix}\right\} = \frac{\text{slide}}{\frac{1}{2}\text{ time}}$$

$$= \left\{\begin{matrix}\text{vert accel acquired in falling}\\ \text{freely thro the corresponding}\\ \text{vert ht A E}\end{matrix}\right\} = \sqrt{2\,g.\,\text{A E}}$$

$$= \sqrt{2\,g.\sin a \times \text{slide}}$$

Time required

$$\text{for a given sliding acceleration} = \frac{\text{sliding acceleration}}{g.\sin a}$$

$$\text{for a given slide, as A C, from rest} = \frac{\text{slide}}{\frac{1}{2}\text{ final sliding velocity}}$$

$$= \sqrt{\frac{\text{slide}}{\frac{1}{2}\,g.\sin a}}$$

$$= \frac{\text{time reqd to fall freely thro the corresponding vert ht A E}}{\sin a}.$$

$$\left.\begin{matrix}\text{for a given slide, from}\\ \text{rest or otherwise}\end{matrix}\right\} = \frac{\text{slide}}{\text{mean sliding vel}} = \frac{\text{slide}}{\frac{1}{2}(\text{initial} + \text{final sliding vels})}$$

$$\textbf{Cosine } a = \frac{\text{base E C}}{\text{length A C}} = \frac{\text{horizontal stretch, as E C, of any length, as A C}}{\text{that length}} = \frac{\sqrt{\text{A C}^2 - \text{A E}^2}}{\text{A C}}$$

$$\textbf{Sine } a = \frac{\text{height A E}}{\text{length A C}} = \frac{\text{fall, A E, in any given length, A C}}{\text{that length}} = \frac{\sqrt{\text{A C}^2 - \text{E C}^2}}{\text{A C}}$$

* Because o and a are equal.

Slide, as A C

in a given time, starting from rest = time × ½ final sliding vel
= time ^{2} × ½ g. sin a.

in a given time, starting from rest or otherwise = time × mean sliding vel
= time × ½ (initial + final, sliding vels)

required for a given sliding acceleration (starting from rest) = $\frac{\text{sliding acceleration}^2}{2\ g.\ \sin a}$

But in practice the sliding on the plane is always opposed by friction. To include the effect of friction, we have only to substitute
"$g \times \left[\sin a - (\cos a.\ \text{coeff fric})\right]$" in place of "$g$. sin a" in the above equations.
Because

Friction = Perpendicular pressure P × coefficient of friction
= weight W × cosine a × coefficient of friction

and
retardation of friction = g × cosine a × coefficient of friction.

(For **table of coefficients** for various substances, see p 373.)

Resultant sliding acceleration
= theoretical sliding accel (due to the sliding force, S) — retardation of fric
= (g. sin a) — (g. cosine a. coeff fric)
= $g \times \left[\sin a - (\text{cosine } a.\ \text{coeff fric})\right]$

If the retardation of friction (= g. cos a × coeff fric) is *not less* than the total or theoretical accel ("g. sin a") the body cannot slide down the plane.

PENDULUMS.

The numbers of vibrations which diff pendulums will make in any given place in a given time, are *inversely* as the square roots of their lengths; thus, if one of them is 4, 9, or 16 times as long as the other, its sq rt will be 2, 3, or 4 times as great; but its number of vibrations will be but ½, ⅓, or ¼ as great. The times in which diff pendulums will make a vibration, are *directly* as the sq rts of their lengths. Thus, if one be 4, 9, or 16 times as long as the other, its sq rt will be 2, 3, or 4 times as great; and so also will be the time occupied in one of its vibrations.

The length of a pendulum vibrating seconds at the level of the sea, in a vacuum, in the lat of London (51½° North) is 39.1393 ins; and in the lat of N. York (40¾° North) 39.1013 ins. At the equator about $\frac{1}{10}$ inch shorter; and at the poles, about $\frac{1}{10}$ inch longer. Approximately enough for experiments which occupy but a few sec, we may at any place call the length of a seconds pendulum in the open air, 39 ins; half sec, 9¾ ins; and may assume that long and short vibrations of the same pendulum are made in the same time; which they actually are, *very* nearly. For measuring depths, or dists by sound, a sufficiently good sec pendulum may be made of a pebble (a small piece of metal is better) and a piece of thread, suspended from a common pin. The length of 39 ins should be measured from the centre of the pebble.

In starting the vibrations, the pebble, or *bob*, must not be *thrown* into motion, but merely *let drop*, after extending the string at the proper height.

To find the length of a pendulum reqd to make a given number of vibrations in a min, divide 375 by said reqd number. The square of the quot will be the length in ins, near enough for such temporary purposes as the foregoing. Thus, for a pendulum to make 100 vibrations per min, we have $\frac{375}{100} = 3.75$; and the square of 3.75 = 14.06 ins, the reqd length.

To find the number of vibrations per min for a pendulum of given length, in ins, take the sq rt of said length, and div 375 by said sq rt. Thus, for a pendulum 14.06 ins long, the sq rt is 3.75; and $\frac{375}{3.75} = 100$, the reqd number.

Rem. 1. By practising before the sec pendulum of a clock, or one prepared as just stated, a person will soon learn to count 5 in a sec, for a few sec in succession; and will thus be able to divide a sec into 5 equal parts; and this may at times be useful for very rough estimating when he has no pendulum.

Centre of Oscillation and Percussion.

Rem. 2. When a pendulum, or any other suspended body, is vibrating or oscillating backward and forward, it is plain that those particles of it which are far from the point of suspension move faster than those which are near it. But there is always a certain point in the body, such that if *all* the particles were concentrated at it, so that all should move with the same actual vel, neither the number of oscillations, nor their angular vel, would be changed. This point is called the *center of oscillation*. It is not the same as the cen of grav, and is always farther than it from the point of suspension. It is also the *centre of percussion* of the suspended vibrating body. The dist of this point from the point of susp is found thus: Suppose the body to be divided into many (the more the better) small parts; the smaller the better. Find the weight of each part. Also find the cen of grav of each part; also the dist from each such cen of grav to the point of susp. Square each of these dists, and mult each square by the wt of the corresponding small part of the body. Add the products together, and call their sum p. Next mult the weight of the entire body by the dist of its cen of grav from the point of susp. Call the prod g. Divide p by g. This p is the **moment of inertia** of the body, and if divided by the wt of the body the sq rt of the quotient will be the **Radius of Gyration.**

Angular Velocity.

When a body revolves around any axis, the parts which are farther from that axis move faster than those nearer to it. Therefore we cannot assign a stated linear velocity in feet per second, or miles per hour etc, that shall apply to *every part* of it. But every part of the body revolves around an entire circle, or through an angle of 360°, in the same time. Hence, all the parts have the same velocity in *degrees* per second, or in *revolutions* per second. This is called the angular velocity. Scientific writers measure it by the length of the arc described by any point in the body in a given time, as a second, the length of the arc being measured by the number of times the length *of its own radius* is contained in it. When so measured,

$$\text{Angular velocity in radii per second} = \frac{\text{linear velocity (in feet etc) per sec}}{\text{length of radius (in feet etc)}}$$

Here, as before, the angular velocity is the same for all the points in the body; because the velocities of the several points are directly as their radii or distances from the axis of revolution.

In each revolution, each point describes the circumference of the circle in which it revolves $= 2\pi r$ ($\pi = 3.1416$ etc; r = radius of said circle). Consequently, if the body makes n revolutions per second, the length of the arc described by each point in one second is $2\pi rn$; and the angular velocity of the body, or *linear* velocity of any point *measured in its own radii*, is

$$\mathbf{a} = \frac{2\pi rn}{r} = 2\pi n = \text{say } 6.2832 \times \text{revs per } \textit{second} = \text{say } .1047 \times \text{revs per } \textit{minute}.$$

Moment of Inertia.

Suppose a body revolving around an axis, as a grindstone; or oscillating, like a pendulum. Suppose that the distance from the axis of revolution (which, in the pendulum, is the point of suspension) to each individual particle of the body, has been measured; and that the square of each such distance has been multiplied by the weight of that particle to which said distance was measured.

The sum of all these products is the moment of inertia of the body. Or

$$\textbf{Moment of inertia} = \left\{\begin{array}{c}\text{the sum,}\\ \text{for all the particles}\end{array}\right\} \text{ of } \left\{\begin{array}{c}\text{weight}\\ \text{of}\\ \text{particle}\end{array}\right. \times \left.\begin{array}{c}\text{square of dist}\\ \text{of particle from}\\ \text{axis of revolution}\end{array}\right.$$

or, $I = \Sigma\, d^2 w.$

Scientific writers frequently use the *mass* of each particle;

i e, $\frac{\text{its weight}}{\text{acceleration } (g) \text{ of gravity, or about } 32.2}$ instead of its weight, in calculating the moment of inertia.

In practice we may suppose the body to be divided into portions measuring a cubic inch (or some other small size) each; and use these instead of the theoretical infinitely small particles. The smaller these portions are taken, the more nearly correct will be the result.

When the moment of inertia of a mere *surface* is wanted (instead of that of a *body*), we suppose the surface to be divided into a number of small *areas*, and use them instead of the *weights* of the small portions of the body. For an example, see p 486.

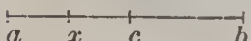
$$\textbf{Moment of inertia} = \begin{array}{c}\text{weight of body,}\\ \text{or}\\ \text{area of surface}\end{array} \times \begin{array}{c}\text{square of}\\ \text{radius of gyration}\end{array}:$$

For definition of Radius of gyration, see p 440. A body may have any number of radii of gyration, depending upon the position of the axis of revolution.

Table of Radii of Gyration.

Body	Revolving around	Radius of Gyration
Any body or figure	any given axis	$\sqrt{\frac{\text{moment of inertia around the given axis}}{\text{weight of body, or area of surface}}}$
Solid cylinder	its longitudinal axis	radius of cylinder $\times \sqrt{\frac{1}{2}}$ = radius of cylinder $\times$ about .7071
ditto	a diam, midway between its ends	$\sqrt{\frac{\text{length}^2}{12} + \frac{\text{radius}^2 \text{ of cylinder}}{4}}$
ditto, infinitely short (circular surface)	a diameter	$\frac{\text{radius of cylinder}}{2}$
Hollow cylinder	its longitudinal axis	$\sqrt{\frac{\text{inner rad}^2 + \text{outer rad}^2}{2}}$
ditto, infinitely thin	ditto	radius of cylinder
ditto, of any thickness	a diam midway between its ends	$\sqrt{\frac{\text{inner rad}^2 + \text{outer rad}^2}{4} + \frac{\text{length}^2}{12}}$
ditto, infinitely thin	ditto	$\sqrt{\frac{\text{radius}^2 \text{ of cylinder}}{2} + \frac{\text{length}^2}{12}}$
ditto, infinitely thin and infinitely short (circumference of a circle)	a diameter	radius of cylinder $\times \sqrt{\frac{1}{2}}$ = radius of cylinder $\times$ about .7071
Solid sphere	a diameter	$\sqrt{\frac{\text{radius}^2 \text{ of sphere}}{2.5}}$ = radius of sphere $\times \sqrt{.4}$ = radius of sphere $\times$ about .63246

Table of Radii of Gyration.—CONTINUED.

Body	Revolving around	Radius of Gyration
Hollow sphere of any thickness	a diameter	$\sqrt{\frac{2\,(\text{outer rad}^5 - \text{inner rad}^5)}{5\,(\text{outer rad}^3 - \text{inner rad}^3)}}$
ditto, thin	ditto	approx (outer rad + inner rad) × .4085
ditto, infinitely thin (spherical surface)	ditto	radius of sphere $\times \sqrt{\frac{2}{3}}$ = radius of sphere × about .8165
Straight line, $a\,b$ 	any point, x, in its length	$\sqrt{\frac{a\,x^3 + x\,b^3}{3\,a\,b}}$
	either end, a or b	length $a\,b \times \sqrt{\frac{1}{3}}$ = length $a\,b$ × about .5775
	its center, c	$a\,c \times \sqrt{\frac{1}{3}}$ = length $a\,b$ × about .2887
Solid cone	its axis	radius of base of cone $\times \sqrt{.3}$ = radius of base of cone × .5477
Circular plate, of rectangular cross section	See Solid cylinder	For the *thickness* of plate or ring, measured perpendicularly to the plane of the circumference, take the *length* of the cylinder.
Circular ring, of rectangular cross section	See Hollow cylinder	
Square, rectangle and other surfaces	For *least* radius of gyration, or that around the *longest* axis, see p 440 and 441.	

CENTRIFUGAL FORCE.

When a body a, Fig 3 moves in a circular path abd, it tends, at each point, as a or b, to move in a tangent at or bt' to the circle at that point. But at each

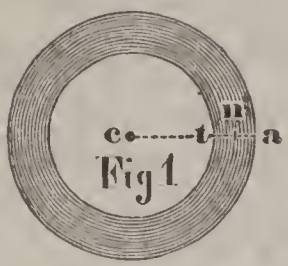


Fig 1.

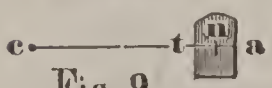


Fig 2

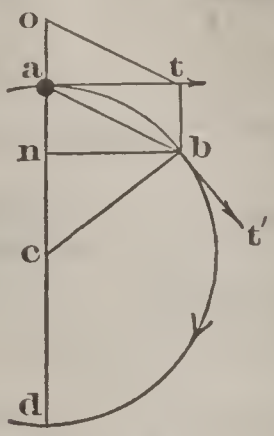
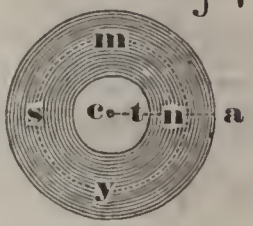


Fig. 3

Fig 4



point, as a etc, in the path, it is *deflected* from the tangent by a force acting toward the center, c, of the circle. This force may be exerted by a string ca, by the rails on a curve etc etc. At each point, therefore, the tangential tendency at is resolved into two components. One of these, $ao = bt$, is its **centrifugal force,** or resistance to the deflecting or **centripetal force** an. The other is a new tangential force ab, which carries the body to the next point b in its path.*†

The centrifugal force, being directly opposed to the centripetal force, necessarily acts directly *from* the center. Hence its name. Being merely a resistance to the centripetal force, it is necessarily equal to it, and ceases when *it* ceases. These two equal and opposite forces, acting against each other through the material of the string, rail, fly-wheel etc, produce among its particles a strain equal to one of them. See Art. 13, p 311. The moment this strain exceeds the strength of the string etc, the latter breaks; the centrip and centrif forces therefore instantly cease; and the tangential force, now undisturbed, carries the body in a tangent, at or bt' etc, to its circular path, or at right angles to the direction of the centrif force.

$$\textbf{Centrifugal force} = \text{weight of body} \times \frac{\text{velocity}^2 \text{ in feet per second}}{\text{radius, } ca \text{ Fig 3, in feet} \times g}$$

$$= \frac{\text{weight of body}}{} \times \frac{\text{No.}^2 \text{ of revs per min} \times \pi^2 \times \text{rad } ca \text{ Fig 3, in ft}}{900\,g}$$

π = about 3.1416. See page 123. π^2 = about 9.8696. See caution, p. 369.
g = 32.2. See page 362.

Demonstration. (1st) Let ab, Fig 3, be the minute space described by the revolving body a in an exceedingly short time t,* so that we may consider the *straight line*, or *chord*, ab, as equal to the *arc*, ab.

Now (Euclid, vi. 8)

$$\text{diameter } ad : ab :: ab : an, \quad \text{or} \quad an = \frac{ab^2}{ad}$$

But an is $= tb$
= the distance through which the body a deviates from the tangent at toward the center c in the time t, or while moving from a to b †
= *half* the *acceleration* toward c produced by the centripetal force in the time t:

---

* We of course cannot draw this correctly. In order to do so, we should have to make ab far shorter than we could represent it by a line. As it is, we may imagine the path of the body a to be, not a circle, but a polygon, of which ab is one of the sides; the ball flying freely in straight lines from point to point, as from a to b, and striking, at each point, a flat surface tangential to the circle. These surfaces take the place of the string ca; each of them, as the ball strikes it, reacting with a centripetal force $= oa = an$.

† When the body reaches the next point, as b, in its path (see foot-note *) a *new* deviation has to take place; *i e*, from the *new* tangent bt'. Hence the acceleration of the velocity of deviation does not *accumulate* as the body revolves, as does that of the velocity of a falling body during its fall.

whence we have

$$\tfrac{1}{2}\text{ centripetal acceleration in time } t = \frac{a\,b^2}{a\,d}.$$

The same proportion holds good if we let

$a\,b$ = the *velocity* of revolution in feet *per second*
$a\,n$ = half the centripetal acceleration *in one second.*

Therefore, since $\quad an = \dfrac{a\,b^2}{a\,d} \quad$ or $\quad 2\,an = \dfrac{a\,b^2}{\frac{1}{2}\,a\,d}$

$$\text{centripetal acceleration given in one second} = \frac{\text{velocity}^2 \text{ of revolution in feet per second}}{\text{radius } a\,c \text{ in feet}}.$$

But (see Art. 9, page 310) forces are proportional to the accelerations which they can produce in a given body in a given time: and gravity (or the weight of the body) would, in one second, give to it an acceleration, g, of about 32.2 feet per second (see page 362). Therefore,

$$g : \text{centrip accel given in 1 sec} :: g : \frac{\text{vel}^2 \text{ of rev in ft per sec}}{\text{radius in feet}} :: \text{weight of body} : \text{centrip or centrif force}$$

or,

$$\text{centrifugal force} = \text{weight} \times \frac{\text{velocity}^2 \text{ of revolution in ft per sec}}{\text{radius} \times g},$$

as in the first formula.

(2d) Velocity in feet per second

$$= \frac{\text{revolutions per minute} \times \text{circumference in feet}}{60 \text{ (seconds per minute)}}$$

$$= \frac{\text{revolutions per minute} \times \text{radius in feet} \times 2\pi}{60}.$$

Hence, velocity^2 in feet per second

$$= \frac{\text{revolutions}^2 \text{ per minute} \times \text{radius}^2 \text{ in feet} \times 4\pi^2}{3600}$$

$$= \frac{\text{revolutions}^2 \text{ per minute} \times \text{radius}^2 \text{ in feet} \times \pi^2}{900}.$$

But, by the first formula, page 368,

$$\text{centrifugal force} = \text{weight} \times \frac{\text{velocity}^2 \text{ in feet per second}}{\text{radius} \times g}.$$

Hence,

$$\text{centrifugal force} = \text{weight} \times \frac{\text{revolutions}^2 \text{ per minute} \times \text{radius}^2 \times \pi^2}{\text{radius} \times g \times 900}$$

$$= \text{weight} \times \frac{\text{revolutions}^2 \text{ per minute} \times \pi^2 \times \text{radius}}{900\,g},$$

as in the second formula.

The centrif force thus found will be in lbs, tons etc, according as the wt of the body is taken in lbs, tons etc.

Caution. If the dimension $a\,t$ of the body, Figs 1 and 2, measd in the direction of the rad $c\,a$, does not exceed about one-third of the entire rad $c\,a$, we may, near enough for ordinary practice, measure the rad from the cen c of the circle to the cen of grav n of the body Fig 2, or to the cen of grav n of cross section of a revolving ring Figs 1 and 4.

When $a\,t$ is equal to one-third of $a\,c$, the rad thus found will be but about one-fiftieth part too short; when one-half of $a\,c$, about one-nineteenth part too short; and when two-thirds of $a\,c$, about one-ninth part too short. But when, as is often the case, as in fly-wheels, etc, $a\,t$ is much less than one-third of $a\,c$, the error is not worth notice in practice.

But if greater accuracy is required, or if, as in Fig 4, $a\,t$ is greater than about one-third of $c\,a$, **use the radius of gyration,** see p 366, **and the velocity of the center of gyration.**

$$\text{Velocity of center of gyration in feet per second} = \text{Velocity of outer particles in feet per second} \times \frac{\text{radius of gyration}}{\text{outer radius } c\,a \text{ Figs 1, 2 and 4}}$$

$$= \frac{\text{Revolutions per minute}}{60} \times \text{radius of gyration} \times 2\pi$$

The ring Fig 1, or the body, Fig 2, must be united to the center c, either by arms, or by a string or wire, etc, and the weight of these arms, etc, will slightly shorten the radius of gyration. But in practice this effect is usually too small to be regarded; or a trifling allowance is made for it by guess.

FRICTION.

Art. 1. When one rough body rests upon another, the projections and depressions forming **the roughnesses** of their surfaces of contact **interlock** to a greater or less extent; and, in order to slide one over the other, we must expend a portion of the sliding force, either in *separating* the bodies (as by lifting the upper one) sufficiently to clear the projections, or in breaking off some of the projections and clearing the others.

Thus, let B Fig 6 p 314 represent one of the minute projections (highly magnified) on the surf of the lower body: *isg* one of those of the upper body; and *Fg* a force tending to slide the upper body hor over the lower one. To do so it must separate the two bodies (by pushing the upper one up the inclined plane *st*) until *s* clears *t*; but it may reduce the height of this lift by grinding or breaking off a part of *t* or of *s*. It is immaterial whether the surf of contact is hor, inclined or vert. The separation of the two bodies must take place in opposition to whatever force, acting perp to the surf of contact, tends to hold them together.

The surfs of all bodies are more or less rough. Hence this interlocking takes place to some extent between even the smoothest surfs. Without it, the most powerful vise could not prevent the lightest wt from falling out of its jaws; and the smallest conceivable force would slide the greatest conceivable wt.

The resistance which the sliding force thus encounters is called **friction.***

Art. 2. Friction always tends to *prevent relative motion of the two bodies between which it acts; i e*, motion of one of the bodies relatively to the other. In doing so, however, it tends equally to *cause* relative motion between each of those two and *a third, or outside body*. Thus, the fric between a belt and the pulley *driven by it*, tends to prevent slipping between *them;* but thus tends to make the belt slip on the *driving* pulley, and sets the *driven* pulley and its shaft in motion *relatively to the bearing* in which the shaft revolves. *This* motion is resisted by the fric *between journal and bearing;* and *this* fric, in turn, tends equally to make the *bearing* revolve with the journal, and to make the belt slip on the driven pulley.

Art. 3. The fric between two bodies *at rest* relatively to each other is called **static friction**, or fric of rest. That between two bodies *in relative motion* is called **kinetic friction** or fric of motion.

Art. 4. (**a**). **The ultimate or maximum static fric** between two bodies, as U and L Fig 1, (or the greatest fric resistance which they are capable of opposing to any sliding force when at rest) is equal to a force (as that of the wt F) which is just upon the point of making U begin to slide upon L.† Thus fric, like other forces, may be expressed in *weights*, as in lbs.

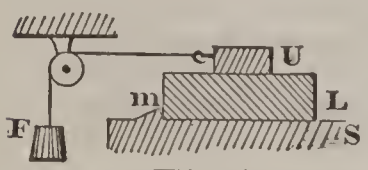


Fig. 1

(**b**) A resistce cannot exceed the force which it resists.‡ Therefore if F is less than the ult static fric between U and L, the *frictional resistce actually exerted* by them is also less. When F is = the ult fric (and U is therefore on the point of sliding) the actual resistce is = the ult stat fric. If F exceeds the ult stat fric, the excess gives motion to U.

Art. 5. If, when a body is in motion, all extraneous forces and resistces are removed or kept in equilib, it moves at a uniform vel. Hence, if the force, F Fig 1, is just = the ult kinetic fric between U and L, their vel is uniform. If F exceeds this, the excess *accelerates* the vel. If the ult kinetic fric exceeds F, the excess *retards* the vel. Thus **the actual fric resistce** exerted by two bodies *in relative motion* is = their **ult kinetic fric** = that force (as F) which can just maintain their relative vel uniform.

Hence, if the hor surf S upon which L rests, could be made perfectly frictionless, the pres of L against the lug *m* (which would then always be = the *actual* fric resistce between U and L) would also be = their *ult* fric so long as U continued in motion over L, and might therefore be greater or less than or = F; but when

---

* "Friction" (meaning rubbing) is a misnomer in so far as it implies that rubbing must take place in order to produce the resistance. For we meet this resistance, not only during rubbing, but also before motion (or rubbing) takes place. "Resistance of roughness" would better express its nature.

† We here neglect the fric of the string and pulley, and assume that *all* the force of the wt F is transmitted by the string to U.

‡ If a resisting *force* exceeds the force resisted, the excess is not resistce, but motive force.

U was at rest the pres against m would be = F, and less than (or at most just =) the ult fric.

Art. 6. Since no surface can be made *absolutely* smooth, some separation of the two bodies must in all cases take place in order to clear such projections as exist. Hence the fric is always more or less affected by the amount of the perp pres which tends to keep them together.

The *proportion* which the ult fric, in a given case, bears to the perp pres, is called the **coefficient of friction** for that case. Or,

$$\text{Coefficient of friction} = \frac{\text{ultimate friction}}{\text{perpendicular pressure}}$$

and

$$\text{Ultimate friction} = \text{perp pres} \times \text{coeff of fric}$$

Thus, if a force F Fig 1, of 10 lbs, just balances the ult fric between U and L, and if the wt of U (the perp pres in this case since the surf between U and L is *hor*) is 50 lbs, then the coeff of fric between U and L is $= \frac{10 \text{ lbs}}{50 \text{ lbs}} = .2$.

The coeff is usually expressed decimally, or by a common fraction; but sometimes, as in the case of railroad cars and engines, in lbs (of fric) per ton (of perp pres). Or by the "angle of fric" in degs and mins. (Art. 61, p 353.)

The coeff is diff for diff materials; and, in a given material, varies with the smoothness, cleanness, dryness etc of the surf.

Art. 7. The coeff of static fric may be found experimentally, either as in the above example, or by inclining the surf of contact, as in Art. 61, p 353. Expts on fric with unguents cannot well be made on a small scale in the latter way; on account of the stickiness or cohesion of the unguent.

Art. 8. (a) To find the coeff of kinetic fric, allow one of the bodies, U Fig 2, to slide down an inclined plane A C formed of the other one and having any convenient known steepness A C E greater than the angle of fric (Art. 61, p 353). Note the *vert* dist A E through which U descends in sliding any dist as A C, (A E = A C × sine of A C E, table of sines etc, pp 60 etc); also its actual *sliding* vel in ft per sec on reaching C. Calculate the *vert* dist A D through which it would have to descend *along the plane* (from A to B) to acquire that vel *if there were no fric.* $\left(\text{A D} = \frac{\text{velocity}^2 \text{ in ft per sec}}{\text{twice the accel } g \text{ of grav}^*}\right)$.

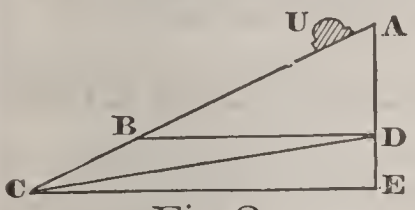


Fig. 2

Find D E (= A E − A D), and the hor dist E C corresponding to A C (E C = A C × cosine A C E = $\sqrt{\text{A C}^2 - \text{A E}^2}$). Then

$$\text{Coeff of the average fric in sliding from A to C} = \frac{\text{D E}}{\text{E C}}$$

because, if we let A E represent the *total* sliding force expended (in moving U from A to C and in overcoming the fric); then A D represents the portion of A E expended on *vel;* and D E that expended on *fric,* and therefore = the fric itself. And (Art. 62, p 353) $\frac{\text{A E}}{\text{E C}} = \frac{\text{sliding force}}{\text{perp pres}}$.

Hence $\frac{\text{D E}}{\text{E C}} = \frac{\text{friction}}{\text{perp pres}} = \text{coeff.}$

(b) Or, find the sine and the tangent (table pp 60 etc) of A C E; and the dist A C (= time^2 in secs × $\frac{1}{2}$ g^* × sine of A C E) through which U *would* slide in a given time *if there were no fric.* Measure the dist A B through which it *actually slides* in that time; and find B C = A C − A B. Then

$$\left.\begin{matrix}\text{coeff of the average} \\ \text{fric in sliding from A to B}\end{matrix}\right\} = \tan \text{D C E} = \tan \text{A C E} \times \frac{\text{B C}}{\text{A C}}$$

because (1st) A C : A B : B C : : A E : A D : D E : : theoretical vel due to total slid-

* g = about 32.2; 2 g = about 64.4.

ing force : actual vel : frictional retardation :: total sliding force : sliding force used in giving the actual vel : fric, or the sliding force reqd to balance it. And if AE = the total sliding force, then EC = the perp pres; and (Art. 62, p 353) $\frac{DE}{EC}$ = coeff = tan DCE. (2d) Owing to the similarity of the two triangles ABD and ACE, AC : BC :: AE : DE :: $\frac{AE}{EC} : \frac{DE}{EC}$ :: tan ACE : tan DCE.

Art. 9. In 1831 to 1834, **Gen'l Arthur Morin*** experimented with pressures not exceeding about 30 lbs per sq in; and arrived at the following conclusions in regard to sliding fric where the perp pres is considerably less than would be necessary to abrade the surfs appreciably. These were for a long time generally regarded as constituting the **three fundamental laws of fric.** But see Art. 11, p 374.

1st. The ult fric between two bodies is proportional to the total perp force which presses them together; *i e*, the **coeff is independent of the perp pres** and of its *intensity* (pres *per unit of surf*). Hence

2d. For any given *total* perp pres, **the coeff is independent of the area of surf in contact.**

If upon a hor support we lay a brick, measuring 8 × 4 × 2 ins, first upon its long edge (8 × 2 ins) and then upon its side (8 × 4 ins), we double the area of contact, while the total pres (the wt of the brick) remains the same, and *thus reduce the pres per sq in* by one-half. Consequently (the coeff remaining practically the same) we have only half the *fric per sq in*. But we have twice as many sq ins of contact, and therefore the same *total* fric.

But if we can increase or diminish the area of contact *without affecting the pres per sq in*, the *total* pres will of course *vary as the area*, and the total fric will vary in the same proportion, for the coeff remains the same. Thus, if we place two similar sheets of paper between the leaves of a book (taking care not to place both sheets between the same two leaves) and then squeeze the book in a letter-copying press, it will require about twice as much force to pull out both sheets as to pull out only one of them.

3d. Although the coeff of *static* fric between two bodies is often much greater than their coeff of kinetic fric; yet **the coeff of kinetic fric is independent of the vel.**

This applies also (approx) to the *fric*, and hence to the *work* (in *foot*-pounds etc) of overcoming fric through a given *dist;* for then the work (= resistce × dist) is independent of the vel. But in a given *time*, the dist (and consequently the work also) of course varies as the vel. See Art. 21, p 374 *f*.

Art. 10. (**a**) Some kinds of surfaces appear to interlock their projections much more perfectly when at rest relatively to each other, than when in even very slow motion; and in some cases the degree of interlocking seems to increase with time of contact. Hence there is often a great diff in amount between fric of rest and fric of motion. Thus, Gen'l Morin found that with oak upon oak, fibres of the two pieces at right angles, the resistce to sliding while still at rest, and after being for "some time in contact," was about one eighth greater than when the pieces had a relative vel of from 1 to 5 ft per sec.

(**b**) But experience shows that even very slight jarring suffices to remove this diff; and since all structures, even the heaviest, are subject to occasional jarring, (as a bridge, or a neighboring building, or even a hill, during the passage of a train; or a large factory by the motion of its machinery; or in numberless cases, by the action of the wind) it is expedient, in construction, not to rely on fric for *stability* any further than the coeff for *moving* fric will justify. When it is to be regarded as a resistce, which we must provide force for overcoming, it should be taken at considerably *more* than our tabular statement, p. 373.

* See his "Fundamental Ideas of Mechanics", translated by Jos. Bennett; D. Appleton & Co., New York, 1860.

Table of moving friction, of perfectly smooth, clean, and dry, plane surfaces, chiefly from Morin.

Materials Experimented with.	Coeff of Fric: or Proportion of Fric to the Pres.	Angle of Fric. Deg.	Min.
Oak on oak; all the fibers parallel to the motion	.48	25	38
" " moving fibres at right angles to the others; and to the motion	.32	17	45
" " all the fibres at right angles to the motion	.34	18	47
" " moving fibres on end; resting fibres parallel to the motion	.19	10	46
" cast iron, fibres at right angles to motion	.37	20	19
Elm on oak, fibres all parallel to motion	.43	23	17
Oak on elm, " " "	.25	14	3
Elm on oak, moving fibres at right angles to the others, and to motion	.45	24	16
Ash on oak, fibres all parallel to motion	.40	21	49
Fir on oak, " " " "	.36	19	48
Beech on oak " " " "	.36	19	48
Wrought iron on oak, fibres parallel to motion	.62	31	47
Wrought iron on elm, " " " "	.25	14	3
Wrought iron on east iron, fibres parallel to motion	.19	10	46
" " on wrought iron, fibres all parallel to motion	.14	7	58
Wrought iron on brass	.17	9	39
Wrought iron on soft limestone, well dressed	.49	26	6
" " " hard " " " "	.24	13	30
" " " " " " " wet	.30	16	42
" " or steel on hard marble, sawed. By the writer...about..	.17	9	39
" " " " " smoothly planed, and rubbed mahogany, fibres parallel to motion	.18	10	12
" " " " " smoothly planed wh pine	.16	9	6
Cast iron on oak, fibres parallel to motion	.49	26	6
" " " elm, " " " "	.20	11	19
" " " east iron	.15	8	32
" " " brass	.15	8	32
Steel on east iron	.20	11	19
Steel on steel. By the writer	.14	7	59
Steel on brass	.15	8	32
Steel on polished glass. By the writer...about..	.11	6	17
" quite smooth, but not polished; on perfectly dry planed wh pine, fibres parallel to motion...about..	.16	9	6
" quite smooth, but not polished; on perfectly dry planed and smoothed mahogany, fibres parallel to motion...about..	.18	10	12
Yellow copper on cast iron	.19	10	46
" " on oak	.62	31	48
Brass on east iron	.22	12	25
" on wrought iron, fibres parallel to motion	.16	9	6
" on brass	.20	11	19
" on perfectly dry planed wh pine, fibres parallel to motion...about..	.19	10	46
" " " " " and smoothed mahogany, fibres parallel to motion...about..	.24	13	30
Polished marble on polished marble. By the writer...average	.16	9	6
" " on common brick "	.44	23	45
Common brick on common brick "	.64	32	38
Soft limestone well dressed, on the same	.64	32	38
Common brick, on well-dressed soft limestone	.65	33	2
" " " " " hard "	.60	31	00
Oak across the grain, on soft limestone, well dressed	.38	20	48
" " " " " hard " " "	.38	20	48
Hard limestone on hard limestone, both " "	.38	20	48
" " " soft " " " "	.67	33	50
Soft " " hard " " " "	.65	33	2
Wood on metal, generally, .2 to .62...mean..	.41	22	18
Wood, *very smooth*, on the same, generally, .25 to .5 " ..	.38	20	48
Wood, " " on metal, " .2 to .62 " ..	.41	22	18
Metal on metal, very smooth, dry " .15 to .22 " ..	.18	10	12
Masonry and brickwork, dry " .6 to .7 " ..	.65	33	2
" " " with wet mortar...about..	.47	25	30
" " " " slightly damp mortar " ..	.74	36	30
" on dry clay " ..	.51	27	00
" " moist " " ..	.33	18	15
Marble, sawed; on the same; both dry. By the writer.*.....average " ..	.4	21	49
" " " " " both damp " ..*..... " " ..	.55	28	49
" " on perfectly dry planed wh pine. " ..*..... " " ..	.45	24	14
" " on damp planed wh pine " ..*..... " " ..	.6	31	00
" polished, on perfectly dry planed wh pine " " ..	.26	14	35
White pine, perfectly dry; planed; on the same; all the fibres parallel to motion...about..	.4	21	48
" " damp, planed; on the same " ..	.6	31	00

* But after a few trials the surfaces become so much smoother as to reduce the angles as much as from 2° to 5°; the sliding blocks weighing about 30 lbs each.

Art. 11. Recent experiments, with much greater variations of pres and of vel, and with more delicate apparatus for detecting slight changes in the coeff, although giving conflicting results,* show that **the three laws in Art. 9** are far from correct for surfs moving at high vels, and under great pres; and that they **are only approximately correct** for ordinary vels and pressures; for the coeff is found to vary both with the intensity of the pres and with the vel, as also with the temperature.* But in the cases with which the civil engineer has mostly to deal, slight diffs in the character of the surfs, or even in the dampness of the air, will often cause much greater changes of coeff than those due to any probable changes of pres, vel and temp: so that, within the limits of abrasion, we may generally take Morin's rules as sufficiently correct for such cases.

Art. 12. Prof. A. S. Kimball, of the Worcester (Mass) Inst of Industrial Science, has made some very delicate experiments upon the fric between surfs of pine wood.† The results are given in Fig 3, merely to show how the coeff varied with vel and pres. Our table gives a coeff of .4 for pine on pine.

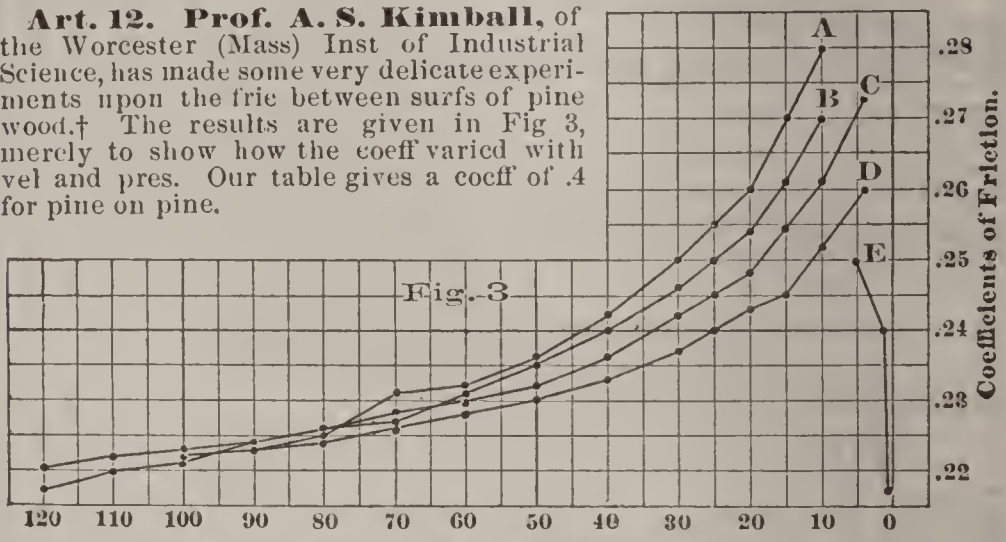


Fig. 3

Line A shows coeffs at diff vels under a pres of 1.58 lbs per sq in.
" B " " " 1.59 " "
" C " " " 1.60 " "
" D " " " 1.61 " "
" E " " " 4.17 " "

It will be seen that at low vels the coeff decreased when the pres per sq in was almost imperceptibly increased; but this diff disappeared as the vel increased. At vels from 4 to 120 ins per sec, the coeff generally decreased as the vel increased; rapidly at first, but more slowly as the vel became greater. This agrees with other recent expts. But at very low vels (.08 to 5 ins per sec) Prof. Kimball found the coeff (line E) *increasing very rapidly with the vel.*

We have made the scale of coeffs large in order to show their variations, which are so slight that they would otherwise be scarcely perceptible. Less delicate expts would have failed to show them at all.

Art. 13. (a) In 1878 **Capt. Douglas Galton** and Mr. Thos Westinghouse made careful expts in England upon fric as affecting the action of **railway brakes.**‡ The fric and pres were automatically recorded by means of

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* This is not surprising in view of the extent to which the coeff is affected by the nature of the surf. If the shape of the minute projections is such that they fit into each other as perfectly under small pressures as under great ones, and if they are too strong to be broken by the pressures applied, the coeff, as stated in the 1st law, should be independent of the pres. But if high pres wedges the projections of one body more closely between those of the other, the coeff should increase under such pres. On the other hand, if the higher pres breaks down the projections while the lower ones are unable to do so, the coeff should decrease under the higher pres. The particles thus broken off may either act as a lubricant and thus still further *reduce* the fric and its coeff, or (if angular and hard) may *increase* it. Change of area of contact, under a given total pres, may, by affecting the *intensity* of the pres, make changes in the coeff similar to those just mentioned.

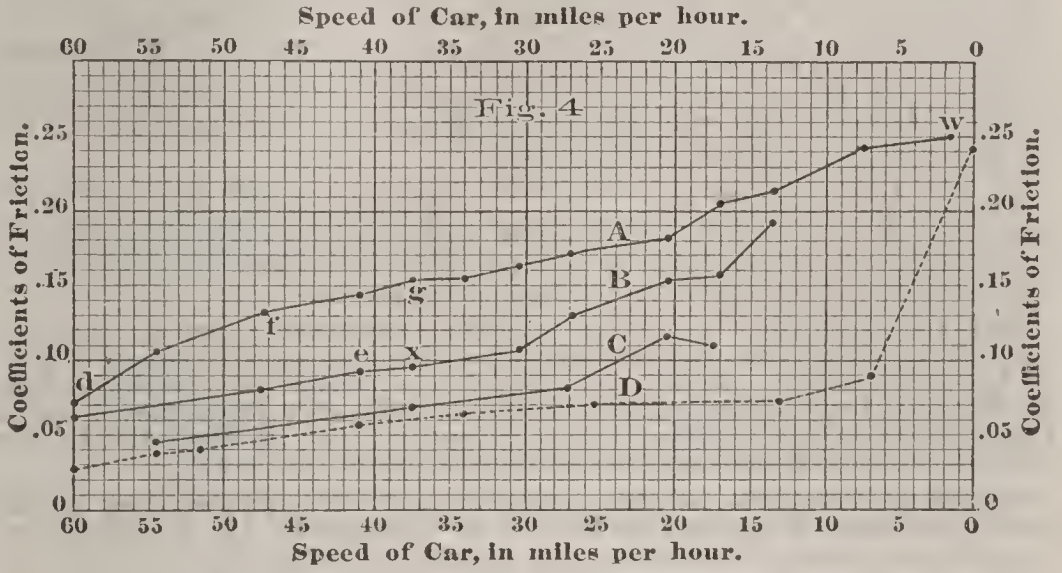
At high vels the roughnesses have not time to interlock as perfectly as at low vels. Hence we should expect a less coeff at high vels. But high vel generally increases the number of projections broken away; and these may either increase or diminish the coeff, as explained above. High vel often *indirectly* affects it by increasing the *temperature.*

† Silliman's Journal (American Journal of Science) March 1876 and May 1877.

‡ See Proc, Instn of Mechl Engrs, London, June and Oct 1878 and April 1879; and "Engineering," London, 1878; vol. 25, pp 432, 469, 490; vol. 26, pp 153, 386, 395.

hydraulic gauges. With cast iron brake-blocks and steel-tired wooden wheels, 43½ ins diam, they found coeffs about as shown in Fig. 4.

The points in lines A, B and C show the average *brake* coeffs, or coeffs of sliding fric between the tread of a *rolling* wheel and the *brake-block.*



Line A shows brake coeffs obtained immed'y after application of brake
" B " " 5 secs " "
" C " " 15 " " "
" D shows *rail* coeffs or coeffs of sliding fric between the tread of a *sliding* or "*skidding*" wheel (held fast by the brake) and the *rail.*

(**b**) From lines A B and C it appears that the **brake coeff** obtained at a given length of time after the application of the brake was generally **greater at low than at high vels.** But where the vel was maintained uniform the **brake coeff diminished as block and wheel remained longer in contact.** Thus, lines A and B show that at 37½ miles per hour the brake coeff was .154 when the brake was first applied (point *g*), but fell to .096 in 5 secs (*x*). Line A (immed'y after application) shows a higher brake coeff (.132 at *f*) at 47½ miles than line B (5 secs after application) shows at 37½ miles (.096 at *x*).

The diminution of the *rail* coeff with length of time of application of brake, was scarcely noticeable.

(**c**) When the brake fric (owing to the reduction of vel and consequent increase of coeff) becomes = the "adhesion" or static fric between the rail and the tire of the rolling wheel, the vel of rotation rapidly falls below that due to the vel of the car; *i e*, **the wheel begins to "skid"** or slide along the rail; and in from .75 to 3 secs the rotation of the wheel ceases entirely.

(**d**) **The rail coeff,** line D, **is generally much less than the brake coeff,** lines A, B and C. The pres on the rail (= the wt on a wheel) was about 5000 lbs per sq in, or greatly in excess of the limit of abrasion. That at the brake was about 200 lbs per sq in. A few expts were made with brake blocks having but ⅓ of the usual area of contact, and therefore 3 times the pres per sq in under a given total pres. They failed to show conclusively that this caused any marked change in the coeff.

(**e**) **The rail coeff,** line D, like the brake coeff, **increases as the vel diminishes:** slowly at first, but much more rapidly as the speed becomes less; until, at the moment of stopping, it is generally even greater than the brake coeff just before skidding. With steel tires on *iron* rails at high vels it was somewhat greater than on steel rails, but this diff disappeared as the vel diminished.

(**f**) **Locomotives** overcome resistces = from ⅙ to ⅓ or more of the wt on all the drivers; *i e*, they have a coeff of .33 or more, although the experimental coeff for steel on steel in motion at low pres, is only about .15. But the cases are so diff that a similarity in their coeffs could hardly be expected. The great wt, say from 2 to 6 or even 7 tons, on a driver, is concentrated on a surf (where the wheel touches the rail) about 2 ins long × about ½ inch wide, or = say 1 sq in. The pres per sq in thus greatly exceeds not only that upon which the tables are based, but also the limit of abrasion. Besides, any point in the tread, during

the instant when it is acting as the fulcrum for the steam pres in the cyl, is *stationary* upon the rail. Its fric (miscalled "adhesion") is therefore *static.*

Capt. Galton found that **the coeff of "adhesion"** was independent of the vel, and depended only on the character of the surfs in contact. With a four-wheeled car having about 5000 lbs load on each wheel, it was generally over .20 on dry rails; in some cases .25 or even higher. On wet or greasy rails, without sand, it fell as low as .15 in one case, but averaged about .18. **With sand** on wet rails it was over .20. Sand applied to dry rails before starting gave .35 and even over .40 at the start, and an average of about .28 during motion; but sand applied to dry rails while the car was in motion was apt to be blown away by the movement of the car and wheels.

(g) Owing to the constancy of the coeff of "adhesion" under given conditions of tire and rail, the brake fric necessary to "skid" the wheels in any case was also practically constant for all vels. But at high vels, owing to the *lower* brake *coeff*, a *higher* brake *pres* was reqd to produce this fixed amount of brake *fric.* The skidding also reqd a longer *time* than at low speeds.

Art. 14. If the pres is sufficient to produce **abrasion** (indeed, while it is much less) the fric often varies greatly, but no precise law has yet been discovered for estimating it. Rennie gives the following **table of coeffs of fric of dry surfaces, under pressures gradually increased up to the limits of abrasion.** It will be noticed that **in this table the coeff generally increases** with the *intensity* of the pres:

Coeffs of friction of dry surfaces, under pressures gradually increased up to the limits of abrasion. (By G. Rennie, C E.)

Pres. in Lbs. per Square Inch.	Wrought Iron on Wrought Iron.	Wrought Iron on Cast Iron.	Steel on Cast Iron.	Brass on Cast Iron.
32.5	.140	.174	.166	.157
186	.250	.275	.300	.225
224	.271	.292	.333	.219
336	.312	.333	.347	.215
448	.376	.365	.354	.208
560	.409	.367	.358	.233
672	.....	.376	.403	.233
709	.....	.434	.....	.234
784	.....	.....	.....	.232
821	.....	.....	.....	.273

Art. 15. (a) **Rolling friction,** or that between the circumf of a rolling body and the surf upon which it rolls, is somewhat similar to that of a pinion rolling upon a rack. In disengaging the interlocking projections, or in lifting the wheel over an obstacle *o*, Figs 5 and 6, the motive force F, instead of *dragging* one over the other, as in Fig 6, p 314, acts at the end of a bent lever F R W Figs 5 and 6, the other end W of which acts in a direction *perp* to the contact surf; and in practical cases of rolling fric proper the leverage R W of the resisting wt of the wheel and its load is very much less, in proportion to that (F R) of the force F, than in our exaggerated figs. Hence the force F reqd to *roll* a wheel etc is usually very much less than would be necessary to *slide* it.

(b) There are usually **two ways of applying the force** in overcoming rolling fric: 1st (Fig 5) at the *axis* of the rolling body; as the force of a horse is applied at the axle of a wagon-wheel; or that of a man at the axle of a wheel-barrow: 2d (Fig 6) at the *circumf;* as when workmen push along a heavy timber laid on top of two or more rollers; or as the ends of an iron bridge-truss play backward and forward by contraction and expansion, on top of metallic rollers or balls (p 614). In Fig 5 we have, in addition to the rolling fric of the circumf of the wheel on its support, the sliding fric of the axle in its bearing. In Fig 6 we have only rolling fric, but at both top and bottom of the wheel.

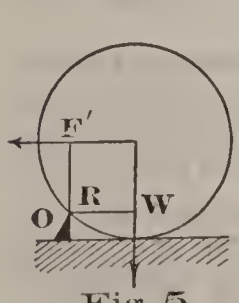


Fig. 5

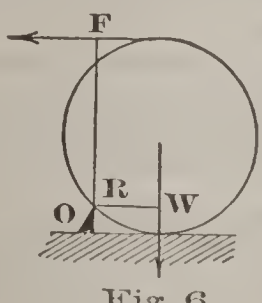


Fig. 6

(c) When the obstacles *o* are very small, as in the case of cart-wheels on smooth hard roads, or of car-wheels on iron or steel rails, the leverage

(F R) of F becomes, practically, in Fig 5 the *radius*, and in Fig 6 the *diam*, of the wheel; while that (R W) of the resistce is very small. Hence, neglecting axle fric in Fig 5, the force F reqd to overcome rolling fric in such cases is directly as the wt W of and on the wheel, and inversely as the diam of the wheel.

The few expts that have been made upon the coeffs of rolling fric, apart from axle fric, are too incomplete to serve as a basis for practical rules. See Art 20, and "Traction," p 375.

(d) The fric (or **"adhesion"**) between wheel and rail, which enables a locomotive to move itself and train, or which tends to make a car-wheel revolve notwithstanding the pres of the brake, is a resistce to the *sliding* of the wheel on the rail; and is therefore not rolling but *sliding* fric; *static* when the wheels either stand still or roll perfectly on the rails; and *kinetic* when they slip or "skid". See Art. 13 (*c*, *d*, *e* and *f*).

Art 16. The friction of liquids moving in contact with solid bodies **is independent of the pressure,** because the "lifting" of the particles of the fluid over the projections on the surf of the solid body, is aided by the pres of the surrounding particles of the liquid, which tend to occupy the places of those lifted. Hence we have, for liquids, no coeff of fric corresponding with that (= resistce ÷ pres) of solids. The resistce is believed to be directly as the area of surf of contact. Recent researches indicate that Resistce = a coeff × area of surf × vel^n, in which both n and the coeff depend upon the vel and upon the character of the surf: and that at low vels $n = 1$, but that at a certain "critical" vel (which varies with the circumstances) n suddenly becomes = 2, owing to the breaking up of the stream into marked counter currents or eddies. The resistance of fluid fric arises principally from the counter currents thus set in motion, and which must be brought into compliance with the direction of the force which is urging the stream forward.

Art. 17. Table of coefficients of moving friction of smooth plane surfaces, when kept perfectly lubricated. (Morin.)

Substances.	Dry Soap.	Olive Oil.	Tallow.	Lard.	Lard & Plumbago.
Oak on oak, fibres parallel to motion	.164	....	.075	.067	
" " " fibres perpendicular to motion	....	....	.083	.072	
" on elm, fibres parallel to motion	.136	....	.073	.066	
" on cast iron, fibres parallel to motion	....	....	.080		
" on wrought iron, " " "	....	....	.098		
Beech on oak, fibres " " "	....	....	.055		
Elm on oak, " " " "	.137	....	.070	.060	
" on elm, " " " "	.139				
" cast iron, " " " "	....	....	.066		
Wrought iron on oak, fibres parallel, greased and wet, .256.					
" " " " fibres parallel to motion	.214	....	.085		
" " on elm, " " " "	....	.055	.078	.076	
" " on cast iron, " " "	....	.066	.103	.076	
" " on wrought iron, " " "	....	.070	.082	.081	
" " on brass, fibres " " "	....	.078	.103	.075	
Cast iron on oak, fibres parallel to motion	.189				
" " " " " " " greased and wet, .218	....	.075	.078	.075	
" " on elm, " " " "	....	.061	.077	....	.091
" " on cast iron, with water, .314	.197	.064	.100	.070	.055
" " on brass	....	.078	.103	.075	
Copper on oak, fibres parallel to motion	....	....	.069		
Yellow copper on cast iron	....	.066	.072	.068	
Brass on cast iron	....	.077	.086		
" on wrought iron	....	.072	.081	....	.089
" on brass	....	.058			
Steel on cast iron	....	.079	.105	.081	
" on wrought iron	....	....	.093	.076	
" on brass	....	.053	.056	....	.067
Tanned oxhide on cast iron, greased and very wet, .365	....	.133	.159		
" " on brass	....	.191	.241		
" " on oak, with water, .29					

The launching friction of the wooden frigate Princeton was found by a committee of the Franklin Institute in 1844. to average about .067 or one-fifteenth of the pressure during the first .75 of a sec, and .022 or one forty-fifth for the next 4 secs of her motion. The slope of the ways was 1 in 13, or 4 deg, 24 mins. They were heavily coated with tallow. Pressure on them = 15.84 lbs per sq inch, or 2280 lbs per sq ft. In the first .75 of a sec the vessel slid 2.5 ins; in the 4 next secs 15 ft. 6.5 ins; total for 4.75 sec 15.75 ft.

Art. 18. The friction of lubricated surfaces varies greatly with the character of the surfs and with that of the lubricant and the manner of its application. If the lubricant is of poor quality, and scantily and unevenly applied under great pres, it may wear away in places and leave portions of the dry surfs in contact. The conditions then approximate to those of unlubricated surfaces. But if the best lubricants for the purpose are used, and supplied regularly and in proper quantity, so as to keep the surfs always perfectly separated, the case becomes practically one of liquid fric (Art. 16), and the resistce is very small. Between these two extremes there is a wide range of variations (see table, Art. 19 (*d*)), the coeff being affected by the smallest change in the conditions. Where any degree of accuracy is reqd, we would refer the reader to the experimental results given in Prof. Thurston's very exhaustive work,* devoted exclusively to this intricate subject.

Art. 19. (**a**) Expts by Mr. Arthur M. Wellington upon the **fric of lubricated journals**† gave a gradual and continuous increase of coeff as the vel of revolution diminished from 18 ft per sec (= a car speed of 12 miles per hour) to a stop. This increase was very slight at high vels, but much more rapid at low ones; as in Figs 3 and 4. At vels from 2 to 18 ft per sec the coeff was much less under high pressures than under low ones; but at starting there was little diff in this respect. The coeff increased rapidly as the **temperature** rose from 100° to 120° and 150° Fahr.

(**b**) Prof. Thurston, also experimenting with **lubricated journals,**‡ found that at starting, the coeff increased with increase of pres, as it did also when in motion, if the pres greatly exceeded the max (say 500 to 600 lbs per sq in) allowable in machinery. He also found that at high vels the coeff *increased* very slowly (instead of continuing to decrease) as the vel increased.

(**c**) Prof. Thurston gives the following **approx formulæ for journal friction** at ordinary temperatures, pressures and speeds, with journal and bearing in good condition and well lubricated:

$$\textbf{Coeff for starting} = (.015 \text{ to } .02) \times \sqrt[3]{\text{pres in lbs per sq in.}}$$

$$\textbf{Coeff when the shaft is revolving} = (.02 \text{ to } .03) \times \frac{\sqrt[5]{\text{vel in ft per } min}}{\sqrt[2]{\text{pres in lbs per sq in.}}}$$

At pressures of about 200 lbs per sq in:

$$\textbf{Temperature of minimum fric; in Fahr degs} = 15 \times \sqrt[3]{\text{vel in ft per } min}$$

Caution. The leverage, with which journal fric resists motion, **increases with the diam** of the journal.

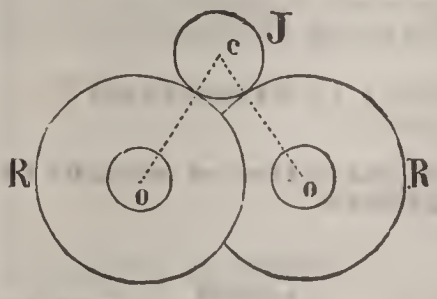
(**d**) The following figures, selected from a table of experimental results given by Prof. Thurston, merely show **the extent to which the coeff of journal fric is affected by pres, vel and temperature;** and hence the risk incurred in rigidly applying general rules to such cases. In these expts the character of journal and bearing, the lubricant and its method of application, remained the same throughout. Where these vary, still further, and much greater, variations in the coeff may occur.

Steel journal in bronze bearing, lubricated with standard sperm oil.

Temperature. Fahrenheit.	Speed of revolution									
	30 feet per *minute*			100 feet per *minute*			500 ft per *min*		1200 ft per *min*	
	Pressures.									
	200 lbs per sq in	100 lbs per sq in	4 lbs per sq in	200 lbs per sq in	100 lbs per sq in	4 lbs per sq in	200 lbs per sq in	100 lbs per sq in	200 lbs per sq in	100 lbs per sq in
	Coeff	Coeff	Coeff	Coeff	Coeff	Coeff	Coeff	Coeff	Coeff	Coeff
130°	.0160	.0044	.125	.0087	.0019	.0630	.0053	.0037	.0065	.0075
90°	.0056	.0031	.094	.0040	.0019	.0630	.0075	.0061	.0100	.0150

* Friction and Lost Work in Machinery and Mill Work. John Wiley & Sons, New York, 1885.
† Trans Amer Soc of Civil Engrs, New York, Dec. 1884.

(e) Where the force is applied first on one side of the journal and then on the opposite side, as in crank pins, the fric is less than where the resultant pres is always upon one side, as in fly-wheel shafts; because in the former case the oil has time to spread itself alternately upon both sides of the journal.



(f) Friction rollers. If a journal J, instead of revolving on ordinary bearings, be supported on friction rollers R, R, the force required to make J revolve will be reduced in nearly the same proportion that the diam of the axle *o* or *o* of the rollers, is less than the diam of the rollers themselves.

Mr. Wellington experimented with a patent bearing on this principle, invented by Mr. A. Higley. Diam of rollers R R, 8 ins; of their axles *o o* $1\frac{3}{4}$ ins; of the journal *c*, $3\frac{1}{2}$ ins. Here, theoretically,

$$\text{fric of patent journal} = \text{fric of } 3\tfrac{1}{2} \text{ in journal} \times \frac{\text{diam of axles } o\,o}{\text{diam of rollers R R}} = \frac{1\frac{3}{4} \text{ ins}}{8 \text{ ins}}$$

or as 1 to 4.6. Under a load of 279 lbs per sq in, Mr. Wellington found it about as 1 to 4 when starting from rest; and about as 1 to 2 at a car speed of 10 miles per hour.

Art. 20. (a) Resistance of railroad rolling stock. This consists of rolling fric between the treads of the wheels and the rails (the treads also sometimes *slide* on the rails, as in going around curves); of sliding fric between the journals and their bearings, and between the wheel flanges and the rail heads; of the resistce of the air; and of oscillations and concussions, which consume motive power by their lateral and vert motions, and also increase the wheel and journal frics.

Its amount depends greatly upon the condition of the road-bed and rails (as to ballast, alignment, surf, spaces at the joints, dryness etc); upon that of the rolling stock (as to wt carried, kind of springs used, kind and quantity of lubricant, condition and dimensions of wheels and axles etc); upon grades and curvature; upon the direction and force of the wind; and upon many minor considerations. Experiments give very conflicting results.

(b) During the summer of 1878, **Mr. Wellington** experimented with loaded and empty box and flat freight cars, passenger and sleeping cars, and at speeds varying from 0 to 35 miles per hour. The cars were started rolling (by grav) down a nearly uniform grade of .7 foot per 100 feet, or 36.5 feet per mile, and 6400 ft long. Their resistces were calculated as in Art. 8 (a). "The rails were of iron, 60 lbs per yd, and the track was well ballasted and in good line and surf, but not strictly first class." The following approx figures are deduced from Mr. Wellington's expts upon cars fitted with ordinary journals:*

Car Resistance in pounds per ton (2240 lbs) of weight of train, on straight and level track in good condition.

Speed of train in miles per hour	Empty cars				Loaded cars			
	Axle, tire and flange	Oscillation and concuss'n	Air	Total	Axle, tire and flange	Oscillation and concuss'n	Air	Total
0	14	0	0	14	18	0	0	18
10	6	.6	.4	7	4	.6	.4	5
20	6	2.7	1.3	10	4	2.	1.	7
30	6	5.3	2.7	14	4	4.7	2.3	11

(c) With the Higley patent anti-fric roller journal, the resistce to *starting* was but about 4 lbs per ton. But sec Art. 19 (f).

(d) About midway in the track experimented upon, was a **curve** of 1° deflection angle (5730 ft rad) 3000 ft long, with its outer rail elevated 3 to 4 ins

* Transactions, American Society of Civil Engineers, Feb 1879.

above the inner one. The rise of the outer rail was begun on the tangent, about 500 ft before reaching the curve. In the first 500 ft of the curve the resistce was greater than that encountered just before reaching the curve, by from .6 to 2.1 (average 1.1) lbs per ton. In the last 500 ft of the curve this excess had diminished to from .2 to .9 (average .6) lbs per ton. Owing to the continuance of the down grade on the curve, the vel increased as the train traversed the curve; but it does not clearly appear whether the decrease in curve resistce was due to the increase in vel, or to the fact that the oscillations caused by entering the curve gradually ceased as the train went on.

(c) **Mr. P. H. Dudley,** experimenting with his "**dynagraph**"* obtained results from which the following are deduced:

Train Resistance in pounds per ton (2240 lbs) of weight of train, including grades.

Description of train			Trip	Average **speed.** Miles per hour	Average **resistance.**
Loaded cars	Empty cars	Weight tons (2240 lbs)			
29	2	526	Toledo to Cleveland. 95 miles	20	8.34
37	0	633	Cleveland to Erie 95.5 miles	20	7.67
25	2	458	Erie to Buffalo. 88 miles	20	8.89

"With the long and heavy trains of the L. S. & M. S. Ry, of 600 to 650 tons, it reqd less fuel with the same engine to run trains at 18 to 20 miles per hour than it did at 10 to 12 miles per hour", owing to the fact that at the higher speeds steam was used expansively to a greater extent, and hence more economically.

Art. 21. The work, in ft-lbs, reqd to overcome fric through any dist, is = the fric in lbs × the dist in ft. In order that a body, started sliding or rolling freely on a hor plane and then left to itself, may do this work; *ie*, may slide or roll through the given dist, its kinetic energy (= its wt in lbs × its vel² in ft per sec ÷ 2*g*†) must = the first-named prod. Conversely, **the dist in ft** through which such a body will slide or roll on a hor plane, is

$$= \frac{\text{its kinetic energy in ft-lbs, at start}}{\text{fric in lbs}}$$

$$= \frac{\text{wt of body in lbs} \times \text{initial vel}^2 \text{ in ft per sec}}{\text{wt of body in lbs} \times \text{coeff of fric} \times 2g\dagger} = \frac{\text{initial vel}^2 \text{ in ft per sec}}{\text{coeff of fric} \times 2g\dagger}$$

The time reqd, in secs, is $= \frac{\text{dist in ft, so found}}{\text{mean vel, in ft per sec}} = \frac{\text{dist in ft}}{\frac{1}{2} \text{ initial vel in ft per sec}}$

Suppose two similar locomotives, A and B, each drawing a train on a level straight track; A at 10 miles, and B at 20 miles, per hour. The total resistce of each eng and train (which, for convenience, we suppose to be independent of vel) is 1000 lbs. Hence the force, or total steam pres in the two cyls reqd to balance the fric and thus maintain the vel, is the same in each eng. In traveling *ten miles* this force does the same amount of work (1000 lbs × 10 miles = 10000 pound-miles) in each eng, and with the same expenditure of steam in each; although B must supply steam to its cyls *twice as fast* as A, in order to *maintain* in them the same pres. In *one hour* the force in A does 10000 lb-miles as before, but that in B does (1000 lbs × 20 miles =) 20000 lb-miles, and with twice A's expenditure of steam.

But in fact the resistce of a given train is much greater at higher vels. See table Art. 20 (*b*) And even if we still assumed the resistce to be the same at both vels, B must exert more force than A in order to *acquire* a vel of 20 miles per hour while A is acquiring 10 miles per hour.

* An inst for measuring the strain on the draw-bar of a locomotive, or the force which the latter exerts upon the train.

† g = acceleration of gravity = say 32.2; $2g$ = say 64.4. See p 362.

A double purchase crane, with a weight of 7000 lbs suspended from it, showed a fric of $\frac{1}{5}$ the weight, or nearly 800 lbs. One ton suspended at each end of a chain passing over 2 cast-iron sheaves of 2 feet diam; with wrought-iron journals, working in brass bearings, well oiled, gave $\frac{1}{14}$ of the weight; or $\frac{2 \text{ tons}}{14} = 320$ lbs; or 160 lbs per ton.

Morin says the fric of a sled on dry ground is ⅜ of the pres. Babbage states that a block of stone of 1080 lbs was drawn aloug a rock surfacc by 758 lbs; or fric 70 per ct: on a wooden sled on a wooden floor, 60 per cent; with both wooden surfaces greased, only 6 per cent; and with the block on top of wooden rollers 3 ins diam, only 2.6 per cent. Rubble masonry on wet clay .2 to .35.

TRACTION.

Traction on common roads, and canals; or the power reqd to draw vehicles and boats along them. In couuectiou with this subject read the preceding and the following one.

The following table shows tolerable approximations to the force in lbs per ton, reqd to draw a stage coach and passeugers, up ascents on the Holyhead turnpike road in England, (a fine road,) by horses; as ascertaiued by means of a dynamometer. The entire weight was 1⅛ tons; but in the table, the results are giveu per single ton. From the nature of such cascs, no great accuracy is attaiuable.

Proportional Ascent.	Ascent in Ft. per Mile.	At 4 Miles per Hour.	At 6 Miles per Hour.	At 8 Miles per Hour.	At 10 Miles per Hour.
		Lbs.	Lbs.	Lbs.	Lbs.
1 in 15½	340.7	210	216	225	240
1 " 20	264.	196	202	212	229
1 " 26	203.1	155	160	166	175
1 " 30	176.	137	142	147	154
1 " 40	132.	114	120	124	130
1 " 64	82.5	109	115	120	126
1 " 118	44.7	102	107	113	120
1 " 138	38.3	99	103	109	117
1 " 156	33.9	98	101	106	112
1 " 245	21.6	93	96	101	107
1 " 600	8.8	81	85	91	96
Level.	0.	76	80	85	91

The following results, most of them with the same instrument, are also in lbs per ton; with a four-wheeled wagon, at a slow pace, on a level; and the roads in fair condition.

On a cubical block pavement..... 32 lbs per ton.....to 50.
" McAdam road, of small broken stone..... 62 " " " probably to 75.
" gravel road..... 140 " " "
" Telford road, of small stone on a paving of spawls 46 " " " " " 75.
" broken stone, on a bed of cement concrete..... 46 " " " " " 75.
" common earth roads..... 200 to 300. On a plank road 30, to 50 lbs.

The tractive power of a horse diminishes as his speed increases; and perhaps, within certain limits, say from ¾ to four miles per hour, nearly in inverse proportion to it. Thus, the average traction of a horse, *on a level*, and actually pulling for 10 hours in the day, may be assumed approximately as follows:

Miles per hour.	Lbs. Traction.	Miles per hour.	Lbs. Traction.
¾	333.33	2¼	111.11
1	250.	2½	100.
1¼	200.	2¾	90.91
1½	166.66	3	83.33
1¾	142.86	3½	71.43
2	125.	4	62.50

If he works for a smaller number of hours, his traction may increase as the hours diminish; down to about 5 hours per day and for speeds of about from 1¼ to 3 miles per hour. Thus, for 5 hours per day his traction at 2½ miles per hour will be 200 lbs, &c. When ascending a hill, his power diminishes so rapidly, from having partially to raise his own weight, (which averages about 1000 to 1100 lbs,) that up a slope of 5 to 1, he can barely struggle along without any load. On such an ascent, (see table, p 354, of Force in Rigid Bodies,) he must exert a force equal to 439 lbs per ton, or of 196 lbs for the 1000 lbs of his own weight. Assuming that on a level piece of good turnpike, he would when hauling a cart and load, together weighing 1 ton, have to exert a traction of 60 lbs: then on ascending a hill of 4° inclination, (or 1 in 14.3; or 369¼ ft per mile,) he would have to exert 156 lbs, against the gravity of the 1 ton: and 67 lbs, against that of his own weight; or 223 lbs in all. He may, for a few mins, exert without injury, about twice his regular traction. This calculation shows that up a hill of 4°, an average horse is fully tasked in drawing a total load of one ton; and should, therefore, be allowed, in such a case, to choose his own gait; and to rest at short intervals. A fair load for a single horse with a cart, at a variable walking pace, working 10 hours per day, on a common undulating road in good order, is about half a ton, in addition to the cart, which will be about half a ton more. With two horses to this same cart, the *load* alone may be about 1¼ tons.

Rem. Since the action of gravity is the same on good roads and bad ones, it follows that **ascents become more objectionable the better the road is.** Thus, on an ascent of 2°, or 184.4 ft per mile, gravity alone requires a traction of 78 lbs per ton;

which is about 10 times that on a level railroad at 6 miles an hour; but only about equal to that on a level common turnpike road, at the same speed. Therefore, (to speak somewhat at random,) it would require 10 locomotives instead of 1; but only 2 horses instead of 1. A grade of 1 in 35; or 150 ft to a mile; or 1° 38′, is about the steepest that permits horses to be driven down a hard smooth road, in a fast trot, without danger. It should, therefore, not be exceeded except when absolutely necessary, especially on turnpikes.

On canals and other waters, the liquid is the resisting medium that takes the place of friction on level roads. But unlike friction, its resistance varies as the squares of the vels; (see Art 26 of page 280,) at least from the vel of 2 ft per sec, or 1.364 miles per hour; to that of 11½ ft per sec, or 7.84 m per h. As the speed falls below 1½ m per h, the resistance varies less and less rapidly; and this is the case whether the moved body floats partly above the surface; or is entirely immersed. In towing along stagnant canals, &c, the vel is usually from 1 to 2½ m per h; for freight most frequently from 1½ to 2. Less force is required to tow a boat at say 2 m per h, where there is no current, than at say 1¼ m per h, against a current of ¾ m per h, because in the last case the boat has to be *lifted up* the very gradual inclined plane or slope which produces the current.

The force required to tow a boat along a canal depends *greatly* upon the comparative transverse sectional areas of the channel, and of the immersed portion of the boat. When the width of a canal at water-line is at least 4 times that of the boat; and the area of its transverse section as great as at least 6½ times that of the *immersed* transverse section of the boat, the towing at usual canal vels will be about as easy as in wider and deeper water. With less dimensions, it becomes more difficult. (D'Aubuisson.) Much also depends on the shape of the bow and other parts of the boat; and on the proportion of its length to its breadth and depth. Hence it is seen that the mere weight of the load is by no means so controlling an element as it is on land. The whole subject, however, is too intricate to be treated of here. Morin states that naval constructors estimate the resistance to sailing and steam vessels at sea, at but from about .5 to .7 of a lb for every sq ft of immersed transverse section, when the vel is 3 ft per sec, or 2.046 miles per hour. It is far greater on canals.

On the Schuylkill Navigation of Pennsylvania, of mixed canal and slackwater, for 108 miles, the regular load for 3 horses or mules, is a boat of very full build; and no keel: 100 ft long, 17½ ft beam; and 8 ft depth of hold; drawing 5½ ft when loaded.* Weight of boat about 65 tons; load 175 tons of coal, (2240 lbs;) total weight 240 tons, or 80 tons per horse or mule. On the down trip with the loaded boats, for 4 days, the animals are at work, *actually towing*, (except at the locks,) for 18 hours out of the 24; thus exceeding by far the limits of time usually allowed for continuous effort.

On the canal sections, (which have 60 ft water-line; and 6 ft depth,) the speed is 1¾ miles per hour; and on the deep wide pools, 2 miles.

On the up trip with the empty 65-ton boats, the average speed is about 2½ miles per hour. The empty boats draw 16 to 18 ins water; and frequently keep on without stopping to rest day or night through the entire distance of 108 miles. The animals generally have 2 or 3 days' rest at each end of the trip; but are materially deteriorated at the end of the boating season.

If our preceding assumption of 143 lbs traction of a horse at 1¾ miles per hour, is correct, the traction of the loaded boats on the canal sections is $\frac{143 \text{ lbs}}{80 \text{ tons}} = 1.83$ lbs per ton.

The intelligent engineer and superintendent of the Sch Nav, James F Smith, gives as the results of his own extensive observation, that one of these large boats loaded (240 tons in all) may, without distressing the animals, be drawn along the canal sections, for 10 hours per day, as follows: By one average horse or mule, at the rate of 1 mile: by two animals, at 1½ miles; and by three, at 1¾ miles per hour. When four animals are used the gain of time is very trifling. At a time of rivalry among the boatmen, one of them used 8 horses; but with these could not exceed 2½ miles per hour in the canal portions. Two or more horses together cannot for hours pull as much as when working separately.

If our preceding short table of the traction of a horse at diff vels for 10 hours is correct, then the traction of the above loaded coal boats (240 tons) on the canal sections of the navigation, is as follows: The last column shows the traction in lbs per sq ft of area of immersed transverse section where largest; viz, about 95 sq ft.

Horses.	Miles per Hour.		Lbs. per Ton.	Lbs. per Sq Ft.
1	1	$\frac{250}{240}$	1.04	2.63
2	1½	$\frac{333}{240}$	1.39	3.50
3	1¾	$\frac{428}{240}$	1.78	4.50
3 on pools	2	$\frac{375}{240}$	1.56	3.95
8	2½	$\frac{800}{240}$	3.33	8.42
3 up-trip	2½	$\frac{300}{65}$	4.61	12.50

Lachine Canal, Canada, 120 ft wide at water-line; 80 ft at bottom; depth on mitre sills 9 ft; 6 horses tow loaded schooners with ease.

Before the enlargement of the **Erie canal,**† its dimensions were 40 ft water-line; 28 ft bottom; 4 ft depth of water. The average weight of the boats was about 30 tons. With 75 tons of load, or 105 tons total, they were towed by 2 horses, at the rate of about 2 miles per hour; which by our table gives a traction of nearly 2.4 lbs per ton. The boats were about 80 ft long; 14 ft beam; full 3¼ ft draught loaded; hence the traction by our table would be about 5.7 lbs per sq ft of immersed transverse section.

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* **Cost of boats,** 1884, (Schuylkill Canal) about $1800. Annual repairs about $85. Boats last 16 to 20 years. Length, 102 ft; beam, 17½ ft; draft, 1½ to 5½ ft; capacity, 180 tons; weight, about 58 tons; speed, with 3 mules, 1¾ miles per hour.

† Length 363 miles; cost $19680 per mile. The enlarged canal has 70 ft; 42 ft; and 7 ft of water; and cost $90800 per mile for the enlargement only. The cost of the several canals in Pennsylvania has ranged between $23000 and $50000 per mile.

While, for 82-ton loaded boats on a smaller canal, (the boats nearly touching bottom,) the traction at 1¾ miles, would be 3½ lbs per ton; or about twice as great as the above 1.78 lbs. It also would be 5.7 lbs per sq ft of immersed section.

For **traction on railroads**, see Art 20, pp 374 *e* and 374 *f*.

ANIMAL POWER.

Art. 1. So far as regards horses, this subject has been partially considered under the preceding head, Traction. All estimates on this subject must to a certain extent be vague, owing to the diff strengths and speeds of animals of the same kind; as well as to the extent of their training to any particular kind of work. Authorities on the subject differ widely; and sometimes express themselves in a loose manner that throws doubt on their meaning. We believe, however, that the following will be found to be as close approximation to practical averages as the nature of the case admits of with our present imperfect knowledge. We suppose a good average trained horse, weighing not less than about ½ a ton, well fed and treated. Such a one, when actually walking for 10 hours a day, at the rate of 2½ miles per hour, on a good *level* road, such as the tow-path of a canal, or a circular horse-path,* **can exert a continuous pull, draught, power, or traction, of 100 lbs.**

Now, 2½ miles per hour, is 220 ft per min, or 3⅔ ft per sec; and since 10 hours contain 600 min, his day's work of actual hauling on a level, at that speed, amounts to

$$\underset{\text{min}}{600} \times \underset{\text{ft}}{220} \times \underset{\text{lbs}}{100} = 13\,200\,000 \text{ ft-lbs per day.}$$

Or, 22000 ft-lbs per min, or 366⅔ ft-lbs per sec.† Which means that he exerts force enough during the day to *lift* 13 200 000 lbs 1 foot high; or 1 320 000 lbs 10 feet high; or 132 000 lbs 100 ft high, &c. He may exert this force either in *traction* (hauling) or in *lifting* loads. If he has to raise a small load to a great height, the machinery through which he does it must be so geared as to gain speed, at the loss (commonly but improperly so expressed) of power. Whether he lifts the great weight through a small height, or the small weight through a great height, he exerts precisely the same amount of force or power. In connection with this subject, the student should read Arts 5, 9, 11, &c, of Force in Rigid Bodies. Also, see Hauling by Horses and Carts.

Experience shows that within the limits of 5 and 10 hours per day, (the speed remaining the same,) **the draft of a horse may be increased in about the same proportion as the time is diminished;** so that when working from 5 to 10 hours per day, it will be about as shown in the following table. Hence, the total amount of 13 200 000 ft-lbs per day may be accomplished, whether the horse is at work 5, 6, or 8, &c, hours per day.‡ This, of course, supposes him to be actually lifting or hauling *all the time;* and makes no allowance for stoppages for any purpose.

Table of draft of a horse, at 2½ miles per hour, on a level.

Hours per day.	Lbs.	Hours per day.	Lbs.
10	100	7	$142\frac{6}{7}$
9	$111\frac{1}{9}$	6	$166\frac{2}{3}$
8	125	5	200

Experience also shows that at speeds between ¾ and 4 miles an hour, his force or draught will be inversely in proportion to his speed. Thus, at 2 miles an hour, for 10 hours of the day, his draught will be

$$\underset{\text{miles}}{2} : \underset{\text{miles}}{2\frac{1}{2}} :: \underset{\text{lbs}}{100} : \underset{\text{lbs}}{125} \text{ draught.}$$

At 1½ miles, it would be 166⅔ lbs; at 3 miles, 83⅓ lbs; and at 4 miles, 62½ lbs; as per table in Traction.

Therefore, in this case also, the entire amount of his day's work remains the same;§ and within

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* To enable a horse to work with ease **in a circular horse-walk**, its diam should not be less than 25 ft; 30 or 35 would be still better.

† **A nominal horse-power** is 33000 ft-lbs per minute; this being the rate assumed by Boulton and Watt in selling their engines; so that purchasers wishing to substitute steam for horses, should not be disappointed. Their assumption can be carried out by a very strong horse day after day for 8 or 10 hours; but as the engine can work day and night for months without stopping, which a horse cannot, it is plain that a one-horse engine can do much more work than any one such horse. Hence many object to the term horse-power as applied to engines; but since everybody understands its plain meaning, and such a term is convenient, it is not in fact objectionable. Boulton and Watt meant that a one-horse engine would at any moment perform the work of a very strong horse. An *average* horse will do but 22000 ft-lbs per min.

‡ It is plain that although the *day's* labor will be the same, that of an hour, or of a min, will vary with the number of hours taken as a day's work. It must be remembered that a *working day* of a given number of hours, by no means implies, in every case, that number of hours of *actual work;* but includes intermissions and rests.

§ **This remark about speed will not apply to loads towed through the water.** Thus, if his draught at 2 miles an hour be 125 lbs; and at 4 miles, 62½ lbs; he will *on land* draw loads in these proportions; but in hauling a boat *through the water* at the greater speed, he has to encounter the increased resistance of the *water* itself; which resistance at 4 miles is much more than twice as great as at 2 miles; probably 4 times as great. Therefore, at 4 miles on a canal, his draught of 62½ lbs would not suffice for *a load* half as great as he could tow with his draft of 125 lbs at 2 miles.

all the foregoing limits of hours and speed, may be practically taken to be about 13 200 000 ft-lbs per day; or 22000 ft-lbs per min of a day of 10 hours. But it does not follow that the horse can always in practice actually *lift loads* at that rate; because generally a part of his power is expended in overcoming the *friction* of the machinery which he puts in motion; and moreover, the nature of the work may require him to stop frequently; so that in a *working day* of 8 or 10 hours, the horse may not actually be *at work* more than 5, 6, or 7 hours.

As a rough approximation, to allow for the waste of force in overcoming the friction of hoisting machinery, and the weight of the hoisting chains, buckets, &c, we may say that **the useful or paying daily net work of a horse, in hoisting by a common gin,** is about 10 000 000 ft-lbs. That is, he will raise equivalent to 10 000 000 lbs net of water, or ore, &c, 1 foot. The load which he can raise at once, including chains, bucket, and an allowance for friction, will be as much greater than his own direct force, as the diam of the horse-walk is greater than that of the winding drum; and it will move that much slower than he does. His own direct force will vary according to the number of hours per day that he may be required to work, as in the foregoing table. With these data, the size of the buckets can be decided on; and of these there should be at least two, so that the empty one at the bottom may be filled while the full one at top is being emptied; so as to save time. The same when the work is done by men.

Art. 2. A practised laborer hauling along a level road, by a rope over his shoulders; or in a circular path, pushing before him a hor lever, at a speed of from 1½ to 3 miles per hour, exerts about ⅛ part as much force as a horse; or 2 200 000 ft-lbs per day; or 36663⅓ ft-lbs per min of a day of 10 hours of actual hauling or pushing.

But laborers frequently have to work under circumstances less advantageous for the exertion of their force than when hauling or pushing in the manner just alluded to; and in such cases they cannot do as much per day. Thus in turning a winch or crank like that of a grindstone, or of a crane, the continual bending of the body, and motion of the arms, is more fatiguing. **The size of a winch should not exceed 18 ins,** or the rad of a circle of 3 ft diam; and against it a laborer can exert a force of about 16 lbs, at a vel of 2½ ft per sec, or 150 ft per min, making very nearly 16 turns per min; for 8 hours per day. To these 8 hours an additiou must be made of about ¼ part, for short rests. Or if a *working day* is taken at 8, or 10, &c, hours, $\frac{1}{5}$ part must generally be taken from it for such rests. On the foregoing data an hour's *work* of 60 min of *actual hoisting* would be

$$16 \text{ lbs} \times 150 \frac{\text{ft}}{\text{min}} \times 60 \text{ min} = 144000 \text{ ft-lbs};$$

or, deducting $\frac{1}{5}$ part for rests, 115200 ft-lbs per hour of *time, including rests.* In practice, however, a further deduction must be made for the fric of the machine, and for the wt of the hoisting chains; and in case of raising water, stone, ore, &c, from pits, for the wt of the buckets also. As a rough average we may assume that these will leave but 100 000 ft-lbs of paying, or useful work per hour; that is, that **a man at a winch will actually lift equivalent to 100000 lbs of water, ore, &c, 1 foot high per hour's time, including rests.** This is equal to 1666⅔ ft-lbs per min of a day of 10 hours, including rests. Therefore, in a day of 10 working hours he would raise 1 000 000 lbs net, 1 foot high; **or just $\frac{1}{10}$ part of what a horse would do with a gin** in the same time. We have before seen that in hauling along a level road, he can at a slow pace perform about ⅛ of the daily duty of a horse. He may also work the winch with greater force, say up to 30 or even 40 lbs; but he will do it at a proportionately slower rate; thus, accomplishing only the same daily duty.

With a gin, like those for horses, but lighter, with 2 or more buckets, a practised laborer will in a working day of 10 hours, raise from 1 200 000 to 1 400 000 ft-lbs net of water, ore, &c. With a shallow well or pit, more time is lost in emptying buckets than in a deep one; but the deep one will require a greater wt of rope. To save time in all such operations on a large scale, there should be at least two buckets; the empty one to be filled while the full one is being emptied. It is also best to employ 2 or more men to hoist at the same time, by winches, at both ends of the axis; and the men will work with more ease if the winches are at right angles to each other. Each winch handle may be long enough for 2 or 3 men. An extra man should be employed to empty the buckets. He may take turns with the hoisters. The same remarks apply in some of the following cases.

On a treadwheel a practised laborer will do about 40 per cent more daily duty than at a winch; or in a working day * of 10 hours, including rests, he will do about 1 400 000 ft-lbs. And he can do this whether he works at the outer circumf of the wheel, stepping upon foot-boards, or tread-boards, on a level with its axis; or walks inside of it near its bottom. In both cases he acts by his wt, usually about 130 to 140 lbs; and not by the muscular strength of his arms. When at the level of the axis, his wt acts more directly than when he walks on the bottom of the wheel; but in the first case he has to perform a slow and fatiguing duty resembling that of walking up a continuous flight of steps; while in the second he has as it were merely to ascend a very slightly inclined plane; which he can do much more rapidly for hours, with comparatively little fatigue: and this rapidity compensates for the less direct action of his wt. Therefore, in either case, as experience has shown, he accomplishes about the same amount of daily duty. Treadwheels may be from 5 to 25 ft in diam, according to the nature of the work. They are generally worked by several men at once, and may at times be advantageously used in pile-driving, as well as in hoisting water, stone, &c.

By a good common pump, properly proportioned, a practised laborer will in a day of 10 working hours, raise about 1 000 000 ft-lbs of water, net.†

Bailing with a light bucket or scoop, he can accomplish about 200 000 ft-lbs net of water. **By a bucket and swape,** (a long lever rocking vertically; and weighted at one end so as to balance the full bucket hung from the other; often seen at country

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* The working day must be understood to include necessary rests, and such intermissions as the nature of the work demands; but does not include time lost at meals. A *working day* of 10 hours may, therefore, have but 8, 7, or 6, &c hours of *actual labor.* This will be understood when we hereafter speak of a working day, or simply a day.

† Desaguliers's estimates of daily work of men and horses exceed the above, but are entirely too great.

wells,) 600 000 to 800 000. In the last he has only to pull down the empty bucket, and thereby raise the counterweight. **By 2 buckets at the ends of a rope suspended over a pulley,** 500 000 to 600 000. Here he works the buckets by pulling the rope by hand.

By a tympan, or tympanum,* worked by a treadwheel, about 1 200 000 to 1 400 000.

By a Persian wheel,† a chain-pump, a chain of buckets,‡ or an Archimedes screw, all worked by a treadwheel, from 800 000 to 1 000 000 ft-lbs. Of these four, the first three lose useful effect by either spilling, leaking, or the necessity for raising the water to a level somewhat higher than that at which it is discharged.

When any of the five foregoing machines are worked by men at winches, the result will be about ⅓ less than by treadwheels. They are all frequently worked also by either steam, water, or horse-power.

By walking backward and forward, on a lever which rocks on its center, a man may, according to Robison's Mech Philosophy, perform a much greater duty than by any of the preceding modes. He states that a young man weighing 135 lbs, and loaded with 30 lbs in addition, worked in this manner for 10 hours a day without fatigue; and raised 9¼ cubic feet of water, 11½ ft high per min. This is equal to 3 984 000 ft-lbs per day of 10 hours; or 6640 ft-lbs per min; or nearly $\frac{4}{10}$ of the net daily work of a horse in a gin.

A laborer standing still, can barely sustain for a few min, a load of 100 lbs, by a rope over his shoulder, and thence passing off hor over a pulley. And scarcely as much, when (facing the load and pulley) he holds the end of a hor rope with his hands before him. He cannot push hor with his hands at the height of his shoulders, with more than about 30 lbs force.

Weisbach states from his own observation, that 4 practised men raised a **dolly** (a wooden beetle or rammer, of wood; with 4 hor projecting round bars for handles) weighing 120 lbs, 4 ft high, at the rate of 34 times per min, for 4½ min; and then rested for 4½ min; and so on alternately through the 10 hours of their working day. Therefore, 5 of these hours were lost in rests; and the duty performed by each man during the other 5 hours, or 300 mins, was

$$\frac{120 \times 4 \times 34 \times 300}{4} = 1\,224\,000 \text{ ft-lbs.}$$

In the old mode of driving piles, where the ram of 400 to 1200 lbs suspended from a pulley, was raised by 10 to 40 men pulling at separate cords, from 35 to 40 lbs of the ram were allotted to each man, to be lifted from 12 to 18 times per min, to a height of 3¾ to 4¾ feet each time, for about 3 min at a spell, and then 3 min rest. It was very laborious; and the gangs had to be changed about hourly, after performing but ½ an hour's actual labor.

Hauling by horses. See Traction. When working all day, say 10 working hours, the average rate at which a horse walks while hauling a full load, and while returning with the empty vehicle, is about 2 to 2¼ miles per hour; but to allow for stoppages to rest, &c, it is safest to take it at but about 1.8 miles per hour, or 160 ft per min. The time lost on each trip, in loading and unloading, may usually be taken at about 15 min. Therefore, to find the number of loads that can be hauled to any given dist in a day, first find the time in min reqd in hauling one load, and returning empty. Thus: div twice the dist in ft to which the load is to be hauled; or in other words, div the length in ft, of the round trip, by 160 ft. The quot is the number of min that the horse is in motion during each round trip. To this quot add 15 min lost each trip while loading and unloading; the sum is the total time in min occupied by each round trip. Div the number of min in a working day (600 min in a day of 10 working hours) by this number of min reqd for each trip; the quot will be the number of trips, or of loads hauled per day.

Ex. How many loads will a horse haul to a dist of 960 ft, in a day of 10 working hours, or 600 min? Here, $960 \times 2 = 1920$ ft of round trip at each load. And $\frac{1920}{160} = 12$ min, occupied in walking. And $12 + 15$ in loading, &c) $= 27$ min reqd for each load. Finally, $\frac{600}{27} = \frac{\text{min in 10 hours}}{\text{min per trip}} = 22.2$, or say 22 trips; or loads hauled per day.

Table of number of loads hauled per day of 10 working hours. The first col is the distance to which the load is actually hauled; or half the length of the round trip. The cost of hauling per load, is supposed to be for one-horse carts; the driver doing the loading and unloading; rating the expense of horse, cart, and driver at $2 per day. See Cost of Earthwork, page 742.

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* The tympan revolves on a hor shaft; and is a kind of large wheel, the spokes, arms, or radii of which are gutters, troughs, or pipes, which at their outer ends terminate in scoops, which dip into the water. As the water is gradually raised, it flows along the arms of the wheel to its axis, where it is discha. The scoop wheel is a modification of it. It is an admirable machine for raising large quantities of water to moderate heights. We cannot go into any detail respecting this and other hydraulic machines.

† A kind of large wheel with buckets or pots at the ends of its radiating arms; revolves on a hor axis; discharges at top. The buckets are attached loosely, so as to hang vert, and thus avoid spilling until they arrive at the proper point, where they come into contact with a contrivance for tilting and emptying them. The noria is similar, except that the buckets are firmly held in place, and thus spill much water. It is therefore inferior to the Persian wheel.

‡ An endless revolving vert chain of buckets. D'Aubuisson and some others erroneously call this the noria. It is an effective machine.

Dist. Feet.	No. of Loads.	Cost per Load.	Dist. Feet.	No. of Loads.	Cost per Load.	Dist. Miles.	No. of Loads.	Cost per Load.
		Cts.			Cts.			Cts.
50	38	5.26	1500	18	11.11	1	7	28.57
100	37	5.41	2000	15	13.33	1¼	6	33.33
200	34	5.88	2500	13	15.39	1½	5	40.00
300	32	6.25	3000	11	18.18	2	4	50.00
400	30	6.67	3500	10	20.00	3	3	66.67
600	27	7.41	4000	9	22.22	4	2	100.00
1000	22	9.09	5000	7	28.57	9	1	200.00

If the loading and unloading is such as cannot be done by the driver alone; but requires the help of cranes, or other machinery, an addition of from 10 to 50 cts per load may become necessary. Hauling can generally be more cheaply done by using 2 or 3 horses, and one driver, to a vehicle. The neat load per horse, in addition to the vehicle, will usually be from ½ to 1 ton, depending on the condition, and grades of the road. From 13 to 15 cub ft of solid stone; or from 23 to 27 cub feet of broken stone, make 1 ton. **In estimating for hauling rough quarry stone for drains, culverts, &c,** bear in mind that each cub yard of common seabbled rubble masonry, requires the hauling of about 1.2 cub yds of the stone as usually piled up for sale in the quarry; or about ⅔ of a cub yd of the original rock in place. **A cub yd of solid stone, when broken into pieces, usually occupies about 1.9 cub yds perfectly loose;** or about 1¾ when piled up. A strong cart for stone hauling, will weigh about ⅔ ton; or 1500 lbs; and will hold stone enough for a perch of rubble masonry; or say 1.2 pers of the rough stone in piles. The average weight of a good working horse is about ½ a ton.

Morin gives the following results from careful experiments made by him for the French Government. The draft of the same wheeled vehicle on a road, may in practice be considered to be,

1st. On hard turnpikes, and pavements: in proportion to the loads; inversely as the diams of the wheels; and nearly independent of the width of tire. It increases to uncertain extents with the inequalities of the road; the stiffness (want of spring) of the vehicle; and the speed; (considerably less than as the square roots of the last.)

2d. On soft roads, the draft is less with wide tires than with narrower ones; and for farming purposes he recommends a width of 4 ins. With speeds from a walk to a fast trot, the draft does not vary sensibly.

SPECIFIC GRAVITY.

The sp grav of a body, is its weight as compared with that of an equal bulk of some other body, which is adopted as a standard of comparison. For other substances than air and gases generally, pure water is the usual standard; and since the weight of a given bulk of water varies somewhat with its temperature; and also with the state of the air, the former is assumed to be 62° Fah; and the latter at 30 ins, at sea-level. On the continent of Europe, water at its greatest density, or at a temperature of 4° Centigrade, = 39.2° Fahrenheit, is taken as the standard. But where extreme scientific accuracy is not aimed at, all these considerations may be neglected; and any clear fresh water, at any ordinary temperature, say from 60° to 80°, may be used. At 70°, the resulting sp gr is but 1 part in 1176 greater than at 62° F: at 75°, 1 to 670; at 80°, 1 in 454; at 85°, 1 in 336; at 90°, 1 in 264. At 62° pure water weighs 62.355 lbs avoir per cub ft.

To find the sp grav of a body, heavier than water. Weigh it first in the air; and then in water; and find the diff. The diff is what the body loses in water; and is the weight of a bulk of water equal to the bulk of the body. Then say, Diff : wt in air : : 1 : sp grav of body.

The weight of a given bulk of a substance which is either porous, or absorbent of water, cannot be inferred from its sp gr. Thus pure river sand, is pure quartz; and of course has the same sp gr; yet, a solid cub ft of quartz, weighs nearly twice as much as a cub ft of sand; on account of the interstices of the latter. A brick, some sandstones, &c. absorb water; so that their sp gr will not furnish the weight of a dry mass of the same. In such cases, the engineer will generally first measure the contents of a piece of the substance, if a solid; and then weigh it; thus ascertaining its weight per cub ft. &c. If it is in grains, or dust, he will measure, and then weigh, a cub ft of it. Footnote p 384.

To find the sp grav of a liquid. First carefully weigh some solid body, as a piece of metal, in the air. Then weigh it in water, and note the loss, say L. Then weigh it in the other liquid: and note the loss, say *l*. Then as loss L, is to loss *l*, so is 1, or the sp gr of water, to the sp gr of the liquid. Or, if the sp gr, and weight of the solid body, are already known, merely weigh it in the liquid. Then as its weight in air, is to its loss in the liquid, so is its sp gr, to that of the liquid.

Timber, when first purchased from lumber yards, even under shelter, is rarely, if ever, perfectly dry; but its weight, if tolerably seasoned, will be about ⅛ part greater than given in our tables, or about ¼ to ⅓ part, if green.

Table of specific gravities, and weights.

In this table, the sp gr of air, and gases also, are compared with that of water, instead of that of air; which last is usual.

The specific gravity of any substance is = **its weight in grams per cubic centimetre.**	Average Sp Gr.	Average Wt of a Cub Ft. Lbs.
Air, atmospheric; at 60° Fah, and under the pressure of one atmosphere or 14.7 lbs per sq inch, weighs $\frac{1}{815}$ part as much as water at 60°	.00123	.0765
Alcohol, pure	.793	49.43
" of commerce	.834	52.1
" proof spirit	.916	57.2
Ash, perfectly dry. (See footnote, p 383.) average.. 1000 ft board measure weighs 1.748 tons.	.752	47.
Ash, American white, dry " .. 1000 ft board measure weighs 1.414 tons.	.61	38.
Alabaster, falsely so called; but really Marbles	2.7	168.
" real; a compact white plaster of Paris average ..	2.31	144.
Aluminium	2.6	162.
Antimony, cast, 6.66 to 6.74 average ..	6.79	418.
" native " ..	6.67	416.
Anthracite, 1.3 to 1.84. Of Penn'a, 1.3 to 1.7 usually .. A cubic yard solid, averages about 1.75 cub yds, when broken to any market size; and loose.	1.5	93.5
Anthracite, broken, of any size. Loose average..	.....	52 to 56
" " moderately shaken " ..	.....	56 to 60
" heaped bushel, loose, 77 to 83. A ton, loose, averages from 40 to 43 cub ft " .. at 54 lbs per cub ft, a cub yard weighs .651 ton.		
Asphaltum, 1 to 1.8 " ..	1.4	87.3
Basalt. See Limestones, quarried " ..	2.9	181.
Bath Stone, Oolite " ..	2.1	131.
Bismuth, cast. Also native " ..	9.74	607.
Bitumen, solid. See Asphaltum.		
Brass, (Copper and Zinc,) cast, 7.8 to 8.4 " ..	8.1	504.
" rolled " ..	8.4	524.
Bronze. Copper 8 parts; Tin 1. (Gun metal.) 8.4 to 8.6 " ..	8.5	529.
Brick, best pressed	.....	150.
" common hard	.....	125.
" soft, inferior	.....	100.
Brickwork. See Masonry.		
Boxwood, dry " ..	.96	60
Calcite, transparent " ..	2.722	169.9
Carbonic Acid Gas, is 1½ times as heavy as air " ..	.00187	
Charcoal, of pines and oaks " ..	.....	15 to 30
Chalk, 2.2 to 2.8. See Limestones, quarried " ..	2.5	156.
Clay, potter's, dry, 1.8 to 2.1 " ..	1.9	119.
" dry, in lump, loose " ..	.....	63.
Coke, loose, of good coal " ..	.....	23 to 32
" a heaped bushel, loose, 35 to 42 lbs " ..		
" a ton occupies 80 to 97 cub ft " .. In coking, coals swell from 25 to 50 per cent. Equal weights of coke and coal, evaporate about equal wts of water: and each abt twice as much as the same wt of dry wood.		
Corundum, pure, 3.8 to 4	3.9	
Cherry, perfectly dry average .. 1000 ft board measure weighs 1.562 tons.	.672	42.
Coal, bituminous, 1.2 to 1.5 " ..	1.35	84.
" " broken, of any size; loose " ..	.....	47 to 52
" " moderately shaken " ..	.....	51 to 56
" " a heaped bushel, loose, 70 to 78 lbs.		
" " a ton occupies 43 to 48 cub ft. A cubic yard solid, averages about 1.75 yards when broken to any market size, and loose.		

Table of specific gravities, and weights — (Continued.)

The specific gravity of any substance is = **its weight in grams per cubic centimetre.**	Average Sp Gr.	Average Wt of a Cub Ft. Lbs.
Chestnut, perfectly dry. (See footnote, p 383.)..... average..	.66	41.
1000 feet board measure weighs 1.525 tons.		
Cement, hydraulic. American, Rosendale; ground. loose..... average..	.....	56.
" " " " U.S. Struck bush, 70 lbs.....		
" " " Louisville, " " 62.....	.....	49.6
" " " Copley, " " 67.....	.....	53.6
" " English Portland, U.S. struck bush, by Gillmore, 100 to 128	.....	81 to 102
" " " " Various, weighed by writer, 95 to 102.....	.....	76 to 81.6
" " " a barrel 400 to 430 lbs.		
" " French Boulogne Portland, struck bush, 95 to 110.....	.....	76 to 88
Differences of 4 or 5 pounds either more or less than we here give per loose struck U.S. bush, often occur in the cement from the same manufactory, owing not only to the difficulty of measuring exactly, but to the want of uniformity in the composition of the stone, degree of burning, grinding, dryness, &c. Moreover, the term "loose" is indefinite. We mean by it the average looseness which it has when thrown by a scoop into a half bushel when measuring that quantity for sale. By shaking it may easily be compacted about ⅛ part, so as to weigh $\frac{1}{7}$ more per bush, or cub ft. And by ramming, about ¼ part, so as to weigh about ⅓ more. So with lime, plaster, &c.		
Copper, cast,.....8.6 to 8.8.....	8.7	542.
" rolled,.....8.8 to 9.0.....	8.9	555.
Crystal, pure Quartz. See Quartz.		
Cork.....	.25	15.6
Diamond, 3.44 to 3.55; usually 3.51 to 3.55.....	3.53	
Earth; common loam, perfectly dry. loose.....	.....	72 to 80
" " " " " shaken.....	.....	82 to 92
" " " " " moderately rammed.....	.....	90 to 100
" " " slightly moist, loose.....	.....	70 to 76
" " " more moist, "	.....	66 to 68
" " " " shaken.....	.....	75 to 90
" " " " moderately packed.....	.....	90 to 100
" " " as a soft flowing mud.....	.....	104 to 112
" " " as a soft mud, well pressed into a box.....	.....	110 to 120
Ether.....	.716	44.6
Elm, perfectly dry. (See footnote, p 383.).....average..	.56	35.
1000 ft board measure weighs 1.302 tons.		
Ebony, dry..... " ..	1.22	76.1
Emerald, 2.63 to 2.76..... " ..	2.7	
Fat..... " ..	.93	58.
Flint..... " ..	2.6	162.
Feldspar, 2.5 to 2.8..... " ..	2.65	166.
Garnet, 3.5 to 4.3; Precious, 4.1 to 4.3..... " ..	4.2	
Glass, 2.5 to 3.45..... " ..	2.98	186.
" common window..... " ..	2.52	157.
" Millville, New Jersey. Thick flooring glass..... " ..	2.53	158.
Granite. 2.56 to 2.88. See Limestone, 160 to 180..... " ..	2.72	170.
Gneiss, common, 2.62 to 2.76..... " ..	2.69	168.
" in loose piles..... " ..	.....	96.
" Hornblendic..... " ..	2.8	175.
" " quarried, in loose piles..... " ..	.....	100.
Gypsum, Plaster of Paris, 2.24 to 2.30..... " ..	2.27	141.6
" in irregular lumps..... " ..	.....	82.
" ground. loose, per struck bushel, 70..... " ..	.....	56.
" " well shaken. " " 80..... " ..	.....	64.
" " Calcined, loose, per struck bush, 65 to 75..... " ..	.....	52 to 60
Greenstone, trap, 2.8 to 3.2..... " ..	3.	187.
" " quarried, in loose piles..... " ..		107.
Gravel, about the same as sand, which see.		
Gold, cast, pure, or 24 carat..... " ..	19.258	1204.
" native, pure, 19.3 to 19.34..... " ..	19.32	1206.
" " frequently containing silver, 15.6 to 19.3..... " ..		
" pure, hammered, 19.4 to 19.6..... " ..	19.5	1217.
Gutta Percha..... " ..	.98	61.1
Hornblende, black, 3.1 to 3.4..... " ..	3.25	203.
Hydrogen Gas, is 14½ times lighter than air; and 16 times lighter than oxygen .....average..	.....	.00527
Hemlock, perfectly dry. (Footnote, p 383.)..... " ..	.4	25.
1000 feet board measure weighs .930 ton.		
Hickory, perfectly dry. (See footnote, p 383.)..... " ..	.85	53.
1000 feet board measure weighs 1.971 tons.		
Iron, cast, 6.9 to 7.4..... " ..	7.15	446.
" " usually assumed at..... " ..	7.21	450.
At 450 lbs. a cub inch weighs .2604 lb; 8601 6 cub inches a ton; and a lb = 3 8400 cub inches; cast-iron **gun metal**.....	7.48	467.

Table of specific gravities, and weights — (Continued.)

The specific gravity of any substance is = **its weight in grams per cubic centimetre.**	Average Sp Gr.	Average Wt of a Cub Ft. Lbs
Iron, wrought, 7.6 to 7.9; the purest has the greatest sp gr .....average..	7.77	485.
" large rolled bars..... " ..	7.6	474.
" " " usually assumed at..... " ..	7.69	480.
" sheet..... " ..	.....	485.
At 480 lbs, a cub inch weighs .2778 lb; and a lb = 3.6000 cub ins.		
Light iron indicates impurity.		
Ivory.....average..	1.82	114.
Ice, .917 to .922..... " ..	.92	57.4
India rubber..... " ..	.93	58.
Lignum vitæ, dry..... " ..	1.33	83.
Lard..... " ..	.95	59.3
Lead, of commerce, 11.30 to 11.47; either rolled or cast..... " ..	11.38	709.6
Limestones and Marbles, 2.4 to 2.86, 150 to 178.8.....	2.6	164.4
" " " ordinarily about.....	2.7	168.
" " " quarried in irregular fragments, 1 cub yard solid, makes about 1.9 cub yds perfectly loose; or about 1¾ yds piled. In this last case, .571 of the pile is solid; and the remaining .429 part of it is voids.....piled..	.....	96.
Lime, quick, of ordinary limestone and marbles 92 to 98 lbs per cub ft....	1.5	95.
" " either in small irregular lumps; or ground, loose 50 to 58....	.....	53.
In either case 1 *solid* measure makes about 1.8 meas loose; and then .555 of the mass is solid, and .445 is voids.		
To measure correctly, none of the lumps should exceed about ⅛ or $\frac{1}{10}$ of the smallest dimension of the vessel used for measuring.		
Lime, quick, ground, loose, per struck bushel 62 to 70 lbs	.....	53.
" " " well shaken, " " ....80 "	.....	64.
" " " thoroughly shaken, " ....93¾ "	.....	75.
Mahogany, Spanish, dry *.....average..	.85	53.
" Honduras, dry..... " ..	.56	35.
Maple, dry*..... " ..	.79	49.
Marbles, see Limestones.		
Masonry, of granite or limestones, well dressed throughout.....	.....	165.
" " " well-scabbled mortar rubble. About $\frac{1}{5}$ of the mass will be mortar.....	.....	154.
" " " well-scabbled dry rubble.....	.....	138.
" " " roughly scabbled mortar rubble. About ¼ to ⅓ part will be mortar.....	.. ..	150.
" " " roughly scabbled dry rubble.....	.....	125.
At 155 lbs per cub ft, a cub yard weighs 1.868 tons; and 14.45 cub ft, 1 ton.		
Masonry of sandstone; about ⅛ part less than the foregoing.		
" " brickwork, pressed brick, fine joints .....average..	.....	140.
" " " medium quality ..... " ..	.....	125.
" " " coarse; inferior soft bricks ..... " ..	.....	100.
At 125 lbs per cub ft, a cub yard weighs 1.507 tons; and 17.92 cub ft. 1 ton.		
Mercury, at 32° Fah.....	13.62	849.
" 60° "	13.58	846.
" 212° "	13.38	836.
Mica, 2.75 to 3.1	2.93	183.
Mortar, hardened, 1.4 to 1.9.....	1.65	103.
Mud, dry, close.....	.....	80 to 110
" wet, moderately pressed.....	....	110 to 130
" wet, fluid	.....	104 to 120
Naphtha.....	.848	52.9
Nitrogen Gas is about $\frac{1}{35}$ part lighter than air.....	.....	.0744
Oak, live, perfectly dry, .88 to 1.02 * .....average..	.95	59.3
" white, " " .66 to .88..... " ..	.77	48.
" red, black, &c*..... " ..	....	32 to 45
Oils, whale; olive..... " ..	.92	57.3
" of turpentine..... " ..	.87	54.3
Oolites, or Roestones, 1.9 to 2.5..... " ..	2.2	137.
Oxygen Gas, a little more than $\frac{1}{10}$ part heavier than air.....	.00136	.0846
Petroleum	.878	54.8
Peat, dry, unpressed.....	.....	20 to 30
Pine, white, perfectly dry, .35 to .45*.....	.40	25.
1000 ft board measure weighs .930 ton.*		
" yellow, Northern, .48 to .62.....	.55	34.3
1000 ft board measure weighs 1.276 tons.*		
" " Southern, .64 to .80.....	.72	45.
1000 ft board measure weighs 1.674 tons.*		

* **Green timbers** usually weigh from one-fifth to nearly one-half more than dry; and ordinary building timbers when tolerably seasoned about one-sixth more than perfectly dry

Table of specific gravities, and weights—(Continued.)

The specific gravity of any substance is = **its weight in grams per cubic centimetre.**	Average Sp Gr.	Average Wt of a Cub Ft. Lbs.
Pine, heart of long-leafed Southern yellow, unseas. (Footnote, p 383.)... 1000 ft board measure weighs 2.418 tons.	1.04	65.
Pitch	1.15	71.7
Plaster of Paris; see Gypsum.		
Powder, slightly shaken	1.	62.3
Porphyry, 2.66 to 2.8	2.73	170.
Platinum..... 21 to 22	21.5	1342.
" uative, in graius..... 16 to 19	17.5	
Quartz, common, pure..... 2.64 to 2.67	2.65	165.
" " finely pulverized, loose		90.
" " " " well shaken		105.
" " " " well packed		112.
" quarried, loose. One measure solid, makes full 1¾ broken and piled		94.
Ruby and Sapphire, 3.8 to 4.0	3.9	
Rosin	1.1	68.6
Salt, coarse, per struck bushel; Syracuse, N. York..... 56 lbs ..		45.
" " " " " Turk's Islaud; Cadiz; Lisbon. 76 to 80 ..		62.
" " " " " St. Barts ..... 84 to 90 ..		70.
" " " " " some well-dried West India.... 90 to 96 ..		74.
" " " " " Liverpool ..... 50 to 55 ..		42.
" Liverpool fine, for table use..... 60 to 62 ..		49.
Sand, of pure quartz, perfectly dried, and loose, usually 112 to 133 lbs per struck bushel	*	90 to 106
At the average of 98 lbs per cub ft, a struck bushel weighs 122½ lbs; and 18.29 bushels, 1 ton; a cub yd = 1.181 tons; 22.86 cub ft. 1 ton. Slight shaking compacts it about 2 to 3 per ct; and ramming about 12 per ct when dry.		
" perfectly wet, voids full of water		118 to 129
" " " at the mean of 124 lbs, a cub yard weighs 1.495 tons; and 18.06 cubic feet = 1 ton.		
" sharp angular sand of pure quartz with very large and very small grains dry may weigh		117.
If any ordinary pure natural sand be sifted into 2 or 3 or more parcels of differently sized grains, a measure of any of these parcels will weigh considerably less than an equal measure of the original sand. Thus, a sand weighing 98 lbs per cub foot, may give others weighing not more than 70 to 80 lbs. At 98 lbs per cub ft. 1 bulk of pure quartz, has made 1.68 bulks of sand; of which the solid occupies .6; and the voids .4. But if this same sand be compacted to 110 lbs per cub ft, then 1 measure of solid quartz makes 1½ measures of sand; of which ⅔ are solid, and ⅓ voids. Sand is very retentive of moisture; and when in large bulks, is rarely as dry as that above in this table. But with its natural moisture, and loose, it is lighter than when dry, its average weight then not exceeding about 85 to 90 lbs per cub ft; or 106¼ to 112½ lbs per struck bushel. See Voids in Sand, p 678.		
Sandstones, fit for building, dry, 2.1 to 2.73, ...131 to 171.	2.41	151.
" quarried, and piled, 1 measure solid, makes about 1¾ piled...	.....	86.
Serpentines, good..... 2.5 to 2.65	2.6	162.
Snow, fresh fallen		5 to 12
" moistened, and compacted by rain		15 to 50
Sycamore, perfectly dry. (See footnote, p 383.) 1000 ft board measure weighs 1.376 tons.	.59	37.
Shales, red or black..... 2.4 to 2.8 ..... average..	2.6	162.
" quarried, in piles ..... " ..	.....	92.
Slate..... 2.7 to 2.9 ..... " ..	2.8	175.
Silver ..... " ..	10.5	655.
Soapstone, or Steatite..... 2.65 to 2.8 ..... " ..	2.73	170.
Steel, 7.7 to 7.9. The heaviest contains least carbon ..... " ..	7.85	490.
Steel is not heavier than the iron from which it is made; unless the iron had impurities which were expelled during its conversion into steel.		
Sulphur ....., average..	2.	125.
Spruce, perfectly dry. Footnote, p 383. ..... " .. 1000 ft board measure weighs .930 ton.	.4	25.
Spelter, or Zinc..... 6.8 to 7.2. ..... " ..	7.00	437.5
Sapphire; and Ruby, 3.8 to 4..... " ..	3.9	
Tallow ..... " ..	.94	58.6
Tar ..... " ..	1.	62.4
Trap, compact, 2.8 to 3.2 ..... " ..	3.	187.
" quarried; in piles..... " ..	.....	107.
Topaz. 3.45 to 3.65..... " ..	3.55	

* The sp gr of pure quartz sand, found as directed near foot of p 380, is of course the same as that of pure quartz, or 2.65. But a cub ft of dry sand weighs, as above, from 90 to 106 lbs, or only from 1.44 to 1.7 times as much as an equal quantity of water. Most authorities give about 1.5 as the sp gr. See first paragraph, p 381.

Table of specific gravities, and weights — (Continued.)

The specific gravity of any substance is = **its weight in grams per cubic centimetre.**	Average Sp Gr.	Average Wt of a Cub Ft. Lbs.
Tin, cast, 7.2 to 7.5 ..... average..	7.35	459.
Turf, or Peat, dry, unpressed.....	.....	20 to 30
Water, pure rain, or distilled, at 32° Fah. Barom 30 ins.....	.....	62.417
" " " " " 62° " " " "	1.	62.355
" " " " " 212° " " " "	.....	59.7
At 60°, a cub inch weighs .03607 lb; or .57712 oz avoir. And a lb contains 27.724 cub ins; equal to a cube of 3.0263 inches on each edge.		
Water, sea, 1.026 to 1.030 ..... average ..	1.028	64.08
Although the wt of fresh water is generally assumed as sixty two and one-third lbs per cub ft, yet 62¼ would be nearer the truth, at ordinary temperatures of about 70°; or a lb = 27.759 cub ins; and a cub in = .5764 oz avoir; or .4323 oz troy; or 252.175 grains. The grain is the same in troy, avoir, and apoth.		
Wax, bees .....average..	.97	60.5
Wines, .993 to 1.04 ..... " ..	.998	62.3
Walnut, black, perfectly dry. (See footnote, p 383.) ..... " ..	.61	38.
1000 ft board measure weighs 1.414 tons.		
Zinc, or Spelter, 6.8 to 7.2 ..... " ..	7.00	437.5
Zircon, 4.0 to 4.9..... " ..	4.45	

WEIGHTS AND MEASURES.

United States and British measures of length and weight, of the same denomination, may, *for all ordinary purposes*, be considered as equal; but the **liquid and dry** measures of the same denomination differ widely in the two countries. **The standard measure of length** of both countries is theoretically that of a pendulum vibrating seconds at the level of the sea, in the latitude of London, in a vacuum, with Fahrenheit's thermometer at 62°. The length of such a pendulum is supposed to be divided into 39.1393 equal parts, called inches; and 36 of these inches were adopted as the standard yard of both countries. But the Parliamentary standard having been destroyed by fire, in 1834, it was found to be impossible to restore it by measurement of a pendulum; and the present British standard yard is, in consequence, shorter than that of the U. S. by the *latest* comparison, about 1 part in 40,000, or .03 inch in 100 feet; or 1.584 inches in a mile. But at a temperature of 62°.25 Fah for the British standard and 59°.62 for the U. S. one, the two are of the same length, and on this basis the U. S. government declares the measures of the two countries to be the same; as in our tables.

Troy Weight.

U. S. and British.

24 grains..... 1 pennyweight, dwt.
20 pennyweights ..... 1 ounce = 480 grains.
12 ounces..... ..... 1 pound = 240 dwts = 5760 grains.

Troy weight is used for gold and silver.
A carat of the jewellers, for precious stones, is in the U. S. = 3.2 grs; in London, 3.17 grs; in Paris, 3.18 grs, divided into 4 jewellers' grs. **In troy, apothecaries', and avoirdupois, the grain is the same.**

Approximate Values of Foreign Coins, in U. S. Money.

The **references** ([1], [2], [3], and [4]) are to foot-notes on next page.

From Circular of U. S. Treasury Department, Bureau of the Mint, Jan. 1, 1887; from "Question Monétaire," by H. Costes, Paris, 1884; and from our 10th edition.

Argentine Repub.—Peso = 100 Centavos, 96.5 cts.[2 3] Argentino = 5 Pesos, $4.82.
Austria.—Florin = 100 Kreutzer, 47.7 cts.,[2] 35.9 cts.[3] Ducat, $2.29. Maria Theresa Thaler, or Levantin, 1780, $1.00.[2] Rix Thaler, 97 cts.[4] Souverain, $3.57.[4]
Belgium.[1]—Franc = 100 centimes, 17.9 cts.,[2] 19.3 cts.[3]
Bolivia.—Boliviano = 100 Centavos, 96.5 cts.,[2] 72.7 cts.[3] Once, $14.95. Dollar, 96 cts.[4]
Brazil.—Milreis = 1000 Reis, 50.2 cts.,[2] 54.6 cts.[3]
Canada.—English and U. S. coins. Also Pound, $4.[4]
Central America.[4]—Doubloon, $14.50 to $15.65. Reale, average 5¾ cts. See Honduras.
Ceylon.—Rupee, same as India.

(**Foreign Coins** *Continued.* **Small figures** [1, 2, 3, 4] *refer to foot notes.*)

Chili.—Peso = 10 Dineros or Decimos = 100 Centavos, 96.5 cts.,[2] 91.2 cts.[3] Condor = 2 Doubloons = 5 Escudos = 10 Pesos. Dollar, 93 cts.[4]

Cuba.—Peso, 93.2 cts.[3] Doubloon, $5.02.

Denmark.—Crown = 100 Ore, 25.7 cts.,[2] 26.8 cts.[3] Ducat, $1.81.[4] Skilling, ¾ ct.[4]

Ecuador.—Sucre, 72.7 cts.[3] Doubloon, $3.86. Condor, $9.65. Dollar, 93 cts.[4] Reale, 9 cts.[4]

Egypt.—Pound = 100 Piastres = 4000 Paras, $4 94,3.[3]

Finland.—Markka = 100 Penni, 19.1 cts.[2] 10 Markkaa, $1.93.

France.[1]—Franc = 100 Centimes, 17.9 cts.,[2] 19.3 cts.[3] Napoleon, $3.84.[4] Livre, 18.5 cts.[4] Sous, 1 ct.[4]

Germany.—Mark = 100 Pfennigs, 21.4 cts.,[2] 23.8 cts.[3] Augustus (Saxony), $3.98.[4] Carolin (Bavaria), $4.93.[4] Crown (Baden, Bavaria, N. Germany), $1.06.[4] Ducat (Hamburg, Hanover), $2.28.[4] Florin (Prussia, Hanover), 55 cts.[4] Groschen, 2.4 cts.[4] Krentzer (Prussia), .7 ct. Maximilian (Bavaria), $3.30.[4] Rix Thaler (Hamburg, Hanover), $1.10[4] (Baden, Brunswick), $1.00[4] (Prussia, N. Germany, Bremen, Saxony, Hanover), 69 cts.[4]

Great Britain.—Pound Sterling or Sovereign (£) = 20 Shillings = 240 Pence, $4.86.65.[3] Guinea = 21 Shillings. Crown = 5 Shillings. Shilling (*s*), 22.4 cts.,[2] 24.3 cts. ($\frac{1}{20}$ pound sterling). Penny (*d*), 2 cts.

Greece.[1]—Drachma = 100 Lepta, 17 cts.,[2] 19.3 cts.[3]

Hayti.—Gourde of 100 cents, 96.5 cts.[2 3]

Honduras.—Dollar or Piastre of 100 cents, $1.01. See Central America.

India.—Rupee = 16 Annas, 45.9 cts.,[2] 34.6 cts.[3] Mohur = 15 Rupees, $7.10. Star Pagoda (Madras), $1.81.[4]

Italy, etc.[1]—Lira = 100 Centesimi, 17.9 cts.,[2] 19.3 cts.[3] Carlin (Sardinia), $8.21.[4] Crown (Sicily), 96 cts.[4] Livre (Sardinia), 18.5 cts.[4] (Tuscany, Venice), 16 cts.[4] Ounce (Sicily), $2.50.[4] Paolo (Rome), 10 cts.[4] Pistola (Rome). $3.37.[4] Scudo[4] (Piedmont), $1.36 (Genoa), $1 28 (Rome), $1.00 (Naples, Sicily), 95 cts. (Sardinia), 92 cts. Teston (Rome). 30 cts.[4] Zecchino (Rome), $2.27.[4]

Japan.—Yen = 100 Sen (gold), 99.7 cts.[3] (silver), $1.04[2], 78.4 cts.[3]

Liberia.—Dollar, $1.00.[3 4]

Mexico.—Dollar, Peso, or Piastre = 100 Centavos (gold), 98.3 cts. (silver), $1.05,[2] 79 cts.[3] Once or Doubloon = 16 Pesos, $15.74.

Netherlands.—Florin of 100 cents, 40.5 cts.,[2] 40.2 cts.[3] Ducatoon, $1.32.[4] Guilder, 40 cts.[4] Rix Dollar, $1.05.[4] Stiver, 2 cts.[4]

New Granada.—Doubloon, $15.34.[4]

Norway.—Crown = 100 Ore = 30 Skillings, 25.7 cts.,[2] 26.8 cts.[3]

Paraguay.—Piastre = 8 Reals, 90 cts.

Persia.—Thoman = 5 Sachib-Kerans = 10 Banabats = 25 Abassis = 100 Scahis, $2.29.

Peru.—Sol = 10 Dineros = 100 Centavos, 96.5 cts.,[2] 72.7 cts.[3] Dollar, 93 cts.[4]

Portugal.—Milreis = 10 Testoons = 1000 Reis, $1.08.[3] Crown = 10 Milreis. Moidore, $6.50.[4]

Russia.—Rouble = 2 Poltinniks = 4 Tchetvertaks = 5 Abassis = 10 Griviniks = 20 Pietaks = 100 Kopecks, 77 cts.,[2] 58.2 cts.[3] Imperial = 10 Roubles, $7.72. Ducat = 3 Roubles, $2.39.

Sandwich Islands.—Dollar, $1.00.[4]

Sicily.—See Italy.

Spain.—Peseta or Pistareen = 100 Centimes, 17.9 cts.,[2] 19.3 cts.[3] Doubloon (new) = 10 Escudos = 100 Reals. $5.02. Duro = 2 Escudos,[4] $1.00.[2] Doubloon (old), $15.65.[4] Pistole = 2 Crowns, $3.90.[4] Piastre, $1.04.[4] Reale Plate, 10 cts.[4] Reale vellon, 5 cts.[4]

Sweden.—Crown = 100 Ore, 25.7 cts.,[2] 26.8 cts.[3] Ducat, $2.20.[4] Rix Dollar, $1.05.[4]

Switzerland.[1]—Franc = 100 Centimes, 17.9 cts.,[2] 19.3 cts.[3]

Tripoli.—Mahbub = 20 Piastres, 65.6 cts.[3]

Tunis.—Piastre = 16 Karobs, 12 cts.[2] 10 Piastres, $1.16.6.

Turkey.—Piastre = 40 Paras, 4.4 cts.[3] Zecchin, $1.40.[4]

United States of Colombia.—Peso = 10 Dineros or Decimos = 100 Centavos, 96.5 cts.,[2] 72.7 cts.[3] Condor = 10 Pesos, $9.65. Dollar, 93 5 cts.[4]

Uruguay.—Peso = 100 Centavos or Centesimos (gold), $1.03 (silver), 96.5 cts.[2]

Venezuela.—Bolivar = 2 Decimos, 17.9 cts.,[2] 19.3 cts.[3] Venezolano = 5 Bolivars.

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[1] France, Belgium, Italy, Switzerland, and Greece form the Latin Union. Their coins are alike in diameter, weight, and fineness.

[2] = 19.3 times the value of a single coin in francs as given by Costes.

[3] Par of exchange, or equivalent value in terms of U.S. gold dollar.—Treasury Circular.

[4] From our 10th edition.

The U. S. gold dollar weighs 25.8 grs; and contains 23.22 grs of pure gold.
" " **10** " " 258 grs; " " 232.20 grs " "
" " **20** " " 516 grs; " " 464.40 grs " "

Perfectly pure gold is worth $1 per 23.22 grs = $20.67183 per troy oz = $18.84151 per avoir oz. **Standard** (U. S. coin) is worth $18.60465 per troy oz = $16.95736 per avoir oz. It consists of 9 parts by wt of pure gold, to 1 part alloy. Its value is that of the pure gold only; the cost of the alloy and of the coinage being borne by Gov. **A cub foot of pure gold** weighs about 1204 avoir lbs; and is worth $362963. A cub inch weighs about 11.148 avoir oz; and is worth $210.04.

Pure gold is called fine, or 24 carat gold; and when alloyed, the alloy is supposed to be divided into 24 parts by wt, and according as 10, 15, or 20, &c, of these parts are pure gold, the alloy is said to be 10. 15. or 20, &c, carat.

Pure silver fluctuates in value; thus during 1878. 1879, it ranged between $1.05 and $1.18 per troy oz. or $.957 and $1.076 per avoir oz. A cub inch weighs about 5.528 troy, or 6.065 avoir ounces. **The U. S. silver dollar** weighs 412.5 grs troy; but its subdivisions weigh at the rate of about 8 per ct less. All consist of 9 parts silver to 1 part alloy.

The average fineness of California native gold, by some thousands of assays at the U. S. mint in Philada, is 88.5 parts gold, 11.5 silver. Some from Georgia, 99 per ct gold.

Apothecaries' Weight.

U. S. and British.

20 grains	1 scruple.
3 scruples	1 dram = 60 grs.
8 drams	1 ounce = 24 scruples = 480 grs.
12 ounces	1 pound = 96 drams = 288 scruples = 5760 grains.

In troy and apoth weights, the grain, ounce, and pound are the same.

Avoirdupois, or Commercial Weight.

U. S. and British.

27.34375 grains	1 dram.
16 drams	1 ounce = 437½ grains.
16 ounces	1 pound = 256 drams = 7000 grains.
28 pounds	1 quarter = 448 ounces.
4 quarters	1 hundredweight = 112 lbs.
20 hundredweights	1 ton = 80 quarters = 2240 lbs.

A stone = 14 pounds. **A quintal** = 100 pounds avoir.

The standard of the avoir pound, which is the one in common commercial use, is the weight of 27.7015 cub ins of pure distilled water, at its maximum density at about 39°.2 Fahr, in latitude of London, at the level of the sea; barom at 30 ins. **But this involves an error** of about 1 part in 1362, for the 1 lb of water = 27.68122 cub ins.

A troy lb = .82286 avoir lb. An avoir lb = 1.21528 troy lb, or apoth.

A troy oz = 1.09714 avoir oz. An avoir oz = .911458 troy oz. or apoth.

Long Measure.

U. S. and British.

By law, the U. S. standards of length, as well as of weight, are made the same as the British

12 inches	1 foot = .3047973 metre.
3 feet	1 yard = 36 ins = .9143919 metre.
5½ yards	1 rod, pole, or perch = 16½ feet = 198 ins.
40 rods	1 furlong = 220 yards = 660 feet.
8 furlongs	1 statute, or land mile = 320 rods = 1760 yds = 5280 ft = 63360 ins.
3 miles	1 league = 24 furlongs = 960 rods = 5280 yds = 15840 ft.

A point = $\frac{1}{72}$ inch. **A line** = 6 points = $\frac{1}{12}$ inch. **A palm** = 3 ins. **A hand** = 4 ins. **A span** = 9 ins. **A fathom** = 6 feet. **A cable's length** = 120 fathoms = 720 feet. **A Gunter's surveying chain** is 66 feet, or 4 rods long. It is divided into 100 links of 7.92 ins long. 80 Gunter's chains = 1 mile.

A nautical mile, geographical mile, sea mile, or knot, is variously defined as being = the length of

	metres	feet	statute miles
1 min of longitude at the equator =	1855.345	6087.15	1.15287
1 " latitude " " =	1842.787	6045.95	1.14507
1 " " " pole =	1861.655	6107.85	1.15679
1 " " at lat 45° =	1852.181	6076.76	1.15090
1 " a great circle of a *true sphere* whose *surface area* is equal to that of the earth =	value adopted by **U. S. Coast and Geodetic Survey** 1853.248	6080.27	1.15157
British **Admiralty knot** =	1853.169	6080.00	1.15152

The above lengths of minutes, in metres and feet, are those published by the U. S. Coast and Geodetic Survey in Appendix No 12, Report for 1881, and are calculated from Clarke's spheroid, which is now the standard of that Survey.

At the equator 1° of lat = 68.70 land miles; at lat 20° = 68.78; at 40° = 69.00; at 60° = 69.23, at 80° = 69.39; at 90° = 69.41.

Lengths of a Degree of Longitude in different Latitudes, and at the level of the Sea.

These lengths are in common land or statute miles, of 5280 ft. Since the figure of the earth has never been *precisely* ascertained, these are but close approximations. Intermediate ones may be found correctly by simple proportion. 1° of longitude corresponds to 4 mins of civil or clock time; 1 min of longitude to 4 secs of time.

Deg of Lat.	Miles.	Deg of Lat.	Miles.	Deg of Lat.	Miles.	Deg of Lat.	Miles.	Deg of Lat.	Miles.	Deg of Lat.	Miles.
0	69.16	14	67.12	28	61.11	42	51.47	56	38.76	70	23.72
2	69.12	16	66.50	30	59.94	44	49.83	58	36.74	72	21.43
4	68.99	18	65.80	32	58.70	46	48.12	60	34.67	74	19.12
6	68.78	20	65.02	34	57.39	48	46.36	62	32.55	76	16.78
8	68.49	22	64.15	36	56.01	50	44.54	64	30.40	78	14.42
10	68.12	24	63.21	38	54.56	52	42.67	66	28.21	80	12.05
12	67.66	26	62.20	40	53.05	54	40.74	68	25.98	82	9.66

See p 34.

Inches reduced to Decimals of a Foot.

No errors.

Ins.	Foot.	Ins.	Foot.	Ins.	Foot.	Ins.	Foot.	Ins.	Foot.	Ins.	Foot.
0	.0000	**2**	.1667	**4**	.3333	**6**	.5000	**8**	.6667	**10**	.8333
1-32	.0026		.1693		.3359		.5026		.6693		.8359
1-16	.0052		.1719		.3385		.5052		.6719		.8385
3-32	.0078		.1745		.3411		.5078		.6745		.8411
1/8	.0104	1/8	.1771	1/8	.3438	1/8	.5104	1/8	.6771	1/8	.8438
5-32	.0130		.1797		.3464		.5130		.6797		.8464
3-16	.0156		.1823		.3490		.5156		.6823		.8490
7-32	.0182		.1849		.3516		.5182		.6849		.8516
1/4	.0208	1/4	.1875	1/4	.3542	1/4	.5208	1/4	.6875	1/4	.8542
9-32	.0234		.1901		.3568		.5234		.6901		.8568
5-16	.0260		.1927		.3594		.5260		.6927		.8594
11-32	.0286		.1953		.3620		.5286		.6953		.8620
3/8	.0313	3/8	.1979	3/8	.3646	3/8	.5313	3/8	.6979	3/8	.8646
13-32	.0339		.2005		.3672		.5339		.7005		.8672
7-16	.0365		.2031		.3698		.5365		.7031		.8698
15-32	.0391		.2057		.3724		.5391		.7057		.8724
1/2	.0417	1/2	.2083	1/2	.3750	1/2	.5417	1/2	.7083	1/2	.8750
17-32	.0443		.2109		.3776		.5443		.7109		.8776
9-16	.0469		.2135		.3802		.5469		.7135		.8802
19-32	.0495		.2161		.3828		.5495		.7161		.8828
5/8	.0521	5/8	.2188	5/8	.3854	5/8	.5521	5/8	.7188	5/8	.8854
21-32	.0547		.2214		.3880		.5547		.7214		.8880
11-16	.0573		.2240		.3906		.5573		.7210		.8906
23-32	.0599		.2266		.3932		.5599		.7266		.8932
3/4	.0625	3/4	.2292	3/4	.3958	3/4	.5625	3/4	.7292	3/4	.8958
25-32	.0651		.2318		.3984		.5651		.7318		.8984
13-16	.0677		.2344		.4010		.5677		.7344		.9010
27-32	.0703		.2370		.4036		.5703		.7370		.9036
7/8	.0729	7/8	.2396	7/8	.4063	7/8	.5729	7/8	.7396	7/8	.9063
29-32	.0755		.2422		.4089		.5755		.7422		.9089
15-16	.0781		.2448		.4115		.5781		.7448		.9115
31-32	.0807		.2474		.4141		.5807		.7474		.9141
1	.0833	**3**	.2500	**5**	.4167	**7**	.5833	**9**	.7500	**11**	.9167
	.0859		.2526		.4193		.5859		.7526		.9193
	.0885		.2552		.4219		.5885		.7552		.9219
	.0911		.2578		.4245		.5911		.7578		.9245
1/8	.0938	1/8	.2604	1/8	.4271	1/8	.5938	1/8	.7604	1/8	.9271
	.0964		.2630		.4297		.5964		.7630		.9297
	.0990		.2656		.4323		.5990		.7656		.9323
	.1016		.2682		.4349		.6016		.7682		.9349
1/4	.1042	1/4	.2708	1/4	.4375	1/4	.6042	1/4	.7708	1/4	.9375
	.1068		.2734		.4401		.6068		.7734		.9401
	.1094		.2760		.4427		.6094		.7760		.9427
	.1120		.2786		.4453		.6120		.7786		.9453
3/8	.1146	3/8	.2813	3/8	.4479	3/8	.6146	3/8	.7813	3/8	.9479
	.1172		.2839		.4505		.6172		.7839		.9505
	.1198		.2865		.4531		.6198		.7865		.9531
	.1224		.2891		.4557		.6224		.7891		.9557
1/2	.1250	1/2	.2917	1/2	.4583	1/2	.6250	1/2	.7917	1/2	.9583
	.1276		.2943		.4609		.6276		.7943		.9609
	.1302		.2969		.4635		.6302		.7969		.9635
	.1328		.2995		.4661		.6328		.7995		.9661
5/8	.1354	5/8	.3021	5/8	.4688	5/8	.6354	5/8	.8021	5/8	.9688
	.1380		.3047		.4714		.6380		.8047		.9714
	.1406		.3073		.4740		.6406		.8073		.9740
	.1432		.3099		.4766		.6432		.8099		.9766
3/4	.1458	3/4	.3125	3/4	.4792	3/4	.6458	3/4	.8125	3/4	.9792
	.1484		.3151		.4818		.6484		.8151		.9818
	.1510		.3177		.4844		.6510		.8177		.9844
	.1536		.3203		.4870		.6536		.8203		.9870
7/8	.1563	7/8	.3229	7/8	.4896	7/8	.6563	7/8	.8229	7/8	.9896
	.1589		.3255		.4922		.6589		.8255		.9922
	.1615		.3281		.4948		.6615		.8281		.9948
	.1641		.3307		.4974		.6641		.8307		.9974

Square, or Land Measure.

U. S. and British.

144 square inches..... 1 sq foot. 100 sq ft = **1 square.**
9 sq feet..... 1 sq yard = 1296 sq ins.
30¼ sq yards..... 1 sq rod = 272¼ sq feet.
40 sq rods..... 1 rood = 1210 sq yds = 10890 sq feet.
4 roods..... 1 acre = 160 rods = 4840 sq yds = 43560 sq feet.

A section of land is 1 mile sq, or 27878400 sq ft; or 3097600 sq yds; or 640 acres. **An acre** contains 10 sq Gunter's chains. **A sq acre** is 208.710 feet; a sq half acre, 147.581 ft; and a sq quarter acre, 104.355 ft on each side. **A circular acre** is 235.504 feet: a circular half acre = 166.527 ft; and a circular quarter acre = 117.752 ft diam. **A circular inch** is a circle of 1 inch diam; a sq ft = 183.346 cir ins. **Also 1 sq inch** = 1.27324 cir ins; and 1 cir inch = .7854 of a sq inch.

Cubic, or Solid Measure.

U. S. and British.

1728 cubic inches..... 1 cubic, or solid foot.
27 cubic feet..... 1 cubic, or solid yard.

A cord of wood = 128 cub ft; being 4 ft × 4 ft × 8 ft. **A perch of masonry** actually contains 24¾ cub ft; being 16½ ft × 1½ ft × 1 ft. It is generally taken at 25 cub ft; but by some at 22, &c; and there is every probability that a payer will be cheated unless the number of cubic ft be distinctly agreed upon in his contract. It is gradually falling into disuse among engineers; and the cub yd is very properly taking its place. **To reduce cub yds** to perches of 25 cub ft, mult by 1.080; and to reduce perches to cub yds, mult by .926. **The Brit rod** of brickwork, of house-builders, is 16½ feet square, by 14 inches (1½ English bricks) thick = 272¼ sq ft of 14 inch wall. It is conventionally taken at 272 sq ft; which gives 317⅛ cub ft. In Brit engineering works the rod is 306 cub ft, or 11⅓ cub yds. The Montreal, (Canada,) **toise** = 261½ cub ft; or 9.6852 cub yds, or 10.46 perches of 25 cub ft. The Canadian **chaldron** = 58.64 cub ft. **A ton** (2240 lbs) of Pennsylvania anthracite, when broken for domestic use, occupies from 41 to 43 cub ft of space; the mean of which is equal to 1.556 cub yds: or a cube of 3.476 ft on each edge. Bituminous coal 44 to 48 cub ft; mean equal to 1.704 cub yd; or a cube of 3.583 ft on each edge. Coke 80 cub ft.

A cubic foot is equal to

1728 cub ins, or 3300.23 spherical ins.
.037037 cub yard, or 1.90985 spherical ft.
.002832 myriolitre, or decastere.
.028316 kilolitre, or cubic metre, or stere.
.283161 hectolitre, or decistere.
2.83161 decalitres, or centisteres.
28.3161 litres, or cub decimetres.
283.161 decilitres.
2831.61 centilitres.
28316.1 millilitres, or cub centimetres.
.803564 U.S. struck bushel of 2150.42 cub ins, or 1.24445 cub ft.
.779013 Brit bushel of 2218.191 cub ins, or 1.28368 cub ft.
3.21426 U. S. pecks.
3.11605 Brit pecks.
7.48052 U. S. liquid galls of 231 cub ins.
6.42851 U. S. dry galls.
6.23210 Brit galls of 277.274 cub ins.
29.92208 U. S. liquid quarts.
25.71405 U. S. dry quarts.
24.92842 Brit quarts.
59.84416 U. S. liquid pints.
51.42809 U. S. dry pints.
49.85684 Brit pints.
239.37662 U. S. gills.
199.42737 Brit gills.
.26667 flour barrel of 3 struck bushels.
.23748 U. S. liquid barrel of 31½ galls.

A cubic inch is equal to

16.38663 millilitres; or 1.638663 centilitres; or .1638663 decilitre; or .01638663 litre; or to .0005787 cub ft; or to .138528 U. S. gill; or 1.90985 spherical ins.

A cubic yard is equal to

27 cub feet, or to 201.974 U. S. galls.
46656 cub ins.
.0764534 myriolitre.
.764534 kilolitre, or cub metre.
7.64534 hectolitres.
7.2 flour barrels of 3 struck bushels.
76.4534 decalitres.
764.534 litres, or cub decimetres.
7645.34 decilitres.
21.69623 U. S. bushels (struck).
21.03336 Brit bushels.

A sphere 1 foot in diameter, contains

.01939 cub yard.
.5236 cub foot.
904.781 cub inches.
.42075 U. S. bushel.
1.6830 U. S. pecks.
13.4639 U. S. dry quarts.
26.9278 U. S. dry pints.
3.9168 U. S. liquid gallons.
15.6672 U. S. liquid quarts.
31.3344 U. S. liquid pints.
125.3376 U. S. liquid gills.
3.2631 Brit imp gallons.
13.6525 Brit imp quarts.
26.1050 Brit imp pints.
104.4201 Brit imp gills.
14.8263 litres.
1.48263 decalitres.
.148263 hectolitres.

A sphere 1 inch in diameter, contains

.000303 cub foot.
.5236 cub inch.
.07253 U. S. gill.
.06043 Brit gill.
8.580 millilitre.
.8580 centilitre.
.08580 decilitre.

A cylinder 1 foot in diameter, and 1 foot high, contains

.02909 cub yard.
.7854 cub foot.
1357.1712 cub inches.
.63112 U. S. dry bushels.
2.5245 U. S. dry pecks.
20.1958 U. S. dry quarts.
40.3916 U. S. dry pints.
5.8752 U. S. liquid gallons.
23.5008 U. S. liquid quarts.
47.0016 U. S. liquid pints.
188.0064 U. S. liquid gills.
4.8947 Brit imp gallons.
19.5788 Brit imp quarts.
39.1575 Brit imp pints.
156.6302 Brit imp gills.
222.395 decilitres.
22.2395 litres.
2.22395 decalitres.
.222395 hectolitre.

A cylinder 1 inch in diameter, and 1 foot high, contains

.005454 cub foot.
9.4248 cub inches.
.2805 U. S. dry pint.
.3264 liquid pint.
1.3056 U. S. gill.
.2719 Brit imp pint.
1.0877 Brit imp gill.
15.4441 centilitres.
1.54441 decilitres.
.154441 litres.

For others, see below; also p 157.

Liquid Measure. U. S. only.

The basis of this measure in the U. S. is the old Brit wine gallon of 231 cub ins; or 8.33888 lbs avoir of pure water, at its max density of about 39°.2 Fahr; the barom at 30 ins. A cylinder 7 ins diam, and 6 ins high, contains 230.904 cub ins, or almost precisely a gallon; as does also a cube of 6.1358 ins on an edge. Also a gallon = .13368 of a cub ft; and a cub ft contains 7.48052 galls; nearly 7½ galls. **This basis however involves an error** of about 1 part in 1362, for the water actually weighs 8.345008 lbs.

cub ins.			
4 gills	1 pint	= 28.875.	
2 pints	1 quart	= 57.750 = 8 gills.	
4 quarts	1 gallon	= 231. = 8 pints = 32 gills.	

63 gallons	1 hogshead.
2 hogsheads	1 pipe, or butt.
2 pipes	1 tun.

In the U. S. and Great Brit. **1 barrel of wine** or brandy = 31½ galls: in Pennsylvania, a half barrel, 16 galls; a double barrel, 64 galls; **a puncheon,** 84 galls: **a tierce,** 42 galls. A liquid measure barrel of 31½ galls contains 4.211 cub ft = a cube of 1.615 ft on an edge; or 3.384 U. S. struck bushels. **A gill** = 7.21875 cub ins. **The following cylinders** contain some of these measures very approximately. For others, see above; also p 157.

	Diam.		Height.
cub ins.	Ins.		Ins.
Gill (7.21875)	1¾		3
½ pint	2¼		3⅝
Pint	3½		3
Quart	3½		6

	Diam.		Height.
	Ins.		Ins.
Gallon	7		6
2 gallons	7		12
8 gallons	14		12
10 gallons	14		15

To reduce U. S. liquid measures to Brit ones of the same denomination, divide by 1.20032; or near enough for common use, by 1.2; or to reduce Brit to U. S. multiply by 1.2.

Dry Measure.

U. S. only.

The basis of this is the old British Winchester struck bushel of 2150.42 cub ins; or 77.627413 pounds avoir of pure water at its max density. Its dimensions by law are 18½ ins inner diam; 19½ ins outer diam; and 8 ins deep; and when heaped, the cone is not to be less than 6 ins high; which makes a heaped bushel equal to 1¼ struck cones; or to 1.55556 cub ft.

	Edge of a cube of equal capacity.
2 pints 1 quart, = 67.2006 cub ins = 1.16365 liquid qt	4.066 ins.
4 quarts 1 gallon, = 8 pints, = 268.8025 cub ins, = 1.16365 liq gal	6.454 "
2 gallons 1 peck, = 16 pints, = 8 quarts, = 537.6050 cub ins	8.131 "
4 pecks 1 struck bushel, = 64 pints, = 32 quarts, = 8 gals, = 2150.4200 cub ins.	12.908 "

A struck bushel = 1.24445 cub ft. A cub ft = .80356 of a struck bushel. The dry **flour barrel** = 3.75 cub ft; = 3 struck bushels. The dry barrel is not, however, a legalized measure; and no great attention is given to its capacity; consequently, barrels vary considerably. A barrel of flour contains by law, 196 lbs. In ordering by the barrel, the amount of its contents should be specified in pounds or galls.

To reduce U. S. dry measures to Brit imp ones of the same name, div by 1.031516; and to reduce Brit ones to U. S. mult by 1.031516; or for common purposes use 1.032.

British Imperial Measure, both liquid and dry.

This system is established throughout Great Britain, to the exclusion of the old ones. Its basis is the imperial gallon of 277.274 cub ins, or 10 lbs avoir of pure water at the temp of 62° Fahr, when the barom is at 30 ins. **This basis involves an error** of about 1 part in 1836, for 10 lbs of the water = only 277.123 cub ins.

		Avoir lbs. of water.		Cub. ins.	Cub. ft.	Edge of a cube of equal capacity. Inches.
4 gills	1 pint	1.25		34.6592	.....	3.2605
2 pints	1 quart	2.50		69.3185	.....	4.1079
2 quarts	1 pottle	5.		138.637	.....	5.1756
2 pottles	1 gallon	10.		277.274	.....	6.5208
2 gallons	1 peck	20.		554.548	.....	8.2157
4 pecks	1 bushel	80.	Dry meas.	2218.192	1.2837	13.0417
4 bushels	1 coomb	320.		8872.768	5.1347	
2 coombs	1 quarter	640.		17745.536	10.2694	

The imp gall = .16046 cub ft; and 1 cub ft = 6.23210 galls. The imp gal = 1.20032, or very nearly $1\frac{1}{5}$ U. S. liquid galls.

The weight of water affords an easy way to find the cubic contents of a vessel. First weigh the vessel by itself; and then full of water. The diff will be the weight of the water; and this divided by 62.3 or by the number in the table opp the temp of water, will be the contents in cub ft.

To obtain the size of commercial measures by means of the weight of water.

At the common temperature of from 70° to 75° Fah, a cub foot of fresh water weighs very approximately $62\frac{1}{4}$ lbs avoir. A cubic half foot, (6 ins on each edge,) 7.78125 lbs. A cub quarter foot, (3 ins on each edge,) .97266 lb. A cub yard, 1680.75 lbs; or .75034 ton. A cub half yd, (18 ins on each edge,) 210.094 lbs; or .0938 ton. A cub inch, .036024 lb; or .576384 ounce; or 9.2222 drams; or 252.170 grains. An inch square, and one foot long, .432292 lb. Also 1 lb = 27.75903 cub ins, or a cube of 3.028 ins on an edge. An ounce, 1.735 cub ins; a ton, 35.984 cub ft, all near enough for common use.

Original.

Liquid Measures.

	Lbs Avoir. of Water.
U. S. Gill	.26005*
U. S. Pint	1.0402
U. S. Quart	2.0804
U. S. Gallon 8 lbs $5\frac{1}{7}$ oz	8.3216
U. S. Wine Barrel, 31½ Gall	262.1310

Dry Measures.

U. S. Pint	1.2104
U. S. Quart	2.4208
U. S. Gallon	9.6834
U. S. Peck	19.3668
U. S. Bushel, struck	77.4670

* Or 4 ounces; 2 drams; 15.6625 grs.

Liquid and Dry.

	Lbs Avoir. of Water.
British Imp Gill	.31214*
" " Pint	1.24858
" " Quart	2.49715
" " Gallon	9.9886
" " Peck	19.9772
" " Bushel	79.9088

* 4.9942; or very nearly 5 ounces.

French Measures.

Centilitre	.02198†
Decilitre	.2198‡
Litre	2.1981
Decalitre, or Centistere	21.9808
Metre, or Stere	2198.0786

† Or 5.6271 drams; or 153.866 grs.
‡ 3.5169 ounces.

METRIC WEIGHTS AND MEASURES.

The French Metre.

The French metre was intended to be the one ten-millionth part of the dist from either pole of the earth to the equator: but after it had been introduced into use, errors were discovered in the calculations employed for ascertaining that dist; so that the French metre, like the Brit standard yard, is not what it was intended to be.

The U. S. Govt adopts for its length 1.093623 yds = 3.280869 ft = 39.370432 ins U. S. or British measure. **But in ordinary business transactions** 39.37 ins are a legal metre. At 3 ft 3⅜ ins, the length is but 1 part in 8616 too great.

French Measures of Length.

By U. S. and British Standard.

	Ins.	Ft.	Yds.	Miles.
Millimetre*	.039370	.003281	.....	.....
Centimetre†	.39370428	.032809	.....	.....
Decimetre	3.9370428	.3280869	.1093623	.....
Metre‡	39.370428	3.280869	1.093623	.....
Decametre	393.70428	32.80869	10.93623	.....
Hectometre	Road	328.0869	109.3623	.0621375
Kilometre	measures.	3280.869	1093.623	.6213750
Myriametre		32808.69	10936.23	6.213750

* Nearly the $\frac{1}{25}$ part of an inch. † Full ⅜ inch.

‡ Very nearly 3 ft, 3⅜ ins, which is too long by only 1 part in 8616.

French Square Measure.

By U. S. and British Standard.

	Sq. Ins.	Sq. Feet.	Sq. Yds.	Acres.
Sq Millimetre	.001550	.00001076	.0000012	.....
Sq Centimetre	.155003	.00107641	.0001196	.....
Sq Decimetre	15.5003	.10764101	.0119601	.....
Sq Metre, or Centiare	1550.03	10.764101	1.19601	.000247
Sq Decametre, or Are	155003	1076.4101	119.6011	.024711
Decare (not used)	.....	10764.101	1196.011	.247110
Hectare	.....	107641.01	11960.11	2.47110
Sq Kilometre	3861090 **sq miles.**	10764101	1196011.	247.110
Sq Myriametre	38.61090 "	.....	.....	24711.0

French Cubic, or Solid Measure.

According to U. S. Standard.

Only those marked "Brit" are British.

	Cub Ins.		
Millilitre, or cub Centimetre	.0610254	Liquid. .0084537 gill. " .0070428 Brit gill. Dry. .0018162 dry pint.	
Centilitre	.610254	Liquid. .084537 gill. " .070428 Brit gill. Dry. .018162 dry pint.	
Decilitre	6.10254	Liquid. .84537 gill = .21134 pint. " .70428 Brit gill = .17607 Brit pint. Dry. .18162 dry pint.	
Litre, or cubic Decimetre	61.0254	Liquid. 1.05671 quart = 2.1134 pints. " .88036 Brit quart = 1.7607 Brit pints. Dry. .11351 peck = .9081 dry qt = 1.8162 dry pt.	
Decalitre, or Centistere	610.254	Liquid. 2.64179 U. S. liquid gal. " 2.20090 Brit gal. Dry. .283783 bush = 1.1351 peck = 9.081 dry qts.	
	Cub Ft.		
Hectolitre, or Decistere	.353156 3.53156	Liquid. 26.4179 U. S. liquid gal. " 22.0090 Brit gal. Dry. 2.83783 bush.	
Kilolitre, or Cubic Metre, or Stere	35.3156	Liquid. 264.179 U. S. liquid gal. " 220.090 Brit gal. Dry. 28.3783 bush.	Cub yds, 1.3080.
Myriolitre, or Decastere	353.156	Liquid. 2641.79 U. S. liquid gal. Dry. 283.783 bush.	Cub yds, 13.080.

French Weights, reduced to common Commercial or Avoir Weight, of 1 pound = 16 ounces, or 7000 grains.

	Grains.
Milligramme	.015432
Centigramme	.15432
Decigramme	1.5432
Gramme	15.432
By law a 5-cent nickel = 5 grammes	Pounds av.
Decagramme	.022046
Hectogramme	.22046
Kilogramme	2.2046
Myriogramme	22.046
Quintal*	220.46
Tonneau; Millier; or Tonne	2204.6

The gramme is the basis of French weights; and is the weight of a cub centimetre of distilled water at its max density, at sea level, in lat of Paris; barom 29.922 ins.

French Measures of the "Systeme Usuel."

This system was in use from about 1812 to 1840, when it was forbidden by law to use even its names. This was done in order to expedite the general use of the tables which we have before given. But as the Systeme Usuel appears in books published during the above interval, we add a table of some of its values.

Measures of Length.

	Yards.	Feet.	Inches.
Ligne usuel, or line	.....	.....	.09113
Pouce usuel, or inch, = 12 lignes	.....	.09113	1.09362
Pied usuel, or foot, = 12 pouces	.36454	1.09362	13.12344
Aune usuel, or ell	1.31236	3.93708	47.245
Toise usuel, = 6 pieds	2.18727	6.56181	78.74172

Weights, Usuel.		Cubic, or Solid, Usuel.	
Grain usuel	.8375 grains.	Litron usuel, or 1 litre	= 1.7608 British pint.
Gros usuel	60.297 "		
Once usuel	1.10258 avoir oz.		
Marc usuel	.55129 avoir lb.	Boisseau usuel	2.7512 British gals.
Livre usuel, or pound,	1.10258 avoir lb.		

Before 1812, or before the "Systeme usuel," the Old System, "Systeme Ancien," was in use.

French Measures of the "Systeme Ancien."

Lineal.	Square.			Cubic.		
	Sq. ins.	Sq. ft.	Sq. yds.	C. ins.	C. ft.	C. yds.
Point ancien, .0148 ins						
Ligne ancien, .0888 ins	.00789	.....	.....	.0007		
Pouce ancien, 1.06577 ins = .0888 ft	1.1359	.....	.....	1.2106		
Pied ancien, 12.7892 ins = 1.06577 ft	.....	1.1359	.....	.....	1.2106	
Aune ancien, 46.8939 ins = 3.90782 ft = 1.30261 yds						
Toise ancien, = 6.3946 ft = 2.1315 yds	.....	40.8908	4.5434	.....	261.482	9.6845
League = 2282 toises = 2.7637 miles						

There is, however, much confusion about these old measures. Different measures had the same name in different provinces.

* The *avoirdupois* quintal is 100 avoirdupois pounds.

Russian.

Foot; same as U. S. or British foot. **Sachine** = 7 feet. **Verst** = 500 sachine = 3500 feet = 1166⅔ yards = .6629 mile. **Pood** = 36.114 lbs avoirdupois.

Spanish.

The castellano of Spain and New Granada, for weighing gold, is variously estimated, from 71.07 to 71.04 grains. At 71.055 grains, (the mean between the two,) an avoirdupois, or common commercial ounce contains 6.1572 castellano; and a lb avoirdupois contains 98.515. Also a troy ounce = 6.7553 castellano; and a troy lb = 81.064 castellano. Three U. S. gold dollars weigh about 1.1 castellano.

The Spanish mark, or marco, for precious metals, in South America, may be taken in practice, as .5065 of a lb avoirdupois. In Spain, .5076 lb. In other parts of Europe, it has a great number of values; most of them, however, being between .5 and .54 of a pound avoirdupois. The .5065 of a lb = 3545½ grains; and .5076 lb = 3553.2 grains. 1 marco = 50 castellanos = 400 tomine = 4800 *Spanish gold*-grains.

The arroba has various values in different parts of Spain. That of Castile, or Madrid, is 25.4025 lbs avoirdupois; **the tonelada** of Castile = 2032.2 lbs avoirdupois; **the quintal** = 101.61 lbs avoirdupois; **the libra** = 1.0161 lbs avoirdupois; **the cantara** of wine, &c, of Castile = 4.263 U. S. gallons; that of Havana = 4.1 gallons.

The vara of Castile = 32.8748 inches, or almost precisely 32⅞ inches; or 2 feet 8⅞ inches. **The fanegada** of land since 1801 = 1.5871 acres = 69134.08 square feet. **The fanega** of corn, &c = 1.59914 U. S. struck bushels. In California, **the vara by law** = 33.372 U. S. inches; and **the legua** = 5000 varas; or 2.6335 U. S. miles.

Civil, or Common Clock Time.

60 thirds, marked ‴	1 second, marked ″.
60 seconds	1 minute ′.
60 minutes	1 hour, = 3600 sec.
24 hours	1 civil day, = 1440 min. = 86400 sec.
7 days	1 week, = 168 hours = 10080 min.
4 weeks	1 civil month, = 28 days = 672 hours.

For Standard Railway Time, see p 396.

13 civil months, (or 52 weeks,) 1 day, 5 hours, 48 min, 49$\frac{7}{10}$ sec; or 365 days, 5 hours, 48 min, 49$\frac{7}{10}$ sec, = 1 civil year. **A solar day** is the time between two successive solar noons, or transits of the sun over the meridian of a place. These intervals are not of equal lengths all the year round. The average length of all the solar days is called the **mean solar day;** and is the same as the common civil day of 24 hours of clock time. Civil noon is at 12 o'clock; but solar, or apparent noon, may be about 14½ min before; or 16¼ min after 12 of correct clock time. **A sidereal day** is the interval between two passages of the same star past the range of two fixed objects; and is the precise time reqd for one complete rev of the earth on its axis. The sidereal day never varies; but is always equal to 23 hours, 56 min, 4.09 sec;* so that a star will on any night appear to set, or to pass the range of any two fixed objects, 3 min, 55.91 sec earlier by the clock, than it did on the night before,† so that the number of sidereal days in a civil year is 1 greater than that of the civil days.

An astronomical day begins at noon, and its hours are counted from 0 to 24. In comparing it with the civil day, the last is supposed to begin at the midnight *before* the noon at which the first began.

Astronomers are now (1884-5) taking measures to make their "day" correspond with the civil day.

TABLE showing how much earlier a star passes a given range, on each succeeding night. — (Original.)

Nights.	Min.	Sec.	Nights.	H.	Min.	Sec.	Nights.	H.	Min.	Sec.
1	3	55.91	11		43	15.01	21	1	22	34.11
2	7	51.82	12		47	10.92	22	1	26	30.02
3	11	47.73	13		51	6.83	23	1	30	25.93
4	15	43.64	14		55	2.74	24	1	34	21.84
5	19	39.55	15		58	58.65	25	1	38	17.75
6	23	35.46	16	1	2	54.56	26	1	42	13.66
7	27	31.37	17	1	6	50.47	27	1	46	9.57
8	31	27.28	18	1	10	46.38	28	1	50	5.48
9	35	23.19	19	1	14	42.29	29	1	54	1.39
10	39	19.10	20	1	18	38.20	30	1	57	57.30
							31	2	1	53.21

***This gives a means of regulating a watch** with much accuracy and by a very simple process. The writer, after having regulated his chronometer watch for a year by this method only, differed but a few seconds from the actual time as deduced from careful solar observations. Even a person not accustomed to ranging objects very accurately, need scarcely err a minute in a period of any number of years. It having occurred to him that the motion of a star in a second or two might be visible to the naked eye, he stuck a pin horizontally into a window-jamb; and placing his eye close to it, sighted along one side of it, at a large star setting behind the top of a roof about 100 feet distant, and found that his conjecture was correct. Those stars which are farthest from the poles appear to move the fastest, and are therefore the best. Those less than of the second magnitude are not satisfactory. If the first observations of a given star be made as late as midnight, that same star will answer for about three months, until at last it will begin to pass the range in daylight. Before this happens, the observer must transfer the time to another star which sets later; if near midnight, the better, as it will serve for a longer time. A window looking west is the best. The longer the range, the greater will be the apparent motion of the star; and, consequently, the observations will be more correct. If such a range can be secured as will strike the heavens at an angle of at least 40° above the horizon, the error from refraction will not appreciably affect an observation; at a much less angle it may do so to the extent of three or four seconds. A candle must be so placed as to render the pin and the watch visible at the same time. A little practice will render the process very easy, and supersede the necessity for more remarks on the subject. Of course, a memorandum must be made and preserved of the date, hour, minute, and (approximately) second, at which the first passage of the star took place. Subsequent passages will occur earlier, as shown in the foregoing table. The watch must be previously known to be right, when taking the first observation. If we require afterward to keep the correct time. Any person who will take the trouble thus to observe, and note down throughout a year, about half a dozen stars following each other at tolerably equal intervals of time, will on almost any clear night afterward be able, after a short calculation, to ascertain the correct clock time. The writer observed the passages of two or three stars behind different ranges, on the same nights, in order to obtain a mean of several observations; his object being to ascertain how pocket chronometers of the best makers would keep time under the vicissitudes of temperature, railroad travelling, &c, &c, to which they are ordinarily exposed. He used two of the best for this purpose, and the result was that their changes of *rate* were at times as great as from three to eight seconds per day. For ordinary purposes, therefore, they are of but little, if any, more service than a *good* common watch, of one-fourth the cost.

† More accurately 3 min, 55.90944 sec.

STANDARD RAILWAY TIME, ADOPTED 1883.

The following arrangement of standard time was recommended by the General and Southern Time Conventions of the railroads of the United States and Canada, held respectively in St. Louis, Mo., and New York city, April, 1883, and in Chicago, Ill., and New York city, in October, 1883, and went into effect on most of the railroads of the United States and Canada, November 18th, 1883. Most of the principal cities of the United States have made their respective local times to correspond with it. This system was proposed by Mr. W. F. Allen, Secretary of the Time Conventions, and its adoption was largely due to his efforts. We are indebted to Mr. Allen for documents from which the following has been condensed. Five standards of time, or five "times," have been adopted for the United States and Canada. These are, respectively, the mean times of the 60th, 75th, 90th, 105th, and 120th meridians west of Greenwich, England. As each of these meridians, in the above order, is 15° west of its predecessor, its time is one hour slower. Thus, when it is noon on the 90th meridian, it is 1 P. M. on the 75th, and 11 A. M. on the 105th. The following gives the name adopted for the standard time of each meridian, and the conventional color adopted, and uniformly adhered to, by Mr. Allen, for the purpose of designating it and its time, &c, on the maps published under his auspices:

Longitude west from Greenwich.	Name of Standard Time.	Conventional color.
60°	Intercolonial.	Brown.
75°	Eastern.	Red.
90°	Central.	Blue.
105°	Mountain.	Green.
120°	Pacific.	Yellow.

Theoretically, each meridian may be said to give the time for a strip of country 15° wide, running north and south, and having the meridian for its center. Thus the meridian on which the change of time between two standard meridians is supposed to take place, lies half-way between them. But it would, of course, not be practicable for the railroads to use an imaginary line in passing from one time standard to another. The changes are made at prominent stations forming the termini of two or more lines; or, as in the case of the long Pacific roads, at the ends of divisions. As far as practicable, points at which changes of time had previously been made, were selected as the changing points under the new system. Detroit, Mich., Pittsburgh, Pa., Wheeling and Parkersburg, W. Va., and Augusta, Ga., although not situated upon the same meridian, are points of change between *eastern* and *central* standard times. A train arriving at Pittsburgh from the east at noon, and leaving for the west 10 minutes after its arrival, leaves (by the figures shown upon its time-table, and by the watches of its train hands) not at 10 minutes after 12, but at 10 minutes after 11.

The necessity for making the changes of time at principal points, instead of on a true meridian line, necessitates also some "overlapping" of the times, or of their colors on the map. Thus, most of the roads between Buffalo and Detroit, on the *north* side of Lake Erie, run by "eastern," or "red," time; while those on the *south* side of the Lake, between Buffalo and Toledo, immediately opposite to and directly south of them, run by "central" or "blue" time.

If the changes of time were made at the meridians midway between the standard ones, it would not be necessary for any town to change its time more than 30 minutes. As it is, somewhat greater changes had to be made at a few points. Thus, standard time at Detroit is 32 minutes ahead, and at Savannah 36 minutes back, of mean local time.

In most cases the necessary change was made upon the railways by simply setting clocks and watches ahead or back the necessary number of minutes, and without making any change in time-tables.

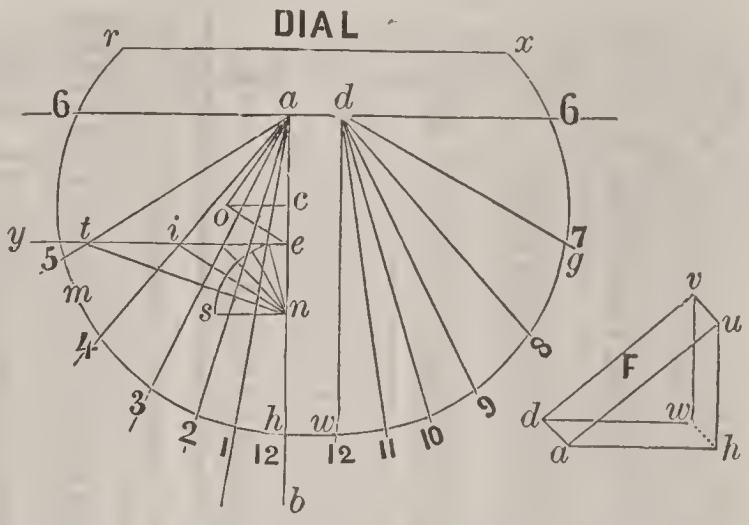
Halifax, and a few adjacent cities, use the time of the 60th meridian, that being the nearest one to them; but the *railroads* in the same district have adopted the 75th meridian, or eastern, time; so that, for railroad purposes, intercolonial time has never come into force.

In 1873 there were 71 time standards in use on the railroads of the United States and Canada. At the time of the adoption of the present system this number had been reduced, by consolidation of roads, &c, to 53. By its adoption, the number became 5, or, practically, 4, owing to the adoption of eastern time by the intercolonial roads, as already explained.

DIALLING.

To make a horizontal Sun-dial,

Draw a line $a\ b$; and at right angles to it, draw 66. From any convenient point, as c, in $a\ b$, draw the perp $c\ o$. Make the angle $c\ a\ o$ equal to the lat of the place; also the angle $c\ o\ e$ equal to the same; join $o\ e$. Make $e\ n$ equal to $o\ e$; and from n as a center, with the rad $e\ n$, describe a quadrant $e\ s$; and div it into 6 equal parts. Draw $e\ y$, parallel to 6, 6; and from n, through the 5 points on the quadrant, draw lines $n\ t$, $n\ i$, &c, terminating in $e\ y$. From a draw lines $a\ 5$, $a\ 4$, &c, passing through t, i, &c. From any convenient point, as c, describe an arc $r\ m\ h$, as a kind of finish or border to half the dial. All the lines may now be effaced, except the hour lines a 6, a 5, a 4, &c, to a 12, or $a\ h$; unless, as is generally the case, the dial is to be divided to quarters of an hour at least. In this case each of the divisions on the quadrant $e\ s$, must be subdivided into 4 equal parts; and lines drawn from n, through the points of subdivision, terminating in $e\ y$. The quarter-hour lines must be drawn from a, as were the hour lines. Subdivisions of 5 min may be made in the same way; but these, as well as single min, may usually be laid off around the border, by eye. About 8 or 10 times the size of our Fig will be a convenient one for an ordinary dial. To draw the other half of the Fig, make $a\ d$ equal to the intended thickness of the gnomon, or style, of the dial; and draw d 12, parallel, and equal to a 12; and draw the arc $x\ g\ w$, precisely similar to the arc $r\ m\ h$. Between x and w, on the arc $x\ g\ w$, space off divisions equal to those on the arc $r\ m\ h$; and number them for the hours, as in the Fig. The style F, of metal or stone, (wood is too liable to warp,) will be triangular; its thickness must throughout be equal to $a\ d$ or $h\ w$; its base must cover the space $a\ d\ h\ w$; its point will be at $a\ d$; and its perp height $h\ u$, over $h\ w$, must be such that lines $v\ d$, $u\ a$, drawn from its top, down to a and d, will make the angles $u\ a\ h$, $v\ d\ w$, each equal to the lat of the place. Its thickness, if of metal, may conveniently be from $\frac{1}{8}$ to $\frac{1}{4}$ inch; or if of stone, an inch or two, or more, according to the size of the dial. Usually, for neatness of appearance, the back $h\ u\ v\ w$ of the style is hollowed inward. The upper edges, $u\ a$, $v\ d$, which cast the shadows, must be sharp and straight. The dial must be fixed in place hor, or perfectly level; $a\ h$ and $d\ w$ must be placed truly north and south; $a\ d$ being south, and $h\ w$ north. The dial gives only sun or solar time; but clock time can be found by means of the "fast or slow of the sun," as given by all almanacs. If by the almanac the sun is 5 min, &c, fast, the dial will be the same; and the clock or watch, to be correct, must be 5 min slower than it; and vice versa.



To make a Vertical Sun-Dial.

Proceed as directed above, except that the angles $c\ a\ o$ and $c\ o\ e$ on the drawing, and the angle $u\ a\ h$ or $v\ d\ w$ of the style, must be equal to the *co*-latitude (= difference between the latitude and 90°) of the place, and the hours must be numbered the opposite way from those in the above figure; *i. e.* from h to y number 12, 11, 10, 9, 8, 7; and from w to g number 12, 1, 2, 3, 4, 5. The dial plate must be placed vertically, in the position shown in the figure, facing exactly south, and with $a\ h$ and $d\ w$ vertical.

TABLE OF WEIGHT OF CAST IRON.*

The weight of a pattern of perfectly dry white pine, if mult by 20, will give approximately the wt of the casting. If well seasoned, but still not perfectly dry, mult by 19, or by 18.

Assuming 450 lbs to a cub ft, a pound contains 3.8400 cubic inches; a ton 5 cub ft; and a cubic inch weighs .2604 lbs.

Thickness or Diameter in Inches.	Thickness or Diam. in decimals of a foot.	Wt. of a *Square Foot.* Lbs.	Wt. of a *Square* bar, 1 ft. long. Lbs.	Wt. of a *Round* bar, 1 ft. long. Lbs.	Wt. of *Balls.* Lbs. †
1-32	.0026	1.173	.003	.002	
1-16	.0052	2.344	.012	.010	
3-32	.0078	3.516	.027	.021	.0001
⅛	.0104	4.687	.048	.038	.0003
5-32	.0130	5.861	.076	.060	.0005
3-16	.0156	7.032	.110	.086	.0009
7-32	.0182	8.203	.150	.118	.0014
¼	.0208	9.375	.195	.154	.0021
9-32	.0234	10.54	.247	.194	.0030
5-16	.0260	11.73	.305	.240	.0042
11-32	.0287	12.89	.370	.290	.0056
⅜	.0313	14.06	.440	.346	.0072
13-32	.0339	15.24	.516	.400	.0092
7-16	.0365	16.41	.598	.470	.0114
15-32	.0391	17.56	.687	.540	.0140
½	.0417	18.75	.781	.610	.0170
9-16	.0469	21.10	.989	.777	.0243
⅝	.0521	23.44	1.221	.959	.0334
11-16	.0573	25.79	1.478	1.161	.0444
¾	.0625	28.12	1.758	1.381	.0575
13-16	.0677	30.47	2.064	1.621	.0732
⅞	.0729	32.81	2.393	1.880	.0913
15-16	.0781	35.16	2.747	2.158	.1124
1.	.0833	37.50	3.125	2.455	.1363
1-16	.0885	39.84	3.528	2.771	.1636
1⅛	.0938	42.19	3.955	3.107	.1942
3-16	.0990	44.53	4.407	3.461	.2284
¼	.1042	46.87	4.883	3.835	.2664
5-16	.1094	49.22	5.384	4.229	.3084
⅜	.1146	51.57	5.909	4.640	.3546
7-16	.1198	53.91	6.461	5.073	.4058
½	.1250	56.26	7.033	5.523	.4603
9-16	.1302	58.60	7.632	5.993	.5204
⅝	.1354	60.94	8.253	6.484	.5852
11-16	.1406	63.28	8.900	6.991	.6555
¾	.1458	65.63	9.572	7.518	.7310
13-16	.1510	67.97	10.27	8.064	.8122
⅞	.1563	70.32	10.99	8.630	.8991
15-16	.1615	72.66	11.73	9.215	.9920
2.	.1667	75.01	12.50	9.821	1.073
⅛	.1771	79.70	14.11	11.09	1.308
¼	.1875	84.40	15.83	12.43	1.554
⅜	.1979	89.07	17.63	13.85	1.827
½	.2083	93.75	19.54	15.34	2.131
⅝	.2188	98.44	21.54	16.56	2.467
¾	.2292	103.2	23.64	18.56	2.835
⅞	.2396	107.8	25.84	20.29	3.241
3.	.2500	112.6	28.13	22.10	3.682
3⅛	.2604	117.3	30.52	23.97	4.162
¼	.2708	121.8	33.01	25.93	4.681
⅜	.2813	126.5	35.60	27.95	5.243
½	.2917	131.2	38.28	30.07	5.846
⅝	.3021	135.9	41.07	32.25	6.498
¾	.3125	140.6	43.95	34.51	7.193
⅞	.3229	145.3	46.93	36.85	7.934
4.	.3333	150.0	50.01	39.27	8.726
⅛	.3438	154.7	53.18	41.77	9.572
¼	.3542	159.3	56.46	44.33	10.47
⅜	.3646	164.0	59.82	46.99	11.42
½	.3750	168.7	63.33	49.71	12.43
⅝	.3854	173.4	66.86	52.52	13.49
¾	.3958	178.1	70.52	55.39	14.62
⅞	.4063	182.8	74.28	58.34	15.81
5.	.4167	187.5	78.12	61.37	17.05
⅛	.4271	192.2	82.10	64.47	18.35
¼	.4375	196.9	86.14	67.65	19.73
⅜	.4479	201.6	90.29	70.52	21.18
½	.4583	206.2	94.54	74.26	22.68
⅝	.4688	210.9	98.89	77.66	24.27
¾	.4792	215.6	103.3	81.16	25.93
⅞	.4896	220.3	107.9	84.72	27.41
6.	.5000	225.0	112.5	88.36	29.44
¼	.5208	234.4	122.1	95.89	33.28
½	.5417	243.8	132.0	103.7	37.44
¾	.5625	253.1	142.4	111.9	41.94
7.	.5833	262.5	153.2	120.2	46.77
¼	.6042	271.9	164.2	129.0	51.97
½	.6250	281.3	175.8	138.1	57.54
¾	.6458	290.7	187.7	147.4	63.47
8.	.6667	300.0	200.1	157.0	69.82
¼	.6875	309.4	212.7	167.0	76.58
½	.7083	318.8	225.8	177.3	83.74
¾	.7292	328.2	239.3	187.9	91.35
9.	.7500	337.4	253.1	198.8	99.42
¼	.7708	346.8	267.4	210.0	107.9
½	.7917	356.2	282.1	221.5	116.8
¾	.8125	365.6	297.0	233.3	126.3
10.	.8333	375.0	312.5	245.5	136.3
¼	.8542	384.4	328.4	257.8	146.8
½	.8750	393.7	344.5	270.6	157.9
¾	.8958	403.1	361.2	283.7	169.3
11.	.9167	412.5	378.2	297.0	181.5
¼	.9375	421.9	395.5	310.6	194.2
½	.9583	431.2	413.3	324.6	207.3
¾	.9792	440.6	431.4	338.8	219.2
12.	1 Foot.	450.	450.	353.4	235.6

† **Wts of balls** are as the cubes of their diams. See table, p 416.

To find the weight of a spherical shell. From the weight of a ball which has the outer diam of the shell, take the wt of one which has its inner diam.

* **For Copper,** mult by 1.2; Lead, mult by 1.6; Brass, add 1-7th; Zinc, mult by .97. All approximate. See table, 415.

WEIGHT OF CAST-IRON PIPES per running foot.

Assuming the weight of cast-iron at 450 lbs per cub ft. or .2604 lb per cub inch. No allowance is here made for the spigot and faucet-joints used in water-pipes. As these are now commonly made, (see Fig 38, page 295,) they add to the weight of each length or section of pipe of any size, about as much as that of 8 inches in length of the plain pipe as given in the table.

For lead-pipe mult by 1.6; **copper,** mult by 1.2; **brass,** add 1-7th; **welded iron,** mult by 1.0667, or add one fifteenth part.

Inner diam. or bore, in Inches.	THICKNESS OF PIPE IN INCHES.												
	¼	⅜	½	⅝	¾	⅞	1	1⅛	1¼	1⅜	1½	1¾	2
	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.
1.	3.07	5.07	7.38	9.99	12.9	16.2	19.7	23.5	27.7	32.1	36.9	47.4	59.1
¼	3.69	6.00	8.61	11.5	14.8	18.3	22.2	26.3	30.8	35.5	40.6	51.7	64.0
½	4.30	6.92	9.84	13.1	16.6	20.5	24.6	29.1	33.8	38.9	44.3	56.0	68.9
¾	4 92	7.84	11.1	14.6	18.5	22.6	27.1	31.8	36.9	42.3	48.0	60.3	73.8
2.	5.53	8.76	12.3	16.2	20.3	24.8	29.5	34.6	40.0	45.7	51.7	64.6	78.7
¼	6.15	9.69	13.5	17.7	22.2	26.9	32.0	37.4	43.1	49.0	55.4	68.9	86.7
½	6.76	10.6	14.8	19.2	24.0	29.1	34.5	40.1	46.1	52.4	59.1	73.2	88.6
¾	7.37	11.5	16.0	20.8	25.9	31.2	36.9	42.9	49.2	55.8	62 7	77.5	93.5
3.	7.98	12.5	17.2	22.3	27.7	33.4	39.4	45.7	52.3	59.2	66 4	81.8	98.4
¼	8.60	13.4	18.5	23.8	29.5	35.5	41.8	48.4	55.4	62.3	70.1	86.1	103.
½	9.21	14.3	19.7	25.4	31.4	37.7	44.3	51.2	58.4	65.9	73.8	90.4	108.
¾	9.83	15.2	20.9	26.9	33.2	39 8	46.8	54.0	61.5	69.3	77.5	94.7	113.
4.	10.3	16.1	22.2	28.5	35.1	42.0	49.2	56.7	64.6	72.7	81.2	99.0	118.
¼	11.1	17.1	23.4	30.0	36 9	44.1	51.7	59.5	67.7	76.1	84.9	103.	123.
½	11.7	18.0	24.6	31.5	38.8	46.3	54.1	62.3	70.7	79.5	88.6	108.	128.
¾	12.3	18.9	25.8	33.1	40.6	48.5	56.6	65.0	73.8	83.9	92.3	112.	133.
5.	12.9	19.8	27.1	34.6	42.5	50.6	59.1	67.8	76.9	87.2	96.0	116.	138.
¼	13.5	20.8	28.3	36.1	44.3	52.8	61.5	70.6	80.0	90.6	99.6	121.	143.
½	14.2	21.7	29.5	37.7	46.1	54.9	64.0	73.3	83.0	94.0	103.	125.	148.
¾	14.8	22.6	30.8	39.2	48.0	57.1	66.4	76.1	86.1	97.4	107.	129.	153.
6.	15.4	23.5	32.0	40.8	49.8	59.2	68.9	78.9	89.2	99.8	111.	134.	158.
½	16.6	25.4	34.5	43.8	53.5	63.5	73.8	84.4	95.3	107.	118.	142.	167.
7.	17.8	27.2	36.9	46.9	57.2	67.8	78.7	89.4	102.	113.	126.	151.	177.
½	19.1	29.1	39.4	50.0	60.9	72.1	83.7	95 5	108.	120.	133.	159.	187.
8.	20.3	30.9	41.8	53.1	64.6	76.4	88.6	101.	114.	127.	140.	168.	197.
½	21.5	32.8	44.3	56.1	68.3	80.7	93.5	107.	120.	134.	148.	177.	207.
9.	22.8	34.6	46.8	59.2	72.0	85.1	98.4	112.	126.	140.	155.	185.	217.
½	24 0	36.4	49.2	62.3	75.7	89.3	103.	118.	132.	147.	163.	194.	226.
10.	25.1	38.3	51.7	65.3	79.4	93.6	108.	123.	138.	154.	170.	202.	235.
½	26.4	40 1	54.1	68.4	83.0	97.9	113.2	129.	145.	161.	177.	211.	245.
11.	27.6	42.0	56 6	71.5	86.7	102.	118.	134.	151.	168.	185.	220.	255.
½	28.8	43.8	59.1	74 6	90.4	107.	123.	140.	157.	174.	192.	228.	265.
12.	30.0	45.7	61.5	77.7	94.1	111.	128.	145.	163.	181.	199.	237.	275.
13.	32.5	49.4	66.4	83.8	102.	120.	138.	156.	175.	195.	214.	254.	294.
14.	35.0	53.1	71.4	89.4	109.	128.	148.	168.	188.	208.	229.	271.	314.
15.	37.4	56.7	76.3	96.1	116.	137.	158.	179.	200.	222.	244.	289.	334.
16.	39.1	60.4	81.2	102.	124.	145.	167.	190.	212.	235.	258.	306.	353.
17.	42.3	64.1	86.1	108.	131.	154.	177.	201.	225.	249.	273.	323.	373.
18.	44.8	67.8	91.0	115.	139.	163.	187.	212.	237.	262.	288.	340.	393.
19.	47.3	71.5	96.0	121.	146.	171.	197.	223.	249.	276.	303.	357.	412.
20.	49.7	75.2	101.	127.	153.	180.	207.	234.	261.	289.	317.	375.	432.
21.	52.2	78.9	106.	133.	161.	188.	217.	245.	274.	303.	332.	392.	452.
22.	54.6	82.6	111.	139.	168.	196.	227.	256.	286.	316.	347.	409.	471.
23.	57.1	86.3	116.	145.	175.	206.	236.	267.	298.	330.	362.	426.	491.
24.	59.6	89.9	121.	152.	183.	214.	246.	278.	311.	343.	375.	441.	511.
25.	62.0	93.6	126.	158.	190.	223.	256.	289.	323.	357.	391.	461.	531.
26.	64.5	97.3	131.	164.	198.	231.	266.	300.	335.	370.	406.	478.	550.
27.	66.9	101.	135.	170.	205.	240.	276.	311.	348.	384.	421.	495.	570.
28.	69.4	105.	140.	176.	212.	249.	286.	323.	360.	397.	436.	512.	590.
29.	71.8	109.	145.	182.	220.	257.	295.	334.	372.	411.	450.	530.	609.
30.	74.2	112.	150.	188.	227.	266.	305.	345.	384.	424.	465.	547.	629.
31.	76.7	116.	155.	195.	234.	275.	315.	356.	397.	438.	480.	564.	649.
32.	79.1	120.	160.	201.	242.	283.	325.	367.	409.	451.	495.	581.	668.
33.	81.6	123.	165.	207.	249.	292.	335.	378.	421.	465.	509.	598.	688.
34.	84.1	127.	170.	213.	257.	300.	345.	389.	434.	479.	524.	616.	708.
35.	86.5	131.	175.	219.	264.	309.	354.	400.	446.	492.	539.	633.	726.
36.	89.0	134.	180.	225.	271.	318.	364.	411.	458.	506.	554.	650.	746.
42.	104.	156.	210.	262.	315.	370.	423.	478.	532.	588.	644.	753.	864.
48.	119.	178.	239.	298.	359.	422.	482.	544.	605.	669.	733.	856.	982.

For warming buildings by steam it usually suffices to allow 1 sq ft of cast or wrought pipe surface for each 120 cub ft of space to be warmed; and 1 cub ft of boiler for each 2000 cub ft of such space.

Table of Weight of WROUGHT IRON* and STEEL.

Wrought iron is here taken at 485 lbs per cub ft; or a sp gr of 7.77. Very pure soft wrought iron weighs from 488 to 492 lbs per cubic foot. Light weight indicates impurities, and weakness. Steel weighs about 490 lbs per cubic foot; therefore, **for steel, an addition must be made to the tabular amounts, of about 1 pound in 100 lbs.**

At 485 lbs per cub ft, a cubic inch weighs .28067 lb; a lb contains 3.5629 cub ins; and a ton, 4.6186 cub ft; and this is about the average of *hammered* iron The usual assumption is 480 lbs per cub ft; which is nearer the average of ordinary *rolled* iron. At 480 lbs, a cubic inch weighs .2778 of a lb; a lb contains 3.6 cub ins; a ton, 4.6667 cub ft; a rod of 1 sq inch area, weighs 10 lbs per yard; or $3\frac{1}{3}$ lbs per foot, exactly.

Thickness or Diameter in Inches.	Thickness or Diam. in decimals of a foot.	Wt. of a *Square Foot.* Lbs.	Wt. of a *Square* bar, 1 ft. long. Lbs.	Wt. of a *Round* bar, 1 ft. long. Lbs.	Wt. of *Balls.* Lbs.	Thickness or Diameter in Inches.	Thickness or Diam. in decimals of a foot.	Wt. of a *Square Foot.* Lbs.	Wt. of a *Square* bar, 1 ft. long. Lbs.	Wt. of a *Round* bar, 1 ft. long. Lbs.	Wt. of *Balls.* Lbs.
1-32	.0026	1.263	.0033	.0026		3⅛	.2604	126.3	32.89	25.83	4.484
1-16	.0052	2.526	.0132	.0104		¼	.2708	131.4	35 57	27.94	5.045
3-32	.0078	3.789	.0296	.0233	.0001	⅜	.2813	136.4	38.37	30.13	5.649
⅛	.0104	5.052	.0526	.0414	.0003	½	.2917	141.5	41.26	32.41	6.301
5-32	.0130	6.315	.0823	.0646	.0005	⅝	.3021	146.5	44.26	34.76	7.000
3-16	0156	7.578	.1184	.0930	.0009	¾	.3125	151.6	47.37	37.20	7.750
7-32	.0182	8.841	.1612	.1266	.0015	⅞	.3229	156.6	50.57	39.72	8.550
¼	.0208	10.10	.2105	.1653	.0023	4.	.3333	161.7	53.89	42.33	9.405
9-32	.0234	11.37	.2665	.2093	.0033	⅛	.3438	166.7	57.31	45.01	10.32
5-16	.0260	12.63	.3290	.2583	.0045	¼	.3542	171.8	60.84	47.78	11.28
11-32	.0287	13.89	.3980	.3126	.0060	⅜	.3646	176.8	64.47	50.63	12.31
⅜	.0313	15.16	.4736	.3720	.0078	½	.3750	181.9	68.20	53.57	13.39
13-32	.0339	16.42	.5558	.4365	.0098	⅝	.3854	186.9	72.05	56.59	14.54
7-16	.0365	17.68	.6446	.5063	.0123	¾	.3958	192.0	75.99	59.69	15.75
15-32	.0391	18.95	.7400	.5813	.0151	⅞	.4063	197.0	80.05	62.87	17.03
½	.0417	20.21	.8420	.6613	.0184	5.	.4167	202.1	84.20	66.13	18.37
9-16	.0469	22.73	1.066	.8370	.0262	⅛	.4271	207.1	88.47	69.48	19.78
⅝	.0521	25.26	1.316	1.033	.0359	¼	.4375	212.2	92.83	72.91	21.26
11-16	.0573	27.79	1.592	1.250	.0478	⅜	.4479	217.2	97.31	76.43	22.82
¾	.0625	30 31	1.895	1.488	.0620	½	.4583	222.3	101.9	80.02	24.45
13-16	.0677	32.84	2.223	1.746	.0788	⅝	.4688	227.3	106.6	83.70	26.16
⅞	.0729	35.37	2.579	2.025	.0985	¾	.4792	232.4	111.4	87.46	27.94
15-16	.0781	37.89	2.960	2.325	.1211	⅞	.4896	237.5	116.3	91.31	29.80
1.	.0833	40.42	3.368	2.645	.1470	6.	.5000	242.5	121.3	95.23	31.74
1-16	.0885	42.94	3.803	2.986	.1763	¼	.5208	252.6	131.6	103.3	35.88
⅛	.0938	45.47	4.263	3.348	.2093	½	.5417	262.7	142.3	111.8	40.36
3-16	.0990	48.00	4.750	3.730	.2461	¾	.5625	272.8	153.5	120.5	45.19
¼	.1042	50.52	5.263	4.133	.2870	7.	.5833	282.9	165.0	129.6	50.40
5-16	.1094	53.05	5.802	4.557	.3323	¼	.6042	293.0	177.0	139.0	56.00
⅜	.1146	55.57	6.368	5.001	.3820	½	.6250	303.1	189.5	148.8	62.00
7-16	.1198	58.10	6.960	5.466	.4365	¾	.6458	313.2	202.3	158.9	68.40
½	.1250	60.63	7.578	5.952	.4960	8.	.6667	323.3	215.6	169.3	75.24
9-16	.1302	63.15	8.223	6.458	.5606	¼	.6875	333.4	229.3	180.1	82.52
⅝	.1354	65.68	8.893	6.985	.6306	½	.7083	343.5	243.4	191.1	90.25
11-16	.1406	68.20	9.591	7.533	.7062	¾	.7292	353.6	247.9	202.5	98.45
¾	.1458	70.73	10.31	8.101	.7876	9.	.7500	363.8	272.8	214.3	107.1
13-16	.1510	73.26	11.07	8.690	.8750	¼	.7708	373.9	288.2	226.3	116.3
⅞	.1563	75.78	11.84	9.300	.9688	½	.7917	384.0	304.0	238.7	126.0
15-16	.1615	78.31	12.64	9.930	1.069	¾	.8125	394.1	320.2	251.5	136.2
2.	.1667	80.83	13.47	10.58	1.176	10.	.8333	404.2	336.8	264.5	146.9
⅛	.1771	85.89	15.21	11.95	1.410	¼	.8542	414.3	353.9	277.9	158.2
¼	.1875	90.94	17.05	13.39	1.674	½	.8750	424.4	371.3	291.6	170.1
⅜	.1979	95.99	19.00	14.92	1.969	¾	.8958	434.5	389.2	305.7	182.6
½	.2083	101.0	21.05	16.53	2.296	11.	.9167	444.6	407.5	320.1	195.6
⅝	.2188	106.1	23.21	18.23	2.658	¼	.9375	454.7	426.3	334.8	209.2
¾	.2292	111.2	25.47	20.01	3.056	½	.9583	464.8	445.4	349.8	223.5
⅞	.2396	116.2	27.84	21.87	3.492	¾	.9792	474.9	465.0	365.2	238.4
3.	.2500	121.3	30.31	23.81	3.968	12.	1 Foot.	485.	485.	380.9	253.9

To find the weight of a spherical shell. From the weight of a ball which has the outer diam of the shell, take the wt of one which has its inner diam.

* **For Copper,** add 1-7th part. Lead, mult by 1.47. Brass, mult by 1.06. Zinc, mult by .9. Tin, mult by .95. All approximate. See table, p 415.

Weight of 1 ft in length of FLAT ROLLED IRON, at 480 lbs per cubic foot. For cast iron, deduct $\frac{1}{16}$ part; for steel, add $\frac{1}{48}$; for copper, add $\frac{1}{7}$; for cast brass, add $\frac{1}{13}$; for lead, add $\frac{1}{2}$; for zinc, deduct $\frac{1}{12}$.

Width in Ins.	THICKNESS IN INCHES.											
	1-16	⅛	3-16	¼	5-16	⅜	7-16	½	⅝	¾	⅞	1
1.	.2083	.4166	.6250	.8333	1.042	1.250	1.458	1.666	2.083	2.500	2.916	3.333
⅛	.2344	.4688	.7033	.9375	1.172	1.406	1.640	1.875	2.344	2.812	3.280	3.75
¼	.2605	.5210	.7810	1.042	1.303	1.563	1.823	2.083	2.605	3.125	3.646	4.166
⅜	.2865	.5730	.8595	1.146	1.432	1.719	2.006	2.292	2.864	3.438	4.012	4.583
½	.3125	.6250	.9375	1.250	1.562	1.875	2.188	2.500	3.125	3.750	4.375	5.000
⅝	.3385	.6771	1.015	1.354	1.692	2.031	2.370	2.708	3.384	4.062	4.740	5.416
¾	.3646	.7292	1.094	1.458	1.823	2.188	2.550	2.916	3.646	4.375	5.105	5.833
⅞	3906	.7812	1.172	1.562	1.953	2.344	2.735	3.125	3.906	4.688	5.470	6.25
2.	.4166	.8333	1.25	1.666	2.083	2.500	2.916	3.333	4.166	5.000	5.833	6.666
⅛	.4427	.8855	1.328	1.771	2.214	2.656	3.098	3.542	4.428	5.312	6.196	7.083
¼	.4688	.9375	1.406	1.875	2.344	2.812	3.281	3.750	4.688	5.624	6.562	7.500
⅜	.4948	.9895	1.484	1.979	2.474	2.968	3.463	3.958	4.948	5.936	6.926	7.916
½	.5210	1.042	1.562	2.083	2.605	3.125	3.646	4.166	5.210	6.250	7.291	8.333
⅝	.5470	1.094	1 641	2.187	2.735	3.282	3.829	4.375	5.470	6.564	7.658	8.750
¾	.5730	1.146	1.719	2.292	2.865	3.438	4.011	4.583	5.730	6.876	8.022	9.166
⅞	.5990	1.198	1.797	2.396	2.995	3.594	4.193	4.792	5.990	7.188	8.386	9.583
3.	.625	1.250	1.875	2.500	3.125	3.750	4.375	5.000	6.250	7.500	8.750	10.00
⅛	.6515	1.303	1 954	2.605	3.257	3.908	4.560	5.210	6.514	7.816	9.120	10.42
¼	.6770	1.354	2.031	2.708	3.385	4.062	4.739	5.416	6.770	8.124	9.478	10.83
⅜	.7031	1.406	2.109	2.812	3.516	4.218	4.921	5.625	7.032	8.436	9.842	11.25
½	.7291	1.458	2.188	2.916	3.646	4.375	5.105	5.833	7.291	8.750	10.21	11.66
⅝	.7555	1.511	2.266	3.021	3.777	4.533	5.288	6.042	7.554	9.066	10.58	12.08
¾	.7812	1.562	2.343	3.125	3.906	4.686	5.468	6.25	7.812	9.372	10.94	12.50
⅞	.8070	1.614	2.421	3.229	4.035	4.842	5.65	6.458	8.070	9.684	11.30	12.92
4.	.8333	1.666	2.500	3.333	4.166	5.000	5.833	6.666	8.333	10.00	11.66	13.33
⅛	.8595	1.719	2.578	3.438	4.297	5.156	6.016	6.875	8.594	10.31	12.03	13.75
¼	.8855	1.771	2.656	3 542	4.427	5.312	6.198	7.083	8.854	10.62	12.40	14.16
⅜	.9115	1.823	2.734	3.646	4.557	5.468	6.380	7.291	9.114	10.94	12.76	14.58
½	.9375	1.875	2.812	3.750	4.687	5.624	6.562	7.500	9.374	11.25	13.12	15.00
⅝	.9636	1.927	2.891	3.854	4.818	5.782	6.745	7.708	9.636	11.56	13.49	15.42
¾	.9895	1.979	2.968	3.958	4.947	5.936	6.926	7.917	9.894	11.87	13.85	15.83
⅞	1.016	2.031	3.048	4 062	5.080	6.096	7.112	8.125	10.16	12.19	14.22	16.25
5.	1.042	2.083	3.125	4.166	5.210	6.25	7.291	8 333	10.42	12.50	14.58	16.66
⅛	1.068	2.136	3.204	4.271	5.340	6 408	7.476	8.542	10.68	12.81	14.95	17.08
¼	1.094	2.188	3.282	4.375	5 470	6.564	7.658	8.750	10.94	13.13	15.31	17.50
⅜	1.120	2.240	3.360	4.479	5.600	6.720	7.840	8.958	11.20	13.44	15.68	17.92
½	1.146	2.292	3.438	4.584	5.730	6.876	8.022	9.167	11.46	13.75	16.04	18.33
⅝	1.172	2.344	3.516	4.687	5.860	7.032	8.204	9.375	11.72	14.06	16.40	18.75
¾	1.198	2.396	3.594	4.791	5.990	7.188	8.386	9.583	11.98	14.37	16.77	19.16
⅞	1.224	2.448	3.672	4.896	6.120	7.344	8.568	9.792	12.24	14.69	17.13	19.58
6.	1.250	2.500	3.750	5.000	6.250	7.500	8.750	10.00	12.50	15.00	17.50	20.00
⅛	1.276	2 552	3.828	5.104	6.380	7.656	8.932	10.21	12.76	15.31	17.86	20.42
¼	1.302	2.604	3.906	5.208	6.510	7.812	9.114	10.42	13.02	15.62	18.23	20.83
⅜	1.328	2.657	3.984	5.313	6 640	7.968	9.297	10.63	13.28	15.93	18.59	21.25
½	1.354	2.708	4.063	5.417	6.770	8.126	9.480	10.83	13.54	16.25	18.96	21.66
⅝	1.381	2.761	4.143	5 521	6.906	8.286	9.668	11.04	13.81	16.57	19.33	22.08
¾	1.406	2.813	4.218	5.625	7.030	8.436	9.843	11.25	14.06	16.87	19.69	22.50
⅞	1.432	2.864	4.296	5.729	7.160	8.592	10.02	11.46	14.32	17.18	20.04	22.92
7.	1.458	2.916	4.375	5.833	7.291	8.750	10.20	11.66	14.58	17.50	20.42	23.33
⅛	1.484	2.969	4.452	5.938	7.420	8.904	10.39	11.87	14.84	17.81	20.78	23.75
¼	1.511	3.021	4.533	6.042	7.555	9.066	10.58	12.08	15.11	18.13	21.16	24.16
⅜	1.536	3.073	4.608	6.146	7.680	9.216	10.75	12.29	15.36	18.43	21.50	24.58
½	1.562	3.125	4.686	6.250	7.810	9.372	10.93	12 50	15.62	18.74	21.86	25.00
⅝	1.588	3.177	4.764	6.354	7.940	9.528	11.12	12.71	15.88	19.05	22.24	25.42
¾	1.615	3.229	4.845	6 458	8.075	9.690	11.31	12.92	16.15	19.38	22.62	25.83
⅞	1.641	3.281	4.923	6.562	8.205	9.846	11.48	13.13	16.41	19.69	22.96	26.25
8.	1.666	3.333	5.000	6.666	8.333	10.00	11.66	13.33	16.66	20.00	23.33	26.66
⅛	1.693	3.386	5.079	6.771	8.455	10.15	11.85	13.54	16.91	20.30	23.70	27.08
¼	1.719	3.438	5.157	6.875	8.595	10.31	12.03	13.75	17.19	20.61	24.06	27.50
⅜	1.745	3.489	5.235	6.979	8.725	10.47	12.21	13.96	17.45	20.94	24.42	27.92
½	1.771	3.542	5.313	7.083	8.855	10.63	12.40	14 17	17.71	21.26	24.80	28.33
⅝	1.797	3.594	5.391	7.188	8.985	10.78	12.58	14.37	17.97	21.56	25.16	28.75
¾	1.823	3.646	5.469	7.292	9.115	10.94	12.76	14.58	18.23	21.88	25.52	29.17
⅞	1.849	3.698	5.547	7.396	9.245	11.09	12.94	14.79	18.49	22.18	25.88	29.58
9.	1.875	3.750	5.625	7.500	9.375	11.25	13.12	15.00	18.75	22.50	26.24	30.00
⅛	1.901	3.802	5.703	7.604	9.505	11.41	13.31	15.21	19.00	22.81	26.62	30.42
¼	1.927	3.854	5.781	7.708	9.635	11.56	13.49	15.42	19.27	23.12	26.98	30.83
⅜	1.953	3.906	5.859	7.812	9.765	11.72	13.67	15.62	19.53	23.44	27.34	31.25
½	1.979	3.958	5.937	7.916	9.895	11.87	13.85	15.84	19.79	23.74	27.70	31.67
⅝	2.005	4.010	6.015	8.021	10.02	12.03	14.04	16.04	20.04	24.06	28.08	32.08
¾	2.031	4.062	6.093	8.125	10.16	12.18	14.21	16.25	20.32	24.36	28.42	32.50
⅞	2.057	4.114	6.171	8.229	10.29	12.34	14.40	16.46	20.58	24.68	28.80	32.92
0.	2.083	4.166	6.250	8.333	10.41	12.50	14.58	16.66	20.82	25.00	29.16	33.33
⅛	2.109	4.219	6.327	8.438	10.55	12.65	14.76	16.87	21.10	25.30	29.52	33.75
¼	2.135	4.270	6.405	8.541	10.67	12.81	14.94	17.08	21.34	25.62	29.88	34.17

Weight of 1 ft in length of FLAT ROLLED IRON, at 480 lbs per cubic foot—(Continued.)

Width in ins.	THICKNESS IN INCHES.											
	1-16	⅛	3-16	¼	5-16	⅜	7-16	½	⅝	¾	⅞	1
10⅜	2.162	4.323	6.486	8.646	10.81	12.97	15.13	17.29	21.62	25.94	30.26	34.58
½	2.188	4.375	6.564	8.750	10.94	13.13	15.31	17.50	21.88	26.26	30.62	35.00
⅝	2.214	4.427	6.642	8.854	11.07	13.28	15.50	17.71	22.14	26.56	31.00	35.42
¾	2.239	4.479	6.717	8.958	11.20	13.43	15.67	17.92	22.40	26.86	31.34	35.83
⅞	2.266	4.531	6.798	9.062	11.33	13.59	15.86	18.12	22.66	27.18	31.72	36.25
11.	2.291	4.583	6.873	9.166	11.46	13.75	16.04	18.33	22.90	27.50	32.08	36.66
⅛	2.318	4.636	6.954	9.271	11.59	13.91	16.22	18.54	23.18	27.82	32.44	37.08
¼	2.344	4.688	7.032	9.375	11.72	14.06	16.40	18.75	23.44	28.12	32.80	37.50
⅜	2.370	4.740	7.110	9.479	11.85	14.22	16.59	18.96	23.70	28.44	33.18	37.92
½	2.395	4.791	7.185	9.582	11.97	14.37	16.76	19.16	23.94	28.74	33.52	38.33
⅝	2.422	4.844	7.266	9.688	12.11	14.53	16.95	19.37	24.22	29.06	33.90	38.75
¾	2.448	4.896	7.344	9.792	12.24	14.68	17.13	19.58	24.48	29.36	34.26	39.16
⅞	2.474	4.948	7.422	9.896	12.37	14.84	17.32	19.79	24.74	29.68	34.64	39.58
12.	2.500	5.000	7.500	10.00	12.50	15.00	17.50	20.00	25.00	30.00	35.00	40.00

ROLLED IRON AND STEEL.* AVERAGE PRICES. Phila, 1886. **Iron bars.** Base price, or price for "ordinary sizes"; *i e*, from ¾ inch to 2 ins diameter, round and square; and from 1 × ¾ inch to 6 × 1 inch, flat: Ordinary merchant quality, called "refined", 2 cts per lb; "Extra refined" and rivet iron, 3 cts per lb. Sizes larger or smaller than "ordinary" bring higher prices per lb. The "extra", or charge *in addition* to the above base price, increases gradually up to .7 ct per lb for ¼ inch round and square; 2.5 cts for 7 inches round and square; 1.2 cts for ½ × 3/16 inch flat; and 1.1 cts for 12 × 2 ins flat. **Swedish and Norway iron:** ½ × ¼ inch and ⅜ inch square, and larger; in the original bar as imported (called "Swedish"), 4 cts per lb; re-rolled after importation (called "Norway"), 4½ cts per lb. **Hoop iron:** from 1½ inch × No 17, 2½ cts per lb; to ½ inch × No 22, 5 cts per lb. **Sheet iron:** see page 403. **Plate iron.** Rectangular plates, 3/16 inch thick and heavier; and say from 2 to 5 feet wide, and from 4 to 12 feet long. "Common or "puddled", for bridge plates, sheathing of ships etc, where little or no bending is required, 2 cts per lb; "Shell"; for shells of boilers etc, to stand bending cold with the grain to cylinders with radii of say one or two feet, but not flanging, 2½ cts per lb; "Best flange", 3½ cts per lb. Fire-box plates of shell or flange iron, 1 ct per lb extra. Extra charges for plates of unusually great or small widths or lengths. **Angle and T iron:** see pages 525 etc. **I beams, channel beams, deck beams:** see pages 520 etc. **Pig iron.** American foundry, $18 per ton of 2240 lbs; Forge, for conversion into wrought iron by puddling etc, $16 per ton of 2240 lbs; Charcoal foundry and forge, $20 per ton of 2240 lbs. **Cast Steel for tools.** Best American. Ordinary sizes, 8½ cts per lb; **Machinery steel:** for shafting etc, ordinary sizes, 3 cts per lb. The range of "ordinary" sizes is nearly the same as given above for iron. For larger and smaller sizes, extras running up to from 50 to 100 per cent. **Steel plates** for boilers and fire-boxes, 4 cts per lb. **Tire and spring steel.** Bessemer, 3 cts per lb. **Sheet steel.** American.† Not lighter than No 17. Cast, 7 cts per lb; Bessemer, 4½ cts per lb. For each number lighter than No 17, ½ ct per lb extra.

Rolled **Star Iron.** Standard sizes. Carnegie Bros & Co, Limited, Pittsburgh. The thicknesses are those at the end and at the root of one of the four arms, in ins. Rolled in lengths of 20 to 25 ft. Area in sq ins. Wt in lbs per foot run.

Ins.	Thick.	Area.	Wt.	Ins.	Thick.	Area.	Wt.
4 × 4	⅜ — 9-16	3.6	12	2.5 × 2.5	5-16 — 13-32	1.65	5.5
3.5 × 3.5	⅜ — ½	2.85	9.5	2 × 2	¼ — 13-32	1.13	3.75
3 × 3	5-16 — 15-32	2.18	7.25	1.5 × 1.5	3-16 — 5-16	0.69	2.3

* Morris, Wheeler & Co, 16th and Market Sts, Philadelphia.
† William & Harvey Rowland, Frankford, Philadelphia.

Sheet iron.* **Prices,** Phila. 1886; in sheets 24 to 32 inches wide, 6 to 8 feet long; No 14 to No 28 by second table on p 411; common (puddled) iron, annealed, 2½ to 3½ cts per pound, being least for the lower (thicker) numbers; best charcoal bloom iron, 3¾ to 5 cts per lb.

Galvanized sheet iron.* Common (puddled) iron, 4¼ to 6 cts per pound; best charcoal bloom iron, 5 to 6½ cts per pound. The lower (thick) nos are the cheapest.

Weights per square foot of **galvanized sheet iron.** Standard list adopted by the American Galvanized Iron Ass'n, at Pittsburgh, April, 1884.

No.	Ounces avoir per sq ft	Sq ft per 2240 lbs.	No.	Ounces avoir per sq ft.	Sq ft per 2240 lbs.	No.	Ounces avoir per sq ft.	Sq ft per 2240 lbs.
29	12	2987	24	17	2108	19	33	1086
28	13	2757	23	19	1886	18	38	943
27	14	2560	22	21	1706	17	43	833
26	15	2389	21	24	1493	16	48	746
25	16	2240	20	28	1280	14	60	597

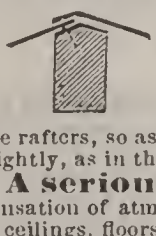
The galvanizing is simply a thin film of zinc on both sides of the sheet, as in what is known as "tinned plates," or "tin;" which are in reality sheet iron similarly coated with tin. Zinc, like tin, resists corrosion from ordinary atmospheric influences, much better than iron; and hence the use of these metals as a protection to the iron. A well galvanized roof, of a good pitch, will suffer but little from 5 to 6 years' exposure without being painted. It will then take paint readily, and should be painted. It is better, however, always to paint tin ones at once.

Paint does not adhere well to new zinc, and this is the principal reason why new galvanized roofs are not painted; but this may be remedied by first brushing the zinc over with the following: One part of chloride of copper, 1 part nitrate of copper, 1 part of sal-ammoniac. Dissolve in 64 parts of water. Then add 1 part of commercial hydrochloric acid. When brushed with this solution, the zinc turns black; dries within 12 to 24 hours, and may then be painted.

Paint of some mineral oxide of a brown color is generally used; one coat being applied to both sides in the shop; and the other after being put on the roof. Repainting every 3 or 4 years will suffice afterward. Ungalvanized iron (called BLACK IRON, for distinction) is also very enduring for roofs, if well painted every 1 or 2 years. The chief advantage of galvanized roofing is that it does not require painting so often as the black. The galvanizing adds about ⅛ of a lb per square foot of surface, or about ¼ lb per sq ft of sheet as coated on both sides; without regard to the thickness of the sheet. Paint for roofs should not have much *dryer*. See Painting, p 429.

The sulphurous fumes from coal are very corrosive of EITHER GALVANIZED OR BLACK IRON; as may be seen in shops, railroad bridges, or engine houses, roofed with either; if efficient means are not provided for carrying off the smoke; and the same with other metals. THE ACID OF OAK TIMBER is said to destroy the zinc of galvanized iron. See pp 418, 419. Flat iron is usually nailed upon a sheeting of boards; but the strength of corrugated iron obviates the necessity for this, and enables it to stretch 5 or 6 ft from purlin to purlin, without intermediate support. The corrugated sheets are riveted together on the roof, by rivets of galvanized wire about one-eighth inch thick, 300 to a pound, well driven (so as to exclude rain) 3 or 4 inches apart, all around the edges. The rivet-holes are first punched by machinery, so as to insure coincidence in the several sheets; and the rivets are driven by two men, one above, and one beneath the roof. For black iron, ungalvanized nails, boiled in linseed oil as a partial preservative from rust, are commonly used; as also in shingling or slating. Galvanized ones, however, would be better in all these cases; or even copper ones for slating because good slate endures much longer than either shingles or iron, and therefore it becomes true economy to use durable metals for fastening it. In none of these cases, however, are the nails fully exposed to the weather.

The sheets of flat iron are put together by overlapping and FOLDING THE EDGES, much the same as shown by the fig page 418, head Tin; the joints which run up and down the roof being the same as at *s a*, and the horizontal ones as at *t t*; except that inasmuch as these are not soldered in the iron sheets, the joint is made about ¾ to 1 inch wide, instead of ½ inch, the better to provide against leaking. Cleats are used as in tin, with 2 nails to a cleat. The iron plates are best laid on sheeting boards; but in sheds, &c, are sometimes laid directly on rafters, not more than about 18 ins apart in the clear; the plates being allowed to sag a little between the rafters, so as to form shallow gutters. In such cases it is well to bevel off the tops of the rafters slightly, as in this fig.



A serious objection to iron as a roof covering, is its rapid condensation of atmospheric moisture; which falls from the iron in drops like rain, and may do injury to ceilings, floors, or articles in the apartments immediately beneath the roof. Painting does not appreciably diminish this: it may, however, be obviated by plastering, as shown at 11, of Figs 21½, of Trusses, p 583. Corrugated iron makes an excellent permanent street or other awning.

Corrugated sheet iron. The size of sheets generally used for corrugating, is 30 inches wide by 96 inches long. Corrugation reduces the width to 27½ inches. When the corrugated sheets are laid upon the roof, the overlapping of about 2½ inches along the sides, and of 4 inches along their ends, diminishes the area of roof covered by a sheet, to about seven-eighths of that of the entire corrugated sheet itself; or, the weight per square foot of roof covered, will be about one-seventh greater than that per square foot of the corrugated sheet; or, the weight of corrugated iron per square foot of roof covered is about one-fifth greater than that of the flat sheets from which it is made.

About 6 inches are usually allowed for the extension over the eaves.

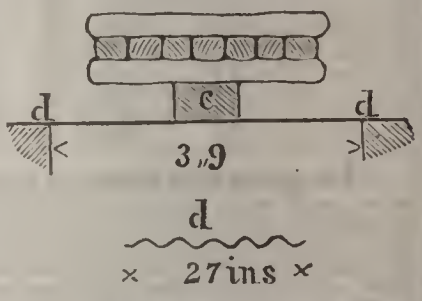
The weights per square foot corresponding to the different numbers of the Birmingham wire gauge, vary somewhat with the different makers. The two styles of corrugation given in the table below, 5 x 1¼ and 2½ x ⅝, are those most frequently used.

* Marshall, Bros & Co, Front St and Girard Ave, Phila. McDaniel & Harvey Co, 16th St and Washington Ave, Phila.

No. Bmghm wire ga.	Thickness in ins.	Wt in lbs per sq ft of *sheets*.		Wt in lbs per sq ft of *roof*.		Approx prices in cts per lb, Phila, 1886.	
Black	Black	Black	Lead coated† or galv'd	Black	Lead coated† or galv'd	Black	Lead coated† or galv'd
20	.035	1.84	2.	2.12	2.3	3¼	5¼
22	.028	1.50	1.6	1.73	1.84	3½	5½
24	.022	1.20	1.25	1.38	1.44	3½	5½
26	.018	1.00	1.12	1.15	1.29	3¾	5¾

Strength of Corrugated Iron. Experiments by the author.

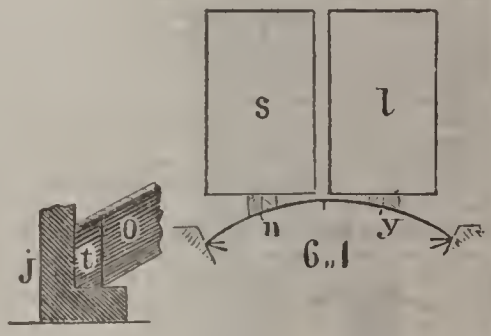
First. A sheet *d d*, of No. 16 iron, (about $\frac{1}{15}$ inch thick,) 27 ins wide, by 4 ft long, with five complete corrugations of 5 ins by 1 inch, was laid on supports 3 ft 9 ins apart. A block of wood *c*, 9 ins wide, by 7 ins thick, and 30 ins long, was placed across the center, and gradually loaded with castings weighing 1600 lbs.



This caused a deflection at the center of precisely ½ an inch. On the removal of the load after an hour, no permanent set was appreciable. The severity of the test was purposely increased by applying the several castings very roughly, jolting the whole as much as possible.* The suspended area of the sheet was 8.44 sq ft; and since the actual *center* load of 1600 lbs is about equivalent to 3000 lbs *equally distributed*, it amounts to $\frac{3000}{8.44} = 355$ lbs per sq ft distributed. But 3000 lbs distributed would produce a deflection of but about full ¼ of an inch. Again, 355 lbs per sq ft is about 4 times the weight of the greatest crowd that could well congregate upon a floor. Consequently this iron, at 3′ 9″ span, is safe in practice for any ordinary crowd. Moreover, such a crowd would produce a center deflection of only the ¼th part of ¼ of an inch; or $\frac{1}{16}$ of an inch; or $\frac{1}{720}$ of the clear span; which is but two-thirds of Tredgold's limit of $\frac{1}{480}$ of the span.

In one experiment the ends of the sheets rested upon supports dressed so as to present undulations corresponding tolerably closely with the shape of the corrugations; but in the other the supports were flat, and each end of the sheet rested only upon the lower points of the corrugations. No appreciable difference was observed in the results

Second. An arch of No. 18 ($\frac{1}{20}$ inch) **iron,** corrugated like the foregoing, but the depth of corrugation increased to 1¼ ins by the process of arching the sheet; clear span 6 ft 1 inch; rise 10 ins; breadth 27 ins. (of which, however, only 25 ins bore against the abutments)



Each foot *o* of the arch abutted upon a casting *j*, the inner portion *t* of which was undulated on top, to correspond with the corrugations of the arch, which rested upon it. At *y*, (one-fourth of the span,) two wooden blocks were placed, occupying a width of 9 inches, and extending across the arch; on them was piled a load, *l*, of castings, to the extent of 4480 lbs, or 2 tons. Under this load the arch descended about half an inch at *y*, becoming flatter on that side, and slightly more curved upward along the unloaded side *n*. Two similar blocks were then placed at *n*, and two tons of load, *s*, were piled upon them, in addition to the 2 tons at *l*; making a total of 8960 lbs, or 4 tons. This brought the arch more nearly back to its original shape; but still slightly straightened at both *n* and *y*, and a little more curved in the center. The load was then increased to 10000 lbs, and left standing for several days. Two iron ties, each ½ by 1¾, which were used for preventing the abutment castings *j* from spreading, were found to have stretched nearly ¼ of an inch. Additional ones were inserted, and the load increased to a total of 6 tons, or 13440 lbs; parts of it on *s* and *l*, and part in the shape of long broad bars of iron at the center of the arch, below the loads *s* and *l*, and between *n* and *y*. So far as could be judged by eye, the shape of the arch was now almost perfect. *The loads s and l did not touch each other.* After standing more than a week, the load was accidentally overturned, crippling the arch. The load was equal to about 1000 lbs per sq ft of the arch. Such arches have since come into common use instead of brick, **for fireproof floors.**

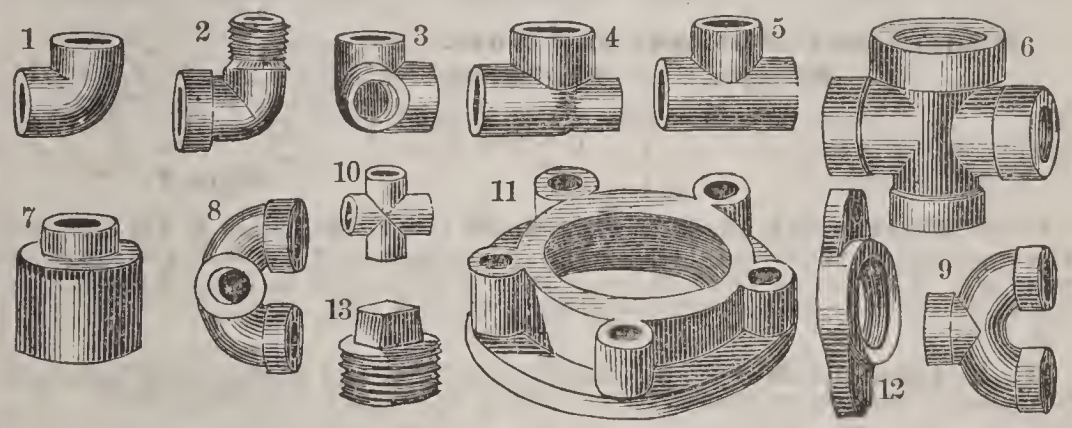
Curved roofs of 25 to 30 ft span, rising about ¼ span, may be made of ordinary corrugated iron of Nos 16 to 13, riveted as usual; and having no accessories except tie-rods a few feet apart; continuous angle-iron skewbacks; and thin vertical rods to prevent the ties from sagging.

* Without letting the deflection exceed ½ inch; which was prevented by a stop under the sheet.
† Marshall, Bros & Co, Front St and Girard Ave, Phila.

Dimensions, weights, and *list*-prices of *standard* sizes of welded **wrought-iron pipe,*** for steam, gas, and water; (**for boiler tubes,** see lower table) usually in lengths of about 18 ft. Other sizes and lengths made to order, at extra prices. Pipes from ⅛ inch to 1¼ ins inner diam ("nominal") are usually "*butt*-welded;" larger sizes, "*lap*-welded." **Discounts,** 1886: on butt-welded, about 25 to 30 per cent; on lap-welded, about 45 to 50 per cent.

Inner Diam.		Thickness.	Wt per foot.	Threads per inch of screw.	Price per ft run. Adopted June 11, 1884.	
Nominal.†	Actual.				Plain.	Galv'd.
Ins.	Ins.	Ins.	Lbs		$	$
⅛	.270	.068	.24	27	.03¼	
¼	.364	.088	.42	18	.03¼	.05
⅜	.494	.091	.56	18	.03¾	.05½
½	.623	.109	.84	14	.04¾	.06
¾	.824	.113	1.12	14	.06	.08
1	1.048	.134	1.67	11½	.08	.10½
1¼	1.380	.140	2.24	11½	.11	.14
1½	1.611	.145	2.68	11½	.21	.24
2	2.067	.154	3.61	11½	.26	.30
2½	2.468	.204	5 74	8	.42	.47
3	3.067	.217	7.54	8	.55	.62
3½	3.548	.226	9.00	8	.67	.83
4	4.026	.237	10.66	8	.83	1.00
4½	4.508	.246	12.34	8	1.00	1.25
5	5.045	.259	14.50	8	1.20	1.50
6	6 065	.280	18.76	8	1.50	2.00
7	7.023	.301	23.27	8	2.00	
8	7.982	.322	28.18	8	2.75	
9	9.001	.344	33.70	8	3.70	
10	10 019	.366	40.06	8	4.75	
11	11.	.375	45.02	8	5.75	
12	12.	.375	49.00	8	6.50	
13	13.25	.375	54.00	8	7.75	
14	14.25	.375	58.00	8	9 00	
15	15.25	.375	61.00	8	10.00	

Fittings for Wrought-iron Pipes. 1, Elbow. 2, Service Elbow. 3, Elbow with side outlet. 4, Reducing T. 5. T. 6, Reducing Cross. 7, Reducing Coupling or Socket. 8, Return Bend with side outlet. 9, Return Bend with back outlet. 10, Cross. 11, Flange Union. 12, Oval Flange. 13, Plug.



Dimensions, weights, and *list*-prices of *standard* sizes of lap-welded **wrought-iron boiler tubes,*** in lengths up to 20 ft. Other sizes and lengths made to order, at extra prices. **Discount,** 1886, about 40 to 45 per cent.

Outer Dia.‡	Ths	Ths.	Wt per ft.	Price§ per ft.	Outer Dia.‡	Ths	Ths.	Wt per ft.	Price§ per ft.	Outer Dia.‡	Ths	Ths.	Wt per ft.	Price§ per ft.
Ins.	Ins.	B'gm w. ga.	Lbs.	$	Ins.	Ins.	B'gm w. ga.	Lbs.	$	Ins.	Ins.	B'gm w. ga.	Lbs.	$
1	.072	15	.70	.23	3	.109	12	3.33	.34	8	.165	8	13.65	1.85
1¼	.072	15	.90	.23	3¼	.120	11	3.96	.38	9	.180	7	16.76	2.25
1½	.083	14	1.24	.23	3½	.120	11	4.28	.43	10	.203	6	21.00	2.75
1¾	.095	13	1.66	.22	3¾	120	11	4.60	.45	11	.220	5	25.00	3.25
2	.095	13	1 91	.22	4	.134	10	5.47	.52	12	.229	4½	28.50	3.55
2¼	.095	13	2.16	.25	4½	.134	10	6.17	.60	13	.238	4	32.06	4.20
2½	.109	12	2.75	.28	5	.148	9	7.58	.72	14	.248	3½	36.00	4.75
2¾	.109	12	3.04	.31	6	.165	8	10.16	1.00	15	.259	3	40.60	5.75
					7	.165	8	11.90	1.45	16	.270	2½	45.20	6 75

* **The Allison Manufacturing Co,** 32d and Walnut Sts, Phila, manufacture pipes, tubes, fittings, railroad cars and furnish appliances for roofs, buildings, and bridges, railroad supplies, &c, &c.

Morris, Tasker & Co, Limited, Pascal Iron Works, Phila, manufacture pipes and tubes, and their fittings, iron and brass valves and cocks, machines and tools for manipulation of pipes &c.

† In ordering *pipes*, give the "nominal" *inner* diameter. It is merely an arbitrary name for the pipe, and, in some cases, tends to mislead. Thus, the pipe whose "*nominal*" inner diam is one-eighth inch, has an "*actual*" inner diam of *full quarter* inch.

‡ In ordering *boiler tubes*, give the *outer* diameter.

§ Adopted December 17, 1883.

Bolts, nuts, and washers. The following dimensions for bolts and nuts were proposed by Mr. Wm. Sellers and adopted by the Franklin Institute, of Philadelphia, as a standard, in 1864, and by the U. S. Navy Dept* in 1868. They have been adopted by the principal machinists of the country, and are known as "**Franklin Institute Standard,**" or "**Sellers,**" **dimensions.** The angle *a*, Fig 1, between the two sides of a thread, is 60°. *w* is the width, (measured lengthwise of the bolt) of the flat top and bottom of each thread. N is the number of threads per inch of length of screw.

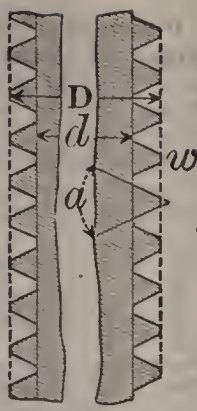
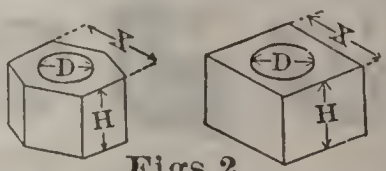


Fig.1

D ins	d ins	w ins	N	D ins	d ins	w ins	N	D ins	d ins	w ins	N	D ins	d ins	w ins	N
1/4	.185	.0062	20	1	.837	.0156	8	2	1.712	.0277	4 1/2	4	3.567	.0413	3
5-16	.240	.0074	18	1 1/8	.940	.0178	7	2 1/4	1.962	.0277	4 1/2	4 1/4	3.798	.0435	2 7/8
3/8	.294	.0078	16	1 1/4	1.065	.0178	7	2 1/2	2.176	.0312	4	4 1/2	4.028	.0454	2 3/4
7-16	.344	.0089	14	1 3/8	1.160	.0208	6	2 3/4	2.426	.0312	4	4 3/4	4.256	.0476	2 5/8
1/2	.400	.0096	13	1 1/2	1.284	.0208	6	3	2.629	0357	3 1/2	5	4.480	.0500	2 1/2
9-16	.454	.0104	12	1 5/8	1.389	.0227	5 1/2	3 1/4	2.879	.0357	3 1/2	5 1/4	4.730	.0500	2 1/2
5/8	.507	.0113	11	1 3/4	1.491	.0250	5	3 1/2	3.100	.0384	3 1/4	5 1/2	4.953	.0526	2 3/8
3/4	.620	.0125	10	1 7/8	1.616	.0250	5	3 3/4	3.317	.0413	3	5 3/4	5.203	.0526	2 3/8
7/8	.731	.0138	9									6	5.423	.0555	2 1/4

Dimensions of Heads and Nuts.

	Rough.	Finished.
X	1 1/2 D + 1/8 inch.	1 1/2 D + 1-16 inch.
H (in *head*)	1/2 X.	D — 1-16 inch.
H (in *nut*)	D.	" "



Figs.2

In the **Whitworth** (English) **standard** thread, the angle *a*, Fig 1, is 55°. The tops and bottoms of the threads are rounded, instead of flat as in the American standards. The number (N) of threads per inch is the same as above for diams of bolt up to three ins, except for D = 1/2 inch; where N = 12.

Plate-iron washers. Standard sizes. Diams of washers and bolt-holes in ins. Approx thickness by Birmingham wire gauge, p 410. Approx number in one lb.

Diams.		Ths.	No.	Diams.		Ths.	No.	Diams.		Ths.	No.
1/2	1/4	18	450	1 1/4	1/2	.14	43	2 1/4	15-16	9	8.6
5/8	5-16	16	210	1 3/8	9-16	12	26	2 1/2	1 1-16	9	6.2
3/4	5-16	16	139	1 1/2	5/8	12	22.5	2 3/4	1 1/4	9	5.2
7/8	3/8	16	112	1 3/4	11-16	10	13.1	3	1 3/8	9	4.
1	7-16	14	68	2	13-16	10	10.1	3 1/2	1 1/2	9	2.8

Price in Philadelphia, 1886, about 5 to 6 cts per lb for outer diams from 1 1/4 to 3 1/2 ins.

* Except that the Navy Dept uses, *for both rough and finished heads and nuts*, the dimensions given above for *rough* heads and nuts. The standard of the Navy Dept is known as "**United States Standard.**"

A *square* head and nut together, weigh about as much as a length of the bolt equal to 7 or 8 times D. *Hexagon*, 6 or 7.

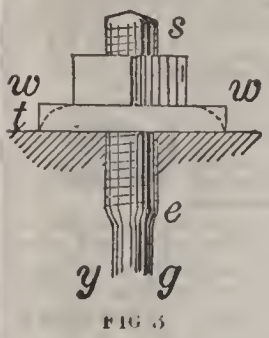


FIG. 3

With the above dimensions a bolt will generally fail by breaking off between the head and the nut, where the diameter is decreased by cutting the thread, rather than by stripping off its threads. **The diam D of the thread** must of course be greater than that required to bear safely the proposed tensile strain, by an amount equal to *twice* the depth of the thread. The waste of iron, which would result from making the *entire bolt* of this greater diam, is frequently avoided by making the bolt from a bar of only sufficient dimensions to bear the strain safely, **and upsetting its ends** as in Fig 3, thus increasing their diam sufficiently to allow for the cutting of the threads. But see **Rem,** p 408.

In carpentry, as well as in ties for masonry, *washers*, *w w*, of either cast or wrought iron, are placed between the timber, or stone, and the head and nut; in order to distribute the pressure over a greater surface, and thus prevent crushing; especially in timber.

When much strained against wood, the side of a square wrought-iron washer; or the diam *w w* of a circular one, should not be less than 4 diams of the *screw*, as in the fig; and its thickness, *t w*, ¼ diam *at least.*

Two such square washers will together weigh as much as 18 diams in length of a round rod of the same diam as the screw. Two round washers will weigh together as much as 14 diams of rod of same diam as screw. In either case, a square head and nut will weigh as much as 6 diameters. Cast-iron washers, being more apt to split under heavy strains, may be made about twice as thick as wrought ones. When the strain is very great, the diam of the washer may be 5 or 6 times that of the screw; and its thickness equal to diam; but 4 diams will suffice for most practical purposes, or even 2.5 when there is but little strain, and the thickness may then be but .1 or .2 diam of bolt.

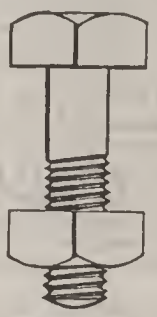


Fig. 4.

Fig. 5.

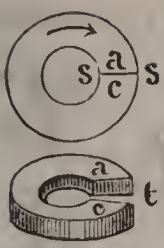
Table of machine and car bolts, with square and hexagon heads and nuts, Figs 4 and 5; made by Hoopes & Townsend, 1330 Buttonwood St, Phila. All their **bolts are cut with U. S. Standard threads,** as per table on p 406, unless otherwise ordered. For **washers,** see p 406.

Diam D, Fig 1, of bolt, ins.	Length ins exclusive of head.		Weight of 100 bolts with nuts, in lbs.		List price* in dollars per 100.		Size of sq head, ins.		Size of hex head, ins.		Size of nut, ins.		Chamfered *Square* Nuts.		Chamfered *Hexagon* Nuts.	
	Min.	Max.	Min.	Max.	Min.	Max.	Width.	Thickn's.	Width.	Thickn's.	Width.	Thickn's.	No. per 100 lbs.	List price† in cts per lb.	No. per 100 lbs.	List price‡ in cts per lb.
¼	1½	8	4	13.75	2.80	4.10	7-16	3-16	7-16	¼	½	¼	7300	25	8200	35
5-16	"	"	7	20.75	3.20	5.15	½	¼	½	5 16	19-32	5-16	4700	23	5400	33
⅜	"	12	10.5	44.5	3.60	7.80	⅝	5-16	⅝	⅜	11-16	⅜	2550	19	3000	26
7-16	"	"	15.2	61.4	4.60	10.90	11-16	⅜	11-16	7-16	25-32	7-16	1700	19	2030	26
½	"	20	22.5	123.2	5.00	16.10	13-16	7-16	13-16	½	⅞	½	1180	15½	1400	20
9-16	"	"	30	159.5	7.20	25.75	⅞	½	⅞	9-16	31-32	9-16	890	15½	1080	20
⅝	"	"	39.5	196.5	7.20	25.75	1	½	1	⅝	1 1-16	⅝	645	14½	780	18½
¾	"	"	63	286.8	10.50	32.70	1 3-16	⅝	1 3-16	¾	1¼	¾	380	13½	470	16½
⅞	"	"	100	415.3	14.90	46.40	1⅜	¾	1⅜	⅞	1 7-16	⅞	260	13½	310	16½
1	"	"	153	558.	22.00	62.70	1½	⅞	1 9-16	1	1⅝	1	172	13	212	16

* For bolts with *square* heads and nuts. Bolts with *hex* heads and nuts are about 20 per cent higher. **Discount,** 1886, about 75 per cent. Bolts are also made (at extra prices) with **button-shaped** and **countersunk** heads. The price per 100 varies with the length. We give the extremes. The bolts have finished points, and chamfered heads and nuts, as shown. From ½ inch to 8 ins **the lengths increase** by ½ ins; from 8 ins to 20 ins, by ins.

† Not tapped. **Discount,** 1886, about 9½ cts per lb.

‡ Not tapped. **Discount,** 1886, about 10 cts per lb.



Lock-nut washers. When bolts are subjected to much rough joltiug, as at rail-joints, &c, the *nuts are liable to wear loose, and unscrew themselves.* Ou railroads this is a source of great annoyance, and innumerable devices for preventing it have been tried. The **Verona** LOCK-NUT washer is a simple circular washer made of steel; with a slit *s s* cut through it, leaving sharp edges. Ou one side, *a*, of the slit, the metal is pressed upward about ⅛ inch; and that on the other side, *c*, downward, the same distance; so that a perspective view would be somewhat as at *t*. Now, when the nut is screwed down over the washer, in the direction of the arrow, the slit offers no obstruction; but if the nut afterward tends to unscrew itself, the sharp upper edge of the slit, along *a*, presents friction against the bottom of the nut, which tends to hold it in place. Besides, the washer, by its elasticity, tends to resume its original shape, and thus presses the threads of the nut against those of the bolt; and the additional friction thus produced, also aids in holding the nut. The same principles are employed in a nut-lock recently (1884) introduced upon the **Houston and Texas Central,** where the lock-nut washer is a long strip of steel, with *two* holes, each of which has its edges formed like those of a Verona washer, and through *each* of which passes one of the bolts of the rail joint. **Another device** is to cut at the end of the screw a few threads of a screw of less diam than the main one, and in the opposite direction. The nut is then screwed upon the larger diam; and after it the lock-nut is screwed in the other direction upon the smaller diam, until it comes into contact with the main nut. In the **Smith** lock-nut bolt, this second nut is only about ⅛ inch thick; and after being driven home, one of its corners is bent over the edge of the main nut. These bolts cost, in 1884, about 5 cts each.

See the **Cambria** nut-lock, p 765,

The **Atwood** lock-nuts take advantage of elasticity in the nut itself, which is obtained either by slitting the nut, or by reducing its thickness near the bolt hole.

It is claimed that **if the threads** of an ordinary bolt and nut are carefully cut, so as to be **in contact with each other throughout,** no lock-nut contrivance is necessary, because the friction between the two threads is distributed over a larger surf, and *abrasion* does not take place so readily as if the threads touched each other at only a few points. The nuts are therefore less apt to wear loose under repeated jarring.

Owing to the difficulty of obtaining such perfect fitting bolts and nuts, due to the wear of the cutting tools used in their mfr, the **Harvey Screw and Bolt Co,** 52 Wall St, New York, furnish bolts and nuts in which the thread on the bolt differs slightly in shape from that in the nut. They also furnish nuts in which the thread, instead of being of uniform shape throughout, gradually becomes *deeper* and *thicker*, by having its side angle *a*, Fig 1, p 406, made more acute, and its top truncated. These nuts are used with bolts having the usual uniform thread. The bolt enters the nut upon the side where the thread is of the same shape as its own; but its thread encounters, and is forced into, the gradually narrowing and deepening path between the threads of the nut. In both devices, the enforced conformity between the two threads, is relied upon to give the desired completeness of contact between them. The greater force required in screwing on the nut, also increases the *friction* between the threads.

Table of diameters, weights, and approximate breaking strains, for round rolled iron bolts, ties, or bars; assuming the breaking strain per square inch of average quality of rolled iron to be as follows: Up to 1 inch square, or 1 inch diam, 20 tons, or 44800 lbs; from 1 to 2 ins sq or diam, 19 tons: 2 to 3 ins, 18 tons; 3 to 4 ins, 17 tons; 4 to 5 ins, 16 tons; 5 to 6 ins, 15 tons. The first 4 columns of the table are to be used when the screw end of the bolt is enlarged or upset, so that the shank or body of the bolt shall not be weakened by the cutting of the screw threads. But when the shank is so weakened, the diam and weight of the bolt must be taken from the last 2 cols.

Rem. But it is **very important** to know that a long upset rod is no stronger than one not upset, against *slowly applied* loads or strains. Both will then break at about midlength, under equal pulls. Therefore in such cases the col of greatest diams in the table should be used.

EXAMPLE 1. To find the diam of a bolt, that shall just break under a strain, or a load of 52.5 tons, we see by the table and opposite 52.5 tons, that it will be 1⅞ ins if the screw end is enlarged, and 2.3 ins if it is not. In the first case, the weight of the bolt will be 9.3 lbs per foot run; and in the second, 13.8 lbs.

EXAMPLE 2. What must be the diam in order to sustain a strain of 52.5 tons, with a safety of 3? Here $52.5 \times 3 = 157.5$ tons. In the table, the nearest we find to 157.5 tons, is 163.6; opposite which we find the diam 3½ ins. A diam a trifle less than this will *break* under a strain of 157.5 tons; and consequently will have a safety of 3 for 52.5 tons.

The breakg strains in this table will also answer for square BARS, by merely increasing them ¼ part; for a round bar has very approximately $\frac{4}{5}$ of the strength of a square one whose side is equal to the diam of the round one; or the square one has 1¼ times the strength of the round one; or, more correctly, as 1 to .7854. **For the strength of COPPER bolts,** multiply the tabular ones by the decimal .8; and for their weight, increase, that of iron ones $\frac{1}{7}$ part. Heads, nuts, and washers, are not included in the table.

WEIGHT AND STRENGTH OF IRON BOLTS. (Original.)

For **square** ones or for **copper** see preceding paragraph.

Ends enlarged, or upset.				Ends not enlarged.		Ends enlarged, or upset.				Ends not enlarged.	
Diam. of shank	Weight per foot run.	Breaking strain.	Breaking strain.	Diam. of shank	Weight per foot run.	Diam. of shank	Weight per foot run.	Breaking strain.	Breaking strain.	Diam. of shank	Weight per foot run.
Ins.	Pds.	Tons.	Pds.	Ins.	Pds.	Ins.	Pds.	Tons.	Pds.	Ins.	Pds.
1/8	.0414	.245	549			1 3/4	8.10	45.7	102368	2.14	12.0
3/16	.093	.553	1239			1 13/16	8.69	49.0	109760	2.22	12.9
1/4	.165	.983	2202	.35	.321	1 7/8	9.30	52.5	117600	2.30	13.8
5/16	.258	1.53	3427	.43	.452	1 15/16	9.93	56.0	125440	2.38	14.7
3/8	.372	2.21	4950	.50	.654	2.	10.6	59.7	133728	2.45	15.7
7/16	.506	3.00	6720	.58	.897	2 1/8	12.0	63.8	142912	2.59	17.5
1/2	.661	3.93	8803	.66	1.14	2 1/4	13.4	71.6	160384	2 73	19.5
9/16	.837	4.97	11133	.73	1.41	2 3/8	14.9	79.7	178528	2.88	21.6
5/8	1.03	6.14	13754	.80	1.67	2 1/2	16.5	88.4	198016	3.02	23.9
11/16	1.25	7.42	16621	.88	2.03	2 5/8	18.2	97.4	218176	3.16	26.1
3/4	1.49	8.83	19779	.96	2.41	2 3/4	20.0	106.9	239456	3.30	28.5
13/16	1.75	10.4	23296	1.04	2.81	2 7/8	21.9	116.8	261632	3.45	31.1
7/8	2.03	12.0	26880	1.12	3.26	3.	23.8	127.2	284928	3.60	33.9
15/16	2.33	13.8	30912	1.20	3.77	3 1/4	27.9	141.0	315840	3.86	39.1
1 in.	2 65	15.7	35168	1.27	4.27	3 1/2	32.4	163.6	366464	4.12	44.4
1 1/16	2.99	16.8	37632	1.35	4.77	3 3/4	37.2	187.7	420448	4.41	51.0
1 1/8	3.35	18.9	42336	1.42	5.28	4.	42.3	213.6	478464	4.70	57.8
1 3/16	3.73	21.1	47264	1.49	5.81	4 1/4	47.8	227.0	508480	4.98	65.2
1 1/4	4.13	23.3	52192	1.55	6.39	4 1/2	53.6	254.5	570080	5.25	72.9
1 5/16	4.56	25.7	57568	1.64	7 04	4 3/4	59.7	283.5	635040	5.53	80.5
1 3/8	5.00	28.2	63168	1.72	7.74	5.	66.1	314.2	703808	5.80	88.1
1 7/16	5.47	30.8	68992	1.80	8.48	5 1/4	72.9	324.7	727328	6.08	97.0
1 1/2	5.95	33.6	75264	1.87	9.20	5 1/2	80.0	356.4	798336	6.36	106.
1 9/16	6.46	36.4	81536	1.94	9 88	5 3/4	87.5	389.5	872480	6.63	116.
1 5/8	6.99	39.4	88256	2.00	10.6	6.	95.2	424.1	949984	6.90	126.
1 11/16	7.53	42.5	95200	2.07	11.3		**See Rem, p 408.**				

BUCKLED PLATES

of iron or steel are usually 3 or 4 feet square, from 1-20th to 3-8th inch thick, with a flat rim about 2 inches wide all around, with rivet or bolt holes for holding the plate firmly down to its intended place. The rest of the plate is stamped into the form of a kind of groined arch rising from 1 to 3 inches in the center. They are very strong, and are used for the floors of fire-proof buildings, and of city iron bridges, covered with asphalt or stone paving, &c. One of 3 feet square, .25 inch thick, curved 1.75 inches, and with a 2-inch rim well bolted down on all sides, required a quiet, equally distributed load of 18 tons to crush it. When unbolted the strength is only half as great. **Buckled plates of soft puddled steel** bear nearly twice as much.

Table of safe, quiet, uniformly distributed loads for buckled iron plates 3 feet square, arched 1.75 inches, and well bolted down on all sides. **Keystone Bridge Co,** Pittsburgh, Pa. Prices, iron 3½ cents per lb at mill; steel, 4.

Thickness.		Weight of one plate.	Safe load on one plate (= one-fourth of ultimate load).	
Birmingham wire gauge.	Inch.	Pounds.	Tons of 2240 lbs.	Pounds.
18	.048	17	.27	604
16	.066	24	.43	963
12	.107	39	.64	1433
	1-8th	45	1.00	2240
	3-16th	68	2.5	5600
	1-4th	90	4.5	10080
	5-16th	113	6.2	13888
	3-8th	135	9.0	20160

The Birmingham **wire gauge** is the one in most general use for iron. The new British w g went into effect March 1st 1884. In the "American" w g of Darling, Brown & Sharpe, Providence R. I., each diam or thick is = the next smaller one × 1.122932. We take the wt of wrot iron per cub ft at 485 lbs in the first two; and at 486 in the last. For the **wt of steel,** mult that of iron by 1.01. For **lead,** mult iron by 1.46. For **zinc,** mult iron by .9. For **brass** (approx), mult iron by 1.06. For **copper,** mult iron by 1.134. See p 411 and Trenton Gauge p 412.

No.	**Birmingham** W. Ga.			**New British** W. Ga.			**American** W. Ga.		
	Diam of wire, or thickness of sheet, ins.	Wt of iron wire, in lbs per lin ft.	Wt of iron sheets, in lbs per sq ft.	Diam of wire, or thickness of sheet, ins.	Wt of iron wire, in lbs per lin ft.	Wt of iron sheets, in lbs per sq ft.	Diam of wire, or thickness of sheet, ins.	Wt of iron wire, in lbs per lin ft.	Wt of iron sheets, in lbs per sq ft
7-0	.....	.....	.....	.500	.661	20.21			
6-0	.....	.....	.....	.464	.569	18.75			
5-0	.....	.....	.....	.432	.494	17.46			
4-0	.454	.546	18.35	.400	.423	16.17	.460000	.561	18.63
3-0	.425	.479	17.18	.372	.366	15.03	.409642	.445	16.58
2-0	.380	.383	15.36	.348	.320	14.06	.364796	.353	14.77
0	.340	.306	13.74	.324	.278	13.09	.324861	.280	13.15
1	.300	.238	12.13	.300	.238	12.13	.289297	.222	11.70
2	.284	.214	11.48	.276	.202	11.15	.257627	.176	10.43
3	.259	.178	10.47	.252	.168	10.19	.229423	.139	9.291
4	.238	.150	9.619	.232	.142	9.377	.204307	.111	8.273
5	.220	.128	8.892	.212	.119	8.568	.181940	.0877	7.366
6	.203	.109	8.205	.192	.0976	7.760	.162023	.0696	6.561
7	.180	.0859	7.275	.176	.0820	7.113	.144285	.0552	5.842
8	.165	.0721	6.669	.160	.0677	6.466	.128490	.0438	5.203
9	.148	.0580	5.981	.144	.0548	5.820	.114423	.0347	4.633
10	.134	.0476	5.416	.128	.0434	5.173	.101897	.0275	4.125
11	.120	.0382	4.850	.116	.0357	4.688	.090742	.0218	3.674
12	.109	.0315	4.405	.104	.0286	4.203	.080808	.0173	3.272
13	.095	.0239	3.840	.092	.0224	3.718	.071962	.0137	2.914
14	.083	.0183	3.355	.080	.0169	3.233	.064084	.0109	2.595
15	.072	.0137	2.910	.072	.0137	2.910	.057068	.00863	2.310
16	.065	.0112	2.627	.064	.0108	2.587	.050821	.00684	2.053
17	.058	.00891	2.344	.056	.00832	2.263	.045257	.00543	1.832
18	.049	.00636	1.980	.048	.00610	1.940	.040303	.00430	1.631
19	.042	.00467	1.697	.040	.00423	1.617	.035890	.00341	1.452
20	.035	.00325	1.415	.036	.00344	1.455	.031961	.00271	1.293
21	.032	.00271	1.293	.032	.00269	1.293	.028462	.00215	1.152
22	.028	.00208	1.132	.028	.00207	1.132	.025346	.00170	1.026
23	.025	.00166	1.010	.024	.00152	.9700	.022572	.00135	.913
24	.022	.00128	.8892	.022	.00128	.8891	.020101	.00107	.814
25	.020	.00106	.8083	.020	.00106	.8083	.017900	.000849	.724
26	.018	.000859	.7225	.018	.000857	.7275	.015941	.000673	.644
27	.016	.000678	.6467	.0164	.000712	.6628	.014195	.000534	.574
28	.014	.000519	.5658	.0148	.000579	.5982	.012641	.000423	.511
29	.013	.000448	.5254	.0136	.000489	.5497	.011257	.000336	.455
30	.012	.000382	.4850	.0124	.000408	.5012	.010025	.000266	.405
31	.010	.000265	.4042	.0116	.000357	.4688	.008928	.000211	.360
32	.009	.000215	.3638	.0108	.000309	.4365	.007950	.000167	.321
33	.008	.000170	.3233	.0100	.000265	.4042	.007080	.000133	.286
34	.007	.000130	.2829	.0092	.000224	.3718	.006305	.000105	.254
35	.005	.0000662	.2021	.0084	.000187	.3395	.005615	.0000837	.226
36	.004	.0000424	.1617	.0076	.000153	.3072	.005000	.0000662	.202
37	.....	.....	.....	.0068	.000122	.2748	.004453	.0000525	.180
38	.....	.....	.....	.0060	.0000952	.2425	.003965	.0000417	.159
39	.....	.....	.....	.0052	.0000714	.2102	.003531	.0000330	.142
40	.....	.....	.....	.0048	.0000608	.1940	.003144	.0000262	.127
41	.....	.....	.....	.0044	.0000513	.1778			
42	.....	.....	.....	.0040	.0000423	.1617			
43	.....	.....	.....	.0036	.0000344	.1455			
44	.....	.....	.....	.0032	.0000271	.1293			
45	.....	.....	.....	.0028	.0000207	.1132			
46	.....	.....	.....	.0024	.0000152	.0970			
47	.....	.....	.....	.0020	.0000106	.0808			
48	.....	.....	.....	.0016	.0000068	.0647			
49	.....	.....	.....	.0012	.0000038	.0485			
50	.....	.....	.....	.0010	.0000026	.0404			

No trade Stupidity is more thoroughly senseless than the adherence to the various Birmingham, Lancashire, &c, gauges; instead of at once denoting the thickness and diameter of sheets, wire, &c, by the parts of an inch; as has long been suggested. Thus, No. 1/8, or No. 1/20 wire, or sheet-metal of any kind, should be understood to mean 1/8 or 1/20 of an inch diam, or thickness. To avoid mistakes, which are very apt to occur from the number of gauges in use; and from the absurd practice of applying the same No. to different thicknesses of different metals, in different towns, it is best to ignore them all; and in giving orders, to define the diameter of wire, and the thickness of sheet-metal, by parts of an inch. Or the weight per hundred ft for wire; or per sq ft for sheets, may be employed. We *believe* that the *foregoing* Birmingham gauge applies to zinc, copper, brass, and lead; although it is generally stated to be for iron and steel only. Another Birmingham gauge is used for sheet-brass, gold, silver, and some other metals; but we have never seen it stated what those others are. There are different gauges even for wire to be used for different *purposes;* and various firms have gauges of their own; not even according among themselves.

As Mr. Stubs makes various English gauges, the term **"Stubs gauge"** *by itself* means nothing. Generally, however, in our machine shops, it applies to the Birmingham gauge of the preceding table.

Birmingham gauge for sheet Brass, Silver, Gold, and all metals except iron and steel?

No.	Thickn's.	No.	Thickn's.	No.	Thickn's.	No.	Thickn's.	No.	Thickn's.	No.	Thickn's.
	Inch		Inch		Inch		Inch		Inch		Inch
1	.004	7	.015	13	.036	19	.064	25	.095	31	.133
2	.005	8	.016	14	.041	20	.067	26	.103	32	.143
3	.008	9	.019	15	.047	21	.072	27	.113	33	.145
4	.010	10	.024	16	.051	22	.074	28	.120	34	.148
5	.012	11	.029	17	.057	23	.077	29	.124	35	.158
6	.013	12	.034	18	.061	24	.082	30	.126	36	.167

The mills rolling sheet iron in the United States generally use the following, which varies slightly from the Birmingham gauge:

No.	lbs per sq ft	No.	lbs per sq ft	No.	lbs per sq ft	No.	lbs per sq ft
1	12 50	8	6.86	15	2.81	22	1.25
2	12.00	9	6.24	16	2.50	23	1.12
3	11.00	10	5.62	17	2.18	24	1.00
4	10.00	11	5.00	18	1.86	25	.90
5	8.75	12	4.38	19	1.70	26	.80
6	8.12	13	3.75	20	1.54	27	.72
7	7.50	14	3.12	21	1.40	28	.64

Price of brass and copper wire, approximate for 1886. **Ansonia Brass and Copper Co,** Nos. 19 and 21 Cliff St, New York. For 100 lbs or more:

Nos. 0 to 25 Copper.....30 to 40 cts per lb.
" " High brass.....22 to 32 " " "
" " Low brass.....26 to 36 " " "

Unannealed or hard **brass wire** has about 3/4ths the strengths of the table p 412, and about 1/9 more weight. If annealed, only full half the strength.

Hard copper wire may be taken at 2/3 of the tabular strengths, and full 1/7 more weight.

Table of Charcoal Iron Wire made by Trenton Iron Co., Trenton, N. J. The numbers in the first column are those of the **Trenton Iron Co's gauge.** The corresponding diameters in the second column will be seen to be somewhat less than those of the Birmingham gauge, p 410.

No.	Diam. ins.	Lineal feet to the Pound.	Tensile Str'gth Approx lbs.	No.	Diam. ins.	Lineal feet to the Pound.	Tensile Str'gth Approx lbs.	No.	Diam. ins.	Lineal feet to the Pound.
00000	.450	1.863	12598	11	.1175	27.340	1010	26	.018	1164.689
0000	.400	2.358	9955	12	.105	34.219	810	27	.017	1305.670
000	.360	2.911	8124	13	.0925	44.092	631	28	.016	1476.869
00	.330	3.465	6880	14	.080	58.916	474	29	.015	1676.989
0	.305	4.057	5926	15	.070	76.984	372	30	.014	1925.321
1	.285	4.645	5226	16	.061	101.488	292	31	.013	2232.653
2	.265	5.374	4570	17	.0525	137.174	222	32	.012	2620.607
3	.245	6.286	3948	18	.045	186.335	169	33	.011	3119.092
4	.225	7.454	3374	19	.040	235.084	137	34	.010	3773.584
5	.205	8.976	2839	20	.035	308.079	107	35	.0095	4182.508
6	.190	10.453	2476	21	.031	392.772	.....	36	.009	4657.728
7	.175	12.322	2136	22	.028	481.234	.....	37	.0085	5222.035
8	.160	14.736	1813	23	.025	603.863	.....	38	.008	5896.147
9	.145	17.950	1507	24	.0225	745.710	.....	39	.0075	6724.291
10	.130	22.333	1233	25	.020	943.396	.....	40	.007	7698.253

The wire in this table is supposed to be hard, bright. or unannealed. Annealing renders wire more pliable, but less elastic; and reduces its strength perhaps 20 or 25 per cent. **The strengths in the last column** are at the rate of 81000 lbs per sq inch for No. 0; 93000 for No. 10; and 111000 for No. 20; and are say from 15 to 25 per ct greater than those of ordinary market wire. The wires in the table are all made of the same specially prepared iron; and the increased strength per sq in of the smaller ones is due to their more frequent passage through the draw-plates. The choice Swedish and Norway irons would not yield wire of much greater strength; while the cost would be about 60 per ct more.

Hard steel wire averages about twice the strength of iron wire.

To find approximately the number of straight wires that can be got into a cable of given diameter.

Divide the diameter of the cable in inches, by the diameter of a wire in inches. Square the quotient. Multiply said square by the decimal .77. The result will be correct within about 4 or 5 per cent at most, in a *cylindrical* cable.

The solidity, or metal area of all the wires in a cable, will be to the area of the cable itself, about as 1 to 1.3. In other words, the area of the voids is nearly ¼ that of the cable; while that of the wires is fully ¾ that of the cable. All approximate.

Price-list of Charcoal Iron and Bessemer Steel Wire.* Bright and annealed. **Trenton Iron Co.**

Nos.	0 to 9,	10 & 11,	12,	13 & 14,	15 & 16,	17,	18,	19,	20,	21,	22,
Cts per lb.	10	11,	11½,	12½,	14,	15,	16,	19,	20,	21,	22,

Nos.	23,	24,	25,	26,	27,	28,	29,	30,	31,	32,	33,	34,	35,	36,
Cts per lb.	23,	24,	25,	26,	28,	29,	30,	32,	33,	35,	37,	40,	45,	55,

Price-list of Cast (crucible) Steel Wire.† Trenton Iron Co.

Nos.	0 to 6,	7 to 9,	10 & 11,	12,	13,	14,	15,	16,	17,	18,	19,	20,	21,	22,	23,
Cts per lb.	23,	24,	25,	26,	28,	30,	32,	33,	34,	36,	38,	40,	50,	60,	75.

The Co make other wire of specified quality to order; also **Wire Ropes,** and fittings for same. See p 413.

* **Discounts** on *iron* and *Bessemer steel* wire (bright or annealed). Nos. 00000 to 36, about 60 to 75 per cent. Nos. 37 to 45, about 50 per cent. 1886.

† **Discount** on *crucible steel* wire, about 50 to 60 per cent. 1886.

Table of Wire Rope manufactured by **John A. Roebling's Sons Co., Trenton, N. J. Prices,** 1886, net, except on large orders. The prices and weights given are for ropes with *hemp* centers. When made with *wire* centers the prices are one-tenth higher, and the weights one-tenth greater.

Rope of 133 Wires (19 wires in a strand).

Trade No.	Diam. Ins.	Circumf. Ins.	Pounds per foot run.	Breaking load, lbs.		Minimum diam of drum, in ft.		Price per ft run, in cents.	
				Iron.*	Cast steel*	Iron.	Cast st'l*	Iron.*	Ca't st*
1	2¼	6¾	8.00	148000	260000	8	9	90	152
2	2	6	6.30	130000	200000	7	8	67	120
3	1¾	5½	5.25	108000	156000	6.5	7.5	59	100
4	1⅝	5	4.10	88000	128000	5	6	50	80
5	1½	4¾	3.65	78000	110000	4.75	5.5	45	71
5½	1⅜	4⅜	3.00	66000	.....	4.5	..	37	...
6	1¼	4	2.50	54000	78000	4	5	31	50
7	1⅛	3½	2.00	40000	60000	3.5	4.5	25	41
8	1	3⅛	1.58	32000	48000	3	4	22	34
9	⅞	2¾	1.20	23000	40000	2.75	3.75	17	27
10	¾	2¼	0.88	17280	26000	2.5	3.5	14	21
10¼	⅝	2	0.70	10260	18000	2	3	12	18
10½	9/16	1⅝	0.44	8540	13000	1.75	2.75	10	17
10¾	½	1½	0.35	6960	11000	1.5	2	8	15
10⅞	3/8	1¼	0.26	5000	.....	1	.....	6.5	.....

Rope of 49 Wires (7 wires to the strand).

Trade No.	Diam. Ins.	Circumf. Ins.	Pounds per foot run.	Breaking load, lbs.		Price per foot run, in cents.	
				Iron.*	Cast steel.*	Iron.*	Cast st'l*
11	1½	4⅝	3.37	72000	134000	41	70
12	1⅜	4¼	2.77	60000	110000	33	60
13	1¼	3¾	2.28	50000	90000	29	50
14	1⅛	3⅜	1.82	40000	72000	23	40
15	1	3	1.50	32000	60000	20	32
16	⅞	2⅝	1.12	24600	44000	16	25
17	¾	2⅜	0.88	17600	34000	12	19
18	11/16	2⅛	0.70	15200	27000	10	16
19	5/8	1⅞	0.57	11600	20000	9	14
20	9/16	1⅝	0.41	8200	16000	7	11
21	½	1⅜	0.31	5660	12000	6	8
22	7/16	1¼	0.23	4260	.....	4.5	...
23	3/8	1⅛	0.19	3300	8000	4	7
24	5/16	1	0.16	2760	6000	3.5	5
25	9/32	⅞	0.125	2060	.....	3	...

Notes on the Use of Wire Rope, by the Roeblings Co.

The ropes with 19 wires per strand are the most pliable, and therefore best adapted for **hoisting** and **running** rope. The others are stiffer and better adapted for **guys,** &c. Ropes of iron or steel, **up to 3 ins diam,** made to order. For the **safe working load** take one-fifth to one-seventh of the breaking load, according to speed. **Hemp center** rope is more pliable than wire center. Wire rope **must not be coiled** or uncoiled like hemp rope. When on a reel, the latter should be mounted on a spindle or flat turu-table to pay off the rope. When forwarded in a small coil without a reel, roll it on the ground like a wheel, and thus run off the rope. Avoid untwisting and short bends. **To preserve wire rope,** apply raw linseed oil (which may be mixed with an equal quantity of Spanish brown or lamp-black) with a piece of sheepskin, keeping the wool against the rope. **If for use in water or under ground,** add 1 bushel of fresh slacked lime, and some sawdust, to 1 barrel of tar. Boil the mixture well, and saturate the rope with it while hot. Never use **galvanized rope** for *running* rope. The grooves of **cast-iron pulleys** should be filled with blocks of well-seasoned hard wood, set on end. Leather or india-rubber is better where the pulleys are large and run very fast. **Galvanized** wire rope for rigging is cheaper and more durable than hemp rope; and does not stretch permanently under great strains. Its bulk is one-sixth, and its weight one-half, that of hemp rope. Roebling's wire rope has been made the standard by the U. States Navy Department. **Shackles, sockets, swivel-hooks,** and **fastenings,** &c. furnished and put on, and splices made. **Pulley wheels** furnished. Also, **galvanized steel cables** for **suspension bridges.** Crucible cast-steel wire ropes are much more durable than iron ones. They should be kept well lubricated. The foregoing is condeused from the circular of the Roeblings Co.

* Ropes of Bessemer steel, and of Siemens-Martin steel, are sold at the same prices as iron ropes. They have stood higher strains; but, in view of the lack of uniformity in those steels, it is not advisable to reduce their diam below that of iron rope for the same work.

On planes in Schuylkill Co, &c, a wire rope generally lasts long enough to raise one million of tons of coal up a plane half a mile long, and rising 1 in 10. The ordinary duration on inclined planes throughout the country is from 1½ to 4 years, according to the amount of service; and also greatly to the care taken of them, and of the sheaves and rollers upon which they move.

On the Mt Pisgah plane, 2500 ft long, rising 660 ft, for raising empty coal cars, and lowering loaded ones, thin iron bands, 7 inches wide, and about ⅛ inch thick, have been used instead of ropes. They scarcely exhibit any sign of wear in several years. They should be riveted; being apt to break if welded. Steel would probably be the best material in many cases.

Table of Manilla rope.

Diam. Ins.	Circ. Ins.	Wt per foot. lbs.	Breaking load. Tons.	Breaking load. lbs.	Diam. Ins.	Circ. Ins.	Wt per foot. lbs.	Breaking load. Tons.	Breaking load. lbs,
.239	¾	.019	.25	560	1.91	6	1.19	11.4	25536
.318	1	.033	.35	784	2.07	6½	1.39	13.0	29120
.477	1½	.074	.70	1568	2.23	7	1.62	14.6	32704
.636	2	.132	1.21	2733	2.39	7½	1.86	16.2	36288
.795	2½	.206	1.91	4278	2.55	8	2.11	17.8	39872
.955	3	.297	2.73	6115	2.86	9	2.67	21.0	47040
1.11	3½	.404	3.81	8534	3.18	10	3.30	24.2	54208
1.27	4	.528	5.16	11558	3.50	11	3.99	27.4	61376
1.43	4½	.668	6.60	14784	3.82	12	4.75	30.6	68544
1.59	5	.825	8.20	18368	4.14	13	5.58	33.8	75712
1.75	5½	.998	9.80	21952	4.45	14	6.47	37.0	82880

The strength of Manilla ropes, like that of bar iron, is very variable; and so with hemp ones. The above table supposes an average quality. Ropes of good *Italian* hemp are considerably stronger than Manilla; but their cost excludes them from general use. **The tarring of ropes** is said to lessen their strength; and, when exposed to the weather, their durability also. We believe that the use of it in standing rigging is partly to diminish contraction and expansion by alternate wet and dry weather. **The common rules** for finding the strength of rope by multiplying the square of the diam or circumf by a given coefficient are entirely erroneous. **Prices** in Philada, 1886, Manilla, 13 to 14 cts per lb; Italian hemp, 20 cts; American hemp, 12 cts; Sisal hemp, 10 cts; jute, (E. Indies,) 7 cts. E. H. Fitler & Co., 23 N. Water St, Phila.

The strengths of pieces from the same coil may vary 25 per ct.
A few months of exposed work weakens ropes 20 to 50 per ct.

WEIGHT AND STRENGTH OF IRON CHAINS.

Table of strength of chains.

Chains of superior iron will require ¼ to ⅓ more to break them. (Original.)

Diam of rod of which the links are made.	Weight of chain per ft run.	Breaking strain of the chain.		Diam of rod of which the links are made.	Weight of chain, per ft run.	Breaking strain of the chain.	
Ins.	Pds.	Pds.	Tons.	Ins.	Pds.	Pds.	Tons.
3-16	.5	1731	.773	1	10.7	49280	22.00
¼	.8	3069	1.37	1⅛	12.5	59226	26.44
5-16	1.	4794	2.14	1¼	16.	73114	32.64
⅜	1.7	6922	3.09	1⅜	18.3	88301	39.42
7-16	2.	9408	4.20	1½	21.7	105280	47.00
½	2.5	12320	5.50	1⅝	26.	123514	55.14
9-16	3.2	15590	6.96	1¾	28.	143293	63 97
⅝	4.3	19219	8.58	1⅞	32.	164505	73.44
11-16	5.	23274	10.39	2	38.	187152	83.55
¾	5.8	27687	12.36	2¼	54.	224448	100.2
13-16	6.7	32301	14.42	2½	71.	277088	123.7
⅞	8.	37632	16.80	2¾	88.	335328	149.7
15-16	9.	43277	19.32	3	105.	398944	178.1

The links of ordinary iron chains are usually made as short as is consistent with easy play, in order that they may not become bent when wound around drums, sheaves, &c; and that they may be more easily handled in slinging large blocks of stone, &c. U. S. Govt. expts, 1878, prove that studs weaken the links.

When so made, their weight per foot run is quite approximately 3½ times that of a single bar of the round iron of which they are composed. Since each link consists of two thicknesses of bar, it might be supposed that a chain would possess about double the strength of a single bar; but the strength of the bar becomes reduced about $\frac{3}{10}$, by being formed into links; so that the chain really has but about $\frac{7}{10}$ of the strength of two bars. As a thick bar of iron will not sustain as heavy a load in proportion as a thinner one, so of course, stout chains are proportionably weaker than slighter ones. In the foregoing table, 20 tons *per sq inch*, is assumed as the average breaking strain of a single straight bar of ordinary rolled iron, 1 inch in diam; or 1 inch square; 19 tons, from 1 to 2 ins; and 18 tons, from 2 to 3 ins. Deducting $\frac{3}{10}$ from each of these, we have as the breaking strain of the two bars composing each link, as follows: 14 tons *per sq inch*, up to 1 inch diam; 13.3 tons, from 1 to 2 ins; and 12.6 tons, from 2 to 3 ins diam: and upon these assumptions the table is based. The wts are approximate; depending upon the exactness of diameter of the iron, and shape of link.

Approximate prices of chains, in cents per pound. Bradlee & Co, 816 Richmond St, Philadelphia, 1886.

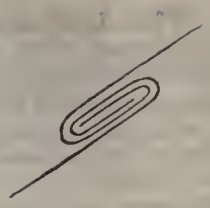
Diameter of rod from which the links are made; ins.....	⅜	½	1
Ordinary proved or coil chain.....	6¾	5	3½
Crane chain.....	8½	6½	4½
Chain of combined iron and steel.....	14½	11	7½

ROLLED LEAD, COPPER, and BRASS: Sheets and Bars.

Thickness or Diameter, or side, in Inches.	LEAD.			COPPER.			BRASS.			Thickness or Diameter, or side, in Inches.
	Sheets, per Square Foot.	Square Bars; 1 Foot long.	Round Bars; 1 Foot long.	Sheets, per Square Foot.	Square Bars; 1 Foot long.	Round Bars; 1 Foot long.	Sheets, per Square Foot.	Square Bars; 1 Foot long.	Round Bars; 1 Foot long.	
	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	
1-32	1.86	.005	.004	1.44	.004	.003	1.36	.004	.003	1-32
1-16	3.72	.019	.015	2.89	.015	.012	2.71	.014	.011	1-16
3-32	5.58	.044	.034	4.33	.034	.027	4.06	.032	.025	3-32
⅛	7.44	.078	.061	5.77	.060	.047	5.42	.056	.044	⅛
5-32	9.30	.121	.095	7.20	.094	.074	6.75	.088	.069	5-32
3-16	11.2	.174	.137	8.66	.135	.106	8.13	.127	.100	3-16
7-32	13.0	.237	.187	10.1	.184	.144	9.50	.173	.136	7-32
¼	14.9	.310	.244	11.5	.240	.189	10.8	.226	.177	¼
5-16	18.6	.485	.381	14.4	.376	.295	13.5	.353	.277	5-16
⅜	22.3	.698	.548	17.3	.541	.425	16.3	.508	.399	⅜
7-16	26.0	.950	.746	20.2	.736	.578	19.0	.691	.543	7-16
½	29.8	1.24	.974	23.1	.962	.755	21.7	.903	.709	½
9-16	33.5	1.57	1.23	26.0	1.22	.955	24.3	1.14	.900	9-16
⅝	37.2	1.94	1.52	28.9	1.50	1.18	27.1	1.41	1.11	⅝
11-16	40.9	2.34	1.84	31.7	1.82	1.43	29.8	1.70	1.34	11-16
¾	44.6	2.79	2.19	34.6	2.16	1.70	32.5	2.03	1.60	¾
13-16	48.3	3.27	2.57	37.5	2.55	1.99	35.2	2.38	1.87	13-16
⅞	52.1	3.80	2.98	40.4	2.94	2.31	37.9	2.76	2.17	⅞
15-16	56.0	4.37	3.42	43.3	3.38	2.65	40.6	3.18	2.49	15-16
1.	59.5	4.96	3.90	46.2	3.85	3.02	43.3	3.61	2.84	1.
1⅛	66.9	6 27	4.92	52.0	4.87	3.82	48.7	4.57	3.60	1⅛
1¼	74.4	7.75	6.09	57.7	6.01	4.72	54.2	5.64	4.43	1¼
1⅜	81.8	9.37	7.37	63.5	7.28	5.72	59.6	6.82	5 37	1⅜
1½	89.3	11.2	8.77	69.3	8.65	6.80	65.0	8.12	6.38	1½
1⅝	96.7	13.1	10.3	75.1	10.2	7.98	70.4	9.53	7.49	1⅝
1¾	104.	15.2	11.9	80.8	11.8	9.25	75.9	11.1	8.68	1¾
1⅞	112.	17 5	13.7	86.6	13.5	10.6	81.3	12.7	9.97	1⅞
2.	119.	19.8	15.6	92.3	15.4	12.1	86.7	14.4	11.3	2.

Approximate prices, Phila, 1886. Copper Sheets, Nos. 1 to 25, 16 cts per lb. Ingots, 10½ to 12 cts. Brass Sheets, No. 28 and heavier, 16 to 50 cts per lb, according to size.

Roof copper is usually in sheets of 2½ ft × 5 ft; or 12½ square feet, weighing 10 to 14 lbs per sheet; and is laid on boards.* No solder is used in the horizontal joints as it is in tin roofs; but both the horizontal and the sloping joints are formed by only overlapping and bending the sheets, much as shown by the figs on page 418; except that the horizontal joints are bent or locked together, as in this figure; and then flattened down close.



Sheet lead. Price, Philadelphia, 1886, about 5½ cts per lb. Tatham Bros, 226 S Fifth St. List of **standard wts** in lbs per sq ft. Thicknesses in decimals of an inch.

Wt.	Th.	Wt.	Th.	Wt.	Th.	Wt.	Th.	Wt.	Th.	Wt.	Th.
2.5	.042	4	.068	6	.102	8	.136	10	.170	14	.237
3	.051	5	.085	7	.119	9	.153	12	.203	16	.271

WEIGHT OF BALLS.

Diameter in Inches.	Cast Lead.	Cast Copper.	Cast Brass.	Cast Iron.	Diameter in Inches.	Cast Lead.	Cast Copper.	Cast Brass.	Cast Iron.
	Lbs.	Lbs.	Lbs.	Lbs.		Lbs.	Lbs.	Lbs.	Lbs.
½	.026	.021	.019	.017	5¼	30.1	24.1	21.5	19.8
¾	.088	.070	.063	.058	½	34.7	27.7	24.7	22.7
1.	.209	.167	.148	.136	¾	39.6	31.7	28.3	25.9
¼	.408	.325	.290	.266	6.	45.0	36.0	32.0	29.4
½	.705	.562	.501	.460	½	57.2	45.8	40.8	37.4
¾	1.12	.893	.795	.731	7.	71.5	57.2	50.9	46.8
2.	1.67	1.33	1.19	1.07	½	88.0	70.3	62.6	57.5
¼	2.38	1.90	1.69	1.55	8.	106.	85.3	76.0	69.8
½	3.25	2.60	2.32	2.13	½	127.	102.	91.2	83.7
¾	4.34	3.47	3.09	2.83	9.	151.	121.	108.	99.4
3.	5.63	4.50	4.01	3.68	½	178.	143.	127.	117.
¼	7.15	5.72	5.10	4.68	10.	208.	167.	148.	136.
½	8.94	7.14	6.36	5.85	½	241.	193.	172.	158.
¾	11.0	8.79	7.83	7.19	11.	277.	222.	198.	182.
4.	13.4	10.7	9.50	8.73	½	317.	253.	226.	207.
¼	16.0	12.8	11.4	10.5	12.	360.	288.	257.	236.
½	18.9	15.2	13.5	12.4					
¾	22.7	17.9	15.9	14.6	The weights of balls are as the cubes of their diams.				
5.	26.0	20.8	18.6	17.0					

Lead pipe. Price, Philadelphia, 1886, about 5½ cts per lb. Tin-lined, 15 cts. Tatham Bros, mfrs, 226 S. Fifth St. List of **standard sizes.**

Inner diam.	Thickness.	Wts per ft (F) and per rod (R) of 16½ ft.	Inner diam.	Thickness.	Wts per ft (F) and per rod (R) of 16½ ft.	Inner diam.	Thickness.	Wt in lbs per ft.	Inner diam.	Thickness.	Wt in lbs per ft.
Ins.	Ins.		Ins.	Ins.		Ins.	Ins.		Ins.	Ins.	
⅜	.06	7 lbs R	¾	.08	16 lbs R	1½	.14	3.5	3	3-16	9
"	.08	10 oz F	"	.10	1¼ lbs F	"	.17	4.25	"	¼	12
"	.12	1 lb F	"	.12	1¾ lbs F	"	.19	5.	"	5-16	16
"	.16	1¼ lbs F	"	.16	2¼ lbs F	"	.23	6.5	"	⅜	20
"	.19	1½ lbs F	"	.20	3 lbs F	"	.27	8	3½	3-16	9.5
½	.07	9 lbs R	"	.23	3½ lbs F	1¾	.13	4	"	¼	15
"	.09	¾ lb F	"	.30	4¾ lbs F	"	.17	5	"	5-16	18.5
"	.11	1 lb F	1	.10	24¾ lbs R	"	.21	6.5	"	⅜	22
"	.13	1¼ lbs F	"	.11	2 lbs F	"	.27	8 5	4	3-16	12 5
"	.16	1¾ lbs F	"	.14	2½ lbs F	2	.15	4.75	"	¼	16
"	.19	2 lbs F	"	.17	3¼ lbs F	"	.18	6	"	5-16	21
"	.25	3 lbs F	"	.21	4 lbs F	"	.22	7	"	⅜	25
⅝	.08	12 lbs R	"	.24	4¾ lbs F	"	.27	9	4½	3-16	14
"	.09	1 lb F	1¼	.10	2 lbs F	2½	3-16	8	"	¼	18
"	.13	1½ lbs F	"	.12	2½ lbs F	"	¼	11	5	¼	20
"	.16	2 lbs F	"	.14	3 lbs F	"	5-16	14	"	⅜	31
"	.20	2½ lbs F	"	.16	3¾ lbs F	"	⅜	17			
"	.22	2¾ lbs F	"	.19	4¾ lbs F						
"	.25	3½ lbs F	"	.25	6 lbs F						

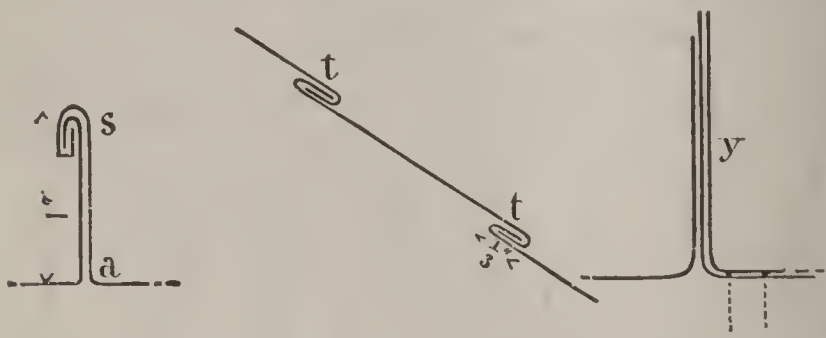
Lead service pipes for single dwellings in Philadelphia are usually of from ½ inch bore, wt 1 to 2½ lbs; to ⅝ inch bore, wt 1½ to 3 lbs per ft run, according to head. They **rarely burst** from sudden closing of stopcocks; but sometimes do so from the freezing of the contained water.

* **To which it is held by copper cleats:** as at Fig *y*, page 418. **Price** of roofing copper in 1886, about 16 cts; and copper nails, 30 cts per lb.

Seamless drawn brass and copper tubes, from ¼ inch to 8 inches outer diameter, and in lengths up to 30 feet, are made by American Tube Works, Boston, Mass.

TIN AND ZINC.

The pure metal is called block tin. When perfectly pure, (which it rarely is, being purposely adulterated, frequently to a large proportion, with the cheaper metals lead or zinc,) its sp grav is 7.29; and its weight per cub ft is 455 lbs. It is sufficiently malleable to be beaten into tin foil, only $\frac{1}{1000}$ of an inch thick. Its tensile strength is but about 4600 lbs per sq inch; or about 7000 lbs when made into wire. It melts at the moderate temperature of 442° Fah. Pure block tin is not used for common building purposes; but thin plates of sheet iron, covered with it on both sides, constitute the *tinned plates*, or, as they are called, the *tin*, used for covering roofs, rain pipes, and many domestic utensils. For roofs it is laid on boards.



The sheets of tin are united as shown in this fig. First, several sheets are joined together in the shop, end for end, as at *t t*; by being first bent over, then hammered flat, and then soldered. These are then formed into a roll to be carried to the roof; a roll being long enough to reach from the peak to the eaves. Different rolls being spread up and down the roof, are then united along their sides by simply being bent as at *a* and *s*, by a tool for that purpose. The roofers call the bending at *s* a *double groove*, or *double lock*; and the more simple ones at *t*, a *single groove*, or lock.

To hold the tin securely to the sheeting boards, pieces of the tin 3 or 4 ins long, by 2 ins wide, called cleats, are nailed to the boards at about every 18 ins along the joints of the rolls that are to be united, and are bent over with the double groove *s*. This will be understood from *y*, where the middle piece is the cleat, before being bent over. The nails should be 4-penny slating nails, which have broader heads than common ones. As they are not exposed to the weather, they may be of plain iron.

Much use is made of what is called **leaded tin,** or **ternes,** for roofing. It is simply sheet-iron coated with lead, instead of the more costly metal tin. It is not as durable as the tinned sheets, but is somewhat cheaper.

The best plates, both for tinning and for ternes, are made of charcoal iron; which, being tough, bears bending better. Coke is used for cheaper plates, but inferior as regards bending. In giving orders, it is important to specify whether charcoal plates or coke ones are required;* also whether *tinned* plates, or *ternes*.

Tinned and leaded sheets of Bessemer and other cheap **steel,** are now much used. They are sold at about the price of charcoal tin and terne plates.

There are also in use for roofing, certain compound metals which resist tarnish better than either lead, tin, or zinc; but which are so fusible as to be liable to be melted by large burning cinders falling on the roof from a neighboring conflagration.

A roof covered with tin or other metal should, if possible, slope not much *less* than five degrees, or about an inch to a foot; and at the eaves there should be a sudden fall into the rain-gutter, to prevent rain from backing up so as to overtop the double-groove joint *s*, and thus cause leaks. Where coal is used for fuel, tin roofs should receive two coats of paint when first put up, and a coat at every 2 or 3 years after. Where wood only is used, this is not necessary; and a tin roof, with a good pitch, will last 20 or 30 years.†

Two good workmen can put on, and paint outside, from 250 to 300 sq ft of tin roof, per day of 8 hours.

Tinned iron plates are sold by the box. These boxes, unlike glass, have *not* equal areas of contents. They may be designated or ordered either by their names or sizes. Many makers, however, have their private brands in addition; and some of these have a much higher reputation than others. See table of sizes, etc., p. 419.

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* **Prices,** Phila, 1886. Tinned plates, charcoal, IC — 10 × 14 and 14 × 20, the standard sizes (see table, p 419) $5.50 to $7 per box of 112 lbs, according to grade. Coke, IC — 14 × 20, $5 to $5.50. Roofing ternes, IC — 14 × 20, $5 to $7. IC, 20 × 28, $10 to $15.50 per box of 224 lbs. Wm. F. Potts, Son & Co, 1225 Market St, Phila; Hall & Carpenter, 709 Market St; Merchant & Co, 525 Arch St.

† **The cost of tin-roofing,** so-called, but actually ternes, in Philadelphia, 1886, is about 7 or 8 cts per sq ft of roof, including ternes, all labor, and one coat of paint on each side. It is often laid on old shingle roofs. Galvanized iron **rain water-pipes,** 3 ins diam, about 20 cts per ft run, put up.

Table of Tinned and Terne Plates.

The *marks* indicate the *thicknesses*, approximately as follows:

Mark.	Number Birmingham wire gauge.	Ins.	Lbs per sq ft.	Mark.	Number Birmingham wire gauge.	Ins.	Lbs per sq ft.
IC	30	.012	.48	DC	27	.016	.64
IX	28	.014	.56	DX	25	.020	.80
IXX	27	.016	.64	DXX	23	.025	1.00
IXXX	26	.018	.72	DXXX	22	.028	1.13
IXXXX	25	.020	.80	DXXXX	21	.032	1.29

Size, inches.	Mark.	No. of sheets in a box	Weight per box, pounds.	Size, inches.	Mark.	No. of sheets in a box	Weight per box, pounds.	Size, inches.	Mark.	No. of sheets in a box	Weight per box, pounds.
9 × 18	IC	225	130	13 × 13	IXX	225	194	16 × 16	IC	225	205
"	IX	"	162	13 × 26	IC	112	135	"	IX	"	256
10 × 10	IC	"	80	"	IX	"	169	"	IXX	"	294
"	IX	"	100	"	IXX	"	194	16 × 19	IX	112	152
10 × 14	IC	"	112	14 × 14	IC	225	156	17 × 17	IX	"	141
"	IX	"	140	"	IX	"	196	"	IXX	"	166
"	IXX	"	161	"	IXX	"	225	17 × 25	DX	100	252
"	IXXX	"	182	"	IXXX	"	254	"	DXX	"	294
"	IXXXX	"	203	14 × 20	IC	112	112	18 × 18	IX	112	162
10 × 20	IC	"	160	"	IX	"	140	"	IXX	"	180
"	IX	"	200	"	IXX	"	161	"	IXXXX	"	235
11 × 11	IC	"	97	"	IXXX	"	182	20 × 20	IX	"	200
"	IX	"	121	"	IXXXX	"	203	"	IXX	"	230
11 × 22	IC	112	97	14 × 22	IX	"	154	"	IXXX	"	260
"	IX	"	121	"	IXX	"	177	"	IXXXX	"	290
"	IXX	"	139	14 × 24	IX	"	168	20 × 28	IC	"	224
12 × 12	IC	225	112	"	IXX	"	193	"	IX	"	280
"	IX	"	140	14 × 25	IC	"	140	"	IXX	"	322
"	IXX	"	161	"	IX	"	175	"	IXXX	"	364
12 × 24	IC	112	115	"	IXX	"	201	"	IXXXX	"	406
"	IX	"	144	14 × 26	IXXX	"	237				
"	IXX	"	166	14 × 28	IC	"	157				
"	IXXX	"	187	"	IX	"	196				
12½ × 17	DC	100	98	"	IXX	"	225				
"	DX	"	120	14 × 30	IXX	"	241				
"	DXX	"	147	14 × 31	IX	"	217				
"	DXXX	"	168	"	IXX	"	249				
"	DXXXX	"	189	15 × 15	IX	225	225				
13 × 13	IC	225	135	"	IXX	"	259				
"	IX	"	169	"	IXXX	"	326				

Terne Plates.

10 × 20	IC	112	80
"	IX	"	100
14 × 20	IC	"	112
"	IX	"	140
20 × 28	IC	"	224
"	IX	"	280

Sheets of larger size may be made to special order; those of tinned iron, in England; but leaded ternes are made in Philada also, and elsewhere.

A box of 225 sheets of 13¾ by 10. contains 214.84 sq ft, but, allowing for overlapping, it will cover but about 150 sq ft of roof; even without any allowance for the waste which occurs in cutting away portions in order to fit at angles, &c.

To find the area of roof covered by any sheet, first deduct 2 ins from its width, and 1 inch from its length.

Zinc, in sheets, and laid in the same manner as slates, is much used in some parts of Europe for roofing. By exposure to the weather, it soon becomes covered by a thin film of white oxide, which protects it from further injury, and renders the roof very durable.* Corrugated sheet zinc is also used. See Galvanized Sheet Iron, page 403.

Zinc sheets are usually about 3 ft by 7 or 8 ft. The gauge differs from that of iron; thus No 13 is .032 of an inch thick, or 1.22 lbs per sq ft; No 14 = .035 inch, and 1.35 lbs; No 15 = .042 inch, and 1 49 lbs; No 16 = 049 inch, and 1.62 lbs per sq ft. Any of these numbers may be used on roofs, for which purpose it should be very pure.

Water kept in zinc vessels is said to become injurious to health; and recently an outcry has on that account arisen against galvanized-iron service-pipes in dwellings. Yet such have been in use for many years in New England, Philada, and elsewhere, without as yet any delcterious effects. This is possibly owing to the fact that service-pipes being short, the water is usually all drawn through them several times a day; and hence does not remain in contact with the zinc or lead long enough to acquire a poisonous character. In taking possession of a house in which the water has remained stagnant in the service-pipes for some considerable time, such water should all be run to waste; otherwise sickness may ensue from its use.

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*__The price of sheet zinc__ does not ordinarily differ much from that of sheet lead, which in Philada in 1886 is about 6 to 7 cts per lb; or in pigs, from 4 to 6 cts.

__The price of block tin,__ made into either pipes or sheets, about 35 cts per lb. Messrs Tatham & Bros, 226 South Fifth St, make both. In bars, 22 to 25 cts, in 1886.

BOARD MEASURE.

Remark on following table. The table extends to 12 ins by 24 ins, but it is easy to find for greater sizes; thus, for example, the board measure in a piece of 19 by 22, will be twice that of a piece of 19 by 11, or $17.42 \times 2 = 34.84$ ft board meas; or that of 19¼ by 22, will be that of 10¼ by 22 added to that of 9 by 22, or $18.79 + 16.50 = 35.29$. A foot of board meas is equal to 1 foot square and 1 inch thick, or to 144 cub ins. Hence 1 cub ft = 12 ft board meas.

Feet of Board Measure contained in one running foot of Scantlings of different dimensions. (Original.)
1000 ft board measure = 83⅓ cub ft.

Width in Inches.	THICKNESS IN INCHES.									Width in Inches.
	1	1¼	1½	1¾	2	2¼	2½	2¾	3	
	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	
¼	.0208	.0260	.0313	.0365	.0417	.0469	.0521	.0573	.0625	¼
½	.0417	.0521	.0625	.0729	.0833	.0938	.1042	.1146	.1250	½
¾	.0625	.0781	.0938	.1094	.1250	.1406	.1563	.1719	.1875	¾
1.	.0833	.1042	.1250	.1458	.1667	.1875	.2083	.2292	.2500	1.
¼	.1042	.1302	.1563	.1823	.2083	.2344	.2604	.2865	.3125	¼
½	.1250	.1563	.1875	.2188	.2500	.2813	.3125	.3438	.3750	½
¾	.1458	.1823	.2187	.2552	.2917	.3281	.3646	.4010	.4375	¾
2.	.1667	.2083	.2500	.2917	.3333	.3750	.4166	.4583	.5000	2.
¼	.1875	.2344	.2813	.3281	.3750	.4219	.4688	.5156	.5625	¼
½	.2083	.2604	.3125	.3646	.4167	.4688	.5208	.5729	.6250	½
¾	.2292	.2865	.3438	.4010	.4583	.5156	.5729	.6302	.6875	¾
3.	.2500	.3125	.3750	.4375	.5000	.5625	.6250	.6875	.7500	3.
¼	.2708	.3385	.4063	.4739	.5416	.6094	.6771	.7448	.8125	¼
½	.2917	.3646	.4375	.5104	.5833	.6563	.7292	.8021	.8750	½
¾	.3125	.3906	.4689	.5469	.6250	.7031	.7813	.8594	.9375	¾
4.	.3333	.4167	.5000	.5833	.6667	.7500	.8333	.9167	1.000	4.
¼	.3542	.4427	.5312	.6198	.7083	.7969	.8854	.9740	1.063	¼
½	.3750	.4688	.5625	.6563	.7500	.8438	.9375	1.031	1.125	½
¾	.3958	.4948	.5938	.6927	.7917	.8906	.9896	1.086	1.188	¾
5.	.4167	.5208	.6250	.7292	.8333	.9375	1.042	1.146	1.250	5.
¼	.4375	.5469	.6563	.7656	.8750	.9844	1.094	1.203	1.313	¼
½	.4583	.5729	.6875	.8020	.9167	1.031	1.146	1.260	1.375	½
¾	.4792	.5990	.7188	.8385	.9583	1.078	1.198	1.318	1.438	¾
6.	.5000	.6250	.7500	.8750	1.000	1.125	1.250	1.375	1.500	6.
¼	.5208	.6510	.7813	.9115	1.042	1.172	1.302	1.432	1.563	¼
½	.5417	.6771	.8125	.9479	1.083	1.219	1.354	1.490	1.625	½
¾	.5625	.7031	.8438	.9844	1.125	1.266	1.406	1.547	1.688	¾
7.	.5833	.7292	.8750	1.021	1.167	1.312	1.458	1.604	1.750	7.
¼	.6042	.7552	.9063	1.057	1.208	1.359	1.510	1.661	1.813	¼
½	.6250	.7813	.9375	1.094	1.250	1.406	1.563	1.719	1.875	½
¾	.6458	.8073	.9688	1.130	1.292	1.453	1.615	1.776	1.938	¾
8.	.6667	.8333	1.000	1.167	1.333	1.500	1.667	1.833	2.000	8.
¼	.6875	.8594	1.031	1.203	1.375	1.547	1.719	1.891	2.063	¼
½	.7083	.8854	1.063	1.240	1.417	1.594	1.771	1.948	2.125	½
¾	.7292	.9114	1.094	1.276	1.458	1.641	1.823	2.005	2.188	¾
9.	.7500	.9375	1.125	1.313	1.500	1.688	1.875	2.062	2.250	9.
¼	.7708	.9635	1.156	1.349	1.542	1.734	1.927	2.120	2.313	¼
½	.7917	.9895	1.188	1.385	1.583	1.781	1.979	2.177	2.375	½
¾	.8125	1.016	1.219	1.422	1.625	1.828	2.031	2.234	2.438	¾
10.	.8333	1.042	1.250	1.458	1.667	1.875	2.083	2.292	2.500	10.
¼	.8542	1.068	1.281	1.495	1.708	1.922	2.135	2.349	2.563	¼
½	.8750	1.094	1.313	1.531	1.750	1.969	2.188	2.406	2.625	½
¾	.8958	1.120	1.344	1.568	1.792	2.016	2.240	2.463	2.688	¾
11.	.9167	1.146	1.375	1.604	1.833	2.063	2.292	2.521	2.750	11.
¼	.9375	1.172	1.406	1.641	1.875	2.109	2.344	2.578	2.813	¼
½	.9583	1.198	1.438	1.677	1.917	2.156	2.396	2 635	2.875	½
¾	.9792	1.224	1.469	1.714	1 958	2.203	2.448	2.693	2.938	¾
12.	1.000	1.250	1.500	1.750	2.000	2.250	2.500	2.750	3.000	12.
½	1.042	1.302	1.563	1.823	2.083	2.344	2.604	2.865	3.125	½
13.	1.083	1.354	1.625	1.896	2.167	2.438	2.708	2.979	3.250	13.
½	1.125	1.406	1.688	1.969	2.250	2.531	2.813	3.094	3.375	½
14.	1.167	1.458	1.750	2.042	2 333	2.625	2.917	3.208	3.500	14.
½	1.208	1.510	1.813	2.115	2.417	2.719	3.021	3.322	3.625	½
15.	1.250	1.563	1.875	2.188	2.500	2.813	3.125	3.438	3.750	15.
½	1.292	1.615	1.938	2.260	2.583	2.906	3.229	3.552	3.875	½
16.	1.333	1.667	2.000	2.333	2.667	3.000	3.333	3.667	4.000	16.
½	1.375	1.719	2.063	2.406	2.750	3.094	3.438	3.781	4.125	½
17.	1.417	1.771	2.125	2.479	2.833	3.188	3.542	3.896	4.250	17.
½	1.458	1.823	2.187	2.552	2.917	3.281	3.646	4.010	4.375	½
18.	1.500	1.875	2.250	2.625	3.000	3.375	3.750	4.125	4.500	18.
19.	1.583	1.979	2.375	2.771	3.167	3.563	3.958	4.354	4.750	19.
20.	1.667	2.083	2.500	2.917	3.333	3.750	4.167	4.583	5.000	20.
21.	1.750	2.188	2.625	3.063	3.500	3.938	4.375	4 812	5.250	21.
22.	1.833	2.292	2.750	3.208	3.667	4.125	4.583	5.042	5.500	22.
23.	1.917	2.396	2.875	3.354	3.833	4.313	4.792	5.270	5.750	23.
24.	2.000	2.500	3.000	3.500	4.000	4.500	5.000	5.500	6.000	24.

Table of Board Measure—(Continued.)

Width in Inches.	Feet of Board Measure contained in one running foot of Scantlings of different dimensions. (Original.) THICKNESS IN INCHES.									Width in Inches.
	3¼	3½	3¾	4	4¼	4½	4¾	5	5¼	
	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M	Ft.Bd.M.	
¼	.0677	.0729	.0781	.0833	.0885	.0938	.0990	.1042	.1094	¼
½	.1354	.1457	.1562	.1667	.1770	.1875	.1979	.2083	.2188	½
¾	.2031	.2187	.2344	.2500	.2656	.2813	.2969	.3125	.3281	¾
1.	.2708	.2917	.3125	.3333	.3542	.3750	.3958	.4167	.4375	1.
¼	.3385	.3646	.3906	.4167	.4427	.4688	.4948	.5208	.5469	¼
½	.4063	.4375	.4688	.5000	.5313	.5625	.5938	.6250	.6563	½
¾	.4740	.5104	.5469	.5833	.6198	.6563	.6927	.7292	.7656	¾
2.	.5417	.5833	.6250	.6667	.7083	.7500	.7917	.8333	.8759	2.
¼	.6094	.6563	.7031	.7500	.7969	.8438	.8906	.9375	.9844	¼
½	.6771	.7292	.7813	.8333	.8854	.9375	.9896	1.042	1.094	½
¾	.7448	.8021	.8594	.9167	.9740	1.031	1.089	1.146	1.203	¾
3.	.8125	.8750	.9375	1.000	1.062	1.125	1.188	1.250	1.313	3.
¼	.8802	.9479	1.016	1.083	1.151	1.219	1.286	1.354	1.422	¼
½	.9479	1.021	1.094	1.167	1.240	1.313	1.385	1.458	1.531	½
¾	1.016	1.094	1.172	1.250	1.327	1.406	1.484	1.563	1.641	¾
4.	1.083	1.167	1.250	1.333	1.416	1.500	1.583	1.667	1.750	4.
¼	1.151	1.240	1.328	1.417	1.504	1.594	1.682	1.771	1.859	¼
½	1.219	1.313	1.406	1.500	1.593	1.688	1.781	1.875	1.969	½
¾	1.286	1.384	1.484	1.583	1.681	1.781	1.880	1 979	2.078	¾
5.	1.354	1.457	1.566	1.666	1.770	1.875	1.979	2.083	2.188	5.
¼	1.422	1.530	1.644	1.750	1.858	1.969	2.078	2.188	2 297	¼
½	1.490	1.603	1.722	1.833	1.947	2 063	2.177	2.292	2.406	½
¾	1.557	1.676	1.800	1.917	2.035	2.156	2.276	2.396	2.516	¾
6.	1.625	1.750	1.875	2.000	2.125	2 250	2.375	2.500	2.625	6.
¼	1.693	1.823	1.953	2.083	2.214	2.344	2.474	2 604	2.734	¼
½	1.760	1.896	2.031	2.167	2.302	2.438	2.573	2.708	2.843	½
¾	1.828	1.969	2.109	2.250	2.391	2.531	2.672	2.813	2 953	¾
7.	1.896	2.042	2.188	2.333	2.479	2.625	2.771	2.917	3.063	7.
¼	1.964	2.115	2.266	2.416	2.568	2 719	2.870	3 021	3.172	¼
½	2.031	2 187	2.344	2.500	2 656	2 813	2.969	3.125	3.281	½
¾	2.099	2 260	2.422	2.583	2.745	2.906	3.068	3.229	3 391	¾
8.	2.167	2 333	2.500	2.667	2.833	3.000	3.167	3.333	3.500	8.
¼	2.234	2.406	2.578	2.750	2.922	3.094	3.266	3.438	3.609	¼
½	2.302	2.479	2 656	2.833	3.010	3.188	3.365	3.542	3.718	½
¾	2.370	2.552	2.734	2.916	3.099	3.281	3.464	3.646	3.828	¾
9.	2.438	2.625	2.813	3.000	3.187	3.375	3.563	3.750	3.938	9.
¼	2.505	2.698	2.891	3.083	3.276	3.469	3.661	3.854	4.047	¼
½	2.573	2.771	2.969	3.167	3.365	3.563	3.760	3.958	4.156	½
¾	2.641	2.844	3.047	3.250	3.453	3.656	3.859	4.063	4.266	¾
10.	2.708	2.917	3.125	3.333	3.542	3.750	3.958	4.167	4.375	10.
¼	2.776	2.990	3.203	3.416	3.630	3.844	4.057	4.271	4.484	¼
½	2.844	3.063	3.281	3.500	3.719	3.938	4.156	4.375	4.594	½
¾	2.911	3.135	3.359	3.583	3.807	4.031	4.255	4.479	4.703	¾
11.	2.979	3.208	3.438	3.666	3.896	4.125	4.354	4.583	4.813	11.
¼	3.047	3.281	3.516	3.750	3.984	4.219	4.453	4.688	4.922	¼
½	3.115	3.354	3.594	3.833	4.073	4.313	4.552	4.792	5.031	½
¾	3.182	3.427	3.672	3.916	4.161	4.406	4.651	4.896	5.141	¾
12.	3.250	3.500	3.750	4.000	4.250	4.500	4.750	5.000	5.250	12.
½	3.385	3.646	3.906	4.167	4.427	4.688	4.948	5.208	5.469	½
13.	3.521	3.792	4.063	4.333	4.604	4.875	5.146	5.417	5.688	13.
½	3.656	3.938	4.219	4.500	4.781	5.063	5.344	5.625	5.906	½
14.	3.792	4.083	4.375	4.667	4.958	5.250	5.542	5.833	6.125	14.
½	3.927	4.229	4.531	4.833	5.135	5.438	5.740	6.042	6.344	½
15.	4.063	4.375	4.688	5.000	5.313	5.625	5.938	6.250	6.563	15.
½	4.198	4.521	4.844	5.166	5.490	5.813	6.135	6.458	6.781	½
16.	4.333	4.667	5.000	5.333	5.667	6.000	6.333	6.667	7.000	16.
½	4.469	4.813	5.156	5.500	5.844	6.188	6.531	6.875	7.219	½
17.	4.604	4.958	5.313	5.667	6.021	6.375	6.729	7.083	7.438	17.
½	4.740	5.104	5.469	5.833	6.198	6.563	6.927	7.292	7.656	½
18.	4.875	5.250	5.625	6.000	6.375	6.750	7.125	7.500	7.875	18.
19.	5.146	5.542	5.938	6.333	6.729	7.125	7.521	7.917	8.313	19.
20.	5.417	5.833	6.250	6.667	7.083	7.500	7.917	8.333	8.750	20.
21.	5.688	6.125	6.563	7.000	7.438	7.875	8.313	8.750	9.188	21.
22.	5.958	6.417	6.875	7.333	7.792	8.250	8.708	9.167	9.625	22.
23.	6.229	6.708	7.188	7.667	8.145	8.625	9.104	9.583	10.06	23.
24.	6.500	7.000	7.500	8.000	8.500	9.000	9.500	10.00	10.50	24.

Table of Board Measure—(Continued.)

Feet of Board Measure contained in one running foot of Scantlings of different dimensions. (Original.)

Width in Inches.	THICKNESS IN INCHES.									Width in Inches.
	5½	5¾	6	6¼	6½	6¾	7	7¼	7½	
	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	
¼	.1146	.1198	.1250	.1302	.1354	.1406	.1458	.1510	.1563	¼
½	.2292	.2396	.2500	.2604	.2708	.2813	.2917	.3021	.3125	½
¾	.3438	.3594	.3750	.3906	.4063	.4219	.4375	.4531	.4688	¾
1.	.4583	.4792	.5000	.5208	.5417	.5625	.5833	.6042	.6250	1.
¼	.5729	.5990	.6250	.6510	.6771	.7031	.7292	.7552	.7813	¼
½	.6875	.7188	.7500	.7812	.8125	.8438	.8750	.9062	.9375	½
¾	.8021	.8385	.8750	.9115	.9479	.9844	1.020	1.057	1.094	¾
2.	.9167	.9583	1.000	1.042	1.083	1.125	1.167	1.208	1.250	2.
¼	1.031	1.078	1.125	1.172	1.219	1.266	1.313	1.359	1.406	¼
½	1.146	1.198	1.250	1.302	1.354	1.406	1 458	1.510	1.563	½
¾	1.260	1.318	1.375	1.432	1.490	1.547	1.604	1.661	1.719	¾
3.	1.375	1.438	1.500	1.562	1.625	1.688	1.750	1.813	1.875	3.
¼	1.490	1.557	1.625	1.693	1.760	1.828	1.896	1.964	2.031	¼
½	1.604	1.677	1.750	1.823	1.896	1.969	2.042	2.115	2.188	½
¾	1.719	1.797	1.875	1.953	2.031	2.109	2.188	2.266	2.344	¾
4.	1.833	1.917	2.000	2.083	2.167	2.250	2.333	2.417	2 500	4.
¼	1.948	2.036	2.125	2.214	2.302	2.391	2.479	2.568	2.656	¼
½	2.063	2.156	2.250	2.344	2.438	2.531	2 625	2.719	2.813	½
¾	2.177	2.276	2.375	2.474	2.573	2.672	2.771	2.870	2.969	¾
5.	2.292	2.396	2.500	2.604	2.708	2.813	2.917	3.021	3.125	5.
¼	2.406	2.516	2.625	2.734	2 844	2.953	3.063	3.172	3.281	¼
½	2.521	2.635	2.750	2.865	2.979	3.094	3.208	3.323	3.438	½
¾	2.635	2.755	2.875	2.995	3.115	3.234	3.354	3.474	3.594	¾
6.	2.750	2.875	3.000	3.125	3.250	3.375	3.500	3.625	3.750	6.
¼	2.865	2.995	3.125	3.255	3.385	3.516	3.646	3.776	3.906	¼
½	2.979	3.115	3.250	3.385	3.521	3.656	3.792	3.927	4.063	½
¾	3.094	3.234	3.375	3.516	3.656	3.797	3.938	4.078	4.219	¾
7.	3.208	3.354	3.500	3.646	3.792	3.938	4.083	4.229	4.375	7.
¼	3.323	3.474	3.625	3.776	3.927	4.078	4.229	4.380	4.531	¼
½	3.438	3.594	3.750	3.906	4.063	4.219	4.375	4.531	4.688	½
¾	3.552	3.714	3.875	4.036	4.198	4.359	4.521	4.682	4.844	¾
8.	3.667	3.833	4.000	4.167	4.333	4.500	4.667	4.833	5.000	8.
¼	3.781	3.953	4.125	4.297	4.469	4.641	4.813	4.984	5.156	¼
½	3.896	4.073	4.250	4.427	4.604	4.781	4.957	5.135	5 313	½
¾	4.010	4.193	4.375	4.557	4.740	4.922	5.103	5.286	5.469	¾
9.	4.125	4.313	4.500	4.687	4.875	5.063	5.249	5.438	5.625	9.
¼	4.240	4.432	4.625	4.818	5.010	5.203	5.395	5.589	5.781	¼
½	4.354	4.552	4.750	4.948	5.146	5.344	5.541	5 740	5.938	½
¾	4.469	4.672	4.875	5.078	5.281	5.484	5.687	5.891	6.094	¾
10.	4.583	4.792	5.000	5.208	5.417	5.625	5.833	6.042	6.250	10.
¼	4.698	4.911	5.125	5.339	5.552	5.766	5.979	6.193	6.406	¼
½	4.813	5.031	5.250	5.469	5.688	5.906	6.125	6.344	6.563	½
¾	4.927	5.151	5.375	5.599	5.823	6.047	6.271	6.495	6.719	¾
11.	5.042	5.271	5 500	5.729	5.958	6.188	6.417	6.646	6.876	11.
¼	5.156	5.391	5.625	5.859	6.094	6.328	6.563	6.797	7.031	¼
½	5.271	5.510	5.750	5 990	6.229	6.469	6.708	6.948	7.188	½
¾	5.385	5.630	5.875	6.120	6.365	6.609	6.854	7.099	7.344	¾
12.	5.500	5.750	6.000	6.250	6.500	6.750	7.000	7.250	7.500	12.
½	5.729	5.990	6.250	6.510	6.771	7.031	7 292	7.552	7.813	½
13.	5.958	6.229	6.500	6.771	7.042	7.313	7.583	7.854	8.125	13.
½	6.188	6.469	6.750	7.031	7.313	7.594	7.875	8.156	8.438	½
14.	6.417	6.708	7.000	7.292	7.583	7.875	8.167	8.458	8.750	14.
½	6.646	6.948	7.250	7.552	7.854	8.156	8.458	8.760	9.063	½
15.	6.875	7.188	7.500	7.812	8.125	8.458	8.750	9.063	9.375	15.
½	7.104	7.427	7.750	8.073	8.396	8.719	9.042	9.365	9.688	½
16.	7.333	7.667	8.000	8.333	8.667	9.000	9.333	9.667	10.00	16.
½	7.563	7.906	8.250	8.594	8.938	9.281	9.625	9.969	10.31	½
17.	7.792	8.146	8.500	8.854	9 208	9.563	9.917	10.27	10.63	17.
½	8.021	8.385	8.750	9.115	9.479	9.844	10.21	10.57	10.94	½
18.	8.250	8.625	9.000	9.375	9.750	10.13	10.50	10.88	11.25	18.
19.	8.708	9.104	9.500	9.896	10.29	10.69	11.08	11.48	11.88	19.
20.	9.167	9.583	10.00	10.42	10.83	11.25	11.67	12.08	12.50	20.
21.	9.625	10.06	10.50	10.94	11.38	11 81	12.25	12.69	13.13	21.
22.	10.08	10.54	11.00	11.46	11.92	12.38	12.83	13.29	13.75	22.
23.	10.54	11.02	11.50	11.98	12.46	12.94	13.42	13.90	14.38	23.
24.	11.00	11.50	12.00	12.50	13.00	13.50	14.00	14.50	15.00	24.

Table of Board Measure — (Continued.)

Feet of Board Measure contained in one running foot of Scantlings of different dimensions. (Original.)

Width in Inches.	THICKNESS IN INCHES.									Width in Inches.
	7¾	8	8¼	8½	8¾	9	9¼	9½	9¾	
	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	
¼	.1615	.1667	.1719	.1771	.1823	.1875	.1927	.1979	.2031	¼
½	.3229	.3333	.3438	.3542	.3646	.3750	.3854	.3958	.4063	½
¾	.4844	.5000	.5156	.5313	.5467	.5625	.5781	.5938	.6094	¾
1.	.6458	.6667	.6875	.7083	.7292	.7500	.7708	.7917	.8125	1.
¼	.8073	.8333	.8594	.8854	.9115	.9375	.9635	.9896	1.016	¼
½	.9688	1.000	1.031	1.063	1.094	1.125	1.156	1.188	1.219	½
¾	1.130	1.167	1.203	1.240	1.276	1.313	1.349	1.385	1.422	¾
2.	1.292	1.333	1.375	1.417	1.458	1.500	1.542	1.583	1.625	2.
¼	1.453	1.500	1.547	1.594	1.641	1.688	1.734	1.781	1.828	¼
½	1.615	1.667	1.719	1.771	1.822	1.875	1.927	1.979	2.031	½
¾	1.776	1.833	1.891	1.948	2.005	2.063	2.120	2.177	2.234	¾
3.	1.938	2.000	2.063	2.125	2.188	2.250	2.313	2.375	2.438	3.
¼	2.099	2.167	2.234	2.302	2.370	2.438	2.505	2.573	2.641	¼
½	2.260	2.333	2.406	2.479	2.552	2.625	2.698	2.771	2.844	½
¾	2.422	2.500	2.578	2.656	2.734	2.813	2.891	2.969	3.047	¾
4.	2.583	2.667	2.750	2.833	2.917	3.000	3.083	3.167	3.250	4.
¼	2.745	2.833	2.922	3.010	3.099	3.188	3.276	3.365	3.453	¼
½	2.906	3.000	3.094	3.188	3.281	3.375	3.469	3.563	3.656	½
¾	3.068	3.167	3.266	3.365	3.464	3.563	3.661	3.760	3.859	¾
5.	3.229	3.333	3.438	3.542	3.646	3.750	3.854	3.958	4.063	5.
¼	3.391	3.500	3.609	3.719	3.828	3.938	4.047	4.156	4.266	¼
½	3.552	3.667	3.781	3.896	4.010	4.125	4.240	4.354	4.469	½
¾	3.714	3.833	3.953	4.073	4.193	4.313	4.432	4.552	4.672	¾
6.	3.875	4.000	4.125	4.250	4.375	4.500	4.625	4.750	4.875	6.
¼	4.036	4.167	4.297	4.427	4.557	4.688	4.818	4.948	5.078	¼
½	4.198	4.333	4.469	4.604	4.740	4.875	5.010	5.146	5.281	½
¾	4.359	4.500	4.641	4.781	4.922	5.063	5.203	5.344	5.484	¾
7.	4.521	4.667	4.813	4.958	5.104	5.250	5.360	5.542	5.688	7.
¼	4.682	4.833	4.984	5.135	5.286	5.438	5.590	5.740	5.891	¼
½	4.844	5.000	5.156	5.313	5.469	5.625	5.782	5.938	6.094	½
¾	5.005	5.167	5.328	5.490	5.651	5.813	5.975	6.135	6.297	¾
8.	5.167	5.333	5.500	5.667	5.833	6.000	6.167	6.333	6.500	8.
¼	5.328	5.500	5.672	5.844	6.016	6.188	6.359	6.531	6.703	¼
½	5.490	5.667	5.844	6.021	6.198	6.375	6.552	6.729	6.906	½
¾	5.651	5.833	6.016	6.198	6.380	6.563	6.745	6.927	7.109	¾
9.	5.813	6.000	6.188	6.375	6.563	6.750	6.938	7.125	7.313	9.
¼	5.974	6.167	6.359	6.552	6.745	6.938	7.130	7.323	7.516	¼
½	6.135	6.333	6.531	6.729	6.927	7.125	7.323	7.521	7.719	½
¾	6.297	6.500	6.703	6.906	7.109	7.313	7.516	7.719	7.922	¾
10.	6.458	6.667	6.875	7.083	7.292	7.500	7.708	7.917	8.125	10.
¼	6.620	6.833	7.047	7.260	7.474	7.688	7.901	8.115	8.328	¼
½	6.781	7.000	7.219	7.438	7.656	7.875	8.094	8.313	8.531	½
¾	6.943	7.167	7.391	7.615	7.839	8.063	8.286	8.510	8.734	¾
11.	7.104	7.333	7.563	7.792	8.021	8.250	8.479	8.708	8.938	11.
¼	7.266	7.500	7.735	7.969	8.203	8.438	8.672	8.906	9.141	¼
½	7.427	7.667	7.906	8.146	8.386	8.625	8.865	9.104	9.344	½
¾	7.589	7.833	8.078	8.323	8.568	8.813	9.057	9.302	9.547	¾
12.	7.750	8.000	8.250	8.500	8.750	9.000	9.250	9.500	9.750	12.
½	8.073	8.333	8.594	8.854	9.115	9.375	9.635	9.896	10.16	½
13.	8.396	8.666	8.938	9.208	9.479	9.750	10.02	10.29	10.56	13.
½	8.719	9.000	9.281	9.563	9.844	10.13	10.41	10.69	10.97	½
14.	9.042	9.333	9.625	9.917	10.21	10.50	10.79	11.08	11.38	14.
½	9.365	9.666	9.969	10.27	10.57	10.88	11.18	11.48	11.78	½
15.	9.688	10.000	10.31	10.63	10.94	11.25	11.56	11.88	12.19	15.
½	10.01	10.33	10.66	10.98	11.30	11.63	11.95	12.27	12.59	½
16.	10.33	10.67	11.00	11.33	11.67	12.00	12.33	12.67	13.00	16.
½	10.66	11.00	11.34	11.69	12.03	12.38	12.72	13.06	13.41	½
17.	10.98	11.33	11.69	12.04	12.40	12.75	13.10	13.46	13.81	17.
½	11.30	11.66	12.03	12.40	12.76	13.13	13.49	13.85	14.22	½
18.	11.63	12.00	12.38	12.75	13.13	13.50	13.88	14.25	14.63	18.
19.	12.27	12.67	13.06	13.46	13.85	14.25	14.65	15.04	15.44	19.
20.	12.92	13.33	13.75	14.17	14.58	15.00	15.42	15.83	16.25	20.
21.	13.56	14.00	14.44	14.88	15.31	15.75	16.19	16.63	17.06	21.
22.	14.21	14.66	15.13	15.58	16.04	16.50	16.96	17.42	17.88	22.
23.	14.85	15.33	15.81	16.29	16.77	17.25	17.73	18.21	18.69	23.
24.	15.50	16.00	16.50	17.00	17.50	18.00	18.50	19.00	19.50	24.

Table of Board Measure—(Continued.)

Feet of Board Measure contained in one running foot of Scantlings of different dimensions. (Original.)

Width in Inches.	THICKNESS IN INCHES.									Width in Inches.
	10	10¼	10½	10¾	11	11¼	11½	11¾	12	
	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	
¼	.2083	.2135	.2188	.2240	.2292	.2344	.2396	.2448	.2500	¼
½	.4167	.4271	.4375	.4479	.4583	.4688	.4792	.4896	.5000	½
¾	.6250	.6406	.6563	.6719	.6875	.7031	.7188	.7344	.7500	¾
1.	.8333	.8542	.8750	.8958	.9167	.9375	.9583	.9792	1.000	1.
¼	1.042	1.068	1.094	1.120	1.146	1.172	1.198	1.224	1.250	¼
½	1.250	1.821	1.313	1.344	1.375	1.406	1.438	1.469	1.500	½
¾	1.458	1.495	1.531	1.568	1.604	1.641	1.677	1.714	1.750	¾
2.	1.667	1.708	1.750	1.792	1.833	1.875	1.917	1.958	2.000	2.
¼	1.875	1.922	1.969	2.016	2.063	2.109	2.156	2.203	2.250	¼
½	2.083	2.135	2.188	2.240	2.292	2.344	2.396	2.448	2.500	½
¾	2.292	2.349	2.406	2.464	2.521	2.578	2.635	2.693	2.750	¾
3.	2.500	2.563	2.625	2.688	2.750	2.813	2.875	2.938	3.000	3.
¼	2.708	2.776	2.844	2.911	2.979	3.047	3.115	3.182	3.250	¼
½	2.917	2.990	3.063	3.135	3.208	3.281	3.354	3.427	3.500	½
¾	3.125	3.203	3.281	3.359	3.438	3.516	3.594	3.672	3.750	¾
4.	3.333	3.417	3.500	3.583	3.667	3.750	3.833	3.917	4.000	4.
¼	3.542	3.630	3.719	3.807	3.896	3.984	4.073	4.161	4.250	¼
½	3.750	3.844	3.938	4.031	4.125	4.219	4.313	4.406	4.500	½
¾	3.958	4.057	4.156	4.255	4.354	4.453	4.552	4.651	4.750	¾
5.	4.167	4.271	4.375	4.479	4.583	4.688	4.791	4.896	5.000	5.
¼	4.375	4.484	4.594	4.703	4.813	4.922	5.031	5.141	5.250	¼
½	4.583	4.698	4.813	4.927	5.042	5.156	5.270	5.385	5.500	½
¾	4.792	4.911	5.031	5.151	5.271	5.391	5.510	5.630	5.750	¾
6.	5.000	5.125	5.250	5.375	5.500	5.625	5.750	5.875	6.000	6.
¼	5.208	5.339	5.469	5.599	5.729	5.859	5.990	6.120	6.250	¼
½	5.417	5.552	5.688	5.823	5.958	6.094	6.229	6.365	6.500	½
¾	5.625	5.766	5.906	6.047	6.188	6.328	6.469	6.609	6.750	¾
7.	5.833	5.979	6.125	6.271	6.417	6.563	6.708	6.854	7.000	7.
¼	3.042	6.193	6.344	6.495	6.646	6.797	6.948	7.099	7.250	¼
½	6.250	6.406	6.563	6.719	6.875	7.031	7.188	7.344	7.500	½
¾	5.458	6 620	6.781	6.943	7.104	7.266	7.427	7.589	7.750	¾
8.	6.667	6.833	7.000	7.167	7.333	7 500	7.667	7.833	8.000	8.
¼	6.875	7.047	7.219	7.391	7.563	7.734	7.906	8.078	8.250	¼
½	7.083	7.260	7.438	7.615	7.792	7.969	8.146	8.323	8.500	½
¾	7.292	7.474	7.656	7.839	8.021	8.203	8.385	8.568	8.750	¾
9.	7.500	7.388	7.875	8.063	8.250	8.438	8.625	8.813	9.000	9.
¼	7.708	7.901	8.094	8.286	8.479	8.672	8.865	9.057	9.250	¼
½	7.917	8.115	8.313	8.510	8.709	8.906	9.104	9.302	9.500	½
¾	8.125	8.323	8.531	8.734	8.939	9.141	9.344	9.547	9.750	¾
10.	8.333	8.542	8.750	8.958	9.167	9.375	9.583	9.792	10.00	10.
¼	8.542	8.755	8.969	9.182	9.396	9.609	9.823	10.04	10.25	¼
½	8.750	8.969	9.188	9.406	9.625	9.844	10.06	10.28	10.50	½
¾	8.958	9.182	9.406	9.630	9.854	10.08	10.30	10.53	10.75	¾
11.	9.167	9.396	9.625	9.854	10.08	10 31	10.54	10.77	11.00	11.
¼	9.375	9.609	9.844	10.08	10.31	10.55	10.78	11.02	11.25	¼
½	9.583	9.823	10.06	10.30	10.54	10.78	11.02	11.26	11.50	½
¾	9.792	10.04	10.28	10.53	10.77	11.02	11.26	11.51	11.75	¾
12.	10.00	10.25	10.50	10.75	11.00	11.25	11.50	11.75	12.00	12.
½	10.42	10.68	10.94	11.20	11.46	11.72	11.98	12.24	12.50	½
13.	10.83	11.10	11.38	11.65	11.92	12.19	12.46	12.73	13.00	13.
½	11.25	11.53	11.81	12.09	12.38	12.66	12.94	13.22	13.50	½
14.	11.67	11.96	12.25	12.54	12.83	13.13	13.42	13.71	14.00	14.
½	12.08	12.39	12.69	12.99	13.29	13.59	13.90	14.20	14.50	½
15.	12.50	12.81	13.13	13.44	13.75	14.06	14.38	14.69	15.00	15.
½	12.92	13.24	13.56	13.89	14.21	14.53	14.85	15.18	15.50	½
16.	13.33	13.67	14.00	14.33	14.67	15.00	15.33	15.67	16.00	16.
½	13.75	14.09	14.44	14.78	15.13	15.47	15.81	16.16	16.50	½
17.	14.17	14.52	14.88	15.23	15.58	15.94	16.29	16.65	17.00	17.
½	14.58	14.95	15.31	15.77	16.04	16.41	16.77	17.14	17.50	½
18.	15.00	15.38	15.75	16.13	16.50	16.88	17.25	17.63	18.00	18.
19.	15.83	16.23	16.63	17.02	17.42	17.81	18.21	18.60	19.00	19.
20.	16.67	17.08	17.50	17.92	18.33	18.75	19.17	19.58	20.00	20.
21.	17.50	17.94	18.38	18.81	19.25	19.69	20.13	20.56	21.00	21.
22.	18.33	18.79	19.25	19.71	20.17	20.63	21.08	21.54	22.00	22.
23.	19.17	19.65	20.13	20.60	21.08	21.56	22.04	22.52	23.00	23.
24.	20.00	20.50	21.00	21.50	22.00	22.50	23.00	23.50	24.00	24.

PRESERVATION OF TIMBER.

Art. 1. (a) **The decay of timber** is caused by the fermentation of its sap. If dry air circulates freely about the sides and ends of the sticks, the sap evaporates. If air is excluded, as when timber is kept constantly and entirely immersed in salt or fresh water, the sap cannot ferment. In either case timber may resist decay for centuries. Sap, confined in timber with air, ferments, producing **dry rot;** as where beams are enclosed air-tight in brickwork etc; and where green timber is painted or varnished, or treated with creosote etc. The sap then not only prevents the thorough penetration of the oil etc, but may cause the greater part of the wood to rot although its firm outer shell gives it a deceptive appearance of strength. (b) Sap should therefore be first removed by **seasoning;** *i e*, either by drying the wood in air at natural or higher temperatures, or by first steaming the wood under pres so as to vaporize the sap, and then removing the latter by means of a vacuum. Thorough seasoning of large timbers in dry air at ordinary temperatures may require years; and too rapid kiln-drying cracks and weakens the wood. But it is questionable whether steaming and vacuum remove sap as thoroughly as do the slower dry processes. (c) Alternate exposure to water and air is very destructive. It causes **wet rot.**

Art. 2. Sea-worms. The *limnoria terebrans* works from near high-water mark to a little below the surface of mud bottom; the *teredo navalis* within somewhat less limits. The teredo is said to be rendered less active by the presence of sewage in water.

Art. 3. (a) The best timber-preserving processes are practically **useless unless thoroughly well done.** If the gain in durability will not warrant the expenditure of time and money reqd for this, it is more economical to use the wood in its natural state. (b) **The woods best adapted to treatment** are those of an open or porous texture, as hemlock etc. They absorb the oil etc better than the denser woods; and their cheapness renders the use of the treatment more economical. (c) Most of the processes in common use seem to render wood less combustible. (d) After treatment by any process, the wood should be well dried before using.

Art. 4. (a) **Creosote oil,** or **dead oil,** is the best known preservative. Against sea-worms it is effective for at least 25 years, and is the only known protection. (b) As temporary expedients, piles are sometimes covered with sheet metal or with broad-headed nails driven close together. These rust or wear away in a few years. Oak piles, cut in January, and driven with the bark on, have resisted the teredo for 4 or 5 years; and cypress piles, well charred, for 9 years. (c) For ordinary exposures on land, 8 to 10 lbs of creosote oil, **per cub ft** are reqd = say 670 to 830 lbs per 1000 ft board measure = 30 to 40 lbs per cross tie of 4 cub ft. For protection against sea-worms 10 to 12 lbs per cub ft suffice in climates like those of Great Britain and the Northern U. S.; but in warmer waters where the teredo is very active, from 14 to 20 lbs per cub ft are used. Large timbers may not require saturation throughout, and thus may take less *per cub ft.* But see (i) and end of Art. 1 (a). (d) Creosote oil **weighs** about 8.8 lbs per U. S. gallon. Its **cost** (1886) is about 1 ct per lb: that of the process, applied to pine and similar woods, including oil, from 18 cts per cub ft of timber for 10 lbs of oil per cub ft, to 35 cts per cub ft for 18 lbs per cub ft. Special prices for hard wood. See (j). It is cheaper in England. (e) The sticks should be reduced to their intended final dimensions and framed (if framing is reqd) before treatment; especially if for exposure to teredo, which is sure to attack any spots which (as by subsequent cutting) are left unprotected. (f) Creosoted ties have remained sound after 22 years' exposure. The creosote protects the spikes from rusting. (g) Spruce, owing to its irregular density, is unsuitable for creosoting. (h) Creosote renders wood stiffer and slightly more brittle. In hot weather it exudes to some extent and discolors the wood. Its smell excludes it from dwellings. (i) It does not wash out from the wood, but often fails to penetrate the heart-wood. Then, if any sap remains, decay begins at the center. See end of Art 1 (a). Burnettizing the cen of the stick (see Art 7) and using a coating of creosote outside, has long been suggested as the best possible method. This is cheaper than thorough creosoting. For cost, etc, of a "zinc-creosote" process, address J. P. Card, Chicago, Ill. (j) **Creosoting is done by** the Eppinger & Russell Creosoting Works, office 160 Water St, New York. Their arrangements are such that the process can, if desired, be confined to a *portion* of the length of the stick. For their prices see (d). The Carolina Oil & Creosote Co, Wilmington N. C. treat timber with creosote oil obtained from *wood.*

Art. 5. (a) **Mineral solutions** are inferior to creosote, even on land; and useless in running water or against sea-worms; but they approximately double the life of inferior timber under ordinary land exposures; and their cheapness permits their use where that of creosote is too expensive. (b) They render wood harder; and brittle if the solution is too strong. They are liable to be washed out by rain etc. Hence the outer wood decays first. See Art 4 (i) Art 8 (b) (c) (d). (c) A committee of the American Soc of Civ Engrs,* after collating a large number of experiments, recommended **Burnettizing** (Art 7) **for damp exposure,** as that of cross ties, damp floors etc; and **Kyanizing** (Art 6) **for comparatively dry situations** with exposure to air and sun-light, as in bridge timbers, for which it is better suited than Burnettizing because it seems to weaken wood less. In such exposures it preserves wood for 20 to 30 years.

Art. 6. (a) **Kyanizing** consists in steeping the wood in a warm solution of 1 lb of **bi-chloride of mercury** (**corrosive sublimate**) in 100 lbs of water. (b) It is usual to allow the wood to soak a day for each inch of the thickness, or least dimension, of the piece, and one day in addition, whatever the size. (c) With the sublimate at 55 cts per lb (1886) and 4 to 5 lbs per 1000 ft bm, it **costs** about $7 per 1000 ft bm, or 8½ cts per cub ft, or 34 cts per tie of 4 cub ft. (d) Gen'l Cram found the process very **unhealthy,** "salivating all the men"; but Mr. J. B. Francis, at Lowell, and Mr. H. Bissell of the Eastern R. R. of Mass, had little or no trouble in this respect. The sublimate, however, which is very poisonous, is apt to effloresce, and the use of the timber is thus rendered dangerous. (e) The wood decays sooner under than above ground; but spruce ties, kyanized in 1840, were perfectly sound in 1855. The sublimate readily washes out, and the process is therefore unsuitable for damp situations. It is **carried on by** the Proprietors of the Locks and Canals on Merrimack River, at Lowell, Mass.

Art. 7. (a) **Burnettizing** consists in immersing the wood for several hours in a solution of 2 lbs **chloride of zinc** in 100 lbs of water, under a pres of from 100 to 300 lbs per sq inch. (b) It seems to render wood more brittle than kyanizing, and is therefore less adapted for timbers bearing tensile or transverse strain. Spikes rust away rapidly in Burnettized ties. (c) It **costs** (1886) about $5 per 1000 ft bm = 6 cts per cub ft = 24 cts per tie of 4 cub ft.

Art. 8. Other preventives. (a) Steeping in a solution of **sulphate of copper** (**blue vitriol**) has been extensively used, but does not seem to have been permanently successful. The blue vitriol washes out readily. (b) In the **Thilmany process,** as practised by the Wisconsin Wood Preserving Co., Milwaukee, Wis., the timber is first steamed. The steam and air are then exhausted by an air-pump, after which are injected first a solution of sulphate of copper (blue vitriol) or of sulphate of zinc (white vitriol) and then one of chloride of barium, both under pressure. It is claimed that this fills the pores with insoluble sulphate of baryta. Railroad ties require about 12 hours. Cost, about 20 cents per tie, or from $4 to $5 per 1000 feet, board measure. (c) The **Wellhouse process,** as employed by the Chicago Tie Preserving Co., injects first a solution of chloride of zinc with glue, and then one of tannin (both under pres), in order to diminish the subsequent washing out of the chloride. The process costs (1886) about $8 per 1000 ft bm = say 10 cts per cub ft. It is not recommended for sub-aqueous use. (d) The "**Gypsum** process" of the American Wood Preserving Co. of St Louis Mo uses gypsum and chloride of zinc in order to retain the latter more perfectly. The wood is then kiln-dried. (e) Fence-posts etc seem to be preserved to some extent by having only their *lower* ends dipped in **tar** well boiled to remove the ammonia, which last is destructive to wood. The upper end must be left untarred to let the sap evaporate. (f) Attempts at wood preservation by means of **vapor of creosote** etc have proved failures. (g) While wood is thoroughly saturated with **petroleum** it does not decay. But unless the supply is kept up the oil evaporates and leaves the wood unprotected. (h) Cottonwood ties laid upon a **soil** containing about 2 per cent carbonate of lime, 1 per cent salt and ½ per cent each of potash and oxide of iron, on the Union Pacific R. R. in 1868, were found in 1882 "as sound and a good deal harder than when first laid," although such ties in other soils lasted but from 2 to 5 years. (i) The use of solutions of **lime** and of **salt;** and **charring** the surface; are sometimes found useful in damp situations.

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* See Transactions, July, Aug and Sept 1885, to which we are indebted for many of the above suggestions.

Price of lumber. Phila, 1886; Spruce joists, $20 to $24 per 1000 ft board meas. Hemlock joists, $13 to $16. Yellow pine floor boards, $20 to $35. White pine boards, $18 to $50, according to quality, degree of seasoning, &c. Sawed W pine timbers, $28 to $35. Heart Y pine, $20 to $35. Hemlock, $16 to $20. Gillingham, Garrison & Co., 943 Richmond St.

Boards of oak or pine, nailed together by from 4 to 16 tenpenny common cut nails, and then pulled apart in a direction lengthwise of the boards, and across the nails, tending to break the latter in two by a shearing action, averaged about 300 to 400 lbs per nail to separate them; as the result of many trials.

Table of Cut Nails, ordinary pattern.

Base price, Phila, 1886, about $2.50 per keg containing 100 lbs.*

Name.	Length. Inches.	Number of Nails per lb.	Extra over base price. cts per keg.	Name.	Length. Inches.	Number of Nails per lb.	Extra over base price. cts per keg.
3 penny	1¼	557	150	10 penny	3	66	0
4 "	1½	336	75	12 "	3¼	50	0
5 "	1¾	210	75	20 "	4	32	0
6 "	2	163	50	30 "	4½	19	0
7 "	2¼	123	50	40 "	5	16	0
8 "	2½	93	25	50 "	5½	13	0

The sizes and weights vary considerably with different makers. Ours are averages. The above are machine-made, or **cut nails,** *cut* by machinery from *plates* of rolled iron or steel. **Wrought nails** are *forged* by a blacksmith, or by machinery, from rolled iron or steel *rods*.

Nails cut from Bessemer and similar **steels** cost about 10 cts per keg more than the above.

* Morris, Wheeler & Co (Pottstown Iron Co), 16th and Market Sts, Phila, manufacturers.

PLASTERING.

THE plastering of the inside walls of buildings, whether done on laths, bricks, or stone, generally consists of three separate coats of mortar. The first of these is called by workmen the *rough* or *scratch coat;* and consists of about 1 measure of quicklime, to 4 of sand; (which latter need not be of the purest kind;) and ⅓ measure of bullock or horse hair; the last of which is for making the mortar more cohesive, and less liable to split off in spots. This coat is about ⅜ to ½ inch thick; is put on roughly; and should be pressed by the trowel with sufficient force to enter perfectly between and behind the laths; which for facilitating this should not be nailed nearer together than ½ an inch. In rude buildings, or in cellars, &c, this is often the only coat used. When this first coat has been left for one or more days, according to the dryness of the air, to dry slightly, it is roughly *scored*, or *scratched*, (hence its name,) with a pointed stick, or a lath, nearly through its thickness, by lines running diagonally across each other, and about 2 to 4 ins apart. This gives a better hold to the second coat, which might otherwise peel off. If the first coat has become too dry, it is well also to dampen it slightly as the second one is put on.

The second coat is put on about ¼ to ⅜ inch thick, of the same hair mortar, or *coarse stuff*. Before it becomes hard, it is roughed over by a hickory broom, or some substitute, to make the third coat adhere to it better.

The third coat, about ⅛ inch thick, contains no hair; and for giving it a still whiter and neater appearance, more lime is used, say 1 of lime, to 2 of sand; and the purest sand is used. This mortar is by plasterers called **stucco;** a name also applied to mortar when used for plastering the outsides of buildings. Or instead of stucco, the third coat may be, and usually is, of *hard finish*, or *gauge stuff;* which consists of 1 measure of ground plaster of Paris, to about 2 of quicklime, without sand. Hard finish works easier; but is not as good as stucco, for walls intended to be painted in oil. The plaster of Paris is for hastening the hardening.

Either of these third coats is smoothed or polished to a greater or less extent, according to whether it is to show, or to be papered, painted, &c. The polishing tools are merely, the trowel; the hand-float, (a kind of wooden trowel;) and the water-brush, (a short-handled brush for wetting the surface part at a time with water, in order to polish more freely.) For finer polishing, a float made of cork is used. The smooth piece of board about 10 to 12 ins square, with a handle beneath, on which the plasterer holds his mortar until he puts it on to the wall with his trowel, is called a *hawk*.

The more thoroughly each coat is gone over with the water-brush and trowel, (which process is called *hand-floating,*) the firmer and stronger will it be. Frequently only two coats of plastering are put on in inferior rooms; or where great neatness of appearance is not needed. The first is of hair mortar, or coarse stuff; this is scratched with the broom, and then covered by the finishing coat of finer mortar, (stucco.) If this last is nearly all lime, or with but very little sand, to make it work easier, it is called a *slipped coat*. Without any sand it is called *fine stuff*. Neither is as good as stucco, if the wall is to be papered. When this is the case, the third coat also may have a little hair, to give it more strength; but this is not absolutely necessary.

A very good effect may be produced in station-houses, churches, &c, by only two coats of plaster in which fine clean screened gravel is used instead of sand. When lined into regular courses, it resembles a buff-colored sandstone, very agreeable to the eye.

In purchasing plastering hair, care must be taken that it has not been taken from salted hides; inasmuch as the salt will make the walls damp. For the same cause sea-shore sand should not be used. It is almost impossible to wash it entirely free from salt.

In brick walls intended to be plastered, the mortar joints should be left very rough, to let the plaster adhere. If it is put on smooth walls, without first raking out the mortar to the depth of nearly an inch, it is very apt to fall off; especially from outside walls; as can be seen daily in any of our cities. As this raking out of brick joints is tedious and expensive, it would generally be better to use paint rather than plaster. The walls should also be washed clean from all dust; and should be slightly dampened as the plaster is put on.

To imitate granite on outer walls: after the second or smooth coat of plaster is dry, it receives a coat of lime wash, slightly tinted by a little umber, or ochre, &c. After this is dry, in case it appears too dark, or too light, another may be applied with more or less of the coloring matter in it. Finally, a wash of lime and mineral-black is *sprinkled* on from a flat brush, to imitate the black specks of granite. By this simple means, a skilful workman can produce excellent imitations. The horizontal and vertical joints of the imitation masonry, may be ruled in by a small brush, using the same black wash, and a long straight-edge.

The rough surfaces of all walls are more or less warped, or out of line; and it is not possible for the plasterer to rectify this perfectly by eye, as may be seen in almost every house. Even in what are called first-class ones, a quick eye can generally detect unsightly undulations of the plastered surfaces.

To prevent this, the process of **screeding** is resorted to. Screeds are a kind of gauge or guide, formed by applying to the first rough coat, when partly dried, horizontal strips of the plastering mortar, about 8 ins wide, and from 2 to 4 ft apart all around the room. These are made to project from the first coat, out to the intended face of the second one; and while soft are carefully made perfectly straight, and out of wind with each other, by means of the plumb-line, straight-edge, &c. When they become dry, the second coat is put on, filling up the broad horizontal spaces between them; and is readily brought to a perfectly flat surface, corresponding with that of the screeds, by means of long straight-edges extending over two or more of the latter.

A day's work at plastering.

A plasterer, aided by one or two laborers to mix his mortar, and to keep his hawk supplied, can average from 100 to 200 square yards a day, of first coat; about ⅔ as much of second; and half as

much of third, which requires more care. The amount will depend upon the number of angles, size of rooms, whether on ceilings or on walls, &c, &c.

Gen Gillmore's estimate of cost of plastering* 100 square yards with 2 or with 3 coats. Common labor $1 per day.

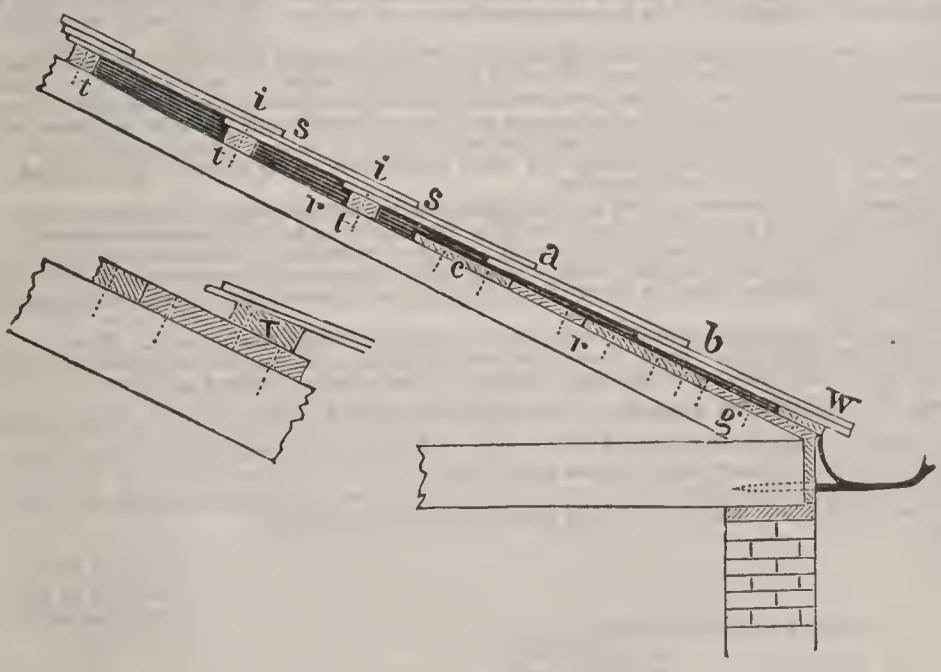
Materials.	Three Coats. Hard finished work.		Two Coats. Slipped coat finish.	
Quicklime	4 casks.	$4.00	3⅓ casks.	$3.33
" for fine stuff	⅔ "	.85		
Plaster of Paris	½ "	.70		
Laths	2000	4.00	2000.	4.00
Hair	4 bushels.	.80	3 bushels.	.60
Common Sand	7 loads.†	2.00	6 loads.	1.80
White Sand	2½ bushels.	.25		
Nails	13 lbs.	.90	13 lbs.	.90
Mason's labor	4 days.	7.00	3½ days.	6.12
Laborer	3 days.	3.00	2 days.	2.00
Cartage	.....	2.00	.....	1.20
Cost of 100 square yards	.....	$25.50	.....	$19.95

This amounts to 25½ cts per sq yd for 3 coats; and say 20 cts for 2 coats. **See Art 5,** p 674.

Plastering laths are usually of split white or yellow pine, in lengths of about 3 to 4 feet; and hence called 3 or 4 ft laths. They are about 1½ ins wide, by ¼ inch thick. They are nailed up horizontally, about ½ inch apart. The upright studs of partitions are spaced at such distances apart, (generally about 15 ins from center to center,) that the ends of the laths may be nailed to them. Laths are sold by the bundle of 1000 each. A square foot of surface requires 1½ four feet laths; or 1000 such laths will cover 666 sq ft. Sawed laths may be had to order, of any required length. A carpenter can nail up the laths for from 40 to 60 sq yds of plastering in a day of 10 hours; depending on the number of angles in the rooms, &c.

SLATING. For prices, see note* p 429.

ROOFING slates are usually from ⅛ to ¼ inch thick; about $\frac{3}{16}$ being a common average. They may be nailed either to a sheeting of rough boards (*c*, *g*, in the fig) from ¾ to 1¼ inch thick, (which should be, but rarely are, tongued and grooved,)



***Average prices of plastering** in Philada, 1886, in cts per sq yard. Three coats, including laths, scaffold, &c, 50 to 55 cts. Two coats, 35 to 40. Three coats on brick or stone (no laths reqd), 30 to 40. Outside plastering, 60; or if to imitate marble, 75. Simple plaster cornices, 1 to 2 cts per inch of girth, per ft run. Plaster center flowers for parlors, $5 to $15 each, put up. The plastering of a 20-ft front, 3-story dwelling, with large 3-story back buildings, $500 to $700. Stipulate *expressly* to pay only for surfaces *actually plastered;* and thus avoid *extras*, even if you have to pay a few cts more per yard.

†**A load** (one-horse), both in the U. S. and in England, usually means a cub yd; but many dealers adopt 20 struck bushels = 25 cub ft = fully a ton.

laid horizontally from rafter to rafter; or sloping, from purlin to purlin, as the case may be; or to stout laths *t t t* about 2 to 3 ins wide, and from 1 to 1¼ thick, nailed to the rafters at distances apart to suit the gauge of the slates. Two nails are used to each slate; one near each upper corner. They may be either of copper, (which is the most durable, but most expensive,) of zinc, or of either galvanized or tinned iron. The last two are generally used; or in inferior work, merely plain iron ones, previously boiled in linseed oil, as a partial preservative from rust. Rust, however, sometimes weakens them so much that they break; and the slates are blown off in high winds, to the danger of passers by. Since good slate endures for a long series of years, it is true economy to use nails that are equally durable. In iron roofs, the slates, instead of being nailed to boards, are sometimes tied directly to the iron purlins, by wire. **A square** of slating, shingling, &c, is 100 sq ft.

In laboratories, chemical factories, &c, subject to **acid fumes**, it is difficult to provide a metal fastening that will not be eaten away. In such cases it is best to depend chiefly upon a layer of mortar between the slates. This will harden before the metal fastenings give way; and will hold the slates in place, while new fastenings are being inserted.

The **least pitch** considered advisable for a roof, to prevent rain or snow from being driven through the interstices between the slates, is about 26½°; or 1 vert to 2 hor; which corresponds to a rise of ¼ the span in a common double pitched roof. But even at steeper pitches, rain, and more particularly snow, will be forced through the roof by violent winds; especially if laths alone be used, or even boarding alone. To avoid this, a layer of mortar about ¼ inch thick, may be spread over the touching surfaces of the slates if on laths. If on boards, the same process may be adopted; or the more common one of first covering the boards with a layer of what is called *slating felt;* but which in reality is merely thick brown paper, soaked in tar. This is sold in long continuous rolls 28 ins wide, and weighing from 40 to 50 lbs. A 50 lb roll will cover about 300 sq ft of roof. With proper precautions against the admission of rain and snow, a pitch as flat as 1 in 2½, or even 1 in 3, may be adopted.

The thickness of slate on a roof is double; except at the *laps i s, i s,* &c, where it is triple. The lap is measured from the nail hole (under *i*) of the lower slate, to the lower edge or *tail, s,* of the upper one; and is usually about 3 ins. In order that the showing lower edges of the slates shall, when laid, form regular straight lines along the roof, the nail holes are made at equal distances from said *lower* edges; so that any irregularity of length is concealed from view at the hidden *heads* of the slates. The slater estimates the length of his slate from the nail hole to the tail: discarding the narrow strip between the nail hole and the head. If from this reduced length the lap be deducted, then one-half of the remainder will be the *gauge, weathering,* or *margin,* of the slating; or, in other words, the *showing* or *exposed* width of the courses of slates. The gauge in ins multiplied by the width of a slate in ins, gives the area in sq ins of finished roof covered by a single slate; and if 144 (the sq ins in a sq foot) be divided by this area, the quotient will be the number of slates required per sq ft of roof. The upper side of a slate is called its *back;* the lower one, its *bed.*

Slating, like shingling, must evidently be commenced at the eaves, and extended upward. Since the beds of the slates are not exactly parallel to the boarding, and consequently do not rest flat upon it, those at the lower edge *w* would easily be broken. To prevent this, a *tilting strip* (a stout wide lath, with its upper side planed a little bevelling, to suit the slope of the slates) is first nailed around near the eaves, for the tails of the lowest course of slates to rest on. This is shown on a larger scale at T.

Slate of the best quality has a glistening semi-metallic appearance, somewhat like that of a surface of paper rubbed with black-lead pencil. That of a dull earthy aspect, is softer, more absorbent, and consequently more liable to yield to atmospheric influences, rain, frost, &c. Iron pyrites frequently occurs in slate; and since it always decomposes and leaves holes, should never be admitted on a roof. Of two qualities of slate, that which absorbs the least weight of water, when pieces of equal size are soaked for an hour or two, is generally the best; being least liable to split by frost, and become weather-worn. This test is easily applied.

In England the **different sizes** are distinguished by absurd names of no meaning. In the United States they are called 6 by 12's; 16 by 24's, &c, according to their measures in inches. They may be cut to order, of almost any prescribed dimensions, or shape. Those in common use vary from about 7 by 14, to 12 by 18. The first forms about 5 to 6 inch courses; and the last about 7 to 8 inch; depending upon how far from the head the nail holes are pierced. The farther this is, the firmer will the slating be.

Slate roofs, like iron ones, heat the rooms immediately below them very much. This is somewhat diminished when the slates are on boards, instead of laths; and still more by a coat of plaster beneath. They are also liable to break when walked on; less so when bedded in mortar.

Weight of slate roofs. Slate weighs about 175 lbs per cub foot; therefore a sq ft, ⅛ inch thick, weighs about 1.8 lbs; $\frac{3}{16}$, 2.7 lbs; and ¼ thick, 3.6 lbs. But owing to the overlapping, a square foot of roof requires about 2½ sq ft of slate of ordinary sizes; and if the slate is laid on boards an inch thick, the weight per sq ft of roof will be increased about 2¼ lbs; or with 1¼ inch boards, 2.8 lbs. Laths will weigh about ¼ lb per sq ft of roof.

Hence,

	Approx Weight of one sq ft of Slating, in lbs.
Slate ⅛ inch thick on laths	4.75
" " on 1 inch boards	6.75
" " on 1¼ " "	7.30
" 3-16 " on laths	7.00
" " " on 1 inch boards	9.00
" " " on 1¼ " "	9.55
" ¼ " on laths	9.25
" " " on 1 inch boards	11.25
" " " on 1¼ " "	11.80

If slating felt is used, add ¼ lb; or if the slates are bedded in ¼ inch of mortar, add 3 lbs.

For the total weight borne by the *roof trusses*, that of the purlins also must be added. This will not vary much from the limits of 1½ to 3 lbs per sq ft in roofs of moderate span. Add for wind and snow, say 20 lbs per sq ft;* and finally add the weight of the truss itself.

For stopping the joints between slates (or shingles, &c) and chimneys, dormer windows, &c, a mixture of stiff white-lead paint, as sold by the keg, with sand enough to prevent it from running, is very good; especially if protected by a covering of strips of lead, or copper, tin, &c, nailed to the mortar-joints of the chimneys, after being bent so as to enter said joints; which should be scraped out for an inch in depth, and afterward refilled. Mortar protected in the same way, or even unprotected, is often used for the purpose; but is not equal to the paint and sand. Mortar a few days old, (to allow refractory particles of lime to slack,) mixed with blacksmith's cinders and molasses, is much used for this purpose, and becomes very hard, and effective.

For prices of slating, see foot note* below.

SHINGLES.

WHITE cedar shingles are the best in use; and when of good quality will last 40 or 50 years in our Northern States. They are usually 27 ins long; by from 6 to 7 ins wide; about ¼ inch thick at upper end; and about ⅝ at lower end or butt; and are laid in courses about 8⅛ ins wide; so that not quite ⅓ of a shingle is exposed to the weather.

They are usually laid in three thicknesses; except for an inch or two at the upper ends, where there are four. They are nailed to sawed shingling-laths of oak or yellow pine; about 16 ft long; 2½ ins wide, and 1 inch thick; placed in horizontal rows about 8½ ins apart. These are nailed to the rafters, or purlins; which, for laths of the foregoing size, should not be more than 2 ft apart from center to center. Two nails are used to each shingle, near its upper end. They should not be of less size than 400 to a lb. Wrought nails being the strongest, are the best; cut ones are apt to break by the warping of the shingles. Two pounds of such nails will suffice for 100 sq ft of roof, including waste. An average shingle 7½ ins wide, in 8½ inch courses, exposes 63¾ sq ins; making 2¼ shingles to a sq ft of roof; but to allow for waste, and narrow shingles, it is better in practice to allow about 3 shingles to a sq ft.

Shingling, like slating, must plainly be begun at the eaves; and extended upward. For closing the joints between the shingles, and chimneys, dormer windows, &c, see at end of Slating.

Cypress and white pine are also much used for shingles, being much cheaper, but scarcely half as durable.† All shingles wear quite thin in time by rain and exposure. In warm damp climates they all decay within 6 to 12 years.

PAINTING.

THE principal material used in house-painting, is either white lead, or oxide of zinc, ground in raw (unboiled) linseed oil, by a mill, to the consistency of a thick paste. In this condition, it is sold by the manufacturers in kegs of 25, 50, and 100 lbs. To prepare it for actual use, merely requires the addition of more linseed oil, say 3 or 4 pints to 10 lbs of the keg paint, for thinning it sufficiently to flow readily under the brush.

Good painting requires 4 or 5 coats; but usually only 4 are used in principal rooms; and 3 in inferior ones. Each coat must be allowed to dry perfectly before the next one is put on. One lb of the keg paint will, after being thinned, cover about 2 sq yds of first coat; 3 yds of second; and 4 yds of each subsequent coat; or 1 sq yd of 3 coats will require in all, 1.08 lbs; of 4 coats, 1¼ lbs; of 5 coats, 1.58 lbs. The reason why the first coats require so much more than the subsequent ones, is that the bare surface of the wood absorbs it more.

When, as is usual, *raw* or unboiled oil is used for thinning, *dryers* must be added to it; otherwise the paint might require several weeks to harden; whereas, with dryers, from 1 to 3 days, according to the weather, suffice for each coat to become hard enough to receive the next one. The dryers most commonly used, are powdered litharge, in the proportion of one heaped teaspoonful; or Japan varnish, 1 table-spoonful, to 10 lbs of the keg paint. Either sugar of lead, or sulphate of zinc, may also be used instead of litharge; and in the same proportion. Although both litharge and Japan varnish are dark-colored, yet the quantity is so small as not to appreciably affect the whiteness of the paint. If the varnish is used in excess, as is often done in the hurry to have work finished, it produces cracks all over the surface. No dryer is necessary if painters' boiled oil be used for thinning. Mere boiling will not cause oil to harden more rapidly; but that intended for painters, has litharge added to it previously to boiling; in the proportion of 1½ lbs to each 10 gallons of raw oil. In some works written for the use of house painters, it is asserted that boiling renders the oil too thick for any but coarse outdoor work. But this is entirely a mistake; for if the boiling be properly done, the oil will be quite thin enough for the best inside work; and will moreover be clearer than while raw; and

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* **Price** of slate, felt, and slating in Philada, in 1886, is from 8 to 9 cts per sq ft, according to quality of slate, kind of nails, &c; but exclusive of boarding. With copper nails add 1 ct per sq ft. The slate from Peach Bottom, York Co, Penna, is the best in the State. It commands 1 or 2 cts per sq ft more than the others. A roof of leaded tin will cost about the same as one of slate; and not much more than half as much as good cedar shingles, (in Philada.) Felt about 2 cts per lb.

† Price of shingles per 1000, in Philada, in 1886: Cedar, 6 ins × 30 ins, $25; 6 ins × 24 ins, $20. Cypress, $18. Shingling laths, $5. Cedar shingles, laths, nails, and shingling complete, 20 cts per sq ft of roof; or about twice as much as slate or leaded tin roofing, exclusive of boards.

will impart to the painted surface a more shining appearance. The heat should be barely sufficient to produce boiling; or about 600° Fah. The boiling should continue about 1½ hours; the oil being thoroughly stirred at short intervals, to prevent the litharge from settling at the bottom. The fire may then be allowed to subside; when the operation will be completed. A sediment will then form at the bottom; which must be left behind when the oil is poured off. Although no dryer is necessary with this oil, still a little litharge may be added when great expedition demands it. Painters rarely use this oil, on account of its trifling increase of cost.

Another substance much used with the thinning oil, (except for the first coat,) is spirits of turpentine; called "turp" by the workmen. The quantity of oil may be diminished, to the extent of the added turp. This being more fluid than oil, causes the paint to work more pleasantly under the brush. It moreover diminishes the tendency of the paint to become yellow; especially in rooms kept closed for some time. It is also much cheaper than oil. It should not be used, or but sparingly, for exposed outdoor work; inasmuch as its tendency is to impair the firmness of the paint; and although its effects are scarcely appreciable indoors, they are quite apparent when the work has to resist the weather. As the fashions change in house-painting, the surface is at times required to present a *shining* or *glossy* finish; at other times a *dead* one is in vogue. The glossy one is that which the paint will naturally have, provided that no more turp than oil be used in the thinning. The dead finish is obtained by using no oil, but turp alone, for the last coat; which in that case is called a *flatting coat*. Although turp is not properly a dryer, still, as it evaporates quickly, it facilitates the hardening of the paint.

In outdoor work it is usually advisable to use more dryer than inside, so that the paint may sooner become hard enough not to be injured by dust or rain. Otherwise less would be better.

When, instead of a white finish, one of some other color is required, the coloring ingredient is mixed with the white paint to be used in the last coat only; although two coloring coats are sometimes found to be necessary before a satisfactory effect is produced. The coloring ingredients may be indigo, lampblack, terra sienna, umber, ochre, chrome yellow, venetian red, red lead, &c, &c; which are ground in oil, ready for sale, by the manufacturers of the white-lead and zinc paints. They are simply well stirred into the white paint.

All surfaces to be painted, should first be thoroughly dry, and free from dust. If on wood, all plane-marks, and other slight irregularities, should first be smoothed off by sand-paper, when the neatest finish is required. Also, all heads of nails must be punched to about ⅛ inch below the surface. To prevent knots from showing through the finished work, (as those in white or yellow pine would do, on account of the contained turpentine,) they must first be *killed*, as it is termed. A usual and effective way of doing this, is by covering them with two coats of shellac varnish; which, when dry, should be smoothed by sand-paper. Another mode, not quite so certain, is by one or two coats of white lead mixed with thin glue-water, or *size*, as it is called.

After these preparations, the first, or *priming* coat, is put on; in which there should be no turp; because it would sink at once into the bare wood, leaving the white lead behind it, in a nearly dry friable condition. After this the nail holes, cracks, &c, must be filled with common glaziers' putty, made of whiting (fine clean washed chalk) and raw linseed oil; boiled oil will not answer; the putty would be friable. The putty would be apt to fall out, if put in before priming; because the wood would absorb the oil, and the putty would then shrink. After the first coat is perfectly dry, the second one is put on; and for it about 1 measure of turp may be mixed with 3 measures of the thinning oil. In the third, and any subsequent coats, equal measures of turp and oil, may be used for thinning, if the work is required to dry *with a gloss;* but if it is to finish *dead*, the last coat must be a *flatting* one; or one in which the thinning oil is entirely omitted, and turp alone substituted for it.*

Painters generally clean their brushes by merely pressing out most of the paint with a knife; and then keep them in water until further use. If to be put away for some time, they may be thoroughly cleaned by turp; or by soap and water. To prevent a hard skin from forming on the top of their paint when not used for some days, they pour on a little oil.

The best paints for preserving iron exposed to the weather, appear to be pulverized oxides of iron, such as yellow and red iron ochres; or brown hematite iron ores finely ground; and simply mixed with linseed oil, and a dryer. White lead applied directly to the iron, requires incessant renewal: and indeed probably exerts a corrosive effect. It may, however, be applied over the more durable colors, when appearance requires it. Red lead is said to be very durable, when pure. An instance is recorded of pump-rods, in a well 200 ft deep, near London, which, having first been thus painted, were in use for 45 years; and at the expiration of that time, their weight was found to be precisely the same as when new; thus showing that rust had not affected them. See p 403.

When the size of the exposed iron admits of it, its freedom from rust may be very much promoted by first heating it thoroughly; and then dipping it into, or washing it well with, hot linseed oil; which will then penetrate into the interior of the iron. For tinned iron exposed to the weather, on roofs, rain pipes, &c, Spanish brown is a very durable color. The tin is frequently found perfectly bright and protected, when this color has been used, after an exposure of 40 or 50 years. White paint washes off in a few years by rain.

Plastered walls should if possible be allowed to dry for at least a year, before being painted in oil; otherwise the paint will be liable to blister. They may, if preferred, be frescoed (water-colors, mixed with size) to the desired tint during the interval.

The painting of unseasoned wood hastens its decay. If the surface to be painted is greasy, the grease must first be removed by water in which is dissolved some lime.

Washes for outside work. Downing, in his work on country houses, recommends the following: *For wood-work;* in a tight bushel, slack half a bushel of fresh lime, by pouring over it boiling water sufficient to cover it 4 or 5 ins deep; stirring it until slacked. Add 2 lbs of sulphate of zinc (white vitriol) dissolved in water. Add water enough to bring all to the consistence of thick whitewash. Apply with a whitewash brush. This wash is white; but it may be colored by adding powdered ochre, Indian red, umber, &c. If lampblack is added to water-colors, it

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* **Average cost of Painting** in Philada, 1886, including scaffold, &c, per square yard. Four coats in plain colors, 30 cts; 3 coats, 25. Graining in imitation of oak, walnut, &c, 50. White lead ground in oil, in kegs, 8 cts per lb. The cost of painting and glazing a 20-ft front, 3-story dwelling, with large 3-story back buildings, $300 to $400. A church of 60 by 80 ft, with basement story, and galleries, $900 to $1000. Avoid extras; or stipulate for them in advance.

should first be thoroughly dissolved in alcohol. The sulphate of zinc causes the wash to become hard in a few weeks.

For brick, masonry, or rough-cast. Slack ½ a bushel of lime as before; then fill the barrel ⅔ full of water, and add a bushel of hydraulic cement. Add 3 lbs of sulphate of zinc, previously dissolved in water. The whole should be of the thickness of paint; and may be put on with a whitewash brush. The wash is improved by stirring in a peck of white sand, just before using it. It may be colored, if desired, like the preceding.

He also gives the following cheap oil-paint for outside work on wood, brick, stone, &c; and says it becomes far harder and more durable than common paint: One measure of *ground* fresh quicklime; add the same quantity of fine white sand, or fine coal ashes; and twice as much fresh wood ashes; all the foregoing to be passed through a fine sieve. Mix well together dry. Mix with as much raw linseed oil as will make the mixture as thin as paint. Apply with a painter's brush. It may be colored like the foregoing, taking care to mix the colors well with oil before adding them. It is best to put on two coats; the first thin, and the second thick.

Also, another, said to stand 15 to 20 years: 50 lbs best white lead; 10 quarts raw linseed oil; ½ lb dryer; 50 lbs finely sifted sharp clean sand; 2 lbs raw umber. Add very little, say ½ pint of turpentine. Apply with a *large* brush.

Cement for stopping joints, such as around chimneys, &c, &c. White lead ground in oil, as sold by the keg; mixed with enough pure sand to make a stiff paste that will not run. It grows hard by exposure, and resists heat, cold, and water. Pieces of stone may be strongly cemented together by it, allowing a few months for proper hardening.

Whitewash for inside work, according to Mr. Downing, "is made more fixed and permanent, by adding 2 quarts of thin *size* to a pailful of the wash, just before using. The best size for this purpose is made of shreds of glove leather; but any clean size of good quality will answer," as thin glue-water. We will add, that the common practice of mixing salt with whitewash, should not be permitted. Paper pasted on a wall which has previously been covered with salt whitewash, is very apt to become wet, and loose, and to fall off during damp weather. The whitewash should be scraped off, and the wall or partition covered with a coat or two of thin size, to protect the paper from the effect of the salt that may still adhere to the plaster.

GLASS, AND GLAZING.

WINDOW glass is sold by the box. Whatever may be the size of the panes, a box contains as nearly 50 sq ft of glass as the dimensions of the panes will admit of.

Panes of any size may be made to order by the manufacturers. The sizes given in the following table, as well as many others, are generally to be had ready made. Ordinary window glass of all the sizes in the table, is about one-sixteenth of an inch thick; and this is the thickness supposed to be intended when a greater one is not specified. *Double-thick* glass is nearly ⅛ inch; and its price is 50 per ct more than the *single* thick. It is of course much stronger than the single.

The panes are confined to the sash by glaziers' putty, made of whiting (powdered chalk) and raw linseed oil; and by small triangular pieces of thin tin, about ¼ inch on a side, which uphold the glass while the putty is being put on; and are allowed to remain afterward, as a protection while the putty continues soft.

TABLE OF NUMBERS OF PANES IN A BOX.

Size in ins.	Panes to a box.	Size in ins.	Panes to a box.	Size in ins.	Panes to a box.	Size in ins.	Panes to a box.	Size in ins.	Panes to a box.
6 × 8	150	12 × 36	17	16 × 42	11	24 × 24	12	30 × 66	4
7 × 9	115	13 × 14	40	48	9	26	12	70	3
8 × 10	90	16	35	54	8	30	10	32 × 34	7
12	75	18	31	60	8	36	9	36	6
9 × 12	67	20	28	18 × 20	20	42	7	42	6
14	57	24	23	22	18	48	6	48	5
16	50	32	17	24	17	54	6	60	4
18	45	14 × 16	32	30	14	60	5	66	3
10 × 12	60	18	29	36	11	66	5	34 × 36	6
14	52	20	26	42	10	26 × 28	10	44	5
16	45	24	22	50	8	32	9	48	5
18	40	30	17	60	7	36	8	54	4
20	36	36	14	20 × 22	17	42	7	60	4
24	30	42	12	24	15	48	6	66	3
30	24	46	11	30	12	54	5	36 × 40	5
11 × 12	55	15 × 16	30	38	10	60	5	44	5
14	47	18	27	42	9	28 × 30	9	48	4
16	41	20	24	48	8	36	7	54	4
18	37	24	20	54	7	42	6	60	3
20	33	30	16	64	6	56	5	70	3
24	27	36	13	22 × 24	14	66	4	38 × 44	4
12 × 14	43	40	12	30	11	30 × 31	7	52	4
16	38	16 × 18	25	36	9	36	7	40 × 46	4
18	34	20	23	42	8	42	6	54	3
20	30	24	19	48	7	48	5	72	3
24	25	30	15	56	6	54	4	44 × 50	3
28	22	36	13	60	5	60	4	56	3
30	20								

The best qualities of American glass made in the vicinity of Philadelphia, Boston, Pittsburg, &c, are for most purely *useful* purposes, as good as those from foreign countries; but when the highest degree of *beauty* is required, as in the lower front windows of first-class dwellings, fancy stores, &c. polished plate-glass of England, France, or Germany, must be used: although the price for moderate sized panes is from 5 to 8 times as great as that of the best quality single-thick American, as given in the following table.* Its perfectly *smooth* surface, free from distorted reflections, also makes it the best for covering pictures; still, if carefully selected American panes be used for this purpose, few except critics in glass will detect the difference.

A thick glass is made expressly for flooring, up to 1 inch thick, and up to 50 inches by 9 feet dimensions. Also, for skylights, from ¼ to ½ inch thick. This can be furnished to order of any size up to 40 inches by 8 or 10 feet. The smaller sizes can also be had *ground*. Grinding prevents the entrance of the full glare of the sun; and, moreover, diffuses the light over a much greater width of space below.

Strength of glass. Tensile 2500 to 9000 lbs per square inch. Boston rods by author, 3500 to 5200. Crushing strength, 6000 to 10000 lbs per square inch. Transversely, (by the writer's trials,) flooring glass, 1 inch square, and 1 foot between the end supports, breaks under a center load of about 170 lbs; consequently, it is considerably stronger than granite, except as regards crushing; in which the two are about equal.

Remark. Window and other glass which contains an excess of potash or of soda is very liable to become dull in time, owing to the decomposition of those ingredients by atmospheric influences.

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* **Price list of American single-thick glass.** adopted by American Window Glass Manufacturers' National Association, July 22, 1886. Double-thick about 50 per cent. more. For ground glass add about $2.50 per box. **Discount,** 1886, about 75 per cent. Benj. H. Shoemaker, dealer in French and American window glass, 205 N. 4th St, Philadelphia; also Malaga Glass and Manufacturing Co., office 400 Chestnut St, Philadelphia.

Size in Inches.		1st Quality.	2d Quality.	3d Quality.	4th Quality.
From	to	Per box.	Per box.	Per box.	Per box.
6 × 8	10 × 15	$8.75	$8.00	$7.50	$7.00
11 × 14	16 × 24	10.00	9.25	8.75	8.00
18 × 22	20 × 30	12.50	11.50	10:25	9.00
15 × 36	24 × 30	13.25	12.00	10.75	9.50
26 × 28	24 × 36	14.50	13.00	11.50	10.25
26 × 36	26 × 44	15.00	13.50	12.25	11.00
26 × 46	30 × 50	16.75	15.25	13.75	11.75
30 × 52	30 × 54	17.50	16.00	14.25	
30 × 56	34 × 56	19.25	17.50	15.75	
34 × 58	34 × 60	20.75	18.75	16.75	
36 × 60	40 × 60	22.25	20.75	18.50	

In small quantities, the following are also approximate prices for American glass: Large plates of ⅛ inch thick, rough, 40 cents per square foot. One inch thick, 75 cents to $1; if either is ground, 10 to 15 cents additional per square foot. Ribbed glass, ⅛ inch thick, 20 cents; ¼ inch, 25 cents per square foot. *Stained* glass, single thickness, ($\frac{1}{16}$ inch,) or *figured* white enameled glass, (single thickness,) 20 to 25 cents per square foot. Superior thicker strong figured glass, first ground, and the transparent figures then formed by polishing away portions of the ground surface, $1.00 per square foot.

Muffled glass is an inferior article of fanciful colored patterns, attached by some imperfect process which allows them to peel off after a year or two of exposure to the weather.

The charge by glaziers for putting the glass into new windows, including putty, tins, and two coats of paint to the sash, (one of which is a priming coat,) is (1886) equal to the cost of the glass *at the above prices.*

For reglazing old sash and removing the broken panes, the charge is about twice as great.

PAPER.*

24 sheets 1 quire. 20 quires 1 ream.

Sizes of drawing papers.

	Ins.	Ins.		Ins.	Ins.
Antiquarian	31	× 52	Super Royal	19	× 27
Double Elephant	26	× 40	Royal	19	× 24
Atlas	26	× 34	Medium	17	× 22
Imperial	21	× 30	Demy	15	× 20
			Cap	13	× 17

The English drawing-papers are stronger and superior to the American. Those by Whatman have a high reputation; they are, however, of different qualities. When paper is pasted on muslin, the difference in quality is not so important. Of paper in rolls, the German makes are the best. There is but little of other makes imported.

Both white and tinted papers, for the use of engineers, are made in continuous rolls, without seams. Widths 36, 42, 54, 58, and 62 ins; usual lengths 40 yds: but can be had to order to 400 yds or more. These may also be purchased ready pasted on muslin, in rolls 10 to 40 yds long. This last, on account of its strength, should be used for all drawings which undergo frequent rolling and unrolling; or other hard usage; particularly working-drawings. For the last purpose, strong *cartridge* or *pattern* paper answers very well. It is for sale in long rolls, of same lengths as white paper, mounted or not; widths up to 54 ins. Color, a light buff.

Tracing paper. Most of that sold, whether domestic or foreign, tears so readily as to be of comparatively little service, except for tracings to be enclosed in letters for mailing. Some of what is called *French vegetable tracing-paper*, is, however, quite stout and strong, and good for line drawings; but it shrivels badly under broad washes of color, even when stretched, forming little puddles, which make it difficult to produce a uniform tint. Sizes 19 × 25, 21 × 26, 28 × 40 ins; also in rolls of 11 and 22 yds. Parchment paper, 37 and 38 ins wide, rolls of 20 and 33 yds, is better, but does not take ink perfectly.

Tracing cloth, usually called *tracing muslin*, and sometimes *vellum cloth*, is altogether preferable to tracing paper, on account of its great strength. Widths 18, 30, 36, and 42 ins; lengths to 24 yds.

Common inks dry pale on either tracing muslin or tracing paper; therefore use India ink. Neither the muslin nor the paper takes colors as kindly as drawing paper.

Profile paper is made in widths of 9 ins and 20 ins, and in single sheets or in long, continuous rolls.

Ruled squares, or **cross-section paper.** Paper carefully ruled in small squares, so that the divisions answer for a scale for the drawing, is exceedingly useful for sketching out plans, &c. It is sometimes ruled on both sides of the sheet.

Colors. A good draughtsman needs but few colors; say India ink, Prussian blue, lake, or carmine, light red, burnt umber, burnt sienna, raw sienna, gamboge, Roman ochre, sap green. Winsor & Newton's colors are among the best in use. Purchase none but the very best India ink. Cakes of colors should always be wiped dry on paper, after being rubbed in water; and but little water should be used while rubbing; more being added afterward.

Lead pencils. *Genuine* A. W. Faber's Nos. 2, 3, and 4, are very good. The hardness increases with the number. Nos. 3 and 4 are good for field-book use: which to prefer, will depend on the character of the paper; No. 3 for smooth, and No. 4 for the coarser or more granular papers. His *lettered* pencils are of a higher grade and better suited for draughting. "H" stands for "hard," "B" for "soft." The degree of hardness or of softness is indicated by the number of H's or of B's. One H corresponds with No. 3. Dixon's American pencils are good. The office draughtsman should have a flat file, or a piece of fine emery paper glued to a strip of wood, upon which to rub his lead to a fine point readily, after using the knife.

* James W. Queen & Co, No 924 Chestnut St, Philadelphia.

STRENGTH OF MATERIALS.

The modulus of Elasticity, and its use. Within the limit of elasticity, a uniform rod of given material lengthens or shortens equally under equal additions of load. If this were also the case beyond said limit, it is plain that there would be some load which would stretch a uniform bar to twice its original length, or shorten it to zero or 0. And this load *in lbs*, for a bar of *one inch square* cross section, is the mod of elas for the given material; or is the E of authors on Strength of Materials. For example, a one-inch square bar of wrought iron will, within the limit of elas, stretch on an average about 1 part in 12000 of its length under each additional load of 2240 lbs. Consequently, if the same rate of stretching continued beyond the limit of elas, it is evident that 12000×2240, or 26880000 lbs, would stretch the bar to twice its original length. Hence these 26880000 lbs are the mod of elas for average bar iron. And so with any other material. Hence the mod of elas is a load which bears the same proportion to the original length of a uniform bar, as the load which will produce any given amount of stretch, is to the length of said stretch. This fact facilitates certain calculations; thus,

$$\left.\begin{array}{l}\text{Load in lbs reqd}\\ \text{to produce a given}\\ \text{stretch within}\\ \text{elas limit, in ins,}\end{array}\right\} = \frac{\text{Reqd stretch in inches}}{\text{orig length in inches}} \times \text{mod of elas.} \times \text{cross sec in sq ins}$$

$$\left.\begin{array}{l}\text{Stretch in ins}\\ \text{produced by any}\\ \text{load within elas}\\ \text{limit,}\end{array}\right\} = \frac{\text{Load in lbs} \times \text{orig length in ins}}{\text{mod of elas} \times \text{cross sec in sq ins}}$$

See Table, below.

E may also be found thus. At the center of a rectang beam supported hor at both ends, apply any load within its elas limit, and measure its deflection. Then as expressed by writers E is = $Wl^3 \div 4 \Delta bd^3$. Which means

$$\text{Coef, const, or Mod E in lbs per sq inch} = \frac{(\text{Load in lbs} + .625 \text{ wt of clear span of beam}) \times \text{cube of span ins,}}{4 \times \text{Def, ins} \times \text{breadth, ins} \times \text{cube of depth ins.}}$$

Moduli of elasticity for different materials. Authors differ considerably in their data on this subject. Where it was possible to adopt an average without error of practical importance, we have done so; but where the differences were too great for this, we have omitted the material entirely. Fortunately it is a matter of but little importance to the engineer: for, with the exception of iron and steel, he rarely need care to know the exact degree to which his materials are lengthened or shortened by their loads.

It must be carefully remembered that in practice the principle of mod of elas applies only within the elas limit of materials. This limit may ordinarily be safely taken at about one-third of the breaking tensile strengths given on pages 463 to 466. This is done in our 5th column. It is, however, but a tolerable approximation. There is reason to doubt the accuracy of many of the items in this table.

MATERIAL.	Modulus or Coeff of Elasticity.	Stretch or Compression in a length of 10 ft, under a load of 1000 lbs per sq in.	Stretch or Compression in a length of 10 ft, under a load of 1 ton per sq in.	Approx elas limit.
	lbs per sq in.	Ins.	Ins.	lbs per sq in.
Ash	1 600 000	.075	.168	4500
Beech	1 300 000	.092	.207	4000
Birch	1 400 000	.086	.192	5000
Brass, cast	9 200 000	.013	.029	6000
" wire	14 200 000	.009	.019	16000
Chestnut	1 000 000	.120	.269	4500
Copper, cast	18 000 000	.007	.015	6300
" wire	18 000 000	.007	.015	10000
Elm	1 000 000	.120	.269	2000
Glass	8 000 000	.015	.034	3200
Iron, cast	12 000 000 to 23 000 000	.010 to .005	.022 to .012	4500 to 8000
" " average	17 500 000	.007	.015	6250
" wrought, in either bars, sheets or plates	18 000 000 to 40 000 000	.006 to .003	.015 to .007	20000 to 40000
" " " average	29 000 000	.004	.009	30000
" wire, hard	26 000 000	.005	.010	27000
" wire ropes	15 000 000	.008	.018	13000
Larch	1 100 000	.109	.244	2300
Lead, sheet	720 000	.167	.....	1100
" wire	1 000 000	.120	.....	1100
Mahogany	1 400 000	.086	.192	2700
Oak	1 000 000 to 2 000 000	.120 to .060	.269 to .134	....
" average	1 500 000	.080	.179	.....
Pine, white or yellow	1 600 000	.075	.168	3300
Slate	14 500 000	.008	.018	3300
Spruce	1 600 000	.075	.168	3700
Steel bars	29 000 000 to 42 000 000	.004 to .003	.009 to .006	3300 to 44000
" " average	35 500 000	.003	.007	39000
Sycamore	1 000 000	.120	.269	4000
Teak	2 000 000	.060	.134	5000
Tin, cast	4 600 000	.026	.....	1500

Fatigue of Materials. In the following articles on Strength of Materials, the ultimate or breaking load is that which will, during its first application, rupture the given piece within a *short* time. But Wöhler's and Spangenberg's experiments show that a piece may be ruptured by **repeated applications** of a load much *less* than this; and that the oftener the load is applied the less it needs to be in order to produce rupture. Thus, wrought iron which required a tension of 53000 lbs per sq inch to break it in 800 applications, broke with 35000 lbs per sq inch applied about 10 million times; the stress, after each application, returning to zero in both cases.

The *diff* between the maximum and minimum tension in a piece subjected to tension only, or between the max and min compression in a piece subjected to comp only; or the *sum* of the max tension and max comp in a piece subjected alternately to tension and comp; is called the **range of stress** in the piece. When this is less than the elastic limit, the application may be repeated an "enormous" number (say about 40 million) of times without rupture.*

For a given number of applications, the load required for rupture is least when the range of stress is greatest. If the stress is alternately comp and tension, rupture takes place more readily than if it is always comp or always tension. That is, it takes place with a less range of stress applied a given number of times, or with a less number of applications of a given range of stress. For a given range of stress and given number of applications, the most unfavorable condition is where the tension and comp are equal.

The above facts are now generally taken into consideration in designing members of important structures subject to moving loads. For instance, Mr. Jos. M. Wilson, C. E., Mem. Inst. C. E. (London Eng.), Mem. Am. Soc. C. E., uses the following formulæ for determining the "permissible stress" in iron bridges, in lbs per sq inch; in order to provide the proper area of cross section for each member.

For pieces subject to *one kind of stress only* (all comp or all tension)

$$\mathbf{a} = u\dagger\left(1 + \frac{\text{min stress in the piece}}{\text{max stress in the piece}}\right)$$

For a piece subject *alternately to comp and tension*, find the max comp and the max tension in the piece. Call the *lesser* of these two maxima "max lesser", and the other or *greater* one, "max greater". Then

$$\mathbf{a} = u\dagger\left(1 - \frac{\text{max lesser}}{2\,\text{max greater}}\right)$$

For a piece whose *max comp and max tension are equal*, this becomes

$$\mathbf{a} = u\dagger\left(1 - \frac{1}{2}\right) = \frac{u}{2}$$

The above **a** is the permissible *tensile* stress in lbs per sq inch on any member; but the permissible *compressive* stress is found by "Gordon's formula" for pillars, p 439, using **a** (found as above) as the numerator, instead of *f*. For *a* in the *divisor* or *denominator* of Gordon's formula (which must not be confounded with the **a** of the *foregoing* formulæ) Mr. Wilson uses for wrought iron:

when both ends are fixed	36000
when one end is fixed and one hinged	24000
when both ends are hinged	18000

Experiments show that materials may fail under a **long continued stress** of much less intensity than that produced by the ult or bkg load.

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* This does not always hold in cases where the elastic limit has been artificially raised by process of manufacture, etc. Oft-repeated alternations between tension and compression below such a limit reduce it to the natural one. A slight flaw may cause rupture under comparatively few applications of a range of stress but little greater, or even less, than the elastic limit. Rest between stresses increases the resisting power of a piece. In many cases, stresses a little beyond the elastic limit, even if oft-repeated, raise that limit and the strength, but render the piece brittle and thus more liable to rupture from shocks; and a little further increase of stress rapidly lessens, or may entirely destroy, the elasticity. A *tensile* stress above the elastic limit greatly lowers, or may even destroy, the *compressive* elasticity, and vice versa. If a tensile stress, by stretching a piece, reduces its resisting area, it may thus reduce its *total* strength, even though the strength *per sq in* has increased. Mr. B. Baker finds that hard steel fatigues much faster under repeated loads than soft steel or iron.

† **u** = 6500 lbs per sq inch for rolled iron in compression
= 7000 lbs " " " tension (plates or shapes).
= 7500 lbs " for double rolled iron in tension (links or rods).

Art. 1. Compressive strengths of American woods, *when slowly and carefully seasoned.* Approximate averages deduced from many experiments made with the U S Govt testing machine at Watertown, Mass, by Mr. S. P. Sharples, for the census of 1880. **Seasoned woods** resist crushing much better than green ones; in many cases, twice as well. This must be allowed for when building bridges, &c, of timber recently cut. Different specimens of the same wood vary greatly; frequently as 5 to 8, 9, or more. See Rem and foot-note p 611.

The strengths in all these tables may readily vary as much as one-third part more or less than our average.	End-wise.* lbs per sq in.	Side-wise.¶ lbs per sq in.	
		.01	.1
Ash, red and white	6800	1300	3000
Aspen	4400	800	1400
Beech	7000	1100	1900
Birch	8000	1300	2600
Buckeye	4400	600	1400
Butternut	5400	700	1600
Buttonwood (sycamore)	6000	1300	2600
Cedar, red	6000	700	1000
" white (arbor vitæ)	4400	500	900
Catalpa (Indian bean)	5000	700	1300
Cherry, wild	8000	1700	2600
Chestnut	5300	900	1600
Coffee tree, Kentucky	5200	1300	2600
Cypress, bald	6000	500	1200
Elm, Am'n or white	6800	1300	2600
" red	7700	1300	2600
Hemlock	5300	600	1100
Hickory	8000	2000	4000
Lignum vitæ	10000	1600	13000
Linden, American	5000	500	900
Locust, black and yellow	9800	1900	4400
" honey	7000	1600	2600
Mahogany	9000	1700	5300
Maple, broad-leafed, Oregon	5300	1400	2600
" sugar and black	8000	1900	4300
" white and red	6800	1300	2900
Oak, white, post (or iron) swamp white, red and black	7000	1600	4000
" scrub and basket	6000	1700	4200
" chestnut and live	7500	1600	4500
" pin	6500	1300	3000
Pine, white	5400	600	1200
" Red or Norway	6300	600	1400
" pitch and Jersey scrub	5000	1000	2000
" Georgia	8500	1300	2600
Poplar	5000	600	1100
Sassafras	5000	1300	2100
Spruce, black	5700	700	1300
" white	4500	600	1?
Sycamore (buttonwood)	6000	1300	2600
Walnut, black	8000	1300	2600
" white (butternut)	5400	700	1600
Willow	4400	700	1400

Hence it appears that *seasoned* white and yellow pines, spruce, and ordinary oaks, which are the woods most employed in the United States for bridges, roofs, etc., crush endwise with from 5000 to 7000 lbs per sq inch, ***in short blocks;*** average, 6000.

But it is well to bear in mind that in practice perfectly equable pressure is rarely secured. In a few trials on sidewise compression, with fairly seasoned white pine blocks, 6 ins high, 5 ins long, and 2 ins wide, we found that under an equally distributed pressure of 5000 lbs total or 500 lbs per sq inch, they compressed about from $\frac{1}{8}$ to $\frac{1}{4}$ inch; which is equal to from $\frac{1}{4}$ to $\frac{1}{2}$ inch per foot of height; or from $\frac{1}{48}$ to $\frac{1}{24}$ of the height; the mean being about $\frac{3}{8}$ inch to a foot, or $\frac{1}{32}$ of the height. Under 10000 lbs total, or 1000 lbs per sq inch, they split badly; and in some cases large pieces flew off. **See Rem, p 500.**

The tensile or cohesive strengths of pine and oak average about 10000 lbs per sq inch, or $\frac{1}{2}$ as much as average cast-iron, or nearly double their resistance to crushing. The tensile strength does not change with the length of the piece; so that in practice we may take its safe strain at from 1000 to 2000 lbs per sq inch, depending upon the character of the structure, &c., without regard to the length, except when this is so great that two or more pieces have to be spliced together to make it; thus weakening the piece very much.

* Specimens 4 centimetres (1.57 inch) square, 32 centimetres (12.6 ins) long. When the length exceeds 10 times the least side, see Wooden Pillars, p 458.

¶ Specimens 4 centimetres (1.57 inch) square, 16 centimetres (6.3 ins) long; laid upon platform of testing machine. Pressure applied at their mid-length, by means of an iron punch 4 centimetres square, or just covering the entire width of the specimen, and one-fourth of its length. The first column (headed ".01") gives the loads producing an indentation of .01 inch. The second column (headed ".1") gives those producing an indentation of .1 inch.

Art. 2. Ultimate average crushing loads in tons, per square foot, for stones, &c. The stones are supposed to be ON BED, and the heights of all to be from 1.5 to 2 times the least side. Stones generally begin to crack or split under about one-half of their crushing loads. In practice, neither stone nor brickwork should be trusted with more than $\frac{1}{6}$ to $\frac{1}{10}$th of the crushing load, according to circumstances. **When thoroughly wet** some absorbent sandstones lose fully half their strength. See head of next page.

	Tons per sq. ft.	Mean. Tons.
Granites and Syenites.	300 to 1200	750
Basalt	.....	700
Limestones and Marbles *	250 to 1000	625
Oolites, good	100 to 250	175
Sandstones fit for building *	150 to 550	350
Sandstone, red, of Connecticut and N. Jersey, to crack	.....	200
Brick*	40 to 300	170
Brickwork, ordinary, cracks with *	20 to 30	25
Brickwork, good, in cement *	30 to 40	35
Brickwork, first-rate, in cement	50 to 70	60
Slate	400 to 800	600
Caen Stone	70 to 200	135
" " to crack	.....	70
Chalk, hard,	20 to 30	25
Plaster of Paris, 1 day old	.....	40
Cement, Portland, neat, U. S. or foreign, 7 days in water	75 to 150	112.5
Common U.S. cements, neat, 7 days in water	15 to 30	22.5
Concrete of Port. cement, sand, and gravel or brok stone in the proper proportions, rammed 1 m old	12 to 18	15
6 months old	48 to 72	60
12 months old	74 to 120	97
With good **common hyd cements,** abt .2 to .25 as much		
Coignet beton, 3 months old	100 to 150	125
Rubble masonry, mortar, rough	15 to 35	25
Glass, green, crown and flint	1300 to 2300	1800
	or 3 times that of granite.	
Ice, firm †	12 to 18	15

Crushing height of Brick and Stone.

If we assume the wt of ordinary brickwork at 112 lbs per cub ft, and that it would crush under 30 tons per sq ft, then a vert uniform column of it 600 ft high, would crush at its base, under its own wt. Caen stone, weighing 130 lbs per cub ft, would require a column 1376 ft high to crush it. Average sandstones at 145 lbs per cub ft, would require one 4158 ft high; and average granites, at 165 lbs per cub ft, one of 8145 feet. But stones begin to crack and splinter at about half their ultimate crushing load; and in practice it is not considered expedient to trust them with more than $\frac{1}{6}$th to $\frac{1}{10}$th part of it, especially in important works; inasmuch as settlements, and imperfect workmanship, often cause undue strains to be thrown on certain parts.

The Merchants' shot-tower at Baltimore is 246 ft high; and its base sustains $6\frac{1}{2}$ tons per sq ft. The base of the granite pier of Saltash bridge, (by Brunel,) of solid masonry to the height of 96 ft, and supporting the ends of two iron spans of 455 ft each, sustains $9\frac{1}{2}$ tons per sq ft. The base of a brick chimney at Glasgow, Scotland, 468 ft high, bears 9 tons per sq ft; and Professor Rankine considers that in a high gale of wind, its leeward side may have to bear 15 tons. The highest pier of Roquefavour stone aqueduct, Marseilles, is 305 ft, and sustains a pressure at base of $13\frac{1}{2}$ tons per sq ft. For greater pressures on arch-stones, see p 694.

* **Trials at St. Louis bridge,** by order of Capt James B. Eads, C. E., showed that some magnesian limestone did not yield under less than 1100 tons per sq ft. A column 8 ins high and 2 ins diam shortened ¼ inch under pressure; and recovered when relieved. **Experiments made with the Govt testing machine at Watertown, Mass, 1882–3,** gave 1400 tons per sq ft ultimate crushg load for white and blue marble from Lee, Mass, 700 for blue marble from Montgomery Co, Pa, 960 for limestone from Conshohocken, Pa, 500 for limestone from Indiana, 840 for red sandstone from Hummelstown, Pa, 260 to 1000 for yellow Ohio sandstone; Phila bricks, flatwise; hard, machine-made, 350 to 700 tons; hand-made, 700 to 1300; pressed, machine-made, 450 to 580; Brickwork columns, 13 ins sq and 13 ins high; in lime, 100 tons; in cement, 150.

† Experiments by Col. Wm. Ludlow, U. S. A., with Govt testing machines, in 1881, gave from 21 to 64 tons per sq ft for pure, hard ice; and 16 to 59 tons for inferior grades. The specimens (6 and 2-inch cubes) compressed ½ to 1 inch before crushing.

Sheet lead is sometimes placed at the joints of stone columns, with a view to equalize the pressure, and thus increase the strength of the column. But experiments have proved that the effect is directly the reverse, and that the column is *materially weakened* thereby. Does this singular fact apply to cast iron and other materials?

Art. 3. Average crushing load for Metals.

It must be remembered that these are the loads for pieces but two or three times their least side in height. As the height increases, the crushing load diminishes. See "Strength of Pillars," p 439.

The crushing load per sq inch, of any material, is frequently called its *constant*, *coefficient*, or *modulus*, of crushing or of compression.	Pounds per sq. inch.	Tons per sq. inch.
Cast Iron, usually. It is usually assumed at 100000 lbs, or say 45 tons per sq inch. Its crushing strength is usually from 6 to 7 times as great as its tensile. Within its average elastic limit of about 15 tons per sq inch, average cast iron shortens about 1 part in 5555; or ⅛ inch in 58 ft under each ton per sq inch of load; or about twice as much as average wrought iron. Hence at 15 tons per sq inch it will shorten about 1 part in 370; or full ⅛ inch in 4 feet. Different cast irons may however vary 10 to 15 per ct either way from this.	85000 to 125000	38 to 56
U. S. Ordnance, or gun metal; Some	175000	78.1
Wrought iron, within elastic limit	22400 to 35840	10 to 16
Its elastic limit under pressure averages about 13 tons per sq inch. It begins to shorten perceptibly under 8 to 10 tons, but recovers when the load is removed. With from 18 to 20 tons, it shortens *permanently*, about $\frac{1}{60}$th part of its length; and with from 27 to 30 tons, about $\frac{1}{16}$th part, as averages. The crushing weights therefore in the table are not those which absolutely mash wrought iron entirely out of shape, but merely those at which it yields too much for most practical building purposes. About 4 tons per sq inch is considered its average safe load, in pieces not more than 10 diams long; and will shorten it ⅛ inch in 30 ft. average.	29120	13
Brass, reduced $\frac{1}{10}$th part in length, by 51000; and ½ by	165000	73.6
Copper, (cast,) crumbles	117000	52.2
(wrought) reduced ⅛th part in length, by	103000	46.0
Tin, (cast,) reduced $\frac{1}{16}$th in length, by 8800; and ⅓ by	15500	6.92
Lead, (cast,) reduced ½ of its length, by 7000 to 7700	7350	3.28
" By writer. A piece 1 inch sq, 2 ins high, at 1200 lbs the compression was 1-200 of the ht; at 2000, 1-29; at 3000, 1-8; at 5000, 1-3; at 7000, 1-2 of the ht.		
Spelter or **Zinc,** (cast.) By writer. A piece 1 inch square, 4 ins high, at 2000 lbs was compressed 1-400 of its ht; at 4000, 1-200; at 6000, 1-100; at 10000, 1-38; at 20000, 1-15; at 40000 yielded rapidly, and broke into pieces.		
Steel, 224000 lbs or 100 tons shorten it from .2 to .4 part.		
" Americau. Black Diamond steel-works, Pittsburg, Penn, experiments by Lieut W. H. Shock. U. S. N., on pieces ½ in square; and 3½ ins, or 7 sides long.		
" Untempered. 100100 to 104000	102050	45.5
" Heated to light cherry red, then plunged into oil of 82° Fah, 173200 to 199200	186200	83.1
" Heated to light cherry red, then plunged into water of 79° Fah; then tempered on a heated plate, 325400 to 340800	333100	148.7
" Heated to light cherry red, then plunged into water of 79° Fah, 275600 to 400000	337800	150.8
" **Elastic limit,** 15 to 27 tons	47040	21
" **Compression.** within elas limit averages abt 1 part in 13300, or .1 of an inch in 111 ft per ton per sq inch; or .1 of an inch in 5.3 ft under 21 tons per sq inch.		
Best steel knife edges, of large R R weigh scales are considered safe with 7000 lbs pres per lineal inch of edge; and solid cylindrical steel **rollers** under bridges, and *rolling on steel.* safe with $\sqrt{\text{diam in ins} \times 3\,100\,000}$, in lbs per lineal inch of roller parallel to axis. **And per the same, for**		

Solid cast iron wheels rolling on wrought iron, $\sqrt{\text{Diam ins} \times 352\,000}$.

" " " " " " cast iron, $\sqrt{\text{Diam ins} \times 222\,222}$.

Solid steel " " " steel, $\sqrt{\text{Diam ins} \times 1\,300\,000}$.

" " " " " wrought iron, $\sqrt{\text{Diam ins} \times 1\,024\,000}$.

" " " " " cast iron, $\sqrt{\text{Diam ins} \times 850\,000}$.

From "Specifications for Iron Drawbridge at Milwaukee," by Don J. Whittemore, C. E.

STRENGTH OF PILLARS.

The foregoing remarks on crushing or compressive strength refer to that of pieces so short as to be incapable of yielding except by crushing proper. Pieces longer in proportion to their diameter of cross section are liable to yield by bending sideways. They are called pillars or columns.

The law governing the strength of pillars is but imperfectly understood; and the best formulæ are rendered only approximate by slight unavoidable and unsuspected defects in the material, straightness and setting of the column. A very slight obliquity between the axis of a pillar and the line of pressure may reduce the strength as much as 50 per cent; and differences of 10 per cent or more in the bkg load may occur between two pillars which to all appearances are precisely similar and tested under the same conditions. Hence a liberal factor of safety should be employed in using any formulæ or tables for pillars. See "factor of safety" p 442, and at foot of p 446.

In our following remarks on this subject, the pillars are supposed to sustain a *constant* load; and the ultimate or breaking load referred to is that one which would, during its first application, cripple or rupture the pillar in a short time. But struts in bridges etc often have to endure stresses which vary greatly in amount from time to time. Their ultimate load is then less. For such cases see p 435.

Long pillars with **rounded ends,** as in Fig 1, have less strength than those with **flat ends,** whether free or firmly fixed. See table p 442, which also shows that in *short* columns the difference in this respect is but slight.

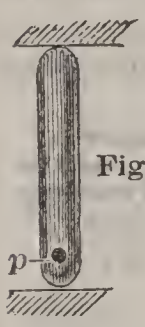

Fig. 1.

In iron bridges and roofs, the ends of the pillars and of oblique struts are frequently sustained by means of pins or bolts passing through (across) them, at either one or both ends, as at p. Fig 1. See also p 612. These we will call **hinged ends.** Our table p 442, shows that pillars so fixed are about intermediate in strength between those with flat and those with round ends. There is much uncertainty about this and all such matters. The strength of a given hinged-end pillar is increased to an important extent by increasing the diameter of the pin.

The **formula** in most general use for the strength of pillars, is that attributed to Prof. **Lewis Gordon** of Glasgow, and called by his name. With the use of the proper coefficients for the given case, it gives results agreeing approximately with averages obtained in practice with pillars of such lengths (say from 10 to 40 diams) as are commonly used.

It is as follows

$$\text{Breaking load in lbs per sq inch of area of cross section of pillar} = \frac{f}{1 + \frac{l^2 \div r^2}{a}}$$

in which

f is a coefficient depending upon the nature of the material and (to some extent) upon the shape of cross section of the pillar. It is often taken, approximately enough, as being the ult crushing strength of short blocks of the given material. For good American wrought iron, such as is used for pillars, 40000 is generally used; for cast iron 80000. Mr. Cleeman* found for mild steel (.15 per cent carbon) 52000; and for hard steel (.36 per cent carbon) 83000 lbs. Mr. C. Shaler Smith gives 5000 for Pine. See p 458.

a, for wrought iron, is usually taken as follows: **a** =

when both ends of the pillar are flat or fixed.....	36000 to 40000
when both ends of the pillar are hinged.....	18000 to 20000
when one end is flat or fixed, and the other hinged...	24000 to 30000

For cast iron about one eighth of these figures is generally used; and for pine about one twelfth.

l is the length of the pillar. If the pillar has, between its ends, supports which prevent it from yielding side-ways, the length is to be measured between such supports. See lines 11 to 18, p 457.

r is the least radius of gyration† of the cross section of the pillar. *l* and *r* must be in the same unit; as both in feet, or both in inches.

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* Proceedings Engineers' Club of Phila, Nov 1884.

† See p 440.

Radius of gyration. Suppose a body free to revolve around an axis which passes through it in any direction; or to oscillate like a pendulum hung from a point of suspension. Then suppose in either case, a certain given amount of force to be applied to the body, at a certain given dist from the axis, or from the point of suspension, so as to impart to the body an angular vel; or in other words, to cause it to describe a number of *degrees* per sec. Now, there will be a certain point in the body, such that if the entire wt of the body were there concentrated, then the same force as before, applied at the same dist from the axis, or from the point of suspension as before, would impart to the body the same angular motion as before. This point is the center of gyration; and its dist from the axis, or from the point of suspension, is the *Radius of gyration*, of the body. In the case of *areas*, as of cross-sections of pillars or beams, the *surface* is supposed to revolve about an imaginary axis; and, unless otherwise stated, this axis is the neutral axis of the area, which passes through its center of gravity. Then

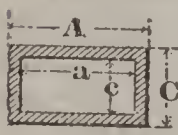
Radius of gyration $= \sqrt{\text{Moment of inertia} \div \text{Area}}$

Square of radius of gyration = Moment of inertia ÷ Area

For moment of inertia, see p 486.

In a circle, the radius of gyration remains the same, no matter in what direction the neutral axis may be drawn. In other figures its length is different for the different neutral axes about which the figure may be supposed to be capable of revolving. Thus, in the I beam Fig 18, p 521, the radius of gyration about the neutral axis X Y is much greater than that about the longer neutral axis W Z. In rules for pillars the *least* radius of gyration must be used.

The following formulæ enable us to find the least radius of gyration, and the square of the least radius of gyration, for such shapes as are commonly used for pillars.

Shape of cross section of pillar		Least radius of gyration	Square of least radius of gyration
	Solid square	$\frac{\text{side}}{\sqrt{12}^*}$	$\frac{\text{side}^2}{12}$
	Hollow square of uniform thickness	$\sqrt{\frac{D^2 + d^2}{12}}$	$\frac{D^2 + d^2}{12}$
	Solid rectangle	$\frac{\text{least side}}{\sqrt{12}^*}$	$\frac{\text{least side}^2}{12}$
	Hollow rectangle of uniform thickness	$\sqrt{\frac{C^3A - c^3a}{12\,(CA - ca)}}$	$\frac{C^3A - c^3a}{12\,(CA - ca)}$
	Solid circle	$\frac{\text{diameter}}{4}$	$\frac{\text{diameter}^2}{16}$
	Hollow circle of uniform thickness	$\sqrt{\frac{D^2 + d^2}{16}}$	$\frac{D^2 + d^2}{16}$

* $\sqrt{12}$ = about 3.4641.

The following are only approximate:

Shape of cross section of pillar		Least radius of gyration	Square of least radius of gyration
	Phœnix column. (See p 449)	$D \times .3636$	$D^2 \times .1322$
	I beam. (See p 521)	$\frac{F}{4.58}$	$\frac{F^2}{21}$
	Channel. (See p 521)	$\frac{F}{3.54}$	$\frac{F^2}{12.5}$
	Deck beam. (See p 521)	$\frac{F}{6}$	$\frac{F^2}{36.5}$
	Angle, with equal legs (See p 525)	$\frac{F}{5}$	$\frac{F^2}{25}$
	Angle, with unequal legs (See p 525)	$\frac{F\,f}{2.6\,(F+f)}$	$\frac{F^2\,f^2}{13\,(F^2+f^2)}$
	T, with $F = f$ (See p 525)	$\frac{F}{4.74}$	$\frac{F^2}{22.5}$
	Cross, with $F = f$		

Table of approximate average ultimate loads in lbs per square inch, as found by experiment with *carefully prepared* specimens. In practice, allowance must be made for the rougher character of actual work, for jarrings etc etc.

Length ÷ least radius of gyration.	Pencoyd Angles, Tees, I beams and Channels.* See pp 521 to 527.						Phœnix columns.† See p 449.	Length ÷ least radius of gyration.
	Steel.		Iron.				Iron.	
	Hard ; .36 per cent carbon	Mild ; .12 per cent carbon						
	Flat ends	Flat ends	Fixed ends	Flat ends	Hinged ends	Round ends	Flat ends	
3	.....	.....	.....	.....	.....	.....	57200	3
17	.....	.....	.....	.....	.....	.....	50400	17
20	100000	70000	46000	46000	46000	44000	48000	20
30	74000	51000	43000	43000	43000	40250	40000	30
40	62000	46000	40000	40000	40000	36500	37000	40
50	60000	44000	38000	38000	38000	33500	37000	50
60	58000	42000	36000	36000	36000	30500	37000	60
70	55500	40000	34000	34000	33750	27750	37000	70
80	53000	38000	32000	32000	31500	25000	36000	80
90	49700	36000	31000	30900	29750	22750	35000	90
100	46500	34000	30000	29800	28000	20500	35000	100
120	40000	30000	28000	26300	24300	16500	34500	120
140	33500	26000	25500	23500	21000	12800	.....	140
160	28000	22000	23000	20000	16500	9500	.....	160
200	19000	14800	17500	14500	10800	6000	.....	200
300	8500	7200	9000	7200	5000	2800	.....	300

The following **simple formula,** by Mr. D. J. Whittemore, was found to agree very closely with the results of the experiments on Phœnix columns :†

$$\text{Breaking load in lbs per sq inch of area of cross section of pillar} = [(1200 - H) \times 30] + \frac{525000}{H^2}$$

$$\text{where } H = \frac{\text{length of pillar}}{\text{diam D, fig p 449}} \text{ both in the same unit.}$$

See also p 443.

Mr. Christie* adopts the following formulæ for obtaining the proper **factor of safety** for pillars of wrought iron or steel :

For flat and fixed ends, $$\text{Factor of safety} = 3 + \left(.01 \frac{\text{length}}{\text{least rad of gyr}}\right)$$

For hinged and round ends, $$\text{Factor of safety} = 3 + \left(.015 \frac{\text{length}}{\text{least rad of gyr}}\right)$$

It will be noticed that the factor of safety, as found by these formulæ, increases with the ratio of the length of the pillar to the least radius of gyration of its cross section ; and is greater for round and hinged ends than for flat and fixed ends. See foot of p 446.

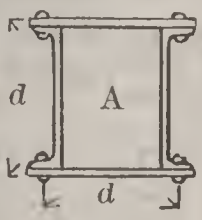
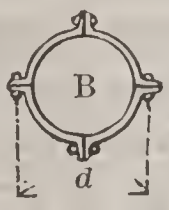
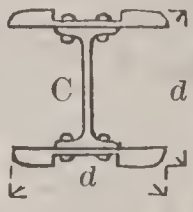
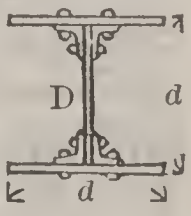
* See "Wrought Iron and Steel in Construction", by Pencoyd Iron Works ; published by John Wiley & Sons, New York, 1884.

† See Transactions, American Society of Civil Engineers ; Jan, Feb and March 1882.

Ultimate crippling strengths in lbs per sq inch of metal section of the four wrought iron pillars below. These formulas are deduced by Chs. Shaler Smith, from many tests by G. Bouscaren, C. E., of large pillars of good American iron. The lower Table is an abridgment of the full ones by C. L. Gates, C. E., in the Trans. Am. Soc. C. E., Oct., 1880.

$H = \frac{\textit{length between end bearings}}{\textit{least diameter } d}$ both in the same measure; and is to be squared.

For safety take from $\frac{1}{3}$ to $\frac{1}{8}$, according to circumstances.

	A 	B 	C 	D 
Flat ends.....	$\frac{38500}{1+\frac{H^2}{5820}}$	$\frac{42500}{1+\frac{H^2}{4500}}$	$\frac{36500}{1+\frac{H^2}{3750}}$	$\frac{36500}{1+\frac{H^2}{2700}}$
One pin end..	$\frac{38500}{1+\frac{H^2}{3000}}$	$\frac{40000}{1+\frac{H^2}{2250}}$	$\frac{36500}{1+\frac{H^2}{2250}}$	$\frac{36500}{1+\frac{H^2}{1500}}$
Two pin ends.	$\frac{37800}{1+\frac{H^2}{1900}}$	$\frac{36600}{1+\frac{H^2}{1500}}$	$\frac{36500}{1+\frac{H^2}{1750}}$	$\frac{36500}{1+\frac{H^2}{1200}}$

Ultimate and safe loads in lbs per sq inch, of the above four pillars, with flat ends, and equally loaded. **Coef of Safety** $= 4 + .05\,H$. By C. L. Gates, C. E.

H.	A. Square Col.		B. Phœnix Col.		C. American Col.		D. Common Col.	
	Ult.	**Safe.**	**Ult.**	**Safe.**	**Ult.**	**Safe.**	**Ult.**	**Safe.**
15	37067	7822	40476	8521	34434	7249	33693	7093
16	36876	7683	40212	8377	34167	7118	33339	6946
18	36470	7443	39645	8091	33597	6856	32589	6651
20	36024	7205	39030	7806	32982	6596	31790	6358
22	35544	6970	38373	7524	32327	6338	30952	6069
25	34767	6622	37317	7110	31285	5959	29639	5646
30	33344	6063	35424	6440	29435	5352	27375	4977
35	31806	5531	33406	5810	27512	4789	25108	4367
40	30198	5033	31352	5226	25584	4264	22919	3820
45	28562	4570	29310	4690	23701	3792	20857	3337
50	26932	4143	27321	4203	21900	3369	18952	2916
55	25333	3728	25415	3765	20203	3004	17214	2550
60	23787	3398	23611	3373	18621	2660	15643	2235

TABLE OF BREAKING LOADS OF IRON PILLARS,

in tons per square inch of metal area. Deduced from Gordon. The ends are supposed to be planed to form perfectly true bearings; and all parts to be equally pressed. The last is rarely the case in practice. **If the pillar is rectangular** instead of square, use the least side for a measure of length. (Original.)

Hollow Round.		Length measd in sides or diams.	Hollow Square.		Solid Round.		Length measd in sides or diams.	Solid Square.	
Breakg loads per sq inch of metal area of transverse section.			Breakg loads per sq inch of metal area of transverse section.		Breakg loads per sq inch of metal area of transverse section.			Breakg loads per sq inch of metal area of transverse section.	
Cast.	Wrt.		Cast.	Wrt.	Cast.	Wrt.		Cast.	Wrt.
Tons.	Tons.		Tons.	Tons.	Tons.	Tons.		Tons.	Tons.
35.7	16.1	1	35.7	16.0	35.6	16.1	1	35.6	16.1
35.5	16.1	2	35.5	16.0	35.2	16.1	2	35.4	16.0
35.2	16.1	3	35.3	16.0	34.6	16.0	3	34.9	16.0
34.8	16.0	4	35.0	16.0	33.9	15.9	4	34.3	16.0
34.3	16.0	5	34.6	16.0	33.0	15.9	5	33.6	16.0
33.7	16.0	6	34.2	16.0	31.9	15.8	6	32.8	15.9
33.0	15.9	7	33.7	16.0	30.6	15.7	7	31.8	15.8
32.3	15.8	8	33.1	15.9	29.3	15.6	8	30.8	15.7
31.5	15.8	9	32.4	15.9	28.0	15.5	9	29.7	15.7
30.6	15.7	10	31.7	15.8	26.7	15.4	10	28.6	15.6
29.7	15.6	11	31.0	15.8	25.4	15.2	11	27.4	15.5
28.8	15.5	12	30.3	15.7	24.1	15.1	12	26.3	15.3
27.9	15.5	13	29.5	15.7	22.8	14.9	13	25.1	15.2
27.0	15.4	14	28.7	15.6	21.6	14.8	14	24.0	15.1
26.0	15.3	15	27.9	15.5	20.5	14.6	15	22.9	15.0
25.1	15.2	16	27.1	15.5	19.4	14.4	16	21.8	14.8
24.2	15.1	17	26.3	15.4	18.3	14.2	17	20.7	14.7
23.3	15.0	18	25.4	15.3	17.2	14.0	18	19.7	14.5
22.3	14.9	19	24.6	15.2	16.2	13.8	19	18.8	14.3
21.4	14.8	20	23.8	15.1	15.3	13.6	20	17.9	14.2
20.6	14.7	21	23.0	15.0	14.5	13.4	21	17.0	14.0
19.8	14.6	22	22.2	14.9	13.7	13.2	22	16.2	13.8
19.0	14.4	23	21.5	14.8	12.9	13.0	23	15.4	13.6
18.3	14.3	24	20.8	14.7	12.2	12.8	24	14.6	13.5
17.5	14.2	25	20.1	14.6	11.6	12.6	25	13.9	13.3
16.8	14.0	26	19.4	14.5	11.0	12.4	26	13.3	13.1
16.2	13.9	27	18.7	14.4	10.4	12.1	27	12.7	12.9
15.5	13.7	28	18.0	14.3	9.90	11.9	28	12.1	12.7
14.9	13.5	29	17.4	14.2	9.41	11.7	29	11.5	12.6
14.3	13.4	30	16.8	14.0	8.93	11.5	30	11.0	12.4
13.8	13.2	31	16.3	13.9	8.50	11.3	31	10.5	12.2
13.2	13.1	32	15.7	13.8	8.10	11.1	32	10.0	12.0
12.7	12.9	33	15.2	13.6	7.70	10.8	33	9.59	11.8
12 3	12.8	34	14.6	13.5	7.36	10.6	34	9.18	11.6
11.7	12.6	35	14.1	13.3	7.04	10.4	35	8.79	11.4
11.3	12.5	36	13.6	13.2	6.71	10.2	36	8.42	11.2
10.9	12.3	37	13.2	13.1	6.41	10.0	37	8.07	11.0
10.5	12.2	38	12.7	13.0	6.11	9.79	38	7.75	10.8
10.1	12.0	39	12.3	12.9	5.85	9.59	39	7.44	10.7
9.75	11.9	40	11.9	12.7	5.62	9.39	40	7.14	10.5
9.40	11.7	41	11.5	12.6	5.40	9.20	41	6.86	10.3
9.07	11.6	42	11.1	12.4	5.19	9.00	42	6.60	10.1
8.76	11.4	43	10.8	12.3	4.99	8.81	43	6.35	9.95
8.46	11.3	44	10.4	12.2	4.79	8.64	44	6.11	9.77
8.16	11.1	45	10.1	12.0	4.59	8.46	45	5.89	9.59
7.88	10.9	46	9.80	11.9	4.42	8.27	46	5.68	9.42
7.61	10.8	47	9.50	11.7	4.26	8.10	47	5.48	9.25
7.36	10.6	48	9.20	11.6	4.11	7.94	48	5.28	9.09
7.13	10.4	49	8.94	11.4	3.97	7.76	49	5.10	8.92
6.91	10.3	50	8.66	11.3	3.84	7.60	50	4.92	8.77
5.90	9.61	55	7.47	10.7	3.22	6.86	55	4.16	8.00
5.10	8.92	60	6.49	10.0	2.75	6.18	60	3.57	7.30
4.44	8.29	65	5.68	9.43	2.37	5.59	65	3.09	6.67
3.90	7.69	70	5.01	8.84	2.06	5.06	70	2.70	6·10
3.44	7.14	75	4.44	8.30	1.80	4.59	75	2.37	5.59
3.08	6.64	80	3.97	7.77	1.59	4.18	80	2.10	5.13
2.46	5.73	90	3.21	6.88	1.27	3.49	90	1.68	4.34
2.02	4.99	100	2.65	6.02	1.04	2.95	100	1.37	3.71

Table 1. HOLLOW CYLIND CAST IRON PILLARS.

Breaking loads, flat ends, perfectly true, and firmly fixed; and the loads pressing equally on every part of the top. By Gordon's formula.

For diams or lengths intermediate of those in the table, the loads may be found near enough by simple proportion.

For thicknesses less than those in the table, the breaking loads may safely be *assumed* to diminish in the same proportion as the thickness, while the outer diam remains the same. But for *greater* thicknesses than those in the table, the loads do not increase as rapidly as the new thickness. Still, in practice, they may be assumed to do so approximately, **if the new thickness does not exceed about ⅛ part of the outer diam.**

Length in feet.	CAST IRON. THICKNESS ¼ INCH. (Original.) Outer Diameter in inches.											Length in feet.
	2	2¼	2½	2¾	3	3½	4	4½	5	5½	6	
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
1	45 2	52.5	59.8	66.8	74.1	88.5	103.0	117.1	131.3	145.5	159.8	1
2	36.2	43.4	51.0	58.6	66.5	81.5	96.7	111.3	125.9	140.5	155.2	2
3	27.2	34.1	41.4	48.8	56.7	71.9	87.6	102.7	117.8	132.9	148.1	3
4	20.2	26.3	32.9	39.7	47.0	61.9	77.5	92.9	108.3	123.7	139.1	4
5	15.1	20.2	25.9	32.0	38.6	52.6	67.4	82.6	97.9	113.4	129.1	5
6	11.6	15.8	20.7	25.9	31.6	44.3	58.2	72.7	87.7	103.0	118.7	6
7	9.1	12.5	16.6	21.0	26.1	37.4	50.1	63.7	78.1	92.9	108.3	7
8	7.3	10.1	13.5	17.3	21.7	31.6	43.2	55.8	69.3	83.4	98.4	8
9	5.9	8.2	11.1	14.3	18.2	26.9	37.3	48.8	61.5	74.9	89.2	9
10	4.9	6.9	9.3	12.0	15.4	23.1	32.4	42.9	54.6	67.1	80.7	10
11	4.1	5.8	7.9	10.3	13.2	20.0	28.3	37.8	48.6	60.3	73.0	11
12	3.5	5.0	6.8	8.9	11.4	17.4	24.9	33.5	43.3	54.2	66.1	12
13	3.0	4.2	5.8	7.6	9.9	15.2	21.9	29.7	38.8	48.8	60.0	13
14	2.6	3.6	5.1	6.7	8.7	13.5	19.5	26.6	34.9	44.1	54.5	14
15	2.3	3.2	4.5	6.0	7.7	11.9	17.4	23.9	31.4	39.9	49.7	15
16	2.0	2.8	4.0	5.3	6.9	10 7	15.6	21.5	28.4	36.4	45.6	16
18	1.6	2.3	3.2	4.2	5.5	8.6	12.7	17.5	23.4	30.2	38.0	18
20	1.3	1.8	2.6	3.4	4.5	7.1	10.5	14.7	19.7	25.6	32.3	20
25	.....	.....	1.7	2.3	3.0	4.7	7.0	9.8	13.3	17.6	22.5	25
30	.....	.....	1.2	1.6	2.1	3.2	5·0	7.0	9.4	12.6	16.1	30
35	.....	.....	.....	.....	1.5	2.4	3.7	5.2	7.1	9.4	12.2	35
40	.....	.....	.....	.....	1.2	1.9	2.8	4.0	5.4	7.2	9.5	40

Weight of 1 foot of length of pillar in pounds.

4.31	4.91	5.53	6.13	6.75	7.97	9.22	10.4	11.7	12.9	14.1

Area of ring of solid metal in square inches.

1.38	1.57	1.77	1.96	2.16	2.55	2.95	3.34	3.73	4.12	4.52

Length in feet.	CAST IRON. THICKNESS ½ INCH. (Original.) Outer Diameter in inches.												Length in feet.
	5	5½	6	6½	7	7½	8	8½	9	10	11	12	
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
2	239	268	297	325	354	383	412	440	469	526	583	640	2
4	205	235	266	296	326	356	387	416	445	504	563	622	4
6	166	196	227	257	288	319	350	380	410	471	532	593	6
8	131	159	188	218	248	279	310	340	371	432	494	557	8
10	103	127	154	182	210	240	270	300	330	391	454	517	10
12	82	103	126	151	177	205	233	262	292	351	413	475	12
14	66	84	104	126	149	174	200	227	255	313	372	434	14
16	54	69	87	106	126	149	173	198	224	277	334	394	16
18	45	58	73	90	108	128	149	172	196	246	300	357	18
20	37	48	62	77	93	111	130	151	173	219	270	323	20
22	32	41	53	66	80	96	113	132	151	194	241	292	22
25	25	34	43	54	65	79	93	109	126	164	206	252	25
30	18	24	31	39	48	59	70	83	96	126	160	199	30
35	13	18	24	30	36	44	53	63	73	98	126	159	35
40	10	14	18	23	28	35	42	50	59	78	102	129	40
45	8	11	15	19	23	28	34	41	48	64	84	107	45
50	6	9	12	15	19	23	28	33	39	53	70	89	50
60	4	6	9	11	14	17	20	24	28	38	50	65	60
70	3	4	6	8	10	12	15	18	21	29	38	50	70
80	3	4	5	6	8	9	11	13	16	22	30	38	80

Weight of 1 foot of length of pillar, in pounds.

22.1	24.5	27.0	29.4	31.9	34.4	36.9	39.4	41.9	46.6	51.6	56.6

Area of ring of solid metal, in square inches.

7.07	7.85	8.64	9.43	10.2	11.0	11.8	12.6	13.4	14.9	16.5	18.1

Table 1. HOLLOW CYLIND CAST IRON PILLARS.

BREAKING LOADS.—(Continued.) By Gordon's Rule.

Length in feet.	CAST IRON. THICKNESS 1 INCH. (Original.) Outer Diameter in Inches.												Length in feet.
	12	13	14	15	16	18	20	22	24	27	30	36	
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
4	1188	1301	1415	1530	1645	1874	2103	2330	2557	2896	3236	3915	4
6	1138	1253	1368	1484	1601	1833	2066	2295	2525	2866	3208	3890	6
8	1065	1184	1303	1423	1543	1779	2015	2247	2479	2824	3170	3860	8
10	989	1110	1231	1355	1475	1716	1957	2193	2430	2780	3128	3823	10
12	909	1030	1152	1275	1399	1644	1889	2129	2369	2723	3076	3778	12
14	829	949	1071	1195	1320	1566	1813	2056	2300	2659	3016	3726	14
16	756	873	992	1114	1237	1484	1733	1979	2226	2589	2951	3668	16
18	683	796	913	1034	1155	1401	1651	1899	2147	2515	2879	3604	18
20	618	727	840	958	1077	1320	1568	1817	2066	2437	2805	3536	20
22	559	663	772	887	1002	1241	1486	1734	1982	2356	2726	3464	22
24	508	606	709	818	929	1163	1404	1650	1899	2272	2644	3387	24
26	459	553	651	756	863	1090	1326	1570	1816	2188	2560	3308	26
28	418	506	598	697	800	1020	1250	1489	1733	2103	2475	3226	28
30	380	462	549	644	743	954	1178	1412	1653	2020	2392	3143	30
32	347	424	506	595	689	893	1110	1338	1575	1938	2308	3059	32
34	318	390	467	552	641	836	1046	1268	1500	1857	2225	2974	34
36	292	359	432	511	596	783	984	1199	1427	1779	2143	2889	36
38	268	331	400	475	556	734	928	1136	1361	1704	2063	2804	38
40	247	305	370	441	518	687	874	1076	1296	1630	1984	2720	40
42	229	283	344	411	484	645	825	1019	1232	1560	1909	2636	42
44	212	263	321	383	452	605	776	963	1169	1491	1834	2552	44
46	197	246	299	358	423	569	734	915	1114	1428	1765	2474	46
48	183	229	280	335	397	536	694	869	1060	1367	1696	2396	48
50	170	213	261	314	373	505	656	824	1008	1306	1627	2319	50
55	144	181	222	269	320	438	573	725	893	1172	1474	2135	55
60	124	157	192	233	278	383	503	641	795	1054	1333	1964	60
65	105	134	165	202	242	335	443	568	709	951	1211	1808	65
70	93	118	146	178	213	297	394	507	636	853	1099	1664	70
80	73	92	113	139	168	235	315	408	516	705	914	1414	80
90	58	73	91	112	136	191	257	335	426	586	768	1209	90
100	48	60	75	92	112	157	213	279	356	491	651	1040	100
Weight of one foot of length of pillar, in pounds.	108	118	128	138	147	167	187	206	226	255	285	344	
Area of ring of solid metal, in square inches.	34.6	37.7	40.8	44.0	47.1	53.4	59.7	66.0	72.2	81.7	91.1	110.0	

Length in feet.	CAST IRON. THICKNESS 3 INCHES. (Original.) Outer Diameter in Feet.											Length in feet.
	3	3½	4	4½	5	5½	6	7	8	10	12	
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
10	10806	12878	14908	16967	18986	21038	23052	27143	31196	39254	47375	10
15	10453	12564	14628	16712	18754	20825	22855	26972	31045	39133	47273	15
20	9996	12150	14250	16369	18440	20533	22585	26737	30837	38962	47131	20
30	8884	11102	13275	15459	17593	19743	21847	26084	30257	38486	46729	30
40	7688	9907	12113	14344	16531	18735	20891	25224	29476	37838	46175	40
50	6554	8702	10888	13126	15341	17580	19780	24107	28532	37037	45484	50
60	5553	7575	9690	11892	14100	16349	18570	23049	27460	36103	44666	60
70	4704	6570	8576	10702	12870	15097	17319	21824	26287	35058	43737	70
80	3998	5699	7570	9595	11692	13873	16070	20566	25048	33924	42712	80
90	3417	4953	6683	8588	10594	12706	14856	19304	23791	32725	41670	90
100	2940	4321	5909	7665	9588	11614	13700	18065	22521	31481	40438	100
110	2547	3788	5238	6888	8677	10606	12613	16869	21270	30213	39220	110
125	1950	2954	4400	5864	7483	9257	11132	14650	19448	27664	36692	125
150	1532	2350	3353	4547	5900	7417	9058	12701	16668	25182	34127	150
Weight of one foot of length of pillar, in pounds.	972	1150	1325	1503	1678	1856	2031	2388	2740	3444	4153	

Coefficients of safety for hollow cast iron pillars.

Mr. James B. Francis, of Lowell, Mass., a high authority, in his "Tables of Cast Iron Pillars," recommends that in order to allow for unequal loading, imperfect casting, bad end bearings, side blows, &c., we should not take the safe load at more than **one-fifteenth** of Hodgkinson's breaking load, if the pillars are roughly cast, and the ends not perfectly planed and adjusted; and **one-fifth** when they are so, and the loads about equally distributed.

It will be seen by the last table on p 462, how our Gordon's loads differ from Hodgkinson's; but we think that the same proportions of ours may be taken as safe; depending on the above conditions.

HOLLOW CYLINDRICAL WROUGHT IRON PILLARS.

Table 4, of breaking loads in tons of hollow cylindrical wrought iron pillars, with flat ends, perfectly true, and firmly fixed, and the loads pressing equally on every part of the top. Calculated by Gordon's formula. No pains have been taken to have the last figure of the loads perfectly correct in every case.

(Original.)

WROUGHT IRON. THICKNESS ⅛ INCH.

Length in feet.	Outer diameter in inches.										Length in feet.
	¾	1	1¼	1½	1¾	2	2¼	2½	2¾	3	
	BREAKING LOAD.										
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
1	3.64	5.27	6.88	8.50	10.1	11.7	13.2	14.8	16.4	18.0	1
2	2.94	4.64	6.32	8.00	9.6	11.2	12.8	14.5	16.1	17.8	2
3	2.30	3.86	5.57	7.28	8.9	10.6	12.2	13.9	15.6	17.3	3
4	1.77	3.13	4.74	6.36	8.1	9.9	11.6	13.3	15.0	16.7	4
5	1.36	2.51	4.07	5.66	7.3	9.1	10.8	12.5	14.2	16.0	5
6	1.04	2.03	3.46	4.91	6.6	8.3	9.9	11.6	13.4	15.2	6
7	.81	1.65	2.91	4.24	5.7	7.4	9.1	10.8	12.6	14.4	7
8	.61	1.36	2.46	3.67	5.1	6.7	8.3	9.9	11.7	13.5	8
9	.50	1.05	2.03	3.18	4.5	6.0	7.5	9.1	10.8	12.6	9
10	.41	.95	1.75	2.77	4.0	5.4	6 9	8.4	10.1	11.8	10
11	.34	.81	1.52	2.41	3.6	4.8	6.2	7.7	9.3	11.0	11
12	.29	.70	1.34	2.14	3.2	4.3	5.6	7.0	8.6	10.2	12
13	.24	.60	1.16	1.88	2.8	3.9	5.2	6.5	8.0	9.5	13
14	.21	.53	1.03	1.69	2.5	3.5	4.7	6.0	7.4	8.9	14
15	.19	.47	.91	1.50	2.3	3.2	4.3	5.5	6.9	8.3	15
16	.18	.42	.84	1.38	2.1	2.9	4.0	5.1	6.4	7.7	16
18	.14	.33	.67	1.11	1.7	2.4	3.4	4.4	5.6	6.8	18
20	.....	.27	.55	.91	1.4	2.0	2.8	3.7	4.7	5.8	20
25	.....	.....	.....	.....	.9	1.4	2.0	2.6	3.4	4.2	25

Weight of one foot of length of pillar, in pounds.

.820	1.15	1.47	1.80	2.13	2.45	2.78	3.11	3.43	3.77

Area of ring of solid metal, in square inches.

.246	.344	.442	.540	.638	.736	.835	.933	1.03	1.13

WROUGHT IRON. THICKNESS ¼ INCH.

Length in feet.	Outer diameter in inches.										Length in feet.
	2	2¼	2½	2¾	3	3½	4	4½	5	5½	6	
	BREAKING LOAD.											
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
1	21.9	25.4	28.3	31.4	34.5	40	47	53	60	66	72	1
2	21.1	24.3	27.6	30.7	33.9	40	47	53	60	66	72	2
3	19.9	23.1	26.4	29.7	33.0	39	46	52	59	65	71	3
4	18.6	21.8	25.3	28.5	31.9	38	45	51	58	64	71	4
5	17.0	20.4	23.5	27.3	30.7	37	44	50	57	63	70	5
6	15.4	18.8	22.1	25.7	29.2	36	43	49	56	62	69	6
7	13.9	17.3	20.5	23.8	27.8	34	41	47	54	61	68	7
8	12.5	15.6	19.1	22.3	25.9	32	40	46	53	60	67	8
9	11.2	14.2	17.5	20.6	24.3	30	38	44	51	58	65	9
10	10.0	13.0	16.1	19.1	22.7	29	37	43	50	57	64	10
11	9.0	10.7	15.7	17.6	21.1	27	35	41	48	55	62	11
12	8.1	10.6	13.5	16.4	19.6	26	33	40	46	54	61	12
13	7.3	9.6	12.4	15.1	18.2	24	31	38	44	52	59	13
14	6.6	8.8	11.3	14.0	17.0	23	30	36	43	51	57	14
15	6.0	8.0	10.4	12.9	15.8	21	28	34	41	49	55	15
16	5.5	7.3	9.5	12.0	14.6	20	27	33	40	47	54	16
18	4.5	6.0	8.0	10.3	12.7	18	24	30	37	43	50	18
20	3.8	5.1	6.8	8.7	11.0	16	21	27	34	40	47	20
25	.....	.....	.....	.....	7.9	12	16	21	27	33	39	25
30	.....	.....	.....	.....	.....	.....	13	17	22	27	32	30
35	.....	.....	.....	.....	.....	.....	10	14	18	22	27	35
40	.....	.....	.....	.....	.....	.....	.....	.....	14	18	23	40
45	.....	.....	.....	.....	.....	.....	.....	.....	11	15	19	45
50	.....	.....	.....	.....	.....	.....	.....	.....	8	12	16	50

Weight of one foot of length of pillar, in pounds.

4.60	5.23	5.90	6.53	7.20	8.50	9.83	11.1	12.4	13.7	15.0

Area of ring of solid metal, in square inches.

1.38	1.57	1.77	1.96	2.16	2.55	2.95	3.34	3.73	4.12	4.51

HOLLOW CYLINDRICAL WROUGHT IRON PILLARS.

Table 4, (Continued.) (Original.)

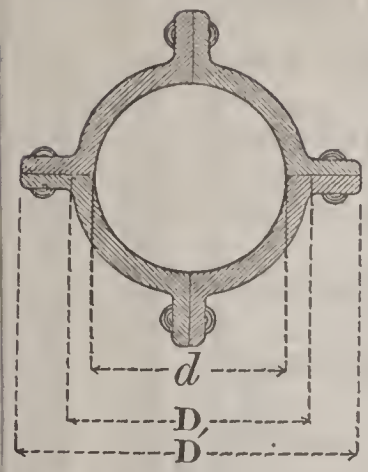
Length in feet.	WROUGHT IRON. THICKNESS ½ INCH.											Length in feet.
	Outer diameter in inches.											
	5	5½	6	6½	7	7½	8	8½	9	10	11	12	
	BREAKING LOAD.												
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
2	112	125	139	152	166	177	189	201	214	238	263	290	2
4	110	123	136	149	163	174	186	199	212	237	262	289	4
6	106	119	132	145	158	171	184	197	210	235	261	288	6
8	101	114	127	140	154	167	181	194	207	232	258	284	8
10	95	108	123	136	149	162	176	189	203	228	254	280	10
12	89	102	116	129	143	157	171	185	199	224	250	276	12
14	82	95	108	122	137	151	165	179	194	219	245	272	14
16	76	89	103	117	131	145	160	173	187	213	240	268	16
18	70	83	97	110	124	138	153	166	180	207	235	263	18
20	64	77	91	104	117	131	145	159	173	201	227	257	20
22	58	70	83	96	109	123	138	151	165	192	220	250	22
25	52	64	76	89	102	115	129	143	157	183	212	241	25
30	42	52	63	74	87	100	113	127	141	167	195	224	30
35	34	43	53	64	75	87	99	112	125	151	178	207	35
40	27	35	44	53	64	75	86	98	110	135	163	190	40
45	23	30	38	46	55	65	76	87	98	123	148	174	45
50	19	24	32	38	47	56	66	76	87	109	133	158	50
60	15	19	24	29	36	43	51	60	69	88	109	132	60
70	11	14	18	23	28	34	40	48	56	73	91	111	70
80	9	11	14	18	22	27	32	37	44	57	74	93	80
90	7	9	11	14	18	22	26	31	36	49	63	78	90
100	6	7	9	12	15	18	22	26	30	41	53	66	100
Weight of one foot of length of pillar, in pounds.	23.6	26.2	28.8	31.4	34.0	36.6	39.3	42.0	44.7	49.7	55.0	60.3	
Area of ring of solid metal, in square inches.	7.07	7.85	8.64	9.43	10.2	11.0	11.8	12.6	13.4	14.9	16.5	18.1	

Table 4, (Continued.) (Original.)

Length in feet.	WROUGHT IRON. THICKNESS 1 INCH.												Length in feet.
	Outer diameter in inches.												
	13	14	15	16	17	18	20	22	24	26	28	30	
	BREAKING LOAD.												
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
1	603	653	704	753	805	854	955	1056	1157	1257	1357	1458	1
10	588	638	691	742	795	846	949	1049	1149	1248	1354	1457	10
20	543	595	651	702	759	810	913	1016	1120	1223	1327	1430	20
30	479	538	594	645	699	758	866	973	1077	1186	1289	1394	30
40	415	470	528	584	636	691	806	912	1027	1130	1237	1348	40
50	355	405	462	516	570	627	740	848	961	1067	1179	1294	50
60	300	348	400	452	505	559	669	781	891	1005	1115	1228	60
70	256	300	348	398	448	499	606	715	824	936	1046	1160	70
80	215	255	298	344	392	440	543	649	757	868	978	1092	80
90	185	222	261	303	347	392	489	590	694	800	910	1023	90
100	157	190	225	262	303	345	436	532	631	735	843	955	100
110	134	162	193	227	264	302	386	474	568	666	770	877	110
125	111	135	162	192	225	259	336	416	505	598	697	799	125
150	82	101	122	145	171	198	262	328	405	485	574	666	150
175	62	78	95	112	133	155	208	266	331	400	478	560	175
200	49	60	74	89	106	124	168	216	269	328	395	467	200
Weight of one foot of length of pillar, in pounds.	126	136	147	157	168	178	199	220	241	262	283	304	
Area of ring of solid metal, in square inches.	37.7	40.8	44.0	47.1	50.3	53.4	59.7	66.0	72.3	78.5	84.8	91.1	

The breaking loads for less thicknesses may safely be assumed to diminish at the same rate as the thickness.

Table of rolled-iron segment-columns of the Phœnix Iron Co, 410 Walnut St, Philada. **For their strengths,** see pp 442, 443, or formula, p 439, with the least radius of gyration, as given below, or as obtained by multiplying D by .3636. The **dimensions** given are subject to slight variations which are unavoidable in rolling iron shapes. The **weights** of columns given are those of the 4, 6, or 8 segments, of which they are composed. The *shanks* of the rivets used in joining them together, of course, merely make up the quantity of metal punched or drilled out, in making the holes; but the rivet-*heads* add from 2 to 5 per cent to the weights given. The **rivets are spaced** 3, 4, or 6 ins apart from cen to cen. **Prices** of the finished columns (1886), from 3 to 4½ cts per pound, at the works, according to specifications, and varying with the quantity ordered, the length, the thickness, and the amount of extra work required.



Any desired thickness between the min and max for any given size, can be furnished. We give the dimensions, wts, &c, corresponding to the principal thicknesses. **G** columns have **8 segs. E, 6 segs. All others, 4 segs.**

Mark.	Thickn's, ins.	Diameters, ins. d	D	D′	One column. Area of cross sec, sq ins.	Wt per ft run, lbs.	Least rad of gyr, ins.	Size of Rivets.
A	3/16	3 5/8	4	6 1/16	3.8	12.6	1.45	1 1/8 × 3/8
"	1/4	"	4 1/8	6 3/16	4.8	16.	1.50	1 1/4 × "
"	5/16	"	4 1/4	6 5/16	5.8	19.3	1.55	1 3/8 × "
"	3/8	"	4 3/8	6 7/16	6.8	22.6	1.59	1 1/2 × "
B¹	1/4	4 13/16	5 5/16	8 1/16	6.4	21.3	1.92	1 5/8 × 1/2
"	3/8	"	5 9/16	8 1/4	9.2	30.6	2.02	1 3/4 × "
"	1/2	"	5 13/16	8 7/16	12.	40.	2.11	1 7/8 × "
"	5/8	"	6 1/16	8 5/8	14.8	49.3	2.20	2 1/8 × "
B²	1/4	5 15/16	6 7/16	9 1/8	7.4	24.6	2.34	1 5/8 × "
"	3/8	"	6 11/16	9 5/16	10.6	35.3	2.43	1 3/4 × "
"	1/2	"	6 15/16	9 1/2	13.8	46.	2.52	1 7/8 × "
"	5/8	"	7 3/16	9 11/16	17.	56.6	2.61	2 1/8 × "
C	1/4	7 3/16	7 11/16	11 9/16	10.	33.3	2.80	1 7/8 × 5/8
"	1/2	"	8 3/16	11 15/16	18.	60.	2.98	2 1/4 × "
"	3/4	"	8 11/16	12 3/16	25.2	84.	3.16	2 5/8 × 3/4
"	1	"	9 3/16	12 9/16	33.2	110.6	3.34	3 × "
"	1 1/4	"	9 11/16	12 15/16	41.2	137.3	3.52	3 1/4 × "
E	1/4	11	11 1/2	15 7/16	16.8	56.	4.18	2 × "
"	1/2	"	12	15 7/8	26.4	88.	4.36	2 3/8 × "
"	3/4	"	12 1/2	16 5/16	37.8	126.	4.55	2 3/4 × "
"	1	"	13	16 3/4	49.8	166.	4.73	3 × "
"	1 1/4	"	13 1/2	17 3/16	61.8	206.	4.91	3 1/4 × "
G	5/16	14 3/8	15	19 1/8	24.	80.	5.45	1 7/8 × 5/8
"	1/2	"	15 3/8	19 7/16	36.	120.	5.59	2 1/4 × "
"	3/4	"	15 7/8	19 7/8	52.	173.3	5.77	2 5/8 × 3/4
"	1	"	16 3/8	20 3/8	68.	226.6	5.95	3 × "
"	1 3/8	"	17 1/8	21	92.	306.6	6.23	3 3/8 × "

Table 2, of breaking loads, in tons, of SOLID CYLINDRICAL CAST IRON pillars, with flat ends, firmly fixed, and equally loaded. By Gordon's rule.

No pains have been taken to have the last figure of the loads perfectly correct in every case.

(Original.)

Length in feet.	DIAMETER OF CYLINDRICAL PILLAR IN INCHES.																					Length in feet.
	½	¾	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4¼	4½	4¾	5	5¼	5½	5¾	6	
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
1	1.81	6.92	16.31	30.0	47.8	69 5	95.1	124	157	194	234	278	324	375	429	487	548	614	682	754	829	908	990	1
2	.56	2.58	7.23	15.4	27.7	44.6	65.2	90.7	120	154	191	233	278	328	380	437	497	562	629	701	775	854	935	2
3	.26	1.26	3.75	8.5	16.3	28.8	44.7	63.6	86.1	115	146	184	224	270	319	373	430	492	557	626	698	776	856	3
4	.15	.73	2.24	5.2	10.3	18.0	28.9	43.4	61.6	85	111	143	177	217	260	310	362	421	480	546	614	688	765	4
5	.10	.48	1.48	3.5	7.0	12.5	20.4	31.2	45.2	63	84	110	139	174	211	255	301	353	408	469	532	601	673	5
6	.07	.33	1.04	2.5	5.0	9.1	15 0	23.1	34.1	48	65	87	111	140	171	209	250	296	344	399	457	521	587	6
7	.05	.25	.77	1.8	3.8	6.9	11.4	17.8	26.4	37	51	69	89	113	140	173	208	248	291	341	393	450	510	7
8	.04	.19	.60	1.4	2.9	5.3	9.0	13.9	20.9	30	41	56	73	93	116	144	174	209	247	291	338	389	443	8
9	.03	.15	.47	1.1	2.3	4.2	7.2	11.3	17.0	24	34	46	61	78	97	121	147	177	210	249	290	336	385	9
10	.02	.12	.38	.93	1.9	3.5	5.9	9.3	14.0	20	28	38	50	64	81	102	125	152	181	215	251	292	336	10
12	.....	.08	.27	.65	1.3	2.4	4.2	6.6	10.0	14	20	27	36	47	60	75	93	113	136	163	192	225	260	12
14	.....	.06	.20	.48	.99	1.8	3.1	4.8	7.4	11	15	20	27	35	46	58	72	88	106	127	150	176	205	14
16	.....	.05	.15	.37	.76	1.4	2.4	3.7	5.7	8.4	12	15	21	27	36	45	56	69	84	101	120	141	165	16
18	.....	.04	.12	.29	.63	1.1	1.9	2.9	4.6	6.7	9.4	12	17	22	29	36	45	55	68	81	97	115	135	18
20	.....	.03	.10	.24	.49	.90	1.5	2.4	3.7	5.4	7.6	10	14	18	24	30	38	46	56	67	81	95	112	20
25	.....	.....	.06	.15	.31	.58	1.0	1.6	2.4	3.5	4.9	6.8	9.1	11	15	19	24	29	37	44	54	63	75	25
30	.....	.....	.04	.10	.22	.40	.7	1.1	1.7	2.5	3.5	4.8	6.4	8.4	11	14	17	21	26	31	37	45	53	30
35	.....	.....	.03	.08	.16	.30	.5	.8	1.2	1.8	2.6	3.6	4.7	6.2	8	11	13	16	20	24	29	34	40	35
40	.....	.. ..	.02	.06	.12	.23	.4	.6	1.0	1.4	1.9	2.7	3.6	4.7	6	8	10	12	15	18	22	26	31	40
50	.....	.. ..	.01	.04	.08	.16	.3	.4	.6	0.9	1.3	1.8	2.4	3.1	4	5	7	8	10	12	15	17	20	50

Weight of one foot of length of pillar, in pounds.

.610	1.38	2.46	3.84	5.52	7.52	9.82	12.4	15.3	18.6	22.1	25.9	30.1	34.5	39.3	44.3	49.7	55.4	61.4	67.7	74.3	81.2	88.4

Area of solid metal, in square inches.

.196	.442	.785	1.23	1.77	2.41	3.14	3.98	4.91	5.94	7.07	8.30	9.62	11.0	12.6	14.2	15.9	17.7	19.6	21.6	23.8	26.0	28.3

Table 5, of breaking loads, in tons, of SOLID CYLINDRICAL WROUGHT IRON pillars, with flat ends, firmly fixed, and equally loaded. By Gordon's rule.

No pains have been taken to have the last figure of the loads perfectly correct in every case.

(Original.)

Length in feet.	DIAMETER OF CYLINDRICAL PILLAR, IN INCHES.																						Length in feet.
	½	¾	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4¼	4½	4¾	5	5¼	5½	5¾	6	
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
1	2.51	6.37	11.9	18.9	27.6	37.9	49.7	63.1	78.1	94.6	112.8	132	154	177	201	227	255	284	315	347	381	416	454	1
2	1.56	4.95	10.0	16.8	25.5	35.7	47.4	60.7	75.7	92.3	110.5	129	151	175	199	225	253	282	313	345	379	414	451	2
3	.95	3.61	8.0	14.4	22.6	32.5	44.1	57.3	72.1	88.5	106.7	126	148	171	195	220	248	277	308	340	374	409	447	3
4	.62	2.52	6.2	11.9	19.5	29.0	40.2	53.0	67.7	83.9	102.0	121	143	166	190	215	243	272	303	335	369	404	442	4
5	.43	1.84	4.8	9.8	16.6	25.4	36.0	48.4	62.7	78.7	96.5	115	136	158	183	208	236	265	296	328	362	397	435	5
6	.31	1.40	3.8	8.0	14.0	22.1	32.0	43.9	57.6	73.1	90.4	109	130	152	177	202	230	258	289	320	353	389	427	6
7	.23	1.08	3.0	6.6	11.9	19.2	28.3	39.5	52.7	67.6	84.3	102	123	145	169	194	221	249	280	312	346	381	418	7
8	.18	.86	2.5	5.4	10.0	16.6	25.0	35.4	47.8	61.9	78.1	96	116	137	160	185	212	240	271	302	336	371	408	8
9	.15	.70	2.0	4.6	8.6	14.4	22.0	31.6	43.2	56.6	72.1	89	109	129	152	176	203	231	261	292	326	360	397	9
10	.12	.57	1.7	3.9	7.4	12.5	19.4	28.2	39.0	51.7	66.4	83	102	122	144	169	193	221	251	282	315	350	385	10
12	.08	.41	1.2	2.8	5.6	9.7	15.3	22.7	32.0	43.0	56.1	71	88	107	128	151	176	203	232	262	294	327	362	12
14	.06	.30	.93	2.2	4.3	7.6	12.2	18.3	26.3	35.8	47.4	61	76	93	113	134	158	183	211	240	271	303	337	14
16	.. ..	.23	.72	1.8	3.4	6.1	9.9	15.0	21.8	30.1	40.3	53	66	82	100	120	142	166	192	219	249	279	312	16
18	.....	.19	.58	1.4	2.8	5.1	8.2	12.5	18.3	25.5	34.4	45	57	71	88	106	126	148	173	199	227	256	288	18
20	.....	.15	.47	1.1	2.3	4.1	6.8	10.5	15.5	21.8	29.5	39	50	63	78	94	113	133	156	180	207	235	265	20
25	....		.31	.7	1.5	2.7	4.6	7.1	10.7	15.2	20.9	28	36	46	58	71	87	104	123	143	166	189	215	25
30	....		.22	.5	1.1	2.0	3.3	5.1	7.7	11.0	15.3	20	26	35	44	54	66	79	95	112	132	152	175	30
35	....		.16	.4	.8	1.4	2.4	3.7	5.7	8.3	11.7	15	20	27	34	42	53	63	76	90	106	123	143	35
40	....		.12	.3	.6	1.1	1.9	3.0	4.5	6.5	9.2	12	16	21	27	34	42	52	62	74	87	102	118	40
50	.. ..		.08	.2	.4	.7	1.2	2.0	3.0	4.3	6.0	8	11	14	18	23	29	36	43	52	61	72	83	50
Weight of one foot of length of pillar, in pounds.																								
	.655	1.48	2.62	4.09	5.90	8.02	10.5	13.3	16.4	19.8	23 6	27.7	32.1	36.9	41.9	47.3	53.1	59.0	65.5	72.2	79.2	86.6	94.3	
Area of solid metal, in square inches.																								
	.196	.442	.785	1.23	1.77	2.41	3.14	3.98	4.91	5.94	7.07	8.30	9.62	11.0	12.6	14.2	15.9	17.7	19.6	21.6	23.8	26.0	28.3	

Table 3, of breaking load, in tons, of SOLID SQUARE CAST IRON pillars, with fiat ends, firmly fixed, and equally loaded. By Gordon's rule.

No pains have been taken to have the last figure of the loads perfectly correct in every case.

(Original.)

Length in feet.	ONE SIDE OF SQUARE PILLAR, IN INCHES.																						Length in feet.
	½	¾	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4¼	4½	4¾	5	5¼	5½	5¾	6	
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
1	2.82	10.25	23.2	41.5	64.8	93.0	126.0	163	205	252	303	359	419	483	552	626	704	787	874	965	1061	1162	1266	1
2	.93	4.15	11.3	23.4	41.0	64.7	92.7	127	166	210	259	313	372	435	503	576	653	735	822	913	1008	1108	1213	2
3	.44	2.14	6.1	13.6	25.4	42.2	64.5	92.3	125	164	209	259	313	373	438	509	584	664	748	837	931	1029	1133	3
4	.25	1.23	3.7	8.5	16.6	28.9	45.2	66.8	93.7	126	164	208	257	312	371	437	508	584	665	751	841	937	1037	4
5	.16	.80	2.5	5.8	11.5	21.1	34.9	49 7	70.6	97.0	129	166	208	257	310	370	434	505	581	662	747	839	935	5
6	.11	.56	1.7	4.1	8.3	14.9	26.1	37.3	54.3	75.6	102	134	170	212	258	311	369	433	502	577	657	744	835	6
7	.08	.42	1.3	3.1	6.3	11.3	18.8	29.0	42.6	59.3	81	108	138	174	215	262	314	371	435	503	576	656	741	7
8	.06	.32	1.0	2.4	4.9	8.9	14.8	23.0	34.0	47.9	66	89	115	146	181	222	268	320	376	438	505	578	656	8
9	.05	.25	.80	1.9	3.9	7.1	12.0	18.5	27.4	39.0	54	74	96	123	153	190	230	275	325	381	442	509	580	9
10	.....	.21	.65	1.6	3.2	5.9	9.9	15.5	23.1	33.2	46	62	81	104	130	162	197	238	282	333	388	448	514	10
12	.....	.14	.45	1.1	2.2	4.2	7.0	11.0	16.6	23.8	33	45	59	77	97	122	149	182	217	258	303	352	407	12
14	.....	.11	.34	.81	1.7	3.1	5.2	8.2	12.5	17.9	25	34	45	59	75	94	115	141	170	203	240	281	326	14
16	.....	.08	.26	.63	1.3	2.3	4.0	6.3	9.6	14.0	19	27	35	46	59	74	92	113	137	164	194	228	266	16
18	.....	.06	.20	.49	1.0	1.9	3.2	5.0	7.7	10.7	15	21	28	37	48	60	75	92	112	134	159	187	219	18
20	.....	.....	.17	.40	.83	1.5	2.6	4.1	6.3	9.1	13	18	23	31	39	49	62	76	93	112	133	157	184	20
25	.....	.....	.11	.26	.53	.96	1.7	2.7	4.1	6.2	9	12	15	20	26	33	41	50	62	74	89	105	124	25
30	... ..	.....	.07	.18	.37	.68	1.2	1.9	2.8	4.2	6	8	11	14	18	23	29	36	44	52	63	75	89	30
35	.....	.....	.05	.13	.27	.50	.9	1.4	2.1	3.0	4	6	9	11	13	16	20	26	32	39	46	55	66	35
40	.....	... ..	.04	.10	.21	.38	.7	1.1	1.6	2.3	3	4	6	8	10	12	15	20	25	30	35	43	51	40
50	.....	.....	.03	.06	.13	.25	.4	.7	1.1	1.5	2	3	4	5	7	9	11	13	16	20	24	28	33	50
Weight of one foot of length of pillar, in pounds.																							
	.781	1.76	3.13	4.88	7.03	9.57	12.5	15.8	19.5	23.6	28.1	33.0	38.3	44.0	50.0	56.5	63.3	70.5	78.1	86.1	94.5	103	113	
Area of solid metal, in square inches.																							
	.250	.563	1	1.56	2.25	3.06	4	5.06	6.25	7.56	9	10.6	12.3	14.1	16	18.1	20.3	22.6	25	27.6	30.3	33.1	36	

Table 6, of breaking loads, in tons, of SOLID SQUARE WROUGHT IRON PILLARS, with flat ends, firmly fixed, and equally loaded. By Gordon's rule.

No pains have been taken to have the last figure of the loads perfectly correct in every case.

(Original.)

Length in feet.	ONE SIDE OF SQUARE PILLAR, IN INCHES.																						Length in feet.
	½	¾	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4¼	4½	4¾	5	5¼	5½	5¾	6	
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
1	3.37	8.32	15.3	24.4	35.4	48.4	63.5	80.6	100	121	144	169	196	225	256	289	324	361	401	442	486	531	578	1
2	2.28	6.74	13.5	22.3	33.3	46.3	61.3	78.4	97	118	142	167	194	223	254	287	322	359	398	439	482	528	575	2
3	1.47	5.13	11.2	19.7	30.3	43.1	58.0	74.9	94	115	138	163	190	219	250	283	318	355	395	436	479	524	572	3
4	.99	3.82	9.05	16.8	27.0	39.3	53.9	70.7	89	110	133	158	185	214	245	278	313	350	390	431	474	519	567	4
5	.69	2.88	7.30	14.2	23.6	35.3	49.4	65.8	84	105	128	153	180	208	239	272	307	344	383	424	468	513	560	5
6	.51	2.22	5.89	11.9	20.4	31.5	44.9	60.7	79	99	121	145	172	201	232	265	300	337	376	417	460	505	552	6
7	.39	1.74	4.79	10.0	17.7	27.1	40.5	55.5	73	93	115	139	165	193	224	256	291	328	367	408	451	496	543	7
8	.30	1.40	3.94	8.4	15.3	24.6	36.4	50.6	67	86	108	132	158	186	216	248	283	319	358	398	441	486	533	8
9	.24	1.14	3.29	7.2	13.2	21.7	32.6	46.0	62	80	101	124	150	177	207	239	273	309	348	388	431	475	522	9
10	.20	.95	2.77	6.2	11.5	19.2	29.2	41.8	57	75	94	116	141	168	198	230	264	300	338	378	420	464	511	10
12	.14	.68	2.03	4.6	8.9	15.1	23.6	34.4	48	64	82	103	127	152	180	210	243	278	315	354	396	439	485	12
14	.10	.51	1.54	3.6	6.9	12.1	19.2	28.5	40	54	71	90	112	136	162	192	223	256	292	330	371	414	459	14
16	.08	.40	1.21	2.8	5.6	9.8	15.8	23.7	34	46	61	79	99	121	145	173	203	235	269	306	346	387	431	16
18	.06	.31	.98	2.3	4.6	8.1	13.1	20.0	29	40	53	69	87	107	130	156	185	215	248	284	322	362	404	18
20	.....	.26	.80	1.9	3.8	6.8	11.1	17.0	25	34	46	60	77	96	117	141	168	196	227	261	297	336	377	20
25	.....	.17	.52	1.2	2.5	4.6	7.6	11.7	17	24	33	44	57	72	89	109	132	156	183	213	245	279	316	25
30	.....	.12	.36	.9	1.8	3.3	5.4	8.5	13	18	25	33	43	55	69	85	104	124	147	173	201	231	263	30
35	.....	.09	.27	.6	1.3	2.4	4.1	6.5	10	13	19	25	33	43	55	68	83	100	120	142	166	192	220	35
40	.....	.06	.21	.5	1.0	1.9	3.2	5.0	8	11	15	20	27	35	44	55	67	82	99	117	137	160	185	40
50	.....	.04	.13	.3	.7	1.2	2.1	3.3	5	7	10	14	18	24	30	38	46	57	69	83	98	115	133	50
Weight of one foot of length of pillar, in pounds.																							
	.83	1.88	3.33	5.21	7.51	10.2	13.4	16.9	20.9	25.2	30.0	35.3	40.9	46.9	53.4	60.2	67.5	75.3	83.4	91.9	101	110	120	
Area of solid metal, in square inches.																							
	.250	.563	1.00	1.56	2.25	3.06	4.00	5.06	6.25	7.56	9.00	10.6	12.3	14.1	16.0	18.1	20.3	22.6	25.0	27.6	30.3	33.1	36.0	

Table of quiescent breaking loads in short tons (2000 lbs) of rolled I beams as pillars or struts, from Carnegie Bros & Co, (Union Iron Mills, Pittsburgh. Penn,) "Tables and Information on Wrought Iron." **Example 1 of use.** The table shows that a heavy 12 inch beam as a pillar will if 32 ft long, and both ends fixed, just give way edgeways (in the direction of the length of the web) with a load of 283 short tons; or sideways (across the web) with 78 short tons. And the same if it is 24 ft long, with one end fixed, and the other end hinged; or if it is 16 ft long, and both ends hinged. **Example 2 of use.** The table shows that if the first beam 32 ft long, had been but 7 ft, it would require about 283 short tons to break it even sideways, which shows that if the 32 ft one is perfectly braced at about every 7 ft against yielding sideways, it will be equally strong in both directions. This principle of course applies to every load in the table. In practice from one-third to one-tenth of the breaking load may be used according to the case.

Lengths.			15 Hvy.		15 Lgt.		12 Hvy.		12 Lgt.		10½ Hvy.		10½ Lgt.		10 Hvy.		10 Lgt.		9 Hvy.		9 Lgt.	
Both ends fxd. Ft.	1 fxd. 1 hgd. Ft.	Both ends hgd. Ft.	Sideway. Sht tons.	Edgeway. Sht tons.	Sideway. Sht tons.	Edgeway. Sht tons.	Sideway. Sht tons.	Edgeway. Sht tons.	Sideway. Sht tons.	Edgeway. Sht tons.	Sideway. Sht tons.	Edgeway. Sht tons.	Sideway. Sht tons.	Edgeway. Sht tons.	Sideway. Sht tons.	Edgeway. Sht tons.	Sideway. Sht tons.	Edgeway. Sht tons.	Sideway. Sht tons.	Edgeway. Sht tons.	Sideway. Sht tons.	Edgeway. Sht tons.
1			360	362	269	270	323	324	227	227	188	189	170	170	204	205	161	162	161	162	128	128
2	1.5	1	358	361	266	270	320	323	223	227	186	188	167	170	202	205	159	161	160	162	126	128
4	3.	2	347	361	258	269	309	323	213	227	177	188	160	169	193	204	153	161	153	161	120	127
6	4.5	3	330	360	244	269	292	322	198	226	165	187	150	168	180	204	144	160	143	161	111	127
8	6.	4	309	359	227	268	270	321	180	226	150	187	136	167	166	203	131	160	131	160	101	126
10	7.5	5	286	358	210	268	247	320	161	226	134	186	112	166	150	203	121	159	120	159	91	125
12	9.	6	262	357	190	267	222	318	143	225	119	184	108	165	132	200	108	158	108	157	80	124
14	10.5	7	236	356	172	266	202	315	127	224	106	182	96	164	117	197	94	156	95	156	75	123
16	12.	8	215	354	155	265	180	313	111	222	93	180	85	162	104	195	83	154	85	154	63	121
18	13.5	9	194	352	138	263	162	310	98	220	82	178	75	160	92	193	72	152	74	152	55	120
20	15.	10	170	350	124	261	145	307	87	218	72	176	67	159	81	191	65	151	64	150	49	118
22	16.5	11	158	347	112	259	130	303	77	216	64	174	58	157	72	188	59	150	59	147	42	116
24	18.	12	138	345	100	257	117	299	68	213	58	172	54	154	65	185	52	149	52	144	38	114
26	19.5	13	125	341	92	255	105	295	61	210	50	170	47	152	57	182	46	146	47	141	34	112
28	21.	14	117	338	82	254	95	291	54	207	45	167	42	149	52	179	41	143	42	138	30	110
30	22.5	15	106	335	74	252	85	288	49	204	40	164	38	147	46	176	37	141	38	135	27	108
32	24.	16	97	332	68	249	78	283	45	201	36	161	34	145	42	173	34	138	34	132	25	106
34	25.5	17	89	329	62	246	71	279	40	198	33	157	30	142	38	169	30	136	29	129	22	104
36	27.	18	80	325	57	244	65	274	36	195	30	154	27	139	35	166	27	133	27	126	20	102
38	28.5	19	74	321	51	242	59	270	33	192	27	151	25	136	32	162	25	130	26	123	17	100
40	30.	20	69	318	47	239	54	265	30	188	25	148	23	133	28	158	23	127	24	120	15	97

Lengths.			8 Hvy.		8 Lgt.		7 Hvy.		7 Lgt.		6 Hvy.		6 Lgt.		5 Hvy.		4 Hvy.		3 Hvy.	
Both ends fixd. Ft.	1 fxd. 1 bgd. Ft.	Both ends hgd. Ft.	Sideway. Sht tons.	Edgeway. Sht tons.	Sideway. Sht tons.	Edgeway. Sht tons.	Sideway. Sht tons.	Edgeway. Sht tons.	Sideway. Sht tons.	Edgeway. Sht tons.	Sideway. Sht tons.	Edgeway. Sht tous.	Sideway. Sht tons.	Edgeway. Sht tons.	Sideway. Sht tons.	Edgeway. Sht tons.	Sideway. Sht tons.	Edgeway. Sht tons.	Sideway. Sht tons.	Edgeway. Sht tons.
1			145	146	118	119	107	108	97	98	86	87	73	73	64	64	54	54	43	43
2	1.5	1	143	145	117	119	106	108	96	98	85	87	71	73	62	64	51	53	41	42
4	3.	2	136	144	110	118	100	107	91	97	80	86	66	72	55	63	45	52	37	40
6	4.5	3	126	143	100	117	92	106	84	97	72	85	60	72	47	62	37	51	32	38
8	6.	4	113	142	90	116	80	105	75	96	63	84	53	71	39	61	29	50	26	36
10	7.5	5	102	141	79	115	73	104	66	95	55	83	46	70	32	60	24	49	20	34
12	9.	6	89	139	70	114	63	103	58	94	47	81	42	69	26	58	20	46	18	32
14	10.5	7	80	137	60	113	55	101	50	93	40	79	37	67	21.5	56	16	44	15	30
16	12.	8	70	135	52	111	48	99	45	91	35	77	32	65	17.8	54	12	42	12	28
18	13.5	9	61	133	45	109	42	97	39	89	30	75	27	63	14.2	52	10	40	9	26
20	15.	10	53	131	39	107	36	95	34	87	26	73	22	61	12.1	50	9	38	8	24
22	16.5	11	47	129	35	104	32	92	30	85	23	70	21	59	10.6	48	8	35	7.1	22
24	18.	12	42	126	31	101	28	89	26	83	20	68	18	57	9.3	46	7	33	6.4	20
26	19.5	13	37	123	27	98	25	87	23	81	17	65	16	55	8.2	44	6.1	31	5.7	18
28	21.	14	33	120	24	95	22	84	20	78	15	62	14	53	7.1	41	5.2	29	5.0	16
30	22.5	15	30	117	22	93	20	82	18	75	14	60	13	51	6.3	39	4.3	27	4.3	15
32	24.	16	27	116	19	91	18	79	16	72	12	58	11	49	5.6	36	3.8	26	3.9	14
34	25.5	17	24	114	17	89	16	77	14	70	11	56	10	47	5.0	33	3.5	24	3.5	12
36	27.	18	22	112	15	87	14	75	13	68	10	54	9	45	4.5	30	3.2	23	3.1	11
38	28.5	19	20	110	14	85	13	72	12	66	9	52	8	43	4.0	27	2.8	21	2.8	9
40	30.	20	18	108	13	83	12	69	11	64	8	50	7	41	3.7	24.5	2.5	20	2.5	8

Dimensions of the above Carnegie I beams from out to out both ways; also wts per ft, and areas.

Depth. Ins.	Wt per ft lbs.	Width of flge. Ins.	Av ths of flge. Ins.	Ths of web. Ins.	Area. Sq Ins.	Depth. Ins.	Wt per ft. lbs.	Width of flge. Ins.	Av ths of flge. Ins.	Ths of web. Ins.	Area. Sq Ins.	Depth. Ins.	Wt per ft. lbs.	Width of flge. Ins.	Av ths of flge. Ins.	Ths of web. Ins.	Area. Sq Ins.
Heavy....15	67	5.5	1.09	.65	20.1	Light....10	30	4.3	.75	.32	9.00	Heavy....6	16	3.38	.56	.24	4.81
Light.....15	50	5.0	.94	.44	15.0	Heavy....9	30	4.34	.75	.35	9.00	Light.....6	13½	3.22	.50	.20	4.05
Heavy....12	60	5.15	1.00	.78	18.0	Light.....9	23¾	4.06	.66	.26	7.13	Heavy....5	12	2.88	.38	.35	3.60
Light.....12	42	4.5	.81	.52	12.6	Heavy....8	27	4.	.72	.36	8.10	Light.....5	10	2.75	.38	.23	3.00
Heavy....10½	35	4.63	.66	.50	10.5	Light.....8	22	3.75	.63	.28	6.61	Heavy....4	10	2.63	.34	.39	3.00
Light.....10½	31½	4.53	.66	.41	9.46	Heavy....7	20	3.63	.63	.26	6.00	Light.....4	8	2.50	.34	.24	2.40
Heavy....10	38	4.5	.81	.51	11.4	Light.....7	18	3.56	.63	.18	5.41	Heavy....3	8	2.34	.34	.34	2.40

Table of quiescent breaking loads in short tons (2000 lbs) of rolled channel iron ⊔ . as pillars or struts. From Carnegie Bros & Cos. (Union Iron Mills, Pittsburgh, Penn) "Tables and Information on Wrought Iron."

For manner of use see the paragraph above the preceding table. Hgd, fxd, mean hinged, fixed.

The headings **12 Hy, 12 Mm.** &c, mean 12 inch heavy, and 12 inch medium channels. **Sideway** means **across** the web; **Edgeway** means **along** the web.

Length of Strut.			12 Hy.		12 Mm.		10 Hy.		10 Mm.	
Both ends fixd. Ft.	1 fxd. 1 hgd. Ft.	Both ends hgd. Ft.	Side Way. Sht Tons.	Edge Way. Sht Tons.	Side Way. Sht Tons.	Edge Way. Sht Tons.	Side Way. Sht Tons.	Edge Way. Sht Tons.	Side Way. Sht Tons.	Edge Way. Sht Tons.
1			268	270	160	160	187	189	121	124
2	1.5	1	260	268	154	160	181	189	118	123
4	3.	2	235	266	135	160	161	188	104	123
6	4.5	3	200	264	112	160	135	187	87	122
8	6.	4	168	262	90	159	111	186	71	122
10	7.5	5	138	260	72	159	90	185	57	121
12	9.	6	114	258	60	159	75	183	46	121
14	10.5	7	95	256	47	158	60	181	38	120
16	12.	8	79	254	38	156	50	178	31	118
18	13.5	9	66	253	32	155	41	176	26	116
20	15.	10	56	252	27	153	35	174	22	115
22	16.5	11	48	249	23	151	31	171	19	113
24	18.	12	41	246	20	149	27	167	16	111
26	19.5	13	36	243	17	147	24	163	14	109
28	21.	14	32	239	15	145	20	159	12	107
30	22.5	15	28	235	13	143	17	156	11	105
32	24.	16	25	230	11	140	15	153	10	103
34	25.5	17	23	226	10	138	14	149	9	100
36	27.	18	21	222	9	136	13	146	8	98
38	28.5	19	19	218	8	134	11	143	7	96
40	30.	20	17	213	7	132	10	140	6	94
			9 Hy.		**9 Mm.**		**8 Hy.**		**8 Mm.**	
1			160	162	96	97	134	135	85	86
2	1.5	1	154	161	93	97	130	134	83	85
4	3.	2	134	160	80	96	112	133	70	85
6	4.5	3	110	159	65	96	91	132	57	84
8	6.	4	90	158	53	95	72	131	45	84
10	7.5	5	72	157	42	95	58	130	36	83
12	9.	6	58	154	33	94	46	127	28	83
14	10.5	7	47	150	27	93	30	125	23	82
16	12.	8	38	147	22	92	25	123	19	81
18	13.5	9	32	143	18	91	21	120	15	80
20	15.	10	27	140	15	90	19	118	13	78
22	16.5	11	23	138	13	88	17	115	11	76
24	18.	12	20	136	11	86	15	112	10	74
26	19.5	13	17	134	9	84	13	109	8	72
28	21.	14	15	131	8	82	12	105	7	70
30	22.5	15	13	129	7.3	80	10	102	6	68
32	24.	16	12	126	6.6	78	9	99	5.5	66
34	25.5	17	11	122	5.9	76	8	96	5	64
36	27.	18	10	118	5.2	74	7.4	93	4.5	62
38	28.5	19	9	115	4.5	72	6.7	89	4	60
40	30.	20	8	111	3.8	70	6.0	86	3.5	58
			7 Hy.		**7 Mm.**		**6 Hy.**		**5 Hy.**	
1			105	106	75	76	54	55	53	54
2	1.5	1	103	106	72	75	51	54	50	54
4	3.	2	87	105	61	75	43	53	41	53
6	4.5	3	70	104	49	74	33	52	31	52
8	6.	4	55	104	39	74	25	51	24	51
10	7.5	5	43	103	30	73	20	50	18	50
12	9.	6	34	100	24	72	16	49	14	49
14	10.5	7	27	98	19	71	13	48	11	47
16	12.	8	23	96	16	70	10	47	8.8	45
18	13.5	9	18.2	94	13	68	8	46	7.2	43
20	15.	10	15.5	92	11	66	7	45	6	41
22	16.5	11	13.5	89	9	64	6	44	5	40
24	18.	12	12	86	8	62	5	42	4.3	38
26	19.5	13	10.5	83	7	60	4.5	40	3.7	36
28	21.	14	9	80	6	58	4.	38	3.2	34
30	22.5	15	7.5	77	5	56	3.6	37	2.8	32
32	24.	16	6.9	75	4.6	54	3.	36	2.5	31
34	25.5	17	6.2	72	4.2	52	2.6	35	2.2	30
36	27.	18	5.5	69	3.7	50	2.3	34	2.0	28
38	28.5	19	5.0	66	3.5	48	2.1	33	1.8	26
40	30.	20	4.5	63	3.1	46	1.9	30	1.6	24

In arches of cast iron for bridges, &c, it is usual among English engineers not to allow more than 2½ tons (5600 lbs) of compression, or thrust, per sq inch. Brunel never subjected cast-iron pillars to more than 1½ tons (3360 lbs) per sq inch. C. Shaler Smith employs as maximum working strains, $\frac{1}{5}$ of the calculated breaking strain for such hollow chords and posts of bridges as are 1 inch or more in thickness, and not more than 15 diams long. For posts, only $\frac{1}{7}$; when not less than ⅝ inch thick, nor more than 25 diams long; or from $\frac{1}{10}$ to $\frac{1}{40}$, when ⅝ thick or less, and more than 25 diams long.

The young engineer must bear in mind that the breakg and the safe loads per sq inch, of pillars of any given material, are not constant quantities; but diminish as the piece becomes longer in proportion to its diam. If a very long piece or pillar be so braced at intervals as to prevent its bending at those points, then its length becomes virtually diminished, and its strength increased. Thus, if a pillar 100 ft long be sufficiently braced at intervals of 20 ft, then the load sustained may be that due to a pillar only 20 ft long. Therefore, very long pillars used for bridge piers, &c, are thus braced; as are also long horizontal or inclined pieces, exposed to compression in the form of upper chords of bridges; or as struts of any kind in bridges, roofs, or other structures.

Mistakes are sometimes made by assuming, say 5 or 6 tons per sq inch, as the safe compressing load for cast iron; 4 tons for wrought; 1000 pounds for timber; without any regard to the length of the piece.

But although the final crushing loads, as given in tables of strengths of materials, are usually those for pieces not more than about 2 diams high, they will not be much less for pieces not exceeding 4 or 5 diams.

Cautions. Remember a heavily loaded cast-iron pillar is easily broken by a side blow. Cast-iron ones are subject to hidden voids. All are subject to jars and vibrations from moving loads. It very rarely happens that the pres is equally distributed over the whole area of the pillar; or that the top and bottom ends have perfect bearing at every part, as they had in the experimental pillars † Cast pillars are seldom perfectly straight, and hence are weakened.

Hollow pillars intended to bear heavy loads should not be cast with such mouldings as *a a*; or with very projecting bases or caps such as *g*, Fig 19. It is plain that these are weak, and would break off under a much less load than would injure the shaft of the pillar. When such projecting ornaments are required, they should be cast separately, and be attached to a prolongation of the shaft, as *c d*, by iron pins or rivets.

Ordinarily, it is better to adopt a more simple style of base and cap, which, as at *b*, can be cast in one piece with the pillar, without weakening it.

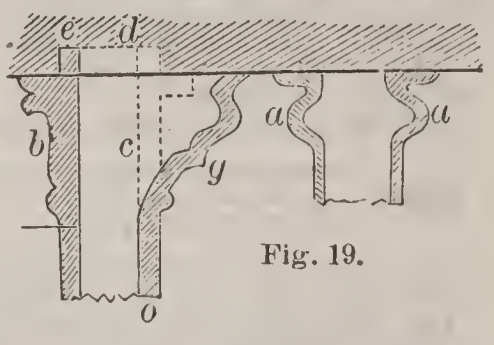


Fig. 19.

Hodginson states that *while the quantity of material is the same in both pillars*, no strength is gained in *hollow* ones by making **the diams greater at the middle than at the ends;** but that in *solid* ones, *with rounded ends*, there is a gain of about $\frac{1}{7}$th part; and in those *with flat ends*, of about $\frac{1}{8}$th or $\frac{1}{9}$th part, by making the diam at the middle about 1½ or 2 times that at the ends. Also that a uniform round pillar has the same strength as a moderately tapering one whose diam at half-way up is equal to the uniform diam of the cylindrical one.

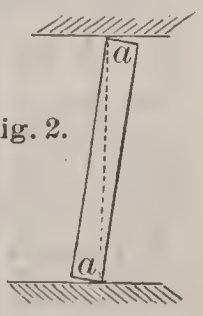


Fig. 2.

Also, that when a flat-ended pillar, Fig. 2, is so irregularly fixed, that the pressure upon it passes along its diagonal *a a*, it loses two-thirds of its strength. Hence the necessity for equalizing, as far as possible, the pressure over every part of the top and bottom of the pillar; a point very difficult to secure in practice.

† In important cases, **both ends should be planed perfectly true;** as is done in iron bridges, &c.

Steel pillars. Mr. Kirkaldy experimented with a small steel pillar or tube of Shortridge, Howell & Co's homogeneous metal. Its length was 4 ft, or 25.6 diams; outer diam $1\frac{7}{8}$ inch: inner diam $1\frac{1}{8}$; thickness $\frac{3}{8}$ inch. Area of cross-section $1\frac{3}{4}$ sq ins. Flat ends. Under 67300 lbs, or 30 tons total pressure, or 17.14 tons per sq inch of solid metal section, it bent very slightly. On increasiug the pressure considerably, the pillar sprang out from under the load. Our table, page 444, gives $24\frac{1}{2}$ tons total, or 13 tons per sq inch, as the ultimate load for a wrought-iron tube of the same size.

Mr. M. G. Love, Paris, as the result of a trial with **small steel rods,** about .4 inch diam. and which had a tensile strength of 48 tons per sq inch, suggests that the comparative strength of pillars of wrought iron, cast-iron, and steel, are **probably** about as follows: At from $1\frac{1}{2}$ to 5 diams in length, steel and cast-iron ones have equal strengths; and either of them is about twice as strong as wrought iron. At 10 diams, steel is $1\frac{1}{3}$ times as stroug as cast; and 2.2 times as strong as wrought iron. At 40 diams, steel is 4 times as strong as cast; and 2.7 as strong as wrought. But this needs confirmatiou. Now that powerful and accurate testing machines are coming more into use, we may hope that the doubts at present existing on such subjects will be set at rest.

Mr Stoney advises that until then steel pillars should not be trusted with more than 1.5 the loads of wrought iron ones.

WOODEN PILLARS.

The strengths of pillars, as well as of beams of timber, depend much on their **degree of seasoning.** Hodgkinson found that perfectly seasoned blocks, 2 diams long, required, in many cases, twice as great a load to crush them as when only moderately dry. This should be borne in mind when building with green timber.

In important practice, timber should not be trusted with more than $\frac{1}{8}$ to $\frac{1}{6}$ of its calculated crushiug load; aud for temporary purposes, not more than $\frac{1}{6}$ to $\frac{1}{4}$.

Mr. Charles Shaler Smith, C. E., of St. Louis, prepared the following formula for the breaking loads of either square or rectangular pillars or posts, of moderately seasoned white, and common yellow pine, with flat ends, firmly fixed, and equally loaded, based upon experiments by himself.

It is Gordon's formula adapted to those woods; and gives results considerably smaller than Hodgkinson's, It is therefore safer.

Call either side of the square, or the least side of the rectangle, the *breadth.* Then,

$$\textbf{Rule.}\quad \left.\begin{array}{c}\text{Breakg load in lbs, per}\\ \text{sq inch of area, of a}\\ \text{pillar of W or Y pine}\end{array}\right\} = \frac{5000\dagger}{1+\left(\frac{\text{sq of length in ins}}{\text{sq of breadth in ins}}\times .004\right)}$$

Or in words, square the length in ins; square the breadth in ins; div the first square by the second one; mult the quot by .004; to the prod add 1; div 5000 by the sum.

Ex. Breakg load per sq inch, of a white pine pillar 12 ins square, and 30 ft, or 360 ins long. Here the sq of length in ins is $360^2 = 129600$. The square of the breadth is $12^2 = 144$; and $\frac{129600}{144} = 900$; and $900 \times .004 = 3.6$; and $3.6 + 1 = 4.6$. Finally, $\frac{5000}{4.6}$ $= 1087$ lbs, the reqd breakg load per sq in. As the area of the pillar is 144 sq ins, the entire breakg load is $1087 \times 144 = 156528$ lbs, or 69.9 tons.

Recent experiments on wooden pillars 20 ft long, and 13 ins square, by Mr. Kirkaldy, of England, confirm the far greater reliability of Mr. Smith's formula. Hence we present the following new set of original tables based upon it.

For solid pillars of cast iron and of pine, whose heights range from 5 to 60 times their side or diam, we may say, near enough for practice, that a cast iron one is abont $16\frac{1}{2}$ times as strong as a pine one; but no such approximate ratio holds good between wrought iron and piue, or between cast and wrought iron.

† The breaking load in lbs per sq inch in short blocks, by Mr. Smith.

Table of breaking loads in tons of square pillars of half seasoned white or common yellow pine firmly fixed and equally loaded. By C. Shaler Smith's formula. (Original.)

Height in feet.	Side of square pine pillar, in inches.													Height in feet.
	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	
	BREAKING LOAD.													
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
1	1.42	2.54	3.99	5.73	7.80	10.1	12.8	15.7	18.9	22.3	26.1	30.1	34.5	1
¼	1.17	2.22	3.59	5.26	7.25	9.6	12.2	15.1	18.3	21.7	25.4	29.2	33.7	¼
½	.97	1.93	3.19	4.80	6.74	9.0	11.6	14.5	17.6	21.0	24.7	28.6	33.0	½
¾	.81	1.66	2.81	4.35	6.19	8.4	10.9	13.7	16.8	20.2	23.9	27.8	32.1	¾
2	.68	1.44	2.48	3.92	5.66	7.8	10.2	12.9	15.9	19.3	23.0	26.9	31.2	2
¼	.57	1·24	2.19	3.53	5.17	7.2	9.6	12.3	15.2	18.5	22.0	25.8	30.1	¼
½	.49	1.07	1.93	3.16	4.70	6.7	8.9	11.5	14.3	17.6	21.1	24.9	29.1	½
¾	.42	.93	1.71	2.85	4.29	6.2	8.3	10.8	13.5	16.7	20.1	23.8	28.0	¾
3	.36	.82	1.52	2.55	3.89	5.6	7.6	10.0	12.7	15.8	19.2	22.9	27.0	3
½	.28	.63	1.21	2.07	3.23	4.8	6.6	8.8	11.3	14.2	17.4	20.9	24.8	½
4	.22	.50	.98	1.70	2.70	4.0	5.7	7.7	9.9	12.7	15.7	19.0	22.7	4
½	.18	·40	.81	1.42	2.28	3.4	4.9	6.7	8.8	11.4	14.1	17.2	20.7	½
5	.15	.34	.68	1.19	1.94	3.0	4.2	5.8	7.7	10.0	12.6	15.5	18.8	5
½	.12	.28	.57	1.02	1.67	2 6	3.7	5.1	6.8	8.9	11.3	14.0	17.1	½
6	.10	.24	.49	.86	1.44	2.3	3.3	4.6	6.1	8.0	10.2	12.7	15.6	6
½	.09	.21	.43	.74	1.26	2.0	2.9	4.1	5.4	7.2	9.2	11.6	14.2	½
7	.08	.18	.37	.66	1.11	1.8	2 6	3 6	4.9	6.5	8.3	10.5	12.9	7
½	.07	.16	.33	.59	.98	1.6	2.3	3.3	4.4	5.9	7.6	9.6	11.8	½
8	.06	.14	.29	.52	.87	1.4	2.0	2.9	3.9	5.2	6.8	8.7	10.8	8
½	.05	.12	.26	.47	.78	1.2	1.8	2.6	3.5	4.8	6.2	7.9	9.9	½
9	.05	.11	.23	.42	.71	1.1	1.6	2.3	3.2	4.3	5.6	7.2	9.1	9
½	.....	.10	.21	.37	.64	1.0	1.5	2.1	2.9	3.9	5.1	6.6	8 4	½
10	.....	.09	.19	.34	.58	.93	1.4	2.0	2.7	3.6	4.7	6.1	7.8	10
½	.....	.....	.17	.31	.53	.86	1.3	1.8	2.5	3.4	4.4	5.7	7.2	½
11	.....	.....	.16	.28	.48	.79	1.2	1.7	2.3	3.1	4.1	5.3	6.7	11
½	.....	.....	.14	.26	.44	.72	1.1	1.5	2.1	2.9	3.8	4.9	6.2	½
12	.....	.....	.13	.24	.41	.65	1.0	1.4	2.0	2.7	3.4	4.5	5.8	12
13	.....	.....	.....	.21	.35	.55	.84	1.2	1.7	2.3	3.1	4.0	5.0	13
14	.....	.....	.....	.18	.31	.46	.70	1.0	1.4	2.0	2.7	3.5	4.4	14
15	.....	.....	.....	.....	.27	.41	.63	.91	1.2	1.7	2.4	3.1	3.9	15
16	.....	.....	.....	.....	.24	.37	.57	.78	1.1	1.5	2.1	2.7	3.5	16
17	.....	.....	.....	.....	.....	.....	.50	.70	1.0	1.4	1.9	2.4	3.1	17
18	.....	.....	.....	.....	.....	.....	.45	.66	.92	1.3	1.7	2.2	2.8	18
20	.....	.....	.....	.....	.....	.....	.....	.....	.76	1.0	1.4	1.8	2.3	20

Height in feet.	Side of square pine pillar, in inches.													Height in feet.
	4¼	4½	4¾	5	5¼	5½	5¾	6	6¼	6½	6¾	7	7¼	
	BREAKING LOAD.													
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
2	35.8	40.6	45.7	51.1	56.8	62.8	69.0	75.5	82.3	89.4	96.8	104.5	112.4	2
3	31.4	36.1	41.1	46.2	51.8	57.7	63.8	70.2	76.9	84.0	91.3	98.9	106.7	3
4	26 8	31.2	35.9	40.8	46.1	51.7	57.6	63.8	70.4	77.3	84.5	92.1	99.9	4
5	22.6	26.5	30.8	35.4	40.5	45.8	51.5	57.4	63.7	70.3	77.2	84.5	92.1	5
6	18.9	22.5	26.4	30.5	35.2	40.1	45.4	51.0	57.0	63.3	69.9	76.8	84.1	6
7	15.8	19.0	22.6	26.2	30.5	35.0	39.9	45.0	50.5	56.5	62.8	69.4	76.3	7
8	13.3	16.1	19.2	22.5	26.3	30.4	34.9	39.7	45.9	50.4	56.2	62.4	69.0	8
9	11.3	13.7	16.5	19.5	22.9	26.6	30.7	35.0	39.9	44.8	50.2	56.0	62.1	9
10	9.7	11.8	14.2	16.9	19.9	23.2	26.9	30.9	35.2	39.9	44.9	50.3	56.0	10
11	8.3	10.2	12.4	14.8	17.5	20.4	23.8	27.4	31.3	35.6	40.2	45.1	50.4	11
12	7.2	8.8	10.7	12.9	15.4	18.0	21.1	24.3	27.9	31.8	36.0	40.6	45.5	12
13	6.2	7.7	9.4	11.4	13.6	16.0	18.8	21.7	24.9	28.5	32.4	36.6	41.1	13
14	5.5	6 8	8.3	10.1	12.1	14.2	16.7	19.4	22.4	25.7	29.2	33.1	37.3	14
15	4.8	6.0	7.4	9.0	10.8	12.7	15.0	17.5	20.2	23.2	26.4	30.0	33.9	15
16	4.4	5.4	6.7	8.1	9·8	11.5	13.6	15.8	18.3	21.0	24.0	27.3	30.8	16
17	4.0	4.9	6.1	7.3	8.8	10.4	12.3	14.3	16.6	19.1	21.9	24.9	28.3	17
18	3.6	4.4	5.5	6.6	8.0	9.4	11.2	13.0	15.1	17.4	19.9	22.7	25.8	18
19	3.3	4.0	5.0	6 0	7.3	8.6	10 2	11.9	13.8	16.0	18.3	20.9	23.7	19
20	3.0	3.7	4.6	5.5	6.6	7.8	9.3	10.9	12.6	14.6	16.8	19.2	21.8	20
22	2.5	3.0	3.8	4.6	5.6	6.6	7.9	9.2	10.7	12.4	14.3	16.3	18 6	22
24	2.1	2 6	3.2	3.9	4.7	5.6	6.7	7.9	9.1	10.6	12.2	14.1	16.0	24
26	1.8	2.2	2.8	3.4	4.1	4.9	5.8	6.8	7.9	9.2	10.6	12.2	13.9	26
28	1.5	1.9	2.4	2.9	3.5	4.2	5.1	5 9	6.9	8.0	9.3	10.7	12.2	28
30	1 3	1.7	2.1	2.6	3.1	3.7	4.4	5.2	6.1	7.1	8.2	9.4	10.8	30
32	1.2	1.5	1.9	2.3	2.7	3.2	3.9	4.6	5.4	6.3	7.3	8.4	9.6	32
34	1.1	1.3	1.7	2.0	2.4	2.9	3.5	4.1	4.8	5.6	6.5	7.5	8.6	34
36	1.0	1.2	1.5	1.8	2.2	2.6	3.1	3.7	4.3	5 0	5.8	6.7	7.7	36
38	.9	1.1	1.3	1.6	2.0	2 4	2.8	3.3	3.9	4.5	5.3	6.1	7.0	38
40	.8	1.0	1.2	1.5	1 8	2.1	2.6	3.0	3.5	4.1	4.8	5.5	6.3	40

Continued on next page.

Table of breaking loads in tons of square pine pillars, with flat ends firmly fixed, and equally loaded. (Continued.)

Original.

Height in feet.	Side of square pine pillar, in inches.													Height in feet.
	7½	7¾	8	8¼	8½	8¾	9	9¼	9½	9¾	10	10¼	10½	
	BREAKING LOAD.													
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
2	120.6	129.1	137.9	147.0	156.3	165.9	175.8	186·0	196.4	207.2	218.2	229.5	241.0	2
4	107.9	116·3	125.0	133.9	143.0	152.6	162.4	172.5	182.8	193.4	204.2	215.3	226.6	4
6	91.7	99.7	108.0	116.5	125.3	134.5	143.5	153.0	162.8	173.5	184.4	195.6	207.0	6
8	75.9	83.2	90.8	98.7	106.8	115.5	124.4	133.6	143.0	153.0	163.2	173.7	184.4	8
10	62.0	68.5	75.3	82.4	89.7	97.7	105.8	114.3	123.0	132.3	141.8	151.6	161.6	10
12	50.7	56.5	62.5	68.7	75.1	82.2	89.6	97.2	105.0	113.5	122.2	131.2	140.5	12
14	41.8	46.7	51.9	57.3	62.9	69.2	75.7	82.4	89.4	97.1	105.0	113.2	121.6	14
16	34.7	38.9	43.4	48.1	53.0	58.5	64.3	70.3	76.5	83.3	90.4	97.7	105.3	16
18	29.1	32.7	36.6	40.7	45.0	49.8	54.9	60.1	65.6	71.7	78.0	84.6	91.4	18
20	24.6	27.7	31.1	34.7	38.5	42.7	47.2	51.8	56.7	62.0	67.6	73.5	79.6	20
23	19.6	22.1	24.8	27.7	30.9	34.4	38.0	41.9	46.0	50.5	55.2	60.2	65.4	23
26	15.8	17.8	20.1	22.5	25.2	28.1	31.1	34.4	37 9	41.6	45.6	49.8	54.3	26
29	13.1	14.8	16.7	18.7	20.9	23.4	25.9	28.6	31.6	34.9	38.2	41.9	45.6	29
32	10.9	12.3	13.9	15.7	17.6	19.7	21.8	24.2	26.7	29.5	32.3	35.5	38.7	32
35	9 3	10.6	11.9	13.4	15.0	16.8	18.7	20.7	22.8	25.2	27.7	30.5	33.3	35
38	8.0	9.1	10.2	11.5	12.9	14.6	16.3	18.0	19.7	21.8	23.9	26.3	28 8	38
41	6.9	7.9	8.9	10.0	11.2	12.6	14.1	15.6	17.2	19.1	21.0	23.1	25.2	41
44	6.0	6.9	7.8	8.8	9.8	11.0	12.3	13.6	15.0	16.7	18.4	20.2	22.1	44
50	4.7	5.4	6.1	6.9	7.7	8.8	10.0	11.3	12.6	13.7	14.9	16.2	17.5	50

Height in feet.	Side of square pine pillar, in inches.													Height in feet.
	10¾	11	11¼	11½	11¾	12		10¾	11	11¼	11½	11¾	12	
	BREAKING LOAD.							BREAKING LOAD.						
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.		Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
4	238.9	251.0	263.2	275.9	288.8	302.1		26.5	28.6	31.1	33.8	36.7	39.9	42
6	218.8	230.6	242.7	255.2	267.9	281.0		24.2	26.4	28.7	31.2	33.9	36.8	44
8	195.5	207.0	218.5	230.4	242.5	255.1		22.4	24.3	26.4	28.7	31.2	33.9	46
10	172.2	183.0	194.1	205.6	217.3	229.5		20.7	22.6	24.5	26.7	28.9	31.4	48
12	150.3	160.3	170.7	181.5	192.5	204.0		19.2	20.9	22.7	24.7	26.8	29.2	50
14	130.5	139.8	149 4	159.4	169.6	180.2		17.8	19.5	21.1	23.0	25.0	27.2	52
16	113.2	121.7	130 4	139.6	149.0	158.8		16.6	18.2	19.7	21.5	23.3	25.4	54
18	98.7	106.2	114.1	122.5	131.0	140.0		15.5	17.0	18.4	20.1	21.8	23.7	56
20	86.2	92.9	100.0	107.6	115.4	123.6		14.5	15.9	17.2	18.8	20.4	22.2	58
22	75.6	81.7	88.1	95.0	102.0	109.5		13.6	14.9	16.2	17.7	19.2	20.9	60
24	66.7	72.2	77.9	84.1	90.5	97.3		11.8	12.9	13.9	15.2	16.5	18.0	65
26	59.1	64.0	69.2	74.9	80 6	86.8		10.1	11.1	12.0	13.2	14.3	15.6	70
28	52.6	57.1	61.8	66.9	72.1	77.7		8.9	9.8	10.6	11.6	12.5	13.7	75
30	47.0	51.1	55.3	60.0	64.7	69.9		7 8	8.6	9.4	10.2	11.1	12.1	80
32	42.1	46.0	49.9	54.0	58.4	63.0		7.0	7.7	8.4	9.1	9.9	10.7	85
34	38.2	41.5	45.0	48.8	52.8	57.1		6.2	6.8	7.4	8.1	8.8	9.6	90
36	34.6	37.7	40.9	44.3	48.0	51.8		5.6	6.1	6.7	7.3	8.0	8.7	95
38	31.5	34.2	37.2	40.4	43.9	47.4		5.1	5.6	6.1	6.6	7.2	7.8	100
40	28.8	31.3	34.0	37.0	40.1	43.5		3.5	3.9	4.2	4.6	5.0	5.5	120

Continued on next page.

Remarks. Mr Kirkaldy found for Riga and Dantzic firs, 20 ft long, and 13 ins square, (or 18½ sides high,) 148 and 138 tons total; or .876 and .817 tons, (1963 and 1829 lbs,) per sq inch. Mr Smith's rule gives for common pine, 160 tons total; or .947 ton, or 2121 lbs, per sq inch. Hodgkinson would give for Riga about 297 tons total.

Each of Mr Kirkaldy's 20-ft pillars shortened about .6 of an inch total; or .03 inch per ft; or ⅛ of an inch in 4 ft 2 ins, under a mean of 1900 lbs per sq inch.

The writer has known 8 unbraced **pillars of hemlock,** tolerably seasoned, 12 ins square, and 42 ft high, to be gradually loaded each with 32 tons, or 71680 lbs total; (or .2222 ton, or 498 lbs per sq inch) without appreciable yielding. Assuming their strength and stiffness to be about as for Mr Smith's pine, (as in all our tables,) they should by him yield at 39.9 tons total. With these same *data*, but with Hodgkinson's **formula,** they should yield at 69.3 tons; and with Hodgkinson's own data, for seasoned red deal, at 91.6 tons. See **Remarks,** p 462.

Table of breaking loads in tons of square pillars of half-seasoned white or common yellow pine, with flat ends firmly fixed, and equally loaded. By C. Shaler Smith's formula. (Continued.)

As this table was partly made by interpolation, the last figure is not always precisely correct.

Original.

Height in feet.	Side of square pine pillar in inches.												Height in feet.
	13	14	15	16	17	18	19	20	21	22	23	24	
	BREAKING LOAD.												
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
4	358	418	482	552	625	703	786	872	964	1060	1161	1265	4
6	335	394	456	526	599	676	760	847	938	1033	1134	1236	6
8	308	367	429	500	572	649	732	818	910	1005	1106	1208	8
10	281	339	400	466	537	612	694	780	870	964	1064	1166	10
12	252	307	365	432	502	576	656	740	829	922	1022	1124	12
14	225	277	333	397	464	536	614	696	784	876	973	1074	14
16	201	250	303	363	428	497	573	652	739	829	925	1024	16
18	179	224	274	331	392	458	531	608	692	780	873	972	18
20	160	201	248	301	359	422	492	566	647	732	822	919	20
22	143	182	224	274	329	388	455	526	604	686	773	866	22
24	127	163	203	249	301	357	421	488	563	642	726	816	24
26	115	148	184	226	275	328	389	453	523	599	680	767	26
28	103	133	167	206	252	302	359	420	490	560	638	721	28
30	93	121	152	189	231	278	332	389	453	522	597	677	30
32	84	109	138	173	212	256	307	361	421	487	558	635	32
34	76	99	126	159	196	237	284	335	392	455	523	597	34
36	69	91	116	146	180	219	264	312	366	426	490	560	36
38	63	84	107	134	166	203	245	290	341	397	458	525	38
40	58	77	99	124	154	188	227	270	318	372	429	494	40
42	54	71	91	115	143	175	212	253	298	349	403	465	42
44	50	66	84	107	133	163	198	236	280	328	380	438	44
46	46	61	78	99	123	152	185	221	263	308	358	413	46
48	43	57	73	92	115	142	173	207	247	290	337	389	48
50	40	53	68	86	107	133	162	194	231	272	317	367	50
52	37	50	64	81	101	124	152	182	217	256	300	347	52
54	35	47	60	76	95	117	144	172	205	242	283	328	54
56	33	44	56	71	89	110	135	162	193	228	267	310	56
58	31	41	52	67	84	103	127	153	182	215	253	294	58
60	29	38	49	63	79	98	120	144	172	204	240	280	60
65	25	33	43	55	69	86	105	126	151	179	211	246	65
70	22	29	37	48	60	74	92	111	134	159	187	218	70
75	19	25	33	42	53	66	82	98	118	141	166	195	75
80	16	22	29	37	46	58	72	87	105	125	148	174	80
85	14	19	26	33	41	52	65	78	94	112	132	156	85
90	13	17	23	30	37	46	58	70	85	102	120	141	90
95	12	16	21	27	33	42	53	64	77	93	108	127	95
100	11	14	19	24	30	38	48	58	70	84	99	117	100
110	10	12	16	20	26	33	40	48	58	70	82	97	110
120	9	11	14	17	22	28	34	41	49	60	71	83	120
130	7	9	12	14	18	23	29	36	43	52	61	72	130
140	6	8	10	12	16	20	25	31	37	44	53	62	140
150	5	7	9	11	14	18	22	27	32	38	46	54	150
160	5	6	8	10	13	16	20	24	29	34	41	48	160
170	4	5	7	9	11	14	17	21	25	30	36	43	170
180	4	5	6	8	10	12	15	19	22	27	32	38	180
190	3	4	5	7	9	11	14	17	20	24	29	34	190
200	3	4	5	6	8	10	12	15	18	22	26	31	200

Table of breaking loads in tons, or in lbs, per sq inch of cross section of half seasoned square pine pillars, whose heights are measured by one of their sides.

Height in sides.	BR LOAD PER SQ IN.		Height in sides.	BR LOAD PER SQ IN.		Height in sides.	BR LD PER SQ IN.		Height in sides.	BR LD PER SQ IN.	
	Tons.	Lbs.		Tons.	Lbs.		Tons.	Lbs.		Tons.	Lbs.
1	2.2232	4980	26	.6027	1350	51	.1960	439	76	.0924	207
2	2.1969	4921	27	.5697	1276	52	.1888	423	77	.0902	202
3	2.1544	4826	28	.5398	1209	53	.1826	409	78	.0879	197
4	2.0978	4699	29	.5116	1146	54	.1763	395	79	.0862	193
5	2.0290	4545	30	.4853	1087	55	.1705	382	80	.0839	188
6	1.9513	4371	31	.4607	1032	56	.1647	369	81	.0821	184
7	1.8665	4181	32	.4379	981	57	.1598	358	82	.0799	179
8	1.7772	3981	33	.4165	933	58	.1545	346	83	.0781	175
9	1.6857	3776	34	.3969	889	59	.1496	335	84	.0763	171
10	1.5942	3571	35	.3781	847	60	.1451	325	85	.0746	167
11	1.5040	3369	36	.3599	809	61	.1406	315	86	.0728	163
12	1.4165	3173	37	.3447	772	62	.1362	305	87	.0714	160
13	1.3317	2983	38	.3295	738	63	.1321	296	88	.0696	156
14	1.2513	2803	39	.3152	706	64	.1286	288	89	.0683	153
15	1.1745	2631	40	.3018	676	65	.1250	280	90	.0670	150
16	1.1027	2470	41	.2889	647	66	.1210	271	91	.0656	147
17	1.0353	2319	42	.2772	621	67	.1179	264	92	.0638	143
18	.9723	2178	43	.2661	596	68	.1147	257	93	.0625	140
19	.9134	2046	44	.2554	572	69	.1112	249	94	.0616	138
20	.8585	1923	45	.2455	550	70	.1085	243	95	.0603	135
21	.8076	1809	46	.2357	528	71	.1054	236	96	.0589	132
22	.7603	1703	47	.2268	508	72	.1027	230	97	.0576	129
23	.7165	1605	48	.2183	489	73	.1000	224	98	.0567	127
24	.6755	1513	49	.2100	472	74	.0973	218	99	.0554	124
25	.6380	1429	50	.2031	455	75	.0951	213	100	.0545	122

Remark.—Gordon and Hodgkinson compared. The difference between them is greater in wooden pillars than in hollow cast iron ones. Moreover, in the latter, Gordon is sometimes greater, sometimes less, than Hodgkinson, as seen per lower table. Mr. Smith's assumed strength and stiffness of pine may safely be taken at about one-fourth less than Hodgkinson's for his red deal; and *with this assumption*, Hodgkinson's rule would make the strength of Smith's pine (**in all our tables for wooden pillars**) greater, to the extent shown by the multipliers **in the following table.** The truth is probably between the two. See **Remark,** p. 460.

Ht in sides.	Multr.	Ht in sides.	Multr.	Ht in sides.	Multr.	Ht in sides.	Multr.	Ht in sides.	Multr.
5	1.04	12.5	1.23	20	1.44	35	1.72	50	1.67
7.5	1.09	15	1.30	25	1.54	40	1.76	60	1.63
10	1.15	17.5	1.37	30	1.64	45	1.71	80	1.59

Gordon's and Hodgkinson's hollow cylindrical cast iron pillars compared.

The thickness is usually from $\frac{1}{16}$ to $\frac{1}{8}$ of the outer diameter; and for these limits, the column G, (Gordon), and H, (Hodgkinson), show the proportions of the breaking loads.

Thickness = $\frac{1}{16}$ of the outer diameter.

Ht in Diams.	G.	H.	Ht in Diams.	G.	H.	Ht in Diams.	G.	H.	Ht in Diams.	G.	H.	Ht in Diams.	G.	H.
5	1	1.25	10	1	1.11	20	1	.97	40	1	.90	70	1	.90
8	1	1.12	15	1	1.02	30	1	.94	50	1	.88	100	1	.97

Thickness = $\frac{1}{8}$ of the outer diameter.

5	1	1.23	10	1	1.08	20	1	.92	40	1	.81	70	1	.82
8	1	1.10	15	1	.98	30	1	.88	50	1	.80	100	1	.88

Art. 4. Ultimate average tensile or cohesive strength of Timber,

Being the least weights in pounds which, if attached to the lower end of a vert rod one inch square, firmly upheld at its upper end, would break it by tearing it apart. For large timbers we recommend to reduce these constants ¼ to ⅓ part.

The strengths in all these tables may readily be one-third part more or less than our averages.	Lbs per sq. inch.		Lbs per sq. inch.
Alder	14000	Mahogany, Honduras	8000
Ash, English	16000	" Spanish	16000
" American (author) abt	16500	Mangrove, white, Bermuda	10000
Birch	15000	Mulberry	12000
" Amer'n black	7000	Oak, Amer'n white	
Bay-tree	12000	" " basket	
Beech, English	11500	" " red	
Bamboo	6000	" Dantzic, seasoned	10000
Box	20000	" Riga	
Cedar, Bermuda	7600	" English	
" Guadaloupe	9500	" live, Amer'n	
Chestnut	13000	Pear	10000
" horse	10000	Pine, Amer'n, white, red, and Pitch, Memel, Riga	10000
Cyprus	6000	Plane	11000
Elder	10000	Plum	11000
Elm	6000	Poplar	7000
" Canada	13000	Quince	7000
Fir, or Spruce	10000	Spruce, or Fir	10000
Hawthorn	10000	Sycamore	12000
Hazel	18000	Teak	15000
Holly	16000	Walnut	8000
Hornbeam	20000	Yew	8000
Hickory, Amer'n	11000		
Lignum Vitæ, Amer'n	11000	**Across the grain,** Oak	2300
Lancewood	23000	" " " Poplar	1800
Larch, Scotch	7000	" " " Larch, 900 to	1700
Locust	18000	" " " Fir, & Pines	550
Maple	10000		

THESE ARE AVERAGES. The strengths vary much with the age of the tree; the locality of its growth; whether the piece is from the center, or from the outer portions of the tree; the degree of seasoning; straightness of grain; knots, &c, &c. Also, inasmuch as the constants are deduced from experiments with good specimens of small size, whereas large beams are almost invariably more or less defective from knots, crookedness of fibre, &c, it is advisable in practice to reduce these constants as recommended above.

Art. 5. Average ultimate tensile or cohesive strength of Metals, per square inch.*

The ultimate tensile or pulling load per square inch of any material is frequently called its *constant*, *coefficient*, or *modulus* of tension, or of tensile strength.	Pounds per sq. inch.	Tons per sq. in.
Antimony, cast	1000	.45
Bismuth, east	3200	1.4
Brass, east 8 to 13 tons, say 18000 to 29000 lbs	23500	10.5
" wire, unannealed or hard, 80000. Annealed	49000	22
Bronze, phosphor wire, hard, 150000. Annealed	63000	28.1
Copper, east 18000 to 30000	24000	10.7
" sheet	30000	13.4
" bolts, 28000 to 38000	33000	14.7
" wire (annealed 16 tons); unannealed	60000	26.8
Gold, cast	20000	8.9
" wire, 25000 to 30000	27500	12.3
Gun metal of copper and tin, 23000 to 55000	39000	17.4
" " cast iron, U. S. ordnance, 36000 to 40000	38000	17
Iron, east, English.....13400 to 22400	17900	8
" " ordinary pig..13000 to 16000	14500	6.47
Amerlean east iron averages one-fourth more than the above.		
Average east iron, when sound, stretches about .00018; or 1 part in 5555 of its length; or $\frac{1}{8}$ inch in 57.9 ft. for every ton of tensile strain per sq inch, up to its elastic limit, which is at about $\frac{1}{2}$ its break-strain. The extent of stretching, however, varies much with the quality of the iron; as in wrought-iron.		
Cast, malleable, annealed 18 to 25 tons	48160	21.5
Iron, wrought, rolled bars, 40000 to 75000, the last exceptional	57500	25.7
" " " " ordinary average.	44800	20
" " " " good "	50400	22.5
" " " " superior	60000	26.8
" " " " best American, (exceptional)	76100	34
" " " " Low Moor, English, average	60000	26.8
" " " plates for boilers, &c, 40000 to 60000	50000	22.3
" " English rivet iron.....55000 to 60000	57500	25.7
" " wire, annealed.....30000 to 60000	45000	20.1
" " " unannealed, or hard...50000 to 100000	75000	33.5
" " " ropes, per sq inch of section of *rope*	38000	16.8
" " large forgings, 30000 to 40000	35000	15.6
In important practice, good bar iron should not be trusted permanently with more than about 5 tons per sq inch; which will stretch it about $\frac{1}{8}$ inch in from 20 to 25 ft.		
Good bar iron stretches about 1 part in 12000 of its length; or about 1 inch in 1000 ft; or $\frac{1}{8}$ inch in 125 ft, for every ton of tensile strain per sq inch of section, up to its elastic limit. This limit usually ranges between 8 and 13 tons per sq inch, or about half the breaking strain, according to quality. The ultimate stretching of rolled bars is from 5 to 30 per ct of the original length; usually 15 to 20 per cent. Plates and angle iron 3 to 17 per cent. Heating, even up to 500° Fah, does not weaken bar iron or steel. For stretch by heat see p 212.		
Lead, cast, 1700 to 2400.....by author	2050	.92
" wire, 1200 to 1600. Pipe 1600 to 1700..... " "	1650	.74
Platinum wire, annealed, 32000. Unannealed	56000	25
Steel, plates, range, 60000 to 103000	81500	36.4
" " of Hussey, Wells & Co, Pittsburg, Pa, 91500 to 97400	94450	42.2
" " Bessemer	98300	44
" Bessemer tool	112000	50
" wire, annealed 30 to 50 tons. Unan, 50 to 90 tons	156800	70

***Large bars of metal** bear less per sq inch than small ones. In cast iron ones 1, 2 and 3 ins sq, the strengths per sq inch were about as 1, .85 and .66; and wrought iron probably averages about the same. See top of next page.

Iron bars re-rolled cold have tensile strength increased 25 to 50 per ct, with no increase of density. They are said to lose this strength if reheated.

Average ultimate tensile, or cohesive strength of Metals per square inch. (CONTINUED.)

	Pounds per sq. inch.	Tons. per sq. in.
Capt. James B. Eads found that forged steel bolts for the St. Louis bridge, 5¾ ins diam, and 22 to 36 ft. long, broke *short* with only 30000 lbs per sq inch; while bolts of but ¾ inch section, *cut from the large ones*, in no instance broke with 100000 lbs per sq inch, but stretched considerably.		
Steel, cast, Bessemer ingots, average	63000	28 1
" " best American Bessemer ingots	86600	38.6
" " " " rolled and hammered, 120000 to 130000	125000	55.8
" " homogeneous, Cammell & Co, England, No 1	58240	26
" " " " " No 2	71680	32
" " " " " No 3	76160	34
" puddled bars, rolled and hammered, 65000 to 135000	100000	44.6
Steel. Experiments by Lieut. W. S. Shock, U. S. N., at Washington, on steel from the Black Diamond Steel-Works, Pittsburg, Pa. All the pieces were cut from the same bar, three pieces for each exp. They were turned down to a diam of .62 of an inch at the intended point of fracture, by a groove, in shape of a circular segment, with a chord of about 1 inch:		
" the bar in its original condition, 109500 to 131900	120700	53.9
" heated to light cherry-red, then plunged into oil of 82° Fah, 201300 to 227500	214400	95.7
" heated to light cherry-red, then plunged into water of 79° Fah. Then tempered on a heated plate, 152500 to 176100	164300	73.3
heated to light cherry-red, then plunged into water of 79° Fah, 132700 to 150500	141600	63.2
Tempering in oil usually increases the strength from 40 to 80 per cent.		
" chrome, made at Brooklyn, N. Y., and tested at West Point Foundry, N.Y., (specific gr 7.816 to 7.956,) 163000 to 199000. Average of 12 specimens	180000	80
" made from very pure Swedish iron, but containing different proportions of carbon. The bars were 21½ ins long, with 14 ins of this length turned down to a uniform diam of 1 inch. The breaking wts, however, in the table, are per sq inch:		
Mark No. 2, carbon .33 per ct, stretched 1.37 ins	68100	30.4
" No. 4 " .43 " " 1 37 "	76160	34
" No 5 " .48 " " 1.25 "	84000	37.5
" No. 6 " .53 " " 1.12 "	95200	42.5
" No. 7 " .58 " " 0.81 "	92960	41.5
" No. 8 " .63 " " 1.00 "	100800	45
" No. 10 " .74 " " 0.69 "	101920	45.5
" No. 12 " .84 " " 1.12 "	123200	55
" No. 15 " 1.00 " " 1.00 "	134400	60
" No. 20 " 1.25 " " 0 62 "	154560	69

With more than about 1.5 per ct of carbon the tensile strength of steel diminishes. A bar of the above No. 15, which broke at 60 tons per sq inch, when turned down for 14 ins of its length; broke with 79½ tons per sq inch when turned down at one point only. This is owing to the fact that the last could not stretch as much as the first, and therefore its diam could not be diminished as much before breaking. All its fibres pulled more unitedly. It will be observed that the steel of greatest strength stretched the least before breaking. This stronger steel would break under a *suddenly* applied force, or impulse, more easily than a weaker one would; because the weaker one, by its stretching, gradually breaks the force of the impulse, on the same principle as a spring. Hence the steel, iron, &c, which is strongest against a gradually applied force or strain, may be unfit for uses where the strain comes upon it suddenly. **The average ultimate tensile strength of steel** is about twice that of wrought iron. **Its deflection as a beam** within the elastic limits is about ⅘ that of wrought, or ⅔ that of cast iron. **Its average stretch**

is about .1 inch in 111 ft for every ton per sq inch of load, up to its elastic limit, which generally ranges at between ½ and ⅔ of its breaking strength; the latter being for the harder, stronger, and less stretchy kinds. A uniform bar of rolled steel, gradually loaded, will stretch from $\frac{1}{400}$ to $\frac{1}{5}$ of its length before breaking; or from $\frac{3}{100}$ of an inch to 2.4 ins per foot, according to quality. The mean of these is nearly $\frac{1}{100}$ of the length, or $1\frac{2}{100}$ inch to a foot. When steel, especially if hard, has to be heated to softness in order to give it a required shape, it is thereby weakened.

Average ultimate tensile or cohesive strength of Metals per square inch. (CONTINUED.)

	Pounds per sq. inch.	Tons per sq. in.
Silver, cast	41000	18.3
Tin, English block	4600	2
" wire	7000	3.1
Zinc, cast...3000 to 3700; (the last by author)	3350	1.5

Art. 6. Average ultimate tensile or cohesive strength of various materials.

The strengths in all these tables may readily be one-third part more or less than our averages.	Pounds per sq. inch.	Tons per sq. ft.		Pounds per sq. inch.	Tons per sq. ft.
Brick, 40 to 400	220	14.1	Marble, strong, wh. Italy.*	1034	66.5
Caen stone, 100 to 200	150	9.7	" Champlain, variegated*	1666	107.1
Cement, hydraulic, Portland, pure, 7 days in water	300	19.3	" Glenn's F'lls, N.Y. blk,* 750 to 1034	892	57.4
" 6 months old	450	28 9	" Montg'y co, Pa, gray*	1175	75.6
" 1 year old	550	35.4	" " white*	734	47.2
Common hyd cements average 1-6 as much. The last, neat, adhere to brick and stone with from 15 to 50 lbs when only 1 month old	32	2	" Lee, Mass, white.*	875	56.3
At end of 1 year, 3 times as much	96	6	" Manchester, Vt,* 550 to 800	675	43.4
See "Cement," p 675, &c			" Tennessee, variegated*	1034	66.5
Glass, 2500 to 9000 (p 432)	5750	385.7	Oolites, 100 to 200	150	9.7
Glue holds wood together with from 300 to 800	550	35	Plaster of Paris, well set.	70	4.5
Horn, ox	9000	579	Rope, Manilla, best	12000	771
Ivory	16000	1029	" hemp, best	15000	965
Leather belts, 1500 to 5000. Good	3000	193	Sandstone, Ohio*	105	6.75
Mortar, common, 6 mos old, 10 to 20	15	.96	" Pictou, N. S.*	434	27.9
			" Conn, red.*	590	37.9
			Slate, Lehigh*	2475	159.1
			" Peach bot'm,* 3025 to 4600	3812	245.1
			Stone, Ransome's artif.	300	19.3
			Whalebone	7600	489

To find the diam in ins of a round rod to bear safely a given pull in lbs.

$$\text{Diam in ins} = \sqrt{\frac{\text{given pull} \times \text{coef of safety}}{\text{ult tensile strength of material in lbs per sq inch} \times .7854.}}$$

Iron is weakened by extreme cold.

The belief (originating with Styff of Sweden,) is gaining ground that iron and steel are not rendered *more brittle by intense cold*, but that the great number of

* **By the author's trials** with one of Riehlé's testing machines. Sections broken 1½ sq inches.

breakages of rails, wheels, axles, &c, in winter, is owing to the more severe blows incident to the frozen and unyielding nature of the earth at that period of the year. But Sandberg's experiments show conclusively that although these metals may perhaps bear as much *steady* force, *gradually* applied, in winter as in summer, yet their resistance to *impulse*, or *sudden force*, is not more than $\frac{1}{2}$ or $\frac{1}{3}$ as great in severe cold; which renders them less flexible and less stretchy. It is probable that this fact does not receive as much attention as it should, in proportioning iron bridges, &c.

Some experiments with good wrought iron showed that even at 23° Fah, or only 9° colder than freezing point, there was a loss of strength of from $2\frac{1}{2}$ to 4 per cent.

RIVETS AND RIVETING.

R, Figs 3, p 469, shows the usual shapes of rivets as sold.*
The **weights** in the **following table** of course include the head; but the **lengths,** as usual, are taken "under the head;" or are those of the shanks only. In practice, discrepancies of 5 or 6 per ct in wt may be expected.
From Carnegie Bros. & Co's "Useful Information," by C. L. Strobel, C E.

Length of Shank. Ins.	Diameters of Rivets in inches.							
	3/8	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4
	Weight of 100 Rivets, in pounds.							
1/2	3.0	8.5	.....	.....	.....	.....	.....	.....
3/4	3.8	9.9	17.3	.....	.....	.....	.....	.....
1	4.6	11.2	19.4	25.6	38.9	.....	.....	.....
1/4	5.4	12.6	21.5	28.7	43.1	65.3	91.5	123
1/2	6.2	13.9	23.7	31.8	47.3	70.7	98.4	133
3/4	6.9	15.3	25.8	34.9	51.4	76.2	105	142
2	7.7	16.6	27.9	37.9	55.6	81.6	112	150
1/4	8.5	18.0	30.0	41.0	59.8	87.1	119	159
1/2	9.2	19.4	32.2	44.1	64.0	92.5	126	167
3/4	10.0	20.7	34.3	47.1	68.1	98.0	133	176
3	10.8	22.1	36.4	50.2	72.3	103	140	184
1/4	11.5	23.5	38.6	53.3	76.5	109	147	193
1/2	12.3	24.8	40.7	56.4	80.7	114	154	201
3/4	13.1	26.2	42.8	59.4	84.8	120	161	210
4	13.8	27.5	45.0	62.5	89.0	125	167	218
1/4	14.6	28.9	47.1	65.6	93.2	131	174	227
1/2	15.4	30.3	49.2	68.6	97.4	136	181	236
3/4	16.2	31.6	51.4	71.7	102	142	188	244
5	16.9	33.0	53.5	74.8	106	147	195	253
1/4	17.7	34.4	55.6	77.8	110	153	202	261
1/2	18.4	35.7	57.7	80.9	114	158	209	270
3/4	19.2	37.1	59.9	84.0	118	163	216	278
6	20.0	38.5	62.0	87.0	122	169	223	287
1/2	21.5	41.2	66.3	93.2	131	180	236	304
7	23.0	43.9	70.5	99.3	139	191	250	321
1/2	24.6	46.6	74.8	106	147	202	264	338
8	26.1	49.4	79.0	112	156	213	278	355
9	29.2	54.8	87.6	124	173	234	306	389
10	32.2	60.3	96.1	136	189	256	333	423
11	35.3	65.7	105	148	206	278	361	457
12	38.4	71.2	113	161	223	300	388	491

The diam of rivets for bridge work is from 1/2 to 1 inch; usually 5/8 to 3/4; and for plates more than .5 inch thick, it is about 1.5 times the thickness; and for thinner ones about twice; but these proportions are not closely adhered to. **The common form of rivets** as sold is shown at R, Figs 3, a **head** and the **shank** in one piece; and S shows the same when after being heated white hot it is inserted into its hole, and a second head (conical) formed on it by rapid hand-riveting as it cools. **When longer than about 6 ins** they are cooled near the middle before being inserted, lest their contraction in cooling should split off their heads. The hemispherical heads often seen, called **snap heads,** are formed by a machine. **The two heads alone require** about as much iron as 3 diams length of shank. **Length of a head** = about 1 diam of shank; and its **width** about 2 diams of shank.

Riveting of Steam and Water Tight Joints.

Joints for boilers and water-tight cisterns are usually proportioned about as per the following table by Fairbairn; and are made as shown either by Fig 1, or Fig 2. Fig 1 is called a **single-riveted,** and Fig 2 a **double-riveted lap-joint.** The dist *a a*, or *c c*, is the lap.
Mr Fairbairn considers the strength of the single-riveted lap-joint to be about .56; and that of the double-riveted, about .7 that of **one** of the full unholed

* **Price in Philada,** 1886, about 3 1/2 cts per lb.

plates, when both joints are proportioned as in his following table. But some later experimenters consider about .5 and .6 as nearer the correct average. Experiments on the subject are quite conflicting; and it is plain that no one set of proportions can precisely suit all the different qualities of plate and rivet iron. With fair qualities of both, there is every reason to rely upon .5 and .6 (or about one-seventh part less than Fairbairn's assumption) as safe for practice. These proportions include **friction** (Art 4), without which they would be *about* .4 and .5.

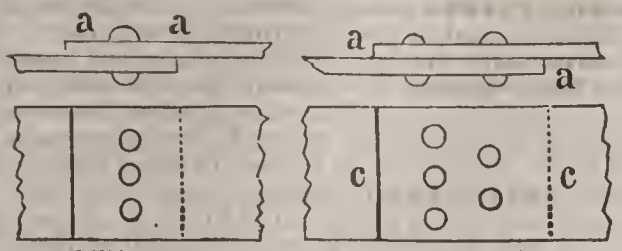
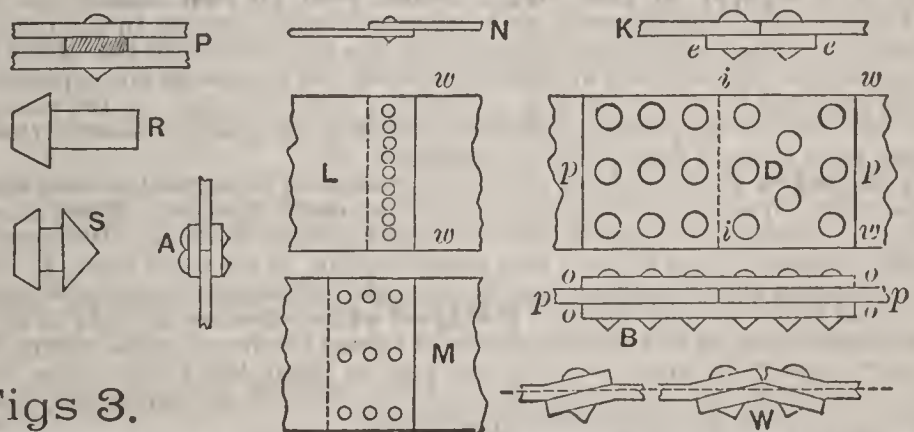


Fig 1. Fig 2.

Fairbairn's table for proportioning the riveting for steam and water-tight lap-joints.

Thickness of each plate.	Diameter of rivets.	Length of shank before driving.	From center to center of rivets.	Lap in single riveting.	Lap in double riveting.
Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
3-16	3/8	7/8	1 1/4	1 1/4	2 1/16
1/4	1/2	1 1/8	1 1/2	1 1/2	2 1/2
5-16	5/8	1 3/8	1 5/8	1 7/8	3 1/16
3/8	3/4	1 5/8	1 3/4	2	3 3/8
1/2	13-16	2 1/4	2	2 1/4	3 3/4
5/8	15-16	2 3/4	2 1/2	2 3/4	4 5/8
3/4	1 1/8	3 1/4	3	3 1/4	5 1/2

Riveting of iron girders, bridges, &c.



Figs 3.

Art. 1. The subject of riveting is abstruse, and involved in much uncertainty; and experimental results are very discrepant. We here propose merely to confine ourselves to what is considered the best joint; and for safety we shall omit friction; see Art 4. In girder and bridge work the lap-joints above described are seldom used. Instead of them, the plates *p*, Figs 3, to be joined, are butted up square against each other, thus forming a **butt-joint,** *i i*, Fig D; and are united by either a single **covering-plate, cover, wrapper, fish-plate,** or **welt** *e e*, Fig K; or the best of all by two of them, as at A, or *o o*, *o o*, Fig B. In what follows, the term *plate* never includes the *covers.* The single cover, like the lap-joint, allows both plates and cover to bend under a strong pull, somewhat as at W, thus weakening them materially; whereas the double cover *o o*, *o o*, Fig B, keeps the pull directly along the axis of the plates, thus avoiding this bending tendency. It also brings the rivets into double shear, thus doubling their strength. When there is but one cover, it should be at least as thick as a plate; and when there are two, experience shows that each had better be about *two-thirds* as thick as a plate, although theory requires each to be but *half* as thick as a plate.

The length w w of covers across the joint is equal to that of the joint.

Butts require twice as many rivets as laps, because in the lap each rivet passes through both the joined plates; and in the butt through only one.

The rivets and plate on one side only (right or left) of the joint-line *i i* of any properly proportioned **butt-joint** D, represent the full strength of the joint, inasmuch as those on one side pull in one direction, against those on the other side, which pull in the opposite direction. Therefore in designing such joints we need keep in mind only those on one side, as is done in what follows. Thus a single, double, or triple-riveted **butt-joint** D implies one, two, or three rows of rivets **on each side** of the joint-line *i i*, and parallel to it. In a properly proportioned *lap* the strength is as **all** the rivets, because one-half of them do not pull against the other half, but one end of **every** rivet pulls in one direction, and its other end in the opposite direction.

The net iron, net plate, or net joint, is that which is left **between** the rivet holes, and **outside** of the two outer ones, all on a straight line drawn through the *centers* of the holes of *one* row. Its width and area are called the net ones of the joint. That between *other* rows does not increase the strength.

In Figs 3, N, and K, the rivets are plainly exposed only to **single shear**; that is, the opposing pulls of the two plates tend to shear each rivet across only one circular section; whereas in Fig B, with two covers, or in Fig A, each rivet is exposed to **double shear,** one just above and one just below the joined plates.

Art. 2. Bridge-joints are not required to be steam or water-tight like those of boilers or cisterns; and, therefore, by increasing the breadth of the overlap, or the length of the covers, the rivets may be placed in several rows behind each other, as the 3 rows of 3 rivets each in M and D, instead of only one row of 9 rivets, as in L. By this means, without losing any of the strength of the 9 rivets, or of the **net** iron, we may narrow the width of the plate to an extent equal to the combined diams (6 in this case) of the holes thus dispensed with in the one row. Moreover, by using more than one row we lessen the weakening effect shown at W. This mode of placing the rivets directly behind each other in several rows, as at M, and at the left-hand half of Fig D, constitutes Mr Fairbairn's **chain riveting**; but the joint will be somewhat stronger if the rivets are placed in **zigzaging** order, as in the right-hand half of Fig D.

The dist apart of the rows from cen to cen should not be *less* than 2 diams. It is questionable to what extent this increase in the number of rows may be carried without an appreciable loss of strength in the rivets consequent upon the impossibility of quite equalizing the strains on the separate rows. But it is probable that if we do not exceed 2 or 3 rows in laps, or the same number *on each side* of the joint-line in butts, we may in practice assume that each row, and each rivet, is nearly equally strained.

Rivet-holes are usually of about one-sixteenth inch greater diam than the original rivet, so as to allow the hot rivet to be easily inserted. The subsequent hammering swells the diam of the rivet until it fills the hole. We may either take this increased diam of rivet into consideration, as we have done, in calculating its shearing and crippling strength, as explained farther on, or with reference to increased safety we may omit it. **Drilled rivet-holes** are said to be better than punched ones, as the drilling does not injure the iron around them; but on the other hand their sharper edges are said to shear the rivets more readily. Hence, such edges are sometimes reamed off. Both these points are, however, disputed; and both modes are in common use.

The dist from the edge of a hole to the end of a plate or cover should not be *less* than about 1.2 diams, to prevent the rivets from tearing out the end of the plate; nor nearer the side edge of a plate than half the clear dist between two holes as given by the Rule in Art 5. The first is rather more than Fairbairn directs.

Rivet holes weaken the net iron left between them, not only by the loss of the part cut out, but either by disturbing the iron around them, or perhaps by changing the shape of the net line of fracture, which may not then resist tension as well as while it was a continuous straight line. Some deny both cause and effect entirely, each party basing its opinion on experiments. But the mass of evidence seems to the writer to show that the net iron loses on an average about one-seventh of the strength due to the net width. With a view to safety, which we consider to be of paramount importance, we shall in what follows assume (until the question is definitely settled) that there **is** such a loss of strength in the net iron.

Riveted joints for resisting compression should depend, not as might be supposed upon their butting ends, but upon either the shearing or the crippling strength of the rivets; for contraction or bad work may throw the

pressure on the rivets. **Machine riveting** is somewhat stronger than that done (as is assumed in our examples) by hand. **The thickness of plates** used in girders, tubular bridges, &c, is usually .25 to .5 inch; with thicker ones up to 1 inch sparingly in large ones. **A packing piece,** as the shaded piece in P, is one inserted between two plates to prevent their being bent or drawn together by the rivets.

Art. 3. A riveted joint may yield in three ways after being properly proportioned, namely, by the shearing of its rivets; or by the pulling apart of the net plate between the rivet holes; or by the **crippling** (a kind of compression, mashing, or crumpling) of the plates by the rivets when the two are too forcibly pulled against each other. It also compresses the rivets themselves transversely, **at a less strain than the shearing one;** and this partial yielding of both plates and rivets allows the joint to **stretch,** and may thus produce injurious unlooked-for strains in other parts of a structure, considerably before there is any danger of actual fracture. Or in steam and water joints it may cause leaks, without farther inconvenience, or danger. For a long time this crippling had entirely escaped notice, and it was supposed that the only important point in designing a riveted joint was that the tensile strength of the net plate, and the shearing strength of the rivets should be equal to each other.

The crippling strength of a joint is as the number of rivets, in a lap, or the number on one side of the joint-line in a butt × diam × thickness of joined plate. This product gives the crippled area of the joint. We shall here call the diam × thickness of plate, the **crippling area** of a rivet. If there are 2 or more plates (not covers) on top of each other at one joint, their united thickness is used for finding the crippling area. **The ultimate crippling unit,** by which the above product is to be multiplied for the actual ultimate crippling strength of the joint, may be safely taken at about 60000 lbs, or 26.8 tons, per sq inch.

The diam of a rivet in ins to resist safely a given single-shearing force is found thus: Mult the shearing force by the coef of safety, that is by the number, 3, 4, or 6, &c, denoting the required degree of safety. Call the product *g*. Mult the ultimate shearing strength per sq inch of the rivet-iron, by the decimal .7854. Call the product *b*. Divide *g* by *b*. Take the sq rt of the quotient. The shearing force and the shearing strength must *both* be in either lbs or tons.

Or by a formula,

$$\text{Diam in ins} = \sqrt{\frac{\text{Shearing force} \times \text{coef of safety}}{\text{Ult shearing strength per sq inch} \times .7854}}$$

If the rivet is to be double-sheared, first mult only half the shearing force by the coef of safety. Then proceed as before.

Or, near enough for practice, mult the diam in single shear by the decimal .7.

The ultimate shearing unit for average rivet-iron may be taken at about 45000 lbs, or 20.1 tons per sq inch of circular sheared section.

Table of ultimate single shearing strength of rivets.

(market sizes), in single shear; at 45000 lbs or 20.1 tons per sq inch.

This table is not to be used when as in our "Example," Art 5, the **crippling** strength of the rivet governs the strength of the joint.

If the rivet is in double shear it will have twice the strength in the table.

For the diam in double shear to equal the strength in the table, mult the diam in the table by the decimal .7; near enough for practice; strictly, .707.

Diam. Ins.	Diam. Ins.	lbs.	Tons.	Diam. Ins.	Diam. Ins.	lbs.	Tons.	Diam. Ins.	Diam. Ins.	lbs.	Tons.
1/8	.125	552	.246		.562	11183	4.99	1	1.000	35343	15.8
	.187	1242	.554	5/8	.625	13806	6.16		1.062	39899	17.8
1/4	.250	2209	.986		.687	16705	7.46	1 1/8	1.125	44731	20.0
	.312	3452	1.54	3/4	.750	19880	8.88		1.187	49838	22.2
3/8	.375	4970	2.22		.812	23332	10.4	1 1/4	1.250	55224	24.6
	.437	6765	3.02	7/8	.875	27060	12.1		1.312	60885	27.2
1/2	.500	8836	3.94		.937	31064	13.9	1 3/8	1.375	66820	29.8

The tensile strength of a properly proportioned joint is equally as either the sectional area of the **net** plate (not covers) across the centers of only **one** row of rivets; or as the shearing or the crippling (as the case may be) areas of **all** the rivets in a lap, or of all the rivets **on one side** of the joint-line in a butt. The tensile strength of fair quality of plate iron, before the rivet holes are made, averages about 45000 lbs, or 20.1 tons per sq inch; but we shall for safety assume, as stated in Art 2, that the making of the holes reduces the strength of the **net** iron that is left about one-seventh part, or to 38500 lbs, or 17.2 tons per sq inch.

Rem. Even this is considerably too great for laps, or for butts with one cover, owing to the weakening of the iron in such by the bending shown at W, Figs 3. But we are not speaking of such.

Art. 4. The friction between the plates in a lap, or between the plates and the covers in a butt, produced by their being pressed tightly together by the contraction of the rivets in cooling, adds much to the strength of a joint while new, perhaps as much as 1.5 to 3 tons per sq inch of circ section of all the rivets in a lap, or of all on one side of a single-cover butt; or 3 to 6 tons of all on one side of a double-cover butt. In quiet structures, this friction might continue to exist, either wholly or in part, for an indefinite period; but in bridges, &c, subject to incessant and violent jarring and tremor, it is probably soon diminished, or entirely dissipated. Hence good authorities recommend not to rely on it, and it is, therefore, omitted in what follows.

Art. 5. We now give rules for finding the number of rivets required for **a double cover butt-joint** (the only kind of which we shall treat), and their clear or net distance apart. This dist + one diam is the **pitch** of the rivets, or their dist from center to center. The principle of the rule will be explained further on, at Art 7, p 474.

First, select a diam of rivet either equal to or greater than .85 times the thickness of the plate. In practice they are generally 1.5 times for plates ½ inch or more thick; and 2 for thinner than ½ in.

Second, mult the greatest total pull in pounds that can come upon the entire joint by the coef (3, 4, or 6, &c) of safety, and call the product p.

Third, multiply the crippling area of the rivet (that is, its diam × the thickness of plate) by 60000. The prod is the ult crippling strength of a rivet. Call it m.

Fourth, divide p by m. The quotient will be the number of rivets to sustain the given pull with the reqd degree of safety.

Then, **the clear distance apart** will be

$$\frac{\text{Number of rows} \times \text{Diam} \times 60000}{38500}$$

Fifth. The clear dist from either end hole of a row to the side edge of the plate, should be not less than half the clear dist between two rivets in a row.

Example. A double-cover butt-joint in .5 inch thick plate is to bear an actual pull of 33750 lbs, with a safety of 4; or not to break with less than 33750 × 4 = 135000 lbs. How many rivets must it have; and how far apart must they be?

First, Here .85 times the thickness of the plate is .5 × .85 = .425 inch; therefore, our rivets must not be less than .425 inch in diam; but we will take .75 inch diam.

Second, The greatest pull × coef of safety = 33750 × 4 = 135000 lbs = p.

Third, The crippling area of a rivet × 60000 = .75 × .5 × 60000 = 22500 = m.

Fourth, $\frac{p}{m} = \frac{135000}{22500} = 6$ rivets required on each side of the joint-line.

And the **clear space** or net width between them will be, **if the 6 rivets are in one row:**

$$\frac{\text{Diam} \times 60000}{38500} = \frac{45000}{38500} = 1.1688 \text{ ins.}$$

And the **pitch** = net space + diam = 1.1688 + .75 = 1.9188 ins, $= \frac{1.9188}{.75}$ = 2.56 diams.

In practice, to avoid troublesome decimals, we might make the net space 1.2 ins; and the pitch 1.95; but to show farther on the working of the rule, we adhere to the more exact ones.

Fifth, The clear dist from each end hole to the side edge of the plate is half of 1.1688 = .5844 ins.

The entire width of net iron is equal to one clear space × number of rivets = 1.1688 × 6 = 7.0128 ins; and the entire width of plate is equal to one pitch × number of rivets, = 1.9188 × 6 = 11.5128 ins.

The area of cross section of unholed plate is $11.5128 \times .5 = 5.7564$ sq ins; its tensile strength **before the holes are made** is $5.7564 \times 45000 = 259038$ lbs. The strength of our joint, omitting friction, is therefore $\frac{135000}{259038} = .52$ of that of the original unholed plate.

If the 6 rivets are in 2 rows of 3 rivets each, the **clear dist between two rivets** in one row will be twice as great as before, or twice 1.1688 = 2.3376 ins. **Pitch** = 2.3376 + .75 = 3.0876 ins = 3.0876 ÷ .75 = 4.12 diams. **Clear dist from** end hole to side edge of plate = half of 2.3376 = 1.1688. **Entire width of net iron** = $2.3376 \times 3 = 7.0128$ ins. **Entire width of plate** = $3.0876 \times 3 = 9.2628$ ins. **Area of cross section of unholed plate** = $9.2628 \times .5 = 4.6314$ sq ins. **Ultimate tensile strength, unholed** = $4.6314 \times 45000 = 208413$ lbs. **Ult strength of riveted joint,** omitting friction = $\frac{135000}{208413} = .65$ of that of the unholed plate.

Thus we see that the arrangement with two rows gives the same strength as one row, with a less total width and area of plate. It of course requires *longer covers.*

If the 6 rivets are in 3 rows of 2 rivets each, the **area of cross section of the unholed plate** is 4.2565 sq ins. Its **tensile strength,** 191542 lbs. Strength of riveted joint = $\frac{135000}{191542} = .7$ of that of the unholed plate.

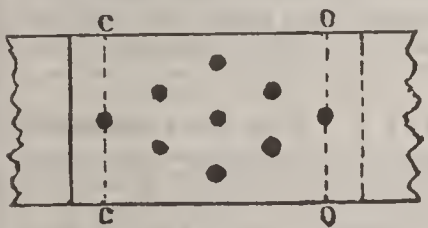
The entire width of net iron (.70128 ins); its area ($.70128 \times .5 = .35064$ sq ins); and its ultimate tensile strength ($.35064 \times 38500 = 135000$ lbs), are the same in each case. The last is the required breaking strength of the joint, as in the beginning of our example; and is equal to the combined crippling strength of the six rivets.

Art. 6. The distance apart of the rows, from center to center of rivets, should not be less than two diameters of a rivet-hole.

Rem. 1. With our constants for tension, shearing, and compression, **the rivets will not yield first by shearing** in a double-cover butt (and of course in double shear), except when the diam is either equal to or **less** than .85 of the thickness of the plate, which will rarely happen. At .85 the crippling and shearing strength of a rivet are equal when using our assumed coeffs of crippling, shearing, and tension.

Rem. 2. Our example was chosen to illustrate the rule. It will rarely happen in practice that the rule will give a number of rivets without a fraction; or that may be divided by 2 and by 3 without a remainder. In case of a fraction, it is plainly best to call it a whole rivet; although the joint thereby becomes a trifle stronger than necessary. Or rivets of a slightly diff diam may be used. If the number of rivets comes out say 7 or 9, we may make 2 rows of 3 and 4, or of 4 and 5, &c. Moreover, the width of the plate is frequently fixed beforehand by some requirement of the structure, and we must arrange the rivets to suit, taking care in all cases to maintain the calculated area of net iron in one row, &c.

Rem. 3. We have (as we at first said we should do) confined ourselves to the simple butt-joint with 2 covers, and with the rivets in either 1, or in 2 or more parallel rows on each side of the joint-line; this being the strongest and the one in most common use in engineering structures. Necessity at times calls for less simple arrangements, for which we cannot afford space, and the strength of which is not so readily calculated. These sometimes yield results which appear strange to the uninitiated; thus, this lap-joint breaks across the net iron of one plate, along either *c c* or *o o, where there is most of it,* and where, therefore, it might be supposed to be the strongest.



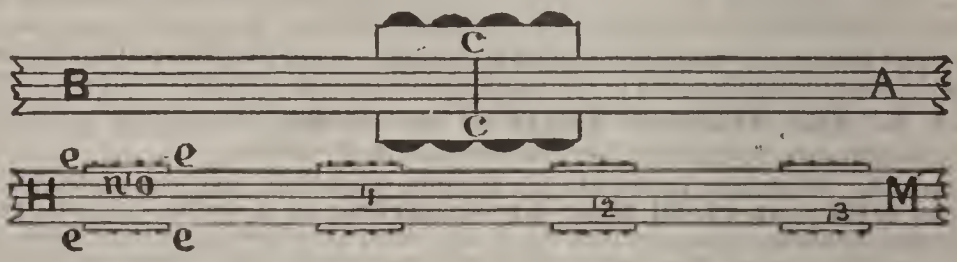
Rem. 4. The following table shows approximately the comparative strengths of the common forms of joints when properly proportioned; varying with quality of sheets, and of rivets:

	With friction.	Without friction.
The original unholed plate	1.00	1.00
Double-riveted butt with two covers	.80	.64
Double-riveted butt with one cover	.65	.52
Single-riveted butt with one cover	.50	.40
Double-riveted lap	.65	.52
Single-riveted lap	.50	.40

Rem. 5. The above tabular strengths for the **lap-joints** will be approximately attained by adopting the following proportions, according as the joint is double- or single-riveted.

	Double riv. zigzag.		Single riv.	
	In thicknesses.	In diams.	In thicknesses.	In diams.
Calling thickness of plate.....	1.	.6	1.	.6
Then make diam of rivet.....	1.67	1.0	1.67	1.0
" " breadth of lap.....	9.0	5.4	5.67	3.4
" " pitch from cen to cen.....	7.0	4.2	4.5	2.7
" " dist from *end* of plate to *edge* of holes.....	2.0	1.2	2.0	1.2
" " dist apart of rows from cen to cen.....	3.33	2.0		

Rem. 6. If two or more plates on top of each other, as the four in A B or M H, are to be jointed together so as to act as one plate of the thickness *c c*, the diams of the rivets, and the thickness of the covers *c c*, *e e* will depend upon whether the junctions of the plates are all in one line with each other as at *c c*, in A B, or whether they break joint with each other as at 0, 1, 2, 3 in M H.



It is plain that the two covers *c c* by means of their connecting rivets convey from A to B, across the joint *c c*, all the strength that partly compensates for the severance of the four plates at that joint; whereas the two covers *e e*, *e e*, and their rivets in like manner convey from *n* of one single plate, to *o* of the adjoining one, across the joint between those two letters, only the strength that partly compensates for the severance of that single plate; and so with the joints at 1, 2, and 3. Therefore the covers *c c*, and their rivets, must be four times as strong as those at any one of the four joints 0, 1, 2, 3. The first, *c c*, are to be regarded as joining two solid plates A and B, each of the fourfold thickness *c c*; and the others as joining two of the single thickness. The covers *c c* will, therefore, each be about two-thirds of the thickness *c c*; and the others each about two-thirds as thick as a single plate. Thus, suppose each of the 4 plates in A B or M H to be ¾ inch thick, making *c c* 3 ins. Then each cover, *c*, is ⅔ of 3 ins, or 2 ins thick; or the two covers, *c c*, together 4 ins, which is thus the effective thickness of the joint, *c c*. But each cover, *e e*, is only ⅔ of ¾ inch, or ½ inch thick; and the effective thickness of joint at either 0, 1, 2, or 3, is that of the 3 unbroken plates plus that of the 2 covers, or $(3 \times \frac{3}{4}) + (2 \times \frac{1}{2}) = 3\frac{1}{4}$ ins.

Art. 7. Principle of the Rule in Art 5. With our constants for shearing (45000 lbs per square inch) and for crippling (60000 lbs per square inch), and with diameter of rivet equal to, or greater than, .85 times the thickness of the plate, as by our rule, the *crippling* strength of a double cover butt joint will be equal to, or less than, its shearing strength. Therefore, to avoid waste of material, either in the plate or in the rivets, we must make

$$\text{Tensile strength of net plate across one row of rivets} = \text{Crippling strength of } \textit{all} \text{ the rivets. Or,}$$

$$\text{Total net width of plate} \times \text{Thickness of plate} \times \text{Tension unit} = \text{Crippling area of one rivet} \times \text{Crippling unit} \times \text{Total number of rivets.}$$

Now, by Art 3, the crippling area of a rivet is = diam of rivet × thickness of plate. We take the crippling unit at 60000 lbs; and the tension unit at 38500 lbs. Therefore (transposing) we must make

$$\text{Total net width of plate} = \frac{\text{Diam of rivet} \times \text{Thickness of plate} \times 60000 \times \text{Total number of rivets}}{\text{Thickness of plate} \times 38500}$$

By making the clear dist between each end rivet of a row and the side edge of plate = half the clear dist between two rivets in a row; and calling the sum of the two end dists one space, we have

$$\text{Number of } \textit{spaces} \text{ in a row} = \text{Number of } \textit{rivets} \text{ in a row.}$$ So that

The clear dist between two rivets in a row, which is

$$= \frac{\text{Total net width of plate}}{\text{Number of } \textit{spaces} \text{ in a row}} \text{ is also } = \frac{\text{Total net width of plate}}{\text{Number of } \textit{rivets} \text{ in a row}}$$

$$= \frac{\text{Diam of rivet} \times \text{Thickness of plate} \times 60000 \times \text{Total number of rivets}}{\text{Thickness of plate} \times 38500 \times \text{Number of rivets in a row.}}$$

But $$\frac{\text{Total number of rivets}}{\text{Number of rivets in a row}} = \text{Number of rows.}$$

Therefore, omitting "thickness of plate," common to both numerator and denominator, we have, as in rule in Art 5,

$$\textbf{Clear dist apart} = \frac{\text{Diam of rivet} \times 60000 \times \text{Number of rows}}{38500}$$

But if the diameter of the rivets is less than .85 times the thickness of the plates, the *shearing* strength of a double-cover butt joint (with our assumed constants for shearing and crippling) is less than its crippling strength. In such cases, for the clear dist between two rivets in a row, say

$$\textbf{Clear dist} = \frac{\text{Circular area of a rivet} \times \text{Shearing unit}}{\text{Thickness of plate} \times \text{Tension unit}} \times 2$$

Rem. 1. Butt joints in double shear, or with 2 covers, being the only ones here considered, and inasmuch as rivets may always be used with a diam greater than .85 of the thickness of the plate, we may in practice always use the Rule in Art 5 for such joints; and, therefore, we gave it alone.

Rem. 2. When using these rules for other kinds of joint, such as laps, or butts with *single* covers, remember that the rivets in such are in **single** shear; and, therefore, we can use Rule in Art 5 (for crippling) only when the diam is either **1.7 or more** times the thickness of plate. **If less,** use Rule above for **shearing;** all on the assumption that our foregoing coefs of crippling and shearing are used.

But the coef for tension must be changed for each kind of these other joints, to allow for the weakening effects of the bending shown at W, Figs 3, as deduced approximately from experiment. The writer believes that the following tension units will give safe approximate results without friction. **For double-cover butts,** double-riveted, 38500 lbs per sq inch, as adopted above. **For double-riveted** laps, or one-cover butts, 28000. **For single-riveted** laps, or one-cover butts, 24000. But, as before remarked, no great certainty is attainable in riveting.

Rem. 3. A joint may fail by crippling without the facts being known or even suspected, for it does not imply that anything **breaks,** but merely that the joint has **stretched;** and this might not be detected even on a *slight* inspection of it. Still it might, and probably often has sufficed to endanger, and even destroy both bridges and roofs by generating strains where none were provided for.

Art. 7. Breaking by shearing. Let *a b c d*, Fig 1, represent a beam, with its ends resting on supports S S; with a load *l*, so heavy as to break it by forcing its entire central part, *o o g g*, away from the two end parts *a d g* and *b c o*; so that while the two latter remain in their places, the central part *slides* out; or is, as it were, *punched* clean out from between them. This peculiar mode of fracture is called *shearing*, or *detrusion*. The force required to produce it, or the resistance which the beam opposes to such a force, may practically be assumed to be in proportion to the area of the sheared section. Thus, since the area of cross section of a beam 1 ft sq is 4 times as great as that of a beam 6 ins sq, the former will present 4 times as great a resistance to shearing; or will, in other words, require 4 times as great a load, or pres, to shear it across. In Fig 1 the total sheared area is equal to twice the transverse area of the beam. See Eye-bars and Pins, page 612. Bridge chords are exposed to great shearing force where they rest on the abuts, but it becomes less toward the center of the span; and so with every equally loaded beam. See p 532.

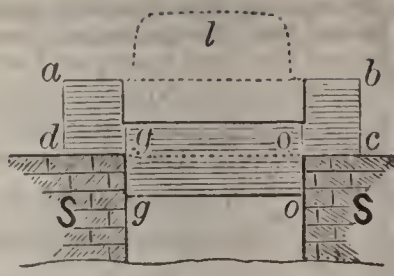


Fig. 1.

We have very few experimental data on this subject.

The shearing strength of white pine, spruce, and hemlock, *parallel to the fibres*, by the author, 250 to 500 lbs per sq inch; oak 400 to 700; and is of use in estimating the resistance along the line *c c*, Fig. 2, at the end of a tie-beam; or at the head of a queen-post, &c.

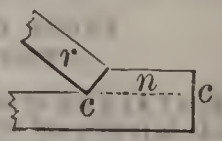


Fig. 2.

Across the fibres the writer found for spruce about 3250 lbs; white pine and hemlock 2500; yellow pine 4300 to 5600; white oak 4400.

Wrought iron is stated at 35000 to 55000 lbs per sq inch; cast iron 20000 to 30000; steel 45000 to 75000 lbs; copper 33000.

Fig. 2½.

The shearing strength of steel and wrought iron is about ¼ part less than the tensile. The punching of rivet-holes in iron or steel plates, is an example of shearing. The rivets in tubular bridges are frequently sheared in two, in time, by the motion of the plates through which they are driven. In punching holes, the area of section is evidently found by mult the circumf of the hole by the thickness of the plate in which it is punched. If a piece of material be supported as shown in Fig 2½, its resistance to shearing will be 3 times as great as in Fig 1, where it is sheared across in 2 places only; whereas in Fig 2½, shearing would have to occur at 6 places, as per the dotted lines.

Art. 8. Breaking by torsion, or twisting. Let *n*, Fig 3, be a vert cylindrical rod of any material, 1 inch diam, the lower end of which is *immovably* fixed; and let *c* be a lever whose leverage *a b*, measd from the axis of the cylindrical rod, is 1 ft. Suppose that with a spring balance attached to the end *b* of the lever, we apply force horizontally, and around the axis of the rod as a center, until the rod breaks by being twisted. Then if we mult together the leverage *a b* (1 foot) and the amount of force shown by the spring balance in lbs, and div the prod by the cube of the diam of the rod in ins the quot will be a certain number of foot-pounds; and will be what is called the *constant*, or *coefficient for torsion*, for all cylindrical bars of that material. If we use a square bar, we shall get the coef for square bars; and so with any other shape. So that if with any other bar, or shaft, we mult the cube of its diam in inches by

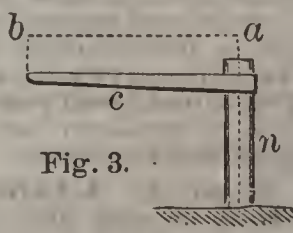


Fig. 3.

said constant, and div the prod by the leverage in feet, the quot will be the force in lbs which will twist the bar in two. In shape of formulas,

$$\frac{\text{Leverage in feet} \times \text{Brkg force in lbs}}{\text{Cube of diam in ins}} = \text{Constant.} \quad \text{And} \quad \frac{\text{Cube of diam in ins} \times \text{Constant}}{\text{Leverage in feet}} = \text{Breakg force in lbs.} \quad \text{Also}$$

$$\frac{\text{Cube of diam in ins} \times \text{Constant}}{\text{Brkg force in lbs}} = \text{Leverage in feet} \quad \text{And} \quad \frac{\text{Leverage in feet} \times \text{Brkg force in lbs}}{\text{Constant}} = \text{Cube of diam in ins.}$$

The constant for solid cylinders of average cast iron is about 600; and for wrought iron 800. For puddled steel about 700; cast steel 1000 to 1700. Wrought copper 400. All may vary one fourth part of these more or less.

For woods, rough averages. W pine or spruce 20 to 25 ft-lbs. Y pine 35. Ash 40. W oak 50. Locust 75. Hickory 85.

To find, by the last formula, the diam of a rod to have a safety of 3, 4. 5, &c, against a given twisting force in lbs, first mult said force by 3, 4, 5, &c, as the case may be, and use the prod as the breaking force. The diam will then be the *safe* one.

Any angle described by the force at *b*, when made to revolve around the axis of the rod as a center, during the twisting process, is called the ANGLE OF TORSION. The length of the twisted rod or shaft does not affect the *amount* of force reqd to produce rupture; but the longer it is, the greater will be the angle of torsion; or in other words, the greater will be the *dist* through which the force must revolve around the axis before fracture takes place. On the other hand, a long shaft will *twist* through a *given angle* under a less force than a short one of same diam and material; and will more readily *bend* under torsion. Authorities say that a working shaft should not twist more than 1°. We should not expose it to more than .1 of its ult strain.

If we know the force in lbs per sq inch reqd for shearing any material, see preceding Art, then the force required to break a **cylinder** of it by torsion, is

$$\text{Torsion force in lbs} = \frac{\text{One-half the shearing force in lbs per sq inch} \times 3.1416 \times \text{Cube of rad of cylinder in ins}}{\text{Leverage in inches.}}$$

That of a square shaft is about $1\frac{1}{5}$ times that of a round one whose diam is equal to a side of the square; or about $\frac{1}{5}$ less than that of a round one of the same transverse area. **For any solid rectangular shaft**

$$\text{Breaking Torsional force in lbs} = \frac{\text{One-third of the shearing force, in lbs per sq in} \times \text{The square of one side} \times \text{The square of the other side.}}{\text{Square root of the sum of the above two squares} \times \text{Leverage in inches.}}$$

Hollow shafts resist torsion better than solid ones of the same area of metal. Calling the outer and inner diams in ins D and *d*, then

$$\text{Breaking Torsional force in lbs} = \frac{(D^4 - d^4) \times \text{Constant}}{\text{Leverage in ft} \times D}$$

Strength of wrought-iron shafting. The shafting used for the transmission of power to the diff parts of machine-shops, many manufacturing establishments, &c, is subjected to twisting strains. It is usually made cylindrical, and of wrought iron. Experience shows that we may safely use the following for shafts of iron of good quality, bearing but little weight, and well supported at proper intervals, say 8 or 9 ft., by self-adjusting ball and socket hangers.

$$\text{Diam of a wrought iron shaft in ins.} = \sqrt[3]{\frac{\text{Horse-powers}}{\text{Number of revs per minute}}} \times 125.$$

Or in words: for the diameter in inches div the number of horse-powers that are to be transmitted along the shaft, by the number of revs which the shaft is reqd to make per min. Mult the quot by 125. Take the cube root of the product. This cube root will be the diameter itself, at the thinnest part, at its bearings.

The last formula shows that the faster a shaft revolves under the same number of horse-powers, the less is the torsional strain upon it. This may at first seem strange, but less so when we reflect that a horse-power is made up of *pres* and *dist;* therefore, the faster it moves, the less is its pressure. Hence many horse-powers revolving rapidly will require a less diam than a small number revolving slower in proportion than its number.

TRANSVERSE STRENGTH.

Art. 9. Transverse (or across) **Strength.** Sometimes called **Relative Strength.**

When either a load *l*, Fig 1, or any other vert force acts upon a hor beam *f o* fixed at one end, the beam becomes a lever, and with the load has a tendency to move or revolve about the fixed end, *f* as a supporting fulcrum, and in so doing to strain or break the beam at said fulcrum, by forces of tension and compression, acting hor or lengthwise of the beam, pulling apart the upper fibres at *f*, and crushing together the lower ones. The load or other force together with the wt of the beam also tends to break the beam by shearing or cutting it across *verti-*

cally as shown at Fig 1, p 532. But it is only the first of these tendencies of which we speak now. It is called the load's **Moment of Rupture,** or Breaking Moment, or merely its moment, about the fulcrum *f;* and is measured in foot-tons, or foot-lbs, inch-lbs, &c, by mult the load in tons or lbs, &c, by its leverage, or the shortest or perp dist *h e* or *f s* of its line of direction *a m* from the fulcrum in ft or ins.

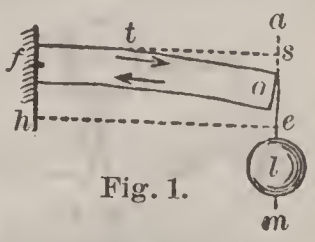


Fig. 1.

If the load instead of being concentrated like *l* is *distributed* in any way along the whole or a part of the beam, its leverage is measured from the fulcrum perp to the *line of direction of its cen of grav;* which is plainly the case also with a concentrated load, because *its* line of direction also passes through its cen of grav. Before the beam bends, its leverage is evidently greater than afterwards, and it becomes less as the bending increases; but as very little bending is allowed in practical cases the leverage may generally be assumed not to change, but to remain as when the beam is hor.

The load evidently tends also to strain, or break the beam at any point whatever as *t*, Fig 1, between itself and the fulcrum *f*, and is assisted in so doing by the wt of beam between *t* and *o*. Therefore any such point *t* may also be assumed to be a fulcrum. The moment of the load will of course be less at such point than at *f* because its leverage *t s* will be shorter.

In the **closed** beam *i a e o*, Fig 2, the load tends to revolve about **the neutral axis** *n* as the supporting fulcrum of its lever the beam, as shown by the dotted lines, and thereby to strain all the fibres from top to bottom of the beam at the section *i n e*, by stretching lengthwise those above *n*, and compressing lengthwise those below *n*. The greatest strain is at the top and bottom fibres; and from them both ways it diminishes until at *n* it is nothing. The load also stretches or compresses the fibres lengthwise at every vert section along the entire length of the beam, more or less according to its leverage and moment at said section; most near the fixed end and least near the free end; so that the extent of stretch indicated by *s i* is the total accumulated stretchings that have taken place in the top fibres at every point from *i* to *a*. The same is the case with the stretches and compressions of the fibres anywhere between *i e* and *a o*, as indicated by the varying hor dists between *n i* and *n s*, or between *n v* and *n e*. The compressed fibres below *n*, and comprised between *n v* and *n e* would as it were vanish, being crushed or mashed flat against the face of the wall.

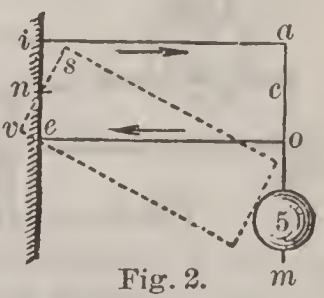


Fig. 2.

A closed beam a a, Fig 3, supported at only one point whether at the center or not, and balanced by two either equal or unequal loads, may plainly be regarded as two levers each of which is essentially in the same condition as Fig 2. Whether the loads are concentrated or distributed their leverages *n e*. *n e* are as before to be measured from *n* and perp to the lines of direction *v o*, *v o* of their centers of grav as in Fig 2. Both the 5 ton loads are manifestly upheld by the support, which of course reacts vert upwards against them in a vert line with their common cen of grav *n*, with a force of 10 tons as per the central arrow.

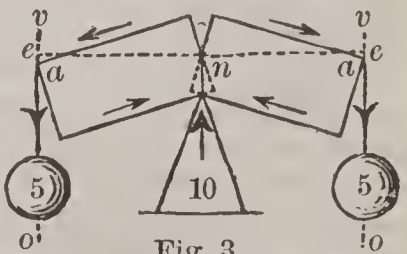


Fig. 3.

Rem. 1. Each end load in Fig 3 being 5 tons, suppose each lever *n e* to be 4 ft. Then the moment of *each* load about the fulcrum *n* would be $= 5 \times 4 = 20$ ft-tons. Hence it might seem that over the support the fibres of the beam near *n* would have to resist a combined moment of 40 ft-tons. But they have actually to present a resistance of but 20 ft-tons, on the same principle that if two men pull against each other at two ends of a rope, each with a *force* of say 30 lbs, the *strain* or *pull* on the rope is not 60 but only 30 lbs, because *strain* is the *reaction* (pressure or pull) *against each other* of two equal opposing forces, and is equal to only one of them. The two above equal *moments* are merely *two forces acting through leverages.*

A closed beam, Fig 4, supported at both ends, and loaded at only one point, whether at the center or not, with a *concentrated* load,

may also like Fig 3 be regarded as two levers with their common fulcrum at *n* in a vert line with the cen of grav of the load. This however is by no means so manifest at first sight as in Fig 3, but needs a little explanation. Let the beam bear 10 tons *concentrated* at its center, then evidently 5 tons of it will rest pressing downwards on each end support; and each support will therefore press upward or *react* against an end of the beam with a force of 5 tons as per the arrows. Now these two 5-ton *reactions* of the *supports* in Fig 4 are to be considered as taking the place of the two 5-ton end *loads* in Fig 3; while the 10-ton *load* in Fig 4 takes the place of the 10-ton *reaction* of the *support* in Fig 3, and hence *in this view of the case* is no longer to be considered at all *as load*, but merely as a fixture for holding the common fulcrum *n* of the two levers in place, or in equilibrium with the upward end reactions. Being no longer regarded *as load*, it of course cannot *in such cases* be assumed to have any moment of rupture; that property being now transferred to the end reactions. Still, to avoid awkwardness of expression we always speak of the moment of the *load* even in such cases, rather than of the moments of the *reactions of the load*. In both Figs 3 and 4 the forces at work are the same *in amount*, but plainly reversed *in direction*.

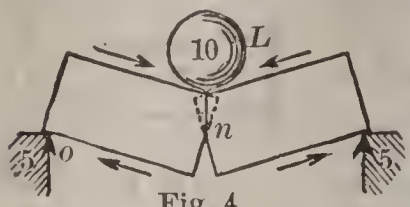


Fig. 4.

Rem. If the load is distributed as the 6 tons in Fig 5, instead of concentrated as in Fig 4, we still consider the beam as consisting of two levers with their common fulcrum *n* in a vert line with the cen of grav *c* of the load. But to find the moment of the load (or more correctly, the moment of the reactions of the supports) about *n* we must proceed a little differently. Thus let the beam be 3 ft span, and the load uniform, weighing 6 tons, and being 1 ft long. Find by rule, p 481, how much of this load rests on each support, (4 tons on *a*, and 2 tons on *o*.) The upward reactions of the supports will therefore also be 4 and 2 tons. Then first find *the moment about n of either one of the reactions*, say of the 4-ton one at *a*. This moment will plainly be (4 tons × 1 ft) = 4 ft-tons. Then find *the moment about n of that part* (3 tons) *of the load that is between n and a*, by mult the wt (3 tons) of that part by the hor dist (*e n* = .25 of a ft) between *its* cen of grav and *n*. This last moment (3 tons × .25 of a ft) = .75 of a ft-ton, being *downward*, evidently diminishes or counteracts the *upward* moment of the 4-ton reaction at *a* about the same fulcrum *n* to the same extent, and is therefore to be subtracted from it, thus leaving 4 — .75 = 3.25 ft-ton for the moment of the 6-ton load about *n*.

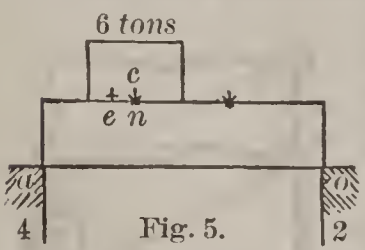


Fig. 5.

The same result will follow if we use the 2-ton reaction of *o*, with the hor leverage *o n*, and the part of the load between *o* and *n*. To find the moment for any other point than *n* see pp 481 to 483.

If the beam is inclined, the moment of rupture may still be found by using the hor span and segments instead of the inclined ones; but the resulting longitudinal strains, as well as the shearing forces become changed, involving much complication. We confine ourselves therefore to hor beams.

In a *cantilever*, Fig 29¼, a load or portion of a load causes a moment or strain in every part of the beam between said load or portion of load and the point *i* of support, but at no other point. Thus *c* causes a moment at each point between *i* and *t*; and *a* causes a moment at each point between *i* and *s*; but *a* causes no moment at any point between *s* and *t*.

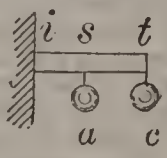


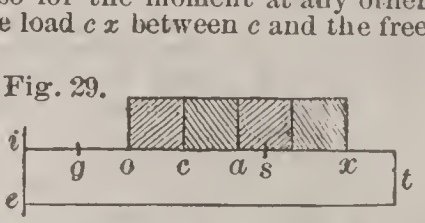
Fig 29¼.

The weight of the beam itself is not here included. When required to be so, consider it as a uniform load, and use Case 3 or Case 10, p 482 and 483, and add the result to that obtained for the load.

The deflection in ordinary cases may be found by the rule on page 506.

General Rule for moments of rupture in hor cantilevers, no matter how irregularly the load or loads may be distributed. Bear in mind that only that part of the load which is *beyond* (towards the free end from) any assumed point tends to break the beam at that point as a fulcrum, and that it does so with a leverage = dist of the cen of grav of that part of the load from the point. The other part of the load has no moment at that point. Thus the whole load *o x* tends to break the beam at *g* or *i* with a leverage = *a g* or *a i* as the case may be, *a* being the cen of grav of the load. And so for the moment at any other point *c*, Fig 29, as a fulcrum, find the wt of all the load *c x* between *c* and the free end *t* of the beam. Also find the cen of grav *s of that part* of the load. Mult the weight just found by its leverage *c s*.

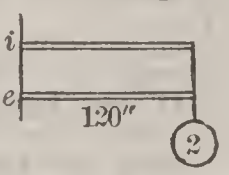
Fig. 29.

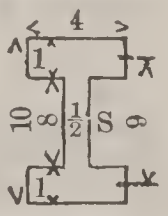


Example 1. We use a *uniform* load in order to illustrate the rule more readily. Let the hor yellow pine beam *i t* be 7 ft long; its breadth and its depth *i e* each 6 ins; the whole load *o x* 4 tons; and *c* the point or fulcrum at which the moment of the load is reqd. Then the wt of the load between *c* and *t* is 3 tons; and its cen of grav *s* is 1.5 ft or 18 ins from *c*. Hence the load's moment at *c* = 3 tons × 18 ins leverage = 54 inch-tons. That is, a load of 3 tons tends with a leverage of 18 ins to rupture the beam at *c*.

Example 2. Let Fig 30 be **a rolled iron I beam** cantilever of the cross-section shown in ins at S, projecting hor 10 ft or 120 ins, and bearing a concentrated load of 2 tons at its free end. The moment of the load at the section *i e* is = 2 × 120 = 240 inch-tons.

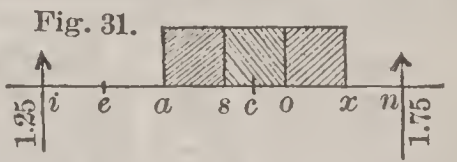
Fig. 30.





General Rule for M of Rup in hor beams supported at each end, no matter how irregularly the load or loads may be distributed. Let *i n*, Fig 31, be such a beam of yellow pine of 6 ft or 72 ins span, 6 ins square, and loaded with 3 tons. First find the cen of grav *c* of the whole load *a x*, and what portion (1.25 and 1.75 tons) of said load rests on each support *i* and *n*, thus, as whole span : whole load : : *either* arm : portion at *other* arm. Consider the upward reactions thus found (1.25 and 1.75 tons) of the two supports to be two forces acting vert upwards against the ends of the beam at *i* and *n* as denoted by the arrows. Let *o* be any point whatever in the beam at which as a fulcrum the load's moment is required. Assume either of the upward end forces, say the 1.25 tons at *i*, to be acting at the outer end *i* of a lever *i o* (4 ft long) of which *o* is the fulcrum. Mult this force (1.25 tons) at *i* by this leverage *i o* (48 ins). Call the product (60) *p*. Find the cen of grav (*s*) of the part load *a o* (2 tons) between *i* and *o*. Mult said part load by the dist *o s* (12 ins) of *its* cen of grav (*s*) from the given point or fulcrum *o*. Deduct the product (24) from *p*. The remainder (36 inch-tons) will be the moment at *o* of the total load *a x*. The same result will follow if we use *n* and the 1.75 tons reaction, but with the load *x o*.

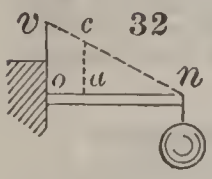
Fig. 31.



Rem. 1. If there is no load between *i* and the fulcrum point, as would be the case if the moment had been reqd at any point between *i* and *a* instead of at *o*, then the above *p* by itself is the moment. Thus *e* is 12 ins from *i*, hence the moment at *e* of the entire load *a x* is 1.25 × 12 = 15 inch-tons.

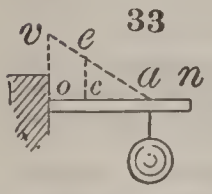
Rem. 2. Although in Fig 31 the load is regarded as having no *moment* at either end *i* or *n* of the beam, yet it produces *shearing forces* there equal to the upward reactions. See "Shearing," p 532.

Although the foregoing general rules apply to all the following cases, still these last will often expedite calculations.

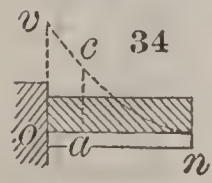


CANTILEVERS.

Case 1. Concentrated load at free end, Fig 32. Greatest moment is at *o*, and = load × *o n*. At any other point *a* it is = load × *a n*. Make *o v* = greatest moment, join *v n*; then *a c* is the moment at any point *a*.



Case 2. Concentrated load at any point a, Fig 33. Greatest moment is at *o*, and = load × *o a*. At *c* it is = load × *c a*. Make *o v* = greatest moment, join *v a*. Then *c e* is the moment at any point *c*. The load has no moment between *a* and *n*.



Case 3. Uniform load throughout, Fig 34. Greatest moment is at *o*, and = whole load × half *o n*. At *n* it is nothing At any point *a* it is = load on *a n* × half *a n*. Make *o v* = greatest moment, draw the dotted parabola with its vertex at *n*. Then *a c* gives the moment at any point *a*.

If the load is not uniform the greatest moment is = whole load × dist from *o* to its cen of grav.

Case 4. Load on one part, Fig 35. Greatest moment is at *o*, and = load × dist from *o* to cen of grav *c* of load. At any point *t* between the load and *o*, moment = load × *t c*. At any point *a* in the load, moment = load on *a s* × dist *a e* of the cen of grav of load on *a s* from *a*.

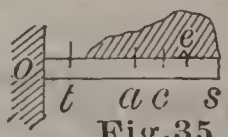


Fig.35

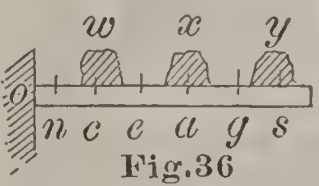


Fig.36

Case 5. Several loads, *w x y*, Fig 36. Find their centers of grav c, *a*, *s*. Greatest moment is at *o*, and = (*w* × *c o*) + (*x* × *a o*) + (*y* × *s o*). Or first find the common cen of grav of all the loads, and mult its dist from *o* by the sum of the three loads. Between the loads the moment at *g* = *y* × *g s*; at *e* = (*y* × *s e*) + (*x* × *a e*); and at *n* = (*y* × *s n*) + (*x* × *a n*) + (*w* × *c n*).

Case 6. One uniform load and one local one, Fig 37. Greatest moment is at *o*. Find that of the uniform one by Case 3; and that of the local one by Case 4, and add them together. No moment between *a* and *n*.

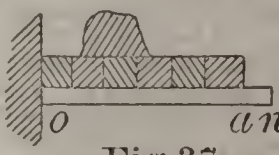
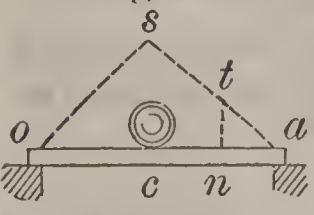


Fig.37

BEAMS SUPPORTED AT BOTH ENDS.

Fig 38



Case 7. Concentrated load at center, Fig 38. Greatest moment is at center, and = half load × half span. At the supports it is nothing. Make *c s* = moment at center, join *s o*, *s a*; then *n t* = moment at any point *n*. Or the moment at any point *n* = half load × *a n*, *a* being the *nearest* support.

Case 8. Concentrated load not at center, Fig 39. Greatest moment is at the load, and is = load × *e o* × *e a* ÷ *o a*. Make *e s* = moment at load, join *s o*, *s a*; then at any point *c* the moment is *c t*. Or at any point *c*, between load and *o*, moment = load × *a e* × *o c* ÷ *o a*. At any point *m*, between load and *a*, moment = load × *o e* × *a m* ÷ *o a*. No moment at *o* or *a*.

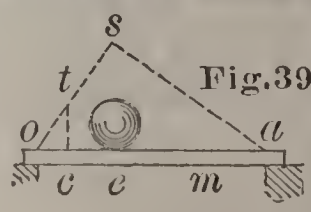
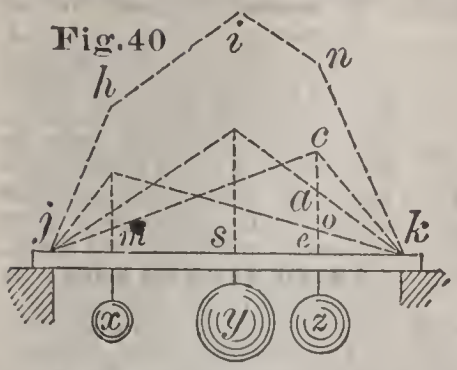


Fig.39

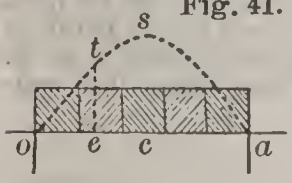
Case 9. Several concentrated loads $x\ y\ z$, Fig 40. By Case 8 find the greatest moment of each load separately, and for each of them draw its dotted vertical and two inclined lines as in this fig. Then for the moment at any point whatever as e, measure the vert dists (in this case $e\ o$, $e\ a$, $e\ c$) to the sloping lines, and add them together. For it is plain that at e we have $e\ o$ for the moment of the load x at that point; $e\ a$ for that of the load y; and $e\ c$ for that of the load z; and so at any other point. Or make $e\ n = e\ o + e\ a + e\ c$; also make $s\ i$ and $m\ h$ respectively equal to the three dists above s and m, and join $j\ h\ i\ n\ k$. Then at any point along the beam $j\ k$ the vert dist to these upper lines gives the moment.

Fig. 40



Case 10. Uniform load from end to end, Fig 41. The greatest moment is at the center c, and is = half load × quarter span. At any other point e moment = half load on $e\ o$ × $e\ a$; or to half load on $e\ a$ × $e\ o$. Make $c\ s$ = half load × quarter span, and draw a parabola $o\ s\ a$, then at any point e the moment is = $e\ t$. Or, moment at any point = half what it would be at that point if the whole load were concentrated there.

Fig. 41.



The shearing or vertical strain at the center is zero or nothing. See Art 11, p 535.

Rem. 1. The weight of the beam itself is usually such a load, but is frequently so small compared with the load that in this and other cases it may be neglected.

Rem. 2. In the case of a uniformly distributed load like that in figure 12, page 535, the *greatest moment of rupture that can occur at any given point* on the span is when the load covers the span from end to end; and in beams or trusses of uniform depth the hor strains at any given section are then also greater than under any partial load; so that if the chords are then strong enough in every part, they will be strong enough for any partial load; which is not the case with web members; any one of which is most strained when the longest segment reaching to it is loaded. See p 536.

Case 11. Uniform load from a support to part way across, Fig 42. Find the cen of grav g of the load, and by p 481 what portion of it rests on each support o and x. Then by General Rule, p 481 the mom at o or x = nothing. At n or at any point a between n and x it is = portion or reaction at x × $x\ n$ (or $x\ a$ as the case may be). At any point c between n and o moment is = reaction at x × $x\ c$ — (load on $c\ n$ × half $c\ n$) or to reaction at o × $o\ c$ — (load on $o\ c$ × half $o\ c$). This plainly **applies to unequal loads also.**†

Fig. 42.

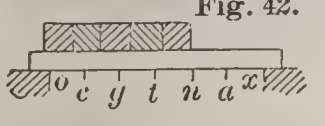


Fig. 43.

To find the place of **greatest moment** *t if the load is uniform* say, as twice $x\ o : n\ o :: n\ o : n\ t$. When the load covers the whole beam it becomes Case 10.

Case 12. Uniform load reaching to neither support, Fig. 43. From either support proceed as from x in Fig 42, except as to greatest moment, which find by trial.*

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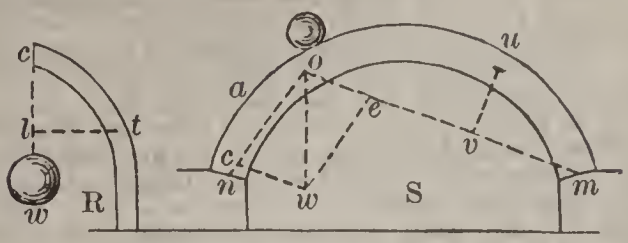
* On this subject see "Humber's Strains in Girders," to which the writer is chiefly indebted for the foregoing.

† Except that then the dist from the given point c to the cen of grav of the part-load $c\ o$ or $c\ n$ is not necessarily equal to half $c\ o$ or half $c\ n$; and if not, said dist must be used instead of said half $c\ o$ or half $c\ n$.

The moment of rupture at any point t **in a bent piece** R with a load upon or suspended from c, is equal to the load $\times$ its leverage $t\,l$, perp to $c\,w$. This moment tends to break the piece R at its cross section at t by tearing apart the fibres to the right of its neutral axis, and by compressing those to the left of it; and to this moment the piece R opposes the moment of resistance of that section as in the case of a beam.



In an arched piece as S loaded at any one point o, draw $o\,n$, $o\,m$, also $o\,w$ vertical and equal by scale to the load, and complete the parallelogram $o\,e\,w\,c$ of forces. Then will $o\,e$ and $o\,c$ by the same scale give two forces into which the load is resolved, and acting in the directions $o\,m$, $o\,n$, much as the two strings of two bows $o\,a\,n$, $o\,u\,m$. The force $o\,e$ tends to break the bow $o\,u\,m$ at any section u with a moment = the force $\times$ its leverage $u\,v$ drawn from the point, and perp to $o\,m$; and the force $o\,c$ tends to break $o\,a\,n$ at any point in the same way with *its* leverage. The section at u or elsewhere resists as in R. The weights of the pieces R and S themselves have not been taken into consideration.

RESISTANCE OF BEAMS.

Having the moment of rupture **of the load,** it is necessary to know whether **the Moment of Resistance,** or simply the Resistance, **of the beam** is sufficient to withstand it.

The **foregoing instructions** for finding moments of *rupture* **apply to horizontal beams of any form of cross-section,** and whether said cross-section is *solid*, as in a common wooden beam; or open, as in a bridge-truss; or whether, as in the rolled I beam or plate girder, the web is solid, but of small cross-section as compared with that of the flanges.

But in treating of the **action of the beam in resisting** this moment of rupture, we shall **first consider only those beams in which each fibre,** throughout the entire cross-section, is to be regarded as **opposing the moment of rupture by a horizontal or longitudinal resistance.** This is always the case in beams of solid rectangular, cylindrical, oval, &c, cross-section; and (strictly speaking) in those with thin solid webs, as I beams and plate girders; but in plate girders it is usual, on the score of safety and in view of the small cross-section of the web, to neglect its share of the resistance, and thus to regard the girder in the same light as a *truss*, or "*open* beam." For the manner of resistance of such beams, see pp 528, &c.

THEORY OF RESISTANCE OF CLOSED BEAMS.

Scientists give the following, which, however, often differs from experiment in beams of composite cross-section, such as I beams, &c. See example, p 489.

In a closed beam, Fig 2, p 479, each of the fibres throughout the entire depth of the yielding section *i n e* opposes the breaking moment of the load by a **Resisting Moment** or Moment of Resistance of its own. As the breaking moment about *n* of the load is made up of its gravity-force or weight mult by its leverage or perp distance *n c* from the fulcrum or neutral ax *n*, so the resisting moment about *n* of each separate fibre, say for instance the one at *i*, is made up of *its* natural longitudinal resisting force or strength mult by *its* leverage or perp distance *n i* above or below the same fulcrum *n*; and the *sum* of all these separate moments is the **moment of resistance** of the cross-section, *i n e*, of the beam. A line, passing through this fulcrum, transversely of the beam, and at right angles to the action of the breaking force, is called the **neutral axis** of the given cross-section.

The longitudinal resisting force or strength, in pounds per square inch, of those fibres which are farthest from the neutral axis, is equal to the ultimate tensile or compressive strength of the material in pounds per square inch, *plus* the assistance in lbs per square inch, which they receive from their natural adhesion to each other. This adhesion resists the longitudinal sliding of the fibres upon each other, without which the beam cannot break. This *total* resistance, in pounds per square inch, of the fibres farthest from the neutral axis, is called the **Coefficient of Resistance** (frequently, but less aptly, the "coefficient, constant, or **modulus, of rupture**") of the material of which the beam consists, and is usually denoted by "**C.**" It is shown (page 489) to be = 18 times the center breaking load, in pounds, of a beam of the given material, 1 inch square × 1 foot span; as given in the table, page 493.

The *other* fibres of the beam are of course *capable* of exerting a resistance equal to that of the farthest fibres; but, owing to their less distance from the neutral axis, they are less *stretched* or *compressed*, when the beam yields. They are therefore unable to put forth their entire resisting power. The length, through which any fibre in a bending beam is stretched or compressed (*i e*, the extent to which it is lengthened or shortened), is plainly (Fig 2, p 479,) proportional to its perpendicular distance from the neutral axis; and, inasmuch as the longitudinal resistance which it actually *exerts* is proportional to the lengthening or shortening of it, it follows that

Perp dist from neutral axis to *farthest* fibre. : Perp dist from neutral axis to any *given* fibre :: Coefficient of resistance (or longitudinal resistance of *farthest* fibre) in lbs per sq in : Longitudinal resistance of said *given* fibre, in lbs per sq in

or $t : t' :: C : f$;

therefore,

$$\text{Longitudinal resistance of given fibre, in lbs } \textit{per sq in} = \frac{\text{Coefficient of resistance} \times \text{Dist from neutral axis to } \textit{given} \text{ fibre}}{\text{Dist from neutral axis to } \textit{farthest} \text{ fibre.}}$$

or $$f = \frac{C\,t'}{t}$$

and

$$\text{Longitudinal resistance of given fibre, in } \textit{lbs} = \text{Its longitudinal resistance in lbs } \textit{per sq in} \times \text{Its area } \textit{in sq ins}$$

$$(F = f a)$$

$$= \frac{\text{Coefficient of resistance} \times \text{Dist from neutral axis to given fibre} \times \text{Area of given fibre}}{\text{Dist from neutral axis to farthest fibre}}$$

or $$F = \frac{C\,t'\,a}{t}$$

The *moment* with which a given fibre resists the rupture of the beam is plainly

= Its longitudinal resistance × Its perp dist from the neutral axis

or $r = F\,t'$

$$= \frac{\text{Coefficient of resistance} \times \text{Dist from neutral axis to given fibre} \times \text{Area of given fibre}}{\text{Dist from neutral axis to farthest fibre}} \times \text{Dist from neutral axis to given fibre}$$

or $$r = \frac{C\,t'\,a}{t}\,t'$$

$$= \frac{\text{Coefficient of resistance}}{\text{Dist from neutral axis to farthest fibre}} \times \text{Area of given fibre} \times \text{*Square* of dist from neutral axis to given fibre}$$

or $$r = \frac{C}{t}\,t'^2\,a$$

It follows from the above that the strengths of *solid* or "*closed*" beams are as the *squares* of their depths; although in "*open*" beams, page 528, &c, it is simply as the *depths*.

The **Moment of Resistance** of the *entire cross-section* of the beam is

$$= \frac{\text{Coefficient of resistance}}{\text{Dist from neutral axis to farthest fibre}} \times \left[\left\{\begin{matrix}\textbf{The sum} \text{ of} \\ \text{the products for} \\ \textit{all} \text{ the fibres}\end{matrix}\right\} \text{ of } \left(\begin{matrix}\text{Area} \\ \text{of the} \\ \text{fibre}\end{matrix} \times \begin{matrix}\text{Square of} \\ \text{its dist from} \\ \text{neutral axis}\end{matrix}\right)\right]$$

or, $R = \frac{C}{t} \times \left\{\begin{matrix}\textbf{sum,} \text{ for all} \\ \text{the fibres}\end{matrix}\right\}$ of $t'^2\,a$; or, $R = \frac{C}{t}\,I$

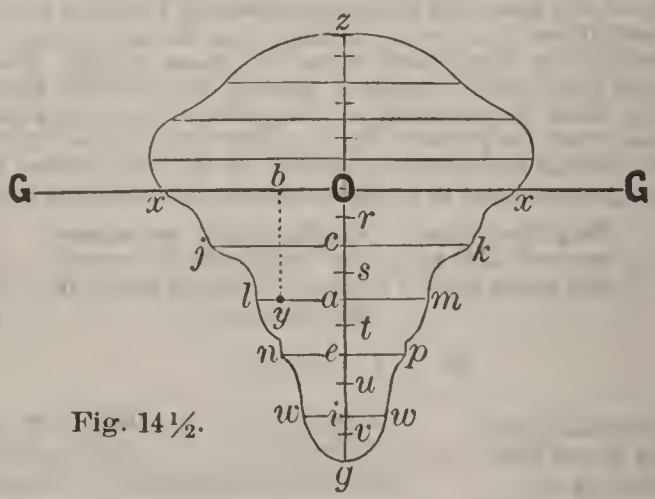


Fig. 14½.

This **sum,** or **"I,"** is, for convenience, called the **Moment of Inertia** of the cross-section of the beam about its neutral axis G G. For beams of rectangular cross-section, such as Figs p 487, it may be found by the rules on that page. For irregular sections, as Fig 14½ above, it cannot be found by ordinary arithmetic, but an approximation, sufficiently close for all practical purposes, may readily be made thus: Both above and below G G, and parallel to it, draw lines *j k*, *l m*, &c, dividing the section into narrow strips. If these lines are equidistant, the subsequent calculations will in some cases be easier; but otherwise it is immaterial whether they are so or not. If they are drawn no closer together, *proportionally to the size of the figure*, than in Fig 14½, the approximation will be near enough for practical purposes. The closer they are the more accurate will be the result; but however close they may be, it will always be a *trifle too small*. Begin by finding the area in sq ins, of the first strip *x x j k*, below G G. Mult this area by the *square* of the dist *Or* to the cen of grav of the strip. Then proceed to the next strip *j k l m*; find its area; and mult it by the square of the dist *o s* to its cen of gr *s*. So with each strip below G G. Add all the prods together. If the section has the same shape, size, and position above G G as below it, (as would be the case with a square, I beam, or circle,) mult their sum by 2. The prod will be the reqd moment. But if, as in Fig 14½, the section above G G differs from the portion below it, we must div it also into strips, and proceed as with the lower part. The sum of *all* the products on both sides of the neutral axis will then be the moment of inertia.

The position of the hor neutral axis G G, may be found by cutting out a correct figure, 4 or 5 ins long, of the section, drawn on thick paper or tin, and balancing it over a straight edge. The line at which it balances is G G. When this has been done, the dist *o g*, in ins, to the farthest fibre, (which may be either above or below G G, according to the shape of the Fig,) can be measured in ins.

The *true* moment of inertia = The approx moment found as above + The moment of inertia of each strip *l m n p*, &c, about *its own* neutral axis

the neutral axis of each strip to be taken parallel to that of the whole figure.

Or (which amounts to the same thing)

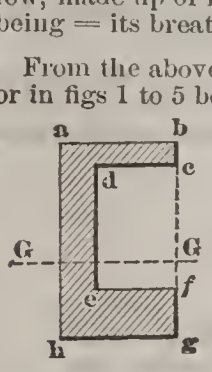
The moment of inertia of the whole figure about any given neutral axis = The sum of the moments of inertia of the several strips about the same neutral axis,

in which

Moment of inertia of each strip about the neutral axis *of the whole fig* = Its moment of inertia about *its own* neutral axis + (Its area × Dist² from neutral axis of fig to that of strip)

This affords a convenient method of finding the moment for figures, like those below, made up of rectangles; the moment of each rectangle about its own neutral axis being = its breath × its depth³ ÷ 12.

From the above it follows that in any **hollow figure,** as A, B, D or G, p 495, or in figs 1 to 5 below



The moment of inertia about any given neutral axis = Mom of in of the *entire* fig (including the missing parts) about the same axis − Mom of in of the missing parts about the same axis

Thus

Mom of in of channel *a b c d e f g h* about G G = Mom of in of rectangle *a b g h* about G G − Mom of in of rectangle *c d e f* about G G

The moment of inertia is plainly independent of the *material* of which the beam consists, of the span, and of the manner in which the beam is supported or loaded; and is the same for all beams of a *given cross-section.* It follows from the foregoing that it is proportional to the *cube* of the depth of the beam.

Moments of Inertia of a few well-known figs are given below. Those of similar figs are to each other as their breadths × cubes of depths.

Square. (Fourth power of side) ÷ 12, whether any side or diagonal is vert.

Parallelogram, rectangular or otherwise; neutral axis parallel to either two of the sides. Breadth × (cube of depth) ÷ 12. The breadth must be measured parallel to the neutral axis; and the depth at right angles to it.

Hollow square or rectangle. $[(B \times D^3) - (b \times d^3)] \div 12$.*

Circle. Rad⁴ × .7854. **Semicircle.** Rad⁴ × .1098.

Ring. (Outer rad⁴ − Inner rad⁴) × .7854.

Ellipse. Long diam vert. Half short diam × (half long diam)³ × .7854.

Elliptic ring. Long diam vert. Let L, S, *l*, *s*, be half the long and half the short diams. Then $[(S \times L^3) - (s \times l^3)] \times .7854$.

Triangle. Base × Perp Ht³ ÷ 36. The *base* is that side which is *parallel* to the neutral axis. This does not apply to *hollow* triangles.

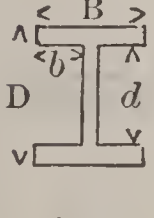


Fig 1.

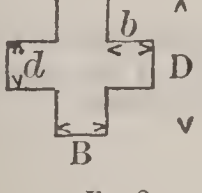


Fig 2.

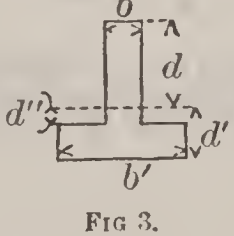


Fig 3.

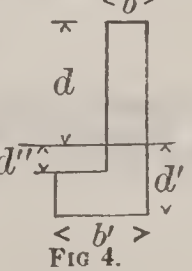


Fig 4.

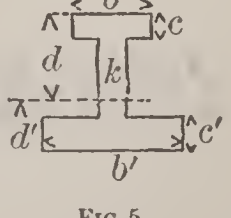


Fig 5.

1. $(B.D^3 - 2\ b.d^3) \div 12$. **2.** $(B.D^3 + 2\ b.d^3) \div 12$. **3** and **4.** $[b.d^3 + b'.d'^3 - (b' - b)\ d''^3] \div 3$. **5.** $[b.d^3 - (b - k).(d - c)^3 + b'.d'^3 - (b' - k).(d' - c')^3] \div 3$.

* B and *b* are respectively the outer and inner dimensions *parallel* to the neutral axis, whether said axis be lengthwise or crosswise of the figure. D and *d* are the outer and inner dimensions *perpendicular* to the neutral axis.

From the last formula on page 486, we have

$$\textbf{Moment of Resistance} = \frac{\text{Coefficient of Resistance}}{\text{Dist from neutral axis to farthest fibre}} \times \text{Moment of inertia}$$

$$\left(\mathbf{R} = \frac{\mathrm{C}}{t}\ \mathrm{I}\right)$$

For beams of square or rectangular cross-section, this becomes

$$\textbf{Moment of Resistance} = \frac{\text{Coefficient of Resistance} \times \text{Area of cross-section in sq ins} \times \text{depth in ins}}{6}$$

$$\left(\mathbf{R} = \frac{\mathrm{C\ A}\ d}{6}\right)$$

In rolled I beams, the moment of resistance is approximately found thus: all the dimensions in inches.

$$\textbf{Moment of resistance} = \left(\begin{matrix}\text{Area of cross-section of one flange} \\ + \\ \text{⅙ area of cross-section of web}\end{matrix}\right) \times \text{Depth of beam} \times \text{Elastic limit of iron in lbs per sq inch}$$

the area of wob being = its thickness × extreme depth of beam; and the area of one flange being = (area of whole beam — area of web) ÷ 2. For average rolled iron, the elastic limit may be taken at 22400 lbs or 10 tons; or about half the ultimate tensile strength.

When a beam is upon the point of failing, its

Moment of rupture is = its moment of resistance.

In other words

$$\frac{\text{Load in lbs} \times \text{span in ins}}{\mathbf{m}} = \frac{\text{Co-efficient of resistance in lbs per sq in}}{\text{Dist from neutral axis to farthest fibre, ins}} \times \text{Moment of inertia, ins}$$

$$\left(\frac{\mathrm{W}\ l}{m} = \frac{\mathrm{C}}{t}\ \mathrm{I}\right)$$

Here **m** is, according to circumstances, 1, 2, 4, 8, &c, as follows:

When the beam is firmly fixed		at one end, and loaded		at the other,	**m**	= 1
"	"	"	"	"	uniformly,	**m**	= 2
"	"	merely *supported* at both ends		"	at the center,	**m**	= 4
"	"	"	"	"	uniformly,	**m**	= 8
"	"	firmly fixed	"	"	at the center,	**m**	= 8
"	"	"	"	"	uniformly,	**m**	= 16

By some authorities the last two are given at 6 and 12. Prof De Volson Wood gives 12 for the last. The term "fixed" in such cases is rather indefinite.

Therefore,

$$\textbf{Total breaking load in lbs} = \frac{\text{Moment of inertia} \times \text{Co-efficient of resistance} \times m}{\text{Dist in ins from neutral axis to farthest fibre} \times \text{span in ins}}$$

$$\text{or}\quad \mathrm{W} = \frac{\mathrm{I\ C}\ m}{t\ l}$$

For the *neat*, or *extraneous*, breaking load; if uniformly loaded deduct the weight of the beam itself. If supported at both ends and loaded at the center, or if fixed at one end and loaded at the other, deduct *half* the wt of the beam.

On page 493 is a table of center breaking loads in pounds, for beams 1 inch square, and of 1 foot span, supported at both ends. In such beams

$$\text{Moment of inertia} = \frac{\text{Depth}^3 \times \text{Breadth}}{12} = \frac{1}{12}$$

m (p 488) is 4; the dist in ins from the neutral axis to the farthest fibre is $\frac{1}{2}$; and the span in inches is 12. The last formula on page 488 becomes, in such cases,

$$\text{Total center breaking load in lbs} = \frac{\text{Coefficient of resistance} \times \frac{1}{12} \times 4}{\frac{1}{2} \times 12} = \frac{\text{Coefficient of resistance}}{18}$$

$$\left(W = \frac{C}{18} \quad C = 18\,W\right)$$

or, in other words,

Coefficient of resistance of any given material = 18 times the center breaking load of a beam of the given material, 1 inch square, 1 foot span, supported at both ends

We thus see that the foregoing theory is identical, in principle, with the practical method for beams of solid cross-section, given on pages 491, &c. For beams of composite cross-section, such as hollow beams, I beams, &c, the theory, as already remarked, gives results differing from those of experiment. Thus:

What is the center breaking load of a solid cast-iron beam 4 ins square, and 6 ft, or 72 ins, clear span, supported at both ends?

Here the moment of inertia is $\frac{4 \times 4^3}{12} = \frac{4 \times 64}{12} = \frac{256}{12} = 21.333$. The coeff of resistance, or 18 times our constant for cast-iron on p 493, is $2025 \times 18 = 36450$. Since the beam is supported at both ends, and loaded at the middle, m is 4. The dist $o\,g$ of the farthest fibre from the neutral axis must in a square be equal to $\frac{1}{2}$ of one side; consequently it is here 2 ins. The clear span is 72 ins. Hence,

$$\text{Breakg load} = \frac{\text{Mom of In} \times \text{Coeff of res} \times m}{\text{Dist } o\,g \text{ of farthest fibre} \times \text{span}} = \frac{21.333 \times 36450 \times 4}{2 \times 72} = \frac{3110352}{144} = 21600 \text{ lbs} = 9.64 \text{ tons.}$$

By our table, p 502, a beam of average cast-iron, 4 ins deep, 1 inch broad, and 6 ft span, breaks with 2.41 tons; consequently, four such, or one 4 ins square, would break with $2.41 \times 4 = 9.64$ tons; thus confirming the accuracy of the foregoing. Applied in the same way to solid cylinders, the result corresponds equally well with experiment and with our table on p 503.

But for Mr Clark's hollow squares, p 516, the formula gives 3.06 tons instead of the actual 2.15; and for his hollow cylinders 2.980 instead of 2.287. **A true Hodgkinson beam,** p 518, with top flange of 1 by 3 ins, bottom flange 1.5 by 12 ins, vert web .75 inch thick, total depth 15 ins, clear span 20 ft, has a moment of inertia of 780; dist from neutral axis to upper fibre 10.7 ins, and to the lowest one 4.3 ins. By Hodgkinson it would yield at the lower flange, and by his rule with a center load of 29.24 tons. By the formula it would be 19.8 tons. **Beam *l*,** p 520, actually broke with about 52 tons; by the formula it would be 40.

The Coeff of resistance for average rolled iron is about 45000 lbs or say 20 tons per sq inch. Cast-iron 36000 lbs or 16 tons. Good straight-grained, well-seasoned white pine or spruce 8100 lbs or 3.6 tons; yellow pine 9000 or 4; good oaks 10000 or nearly 4.5. But as large beams are liable to defects and imperfect seasoning, not more than about two-thirds of these constants should be used in practice.

Comparison between models and actual structures. Many practical men imagine that if a model is strong, an actual bridge, roof, &c, constructed with precisely the same proportions, must be equally strong in proportion to its size. This arises from their ignorance of the fact that the **strength** of similar beams, trusses, &c, increases only in proportion to the **squares** of their spans; while their **weight** increases as the **cubes** of the spans; so that a model 5 or 10 ft long may show a great surplus of strength; while the roof or bridge of 50 or 100 ft span, constructed like it in every respect, may break down under its own weight.

We may compare the two in the following manner: Let us suppose a model 4 feet long of a bridge truss, its wt 6 lbs, and the extraneous center load reqd to break it 120 lbs, or 20 times its own wt. Then its entire center breakg load, including half its own wt, is $120 + 3 = 123$ lbs. Now suppose we are going to build a bridge truss of 200 ft, or 50 times the span of the model. The strength of the truss will be 50^2, or 2500 times that of the model; that is, it will require for its entire center breakg load, $50^2 \times 123 = 2500 \times 123 = 307500$ lbs. Its wt, however, will be 50^3, or 125000 times that of the model, or $125000 \times 6 = 750000$ lbs; and one-half of this weight, or 375000 lbs, must be deducted from its entire center strength, in order to find its extraneous center load. But in this case the half *weight* is greater than the entire center *strength;* consequently the truss would break under its own wt. If, instead of a center load in the model, we had broken it by an equally distributed one, the calculation would plainly be the same, except that in the model the entire weight, instead of $\frac{1}{2}$ of it, would be added to the extraneous load for the entire distributed breakg load; and in the truss, its whole weight must be deducted from its breaking strength, to get the extraneous distributed load.

If the breaking load of a model is 2, 3 or 4, &c, times as great as its weight, then a similar structure 2, 3 or 4, &c, times as large *in every particular* will break under its own weight.

PRACTICAL METHODS FOR FINDING STRENGTHS OF BEAMS.

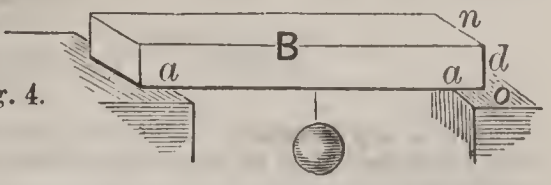


Fig. 4.

To find constants. In beams of the same material, and exactly alike, except in their breadths, *n d*, the strengths vary in the same proportion as those breadths; that is, if one is 2, 3, or 10 times broader than the other, its strength will be 2, 3, or 10 times as great. If they are alike, except in their clear lengths or spans, *a a*, between the points of support, their strengths will be *inversely* as those lengths; that is, if one is 2, 3, or 10 times longer than the other, it will be but ½, ⅓, or $\frac{1}{10}$ part as strong. If they are alike, except in point of depth, *o d*, measured vert, their strengths will be *directly* as the *squares* of their depths; that is, if one is 2, 3, or 10 times as deep as the other, it will also be 4, 9, or 100 times as strong; or in other words, will require 4, 9, or 100 times as great a load to break it. See Art. 11. It must be remembered that we are now speaking only of *strength*, or resistance to *breaking;* and not of *stiffness*, or resistance to *bending*, or *deflecting*. **Stiffness follows laws very diff from those of strength.** See Art 26, &c.

Now, if we combine all the three foregoing elements of size, namely, length, breadth, and depth, we have the fact, that the strength of any beam, of any size, of any given material, is in proportion to its $\frac{\text{breadth} \times \text{the square of its depth.}}{\text{its length}}$ Therefore, if we find by actual trial, what center load will break any beam of known size; and then find what is the proportion between its $\frac{\text{breadth} \times \text{sq of depth}}{\text{length}}$, and its breakg load, said proportion (or, more strictly speaking, *ratio*) will also be that which any similar beam has to *its* breakg load, and will therefore serve to calculate the breakg load of any other similar beam of the same material. For instance, if we take any piece of average good white pine, say 6 ins broad, 10 ins deep, and 12 feet clear span, we find that its $\frac{\text{breadth} \times \text{sq of its depth}}{\text{length}}$ is equal to $\frac{6 \times 100}{12} = 50$. And if we gradually load this at its center until it breaks, we shall find that the breakg load, including half the wt of the clear span of the beam itself, amounts to 22500 lbs. Therefore, the proportion between the $\frac{\text{breadth in ins} \times \text{sq of depth in ins}}{\text{length in feet}}$, and the breakg load, is as 50 to 22500; which is the same as 1 to 450; that is, the breakg load of the beam, including half its own weight, may be found by mult its $\frac{\text{breadth in ins} \times \text{sq of depth in ins}}{\text{length in feet}}$, by 450. And in this same manner may be found the total center breakg load of any rectangular beam of average quality of white pine. For the *neat* load one-half the wt of the clear span must be deducted. It is self-evident that the weight of the beam assists to break it, as well as the neat load; and the extent to which it does so, is the same as if *one-half* of its unsupported wt were concentrated at its center. Hence the rule. On this principle the rule in Art 12 is based. The ratio thus found for any material, is called its **coef for cen breakg loads,** or its **constant** for the same, as it does not vary with the size of the beam. If we take a piece, all of whose dimensions are 1, as 1 inch wide, 1 inch deep and 1 ft span; then the $\frac{\text{breadth in ins} \times \text{sq of depth in ins}}{\text{length in feet}}$ will be $\frac{1 \times \text{sq of } 1}{1} = 1$; and the breakg load (including half its own weight) of such a piece is, at once, the constant reqd. See Remarks 1 to 4. In an average piece of white pine, this load will be found to be about 450 lbs; or the same as the constant obtained from the large beam. **So the student may find them for himself;** and if he uses materials not included in our table, Art 10, it will be well to supply the deficiency, by inserting his own results.

The foregoing directions for finding coefficients may be more briefly expressed

by a formula. After finding the neat center breakg load, by experiment, add to it one-half the wt of the clear span of the beam, for a total center load; then the

$$\left.\begin{matrix}\text{Coef for breakg}\\ \text{strength}\end{matrix}\right\} = \frac{\begin{matrix}\text{Span}\\ \text{in feet}\end{matrix} \times \begin{matrix}\text{Total load}\\ \text{in lbs}\end{matrix}}{\begin{matrix}\text{Breadth}\\ \text{in ins}\end{matrix} \times \begin{matrix}\text{Square of depth}\\ \text{in inches.}\end{matrix}}$$

In a cylinder, as the breadth and the depth are each equal to the diam, it is plain that $B \times D^2$, amounts to the same thing as $diam^3$; and it is always so expressed.

Rem. 1. The variation in strength of equal beams of the same material is so great, that it is necessary to experiment with several pieces, in order to find an average for a constant. The loads given in the preceding tables are also constants, but for crushing and tension. They are averages of the strengths of the materials, derived from experiment. The actual strength of any particular specimen, if of superior quality, may be considerably greater than the average; or on the other hand, if of very poor quality, it may fall as much below it. We should always keep this in mind when referring to any table of constants; and if we have doubts as to the quality of the piece of material which we are about to employ, we should make a corresponding deduction from the constant in the table.

Rem. 2. If, instead of pine, we had experimented with oak, iron, stone, the process for finding the constant would have been precisely the same. **If instead of a square** beam, we use cylindrical, or triangular ones, or any other shape, such as hollow cylinders, H, T, or U beams, &c, we shall in the same way establish constants for either larger or smaller beams of those *shapes*, and of precisely the same *proportions* in every part. See Remark, p 509. Or if, instead of supporting the beam at both ends, we secure it firmly at one end, and load it at the other end until it breaks, we shall obtain the *constant for beams fixed at one end, and loaded at the other*, &c. **Remember that the constants are for loads at rest.** If they are liable to jars, jolts, vibrations, &c, a large margin must be left for safety. Moreover, the constants given in tables are generally deduced from small specimens free from important defects; whereas large beams of any kind of material usually contain irregularities, which diminish their strength; and on this account larger allowances for safety should be made as the dimensions of the beam increase.

Rem. 3. It is not necessary that *b* and *d* be taken in ins, and lengths in ft. They may all be in ins, ft, yds, or any other measure, but since in every-day practice we usually speak of the breadths and depths of beams in ins, and of their lengths in ft, it becomes more *convenient* so to consider them. If other measures be used, the constant will of course be diff; but it will still be such that if the same measure be used for calculating the strength of another beam, the final result will be the same as before. In like manner, the loads may all be taken in tons, &c, instead of lbs; but in giving the rule, it must be stated what measures have been employed.

Rem. 4. There are **peculiarities in some materials,** which lessen the reliability of constants derived from experimenting with small pieces. Thus, a large beam of cast iron will break with a less load in proportion than a small one; because, in the interior of thick masses of that material, more time to cool is required than in the outer surfaces; in consequence of which, there is a want of uniformity in the arrangement of the particles of iron, and this conduces to weakness. All we can do in such cases, is to exercise judgment and caution in making sufficient allowance for safety.

Art. 10. Table of constants or coefficients for the quiescent breaking loads of rectangular beams, supported horizontally at both ends, and loaded at the center; being the average quiescent breaking loads in lbs (including one-half the weight of the beams themselves) for beams 1 inch square, and 1 foot *clear* length between the supports. For *safety in practice*, not more than about $\frac{1}{3}$ to $\frac{1}{8}$ of these constants should be employed; depending upon the importance of the structure, its temporary or permanent character, and the degree of vibration to which it will be exposed. Thus a roof will probably be as safe at $\frac{1}{3}$, as a bridge at $\frac{1}{6}$. Even with a perfectly *safe* load, a beam may *bend* too much. See Art 26.

If any of these coefficients be mult by .589 (or say .6) it will give that for a cylindrical beam whose diam = side of the square. Or if mult by .71 it will give that for a square beam with its diagonal vertical. **Any of these constants may vary one-third part either more or less.** For any beam,

$$\begin{matrix}\textbf{Cen. Breakg}\\ \textbf{load} \text{ in lbs.}\end{matrix} = \frac{\text{Breadth (ins)} \times \text{Square of depth (ins)}}{\text{clear span in feet}} \times \text{Constant.}$$

One Third part of any of these constants (except those for wrought iron and steel), may be taken in ordinary practice as about the average **constant for the greatest center load within the elastic limit.** The loads here given for wrought iron and steel, are already the greatest within elastic limits.

Transverse Strengths, in lbs. See explanation, Art 10, p 492.

In wooden beams in practice deduct one-third part, to allow for knots, crooked grain, &c.

WOODS.	
Ash, English	650
" Amer White (Author).	650
" Swamp	400
" Black	600
Arbor Vitæ, Amer	250
Balsam, Canada	350
Beech,	
" Amer	850
Birch, Amer Black	550
" Amer Yellow	850
Cedar, Bermuda	400
" Guadaloupe	600
" Amer White, or Arbor Vitæ	250
Chestnut	450
Elm, Amer White	650
" Rock, Canada	800
Hemlock	500
Hickory, Amer	800
" " Bitter nut	800
Iron Wood, Canada	600
Locust	700
Lignum Vitæ	650
Larch	400
Mahogany	750
Mangrove, White	650
" Black	550
Maple, Black	750
" Soft	750
Oak, English	550
" Amer White (by Author).	600
" " Red, Black, Basket	850
" Live	600
Pine, Amer White...(by Author).	450
" " Yellow. " " .	500
" " Pitch. " " .	550
" Georgia	850
Poplar	550
Poon	700
Spruce (by Author).	450
" Black	550
Sycamore	500
Tamarack	400
Teak	750
Walnut	550
Willow	350
METALS.	
Brass	850
Iron, cast, 1500 to 2700....average	2100
" " common pig	2000
" " castings from pig	2300
" " employed in our tables	2025
" " for castings 2½ or 3 ins thick	1800?
Iron, wrought, 1900 to 2600.....av	2250

Wrought iron does not *break*: but at about the average of 2250 lbs its elas limit is reached.

Steel, hammered or rolled; elas destroyed by 3000 to 7000..	5000

Under heavy loads hard steel snaps like cast iron, and soft steel bends like wrought iron.

STONES, ETC.	
Blue stone flagging, Hudson River	125
Brick, common, 10 to 30..average	20
" good Amer pressed, 30 to 50 .....average	40
Cuen Stone	25
Cement, Hydraulic, English Portland, artificial, 7 days in water	30
1 year in water	50
" " Portland, Kingston, N. Y., 7 days in water.	30
" " Saylor's Port., 7 days in water.	26
" " Common U. S. cements, 7 dys in water	5

The following hydraulic cements were made into prisms, in vertical moulds, under a pressure of 32 lbs per sq inch, and were kept in sea water for 1 year.

Portland Cement, English, pure, 1 year old	64
Roman Cement, Scotch, pure	23
American Cements, pure, av about	25
Granite, 50 to 150.....average	100
" Quincy	100
Glass, Millville, N. Jersey, thick flooring ....(by Author).	170
Mortar, of lime alone, 60 days old	10
" 1 measure of slacked lime in powder, 1 sand	8
" 1 measure of slacked lime in powder, 2 sand	7
Marble, Italian, White (Author)	116
" Manchester, Vt, " "	95
" East Dorset, Vt, " "	111
" Lee, Mass, " "	86
" Montg'y Co, Pa, Gray "	103
" " " Clouded "	142
" Rutland, Vt, Gray "	70
" Glenn'sFalls, N.Y. Black "	155
" Baltimore, Md, white, coarse "	102
Oolites, 20 to 50	35
Sandstones, 20 to 70 .....average	45
" Red of Connecticut and New Jersey	45
Slate, laid on its bed, 200 to 450, av	325

Art. 11. General facts respecting the breakg loads of a uniform beam of any form of section. Calling the breakg load,

When the beam is firmly fixed at one end, and loaded at the other 1
Then when so fixed, and uniformly loaded, it will be..... 2
When merely *supported* at both ends, and loaded at the center..... 4
" " " " " and uniformly loaded 8
Firmly *fixed*, or tightly *confined* at both ends, and loaded at the center... 8
" " " " " and uniformly loaded..... 16

By some authorities the last two are given at 6 and 12. The term "fixed" in such cases is rather indefinite Prof. De Volson Wood gives 12 for the last.

Rem. 1. When one beam of any form of section is 2, 3, or 4, &c, times as long, broad, and deep, as another, its weight will be 8, 27, or 64, &c, times as great, or as the *cubes* of the linear dimensions; but its breaking load will be only 4, 9, or 16, &c, times as great; that is, the strengths of similar beams are to each other as the *squares* of their respective linear dimensions. But in these breakg loads are included the weights of the beams themselves; one-half of which must be deducted when the load is all at the middle of the beam; or the whole of it when equally distributed. When beams are of the moderate dimensions usually employed in buildings, their own weight is usually so small in comparison with their loads, that at times it may safely be neglected; but as they become longer, since their weight increases much more rapidly than their strength, it at last constitutes too important an item to be overlooked; for the beam may become so great as to break under its own weight; although proportioned precisely like a small one which may safely bear many times *its* own weight.

When a square beam is supported on its edge, instead of on a side (or in other words has its diag vert), it will bear but about $\frac{7}{10}$ths as great a breakg load.

The deflections or bendings of beams (see p 505) are directly in proportion to the load, and to the cube of the length; and inversely to the breadth, and to the cube of the depth, all while within limits of elasticity.

Rem. 2. It is very important to remember that a beam will bear a much greater load placed upon it in small amounts at a time; or (in case it is all applied together) if its pres be allowed to come upon the beam very gradually, than if it be placed upon it suddenly, even without any jarring, and without any previous momentum. When applied in small amounts, or gradually, the bending takes place slowly and with slight momentum; but when applied all at once, the great load descends immediately and rapidly to the full extent of the bending due to the load itself, and in so doing acquires a momentum which carries it still further; thus producing a strain which authorities maintain to be just twice as great as in the former case. A heavy train coming very rapidly upon a bridge, presents a condition intermediate between the two. The tables of constants always suppose the load to be applied very gradually, but as this is frequently not the case in practice, an allowance must be made accordingly.

Art. 12. To find the quiescent breakg load of a hor square, or rectangular beam B. p 491, of any material, supported at both ends, and loaded at the centre.

Rule. Mult together the square of its depth d *o* in ins, its breadth n d in ins, and the coef from the table p. 493. Div the prod by the clear length a a between the supports, in ft. The quot will be the reqd breakg load in lbs; including, however, half the wt of that portion of the beam which lies *between* the supports, and which must be deducted in order to get the neat load.

Ex. What will be the center breakg load of a beam of Connecticut red sandstone, 15 ins wide, 10 ins deep, and 12 ft long between its supports? Here

$$\frac{\text{Depth}^2 \text{ in ins} \times \text{Breadth in ins} \times \text{Constant}}{\text{Clear length in ft}} = \frac{100 \times 15 \times 45}{12} = \frac{67500}{12} = 5625 \text{ lbs};$$

the breakg load reqd.

But we must deduct *one-half* the wt of the clear length of the beam. Now a beam of red sandstone, of 15 ins, by 10 ins, by 12 ft, contains 21600 cub ins = 12½ cub ft; and a cub ft of red sandstone weighs about 140 lbs; therefore the beam weighs $12.5 \times 140 = 1750$ lbs; one-half of which, or 875 lbs, must be taken from the 5625 lbs of breakg load, leaving 4750 lbs as the actual extraneous, or neat breakg load.

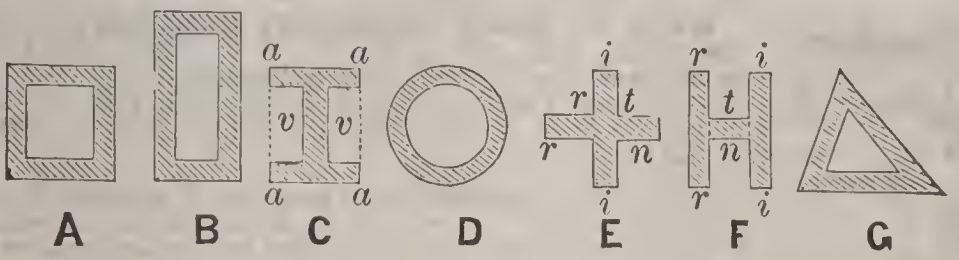
Rem. 1. If cylindrical, first find the breakg load of a square beam, of which each side is equal to the diam of the cylinder. Mult this load by the dec .6, or, more correctly, .589. Hence a square one is 1.7 times as strong as a cylindrical one.

If oval, or elliptic, first find the load for a rectangular beam, whose sides are respectively equal to the two diams, and mult it by .6.

If of wood, and triangular, and its base (whether up or down) hor, first find the breakg load for a rectangular beam, whose breadth is equal to the base; and its depth equal to the perp height of the triangle; and take one-third of the result as an approximation. When the edge is down, the ends must rest in triangular notches in the supports; otherwise, they will be crushed when loaded. See Art 16.

For beams of such sections as A to G, the following rude rules of thumb will often be preferred to more intricate ones, being sufficiently approximate for ordinary purposes, and for any material. See near end of Art 33, p 516.

For the closed Figs A, B, D, G, (each one supposed to be of equal thickness throughout,) first find the load for a solid beam of the same size and shape.



Then find that of a beam of the size of the hollow part. Subtract the last from the first.

For C, (its top and bottom being of equal size,) first find for a rectangular beam $a\ a\ a\ a$. Then for two beams corresponding to the two hollows $v\ v$. Subtract these last from the first.

For E or F, find for three separate beams $r\ r$, $i\ i$, $t\ n$, and add them together.

For angle and Tiron, see p 525.

For U and other shapes in common use, we may use the formula, p 488, or experiment with a model made of the given material, and thus find the necessary constant, as directed, p 491. See Remark, p 509; also Art 33, p 516.

For I beams, see Art 37, and **for Hodgkinson beams,** see Art 35.

Rem. 2. In this case we may remove ¼ part of the material of the solid square, or rectangular beam, without diminishing its breaking strength, although it will bend more. The width may remain uniform, and the depth be reduced either at top or bottom, as shown by the dotted lines at m, Fig 5, strictly two parabolas with bases at load, but the straight ones are best in practice. Or the depth may remain uniform, and the breadth be reduced, as shown by the dots at n, which is a top view of the beam. Theoretically, the dotted lines in n might meet at the ends of the beam; but in practice this would not generally leave sufficient material at the ends for the beam to rest upon securely.

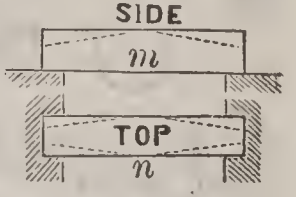


Fig. 5.

Such reductions of beams are rarely made when they are of wood; but in iron ones much expense may be saved thereby.

Rem. 3. Load at one end of the beam, Fig 6, the other end fixed, imagine the load to be at the center, and calculate it by the foregoing rule. Then div the result by 4.

In this case the lower side of the beam may be cut away in the form of a parabola, as shown by the dots. To draw this curve, see page 153. Or the depth may be left uniform, and the sides be cut away, as shown by the dots at t, which is a top view.

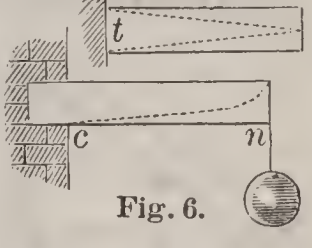


Fig. 6.

Art. 13. When the load is equally distributed along the entire clear length of a horizontal beam, supported at both ends, as in Fig 7, instead of being all applied at the center, assume it to be at the center, and proceed precisely as in the foregoing rule, Art 12. Then mult the load by 2. But in this case the wt of the *entire* clear length of the beam is to be deducted for the neat load.

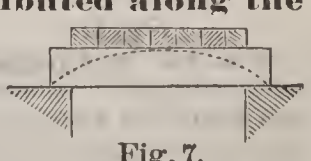


Fig. 7.

Ex. What will be the equally distributed breakg load of the beam of sandstone in the last example? Here the center breakg load has already been found to be

5625 lbs; and $5625 \times 2 = 11250$ lbs, tho reqd distributed load. From this subtract the wt of the entire 12 feet clear length of beam, or 1750 lbs; and the rem, 9500 lbs. is the neat extraneous breakg load. About $\frac{1}{10}$ part of this, or 950 lbs, is quite as much as should be trusted upon so variable and treacherous a material as red sandstone.

Rem. 1. A beam requires twice as much breaking load, *equally distributed*, as it will at its center. In this case the breakg strength of the beam will not be diminished if the top be cut away in the form of a true semi-ellipse, as shown by the dots in Fig 7. Or if the depth must be kept uniform, the sides may be trimmed to two parabolas $o\,c\,o$, $o\,t\,o$, Fig 8. The mode of drawing these figs to any span and height will be found under "Mensuration," pp 150, 153; bnt in practice circular segments will answer.

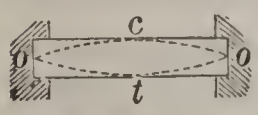


Fig. 8.

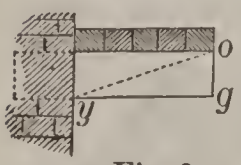
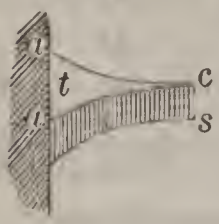


Fig. 9.

Rem. 2. Load uniformly distributed along the entire clear length, $y\,g$, Fig 9, of a hor rectangular beam, firmly fixed at one end only, assume it to be at the center, as in Art 12, and calculate it by the rule in that Art. Then div the result by 2. From the quot deduct *entire* wt of beam for *neat* load.

In this case, theoretically, we may cut off one-half the projecting part $y\,o$ of tho beam, as by the dotted line $y\,o$, without diminishing its breakg strength. But in practice it will rarely be advisable to reduce it to a mere thin edge at o. Or the depth $c\,s$ of tho beam may be left uniform, and the sides be cut away, as shown by the two semi-parabolas $a\,c$, $a\,c$, at t, which is the top of the beam. If $a\,c$, $a\,c$, be even made straight, instead of parabolas, it is plain that there would still be a considerablo saving of expense, if the beam is of iron.



Art. 14. When the entire breakg load is applied at any point o, Fig 10, not at the center: first find by Art 12, what would be the center breakg load; including half the weight of the beam. Then; making c the center of the span, and having the lengths $a\,o$, $o\,g$, $a\,c$ and $c\,g$;

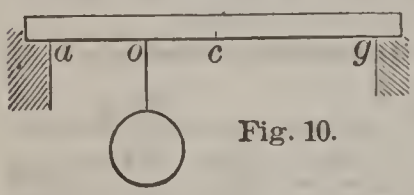


Fig. 10.

$$\text{extraneous concentrated breaking load at } o = \left(\text{said total center breaking load} \times \frac{ac \times cg}{ao \times og}\right) - \text{half the weight of the beam}$$

This rule is not exact if the load rests upon the beam for a short distance on either or on both sides of the point o, but only when it all rests at that very point alone; if it does not the load may be increased. See p 483.

Rem. 1. As a given load approaches either support, its *breaking moment decreases;* but its tendency to *shear* the beam between itself and the nearest support *increases.* See Rem, p 533.

Rem. 2. This beam will bear to be reduced, as at m or n. Fig 5; except, that instead of reducing from its center, as in Fig 5, we must do so from where the load is applied.

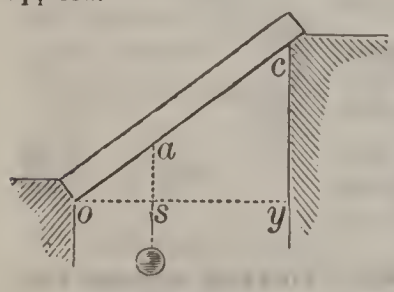


Fig. 11.

Art. 15. When the beam, instead of being hor, is inclined, as in Fig 11, in any of the foregoing cases, the *hor* dist $o\ y$ must be taken as its span, instead of the actual clear length $o\,c$; and $s\,o$, $s\,y$ instead of $a\,o$ and $a\,c$. This applies also to beams fixed at one end, and whether the inclination is upward or downward from the fixed end.

Note. The quantity of material in inclined beams may be reduced, in the same manner as in hor ones.

Art. 16. Triangular beams of wood, according to Barlow's experiments with pine, require abont $\frac{1}{6}$ greater breakg loads with the base up, than when it is down. Or with the base down, about $\frac{1}{7}$ less than when up. Tredgold considers them about equally strong in either position; and that to find the center breakg load, we may first calculate it by Art 12, as if the beam were a rectangular one with the same base and perp height as the triangle; and take $\frac{1}{3}$ of the result. Hence, the triangle is not an economical shape for a beam; for with only $\frac{1}{3}$ the strength of a rectangular one, it has half as much material.

Hodgkinson, with **cast-iron triangular beams,** base up,

made the breakg loads equal to ⅓ of those of rectangular bars, as in wood. Rennie's experiments give about the same proportion, with the base up; but with the base down, he made the strength nearly twice as great, or about $\frac{6}{10}$ that of a rectangular beam of the same width and vertical height. The comparative strengths in the two positions will vary in diff materials, inasmuch as it is affected by the comparative resistances which any given material presents to tension and compression. Within the limit of elasticity the beam will be equally strong, whether the edge or base be up; and will bend equally in either case; so also with the Hodgkinson, or any other form of beam.

Art. 17. To find the side of a square hor beam supported at both ends, and reqd to break under a given quiescent center load.

RULE. Mult the clear bearing in ft, by the given breakg load in pounds. Div the prod by the corresponding constant p 493. Take the cube root of the quot. This cube root will be the reqd depth or breadth of the beam, approximately, in ins. When the size of the beam is so great that its wt must be taken into consideration, increase either its breadth, as directed, in Art 20; or its depth, as per Art 21, or, first find the approx side as before. Then calculate the wt of a sq beam having that side. Add half this wt to the given cen load, and with this increased cen load, repeat the whole calculation. The resulting side will be the reqd one very approx, but still a mere trifle too small.

The breakg, or the safe load of a square beam, if mult by .6, will give that of a **cylinder,** whose diam is equal to a side of the square one.

Art. 18. When the beam is reqd to bear its center load safely, mult the given safe load by the number of times it is exceeded by the breakg load. Then find, by Art 17, the side of a square beam to *break* under this increased load. The beam thus found will evidently be approximately the *safe* one for the actual load; exclusive, however, of the wt of the beam. When this must be included, increase the dimensions as directed in Art 17.

If the load is equally distributed, first div it by 2, then proceed precisely as before.

Art. 19. When the beam is cylindrical, and reqd to break under its center load, to find its diam, mult the load by 1.7, and by Art 17 find the side of a square beam, to break under this increased load. The side thus found will also be approximately the reqd diam. See Rems 1 and 2.

If to be borne safely, first mult it by the number of times it is to be exceeded by the breakg load. Then mult the prod by 1.7, and proceed precisely as before. See Rems 1 and 2.

REM. 1. In neither case, however, is the wt of the beam itself included. When this is necessary, first find the approximate diam as before. Then calculate the wt of a beam having this diam Add half this wt to the given cen load, in either case; and with this increased center load, repeat the whole calculation. The resulting diam will be the required one *very* approximately, but still a mere trifle too small.

REM. 2. **If the load is equally distributed,** first take one-half of it as being a center load, and with this proceed precisely as before.

Art. 20. To find the breadth of a hor rectangular beam, supported at both ends, to break under a given quiescent center load; mult the center load in lbs by the span in feet. Mult the square of the depth in ins by the constant p 493. Div the first prod by the last. The quot will be the breadth approximately. Calculate the wt of a beam having this breadth. Then say, as the center load is to *half* this wt, so is the breadth found, to a new breadth to be added to it. It will still be somewhat too small, owing to the neglect of the wt of the breadth last added. This may readily be found, and its corresponding breadth added.

REM. 1. **If the load is to be borne safely,** (without any regard to the amount of deflection,) first mult it by the number of times it is exceeded by the breakg load.

REM. 2. If in either case **equally distributed,** take half of it as if a center load, and proceed precisely as before.

Art. 21. To find the depth, when the breadth is given, mult the load in lbs by the span in feet. Mult the breadth in ins by the constant p 493. Div the first prod by the last; take the sq rt of the quot for an approximate depth. Calculate the wt of a beam having the depth just found; add *half* of it to the given center load, and with this new load repeat the whole calculation; for a more approximate depth, but still somewhat too small, owing to the neglect of the wt of the depth last added. We may find this, and repeat the whole calculation, or we may merely increase the breadth by Art 20.

REM. If the load is to be borne safely, or if it is equally distributed, see Remarks, Art 20.

Art. 22. To find the safe dimensions to be given to a rectangular beam of given span, supported at both ends, and which is at the same time exposed both to a transverse strain and to a longitudinal tensile or pulling one, or a longitudinal compressive one. The writer is unable to suggest any better rules than the following, which are at least safe. Namely, when the longitudinal strain is tensile, find separately the safe dimensions as if for a beam alone; and as if for a tie alone; and add the two resulting areas together. When the longitudinal strain is compressive, find separately the safe dimensions as if for a beam alone; and as if for a pillar alone; and add the two resulting areas together.

Example 1. A wrought iron rectangular beam of 10 ft span is to sustain with a safety of 6, an equally distributed transverse load of 100000 lbs; and a pulling strain of 200000 lbs. Of what size must it be?

Here the distributed load of 100000 lbs is equal to a safe center one of 50000 lbs; or to a breaking center one of $50000 \times 6 = 300000$ lbs.

Now first we may assume for the beam some probable approx depth, say 12 ins. Then we find by Art 20 that its breadth as a beam alone will be

$$\frac{\text{Breakg load in lbs} \times \text{span in ft}}{\text{sq of depth in ins} \times \text{coef, p 493}} = \frac{300000 \times 10}{144 \times 2500} = \frac{3000000}{360000} = 8.33 \text{ ins.}$$

Again, a bar to bear a pull of 200000 lbs with a safety of 6, should not break with less than 1200000 lbs; therefore since fair bar iron breaks with about 50000 lbs per sq inch, we have $1200000 \div 50000 = 24$ sq ins as the area of bar for the pull alone. We may add all of this to the width of the beam, making it $24 \div 12 = 2$ ins wider; or 10.33 ins wide in all. Or we may add it all to the depth, thus making the beam $24 \div 8.33 = 2.88$ ins deeper, or 14.88 ins in all. Or part may be added to the breadth, and part to the depth.

Example 2. A wrought iron rectangular beam of 10 ft span, is to sustain with a safety of 6, an equally distributed transverse load of 100000 lbs, and a compressive strain of 200000 lbs. Of what size must it be?

Here first assuming some probable approx depth, say 12 ins, we find as before that its breadth as a beam only will be 8.33 ins. As to the compressive force, it is plain that a pillar for sustaining it should be a hollow one with its sides as wide as possible; and this is to be effected by placing it around the outside of our beam. The pillar will therefore have sides of about 8.33 and 12 ins wide; and its breaking load must be $200000 \times 6 = 1200000$ lbs, or say 536 tons. Now the length of the pillar measured by its narrowest side is $120 \div 8.33 = 14.4$ sides; and by table, p. 444, we find that a hollow square wrought iron pillar 14.4 sides long, breaks with 15.5 tons per sq inch of its metal area. Hence we require $536 \div 15.5 = 34.6$ sq ins metal area for our pillar. Now the circumf of the pillar is $8.33 \times 2 + 12 \times 2 = 40.66$ ins. Hence its thickness must be $34.6 \div 40.66 = .85$ of an inch. Hence both the breadth and the depth of the beam must each be increased twice that much or 1.7 inch; thus making it 10.03 ins broad and 13.7 ins deep.

It is plain that our pillar is thicker than necessary, because the tabular widths are supposed to be by outside measure, whereas our width of 8.33 ins is inside measure. The final outer width of 10.03 ins would make the pillar only 12 sides long; at which it would require 15.7 instead of 15.5 tons per sq inch to break it. Other considerations too abstruse to be explained here, combine to make the resulting dimensions in both examples somewhat in excess.

Art. 23. Table of safe quiescent loads for horizontal rectangular beams of white pine or spruce, one inch broad, supported at both ends, and loaded at the center; together with their deflections under said loads.

The safe load is here one-sixth of the breaking load.

For the neat loads, deduct ½ the wt of the beam itself. The *deflections*, however, are the *actual* ones; the wts of the beams having been introduced in calculating them, by the rule in Art 27.

Loads applied suddenly will double the deflections in the table; as when, for instance, if a load is held by hand, just *touching* a beam, the hold should be *suddenly* loosed.

Caution. Inasmuch as this table was based upon well seasoned, straight grained pieces, free from knots, and other defects, we must not in practice take more than about two-thirds of the loads in the table for a safety of 6 in ordinary building timber of fair quality; and with these reduced loads should not reduce the deflections.

Observe also that our table is for safe **center** loads, but it is plain that in practice we cannot always apply the term in its utmost strictness; otherwise the load would have to be sustained by a mere knife-edge, at the *very* center of the beam. Now, in the instance Rem. p. 500, if we attempted to sustain the center load of 6075 lbs upon such a knife-edge, it would at once cut the beam in two. If we even applied it along 3 or 4 ins of the length, it would cut into it, and we should not have a safety of 6 against crushing the top of the beam until as in the case of the ends we distributed the load along full 46 ins of length, or about 32 ins for a safety of 4.

The safe load is here ⅙ of the breakg one; and the last at 450 lbs at the center of a beam 1 inch square, and 1 foot clear length between its supports. For mere temporary purposes, ½ part may be added to the loads in the table, thus making them equal to the ¼ of the breakg load. But in important structures, subject to vibration, ¼ part should be deducted from the tabular loads, thus reducing them to ⅛ of the breaking load. This is especially necessary if the timber is not well seasoned.

With the safe loads in this table a beam may bend too much for many practical purposes. When this is the case, we may, by reducing the loads, reduce the deflections in nearly the same proportion; or see table, p 512.

All the loads in the Table are superabundantly **safe against shearing. Against crushing at the ends,** &c, see **"Cautions"** below the Table, p 500. Original.

Depth of beam. Ins.	Span 4 ft. load lbs.	Span 4 ft. def. ins.	Span 6 ft. load lbs.	Span 6 ft. def. ins.	Span 8 ft. load lbs.	Span 8 ft. def. ins.	Span 10 ft. load lbs.	Span 10 ft. def. ins.	Span 12 ft. load lbs.	Span 12 ft. def. ins.	Span 14 ft. load lbs.	Span 14 ft. def. ins.	Span 16 ft. load lbs.	Span 16 ft. def. ins.	Wt. of 10 ft of beam. lbs.
1	19	.39	13	.92	10	1.8	8	3.0	6	4.4	.....	.....	.....	.....	2
2	75	.22	50	.45	38	.82	30	1.3	25	1.9	21	2.7	19	3.7	4
3	170	.13	114	.30	85	.53	67	.84	57	1.3	48	1.7	42	2.3	6
4	300	.10	200	.22	150	.39	120	.63	100	.92	86	1.3	75	1.7	8
5	469	.08	312	.18	234	.31	187	.50	156	.72	134	1.0	117	1.3	10
6	675	.06	450	.15	337	.26	270	.41	225	.60	193	.83	168	1.1	12
7	919	.06	612	.12	460	.22	367	.35	306	.51	262	.70	230	.93	14
8	1200	.05	800	.11	600	.19	480	.31	400	.45	343	.61	300	.81	16
9	1520	.04	1014	.10	760	.17	607	.27	507	.40	434	.54	380	.72	18
10	1875	.04	1250	.09	937	.16	750	.24	625	.35	536	.49	468	.64	20
11	2270	.04	1514	.08	1135	.14	907	.22	757	.32	648	.44	567	.58	22
12	2700	.03	1800	.07	1350	.13	1080	.20	900	.29	772	.40	675	.53	24
14	3675	.03	2450	.06	1837	.11	1470	.17	1225	.25	1050	.34	918	.45	28
16	4800	.02	3200	.05	2400	.10	1920	.15	1600	.22	1372	.30	1200	.40	32
18	6075	.02	4050	.05	3037	.09	2430	.14	2025	.20	1736	.27	1518	.35	36
20	7500	.02	5000	.04	3750	.08	3000	.12	2500	.18	2145	.24	1875	.31	40
22	9075	.02	6050	.04	4537	.07	3630	.11	3025	.16	2593	.22	2268	.29	44
24	10800	.02	7200	.04	5400	.06	4320	.10	3600	.15	3088	.20	2700	.26	48

(Continued on next page.)

Table, continued. (Original.)

Depth of beam. Ins.	Span 18 ft. load lbs.	Span 18 ft. def. ins.	Span 20 ft. load lbs.	Span 20 ft. def. ins.	Span 25 ft. load lbs.	Span 25 ft. def. ins.	Span 30 ft. load lbs.	Span 30 ft. def. ins.	Span 35 ft. load lbs.	Span 35 ft. def. ins.	Span 40 ft. load lbs.	Span 40 ft. def. ins.	Wt. of 10 ft of beam. lbs.
6	150	1.4	135	1.8	108	2.9	90	4.5	77	6.5	67	9.2	12
7	204	1.2	184	1.5	147	2.5	122	3.9	105	5 8	92	7.6	14
8	267	1.0	240	1.3	192	2.1	160	3.2	137	4.6	120	6.4	16
9	338	.92	304	1.2	243	1.9	202	2.8	174	4.0	152	5.5	18
10	417	.82	375	1.0	300	1.7	250	2.5	214	3.5	188	4.9	20
11	505	.74	454	.93	363	1.5	302	2.2	259	3.2	227	4.3	22
12	600	.68	540	.85	432	1.4	360	2.0	308	2.9	270	3.9	24
14	817	.58	735	.72	588	1.2	490	1.7	420	2.4	367	3 2	28
16	1067	.50	960	.63	768	1.0	640	1.5	548	2.1	480	2.8	32
18	1350	.45	1215	.56	972	.90	810	1.3	694	1.8	607	2.5	36
20	1666	.40	1500	.50	1200	.79	1000	1.2	857	1.6	750	2.2	40
22	2017	.37	1815	.45	1452	.72	1210	1.1	1037	1.5	907	2.0	44
24	2400	.33	2160	.41	1728	.65	1440	.96	1234	1.3	1080	1.8	48
26	2817	.31	2526	.38	2018	.60	1684	.88	1449	1.2	1263	1.6	52
28	3267	.28	2940	.35	2352	.55	1960	.81	1680	1 1	1470	1.5	56
30	3750	.26	3375	.33	2700	.50	2250	.76	1928	1.1	1687	1.4	60
32	4267	.25	3840	.30	3072	.45	2560	.71	2194	1.0	1920	1.3	64
34	4817	.23	4335	.29	3468	.44	2890	.67	2477	.92	2167	1.2	68
36	5400	.22	4860	.27	3888	.43	3240	.63	2777	.86	2430	1.1	72

White oak, and best Southern pitch pine will bear loads ¼ greater.

For cast iron, mult the **loads** in the table by 4.5; and for **wrought** by 5.3. For these new loads, mult the **defs** by .4 for cast; and by .3 for wrought.

If the load is equally distributed over the span, it may be twice as great as the center one, and the defs will be 1¼ times those in the table. **If the loads in the table** be equally distributed along the whole beam, the defs will be but five-eighths as great as those in the table. See Art 26, p 505. When more accuracy is reqd, half the wt of the beam itself must be deducted from the center load; and the whole of it from an equally distributed load. The wt of the beam, in the last column, supposes the wood to be but moderately seasoned, and therefore to weigh 28.8 lbs per cub ft.

Uses of the foregoing table. Ex. 1. What must be the breadth of a hor rect beam of wh pine, 18 ins deep, supported at both ends, and of 20 ft clear length between its supports, to bear safely a load of 5 tons, or 11200 lbs at its center? Here, opposite the depth of 18 ins in the table, and in the column of 20 feet lengths, we find that a beam 1 inch thick will bear 1215 lbs; consequently, $\frac{11200}{1215} = 9.22$ ins, the reqd breadth; for the strength is in the same proportion as the breadth.

Ex. 2. What will be the safe load at the center of a joist of white pine, 18 ft long, 3 ins broad, and 12 ins deep? Here, in the col for 18 ft, and opposite 12 ins in depth, we find the safe load for a breadth of 1 inch to be 600 lbs; consequently, $600 \times 3 =$ 1800 lbs, the load reqd.

Rem. **Cautions in the use of the above table.** For instance, in placing very heavy loads upon short, but deep and strong beams, we must take care that the beams rest for a sufficient dist on their supports to prevent all danger from *crushing* at the ends. Thus, if we place a load of 6075 lbs at the center of a beam of 4 feet span, 18 ins deep, and only 1 inch thick, each end of the beam sustains a vert crushing force of $\frac{6075}{2} = 3037$ lbs, and that **sidewise of the grain,** in which position average white pine, spruce, and hemlock crush under about 800 lbs per sq inch, and do not have a safety of 6 until the pressure is reduced to about 133 lbs per sq inch. Therefore our beam, in order to have a safety of 6 against crushing at its ends, must rest on each support $3037 \div 133 = 23$ sq ins; or for a safety of 4 nearly 16 sq ins. When a pressure is equally distributed sidewise (that is, at right angles to the general direction of the fibres) over the entire pressed surface of a block or beam (to ensure which, the opposite surface must be supported throughout its entire length) the resulting compression might readily escape detection unless actually measured. But when a considerable pressure is applied to only a portion of the surface, as of caps and sills where in contact with the heads and feet of posts, or at the ends of loaded joists or girders, the compression becomes evident to the eye, because the pressed parts sink below the unpressed ones, in consequence of the bending or breaking of the adjacent fibres. What in the first case (especially if slight) would be called **compression,** would

in the second be called **crushing**; even when neither might be so great as to be unsafe.

Owing to the resistance which said adjacent fibres oppose to being bent or broken, it is plain that a given pressure **per sq inch.** or **per sq foot.** &c., will cause somewhat less compression or crushing when applied to only a part of a surface, than when to the whole of it.

The writer has seen 40 half seasoned hemlock posts, each 12 ins square, footing at intervals of 5 ft from center to center, upon similar 12 × 12 inch hemlock sills, to which they were tenoned, and which rested throughout their entire length on stone steps. Each post was gradually loaded with 32 tons, or equal to say 500 lbs per sq inch; and their feet all crushed into the sills from ¼ to ½ inch. Their heads crushed into the caps to the same extent. **In practice** the pressure at the heads and feet of posts is rarely, if ever, perfectly equable; and the same remark applies to the ends of loaded joists, girders, &c., in which a slight bending will throw an excess of pressure upon the inner edges of their supports.

See other cautions on p. 499.

STONE BEAMS.

Table of safe quiescent extraneous loads for beams of good building granite one inch broad, supported at both ends, and loaded at the center; assuming the safe load to be one-tenth of the breaking one; and the latter to be 100 lbs for a beam 1 inch square, and 1 foot clear span. The half weight of the beams themselves is here already deducted by the rule in Art 12. at 170 lbs per cub ft.

Depth in ins.	CLEAR SPANS IN FEET.											
	1	2	3	4	5	6	7	8	10	12	15	20
	Safe center loads in pounds.											
1	10	5										
2	40	20	13	10								
3	90	45	29	21	17							
4	160	79	52	39	31	26	21					
5	250	124	82	61	48	40	34					
6	360	179	119	89	70	58	48	42	32			
7	490	244	162	120	96	79	67	58	45	36	27	16
8	639	319	212	158	126	104	88	76	59	47	36	22
10	999	499	331	248	197	163	139	120	94	76	58	38
12	1439	718	478	357	284	236	201	174	137	111	85	58
14	1959	978	650	487	388	322	274	238	188	153	118	81
16	2559	1278	850	636	507	421	359	312	246	201	157	109
18	3239	1618	1077	806	643	534	455	396	313	257	200	141
20	3999	1998	1329	995	794	660	563	490	388	319	249	176
22	4839	2417	1609	1205	961	860	682	594	470	387	303	216
24	5758	2877	1916	1434	1145	951	813	708	562	463	362	260
27	7288	3642	2425	1815	1450	1205	1030	898	713	588	462	332
30	8998	4496	2995	2243	1791	1489	1273	1110	882	728	573	415
33	10888	5441	3624	2714	2168	1803	1542	1345	1069	883	696	505
36	12958	6476	4314	3231	2581	2147	1836	1603	1275	1054	832	606

If uniformly distributed over the clear span, the safe extraneous loads will be twice as great as those in the table.

For good slate on bed the safe loads may be taken at about 3 times; **for good sandstone on bed** at about one-half; and for **good marble or limestone on bed** at about the same as those in the table. See table, p 493.

Art. 24. Table of breaking loads, in tons, at the center of horizontal rectangular cast-iron beams, one inch wide; supported at both ends: including the weight of the beam itself; one-*half* of which must be deducted from the loads in the table, to obtain the neat breaking load. To do this use the last column. If equally distributed over the clear bearing, the load will be twice as great; including the weight of the beam itself; all of which must be deducted in that case, for the neat load. **Average rolled iron** is ruined as a beam, by about 1¼ times the breaking load of a cast iron one. For the breaking loads of **W. Pine,** spruce, and ordinary oaks, divide those in the table by 4.5; and for white oak, white ash, and best Southern yellow pine, by 4. **Or in practice, to allow for knots,** &c, say for W. Pine, &c, 6.8; and for W. Oak, &c, 6.

Depths of Beams, in ins.	CLEAR SPANS BETWEEN THE ENDS OF SUPPORTS, IN FEET. (ORIGINAL.)																	Wt. of 10 ft. of Beam, in tons.
	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.	6 ft.	8 ft.	10 ft.	12 ft.	14 ft.	16 ft.	18 ft.	20 ft.	24 ft.	28 ft.	32 ft.	36 ft.	40 ft.	
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
1	.904	.452	.301	.226	.181	.151	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.014
¼	1.41	.706	.471	.353	.283	.235	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.017
½	2.03	1.02	.678	.508	.407	.339	.254	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.021
¾	2.77	1.38	.923	.693	.554	.461	.346	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.024
2	3.62	1.81	1.21	.904	.723	.603	.452	.362	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.028
¼	.....	2.29	1.53	1.14	.915	.763	.573	.458	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.031
½	.....	2.82	1.88	1.41	1.13	.942	.706	.565	.471	.....	.....	.....	.....	.....	.....	.....	.....	.....	.034
¾	.....	3.42	2.28	1.71	1.37	1.14	.854	.684	.570	.....	.....	.....	.....	.....	.....	.....	.....	.....	.036
3	.....	4.07	2.71	2.03	1.63	1.36	1.02	.814	.678	.581	.....	.....	.....	.....	.....	.....	.....	.....	.042
¼	.....	.....	3.18	2.39	1.91	1.59	1 19	.955	.796	.682	.....	.....	.....	.....	.....	.....	.....	.....	.045
½	.....	.....	3.69	2.77	2.21	1.85	1.38	1.11	.923	.791	.692	.615	.....	.....	.....	.....	.....	.....	.049
¾	.....	.....	4.21	3.18	2.54	2.12	1 59	1.27	1.06	.908	.794	.706	.....	.....	.....	.....	.....	.....	.052
4	.....	.....	4.82	3.62	2.89	2.41	1.81	1.45	1.20	1.03	.904	.803	.....	.....	.....	.....	.....	.....	.056
¼	.....	.....	.....	4.08	3.27	2.72	2.04	1.63	1.36	1.17	1 02	.907	.816	.....	.....	.....	.....	.....	.059
½	.....	.....	.....	4.58	3.66	3.05	2 29	1.83	1.53	1.31	1.14	1.02	.915	.....	.....	.....	.....	.....	.063
¾	.....	.....	.....	5.10	4.08	3.40	2.55	2 04	1.70	1.46	1.27	1.13	1.02	.....	.....	.....	.....	.....	.066
5	.....	.....	.....	5.65	4.52	3.77	2.82	2.26	1.88	1.61	1.41	1.26	1.13	.942	.....	.....	.....	.....	.069
¼	.....	.....	.....	.....	4.98	4.15	3.11	2.49	2.08	1.78	1.56	1.38	1.25	1.04	.....	.....	.....	.....	.073
½	.....	.....	.....	.....	5.47	4.56	3.42	2.73	2.28	1.95	1.71	1.52	1.37	1.14	.....	.....	.....	.....	.077
¾	.....	.....	.....	.....	5.98	4.98	3.74	2.99	2.49	2.13	1.87	1.66	1.49	1.25	.....	.....	.....	.....	.080
6	.....	.....	.....	.....	6 51	5.42	4.07	3.25	2.71	2.32	2.03	1.81	1.63	1.36	1.16	.....	.....	.....	.084
½	.....	.....	.....	.....	.....	6.37	4.77	3.82	3.18	2.73	2.39	2.12	1.91	1.59	1.36	.....	.....	.....	.090
7	.....	.....	.....	.....	.....	7.38	5.54	4.43	3.69	3.16	2.77	2.46	2.21	1.85	1.58	.....	.....	.....	.097
½	.....	.....	.....	.....	.....	8.48	6.36	5.09	4.24	3.63	3.18	2.82	2.54	2.12	1.82	.....	.....	.....	.104
8	.....	.....	.....	.....	.....	9.64	7.23	5.79	4.82	4.13	3.62	3.21	2.89	2.41	2.07	.....	.....	.....	.111
½	.....	.....	.....	.....	.....	.....	8.16	6.53	5.44	4.67	4.08	3.63	3.27	2.72	2.33	.....	.....	.....	.118
9	.....	.....	.....	.....	.....	.....	9.15	7.32	6.10	5.23	4.58	4.07	3.66	3.05	2.62	2 29	.....	.....	.125
½	.....	.....	.....	.....	.....	.....	10.2	8.16	6.80	5.83	5.10	4.53	4.08	3.40	2.91	2.55	.....	.....	.132
10	.....	.....	.....	.....	.....	.....	11.3	9.04	7.53	6.46	5.65	5.02	4.52	3.77	3.23	2.82	.....	.....	.139
½	.....	.....	.....	.....	.....	.....	.....	9.97	8.31	7.12	6.23	5.54	4 98	4.15	3.56	3.11	.....	.....	.146
11	.....	.....	.....	.....	.....	.....	.....	10.9	9.12	7.81	6.84	6.08	5.47	4.56	3.91	3.42	3.04	.....	.153
½	.....	.....	.....	.....	.....	.....	.....	12.0	9.96	8.54	7.47	6.64	5.98	4.98	4.27	3.74	3.32	.....	.160
12	.....	.....	.....	.....	.....	.....	.....	.....	10.8	9.30	8.14	7.23	6.51	5.42	4.65	4.07	3.62	.....	.167
14	.....	.....	.....	.....	.....	.....	.....	.....	14.8	12.3	11.1	9 84	8.86	7.38	6.33	5.54	4.92	4.43	.195
16	.....	.....	.....	.....	.....	.....	.....	.....	.....	16.5	14 5	12.9	11.6	9.64	8.26	7.23	6.43	5.78	.223
18	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	18.3	16 3	14.6	12.2	10.5	9.15	8.14	7.32	.251

Art. 25. Table of breaking loads in tons, at the center of solid horizontal CYLINDRICAL beams of cast iron, supported at both ends; including the weight of *half* the beam itself, which must be deducted in order to obtain the neat breaking load. To do this, use the last column. If the load is equally distributed along the entire clear length, it will be twice as great as at the center. In this case, the weight of the entire clear length must be deducted. **Wrt iron cyls** will bear about 1 fourth more than cast. For **W. Pine**, spruce, or ordinary oaks, divide the loads by 4.5 or by 6.8; and for white oak, white ash, or best Southern yel pine by 4 or 6. For the breaking load of a square beam, whose breadth and depth each equals the diameter of the cylinder, multiply the tabular loads by 1.7. (ORIGINAL.)

Diam in Inches.	Clear Spans between the End Supports, in Feet.																	Wt of 10 ft of Beams in Tons.
	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.	6 ft.	8 ft.	10 ft.	12 ft.	14 ft.	16 ft.	18 ft.	20 ft.	24 ft.	28 ft.	32 ft.	36 ft.	40 ft.	
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
1.	.533	.266	.177	.133	.106	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.01
¼	1.04	.520	.347	.260	.208	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.02
½	1.80	.899	.600	.449	.360	.300	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.03
¾	2.85	1.43	.951	.713	.571	.476	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.03
2.	4.26	2.13	1.42	1.06	.852	.710	.533	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.04
¼	6.07	3.03	2.02	1.52	1.21	1.01	.758	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.06
½	8.32	4.16	2.77	2.08	1.66	1.39	1.04	.832	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.07
¾	11.1	5.54	3.69	2.77	2 22	1.85	1.38	1.11	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.08
3.	14.4	7.19	4.79	3.59	2.88	2.40	1.80	1.44	1.20	.....	.....	.....	.....	.....	.....	.....	.....	.....	.10
¼	9.14	6.09	4.57	3.66	3.05	2.29	1.83	1.52	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.11
½	.....	11.4	7.61	5.71	4.57	3.81	2.85	2.28	1.90	1.63	.....	.....	.....	.....	.....	.....	.....	.....	.13
¾	.....	14.0	9.36	7.02	5.62	4.68	3.51	2 81	2.34	2.01	.....	.....	.....	.....	.....	.....	.....	.....	.15
4.	.....	17.0	11.4	8.52	6.82	5.68	4.26	3.41	2.84	2.43	2.13	1.89	.....	.....	.....	.....	.....	.....	.18
¼	.....	.....	13.6	10.2	8.18	6.81	5.11	4 09	3.41	2.92	2.56	2.27	.....	.....	.....	.....	.....	.....	.20
½	.....	.....	16.2	12.1	9.71	8.09	6.07	4.85	4.04	3.47	3.03	2.70	.....	.....	.....	.....	.....	.....	.22
¾	.....	.....	19.0	14.3	11.4	9.51	7.14	5.71	4.76	4.08	3.57	3.17	2.85	.....	.....	.....	.....	.....	.25
5.	.....	.....	22.2	16.6	13.3	11.1	8.32	6.66	5.55	4.76	4.16	3.70	3.33	.....	.....	.....	.....	.....	.28
¼	.....	.....	.....	19.3	15.4	12.8	9.63	7.71	6.42	5.50	4.82	4.28	3.85	3.21	.....	.....	.....	.....	.31
½	.....	.....	.....	22.2	17 7	14.8	11.1	8.86	7.38	6.33	5.54	4.92	4.43	3.69	.....	.....	.....	.....	.33
¾	.....	.....	.....	25.3	20.3	16.9	12.7	10.1	8.44	7.23	6.33	5.63	5.06	4.22	3.62	.....	.....	.....	.37
6.	.....	.....	.....	28.8	23.0	19.2	14.4	11.5	9.59	8.22	7.19	6.39	5.75	4.79	4.11	.....	.....	.....	.40
½	.....	.....	.....	.....	29.2	24.3	18.2	14.6	12.1	10.4	9.11	8.10	7.29	6.08	5.21	4.56	.....	.....	.45
7.	.....	.....	.....	.....	36.5	30.4	22.8	18.3	15.2	13.0	11.4	10.1	9.13	7.61	6.52	5.71	5.07	4.57	.54
½	.....	.....	.....	.....	44.9	37.4	28.1	22.5	18.7	16.0	14.0	12.5	11.2	9 36	8.02	7.02	6.24	5.62	.62
8.	.....	.....	.....	.....	.....	45.4	34.1	27.3	22.7	19.5	17.0	15.1	13.6	11.4	9.74	8.52	7.57	6.82	.70
½	.....	.....	.....	.....	.....	54.5	40.9	32.7	27.3	23.4	20.4	18.2	16.4	13.6	11.7	10.2	9.08	8.18	.80
9.	.....	.....	.....	.....	.....	64.7	48.5	38.8	32.4	27.7	24.3	21.6	19.4	16.2	13.9	12.1	10.8	9.71	.89
½	.....	.....	.....	.....	.....	.....	57.1	45.7	38.1	32.6	28.5	25.4	22.8	19.0	17.3	14.3	12.7	11.4	.99
10.	.....	.....	.....	.....	.....	.....	66.6	53.3	44.4	38.0	33.3	29.6	26.6	22.2	19.0	16.6	14.8	13.3	1.1
½	.....	.....	.....	.....	.....	.....	77.1	61.7	51.4	44.0	38.5	34.3	30.8	25.7	22.0	19.3	17.1	15.4	1.2
11.	.....	.....	.....	.....	.....	.....	.....	70.9	59.1	50.6	44.3	39.4	35.4	29.5	25.3	22.1	19.7	17.7	1.3
½	.....	.....	.....	.....	.....	.....	.....	81.0	67.5	57.9	50.6	45.0	40.5	33.8	28.9	25.3	22.5	20.2	1.5
12	.....	.....	.....	.....	.....	.....	.....	.....	76.7	65.7	57.5	51.1	46.0	38.3	32.9	28.8	25.6	23.0	1 6
14	.....	.....	.....	.....	.....	.....	.....	.....	122	104	91.3	81.2	73.0	60.9	52.2	45.7	40.6	36.5	2.2
16	.....	.....	.....	.....	.....	.....	.....	.....	.....	156	136	121	109	90.9	77.9	68.1	60.6	54.5	2.8
18	.....	.....	.....	.....	.....	.....	.....	.....	.....	222	194	173	155	129	111	97.1	86.3	77.7	3.6

LIMIT OF ELASTICITY IN BEAMS.

The limit of elasticity of a beam of any particular form, or material, is determined by experiment with a similar beam, as in the case of constants for breakg loads, &c. Thus, load a beam at the center, by the careful gradual addition of small equal loads; carefully note down the def that takes place within some mins (the more the better) after each load has been applied; in order to ascertain when the defs begin to increase more rapidly than the loads; for when this takes place, the load for elastic limit has been reached.

It is not the defs of the *whole* beam that are to be noted, but those of its clear span only. **Several beams should be tried** to get an average constant; for even in rolled iron beams of the same pattern, and same iron, there is a very appreciable diff of strengths and defs.

Then, to get the constant, so as to apply it to similar beams, using the *total load* applied during the equal defs, including $\frac{1}{2}$ wt of beam,

$$\text{Constant for greatest center loads within elas limit} = \frac{\text{Span in ft} \times \text{Total load in lbs}}{\text{Breadth in ins} \times \text{Square of depth in ins}}$$

REM. Of course, in practice, it is frequently difficult to ascertain with precision when, or under what load, the defs actually do begin to increase more rapidly than the successive loads. For although *theoretically* the defs are equal for equal loads, until the elastic limit is reached, yet in *practice*, they are only *nearly* equal, up to that point. This is owing to the fact that no material composing a beam is perfectly uniform throughout in texture and strength; so that instead of perfect equality of defs, we shall have an alternation of larger and smaller ones. Therefore, some judgment is reqd to determine the final point; in doing which, it is better, in case of doubt, to lean to the side of *safety*.

REM. Within the limits of elasticity, a beam of irregular shape, such as a T, or a Hodgkinson beam, a triangle, &c, will bend to the same extent, whether its top or its bottom be uppermost.

REM. It is assumed always that the load is not subject to jars or vibrations. These would increase the deflections.

Constants for greatest center loads within limits of elasticity, may be had near enough for common practice by taking one-third of the breaking constants in the table on p 493; except those for rolled iron and steel.*

Then,

$$\text{Greatest center load within elas limit of rectangular beam in lbs} = \frac{\text{Breadth in ins} \times \text{Square of depth in ins} \times \frac{1}{3}\text{ of the constant on page 493.*}}{\text{Span in feet}}$$

Art. 26. Deflections, or bendings, of beams, under their loads. The foregoing relates to the *strength* of beams, or their resistance to *breakg;* the following to their *stiffness*, or resistance to *bendg*. The two follow very diff laws.

It is with the defs *within the elastic limit* that the engineer is chiefly interested. They then are directly as the load and as the cube of the span; and inversely as the breadth, and as the cube of the depth; and this, with the following, applies not only to all **rectangular** beams, but to all others of whatever cross section, provided the sections are **similar.**

With same span, breadth, and load, the deflections within elas limits are in all cases **inversely** as the **cubes** of the depths. Hence the depths are inversely as the **cube roots** of the deflections.

With same span, breadth, and deflection, the depths are directly as the **cube roots** of the loads. Hence the loads for equal deflections are as the **cubes** of the depths.

If the deflection of a beam *supported* at both ends and loaded at the center be called	1.
Then that of the same beam, with the same load uniformly distributed will be	.625, or $\frac{5}{8}$
Firmly *fixed* at both ends, and loaded at the center, by Mosely	.2, or $\frac{1}{5}$
" " " " " " uniformly loaded	.125, or $\frac{1}{8}$
Fixed at one end, and loaded at the other	16.
" " " uniformly loaded	6.

The extent to which a beam may bend under even a perfectly safe load, may be too great for many purposes in every-day practice. Tredgold and others assume, that in order not to be observed, or that it may not cause the plaster of ceilings to crack, &c, a beam should not deflect at its center more than the $\frac{1}{480}$ part of its span, or $\frac{1}{40}$ of an inch per ft. Thus, if its span be 20 ft, it should not bend more than $\frac{20}{40}$ths, or $\frac{1}{2}$ of an inch, which is also $\frac{1}{480}$th of 20 ft. For such cases see Art 29.

Shafts of wheels in machinery should not deflect more than half of this, nor a bridge more than say $\frac{1}{1200}$ of its span, or $\frac{1}{100}$ inch per foot, under its heaviest load.

We shall allude first to defs within the limits of safety, or of the elasticity of the beam; and afterward to those not exceeding $\frac{1}{480}$ of the span. After the elastic limit is passed, the defs increase irregularly, and more rapidly than before; and the beam becomes unsafe. As a general rule, the elasticity of a wooden beam is not injured for practical purposes, if the quiescent load does not *exceed* about $\frac{1}{3}$ of the breaking one.

***Except for wrought iron and steel;** for which take the whole constant.

The **Constant for Deflection** for any given material, within the limit of elasticity, is the deflection, in inches, of a beam of that material, 1 inch square, and 1 foot span, supported at both ends, and loaded at the center with a *total* weight (including five-eighths of the weight of the clear span of the beam) of one pound. Such constants may, like those of transverse strength, (see p 491,) be readily found by experiment. Thus, at the center of any rectangular beam, placed hor upon supports at each end, place any load that is within its elastic limit, and measure the resulting def in ins. Mult the wt of the span of the beam by .625, add the prod to the neat load, for a total load. Mult together the total load in lbs. and the *cube* of the span in feet. Also mult together the breadth in ins, and the cube of the depth in ins. Div the first prod by the last one. Div the def by the quot. The last quot will be the reqd constant for any rectangular beam of the same kind and quality of material, whether wood, metal, stone, &c. That is,

$$\left.\begin{matrix}\text{The constant for def within elastic, or safe limits of beam}\end{matrix}\right\} = \text{Def in ins divided by } \left\{\frac{\text{Total load in lbs} \times \text{Cube of span in feet}}{\text{Breadth in ins} \times \text{Cube of depth in ins.}}\right.$$

We add to the experimental neat load, the .625. or $\frac{5}{8}$ of the wt of the clear span of the beam itself, because the wt of the beam equally distributed throughout its span, also aids in producing the def; and it does so to the same extent that $\frac{5}{8}$ of it would do, if collected at the center of an imaginary beam having the same strength throughout as the real one, but having itself no weight. Therefore, in applying the constants for def to beams intended for actual use, we must not omit to add $\frac{5}{8}$ of the wt of the span, to the intended center load, for an equivalent total center load, before making the calculations for def. The weights of *similar* beams (that is, beams *proportioned* exactly alike in every part, but of diff sizes) increase so much more rapidly than their clear spans, that although a small one may safely bear a load of many times its own wt, a much larger one will break down without any load. Having by experiment found the constant of def for any given material, the def of any similar beam of the same material, whether larger or smaller, and loaded at the center, may be found thus:

$$\text{Def within safe or elastic limit in ins} = \frac{\text{Total equivalent center load in lbs} \times \text{Cube of span in ft} \times \text{Constant}}{\text{Breadth in ins} \times \text{Cube of depth in ins.}}$$

The def of a beam of any form whatever of cross section, if within the limit of elasticity, may be found approximately thus,

$$\text{Def in ins.} = \frac{\text{load in lbs} \times \text{Cube of span in inches} \times \text{coef } d}{\text{mod of elas p 434} \times \text{moment of inertia in ins, p 486}}$$

Coef *d*. Beams supported at both ends; center load..... .02083
" " " " uniform " ..... .01302
" fixed at one end; loaded at the other..... .33333
" " " " ; load uniform..... .12500

This formula gives the def produced by the load only. To find that arising from the weight of the beam itself, consider said wt as a uniform load; then find the resulting def by the same formula, and add it to that of the load.

In a rectangular beam supported hor at both ends and loaded at the middle within elas limit, the def in ins will be

$$\textbf{Def} \text{ in ins} = \frac{\left(\text{Load in lbs} + \text{.625 wt of clear span of beam in lbs}\right) \times \text{cube of clear span in ins}}{4 \times \text{coef elas} \times \text{breadth in ins} \times \text{cube of depth in ins}}$$

And the center load in lbs, (including .625 wt of clear span of beam) required to produce any given def in ins within elas limit of such a beam will be

$$\textbf{Load in lbs} = \frac{4 \times \text{coef elas} \times \text{breadth in ins} \times \text{cube of depth in ins} \times \text{given def in ins}}{\text{cube of clear span in ins.}}$$

Table of constants for the deflections, within the safe, or elastic limits. of hor rectangular beams, supported at both ends and loaded at the center. The timbers are supposed to be well seasoned; if not, the constant should be increased.

White oak	.00023 *	White pine	.00032*
Best southern pitch pine, and white ash	.00027 *	Ordinary yellow pine	
Hickory	.00016 *	Spruce	
		Good straight-grained hemlock.	
		Ordinary oaks	

Cast iron..... .000018 to .000036.....Mean .000027 *
Bar iron..... .000012 to .000024..... Mean .000018
Steel, rolled.... .000010 to .000020.....Mean .000015

Full and reliable experiments on the strength and deflections of the various steels are much needed.

It is evident that the stiffer the material is, the smaller will be its constant for *bending*. All these constants vary somewhat with the quality of the metal. The defs also of timber of the same kind, vary so much with the degree of seasoning, the age of the tree, the part it is cut from, &c, that the writer considers it mere affectation to pretend to assign constants for practical use, more nearly approximate than he has here done. They are averages deduced from his own experiments on good pieces, well seasoned; and the loads were allowed to remain on for months, instead of minutes, as usual. Every structure is more or less exposed to vibrations and jars, which in time increase the deflections. In several instances, our experimental timbers bore their breakg loads for months before they actually gave way. And in all kinds, less than $\frac{1}{50}$ of the breakg load produced in a few months a permanent set, or def.

The following are deduced from single experiments only. An allowance is made for the weight of the beam.

Rolled iron beams proportioned exactly as the 7-inch Phœnix beam,						.0000303†
"	"	"	"	30 lbs, 9 inch,	"	" ..... .0000321†
"	"	"	"	50 " heavy 9 inch	"	" .0000264
"	"	"	"	41⅔ lbs, 12 inch	"	" ..... .0000313†
"	"	"	"	51⅔ lbs, 15 inch	"	" ..... .0000365†
"	"	"	"	66⅔ lbs, 15 inch	"	" ..... .0000438

Art. 27. To find the def in inches, of a hor rectangular beam, supported at both ends, and loaded at its center, with any given load within its elasticity; mult the weight of the clear beam itself, in lbs, by the decimal .625. Add the prod to the given center load in lbs. Call the sum the total load. Mult together this total load, the cube of the span in ft, and the constant from the upper table. Also mult together the breadth in ins, and the cube of the depth in ins. Div the first prod by the last one.

Ex. What will be the def of such a beam of average white pine, 9 ins broad, 12 ins deep, 21 feet clear span, and weighing 450 lbs; with a neat center load of 1218.75 lbs?

Here first, $450 \times .625 = 281.25$ lbs. And $281.25 + 1218\,75 = 1500$ lbs total load. Hence,

$$\frac{1500 \times 21^3 \times .\text{Const.}}{9 \times 12^3} = \frac{1500 \times 9261 \times .00032}{9 \times 1728} = \frac{4445.2}{15552} = .286 \text{ inch; reqd def.}$$

Rem 1. **When the load is all at one point not at the center,** *o*, Fig 10 p 496, mult together the two dists *o a*, *o g*, from the load to the points of support. Mult the prod by 4. Div the result by the clear span. Use the quot as if it were the span, in the last rule. The wt of the beam is not here taken into account; it will of course somewhat increase the def.

---

* Averages near enough for ordinary practice by the writer's own trials. **Calling the average elastic def of a steel beam, 1,** that of a similar average wrought one will be 1.2; and that of a cast one 1.8. If that of an average cast beam be 1, that of a wrought one will be .67; and that of a steel one .56. If that of a wrought one be 1, cast will be 1.5; and steel .83.

† We believe that these four beams have the same proportions, as nearly as the process of making them will admit of; so that .000033 may be taken as a near enough average for all four. As before remarked, extreme accuracy must never be expected in such matters. Two halves of the same identical beam will often give differences greater than this.

Rem. 2. **When the neat load is equally distributed along the span,** instead of all being at the center, then for an equivalent total center load, add together the neat load, and the entire wt of the clear span of beam; and mult the sum by the dec .625. With the resulting equivalent center load, proceed precisely as in the foregoing example.

Ex. The def of the foregoing beam of white pine, 9 ins broad, 12 ins deep, 21 feet span, weighing 450 lbs, and bearing an equally distributed load of 1218.75 lbs? Here first 450 + 1218.75 = 1668.75. And 1668.75 × .625 = 1042.97 lbs = equivalent center load. Hence

$$\frac{1042.97 \times 21^3 \times .00032}{9 \times 12^3} = \frac{3090.862}{15552} = .1987 \text{ ins, reqd def.}$$

Rem. 3. With an **equally distributed** load, including the wt of the beam, the def is only $\frac{5}{8}$, or the .625 part as great as it would be if the same total load, including the entire wt of the beam, were all applied at the center.

Rem. 4. If the beam in any of these, or the following cases, **is inclined,** Fig. 11, p 496, use the hor dist *o y*, instead of the actual span *o c*.

Art. 28. Rule 1. **To find the neat center load which will (together with the wt of the beam itself) produce any given def within the elastic limit of the beam:** find the cube of the clear length in feet; mult this cube by the constant from the table on p 507. Also mult the breadth in ins, by the cube of the depth in ins. Div the first prod by the last one. Div the given def in ins, by the quot, for the total reqd load in lbs. Mult the wt of the clear length of the beam in lbs by .625, and deduct the prod from the load so obtained, for the *neat* load. By formula,

$$\text{Total load, including wt of beam} = \text{Deflect in ins} \div \frac{\text{Cube of length in feet} \times \text{Constant in p 507}}{\text{Breadth in ins} \times \text{Cube of depth in ins.}}$$

Ex. What center load in lbs will (together with the wt of the beam itself) produce a def of .286 of an inch, in a beam of white pine, 21 ft span, 9 ins broad, 12 ins deep, and which weighs 450 lbs? **See table, p 499.**

Here $\overset{\text{Cube of 21.}}{9261} \times \overset{\text{Const.}}{.00032} = 2.9635.$ And $\overset{\text{Breadth.}}{9} \times \overset{\text{Cube of 12.}}{1728} = 15552.$

And $\frac{2.9635}{15552} = .0001906.$ And $\frac{.286}{.0001906} = 1500$ lbs.

For the neat load we must deduct .625 of the wt of the beam; or 450 lbs × .625 = 281.25 lbs: so that the neat load is 1500 — 281.25 = 1218.75 lbs, as in Ex 1, Art 27.

If the load is **uniformly distributed,** use precisely the same rule for getting the total load. Then mult this load by 1.6. Deduct the *entire* wt of the clear length of beam.

Ex. What equally distributed load will deflect the foregoing beam .1987 ins? Here, proceeding as before, the only diff is that instead of .286 def, we have .1987 def to be div by .0001906. And $\frac{.1987}{.0001906} = 1042.5$ lbs, as the equivalent *center* load. And 1042.5 × 1.6 = 1668 lbs for the total distributed load, including the entire wt of the beam, or 450 lbs. Hence 1668 — 450 = 1218 lbs, the neat distributed load reqd; agreeing with the preceding example within $\frac{3}{4}$ of a lb; the diff being owing to a neglect of small decimals in the calculation.

Rule 2. **The length, depth, neat center load, and def being given, to find the breadth.**

$$\frac{\text{Neat cen load in lbs} \times \text{Cube of length in feet} \times \text{Constant in Art 26}}{\text{Cube of depth in ins} \times \text{Def in ins}} = \text{Breadth in ins approx.}$$

Or sufficient for the neat load alone.

Now calculate the wt of a beam with the breadth already found. Mult this wt by .625, then say, as

Neat center load : Breadth first found :: .625 of the weight of the beam : Additional breadth reqd.

Add these two breadths together, and their sum will be the total breadth reqd, more approximately; but still somewhat too small, inasmuch as it provides only for the

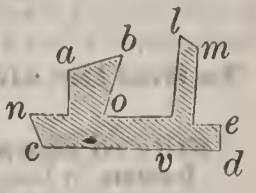
wt of the beam of the breadth first found, and not for that having the additional breadth. This may readily be calculated and added.

RULE 3. **The length, breadth, neat center load, and def, being given, to find the depth.**

$$\frac{\text{Neat cen load in lbs} \times \text{Cube of length in feet} \times \text{Constant in Art 26}}{\text{Breadth in ins} \times \text{Def in ins}} = \text{Cube of depth in ins approx.}$$

Take the cube root of this for the depth itself, approximately. REM. This, like the breadth given by the preceding formula, is too small, inasmuch as it does not allow for the wt of the beam. Therefore, when greater accuracy is required, proceed thus: Calculate the wt of a beam having the depth just found. Mult this wt by .625. Add the prod to the neat center load. Consider the sum as a new neat center load; and using it instead of the neat center load first given, go through the whole calculation again, to obtain a new cube of depth. The cube root of this will be more nearly correct; but still a trifle too small, for the same reason as in the foregoing case.

REM. **In experimenting for constants** of any kind, with beams of irregular cross-sections, this, for instance, it is quite immaterial which breadths and depths are measd; thus, for the breadth we may take *a b*, *l m*, *c d*, or *o e*, &c; and for the depth, either *n c*, *l v*, *m d*, *b o*, &c. It is only necessary to state what parts actually have been taken, so that the corresponding ones may be measd in any other beam which is to be calculated by the constant derived from the experiment. This remark applies to all constants involving the breadth and the depth. The constant itself will of course vary according to which dimensions are taken in the experiment; but the results derived from it when applied to other beams of similar forms, will not be affected thereby, if the corresponding parts be measd in both cases.*



* We may even take *any single oblique* measurement, as *a b*, *l m*, *n c*, *a d*, &c. and call it both the breadth and the depth. This applies to rectangular, or to any other shaped beams.

Art. 29. It is often **required that the deflection shall not exceed a certain amount.** In such cases, if we call the greatest allowable deflection in inches per foot of span, "**D**,"* the formula p 508 for **total center load** (including .625 of the weight of the clear span of the beam) becomes

$$\textbf{load}^* \text{ in lbs} = \frac{D \times \text{span in feet} \times \text{depth}^3 \text{ in inches} \times \text{breadth in inches}}{\text{Span}^3 \text{ in feet} \times \text{constant, p 507}}$$

$$= \frac{D \times \text{depth}^3 \text{ in inches} \times \text{breadth in inches}}{\text{Span}^2 \text{ in feet} \times \text{constant, p 507}}$$

$$\textbf{Extraneous center load in lbs} = \text{total load found as above} - \left(\text{weight of clear span of beam} \times .625\right)$$

For the **total uniformly distributed load**

$$\textbf{load}^* \text{ in lbs} = \frac{1.6 \times D \times \text{depth}^3 \text{ in inches} \times \text{breadth in inches}}{\text{Span}^2 \text{ in feet} \times \text{constant, p 507}}$$

$$\textbf{Extraneous uniformly distributed load in lbs} = \text{total distributed load found as above} - \text{weight of clear span of beam}$$

To find the breadth required:

$$\textbf{Breadth in inches approximately} = \frac{\text{extraneous center load in lbs} \times \text{span}^2 \text{ in ft} \times \text{constant p 507}}{D \times \text{depth}^3 \text{ in inches}}$$

For a closer approximation, add to the breadth so found

$$\text{said breadth} \times \frac{.625 \times \text{weight of clear span of beam}}{\text{center load in lbs}}$$

The sum will still be a mere trifle too small.

If the load is uniformly distributed, use distributed load $\times$ .625, instead of center load, in the first formula for breadth. For a closer approximation, add to the breadth so found,

$$\text{said breadth} \times \frac{\text{weight of clear span of beam}}{\text{uniform load}}$$

The sum will still be a mere trifle too small.

To find the depth required

$$\textbf{Depth in inches approximately} = \sqrt[3]{\frac{\text{extraneous center load in lbs} \times \text{span}^2 \text{ in feet} \times \text{constant, p 507}}{D \times \text{breadth in inches}}}$$

Then calculate the wt of the entire clear span of a beam having this depth, mult it by .625, and add the prod to the neat center load. Consider the sum as a new neat center load; and using it instead of the one first given, go through the whole calculation again, for a new depth. This will be the reqd depth more approx, but a little too small.

If the neat load is **uniformly distributed,** first mult it by .625. Use the prod as a center load, and by the foregoing formula find first the approx depth. Then calculate the wt of the entire clear length of a beam having that depth. Mult this wt by .625, and add it to the prod used as a center load. Consider the sum as a new center load; and using it instead of the one first used, go through the whole calculation again, for a new depth. This will be the reqd depth, approx, but a mere trifle too small.

To find the side required for a square beam

$$\textbf{Side of square beam in inches approximately} = \sqrt[4]{\frac{\text{extraneous center load in lbs} \times \text{span}^2 \text{ in feet} \times \text{constant, p 507}}{D}}$$

The **fourth root** is = the square root of the square root.

---

* When $D = \frac{1}{40}$ (*i e* when the greatest allowable deflection is $= \frac{1}{480}$ span) see tables pp 512, 513.

For a closer approximation, find the weight of a square beam with the side just found. Multiply it by .625, and add the product to the extraneous center load. Employ the formula again, using this increased load instead of extraneous center load. The side thus obtained will still be a mere trifle too small.

To find the diameter required for a solid cylindrical beam.

$$\text{Diameter in inches approximately} = \sqrt[4]{\frac{1.7 \times \text{extraneous center load in lbs} \times \text{span}^2 \text{ in feet} \times \text{constant p 507}}{D}}$$

The **fourth root** is = the square root of the square root.

For a closer approximation, find the weight of the clear span of a beam with the diameter just found. Multiply said weight by .625. Add the product to the original given center load. Then repeat the formula, using the sum last obtained, instead of extraneous center load. The resulting diameter will still be a trifle too small.

The stiffness of a cylinder is to that of a square beam, whose breadth and depth are each equal to the diam of the cylinder, as .589 to 1; or that of the square one is to that of the cylinder as 1 to .589, or as 1.698 to 1; in practice we may use .6 and 1.7. Hence, the cylinder will bend 1.7 times as much as a square one, under the same load.

When, in any of the foregoing cases, **the beam is inclined,** Fig 11, p 496, take the horizontal distance *o y* for the span, instead of *o c*.

Art. 11. General facts respecting the breakg loads of a uniform beam of any form of section. Calling the breakg load,

When the beam is firmly fixed at one end, and loaded at the other 1
Then when so fixed, and uniformly loaded, it will be..... 2
When merely *supported* at both ends, and loaded at the center..... 4
" " " " " and uniformly loaded 8
Firmly *fixed*, or tightly *confined* at both ends, and loaded at the center... 8
" " " " " and uniformly loaded..... 16

By some authorities the last two are given at 6 and 12. The term "fixed" in such cases is rather indefinite Prof. De Volson Wood gives 12 for the last.

Rem. 1. When one beam of any form of section is 2, 3, or 4, &c, times as long, broad, and deep, as another, its weight will be 8, 27, or 64, &c, times as great, or as the *cubes* of the linear dimensions; but its breaking load will be only 4, 9, or 16, &c, times as great; that is, the strengths of similar beams are to each other as the *squares* of their respective linear dimensions. But in these breakg loads are included the weights of the beams themselves; one-half of which must be deducted when the load is all at the middle of the beam; or the whole of it when equally distributed. When beams are of the moderate dimensions usually employed in buildings, their own weight is usually so small in comparison with their loads, that at times it may safely be neglected; but as they become longer, since their weight increases much more rapidly than their strength, it at last constitutes too important an item to be overlooked; for the beam may become so great as to break under its own weight; although proportioned precisely like a small one which may safely bear many times *its* own weight.

When a square beam is supported on its edge, instead of on a side (or in other words has its diag vert), it will bear but about $\frac{7}{10}$ths as great a breakg load.

The deflections or bendings of beams (see p 505) are directly in proportion to the load, and to the cube of the length; and inversely to the breadth, and to the cube of the depth, all while within limits of elasticity.

Rem. 2. It is very important to remember that a beam will bear a much greater load placed upon it in small amounts at a time; or (in case it is all applied together) if its pres be allowed to come upon the beam very gradually, than if it be placed upon it suddenly, even without any jarring, and without any previous momentum. When applied in small amounts, or gradually, the bending takes place slowly and with slight momentum; but when applied all at once, the great load descends immediately and rapidly to the full extent of the bending due to the load itself, and in so doing acquires a momentum which carries it still further; thus producing a strain which authorities maintain to be just twice as great as in the former case. A heavy train coming very rapidly upon a bridge, presents a condition intermediate between the two. The tables of constants always suppose the load to be applied very gradually, but as this is frequently not the case in practice, an allowance must be made accordingly.

Art. 12. To find the quiescent breakg load of a hor square, or rectangular beam B, p 491, of any material, supported at both ends, and loaded at the centre.

Rule. Mult together the square of its depth d *o* in ins, its breadth n d in ins, and the coef from the table p. 493. Div the prod by the clear length a a between the supports, in ft. The quot will be the reqd breakg load in lbs; including, however, half the wt of that portion of the beam which lies *between* the supports, and which must be deducted in order to get the neat load.

Ex. What will be the center breakg load of a beam of Connecticut red sandstone, 15 ins wide, 10 ins deep, and 12 ft long between its supports? Here

$$\frac{\text{Depth}^2 \text{ in ins} \times \text{Breadth in ins} \times \text{Constant}}{\text{Clear length in ft}} = \frac{100 \times 15 \times 45}{12} = \frac{67500}{12} = 5625 \text{ lbs};$$

the breakg load reqd.

But we must deduct *one-half* the wt of the clear length of the beam. Now a beam of red sandstone, of 15 ins, by 10 ins, by 12 ft, contains 21600 cub ins = 12½ cub ft; and a cub ft of red sandstone weighs about 140 lbs; therefore the beam weighs $12.5 \times 140 = 1750$ lbs; one-half of which, or 875 lbs, must be taken from the 5625 lbs of breakg load, leaving 4750 lbs as the actual extraneous, or neat breakg load.

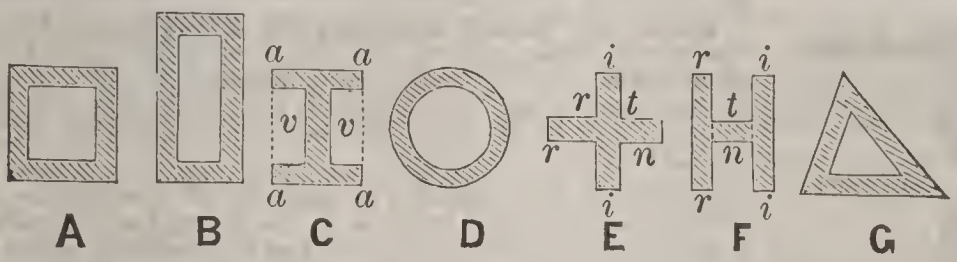
Rem. 1. If cylindrical, first find the breakg load of a square beam, of which each side is equal to the diam of the cylinder. Mult this load by the dec .6, or, more correctly, .589. Hence a square one is 1.7 times as strong as a cylindrical one.

If oval, or elliptic, first find the load for a rectangular beam, whose sides are respectively equal to the two diams, and mult it by .6.

If of wood, and triangular, and its base (whether up or down) hor, first find the breakg load for a rectangular beam, whose breadth is equal to the base; and its depth equal to the perp height of the triangle; and take one-third of the result as an approximation. When the edge is down, the ends must rest in triangular notches in the supports; otherwise, they will be crushed when loaded. See Art 16.

For beams of such sections as A to G, the following rude rules of thumb will often be preferred to more intricate ones, being sufficiently approximate for ordinary purposes, and for any material. See near end of Art 33, p 516.

For the closed Figs A, B, D, G, (each one supposed to be of equal thickness throughout,) first find the load for a solid beam of the same size and shape.



Then find that of a beam of the size of the hollow part. Subtract the last from the first.

For C, (its top and bottom being of equal size,) first find for a rectangular beam *a a a a*. Then for two beams corresponding to the two hollows *v v*. Subtract these last from the first.

For E or F, find for three separate beams *r r*, *i i*, *t n*, and add them together.

For angle and T iron, see p 525.

For U and other shapes in common use, we may use the formula, p 488, or experiment with a model made of the given material, and thus find the necessary constant, as directed, p 491. See Remark, p 509; also Art 33, p 516.

For I beams, see Art 37, and **for Hodgkinson beams,** see Art 35.

Rem. 2. In this case we may remove ¼ part of the material of the solid square, or rectangular beam, without diminishing its breaking strength, although it will bend more. The width may remain uniform, and the depth be reduced either at top or bottom, as shown by the dotted lines at *m*, Fig 5, strictly two parabolas with bases at load, but the straight ones are best in practice. Or the depth may remain uniform, and the breadth be reduced, as shown by the dots at *n*, which is a top view of the beam. Theoretically, the dotted lines in *n* might meet at the ends of the beam; but in practice this would not generally leave sufficient material at the ends for the beam to rest upon securely.

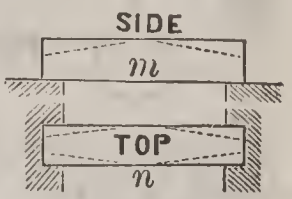


Fig. 5.

Such reductions of beams are rarely made when they are of wood; but in iron ones much expense may be saved thereby.

Rem. 3. Load at one end of the beam, Fig 6, the other end fixed, imagine the load to be at the center, and calculate it by the foregoing rule. Then div the result by 4.

In this case the lower side of the beam may be cut away in the form of a parabola, as shown by the dots. To draw this curve, see page 153. Or the depth may be left uniform, and the sides be cut away, as shown by the dots at *t*, which is a top view.

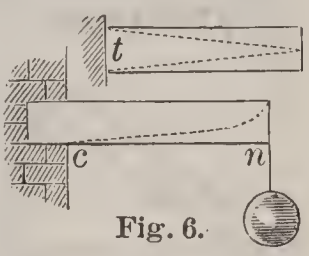


Fig. 6.

Art. 13. When the load is equally distributed along the entire clear length of a horizontal beam, supported at both ends, as in Fig 7, instead of being all applied at the center, assume it to be at the center, and proceed precisely as in the foregoing rule, Art 12. Then mult the load by 2. But in this case the wt of the *entire* clear length of the beam is to be deducted for the neat load.

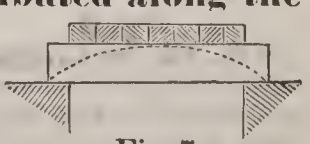


Fig. 7.

Ex. What will be the equally distributed breakg load of the beam of sandstone in the last example? Here the center breakg load has already been found to be

A single beam of wood under each rail, and firmly braced against lateral motion, will suffice for light railroad bridges of very small span. If single beams of sufficient depth cannot be procured, *built-beams* may be used; see *g*, and *i j*, Figs 62, p 613. Assuming the weight of entire bridge and load at two tons per foot, the following dimensions may be used:

Span in Ft.	Size of Beam.	Span in Ft.	Size of Beam.
5	8 × 10 ins.	15	12 × 18
7½	9 × 12 "	17½	13 × 20
10	10 × 14 "	20	14 × 22
12½	11 × 16 "	22½	16 × 24

The greatest dimension to be the depth. The ends should be well bolted down to

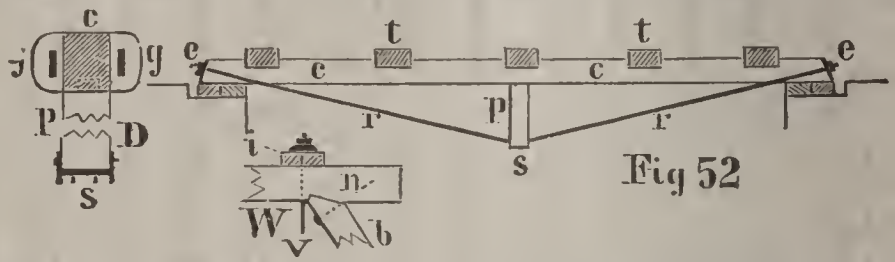


Fig 52

bolsters. These are long stout sticks of timber, from 10 to 15 ins square, (according to the span,) laid across the abuts at the bridge-seat, for the chords to rest on. Frequently two are used at each abut, even in small spans; and we have seen but one, under railroad spans of 150 feet. Large spans may require three or more. They are not necessarily placed in contact with each other; but may be some feet apart, if required.

Or for spans of about 15 to 30 ft, we may use somewhat lighter beams; and truss each of them as in Fig 52, by an iron bar *e s e*; and a center post *p*. In this case the following dimensions will answer; the total deflection of the rod being 1/8 of the clear span. The screw-ends of the bars are supposed to be upset; but the areas are given for the body of the rods.

For each beam.

Span. Ft.	Beam. Ins.	Section of Rod. Sq Ins.	Section of Post. Sq Ins.
15	12 × 15	3½	25
20	13 × 17	4¾	33
25	14 × 18	6	42
30	15 × 20	7	50

It is better to have two rods instead of one under each beam; each rod being of half the section here given; and the two placed several ins apart. This affords a better footing for the post. The ends of the beam should be at right angles to the direction of the rod; and be provided with ample washers *e e*, of wood or iron, for distributing the pressure from the rod, over the whole area of the ends. The ends of these washers may extend a few ins each way beyond the sides of the beam, as shown on a larger scale at *g*. This allows the rods to be outside of the beam; instead of requiring holes to be bored in the latter, for passing the rods through them. They may be nearer together at the foot of the post.

The head of the post may be tenoned into the bottom of the beam; and be further united to it by iron straps. To prevent the foot from being worn by the rods, it should be shod with iron. A cast-iron shoe, as at *s*, may be bolted to it; having ribs for keeping the bars in place. Or a stout wrought-iron shoe may be well secured to it. In either case the rod at *s* should be so united to the shoe as to check any tendency in the foot to slide toward *r* or *r*, under the vibration of passing loads. Perhaps this can be most conveniently done by making each rod *e s e*, in two separate lengths, *r*, *r*; and by uniting their lower ends to the shoe at *s* by hooks and eyes; or by eyes and bolts, &c. Various methods are in use for the heads and feet of the posts of large spans; but we cannot here treat upon details which pertain more to the professional bridge-builder.

This mode of trussing is also well adapted to long floor beams; and has been used in long oblique web-members; as well as in long stretches of chords from one point of support to another.

Continuous beams. When a single beam, as *a b*, Fig 40, is supported not only at its two ends, but at one or more intermediate points, it is said to be continuous. It is stronger than if it were cut into two parts, *a c*, *b c*, each supported at both ends; because the tensile strength of the particles at *o* (lower Fig) assists in counteracting the bendg or breakg tendency of loads on the intermediate parts *o m*, *o n*, of the lower Fig. These particles at *o* must be torn asunder before the beam (if properly proportioned) can fail. Such a beam, *m n*, if very long and flexible, will, under its own wt, assume the shape of the reversed curve *m s o s n*; or if it be stiff, and heavily loaded, the same effect will follow. The points *s s*, at which the curves reverse, are called **the points of contrary flexure**; and the spans are virtually reduced from *m o* and *n o*, to *m s* and *n s*. When the beam is supported at only 3 points, as in the Fig, and *uniformly loaded*, the point of contrary flexure is dist from the central support ¼ of the span; so that each span, *o m*, *o n*, becomes virtually reduced about ¼ part; and the defs will be but about $\frac{4}{10}$ as great as if there were two separate beams. The sections of the beam at *s* and *s* will then experience no hor strain; but merely the vert one arising from half the wt between *m* and *s*, and *n* and *s*. **The position of the point of contrary flexure varies** with the number of intermediate supports, and with the manner of loading; and in bridges, &c, where the load moves along the beam, it changes its place during the transit, so as to bring the points *s s* considerably nearer to the central support *o*; thus reducing materially the advantage commonly supposed to arise from connecting together the ends of adjacent bridge-trusses; if indeed there is any advantage in so doing, which is doubtful. The principle, however, becomes very useful in the case of long rafters or girders, stretching over several points of support, especially when uniformly loaded. Each interval, except the two end ones, will have two points of contrary flexure; and will then have nearly twice as much strength, under an equally distributed load, as a single beam no longer than said interval.

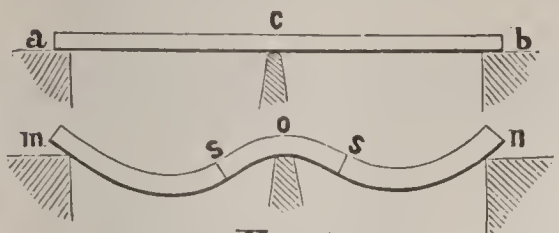


Fig 40

Art. 33. Strength of hollow Beams.

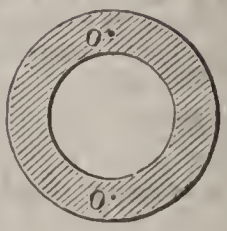


Fig. 12.

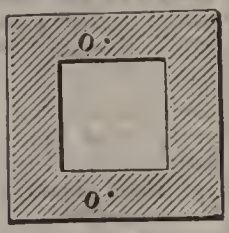


Fig. 13.

During the preliminary investigations relative to the construction of the Menai tubular bridge, a few experiments were made on the strength of hollow cast-iron beams of circular, oval, square, and rectangular cross-sections, supported at both ends, and loaded at the center. The clear span between the supports was in every case 6 ft, the thickness of metal in each beam, $\frac{3}{8}$ inch; area of *solid* cross-section of each, 4.12 sq ins. The mean depth *o o*, Fig 12, of the circular tube, $3\frac{1}{2}$ ins; of the square one, Fig 13, $2\frac{3}{4}$; of the oval, $4\frac{2}{3}$; breadth, $2\frac{1}{3}$; and of the rectangular one, mean depth, $3\frac{2}{3}$; breadth, 1.833 ins. From these experiments Mr. Edwin Clark, assistant engineer in charge, deduced the following constants, and rules for center breakg-loads:

Const for circ tubes, .95; oval, 1; square, 1.14; rectangle, .91. Then, first finding the area of the solid part of the cross-section in sq ins,

$$\text{Center breaking load in tons} = \frac{\text{Area of solid in sq ins} \times \text{Mean depth, } o\ o, \text{ in ins} \times \text{Corresponding constant.}}{\text{Clear span in feet.}}$$

Ex. Circular beam, mean depth *o o*, $3\frac{1}{2}$ ins; area of solid ring, 4.12 sq ins; clear span, 6 ft. Here,

$$\frac{\text{Area. } 4.12 \times \text{Mean depth. } 3.5 \times \text{Const. } .95}{6 \text{ (length.)}} = 2.28 \text{ tons, or } 5107 \text{ lbs, breakg load.}$$

The thickness of the cylinder or tube is about $\frac{1}{10}$ of the diam; and as a mean of 3 trials, it broke with a center load of 2.287 tons, or 5122 lbs; span 6 ft. Hence we derive for similar tubes, the constant 530, to be used in the rule, Art 12; that is, center breakg load in lbs, of circular cast-iron tubes with a thickness of one-tenth of the outer diam $= \frac{\text{Cube of outer diam (in ins)} \times 530}{\text{Clear span in feet}}$; supposing Mr. Clark's iron to have been of average quality.

The average breakg load of 3 square beams was 2.152 tons, or 4820 lbs; of the rectangular ones, 2.3 tons, or 5152 lbs; and of the 6 elliptic ones, 3.207 tons, or 7183 lbs. To all the foregoing extraneous loads must be added half the wt of the beam itself. See Art 9.

Our rule of thumb, p 495, and rule, p 488, give breakg loads about one-third greater than Mr. Clark's results, except for the oval beam, where they agree closely. The discrepancy is probably due to difference of quality of material.

Hollow beams of thin wrought iron were experimented on at the same time; and for these Mr. Clark deduced the following constants, to be used with his foregoing rule for cast-iron ones:

Constants for thin riveted tubes, circular, 1.74; oval, 1.85; rectangular, 1.96.
" " welded tubes, " 1.09; " 1.27; " 1.51.

Art. 34. The following experiments on **riveted sheet-iron cylindrical beams** are by Fairbairn. **1st.** Cylinder 18 ft long; 1 ft outer diam; clear span 17 ft; thickness of iron .037, or $\frac{1}{27}$ of an inch; wt of tube 107 lbs.

Center load. Lbs.	Def. Ins.	Center load. Lbs.	Def. Ins.
1360	.32	2368	.60
1920	.41	2480	.61
2114	.46	2592	.61
2256	.60	2704	.65

After bearing 2704 lbs. for 1½ minutes, failed by crushing at top.

2d. Cyl 16 ft 10 ins long; 12.4 ins outer diam; clear span 15 ft 7½ ins; thickness of iron .113, or full $\frac{1}{9}$ inch; wt of tube 392 lbs.

Center load. Lbs.	Def. Ins.	Center load. Lbs.	Def. Ins.
2000	.17	8000	.84
4000	.34	10000	1.06
6000	.52	11440	Broke.

With 11440 broke by the tearing of the bottom across the shackle-hole from which the load was suspended.

3d. Cyl 25 ft long; 17.68 ins outer diam; clear span 23 ft 5 ins; thickness .0631, or full $\frac{1}{16}$ inch; weight of tube 346 lbs.

Center load. Lbs.	Def. Ins.	Center load. Lbs.	Def. Ins.
1000	.12	5000	.48
2000	.21	5280	.51
3000	.30	5840	.60
4000	.40	6120	.71

With 6400 broke at bottom; 25 ins from center, by tearing through the rivet-holes.

4th. Cyl 25 ft long; 18.18 ins outer diam; clear span 23 ft 5 ins; thickness .119, or scant ⅛ inch; wt of tube 777 lbs.

Center load. Lbs.	Def. Ins.	Center load. Lbs.	Def. Ins.
2000	.15	10000	.82
4000	.30	12000	.95
6000	.43	13000	1.04
8000	.59	14240	Broke.

Broke through the rivet-holes 3 ft 3 ins from center, after sustaining the load for half a min.

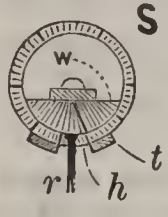


Fig. 14.

The tubes were composed of sheets about 2½ ft wide; and so long that a single sheet sufficed to form the entire circumf of the tube. They were united by double-riveted lap-joints. The loads were placed on a platform, supported by a rod *r*, Fig 14, which passed through a hole *h* in the bottom of the tube *s*. This rod was attached at its upper end to a block of wood *w*, rounded at its lower surface, so as to fit the tube.

Circular blocks of wood were fitted into the ends of the tubes, to prevent them from crushing at those parts under their loads; and the ends rested upon blocks hollowed out to correspond with their cylindrical shape, to a depth equal to about ¼ part of their diam.

Art. 35. Hodgkinson's beams have nearly 1¾ times the strength of a beam of eqnal wt whose top and bottom flanges are equal. Mr. Hodgkinson, having found that on an average, cast iron reqd abont 6½ times as much force to crnsh it as it did to pnll it apart, contrived the beam (of which Fig 15 is a cross-section at the center) in which the upper or compressed rib or flange, *u*, has but ⅙ of the area of the lower or extended one, *b*.* The top flange he therefore assumed to be safe; inasmnch as its area is somewhat greater than the proportion of 1 to 6½: and hence, breaking will take place from the yielding of the bottom flange by extension. As the result of his experiments, he gives the following rule, when the load is applied at top, or equally on both sides of the beam. **See Rem.** next page.

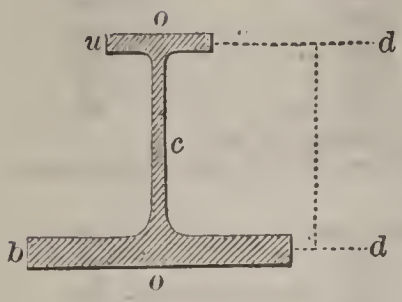


Fig. 15.

$$\text{Center Breakg load in tons} = \frac{\text{Area of bot flange in sq ins} \times \text{depth } o\,o \text{ in ins} \times \text{Constant } 2.166}{\text{Clear length in feet.}}$$

When the lower flange is as much as about 2½ inches thick, experiments show that part of the breakg load thus obtained should be deducted; because thick castings are proportionally weaker than thin ones. Half the wt of the beam itself must be deducted, for the neat breakg load; this, however, is necessary only when the beams are very long; for such as are used for ordinary building purposes, it may be neglected. If the load is equally distributed, it will be twice as great; but the entire wt of the beam must then be deducted.

Ex. The upper rib $u = 3$ ins $\times$ 1 inch $= 3$ sq ins area; bottom rib $b = 1\frac{1}{2}$ ins $\times$ 12 ins $= 18$ sq ins area; total depth $o\,o$, 15 ins; clear span, 20 ft. Here,

$$\frac{18 \times 15 \times 2.166}{20} = \frac{584.82}{20} = 29.241 \text{ tons, the reqd load, including } \tfrac{1}{2} \text{ the beam.}$$

Now to find the wt of half the beam, we may proceed thus: Mult the entire area of its cross-section in sq ins, by the clear span in ins. This gives us the cub ins of iron contained in the beam; and these div by 8600, give the wt of the beam in tons; because 8600 cnb ins of cast iron weigh about 1 ton; or near 4 cubic ins 1 lb. Thus, if the vert rib contains 12 sq ins, then since the two flanges contain 21, the entire section is 33 sq ins; and the span being 240 ins, we have $33 \times 240 = 7920$ cub ins of iron. And $\frac{7920}{8600} = .92$ of a ton, the wt of the beam. One-half of this, or .46 ton, taken from the breakg load 29.241 tons, leaves 28.78 tons as the neat breakg load; showing that in such cases as this it is scarcely worth while in practice to make the deduction. These beams are not always made of the same section throughout, (see Fig 16,) but diminish toward the ends; this method is therefore not always strictly correct, but no great accuracy is needed in such cases.

To find the size of a Hodgkinson beam, reqd to break under a given center load, having the depth. Mult the given load in tons by the clear span in feet. Mult the constant 2.166 by the total depth, *o o*, in ins. Div the first prod by the last; the quot will be the area of the bottom rib in sq ins. This, div by 6, will be the area of the top rib. The bottom rib is nsually made from 6 to 8 times as wide as it is thick; and the top one from 3 to 6 times. The thickness of the stem is usually a little greater at bottom than at top; the average thickness being from $\frac{1}{14}$ to $\frac{1}{24}$ of the depth of the beam. See **Rem.** next page.

To save iron, the *width* of the bottom flange, and of the top one also if thought proper, may be reduced by curves to about ½ as great at each end of the beam as at its center; as shown by the middle sketch of Fig 16, of which the upper sketch is a side view. Or, leaving the dimensions of those flanges unaltered, the *depth* of the vertical rib may be redneed toward the ends, as shown by the lowest sketch. The theoretical curve is here an ellipse. When the width is reduced, the very ends may, for stability, be widened out, as at *e*, which is a top view.

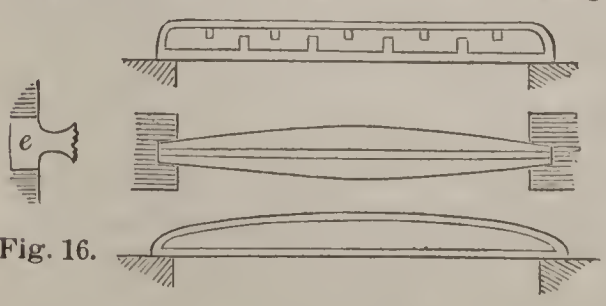


Fig. 16.

The vert rib is generally strengthened by casting brackets

* **In practice** ⅓ is much better and safer than ⅙.

on each side of it, as in the upper sketch. These should not extend entirely to the upper rib, as they then expose the beam to crack as it cools. To prevent this tendency, they may be attached alternately to the top and bottom ribs. The upper ones, however, are rarely needed.

In designing these beams, as well as in all other castings, it is important to avoid sudden transitions from thin to thick parts; and to keep all parts as nearly as possible of the same thickness. Otherwise the castings are apt to warp and crack in cooling. Also, bear in mind that the resistance or strength per sq inch is *considerably less* in thick castings than in thin ones.

Rem. The above rule for breakg loads is safe when the load is equally disposed on top, or on each side of the vert web; and when said web and the flanges are proportioned to each other about the same as those used in Mr. Hodgkinson's experiments. But subsequent investigators have found his beams to break with but little more than half the loads given by the rule, when applied to only one side, as *b o*, or *u o*, Fig 15, of the top or bottom flange. W. H. Barlow, C. E., London, experimenting since Hodgkinson, finds that when a cast-iron beam is liable to be loaded on only one side of the flange, the top flange should have an area equal to ⅓ that of the entire cross-section of the beam; and for beams so proportioned, he gives the following:

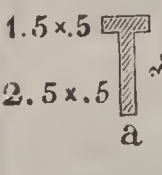
$$\text{Center breaking load, in tons} = \frac{\left(\begin{matrix}\text{Area bot flange}\\ \text{in sq ins}\end{matrix}\right)+\left(\begin{matrix}\text{Half area of}\\ \text{vert rib}\end{matrix}\right)\times\left(\begin{matrix}\text{Depth in ins,*from}\\ \text{center to center of}\\ \text{top and bot flanges}\end{matrix}\right)\times\begin{matrix}\text{Constant}\\ 2.333\end{matrix}}{\text{Clear span in feet.}}$$

Other experimenters recommend that even for loads pressing vertically through the upright rib, the lower flange should have but about 3 instead of 6 times the area of the upper one. **Cast beams should always be tested.**

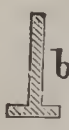
The average ultimate resistance of steel to compression being about twice that to extension, a Hodgkinson beam of that metal should have its lower flange of twice the area of its upper one. **Much uncertainty** exists in the whole matter.

Art. 36. For the purpose of ready reference, we give a few experimental results with cast-iron beams of various shapes: being the actual center breakg loads in tons of sound beams. Some beams of Sterling's toughened cast iron gave results full ⅓ higher than those of common iron.

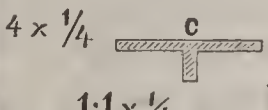
Actual center breakg loads in tons, of cast-iron beams. Clear spans in feet. Breadths and depths in inches.



Span 4½ ft.
Br load 2 tons.

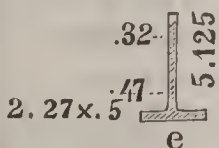


The above inverted.
Br load 2.3 to 2.9 tons.

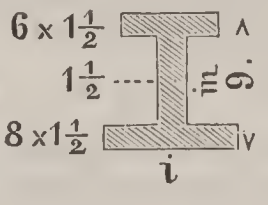


Span 4¼ ft.
Br load .125 ton.

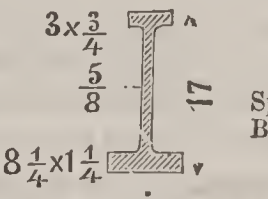
The above inverted.
Br. load .4 ton.



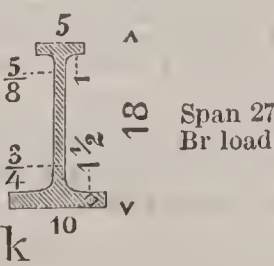
Span 4½ ft.
Br load { 3.7 tons to 4.2.



Span 11⅔ ft †
Br load 20 tons.



Span 18 ft.
Br load 22 to 28 tons.

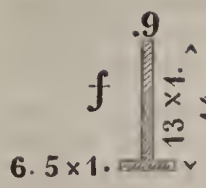


Span 27⅓ ft.‡
Br load 29¼ tons.

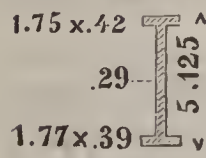
* As shown by *d d*, Fig 15.

† "After hearing 17 tons, the beam was unloaded, and its elasticity appeared to be but little if at all injured." Def under 4½ tons, .15 inch; 8½ tons, .3 inch; 17 tons, 1.1 inches.

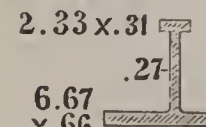
‡ About two hundred of these beams were tested by center loads of 12 tons. Def ⅜ to ⅝ inch.



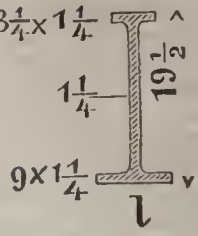
Span 15 ft.
Br load 12½ tons.



Span 4½ ft.
Br load 3 tons.

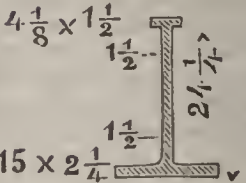


Span 4½ ft.
Br load 10.5 to 11.6 tons.



Span 19 ft.
Br load 50 to 54 tons.

By formula, p 488, it should have been but 40 tons.



Span 30¾ ft.
Br load 58 tons.*

In describing such beams, it is better to give the *entire* depth of the *beam;* for when the depth of the *web* is given, doubts arise whether it is meant to include the thicknesses of the two flanges, or not. Every writer, almost, that we have seen, leaves us in this doubt.

Rem. **In beams either larger or smaller than these,** but whose cross-sections are proportioned exactly as these are, and whose spans are the same that these have, the center breakg loads will be as the cubes of their cross-section lines. Thus, in a beam which is $\frac{1}{10}$, ⅓, ½, 2, 3, or 10 times as large every way, except in span, the breakg load will be $\frac{1}{1000}$, $\frac{1}{27}$, ⅛, 8, 27, or 1000 times as great.

If the spans also differ, first find the load as above, as if they were the same; then say, as the new span, is to the span given in our Figs, so is the breakg load thus found, to the actual breakg load for the new beam. Thus, suppose we wish to make a cast-iron beam, 4 times as large every way as the dimensions given in the first of these Figs; except its span, which is to be, say 10 ft, instead of 4½ ft. Here the first breakg load is found to be $4 \times 4 \times 4 = 64$ times as great; or 2 tons $\times$ 64 = 128 tons. Next,

New Span.		Span in Fig.		First load.		Actual load.
10	:	4.5	::	128	:	57.6 tons.

In such cases we must, however, have regard to Rem 1, Art 11, p 494. The foregoing process applies equally to beams of any other shapes, such as the following ones; or whether solid or hollow, &c; and of any other materials; so that if we have all the dimensions, and the breakg load of any beam whatever, we may find that for another one of the same material, and of the same proportions of cross-section. It may become advisable in important cases, to even make one or more model beams of some hitherto untried form; and to break them in order to find the breakg weight of the actual beam of the same material. In doing this, the defs should also be measd, in order to see whether those of the actual beam may not be too great. See Art 26, &c.

Art. 37. I beams, fig 18; **Channels,** fig 19; and **Deck beams,** fig 20, are made by A. & P. Roberts & Co, Pencoyd Iron Works, office, 261 S 4th St, Philadelphia; by New Jersey Steel & Iron Co, Trenton, N. J. (Agents, Morris, Wheeler & Co, 16th and Market Sts, Philadelphia); by Carnegie Bros & Co, Lim, Union Iron Mills, Pittsburgh Pa; by Phœnix Iron Co, Phœnixville Pa, office 410 Walnut St, Philadelphia; by Passaic Rolling Mill Co, Paterson N. J., office Astor House, New York City; and others. **Price,** Phila. 1886, 3 cts per ℔, cut to specified lengths not exceeding 30 ft. For extra lengths, punching, framing etc address as above. Special discounts on large orders.

Our tables, pp 522, 523, give the **principal sizes** (including the largest and smallest) **made in America.** We give the maximum and minimum thicknesses of web for each pattern. Any desired thickness between these extremes can be furnished. The *width* of flange increases equally with the thickness of web; but the *thickness* of flange, at root and at base, for any given pattern, are the same for all thicknesses of web.

* This iron was "Sterling's toughened," having about 16 per cent of wrought scrap melted in it. Each of the 89 beams was tested by a center load of 20 tons, which produced defs of from 5/8 to 7/8 inch. Entire length, 34½ ft.

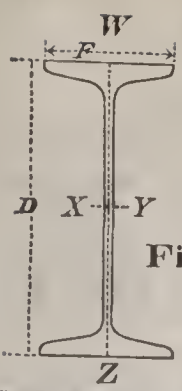


Fig 18.

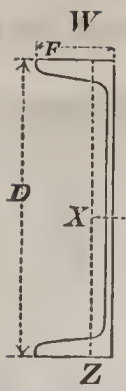


Fig 19.

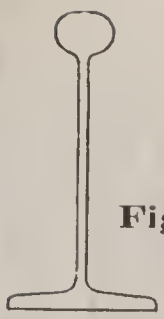


Fig 20.

In Belgium, I beams are rolled as large as 21.5 ins deep; 334 lbs per yd. Agents; J. H. Jackson & Co., 208 Franklin St, New York; Esherick & Co, 263 S. 4th St, Philadelphia.

In any bar, beam etc, of wrought iron, of uniform cross section,

$$\text{Weight per lineal yard, in pounds} = \text{area of cross section, in square inches} \times 10$$

$$\text{Area of cross section, in square inches} = \frac{\text{weight of beam per lineal yard, in pounds}}{10}$$

Strength. Beams of good wrought iron do not *break* under a gradually applied load until after they have *bent* so much as to be useless. **The ultimate load** is that one which so cripples the beam that it continues to yield indefinitely without increase of load. The web is supposed to be placed vert, as in our figs; and the beam supported at both ends, and stayed against yielding sideways; the dist apart of the side supports not exceeding 20 times the width of the flange. Then

$$\text{Center ultimate load in lbs, including half wt of clear span of beam} = \frac{\text{Tabular coefficient for cen ult load* for the given size}}{\text{clear span in ft}}$$

$$\textbf{Extraneous cen ult load} = \text{cen ult load so found} - \text{half wt of clear span of beam}$$

$$\textbf{Distributed ultimate load} = \text{twice cen ult load}$$

$$\textbf{Safe cen or distd load} = \frac{\text{ult cen or distd load}}{\text{required factor of safety}}$$

The **factor of safety** should in no case be less than 3. When the load is subject to variation (see p 435) or vibration, or is liable to be suddenly applied, use from 4 to 6. The factor should be further increased when the length of beam between lateral supports exceeds 20 times the width of flange; until, when it reaches 70 times said width, the factor should be double that for a length = 20 × width.

Caution. With very short spans, the safe loads thus found, although safe as regards a *rupture* of the beam itself, may be so great as to endanger a *crushing* of the ends of the beam, or of the wall etc under them, unless the beam has at its ends a greater length of bearing than would otherwise be needed. See pp 436 to 438.

To find what beam is required to sustain safely a given extraneous load in lbs, whether uniformly distributed or applied at the center. If uniformly distributed, divide it by 2. The quot is the equivalent *center* load. Then, in either case, add half the wt of the clear span of the beam itself. Mult together the sum, the required factor of safety, and the clear span of the beam in ft. Use any desired beam whose coefficient for ult cen load is not less than the prod.

For the deflection in ins at the center of the length of the beam, under any load less than half the ultimate load:

$$\text{Deflection in ins} = \frac{\text{Load in lbs} \times \text{Cube of clear span in ft}}{\text{Wt of beam, in lbs per yd} \times \text{Square of depth D of beam, in ins} \times \text{Constant, given below.}}$$

Constants for the above formula:

For I beam, loaded at center.....11200	For channel beam, loaded at center, 10000
" " uniformly loaded.....18000	" " uniformly loaded, 16000

The strength of a beam or channel when used as a column, may be found by the formula, p 439, using the least radius of gyration, as given in the following tables.

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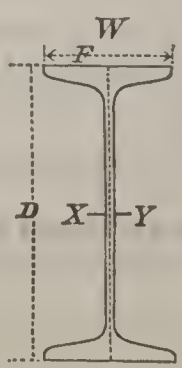
* The coeffs are the cen ult loads in lbs (including half the wt of clear span of beam) for beams *of one ft clear span.*

Pencoyd I Beams.—See page 521.

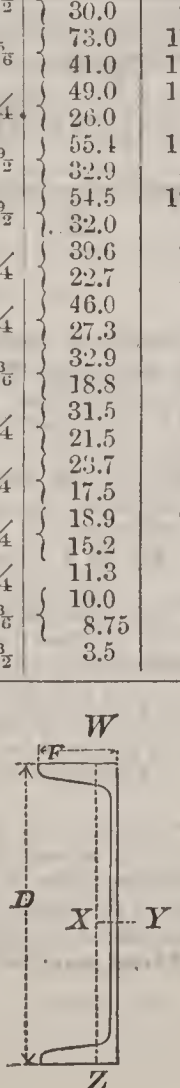
Chart Number.	Depth D, in ins	Width of flange F, in ins.	Thick's of web, in ins.	Thicknesses of flange, in ins.		Weight per yard, in lbs.	Coefficient for ult cen load, in lbs.	Least radius of gyration, in ins.	Moments of Inertia.	
				At root	At edge				About X Y.	About W Z.
	*									
1	15	$5\frac{31}{32}$	$\frac{7}{8}$	$1\frac{5}{16}$	$\frac{3}{4}$	233	1,388,700	1.17	743.60	31.89
	15	$5\frac{3}{4}$	$\frac{21}{32}$			200	1,273,200	1.20	682.08	28.50
2	15	$5\frac{1}{2}$	$\frac{13}{16}$	$1\frac{1}{8}$	$\frac{5}{8}$	201	1,168,800	1.05	626.57	22.16
	15	$5\frac{1}{8}$	$\frac{7}{16}$			145	972,900	1.08	521.19	16.91
3	12	$5\frac{23}{32}$	$\frac{7}{8}$	$1\frac{1}{4}$	$\frac{11}{16}$	194	940,800	1.16	403.48	26.10
	12	$5\frac{1}{2}$	$\frac{21}{32}$			168	867,900	1.17	371.98	23.19
4	12	$5\frac{5}{32}$	$\frac{13}{16}$	1	$\frac{35}{64}$	163	757,200	.99	324.61	17.78
	12	$4\frac{51}{64}$	$\frac{29}{64}$			120	636,600	1.01	272.86	12.22
5	$10\frac{1}{2}$	$5\frac{1}{2}$	$\frac{23}{32}$	$1\frac{1}{8}$	$\frac{21}{32}$	161	708,600	1.17	265.74	22.20
	$10\frac{1}{2}$	$5\frac{1}{4}$	$\frac{15}{32}$			134	644,300	1.19	241.63	19.00
$5\frac{1}{2}$	$10\frac{1}{2}$	$5\frac{1}{8}$	$\frac{21}{32}$	$\frac{15}{16}$	$\frac{17}{32}$	134	585,400	1.05	219.53	14.74
	$10\frac{1}{2}$	$4\frac{7}{8}$	$\frac{13}{32}$			108	521,100	1.07	195.42	12.45
6	$10\frac{1}{2}$	$4\frac{11}{16}$	$\frac{17}{32}$	$\frac{27}{32}$	$\frac{7}{16}$	109	480,900	.94	180.34	9.59
	$10\frac{1}{2}$	$4\frac{1}{2}$	$\frac{11}{32}$			89	432,700	.97	162.26	8.34
7	10	$4\frac{7}{8}$	$\frac{3}{4}$	$1\frac{1}{16}$	$\frac{1}{2}$	137	544,200	.96	194.41	12.63
	10	$4\frac{5}{8}$	$\frac{1}{2}$			112	486,000	.98	173.58	10.64
8	10	$4\frac{17}{32}$	$\frac{1}{2}$	1	$\frac{7}{16}$	106	452,400	.93	161.33	9.17
	10	$4\frac{3}{8}$	$\frac{11}{32}$			90	415,200	.95	148.31	8.09
9	9	$4\frac{23}{32}$	$\frac{3}{4}$	$\frac{29}{32}$	$\frac{1}{2}$	122	436,800	.94	143.70	10.78
	9	$4\frac{3}{8}$	$\frac{11}{32}$			90	369,600	.96	118.81	8.44
10	9	$4\frac{21}{64}$	$\frac{1}{2}$	$\frac{25}{32}$	$\frac{3}{8}$	88	331,500	.87	106.78	6.66
	9	$4\frac{1}{8}$	$\frac{19}{64}$			70	293,700	.89	94.44	5.59
11	8	$4\frac{19}{32}$	$\frac{3}{4}$	$\frac{27}{32}$	$\frac{15}{32}$	109	345,900	.93	98.59	9.43
	8	$4\frac{1}{4}$	$\frac{13}{32}$			81	293,700	.94	83.93	7.23
12	8	$4\frac{1}{8}$	$\frac{7}{16}$	$\frac{3}{4}$	$\frac{3}{8}$	75	260,700	.86	74.50	5.55
	8	4	$\frac{5}{16}$			65	242,100	.88	69.17	5.02
14	7	$4\frac{1}{8}$	$\frac{3}{4}$	$\frac{23}{32}$	$\frac{11}{32}$	88	232,800	.80	58.60	5.63
	7	$3\frac{39}{64}$	$\frac{15}{64}$			51	172,200	.82	43.08	3.43
16	6	$3\frac{3}{8}$	$\frac{5}{8}$	$\frac{21}{32}$	$\frac{5}{16}$	63	144,600	.64	30.77	2.58
	6	3	$\frac{1}{4}$			40	112,200	.66	24.10	1.80
18	5	$2\frac{31}{32}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{1}{4}$	40	81,600	.58	14.69	1.35
	5	$2\frac{3}{4}$	$\frac{7}{32}$			30	69,900	.60	12.50	1.09
19	4	3	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{9}{32}$	38	63,000	.62	9.02	1.46
	4	$2\frac{3}{4}$	$\frac{1}{4}$			28	53,700	.63	7.69	1.17
20	4	$2\frac{21}{64}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{7}{32}$	21.5	39,300	.50	5.55	.54
	4	$2\frac{1}{4}$	$\frac{11}{64}$			18.5	36,000	.51	5.14	.49
21	3	$2\frac{11}{16}$	$\frac{7}{16}$	$\frac{7}{16}$	$\frac{1}{4}$	28.6	34,500	.58	3.91	.96
	3	$2\frac{1}{2}$	$\frac{1}{4}$			23.	30,600	.59	3.29	.77
22	3	$2\frac{13}{32}$	$\frac{5}{16}$	$\frac{13}{32}$	$\frac{13}{64}$	21.7	28,200	.52	3.01	.59
	3 *	$2\frac{1}{4}$	$\frac{5}{32}$			17.	24,600	.53	2.66	.48
*	20	$6\frac{3}{4}$	$\frac{11}{16}$	$1\frac{11}{16}$	$\frac{5}{8}$	272	2,310,000	1.31	1,650.30	46.50
	20	6	$\frac{1}{2}$	$1\frac{11}{32}$	$\frac{17}{32}$	200	1,732,500	1.15	1,238.00	26.62
Steel	$1\frac{1}{4}$	$1\frac{1}{2}$	$\frac{1}{8}$	.16	.11	5.25	4,320	.36	.135	.0685

These three patterns are from the list of the **New Jersey Steel & Iron Co. Trenton, N. J.** The 20 inch are the largest, and the $1\frac{1}{4}$ inch is the smallest, now made in America.



Pencoyd Channels.—See page 521.

Chart Number.	Depth D, in ins.	Width of flange F, in ins.	Thick's of web, in ins.	Thicknesses of flange, in ins.		Weight per yard in lbs.	Coefficient for ult cen load, in lbs.	Least radius of gyration, in ins.	Moments of Inertia.	
				At root	At edge				About X Y.	About W Z.
30	15	4 3/8	1	1	5/8	204.5	1,040,400	1.10	557.44	24.74
	15	4	5/8			148.0	842,700	1.13	451.51	19.05
31	12	3 17/32	1	1	9/16	160.0	626,400	.90	268.51	12.96
	12	2 15/16	13/32			88.5	426,300	.92	182.71	7.42
32	12	2 61/64	5/8	3/4	11/32	101.5	404,700	.72	173.51	5.26
	12	2 35/64	9/32			60.0	274,200	.74	123.71	3.22
34	10	3 1/16	3/4	15/16	7/16	106.0	366,900	.82	131.04	7.13
	10	2 19/32	9/32			60.0	259,500	.84	92.71	4.29
35	10	2 3/4	5/8	3/4	3/8	86.5	294,300	.67	105.16	3.88
	10	2 3/8	1/4			49.0	206,700	.69	73.91	2.33
36	9	2 7/8	3/4	27/32	3/8	93.0	282,300	.67	90.66	4.17
	9	2 7/16	5/16			54.0	200,100	.68	64.34	2.47
37	9	2 13/32	1/2	35/64	19/64	61.0	186,300	.58	59.85	2.05
	9	2 9/64	15/64			37.0	135,600	.59	43.65	1.31
38	8	2 3/4	3/4	11/16	5/16	80.5	210,000	.71	60.00	4.28
	8	2 9/32	9/32			43.0	139,800	.71	40.00	2.17
39	8	2 19/64	1/2	15/32	9/32	54.0	143,400	.60	41.03	1.94
	8	2	13/64			30.0	98,700	.60	28.23	1.06
40	7	2 3/4	3/4	11/16	5/16	73.0	170,100	.65	42.57	3.08
	7	2 19/64	19/64			41.0	117,900	.65	29.51	1.71
41	7	2 9/32	1/2	7/16	1/4	49.0	111,300	.58	27.86	1.65
	7	1 31/32	3/16			26.0	73,800	.58	18.46	.90
42	6	2 5/8	5/8	5/8	9/32	55.4	126,300	.68	25.12	2.56
	6	2 1/4	1/4			32.9	85,500	.67	18.37	1.46
43	6	2 1/2	5/8	5/8	9/32	54.5	124,200	.61	24.35	2.03
	6	2 1/8	1/4			32.0	72,900	.60	17.60	1.15
44	6	2 1/32	1/2	3/8	1/4	39.6	90,200	.52	16.73	1.07
	6	1 3/4	7/32			22.7	54,300	.51	11.67	.59
45	5	2 3/8	5/8	13/32	1/4	46.0	87,400	.58	14.20	1.55
	5	2	1/4			27.3	57,600	.56	10.29	.86
46	5	1 29/32	1/2	3/8	3/16	32.9	62,500	.47	12.54	.73
	5	1 5/8	7/32			18.8	37,200	.45	6.67	.37
47	4	1 31/32	1/2	17/32	1/4	31.5	47,800	.52	6.49	.85
	4	1 23/32	1/4			21.5	36,300	.50	5.16	.54
48		1 23/32	3/8	13/32	1/4	23.7	36,000	.49	4.97	.57
		1 9/16	7/32			17.5	28,800	.48	4.14	.41
49	3	1 21/32	11/32	13/32	1/4	18.9	21,600	.47	2.31	.42
	3	1 17/32	7/32			15.2	18,900	.46	2.03	.32
50	2 1/4	1 3/8	1/4	1/4	1/4	11.3	10,000	.43	.80	.21
51	2	1 5/32	9/32	5/16	3/16	10.0	7,600	.32	.88	.10
	2	1 3/32	7/32			8.75	6,700	.31	.48	.08
52	1 3/4	19/32	3/32	1/4	3/32	3.5	2,300	.17	.28	.01



Rolled iron I beams for railroad bridges of short span. (See table, p 522.) A single 7-inch beam, 88 lbs per yard, under each rail, will suffice for 3 or 4 ft span; one of 15 inch, 200 lbs, for 8 or 10 ft; *two* of 12 inch, 194 lbs, side by side under *each* rail, as in Figs 50 A and 50 B, for 12 to 14 ft; and two of 15 inch, 145 lbs, under each rail, for 15 ft. By employing a greater number of beams, or by introducing a truss-rod, *r r*, Fig 52, p 514, the spans may be increased, or lighter beams be used. Care must be taken to insure lateral stability, by means of hor ties and struts.

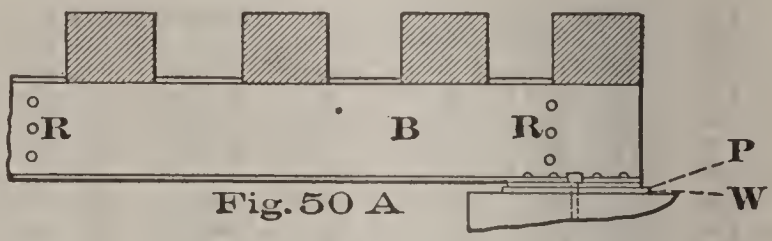


Fig. 50 A

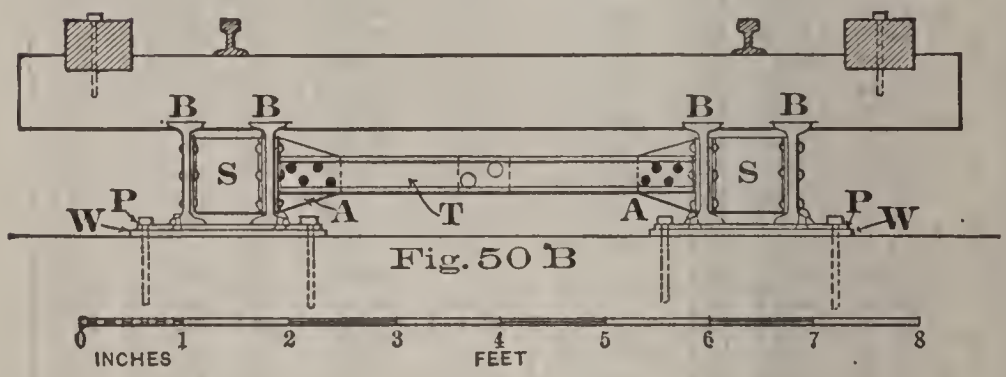


Fig. 50 B

Figs 50 A and 50 B, for which we are indebted to the courtesy of Mr. Jos. M. Wilson, C E, Engr of Bridges and Buildings, **Penna R R,** illustrate a standard form of rolled beam girder in use on that road. For spans of 14 ft, each beam is 12 inch, 180 lbs per yd; for 15 ft, 15 inch, 150 lbs. The two beams, B B, forming a girder, are held at the proper dist apart from each other by separators, S, each of which consists of a short piece of channel iron placed vertically, and having its flanges riveted to the webs of the beams. On some roads, cast-iron separators, and bolts which pass through them and through the webs of the beams, are used instead. The longitudinal dist, R R, Fig 50 A, between the separators, is about 5 times the depth of the beam.

At the same points, R R, &c, of the span, are placed transverse tie-struts, T, Fig 50 B, each composed of two 4-inch 21 lb channel bars placed back to back (so as to form an I) but ¾ inch apart. Between the two channels are riveted angle-plates, A A, ⅜ inch thick, the flanges of which are fastened to the webs of the inner beams of the girders by the same rivets which hold the separators. At their centers, the two channel-bars are separated by a piece of bar-iron, ¾ inch thick × 4 ins square, riveted between them, as shown. The cross-ties, notched to the girders as shown, give additional lateral support.

At the ends of the span, the lower flanges of the beams are riveted to rectangular **"bolster-plates,"** P, which rest upon slightly larger **wall-plates,** W. The rivets are counter-sunk under the plate, P. Both plates are of rolled-iron, ⅝ inch thick. They are held in place on the abuts by bolts passing through both plates as shown. At one end of the span, the bolt holes in the bolster plate are slightly elongated, to allow for **contraction and expansion.**

Angle and T Iron.

The following tables give the sizes made by **A. & P. Roberts & Co, Pencoyd Iron Works, Phila.** They also make "*Square-root*" Angles, Fig 1, from 1 in × 1 in × 1/8 in to 4 in × 4 in × 5/8 in. Many of the sizes in these tables, and others, are made by other mills. **Prices,** Phila, 1886, approximate; Angle iron, 2¼ cts per lb; T iron, 2¾ cts.

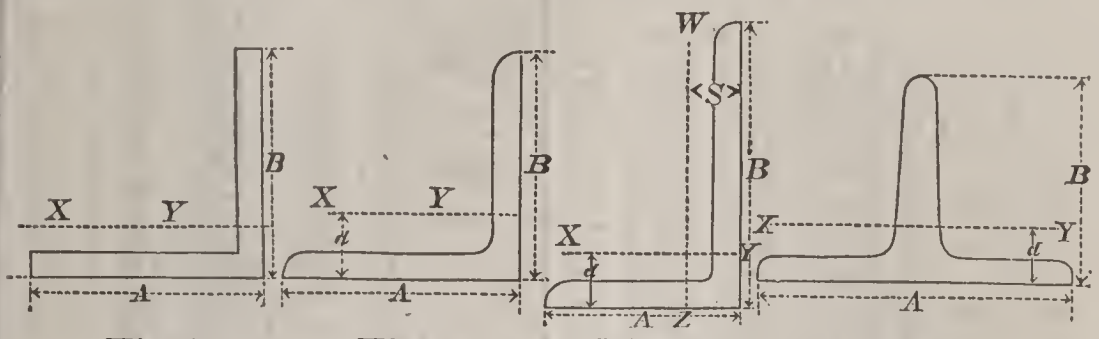


Fig 1. **Fig 2.** **Fig 3.** **Fig 4.**

The sizes A and B are, in all cases, **measured from out to out,** as indicated in the Figs.

Angle iron, of any given dimensions, A and B, in the tables, **can be rolled to any desired thickness** between the maximum and minimum thicknesses given for that size. The dimensions A and B vary slightly with the thickness. Those given are the dimensions corresponding to the *minimum* thickness. In ordering angle iron of thicknesses between the max and min, give *either* the thickness in ins, or the wt in lbs per yard, wanted; but *not both*.

The thickness of each size of T iron is fixed as given in the table, and cannot be varied except by changing the shape of the rolls, or making new ones. This is always expensive. It is important, in designing, or in ordering rolled iron of any kind, to bear this in mind, and not to introduce sizes that have to be specially made.

The area in square ins of cross section of any bar of rolled iron of uniform dimensions throughout, is $= \frac{\text{its weight in lbs per yd}}{10}$.

The ultimate or crippling center load, for a beam consisting of a single bar of angle or T iron, supported at both ends, may be found by the formula, p 488, using the moment of inertia as given in the following tables. **Or, approximately,** and much more simply,

$$\left.\begin{matrix}\textbf{Ult cen load}\\ \text{in lbs}\end{matrix}\right\} = \frac{2800 \times \text{Area of Cross-section in sq ins} \times \text{Depth of Beam in ins}}{\text{Clear span in ft.}}$$

The ultimate distributed load is *twice* the ult *center* load.

For the safe center or distributed load, divide the *ultimate* center or distributed load (as the case may be) by the required *factor of safety*, which should in no case be less than 3, and should generally be from 4 to 6, according to circumstances.

Under any load less than half the ultimate load;

$$\left.\begin{matrix}\textbf{Deflection} \text{ in ins}\\ \text{at center}\\ \textbf{with load at center}\end{matrix}\right\} = \frac{\text{Load in lbs} \times \text{Cube of clear span in ft}}{68000 \times \text{Area in sq ins} \times \text{Square of depth in ins.}}$$

$$\left.\begin{matrix}\textbf{Deflection} \text{ in ins}\\ \text{at center}\\ \textbf{with distributed load}\end{matrix}\right\} = \frac{\text{Load in lbs} \times \text{Cube of clear span in ft}}{108000 \times \text{Area in sq ins} \times \text{Square of depth in ins.}}$$

The breaking load, for a column consisting of a single bar of angle or T iron, with flat ends, firmly fixed, may be found by the formula, p 439, using the least radius of gyration as given in the following tables.

Angles with equal legs, Fig. 2.

Dimensions, A and B, in ins.	Thickness, ins.	Wt. per yd, in lbs.	Moment of Inertia about X Y	Least Rad of Gyration in ins.	Dist *d* from base to neutr'l axis, ins.	Dimensions, A and B, in ins.	Thickness, ins.	Wt per yd, in lbs.	Moment of Inertia about X Y	Least Rad of Gyration in ins.	Dist *d* from base to neutr'l axis, ins.
1 × 1	1/8	2.3	.02	.20	.30	2½ × 2½	1/2	22.5	1.23	.49	.81
" "	1/4	4.4	.04	.20	.35	2¾ × 2¾	1/4	13.1	.95	.55	.78
1¼ × 1¼	1/8	3.0	.05	.26	.36	" "	1/2	25.0	1.67	.54	.87
" "	1/4	5.6	.08	.26	.40	3 × 3	1/4	14.4	1.24	.60	.84
1½ × 1½	3/16	5.3	.11	.31	.44	" "	5/8	33.6	2.62	.59	.98
" "	3/8	9.8	.19	.31	.51	3½ × 3½	3/8	24.8	2.87	.70	1.01
1¾ × 1¾	3/16	6.2	.18	.36	.51	" "	5/8	39.8	4.33	.69	1.10
" "	3/8	11.7	.31	.35	.57	4 × 4	3/8	28.6	4.36	.81	1.14
2 × 2	3/16	7.1	.27	.40	.57	" "	3/4	54.4	7.67	.80	1.27
" "	3/8	13.6	.50	.39	.64	5 × 5	7/16	41.8	10.02	1.00	1.41
2¼ × 2¼	1/4	10.6	.50	.45	.65	" "	1	90.0	19.64	.98	1.61
" "	7/16	17.8	.79	.44	.72	6 × 6	7/16	50.6	17.68	1.19	1.66
2½ × 2½	1/4	11.9	.70	.50	.72	" "	1	110.0	35.46	1.17	1.86

Angles with unequal legs, Fig. 3.

Dimensions, B and A, in ins.	Thickness, ins.	Wt per yd, in lbs.	Moment of Inertia. About X Y.	Moment of Inertia. About W Z.	Least Rad of Gyration, in ins.	Dist in ins from base to neutral axis. *d*	Dist in ins from base to neutral axis. *s*	Dimensions, B and A, in ins.	Thickness, ins.	Wt per yd, in lbs.	Moment of Inertia. About X Y.	Moment of Inertia. About W Z.	Least Rad of Gyration, in ins.	Dist in ins from base to neutral axis. *d*	Dist in ins from base to neutral axis. *s*
3 × 2	1/4	11.9	1.09	.39	.46	.99	.49	5 × 3	3/4	54.4	13.15	3.51	.69	1.84	.84
" "	1/2	22.5	1.92	.67	.46	1.08	.58	5 × 3½	3/8	30.5	7.78	3.23	.80	1.61	.86
3 × 2½	5/16	16.2	1.42	.90	.54	.93	.68	" "	3/4	58.1	13.92	5.55	.79	1.75	1.00
" "	1/2	25.0	2.08	1.30	.54	1.00	.75	5 × 4	3/8	32.3	8.14	4.66	.87	1.53	1.03
3½ × 2½	5/16	17.8	2.19	.94	.56	1.14	.64	" "	1	80.0	18.17	10.17	.86	1.75	1.25
" "	1/2	27.5	3.24	1.36	.56	1.20	.70	5½ × 3½	3/8	32.3	10.12	3.27	.81	1.82	.82
3½ × 3	11/32	21.2	2.53	1.72	.64	1.07	.82	" "	5/8	52.3	15.73	4.96	.80	1.91	.91
" "	5/8	36.7	4.11	2.81	.64	1.17	.92	6 × 3½	7/16	39.6	14.76	3.81	.82	2.06	.81
4 × 3	3/8	24.8	3.96	1.92	.67	1.28	.78	" "	1	85.0	29.24	7.21	.81	2.26	1.01
" "	5/8	39.8	6.03	2.87	.65	1.37	.87	6 × 4	7/16	41.8	15.46	5.60	.92	1.96	.96
4 × 3½	3/8	26.7	4.17	2.99	.74	1.20	.95	" "	1	90.0	30.75	10.75	.91	2.17	1.17
" "	5/8	43.0	6.37	4.52	.73	1.29	1.04	6½ × 4	7/16	44.0	19.29	5.72	.94	2.18	.93
4½ × 3	3/8	26.7	5.50	1.98	.69	1.49	.74	" "	1	95.0	38.66	11.00	.93	2.38	1.13
" "	5/8	43.0	8.44	2.98	.68	1.58	.83	7 × 3½	5/8	61.7	30.25	5.28	.85	2.57	.82
5 × 3	3/8	28.6	7.37	2.04	.70	1.70	.70	" "	1	95.0	45.37	7.53	.84	2.71	.96

T iron with equal legs, Fig 4.

Dimensions, ins A and B.	Thicknesses, ins. Stem, B. At root.	Stem, B. At edge.	Base, A. At root.	Base, A. At edge.	Wt per yd, lbs.	Moment of Inertia about X Y.	Least Rad of Gyr.	Dist *d*, from base to neutral axis, ins.
1 × 1	1/4	5/32	1/4	5/32	3.	.03	.26	.30
1¼ × 1¼	"	3/16	"	3/16	4.5	.07	.27	.37
1½ × 1½	"	"	"	"	6.	.13	.32	.45
1¾ × 1¾	"	"	"	"	7.1	.21	.37	.50
2 × 2	5/16	1/4	5/16	1/4	10.5	.38	.43	.60
2¼ × 2¼	1/4	"	3/8	"	11.75	.52	.50	.61
" "	5/16	"	5/16	"	12.	.54	.47	.65
2½ × 2½	13/32	11/32	13/32	11/32	17.52	.97	.53	.75
" "	15/32	13/32	15/32	13/32	19.5	1.12	.55	.75
3 × 3	1/2	7/16	1/2	7/16	26.0	2.10	.62	.90
3½ × 3½	"	"	"	"	31.9	3.47	.74	1.00
4 × 4	"	"	9/16	"	36.5	5.26	.84	1.14

T iron with unequal legs. (Using the lettering of Fig. 4.)

Dimensions in ins.		Thicknesses, ins.				Wt per yd, lbs.	Moment of Inertia about X Y.	Least Rad of Gyr.	Dist *d* from base to neutral axis, ins.
		Stem B.		Base A.					
A.	B.	At root.	At edge.	At root.	At edge.				
2	9/16	5/16	5/16	1/4	1/4	5.88	.01	.13	.17
"	1	9/32	1/4	"	"	7.	.05	.26	.27
"	1 1/2	5/16	"	9/32	"	8.75	.16	.43	.43
2 1/4	9/16	"	5/16	1/4	"	6.5	.01	.12	.18
2 1/2	1 1/4	"	1/4	"	"	9.1	.10	.33	.32
2 3/4	1 3/4	3/4	3/4	11/32	5/16	18.75	.56	.55	.66
"	2	"	"	"	"	21.	.83	.55	.75
3	1 1/2	11/32	1/4	1/4	1/4	11.2	.19	.41	.37
"	2 1/2	5/8	1/2	7/16	3/8	23.8	1.38	.63	.82
"	3 1/2	1/2	7/16	1/2	7/16	28.25	3.12	.61	1.10
4	2	9/16	3/8	5/16	5/16	20.4	.68	.58	.54
"	3	5/8	5/16	"	"	25.25	2.09	.82	.84
"	"	3/8	3/8	7/16	3/8	25.9	1.94	.86	.77
"	3 1/2	3/4	9/16	3/4	7/16	41.8	4.65	.88	1.09
4 1/2	"	7/8	11/16	1/2	"	44.5	5.27	.91	1.16
5	2 1/2	11/16	7/16	3/8	3/8	30.7	1.61	.72	.67
"	"	"	"	7/16	7/16	33.	1.63	.70	.64
"	3 1/2	7/8	11/16	1/2	1/2	48.44	5.37	1.04	1.05
"	4	19/32	1/2	"	"	44.1	6.24	1.09	1.08

RESISTANCE OF OPEN BEAMS.

Beams, &c, in which all of the longitudinal resistance is regarded as being exerted by the *flanges*.

Art. 1. On page 486 we saw that the strength of "*closed*" beams, or beams of solid cross-section, is proportional to the *squares* of their depths: because the resisting moment of each fibre is proportional to the *square* of its distance from the neutral axis.

Art. 2. But, as in an *open* beam, or *truss*, $i\,x\,e\,a$ Fig 6 (or in a closed beam with a thin web w, Fig 22, p 537) we may place as many of the fibres as possible in the *chords*, $i\,x$ and $e\,a$ Fig 6 (a and b Fig 22) or as far as possible from the neutral axis n Fig 6, so that they may exert their maximum resistance. The chords are thus made to bear most of the horizontal or longitudinal strain. For convenience's sake we assume that they bear it *all*. For the resistance of the *web*, see Art. 5, p 529.

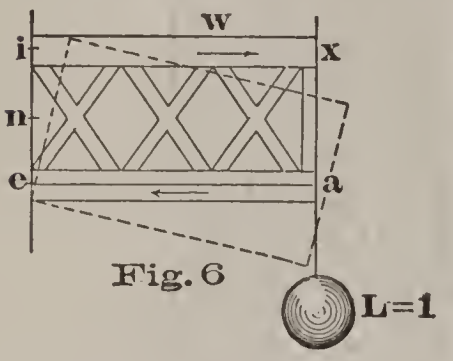


Fig. 6

It is plain that the *breaking* moment of the *load* is the same as in the closed beam Fig 2, p 479, = (Fig 6) the load × its leverage (= $e\,a$ or $i\,x$) about the neutral axis n. But the *resisting* moment of the *beam* now consists of

$$\left(\begin{array}{c}\text{longitudinal tensile}\\ \text{resistance of the}\\ \text{upper chord } i\,x\end{array} \times \begin{array}{c}\text{its leverage } i\,n^*\\ \text{about the neu-}\\ \text{tral axis } n\end{array}\right) + \left(\begin{array}{c}\text{longitudinal compres-}\\ \text{sive resistance of the}\\ \text{lower chord } e\,a\end{array} \times \begin{array}{c}\text{its leverage}\\ ne^* \text{ about the}\\ \text{neutral axis } n\end{array}\right)$$

$$= \begin{array}{c}\text{sum of the longitudinal re-}\\ \text{sistances of the two chords}\end{array} \times \text{half depth* of beam}$$

But we may express this more simply by regarding the neutral axis or fulcrum as being at the inner end i or e of one of the chords; and by writing

$$\textbf{Moment of resistance} \text{ of beam} = \begin{array}{c}\text{longitudinal resistance of}\\ \textit{either one} \text{ chord } i\,x \text{ or } e\,a\end{array} \times \begin{array}{c}\textit{whole} \text{ depth}\\ ie^* \text{ of beam}\end{array}$$

So long as the beam sustains its load, the longitudinal *resistances* of the two chords are equal, although the longitudinal *strengths* of one or of both may be much greater than this resistance. If the strength of either chord becomes *less* than

$$\text{Resistance} = \frac{\text{moment of rupture}}{\text{depth* of beam}}$$

then that chord fails, and the beam gives way.

Hence the

$$\textbf{maximum or ultimate moment of resistance} \text{ of beam} = \begin{array}{c}\text{ultimate longitudinal}\\ \textit{strength} \text{ of the } \textit{weaker}\\ \text{chord } i\,x \text{ or } e\,a\end{array} \times \begin{array}{c}\text{whole depth*}\\ i\,e \text{ of beam}\end{array}$$

In "closed beams," as explained on p 485, the longitudinal tensile and compressive resistances exerted by the fibres against rupture, are aided, to an important extent, by the mutual adhesion of the fibres, which resists their sliding upon each other; for without such sliding, rupture cannot occur. But in "open beams," such as we are now treating of, the thickness of the flanges or chords is so slight, compared with the depth of the beam or truss, that the sliding between their fibres becomes insignificant. The resistance due to the adhesion of the fibres to each other is therefore neglected, and the flanges are regarded as sustaining only longitudinal tensile or compressive strains.

Art. 3. Let Fig 6 be a hor open beam 1.5 ft deep from i to e, and projecting ft from a wall into which it is firmly fixed by its flanges or chords i and e; and let the concentrated load **L** at its outer end be 1 ton. This load tends to pull the beam into the dotted position by stretching or tearing apart the fibres of the upper chord at i. Now with how great a moment of rupture does it tend to do this, and how strong must the fibres of the chord be at i in order that their moment of resistance may oppose it safely? We shall here leave the wt of the beam itself out of consideration. When required to be included see Case 6, p 482.

Regard the lines $i\,e$ and $e\,a$ as the two arms of a bent lever resting on its fulcrum e. This lever is plainly acted upon and balanced by two equal moments: one at each end a and i; namely at a the moment of rupture of the load, equal to (1 ton × 6 ft leverage $a\,e$) = 6 ft tons; and at i the resisting moment of the beam equal to the hor pull or strain on the fibres at the chord i × 1.5 ft. leverage i But we do not yet know what amount of hor pull by the fibres at i is required t

... and half-depths are measured from the *centers of gravity* of cross section of th

balance the moment of the load. It is however very easily found by merely dividing the 6 ft tons moment of the load by the 1.5 ft leverage of the fibres, that is, by the depth of the beam. Thus we get $(6 \div 1.5) = 4$ tons pull at i; and we then have the $6 \times 1 = 6$ ft tons moment of the load, balanced by the $1.5 \times 4 = 6$ ft tons moment of the fibres. Therefore in order just to balance the moment of the load, the chord at i must be strong enough to bear a hor pull of 4 tons; or for a safety of 3, 4 or 6, &c, strong enough to bear a pull of 12, 16, or 24, &c, tons. The web members of course carry this 4 ton hor pulling strain from the upper chord to the lower one upon which it acts as an equal hor compressing one.

In shape of a formula the above stands thus.

$$\text{**Hor strain** at any point in a hor flange of an open cantilever} = \frac{\text{Moment of load at that point}}{\text{Depth of beam}} = \frac{\text{Load} \times \text{its leverage at that point}}{\text{Depth of beam}}$$

Hence if we know the size and of course the ultimate longitudinal tensile and compressive strength of the flange or chord, we have by transposition the ultimate or breaking load of the hor open beam, thus,

$$\text{**Breaking load** at any point of a hor open cantilever} = \frac{\text{Ultimate strength of flange} \times \text{Depth of beam.}}{\text{Leverage of load at that point.}}$$

And for a safety of 3, 4 or 6, &c, we have

$$\text{**Safe load**} = \frac{\text{⅓, ¼ or ⅙, &c, the ult strength of flange} \times \text{Depth of beam.}}{\text{Leverage of load at that point.}}$$

Art. 4. Also in a hor open beam or truss supported at both ends, after having found the moment of the load at any point (by "moments," p 479, &c) the strain on the beam as also its load in lbs or tons are found in the same way or by the same formulas.

Rem. 1. The longitudinal strains on the **flanges of hor closed beams** with thin webs such as common rolled I beams, as well as their loads, are also frequently computed in this same ready way, instead of the more troublesome one, p 488. The webs are left entirely out of consideration as regards the hor strains. Although not strictly correct, it is sufficiently so for ordinary practice, and is safe. With these assumptions the dimensions or sectional areas of the top and bottom flanges are proportioned to the safe unit strains of the material. Thus Hodgkinson having found that the ultimate compressive strength of cast-iron averaged about 6 times as great as its tensile one, gave his upper flange only one-sixth the area of the lower one, in order that both should be equally strong. In wrought-iron the tensile strength is somewhat the greatest, which would lead to making the lower flange the smallest, but here this consideration is outweighed by the practical ones of greater ease of manufacture and of handling or placing which require equal flanges.

Rem. 2. If the flanges are not horizontal, although the beam or truss itself may be so, the longitudinal strains on the flanges will be increased; and the transverse or shearing strains on the webs will also be changed as stated in Art 6. **If the beams are inclined,** modifications arise which we shall not treat of. Strangely, most of our standard authorities on bridge building do not even allude to them.

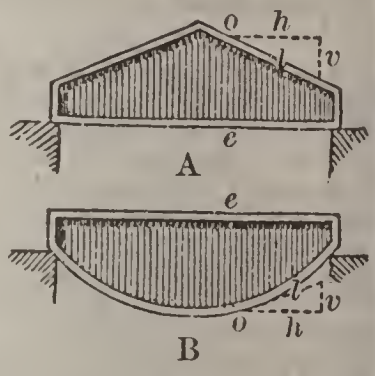
Rem. 3. The principle of the bent lever in open beams explains why **the strength of a truss is as its depth.** (the length of its vert lever-arm) instead of as the *square* of its depth as in closed beams. The strength however is inversely as the length in both kinds.

Art. 5. The web members of an open beam or common truss like Figs 10 and 11, p 558, uniformly loaded, carry the vert or shearing forces of the load and beam from the center each way, up and down alternately from one chord to the other, until finally the end ones deposit it as load on the supports or abutments. For each member receives and carries its share of the shearing force in the shape of *an end load*, thus changing the *shearing* tendency into an alternately *pulling* and *compressing* one according as the alternate members are ties or struts. In doing this any web member that is oblique is (on account of its obliquity) strained to an extent that exceeds its load in the same proportion that the oblique length of the member exceeds the length it would have had if it had been vert, as explained in Art 11, p 557, &c. This excess of strain over the load on the obliques

exhibits itself at their ends as hor pull along one chord, and hor compression along the other; and these hor strains on the chords are the same as those found by moments. Thus it is seen in Figs p 558 that the hor strain at the center of each chord (as there found by tracing up the diff loads or shearing forces in their journey along the obliques) is set down at 16 tons. Now the whole distributed load on one truss of 64 ft span, and 16 ft depth, is 32 tons: and by Case 10, p 483, we find that at the center the moment of this load is $16 \times 16 = 256$ tons; and this divided by the depth of our open truss or 16 ft gives 16 tons for the hor strain at center as before; and so at other points. The *oblique* web members are plainly the only ones that can convey their loads *laterally*, that is in directions tending toward or from the abutments. Vertical members merely convey their loads vert up or down from one chord to the other, at which last they transfer them to oblique members which *can* convey them laterally. If both a pull and a push act at once in opposite directions on a web member, their diff is the actual strain.

Rem. As a matter of economy in *small* spans it is often better not to proportion the sizes of the individual members to the strains they have to bear; but to give to the flanges throughout their entire length the same dimensions as are required at their most strained part, namely, at the center; and to make all the web members as strong as the most strained or end ones. This avoids the extra trouble and expense of getting out and fitting together many pieces of various sizes.

Art. 6. Oblique or curved flanges. We have hitherto supposed the beams and their flanges to be horizontal; but a beam may be hor, and yet have one or both of its flanges oblique or curved as at A and B. In such cases the longitudinal strains along the flanges become greater; and the vert or shearing strains across the web *in most cases* less. See Rem at end of Art. It is plain that such flanges must as it were intercept to some extent (depending on their inclination) the vert force at any point, and convert it into an oblique one along the flanges, somewhat as the oblique web members of an open beam do.

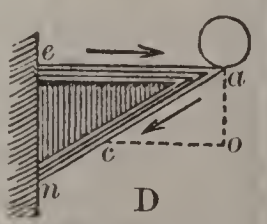


To find these new strains at any point *o*, Figs A, B, of either an upper or lower oblique or curved flange, first ascertain by "Moments," the hor strain at that point for a beam with the depth *o c*; and by "Shearing," the shear also. Then from that point *o* draw a hor line *h* equal by scale to the hor strain; and from its end draw *v* vert and ending either at the flange (produced if necessary) if straight as in A; or at a tangent *l* from *o* if the flange is curved as at B. Then will *l* in either fig give by the same scale the longitudinal strain along the flange at *o*; and *h* and *v* are the components of that strain. As a formula, the Rule reads thus, *o* being the angle formed by *h* and *l* at *o*.

$$\textbf{Strain along oblique flange} = \text{hor strain} \div \text{cosine of } o = \frac{\text{mom of rup}}{\text{depth of beam}} \div \text{cosine of } o.$$

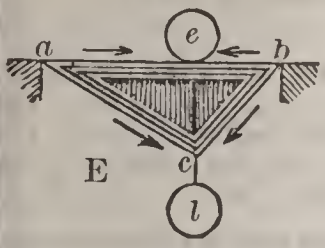
Here *v* shows by the scale how much of the vert or shearing force has been converted into a longitudinal one; and if it be taken from the total shearing force before found, the remainder will show how much of said force still operates on the *web* at *o*. For exceptions, see Rem. The foregoing applies also to oblique flanges of open hor beams.

In the hor triangular flanged beam D with a concentrated load at its free end, draw *a o* vert and equal by scale to the load, and draw *o c* hor. Now here the whole load rests upon the upper end *a* of the oblique flange *a n*, which therefore sustains all of it as an end load, which it deposits as vert pressure at *n*, and thus entirely prevents it from exerting any shearing force whatever upon any part of the beam. The shaded web is therefore of no use here. The line *a c* measures the strain along the oblique flange; *a o* the vert pressure at *n*; and *o c* the hor pull of the load all along the upper flange *a e*. Also *a o* and *o c* are the components of *a c*.



So also in Fig E, with a concentrated load *l* suspended by a string from *c*. The string carries the load up to the two oblique flanges *c a*, *c b*, which convert its shearing tendency into two oblique pulls along themselves. At

the abutments or supports these pulls along $c\,a$ and $c\,b$ become converted into vertical pressures, together equal to the load l; and into hor pressures compressing $a\,b$.



Here also the shaded web is unnecessary; as would likewise be the case if the load were transferred to e, and a single vert post (shown by the dark line) provided to carry it down to c, as the string before carried it up to c. If there is no such post the web acts, and the strain on either oblique flange is found as for A and B. But it is only in a few similar cases that the oblique flange entirely supplants the continuous web.

Humber gives for finding the strain at any point of an oblique or curved web as follows. First find the shear as before as for a horizontal flange. Then

If the **compressed** *flange is inclined down* **to** *the nearest support*, or
If the **stretched** *flange is inclined down* **from** " "
take the diff between the vert component v and the shearing force. But
If the **compressed** *flange is inclined down* **from** *the nearest support*, or
If the **stretched** *flange is inclined down* **to** " " "
take the sum of the vert component v and the shearing force.

Rem. Hence in these last two cases (which do not include any of our above figs) the vertical force on the web is *increased.*

As Humber remarks, in girders or beams with curved or oblique flanges the greatest strain in the web is not always where the greatest shearing strain is produced.

VERTICAL STRAINS IN BEAMS, ETC.

VERTICAL OR SHEARING STRAINS IN BEAMS, ETC.

Art. 1. When a loaded beam, *a v*, Fig 1, rests upon two supports, *l* and *r*, the weight of the beam and load between the supports, and the upward reactions of the two supports, tend not only to *bend* the beam, as in Fig 3, p 550, but also to *cut* or *shear* it across vertically, as in Fig 1.

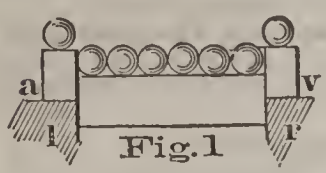


Fig. 1

In practice, **beams rarely fail by shearing,** and then only when heavily loaded close to their points of support, as in Fig 1.

But the vert *forces* which *tend* to produce shearing exist in all beams and trusses. In the latter, they cause all the strain on the verts and a part of that in the obliques (whether web members or flanges); and in beams with thin webs and considerable area of flange, such as box and plate-girders, ps 537, &c, it is usual, and sufficiently (although not strictly) correct, to assume, for convenience, that *all* of the *vert* strains are borne by the *web*, while the *flanges* are regarded as resisting only the *longitudinal* strains. The following instructions are therefore given for finding the amount of vert or shearing strain in the different parts of a beam under different conditions of loading. For the sake of simplicity, we neglect the wt of the beam itself, unless otherwise stated.

Art. 2. Imagine the beam, a v, Fig 2 (supported at both ends, and loaded at its center with 3 tons), **to be divided into a number of slices,** *u s t*, &c, by the vert planes whose edges are shown in the fig. If we take any two adjoining slices, as *m* and *n*, to the *right* of the load, it is plain that we may regard the *left*-hand slice, *m*, as being *pressed downward* by that portion (1½ tons) of the load which goes to the right-hand support, *r*, while its neighboring slice, *n*, on the right, is *upheld* by the equal upward reaction of the right-hand support, *r*. There is, therefore, a vert *strain*, equal to 1½ tons, or to *one* of these forces, tending to shear the beam across on the vert plane separating the two slices; and this tendency must be resisted by the cohesive force of the beam in that vert plane.* Since the downward pressure exerted upon the right-hand support undergoes (in this case) no increase or diminution between the load and the support (there being no intermediate load or support), and since the upward reaction of *r* is also exerted, unchanged, at every point between *r* and *c*, it follows that the shearing strain is equal at all those points. And since the load is at the *center* of the beam, the upward reaction of the *left* abutment, *l*, is equal to that of *r*, and there must, therefore, be an equal shearing strain of 1½ tons at each point in the *left*-hand half of the beam. In other words, **a load, concentrated at the middle of a beam supported at both ends,** exerts a uniform shearing strain, equal to half of said load, throughout the beam.

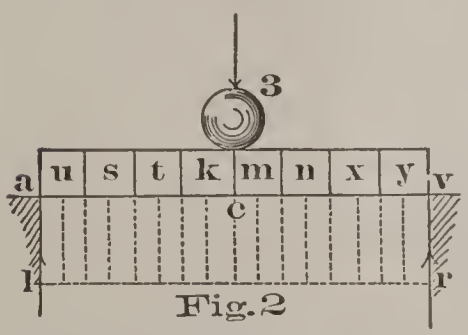


Fig. 2

Art. 3. But while, to the *right* of the load, each slice tended to slide downward past its neighbor on the *right;* the *reverse* is the case to the *left* of the load; each slice there tending to slide downward past its neighbor on the *left.* In other words, to the *right* of the load the vert *downward* strain on each section comes from the *left*, and vice versa.

Art. 4. At the vert section **immediately under or over a concentrated center load** there is, strictly speaking, no *shearing* tendency between the two slices to the right and left of that section, because they evidently have no tendency to slide past each other. But there is, *in* the two slices, a combined *crushing* strain equal to the *entire* load; because *each* of them sustains *half* the load, and is pressed upward by the equal reaction of the abut. If we suppose *a v* to be a truss,

* So long as the joint between *m* and *n* remains intact, *n* is of course also *pressed downward* by the half load, and *m* is also *upheld* by the support. But this does not affect the shearing strain in the joint, because, in order that *n* may receive the downward pres of the half load, said pres must be *transmitted* to it from *m through the joint* in question; and so with the upward reaction transmitted from *n* to *m*. It is the *transmission* of the *original* action and reaction through any given joint that causes the shearing strain in it.

and the vert line between k and m to represent a vert post, then said post will have to bear the combined crushing strain, which, in the beam, comes upon slices k and m (= the entire load of 3 tons), and its pin at c in the lower chord will sustain a *vert* shearing strain of 3 tons in addition to the *hor* shearing strains from the chord.

Art. 5. If, in Fig 2, we make $a\,l$ and $v\,r$ each equal, by scale, to the upward reaction of its abutment, or (in this case) equal to half the load; and draw $l\,r$ hor; then vert lines (of uniform length in this case) drawn between $a\,v$ and $l\,r$ will give the shearing strain at each point in the beam, except of course at the cen, c, as explained in Art 4.

Art. 6. Figs 3 and 4 show the **application of the foregoing to common forms of trusses.** In each fig, a load of 3 tons is supposed to be sustained entirely by only *one truss* of the span. The members sustaining *tension* are shown by *light* lines; and those under *compression*, by *heavy* lines. It is plain

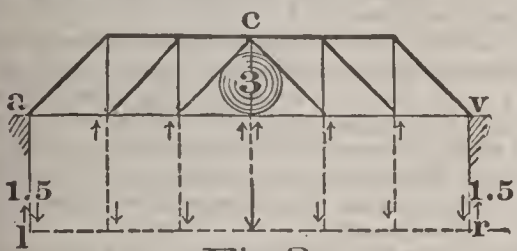


Fig.3

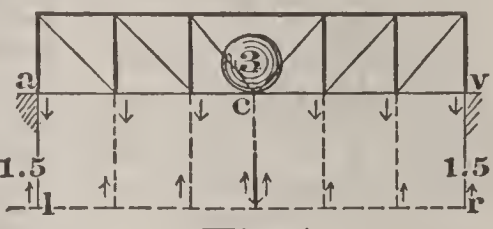


Fig.4

that each vert member, in either fig, is strained 1½ tons.* Each obliqne sustains a *total* strain *greater* thau 1½ tons; but the *vert comp* of each is only 1½ tons. Also, the *downward* strain on each web member to the *right* of the load comes from the *left*, and vice versa. We see, also, that while the two central obliques have no tendency to *slide past each other*, their *combined* vert strain (compression in Fig 3; tension in Fig 4) is eqnal to the entire load; and the pin at c in either fig snstains a *vert* shear equal to the entire load. And the cen vert rod in Fig 3 snstains a pnll equal to the entire load, or 3 tons. Drawing $a\,l$ and $v\,r$ each equal to the npward reaction of its abut; $l\,r$ hor; and the vert dotted lines; the latter give the vert strains (uniform in this case) at the several panel points, except of course at the cen, as just explained.

The arrows pointing *downward* represent the *downward* pressures caused by the *load;* while those pointing *upward* represent the *upward* reactions of the *abuts.* In Fig 3 the *downward* pres at each section is applied at its *foot*, and the *upward* pres at its *head;* and the vert members are therefore in *tension.* In Fig 4 the reverse of all this is the case. The *directions* of the arrows should be carefully noted, as they have an important bearing upon what follows. The *lengths* of the arrows indicate the *amounts* of the forces which they represent; and their *positions* show whether those forces are applied at the upper or lower chord, and whether the force comes to the section from the right or from the left.

Thus, in Fig 3, the arrow pointing downward, *immediately under* the load, and at the *bottom* of the diagram, shows that the force represented by it is applied *immediately at* the joint (coming neither from the right nor from the left) and that said joint is in the *lower* chord.

This force is equal to the entire load, or 3 tons. The two eqnal arrows immediately to the *right* and *left* of the cen line, and at the *top* of the diagram, are made each half as long as the central arrow just referred to, in order to show that the force represented by each is half as great as that represented by the central arrow. These two arrows represent the upward reactions of the two abuts, coming from the *right* and *left*, respectively, and meeting at c in the *upper* chord.

Rem. A single concentrated load produces its greatest shearing strain when placed at one end of the span, immediately over the edge of an abut; at which point the shear is then eqnal to the load; but there is then *no* shearing strain in any *other* part of the beam. As the load moves along the beam, the shear *in front* of it *increases* uniformly, and that *behind* it *decreases* uniformly, until the load reaches the other end of the span. **At whatever point the load may be, the shear at any instant is uniform throughout either one of the two segments** into which the load divides the span. See Figs 5 and 6.

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* Except that the *center* vertical member, in Fig 3, belongs, as it were, to both halves of the truss, and therefore performs the same duty as *two* side verts, by supporting the entire load of 3 tons = *twice* the half load, as explained in Art. 4.

Art. 7. In Figs 5 and 6, **the concentrated load is not at the cen of the beam.** Therefore the upward reactions of the abuts are unequal, as are also the shearing strains in the two portions of the length of the beam.

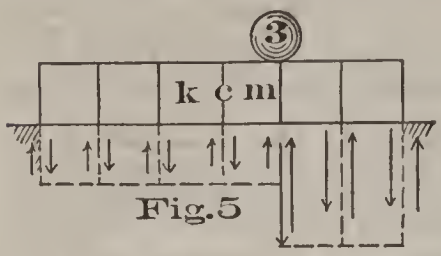


Fig.5

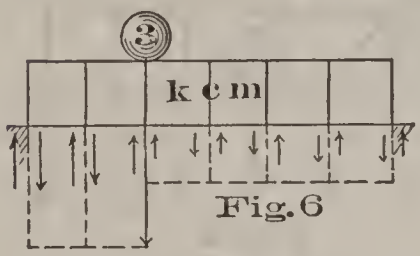


Fig.6

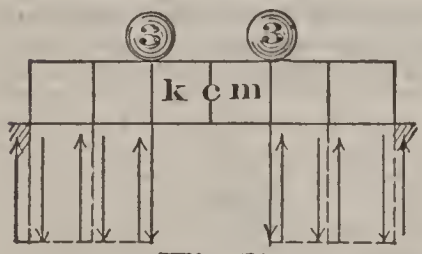


Fig.7

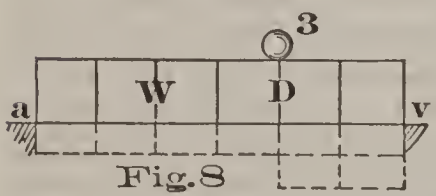


Fig.8

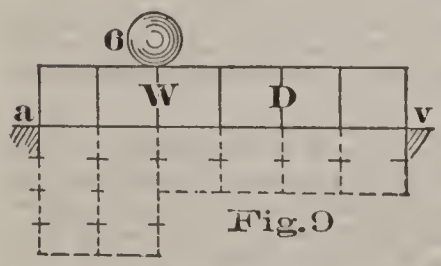


Fig.9

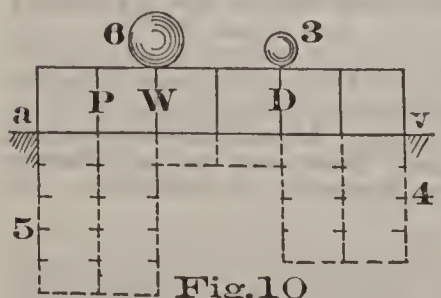


Fig.10

The reaction of either abut $= \dfrac{\text{load} \times \text{Hor dist from cen of grav of load to the } \textit{other} \text{ abut}}{\text{Span.}}$

Art. 8. In Fig 7 we have *two* concentrated loads of 3 tons each, and each placed at a dist from an abut equal to one-third of the span. Here it might be supposed that the shear at any point might be found by simply adding together its shear by Fig 5 and that by Fig 6; but this is not the case, except for those points between an abut and the load nearest to it. At such points the vert forces in Fig 5 and those in Fig 6 act *in the same directions*, and thus *assist* each other when combined as in Fig 7; while at the sections *between the two* loads, they act *in opposite directions*, and consequently *counteract* each other.

Thus, if we compare section *c* in Figs 5 and 6, we will see that in Fig 5 slice *m* tends to slide *downward* past *k*; while in Fig 6 the reverse is the case; so that in Fig 7, which may be regarded as a combination of Figs 5 and 6, these two **equal and opposite shearing tendencies counteract each other,** and there is consequently no shear at *c*.

Art. 9. Similarly, in Fig 10, the shear in W D is equal to the *difference* (1 ton) between those (1 ton and 2 tons, respectively) in W D in Figs 8 and 9; while that in *a* W is equal to the *sum* (5 tons) of those (1 and 4) in *a* W in Figs 8 and 9; and that in D *v* is equal to the sum (4) of those (2 and 2) in D *v* in Figs 8 and 9.

Art. 10. The following **general rules** are illustrated by the foregoing:

Rule 1. The shearing or vert strain at any point of any beam, fixed or supported at one or at both ends, and loaded in any manner, is equal to the diff between the upward vert reaction of either abutment, and any load or part load on the beam between that abut and said point. To find the upward reaction of either abut, see Art 7, above.

Rule 2. Let all that part of the load to the *right* of the given section be called **R**; and that to the left of it, **L**. Then the shearing strain at that section will be equal to the diff between that portion of **R** that goes to the *left*-hand support, and that portion of **L** that goes to the *right*-hand support.

Rem. 1. In applying either of these rules to a **section immediately under or over a concentrated load,** as at W, Fig 9, or concentrated portion of a load, as at W, Fig 10, we must, *theoretically*, consider the section as being the dividing line between the two portions of said concentrated load or part load which go to the two abuts respectively; and must regard said portions as forming parts of the loads on the two portions into which the given section divides the beam.

For instance, in Fig 10, at any point, P, *between a* and W, Rule 1 gives shearing strain = upward reaction of abut at *a* — load between *a* and P = 5 — 0 = 5 tons; or, = load between *v* and P — upward reaction at *v* = 9 — 4 = 5 tons; and Rule 2 gives shear-

ing strain = portion of R going to a — portion of L going to $v = 5 - 0 = 5$ tons. But, *at W*, the wt of 6 tons, resting there, *divides;* two-thirds of it, or 4 tons, going to a, and one-third, or 2 tons, to v. Therefore "L" (or load to the left of W) = $\frac{2}{3}$W = 4 tons; and "R" (or load to the right of W) = the remaining 2 tons of W, + D, 3 tons, = 5 tons. Here, Rule 1 gives shear at W = upward reaction at a — load between a and W = 5 — 4 = 1 ton; or, shear at W = load between v and W — upward reaction at $v = 5 - 4 = 1$ ton; and Rule 2 gives shear at W = one-third of D, or one ton of R going to the left abut at a.

But in practice it is safer to neglect such refinement, and to consider the section W as having a shear equal to the greater of the two shears on each side of it, or, in this case, equal to the shear at any point between a and W, or 5 tons.

The following are applications of Rules 1 and 2:

Rem. 2. The shearing force at any cross section of a **cantilever** (a projecting beam fixed at one end and free at the other), no matter how the load is disposed, is equal to the wt of that part of the beam and its load which is between said section and the free end.

Art. 11. When a **beam,** $a\,v$, Fig 11, **supported at both ends,** is **uniformly loaded throughout,** the shearing strain is greatest at the abuts; at each of which it is equal to half the load. From each support it diminishes uniformly to the center, where it is zero. Therefore, if we make $a\,l$ and $v\,r$ each equal, by scale, to half the load, and from l and r draw straight lines, $l\,c$ and $r\,c$, to the center, c, of the span; then a vert line, as $o\,s$, drawn from any point, o, in the beam, to either line, as $l\,c$, gives the shearing force at said point, o.

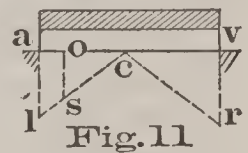


Fig. 11

Art. 12. When a **beam,** $a\,v$, Fig 12, **supported at both ends, is uniformly loaded from one of its supports,** as r, **part way across,** say to n, as when a train, as long as the span, or longer, comes *part way* upon a bridge, the greatest shear is at the abut, r, and is equal to the portion of the load borne by that abut. From that point it decreases uniformly to zero at a zero point, z, which is always within the load itself. **To find the dist, n z, of the zero point, z, from n,** when the load extends from v any given dist, as $v\,n$, toward a,

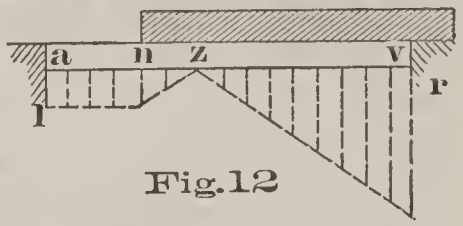


Fig. 12

$$\text{Twice the span} : \text{Length, } v\,n, \text{ of bridge occupied by load} :: \text{Said covered length, } v\,n : n\,z, \quad \textbf{or } n\,z = \frac{v\,n^2}{2\,a\,v}.$$

The *zero* point of *shear* is also the section of *greatest* moment of *rupture.*

From the zero point, the shear again increases uniformly to the end, n, of the load, and at the same rate of increase as from z to v. At n, and from n to the other abut, a, the shearing strain is equal to the portion of the load sustained by said abut, a.

Art. 13. Fig 13 shows the **shearing strains which take place successively at a given point,** 6, in a bridge, 0–8, while a train, as long as the bridge, comes upon it, passes across it, and leaves it, all in the same direction.

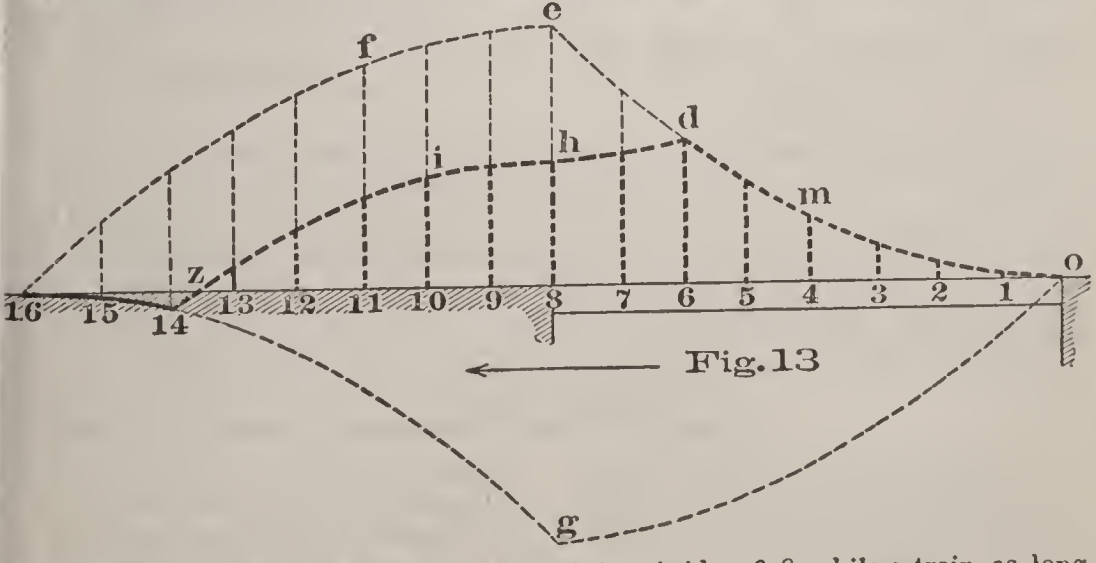


Fig. 13

It also shows the **successive shears at each abut** during the passage of the train. The train is supposed to move in the direction of the arrow, or from right to left, and, for convenience, is supposed to be of uniform wt per ft run throughout. The several shearing strains are found by Art 12. The vertical distances, 8–*e* and 8–*g*, are made each equal, by scale, to half the wt of the train, which is equal to the shear at 0 or at 8 when the head of the train is at 8; and when, consequently, the train just covers the span, as in Fig 11. (It must be borne in mind that we neglect the wt of the bridge itself.)

The vert lines, 6–*d*, 11–*f*, &c, show, by the same scale, the amounts of shearing strain at 8 when the head of the train comes, respectively, to 6, 11, &c. Similar vert lines, drawn from points in the hor line, 16–0, to the *lower* curve, 16–*g*–0, would show the corresponding vert strains at 0; and the *heavy* vert lines, 4–*m*, 6–*d*, 8–*h*, 10–*i*, &c, from 16–0 to the *heavy* curve, 16–14–*d*–0, show the successive shearing strains at the point 6, as the head of the train reaches 4, 6, 8, 10, &c.

It will be noticed that **at each abut** the shear, just before the train touches the bridge at 0, is zero; and that it increases until the train just covers the bridge, when it is equal to half the wt of the train, as in Fig 11. It then decreases, reaching zero again when the rear of the train leaves the bridge at 8.* But at any intermediate point, as 6, the shear increases from zero until the head of the train comes to said point. During this time the shears at the point are the same as those at the abut 8 beyond it (see *n*, Fig 12), and consist solely of shearing strain passing through it *to that abut*. But now the point, 6, begins to pass strains to *both* abuts, and continues to do so until the rear of the train has passed it. These opposing strains partly neutralize each other (see Art 8); and the resultant, or remaining, shear at 6 diminishes until the head of the train reaches such a point, *z*, that 6 becomes the zero point. It then again increases until the head of the train reaches 14. The rear of the train is now at 6. From now on all the shear going to 0 (and no other) passes through 6. Consequently, the shear at 6 is, from now on, the same as that at 0, and, like it, decreases, and becomes zero as the rear of the train leaves the bridge at 8.

The greatest shear that can occur **at any given point** is when the *longer* segment of the span is loaded to that point. It is then greater *at that point* than when the whole span is loaded.

---

* But, as indicated by the two curves, 0–*e*–16 and 0–*g*–16, the shearing strains at the two abutments, 8 and 0, do not increase and decrease *uniformly* or *equally*. Thus: at the *left* abutment 8 (see *upper* curve 0–*e*–16), the increase of shear, as the train comes upon the bridge, is at first slow, and afterward more rapid as the head of the train approaches 8. Similarly, the decrease at 8, as the train leaves the bridge, is at first slow, and afterward more rapid as the rear of the train approaches 8. At the *right* abutment 0, the exact reverse of this is the case. The shearing strains at the two abutments are not *equal* at any given moment except when the train covers the entire bridge, and when there is no train on the bridge.

RIVETED GIRDERS.

Art. 1. Plate girders, Figs 21, 22, and 23: and **box-girders,** Figs 24 and 25; with hor flanges, and supported at both ends. In these, it is usual, and sufficiently correct, to assume for convenience that the *flanges* sustain all of the *hor* strains, and no other; and that the *web* sustains only the *vert* strains.

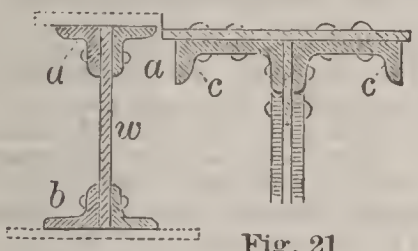


Fig. 21.

Fig. 22.

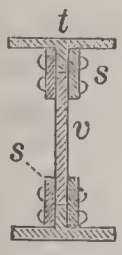


Fig. 23.

Fig. 24.

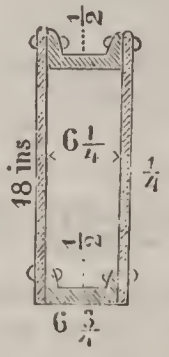


Fig 25.

On this assumption,

The total hor compressive or tensile strain, in lbs, **in either flange,** at any point in the span, is } = Moment of rupture, in ft-lbs, at that point, as found by pp 481, &c / The vert dist in ft between the centers of gravity of cross-section of the two flanges at the same point.

If the moment of rupture is in *inch-tons*, the vert dist must be in *ins*, and the flange strain will be in *tons*, &c, &c.

Flange strain **per square inch** of cross-section of flange } = Total strain in one flange, found as above / Area of cross-section of one flange in sq ins.

Art. 2. The total strains on the two flanges at any given point in the span, are plainly equal to each other; but that on the *upper* flange is of course compressive; and that on the *lower* one, *tensile*.

Inasmuch as the vert dist between the cens of grav of cross-section of the two flanges is approximately the same throughout the span, the strain on either flange at any point may be taken as proportional to the moment of rupture of the girder at that point. Therefore, if we draw a diagram of these moments, as directed for various methods of loading, in pp 482, &c, and let any one of the ordinates equal by scale the flange strain at that point; then the other ords will give, by the same scale, the flange strains at their respective points.

Art. 3. Areas of cross-section of flanges.

Minimum allowable area in sq ins **of *effective* cross-section of lower flange** at any point in the span. } = Tensile strain in lbs on the flange at the given point, as found by Art 1 / Safe tensile strength of the metal in lbs per sq inch

The effective cross-section of the lower flange is = the cross section of the flange plate, *plus* that of both legs of both the angles that fasten it to the web, *minus* the rivet holes. When the rivets are staggered, as in Fig 25 A, it is usual to consider the effective section as equal to a net section taken on a staggered line, X A B C D Y, between the rivet holes, the *oblique* lines, A B and C D, being taken at only *three-fourths* of their actual lengths. But if this should give a greater area than would be obtained by deducting the area lost in all four of the holes, A B C and D, from that of the full cross-section on the *straight* line, X Y, then this last is taken as the effective area. Fig 25 A is a plan of part of the lower flange of a plate girder. F F are the projecting edges of the flange plate. H H are the horizontal legs, and V V the vertical legs, of the angles by which the hor flange plate, F F, is joined to the vert web, W, of the girder.

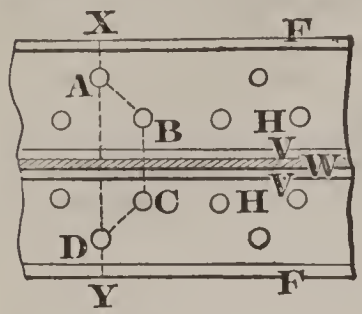


Fig. 25 A.

For girders supporting *quiescent* loads, such as walls, floors, &c, the **safe tensile strength** is usually taken at about 10000 lbs per sq inch for wrought-iron such as is used in girders; but, for *railroad bridges*, from 6000 to 7000 lbs per sq inch.

Having decided upon the *shape* of the **upper flange**, its area of cross-section may be found by means of the rules for iron pillars, p 439 etc.

If the flange is of the usual T shape:

$$\left.\begin{matrix}\textbf{Square of least radius}\\ \textbf{of gyration}\end{matrix}\right\} = \textbf{(Approx)}\ \frac{\text{square of width of flange in ins}}{22.5}.$$

The **effective cross-section** of the *upper* flange is equal to its *entire* section, because the loss of metal occasioned by punching the rivet holes is compensated by the rivets themselves, which also resist compression.

Art. 4. The width of the upper flange is governed by its length of the greatest longitudinal distance between those supports which prevent the girder from yielding *sideways*. Thus, in railroad girders, these supports consist of the transverse bracing; and, in order that the flange may contribute sufficient lateral stiffness to the girder, it is usual, in single-web girders, to make its width not less than about one-twelfth of the greatest longitudinal distance between the points of attachment of that bracing. In buildings, where the load is quiescent, one-twentieth is the usual minimum. In these, however, there is frequently no sideways support between the abutments, so that the proper width of flange becomes one-twentieth of the *span*.

Since the *lower* flange is in *tension*, its *width* is a matter of minor importance (provided sufficient *area* is given), and is generally fixed by considerations of practical convenience.

Art. 5. As the moments of rupture increase as we proceed from the ends of the span toward its center, the total flange strains, and the required area of cross-section of flange to withstand them, increase in the same proportion. The *width* of each flange is usually made uniform throughout the span, and the required increase of area is given by increasing their *thickness*. This is done by increasing (toward the center) the *number of plates* of which each flange is composed. The plates and angles composing a flange are riveted together throughout their length. When a flange-plate, or flange-angle, is too long to be made in one piece, the two lengths which compose it must be joined, where they abut together, by special splices or "covers" (see butt-joints in "Riveting," p 469). In thus splicing the *angles*, *h h*, Fig 25 B, rolled "**angle-covers,**" *c c*, about two feet long, are used.

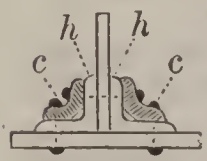


Fig. 25. B.

Art. 6. Rivets. The *vert* rivets in the flanges, which join together the flange-plates and the hor legs of the flange-angles, are generally so spaced that their "**pitch,**" or dist apart from cen to cen in a line parallel with the length of the girder, is from 2½ to 6 ins. Where the flange is made up of two or more plates, as in Art 5, the added plates are made so much longer than the length required by the moments of rupture, as to give room, *outside* of said required length, for a sufficient number of rivets to transmit to the plate its share of the longitudinal flange strains.

The *hor* rivets joining the vert legs of the flange-angles to the web-plate; and those used in splicing together the several lengths of the web-plate; may be spaced by the following rules:

$$\left.\begin{array}{l}\textbf{Greatest allowable strain}\\ \textbf{on each rivet,} \text{ in lbs}\end{array}\right\} = 13000 \times \begin{array}{c}\text{Crippling area of}\\ \text{one rivet, in sq ins.}\end{array}$$

The crippling area of a rivet, in sq ins, is = its diam, in ins × the thickness of the web-plate, in ins.

In buildings, &c, where the load is stationary, 16000 may be used instead of 13000.

$$\left.\begin{array}{l}\textbf{Number of rivets} \text{ in the}\\ \text{depth of the girder between}\\ \text{flange-rivet lines; or in a}\\ \text{length of flange equal to said}\\ \text{depth}\end{array}\right\} = \frac{\begin{array}{c}\text{Total vert or shearing strain}\\ \text{at the joint, in lbs,}\end{array}}{\begin{array}{c}\text{Greatest allowable strain}\\ \text{on each rivet, in lbs}\end{array}}$$

The **vert or shearing strain** may be found by pp 532, &c.
If this, in any case, makes the pitch of the rivets less than about 2½ ins, the thickness of the web-plate should be increased.

Art. 7. The web has to resist the *vert* forces acting upon the girder, and to transmit them to the abuts. In doing so, it is regarded as acting like the web members of the Pratt truss, Fig 31, p 595. In that truss the *compressive* strains are resisted by the vert *posts;* and the *tensile* strains by the main obliques, *c c c.* In the plate girder, vert stiffeners of angle or T iron, riveted to the web, take the place of the vert posts, and resist the crushing strains; while the web itself, in the panels between the stiffeners, takes the place of the obliques, and resists the diag tensile strains. **For the vert strain** at any point in the span, see pp 532, &c. **The diag tensile strain** on the web will be =

$$\frac{\text{Said vert strain} \times \text{length of diagonal drawn across a panel}}{\text{depth of girder.}}$$

This last strain, however, need not be specially considered; because, if the web is made strong enough to resist the crippling tendency of the rivets, it will also be strong enough to resist the diag tensile strain.

Art. 8. The longitudinal hor **dist between two stiffeners** is usually made about equal to the depth of the girder; except that it is seldom less than about 3 ft, or more than 5 ft, whatever the depth of the girder may be. The stiffeners are generally placed somewhat nearer together toward the ends of the span than at its center.

In such railroad bridges as Nos 4 and 6, in our list, p 545, in which the road is carried by transverse floor girders; a heavy stiffener is placed at the end of each floor girder. The stiffeners are thus about 8 ft apart; and, as this exceeds considerably the usual limit, a lighter stiffener is placed between each two of the principal ones. See foot-note ¶, p 545.

Art. 9. Dimensions of stiffeners. Find by pp 532, &c, the vert strain on the girder at the point where the stiffener is to be placed. Then, having decided upon the *shape* of the stiffener, find its area of cross-section by means of the rules for iron pillars, pp 439 etc.

If the stiffener, as usual, is of angle iron, or T iron, riveted to the opposite side of the web plate;

$$\left.\begin{matrix}\textbf{Square of least}\\ \textbf{radius of gyration}\end{matrix}\right\} = (\textbf{Approx})\ \frac{\text{Square of width of stiffener in ins measured transversely of the girder}}{22.5}$$

In order to have as great a radius of gyration as possible, the *narrower* leg of an angle stiffener (if the legs are unequal) is riveted to the web, leaving the wider one projecting.

The area of hor cross section of the small portion of the web between the two angles or T's of the stiffener, is included in that of the stiffener; but that of any packing pieces (see foot-note 2, p 545) is not, because the latter have no firm bearing upon the flanges of the girder, and therefore give comparatively little support to the stiffener.

Art. 10. As already stated, if the web is proportioned as in Art 6, so as to be of sufficient thickness to be safe against crippling by the rivets, it will also be strong enough to resist safely the tensile strain.

Art. 11. The web may also be regarded as resisting the **shearing** and **buckling** tendencies of the vert strains in the girder. The amount of the vert strain, at any point in the span, may be found by pp 532, &c.

The average ult *shearing* strength of rolled bridge plate is about 45000 lbs per sq inch; and its safe strength say 9000 lbs; but, owing to the considerable depth of the web as compared with its thickness, it is more liable to fail through *buckling*. In resisting the buckling tendency, it acts like a flat column; and its ult load may be found approx by the following formula, which is similar, in principle, to that used for columns:

$$\left.\begin{matrix}\textbf{Ult buckling load}\\ \text{in lbs per sq inch of vert}\\ \text{cross-section of web}\end{matrix}\right\} = \frac{30000}{1 + \frac{\text{depth}^2\text{, in ins}}{1000 \times \text{thickness}^2\text{, in ins}}}$$

in which 30000 is taken as the ult crushing load in lbs per sq inch, of wrought iron in short blocks.

$$\textbf{Safe buckling load} = \frac{\text{Ult buckling load}}{\text{Factor of safety, say from 3 to 6 according to circumstances}}$$

Art. 12. In the box girder, Figs 24 and 25, if we could be certain that the strains were equally divided between the **two webs,** that on either one would of course be *half* that as found for the whole girder; but in practice one web is likely, through unavoidable inaccuracies in riveting, &c, to receive more than its share of the strain; and considerable margin should be allowed, according to circumstances, to cover this uncertainty.

For this reason, **plate girders** are **more economical** than box girders. They have, also, the advantage of being more readily accessible for inspection, repairs, painting, &c. On the other hand, **box girders have greater lateral stability.**

Art. 13. Formulæ for the ultimate **crippling strength** of well made plate and box girders, so proportioned as to be secure against buckling and against yielding sideways, and loaded with a quiescent weight; taking the breaking tensile strength of wrought-iron at 44800 lbs, and its elastic limit at half this, or 22400 lbs = 10 tons, per sq inch.

$$\left.\begin{matrix}\text{Quiescent cen crippling load, including } \tfrac{1}{2}\\ \text{the wt of clear span of girder}\end{matrix}\right\} = \frac{\text{Area of cross section of lower flange in sq ins} \times \text{Depth in ins between the cens of grav of cross section of the flanges, at cen of span}}{\text{Span in feet.}} \times 7500$$

The load so found is that which would cripple the girder by bending it beyond recovery. Calling it **W**; then

Quiescent **extraneous** crippling load in lbs = W — half the wt of the clear span of the girder in lbs.

$$\left.\begin{matrix}\text{Quiescent } \textbf{distributed} \text{ crippling load in lbs, including}\\ \text{the wt of the clear span of the beam}\end{matrix}\right\} = \text{twice W.}$$

Quiescent **extraneous distributed** crippling load, in lbs =

(twice W) — weight of entire clear span of girder in lbs.

In railroad bridges, a **factor of safety** of 3 is used with the above loads.

Art. 14. Formulæ for **deflections** within the limit of elasticity:

$$\left.\begin{array}{l}\textbf{Deflection} \text{ in ins at center of span under a } \textbf{uniformly distributed quiescent load}\end{array}\right\} = \frac{\text{Load in tons of 2240 lbs} \times \text{Span}^3\text{, ft}}{220 \times \text{Depth}^2\text{, ins} \times \text{Area of cross section of one flange in sq ins}}$$

$$\left.\begin{array}{l}\textbf{Deflection} \text{ in ins at center of span under a } \textbf{quiescent concentrated center load}\end{array}\right\} = \frac{\text{Load in tons of 2240 lbs} \times \text{Span}^3\text{, ft}}{137 \times \text{Depth}^2\text{, ins} \times \text{Area of cross section of one flange in sq ins}}$$

It is very important that the rivet holes in the web should agree exactly in size and position with the corresponding holes in the vertical legs of the flange-angles; as otherwise considerable deflection may take place while the rivets are coming to their bearings, and undue strains be brought upon the web.

Art. 15. The following are the **results of an experiment** with a box beam like Fig 25 made by Trenton (N J) Iron Co. Channels 6 ins × 2½ ins × ½ inch. Sides ¼ inch thick, 18 ins deep. Weight 207 lbs per yard. Length 20 ft 3 ins; clear span 19 ft 5 ins. The ends steadied sideways, but otherwise unconfined.

Cen load. lbs.	Def. Ins.	Cen load. lbs.	Def. Ins.
0	1/8	76862	1½
12990	3/16	Interval of 26 days.	
19920		81342	1 1/16
24230	¼	85524 at once a	1 1/16
28744	7/16	crackling noise	
32284	½	commenced. In	
37844	9/16	10 min,	2 5/16
42387	⅝	In 1 hour,	2 7/16
46923	11/16	90302	3 7/16
51460	¾	With a *side* defln of	1⅛
55985	13/16	This increased	
60553	15/16	until the side plates gave way at their bottom edges, in an hour.	
65089	1 1/16		
69954	1¼		

Art. 16.

$$\left.\begin{array}{l}\textbf{Weight} \text{ of entire girder of uniform cross section, in lbs}\end{array}\right\} = \left(\begin{array}{l}\text{Area of vert cross section of plates and angles alone, in sq ins}\end{array} \times \begin{array}{l}\text{Length of girder in yards}\end{array} \times 10\right) + \begin{array}{l}\text{Allowance for } \textit{heads alone} \text{ of rivets}\end{array} + \begin{array}{l}\text{Allowance for vert stiffeners, \&c.}\end{array}$$

The **weight of the two heads of a rivet,** after driving, may be roughly taken as averaging about two-thirds of that of the entire rivet. **More exactly:**

If the **diam of rivet** is	⅝	11/16	¾	⅞ inch,
the **weight of its 2 heads** is	.167	.2	.25	.4 lbs.

Riveted girders, erected, **cost,** per pound, about twice as much as the plates. See p 402.

Art. 17. The plates are usually from ¼ to ⅝ inch thick, and from 1 ft wide up to 20 ft long, to 6 ft wide up to 15 ft long. **The angles** (see pp 525, &c) are from 2½ × 2½ × ⅜ to 6 × 6 × 1 inch. **The rivets** (see pp 469, &c) are from ⅝ to 1⅛ ins diam, usually ¾ inch; and are spaced from 2½ to 6 ins apart from center to center. This dist from cen to cen of rivets is called their **pitch.**

Art. 18. Figs 26, 27, and 28 illustrate methods of attaching the vert stiffeners to the webs of the girders. Formerly, they were sometimes bent, both at top and at bottom, as at *i i*, Fig 28, in order to pass the hor angle irons of the flanges. Now, however, their upper and lower ends generally **abut squarely against the hor flanges** of those angles, as in Figs 28 A, 28 B, 28 C, and 28 D; and should be

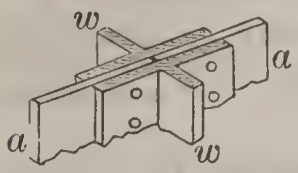


Fig. 26.

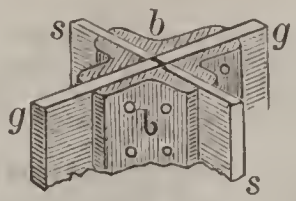


Fig. 27.

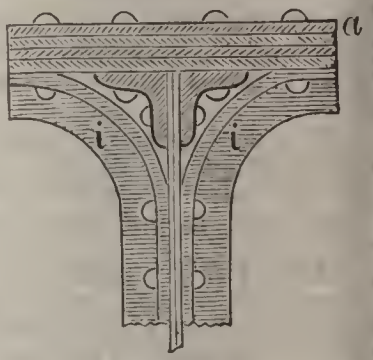


Fig. 28.

trimmed to fit them closely. Different methods are employed **to enable the stiffeners to pass the *vert* flanges of the angles.** Sometimes, as in Figs 28 A and 28 B, "packing pieces" (flat bars of rolled-iron) are placed between each stiffener and the web; sometimes the upper and lower ends of the flange of the stiffener are cut away, as at *a a*, Fig 28 C; and sometimes the stiffeners are crimped or bent slightly, as in Fig 28 D.

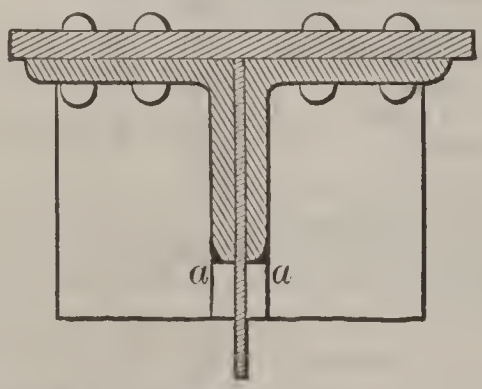


Fig. 28 C

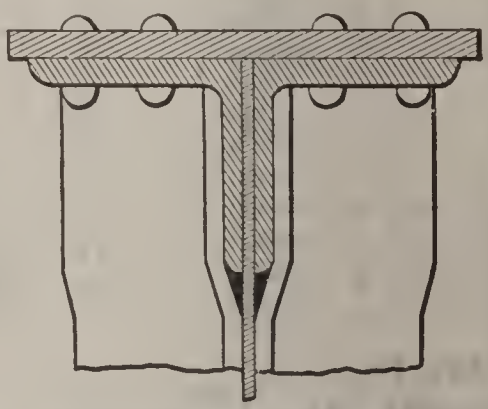


Fig. 28 D

Art. 19. In cases where a girder is placed directly under each rail, the **dist apart of the two girders** of a single track bridge is about 5 ft. Frequently, however, and especially in long spans in order to give the bridge sufficient lateral stability, the main girders are placed further apart; and the cross-ties, T, Fig 28 B, on which the rails rest, are then carried by longitudinal stringers, S, of iron or timber, which rest upon transverse floor girders, F; and these, finally, rest upon the main girders, G. In such cases the latter are generally about 12 ft apart for single track, or where 3 girders are used for double track. Where only two girders are used for double track they are placed about 16 ft apart.

The transverse floor girders are generally about 8 ft apart. They have a vert stiffener under each rail, and frequently others.

Art. 20. Where transverse floor girders are used, they, with light hor diag tension rods, give sufficient **lateral bracing;** but where these are wanting, as in Fig 28 A, special transverse strut-ties, T, &c, are used. They are generally made of angle or T iron.

Art. 21. Figs 28 A and B, kindly furnished by Mr Jos. M, Wilson, C E, Engr of Bridges and Buildings, Penna R R, show standard forms of plate girder bridges in use on that road. **For loads,** see p 546.

Fig 28 A is a cross section of bridge No 3 of the list, p 545, taken near the end of the span, where each flange has but one plate. S S S S are the angle stiffeners. A horizontal, zig-zag, transverse bracing of $3 \times 4 \times \frac{3}{8}$ inch angle-iron (not shown in the fig) connects the two top flanges with each other, and extends the entire length of the girder. From each apex of this bracing, a transverse brace, T, consisting of 2 bars of $3 \times 4 \times \frac{7}{16}$ inch angle-iron, extends to the opposite flange. Similar transverse braces connect the lower flanges. These transverse braces are strengthened by plates P, O, P, O, of $\frac{3}{8}$-inch iron, riveted to them and to the inner vert stiffeners. The plates, P P, also strengthen the diag **"sway-braces,"** D, each of which is a bar of $4 \times 4 \times \frac{1}{2}$-inch T iron.

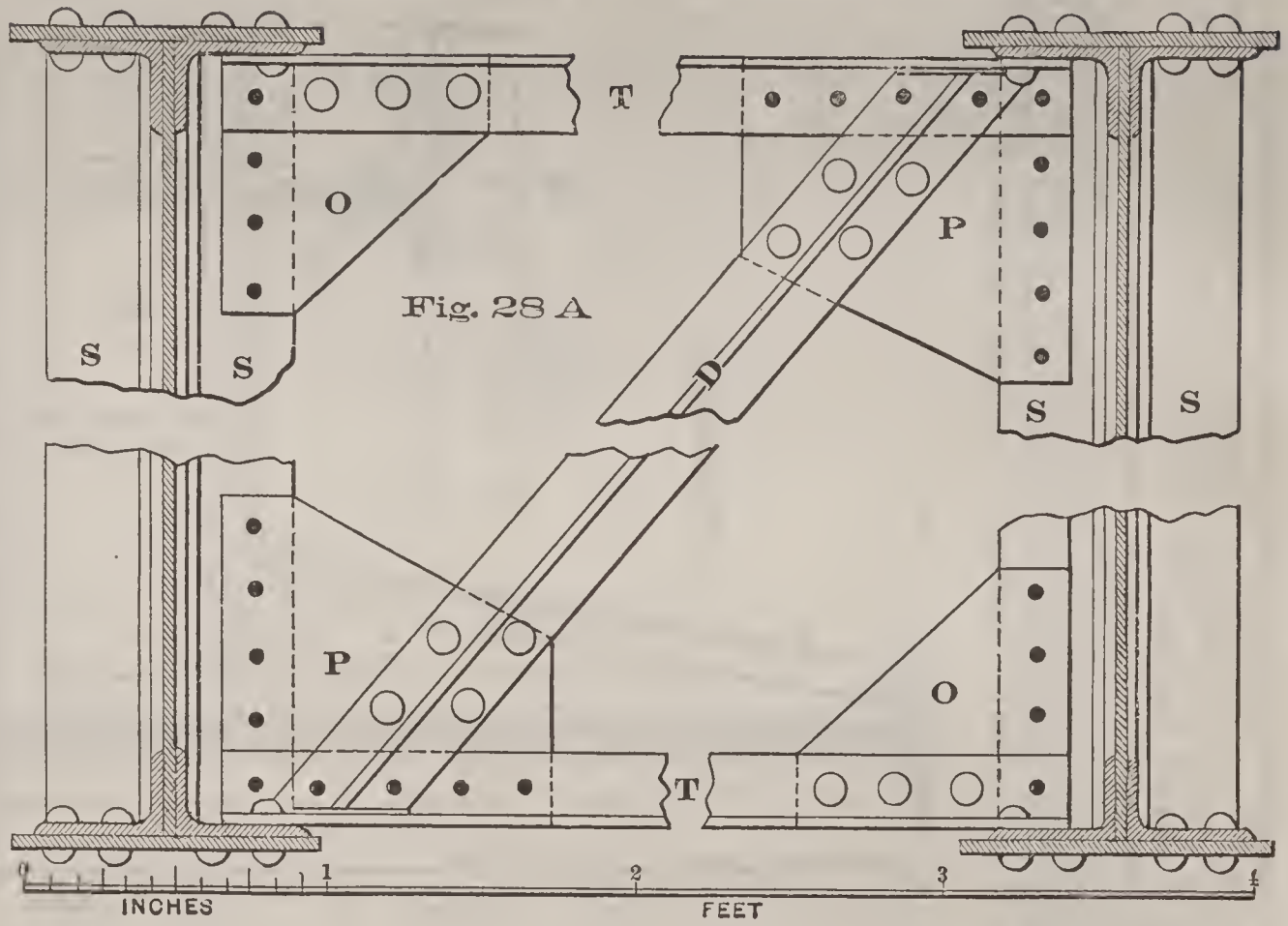
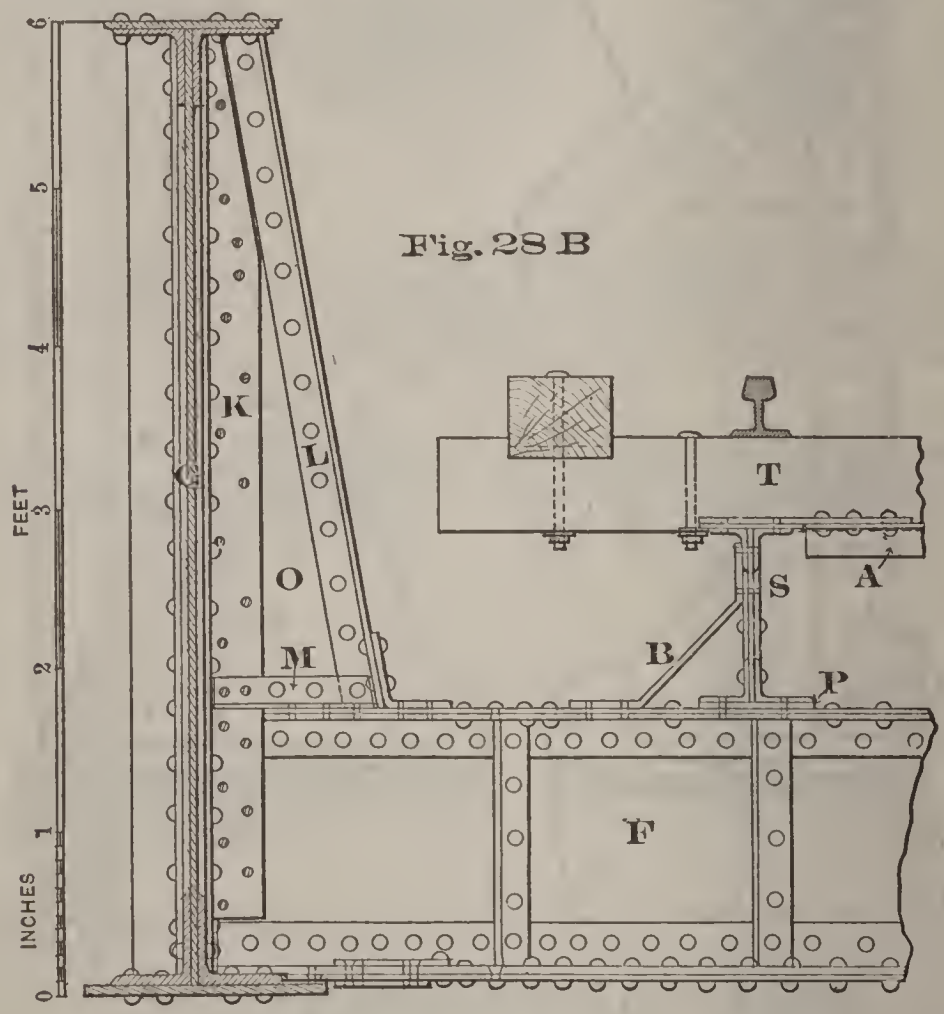


Fig. 28 A

Fig 28 B shows one of the side girders, G, and parts of the transverse girder, F, &c, of bridge No 6. As in No 4, the lower flange of the track stringer, S, consists only of the hor legs of the two angle-bars, and has no flange-plate proper. It rests upon plates, P, $\frac{5}{8} \times 8\frac{1}{2} \times 9$ ins, through which pass the rivets which fasten it to the flange of the transverse girder.

The track stringers are stayed by bent plates, B, which are riveted to them and to the upper flange of each cross girder. A is a transverse brace of $2\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$ inch angle-iron. One of these is riveted, by means of a connecting-plate, to the upper flange of each track-stringer, at its joint with the next one. The plate, O, of $\frac{3}{8}$ inch iron, is placed between, and riveted to, the two inner stiffener-angles, K (only one shown), and two angles, L, $3 \times 3 \times \frac{3}{8}$, and two others, M, $2\frac{1}{2} \times 3 \times \frac{3}{8}$.

Fig. 28 B



On p 545 are given the principal dimensions, weights, &c, for different spans. The numbers (1, 2, 3, 4, 5, 6) are our own, and are used merely for convenience of reference. Where the girders are 5 ft apart (Nos 1, 2, 3, and 5) the bridge is for single track, the cross-ties rest directly upon, and are notched to, the upper chords, and one rail is placed directly over each girder. No 6 is for double track, and the roadway is arranged as shown in Fig 28 B. No 4 is also for double track, with roadway as in Fig 28 B, except that the track stringers, S, rest upon the *lower* chords of the transverse girders.

The lower flange of each main girder is riveted, at each end, to a rectangular rolled-iron **"bolster plate,"** which rests upon a slightly larger **"wall plate."** The two plates are held to the abut by two bolts which pass through both of them. At one end of the span, the bolt holes in the bolster plate are slightly elongated in the direction of the length of the bridge, so that the bolster plate may slide on the wall plate when the girder **expands and contracts** under the influence of **heat and cold.**

Standard Plate Girders, Pennsylvania Railroad.

For loads, see page 546.

	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.
Girders.						
Length	33 ft	49 ft	59 ft 6 ins	61 ft 6¾ ins	64 ft 8 ins	70 ft ¾ ins
Span *	25 to 30 ft	40 to 45 ft	50 to 55 ft	56 ft	55 to 60 ft	65 ft 6 ins
Dist apart, cen to cen.	5 ft	5 ft	5 ft	12 ft 2 ins	5 ft	12 ft 2 ins
Approx gross wt,† lbs						
One girder, alone	6000	12000	17500		21500	
Two girders, with transverse brac'g for single track	14000	27700	39300		49300	
Middle girder, alone				29500		38000
One side girder, alone				20000		25000
Three girders, with transverse brac'g, for double track				115000		141500
Upper flange,‡						
Width	10 ins	12 ins	12 ins	14 ins	16 ins	14 ins
Thickness,						
At cen of span	1⅜ ins	2 3-16 ins.	2 9-16 ins.	4 ins	2⅜ ins	4⅜ ins
At ends "	15-16 in	1 in	1 in	1 in	1 in	1 in
Lower flange,‡						
Width	10 ins	12 ins	12 ins	20 ins	16 ins	20 ins
Thickness,						
At cen of span	1⅜ ins	1⅛ ins	2⅝ ins	3⅛ ins	2½ ins	3 7-16 ins
At ends "	15-16 in	1 in	1 in	1 in	1 in	1 in
Web-plate.						
Depth	36 ins	48 ins	58 ins	60 ins	64 ins	72 ins
Thickness	⅜ in	⅜ in	⅜ in	⅜ in	⅜ in	⅜ in
Angle stiffeners.						
Size § at cen of span	2½×3×⅜	3×4×⅜	3×4×⅜	¶	3×3½×⅜	¶
" at ends "	3×3½×½	3×4×⅝	3×4×⅝	¶	3½×3½×⅝	¶
Dist apart						
At cen of span	3 ft 6 ins	4 ft 3 ins	4 ft 10 ins	4 ft 2 ins	4 ft 11 ins	4 ft 2 ins
Near ends ‖ of span	2 ft 4 ins	3 ft 6 ins	3 ft 11 ins	4 ft 2 ins	3 ft 11 ins	4 ft 2 ins
Transverse girders.						
Dist apart cen to cen.				8 ft 4 ins.		8 ft 3¾ ins
Flanges				9"×¾"		9"×¾"
Web				19"×⅜"		19"×⅜"
Track stringers.						
Upper flange				8"×11-16"		8"×⅝"
Lower "				8⅜"×½"		8⅜"×7-16"
Web				12½"×⅜"		12½"×⅜"

* By **"span"** here is meant the clear distance between the abuts, taken just below the coping. The span to be used in ascertaining moments of rupture, &c, is measured between the centers of the wall-plates on which the girder rests; and is generally from 1 to 2 ft less than the length of the girder.

† These weights include the weights of the *entire* rivets (shanks and heads), and that of the plates and angles as ordered from the mill, and before any subsequent reduction by trimming, or by punching for rivet holes, &c.

‡ Under **"flange"** we include not only the hor flange-*plates*, but also the hor legs of the two angle-bars by which said flange-plates are fastened to the vert web. The thickness of these angles is included in the **flange thickness.** Together they are generally narrower than the flange-plates. By **"width of flange"** we mean its *greatest* width, or the width of the widest flange-plate.

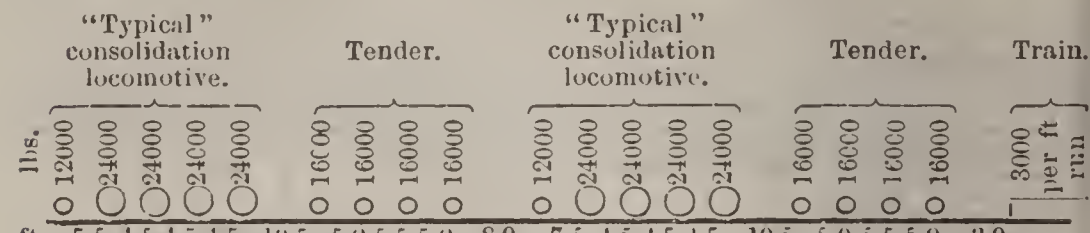
§ Each **vert stiffener** has, between it and the web, a **"packing"** consisting of a flat bar of rolled iron as wide as that leg of the angle stiffener which is fastened to the web of the girder, or wider, and as thick as the angles of the upper and lower flanges. As shown in both figs, the *stiffeners* extend between the *hor* flanges of the upper and lower angles; but the *packing pieces* only between the *edges* of their *vert* flanges. The packing pieces, by keeping the stiffeners away from the web, render it unnecessary to bend them, as at *t t*, Fig 28, or as in Fig 28 D, or to cut away part of their flanges, as in Fig 28 C, in order to enable them to pass the flange-angles.

‖ **At each end of the span,** over the abuts, two or more vert stiffeners are placed on each side of the beam and quite near each other, in order to withstand the severe strains at those points. One wide packing piece is placed under these on each side of the girder. Across each end of the girder a **"cover-plate"** is riveted to the end stiffeners. It is about ⅜ inch thick, as wide as the flanges (tapering when these are of unequal width), and as high as the extreme end depth of the girder.

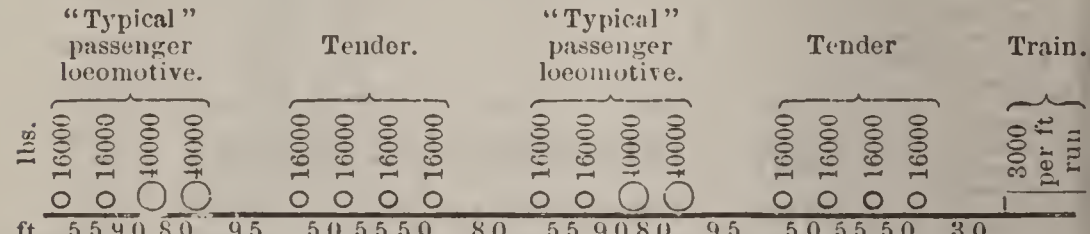
¶ **In Nos 4 and 6,** the stiffeners, at the points where the transverse floor-girders are attached, consist each of two angle-bars placed together with a plate between them, so as to form a T. The angles are about 3½ × 5 × ½ inch, and the plates between them are about ½ × 8 inch. The intermediate stiffeners are single angle-bars with packing pieces, as in the smaller spans. See Art 8.

The bridges in the table are required to carry safely either one of the three following moving loads, A, B, or C. If the bridge is double track, it must carry such a load on *each* track at the same time, the two loads headed in the same direction.

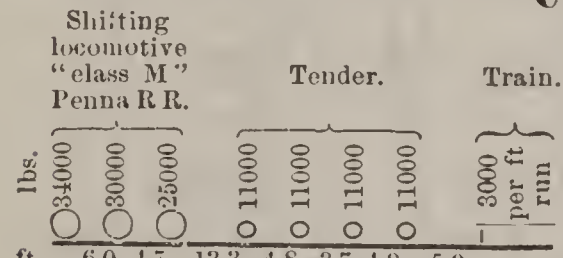
A



B



C



The above "typical" engines were designed (in skeleton) by Mr Wilson, for use in planning bridges for the Penna R R. They were purposely made somewhat heavier than the engines actually in use on the road, in order to provide for the constantly increasing dimensions and weights of the latter, which, however, are fast approaching these "typical" figures. Indeed, the shifting engine **C**, here given ("Class M"), which is in actual use, produces, with certain lengths of span and panel, greater strains in a truss than either of the two typical engines. In calculating the strains on web members, the cross-girder load under the foremost pair of drivers is to be considered as the head of the train; any load upon the preceding cross-girder being neglected.

Equivalent uniform loads. Owing to the great diversity in the design of locomotives and in the distribution of the load upon their several pairs of wheels, the method of specifying the actual or assumed wheel-loads, as above, necessitates much laborious calculation of strains by bridge-builders. To obviate this, Mr. Geo. H. Pegram, C. E., suggests* that the strains in plate girders and in the *chords* (*i a* and *m x*, Fig 13 f, p 564) of trusses, be calculated from an assumed total load of $\left(3000 \text{ lbs} + \frac{60000 \text{ lbs}\dagger}{\text{span in ft}}\right) \times$ span in ft, uniformly distributed over the entire span as in Fig 41, p 483; and that the strains in the *web members* (*a x*, *c r*, etc, Fig 13 f) of trusses be calculated from an assumed load consisting of a train weighing 3000 lbs per ft run and of a single concentrated load of 30000 lbs. In order to find the strains on the several web members in turn, we suppose the train to extend from one end, as *m* Fig 13 f, of the span, first to *o*, then to *p*, etc, and thus to each panel point in succession; the concentrated load being supposed to be placed always at the panel point *n*, *o*, etc, next behind the head *o*, *p*, etc of the train. The first half panel load is to be neglected in making the calculations. Thus, if the train be supposed to extend from *m* to *q*, Fig 13 f, the concentrated load would be assumed to be at *p*, making the original panel load at *n* and at *o* each = 3000 lb × length of one panel in feet; that at *p* = 3000 lbs × panel length *v w* in ft + 30000 lbs; and the half panel load *w q* would be neglected. Mr. Pegram finds that by using 2900 lbs and 25000 lbs respectively, instead of the above 3000 lbs and 30000 lbs, we obtain strains practically equal to the greatest of those caused by any of the above arrangements of wheel loads; and that by using 3500 lbs and 35000 lbs respectively (as would seem to be advisable in view of the rapid increase in the weight of rolling stock), we should add but about 10 per cent to the weights of bridges designed for the above wheel loads.

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* Transactions. American Society of Civil Engineers, June, 1886.

† With 60000 lbs uniformly distributed, the breaking moment at any point is the same as would be caused by 30000 lbs concentrated at that point.

TRUSSES.

Art. 1. When the span of a bridge, roof, &c, becomes so great that single solid beams, supported at their ends, cannot be employed, we resort to compound beams, called **trusses.** composed of several pieces so arranged and united as to furnish the reqd strength. The designing, construction, and erection of trusses of great span, especially when of iron, involve such a multiplicity of important detail, that, like the building of locomotives, cars, &c, they have become a specialty, or a distinct branch of business, to which persons confine themselves to the exclusion more or less of other departments; and thus attain a degree of skill beyond the reach of the general engineer.* The latter, however, should possess a knowledge of the subject sufficient at least to enable him to form a well-grounded opinion of the general merits of a design; and to guard him against the adoption of one involving serious imperfections. In a volume like this we can aim at nothing more than an attempt to illustrate some few general principles. We shall confine ourselves to such trusses as are in common use; showing first the effects of uniform stationary loads, as in the case of roofs; and then those of moving loads, such as an engine and train on a bridge.

Art. 2. Most of the bridge trusses in common use have two long, straight, parallel upper and lower members $l\ l$, $a\ p$; and $l\ t$, $a\ p$, Figs 10, 11, called the **chords;** or in England, the **booms.** Vertical pieces placed between, and connecting the upper and lower chords, are called **posts,** when they sustain compression; and **vertical ties,** or **suspension rods,** &c, when they sustain tension or pull. The oblique pieces seen in these figs are called **braces, strut-braces, main-braces,** &c, when resisting pres or thrust; or **tie-braces, tension-braces, main oblique ties, oblique suspension-rods,** &c, when resisting pulls. Sometimes the same piece is adapted to bear both tension and compression alternately; and may then be called a **tie-strut** or a **strut-tie.** The oblique members alluded to are sometimes called main-braces, whether they are struts or ties; to distinguish them from **counter-braces,** or **counters.** These last are not shown in Figs 10 and 11, but are seen in Figs 28 and 31, crossing the main braces diagonally. These posts, braces, counters, ties, &c, serve not only to keep the two chords asunder, and to prevent them from bending; but to transform the *transverse* strains produced by the wt of the truss and its load, into other strains, acting longitudinally, or *lengthwise,* along the diff members; and to conduct said strains along the truss, to the firm supports of the abuts. A load placed at any one of these members is, of course, partly supported by each abut; one part of it travels up and down alternately between the chords, and along the successive members, until it reaches one abut; and the other part, in like manner, goes to the other abut. These members, therefore, perform the duty of the vert web of the Hodgkinson beam; or of the I rolled beams, or of the tubular girder; and on this account are collectively called the **web members,** in contradistinction from the chords. Each *portion* of any load, while being transferred by the web members, from the spot at which it is placed on the truss, to its final point of support on the abut, produces a strain equal to itself upon every *vert* web member along which it travels between the parallel chords; while upon each *oblique* member encountered on its way, it produces a strain greater than itself, in the same proportion that the oblique member is longer than a vert one.

Whether the web members are strained by compression, or by tension; or, in other words, whether they act as struts, or as ties, the *amount* of strain will be the same. In either case the straining agent is the same identical force, namely the wt, or vert force of gravity of the truss itself, and of its load; and (Art 18, of Force in Rigid Bodies) whether this force exhibits itself as a *push,* or as a *pull,* neither its *amount,* nor its *direction* undergoes any change. So far, therefore, as regards the broad principle involved in the duty performed by the web members, they might be divided simply into **verticals,** and **obliques.** We shall frequently so designate them.

Whatever amount of strain the upper end of an oblique produces in one direction against the upper chord, that same amount will its lower end produce against the parallel lower chord; but in the opposite direction. That is, if the top or head of any oblique, pushes the upper chord toward the right hand; its foot will pull the lower chord to the same extent toward the left hand. This, however, is not *pre-*

* The first writer to whom we are indebted for a knowledge of correct principles on this subject is S. Whipple, C. E., the first edition of whose book (beyond all doubt the pioneer one) bears date, Utica, N. York, 1847. He was followed by Bow, of England, and Haupt, of this country, both in 1851. The Murphy-Whipple bridge (of which Mr. John W. Murphy, C. E., has built several of the best) owes its name to these two gentlemen.

cisely correct, inasmuch as when the oblique is a strut, the pres at its foot is somewhat greater than at its head, because the foot supports also the wt of the strut itself; or if the oblique is a tie, with its head attached to the upper chord, then the strain is a little greater at the head than at the foot; because then the head upholds the wt of the oblique, and the foot sustains none of it. This remark applies, of course, to verts also. Another exception is, when the ends of two obliques meet each other; as those at the center of the trusses, in Figs 10 and 11. If, in such cases, the ends of the obliques abut *against each other*, instead of being separately attached to the chord, they will at that point exert their strains against each other, instead of against the chord.

In any oblique, as *c d*, Fig 1, the vert dist *a c* between its ends; and the hor dist *a d* between the same, are called its vert and hor **spreads,** or **stretches,** or **reaches.**

Art. 3. There is a great diff in principle between two classes of trusses in common use. In some of them, two chords are absolutely essential, as in the Howe truss, p 594; the Pratt, p 595; the Lattice, p 596; the Warren, p 569; and their various modifications, known as the Murphy-Whipple, the Linville, the Latrop, &c, &c, which differ only in certain unessential details. In the Howe and Pratt trusses there is no diff whatever of broad principle. The distinction between them consisting chiefly in the fact that in Howe's the verts are ties, or suspension rods; and the obliques, struts; while in Pratt's, the verts are posts; and the obliques, ties. In all these the strains on the verts and *main* obliques (not on the counters) are least at the center of the truss; and increase gradually toward the end of it: while those on the chords (as in an ordinary wooden beam) are greatest at the center, and least at the ends. Hence, also, such are called **beam trusses.** The strains on the counters are also greatest at the center.

But there is another class, called **suspension trusses,** of which the Fink, Figs 46 and 47; and the Bollman, Figs 44, 45, are the principal representatives. In these but one chord is essential for a perfect truss. From this chord the web members are suspended; and to it alone do they all transfer their strains; and the strain on this hor chord is uniform from end to end. Figs 45 and 47 show perfect bridges, with but one chord each. In Figs 44 and 46, *n n*, *n n*, *appear* to be chords; but strictly speaking they are not; they are merely longitudinal pieces for upholding the cross-beams of the flooring, when the roadway is placed at the bottom of the truss. They have not to resist tension, as in beam trusses.

In all the forementioned trusses the roadway may be placed on either the top or the bottom chord; constituting in the first case a **top road, or a deck bridge:** and in the second, a **bottom road, or a through bridge.**

Art. 4. That part of a truss, such as Figs 10, 28, 31, &c, that is comprised between two adjacent verts, is called a **panel;** thus, in Fig 10, *e i j d*, *d j k c*, &c; and in Fig 31, of Pratt, *t y n w*, is a panel. The Triangular or Warren truss, Fig 11, p 558 has no verts, as essential parts of it; and its subdivisions are called simply triangles; and a panel is a length of truss equal to the width of a triangle. Verts are sometimes added to it when the spaces *a b*, *b c*, *c d*, Fig 11, become too long for safely supporting the roadway without them; thus dividing the truss into half panels. It is not a matter of practical importance as regards strength, whether the number of panels in a truss be odd or even; but it is usually even, with a vert at the center of a truss.

A panel-point, as *a*, *b*, *d*, *c*, *o*, or *n*, Fig 1, is one at which web-members meet a chord, or a rafter in a bridge or roof, as Fig 14, *b*, *c*, *k*, &c,

The length of a panel is its **horizontal** measurement. **The best inclination of obliques,** as regards economy of material in the web, is when their least angle (*t p o*, or *s o n*, &c, Fig 10) with the chord is 45°. This applies also to the admirable Warren truss; in which the triangles are, however, usually made equilateral. When the span is great, and the height of truss correspondingly so, if the panels be made square, or nearly so, with a view to secure this inclination of about 45° for the obliques, the verts (as *t o*, *s n*, &c, Fig 10) will become so far apart, that the stretches or dists *p o*, *o n*, &c, become too long to be safe for upholding their loads of engines, cars, &c, without additional precautions. When, therefore, the expense or inconvenience resulting from this would be too great, the verts may, as in Fig 1, be placed so near together as to make half panel

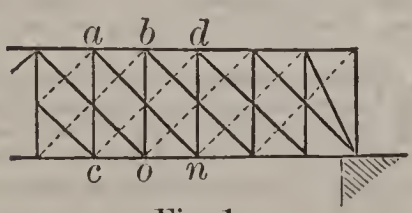

Fig. 1.

much higher than they are long; and the obliques (both the main ones, and the dotted counters) then run across one vert, as in the Fig; or across two, if necessary. In the Warren girder, the expedient is to introduce verts; or else a second set of triangles, as in Fig 1, omitting the verts. From 8 to 12 or 15 ft apart are ordinary dists for verts in bridges of moderately large spans. Frequently panels are made considerably higher than long; disregarding the economical angle of 45°. In large bridges, the main obliques, instead of being each in one piece, are usually made of two or more parallel pieces, disposed in such a manner as to let the counters pass between them diagonally, without mutual interference. Each lower chord (and frequently the upper one also) in large spans, is usually made up of several parallel beams of wood, or bars of iron, side by side.

In European iron bridges, the chords are generally attached to the web members by riveting; but in America this is done by means of cylindrical iron or steel pins as shown for a lower chord in Fig 61, p 612.

The *tension* members in the web, like the lower chord, generally consist of round, flat or square iron bars, with eyes (*s x* Fig 61) at their ends for the passage of the pins.

The upper chord, and the struts or *compression* members in the web, were at first generally made of cast iron; but they are now almost universally of rolled iron or steel. To give them sufficient lateral stability as pillars without an undue expenditure of material, they are made hollow. Many different shapes are used. That shown in Figs 13 and 14 of "Trestles", p 757, is a common one; frequently with channel bars (p 521) instead of the side plates and angles of Fig 14. The Phœnix segment column (p 449) is also very largely used for this purpose. The ends of the struts, like those of the ties, are furnished with eyes for the passage of the pins.

When the web of an iron or steel bridge truss consists of inclined and vertical members they are so arranged that the *vertical*, or *shorter*, members shall bear the *compressive* strains, and the *inclined*, or *longer*, members the *tensile* strains, as in Fig 31, p 595;* because (see lines 11 to 18, p 457) a short *pillar* is stronger than a longer one of the same material and cross section. In great spans, where the truss is necessarily very deep, the obliques are often made to cross *two* panels, as in Fig 1, thus intersecting each vertical post at its center. In such cases the oblique is sometimes fastened to the post by a pin at the point of intersection, in order to further strengthen the post and to prevent the long oblique from sagging.

In wooden bridges, the verts are generally ties, and the obliques, posts, as in Fig 10, p 558. The Howe, Fig 28, p 594, has oblique wooden struts, and vert iron ties.

In long spans provision must be made for the **expansion and contraction** of the truss under the effects of **heat and cold.** See p 614.

If in Fig 10 we imagine lines crossing the panels diag, as the main obliques shown in the Fig do, but in the opposite direction, as shown in Figs 28 and 31, they will represent **counter-braces,** or **counters.** These, like the main obliques, are in some cases struts, and in others ties. Although important members, they are less so than the main obliques. They are unnecessary when the load is uniform and stationary, as is usually assumed to be the case in roofs; and are required only when the load is unequal, or a moving one, as in a train crossing a bridge. In this last case they act chiefly while the span is but partially loaded. If the train at any moment covers the entire span, and is of uniform wt, their action ceases for that time. Their office is solely to counteract the deranging tendency of the unequal loading of diff parts of the truss, as shown in Figs 9½. In Fig 9 *b*, an excess of load along *a o* would tend to derange the main braces *h o* and *t a*; and this would be counteracted by counters across *c o* and *t s*. The same thing may be effected by arranging the main braces, *h o*, *t a*, so as to bear tension as well as compression. The bad effects of unequal loads must plainly become greater in proportion as the load is heavier than the truss itself; and when the bridge becomes very heavy, so that the load must extend over several panels before its effects become serious, but little counterbracing is needed; and that at and near the center only; whereas, in a very light bridge, the counters should extend from the center, where they are most strained; to near the ends, where the strain upon them is least. Inasmuch as we shall first speak of *uniformly loaded* trusses, we shall not here say more respecting counters. See Remark, Art 10.

It would at first sight appear that the several parts of a bridge truss must be most strained when covered from end to end with its maximum load; but this is true only of the chords; and of the main obliques and verts. as *l a*, *t p*, Fig 10, at the *ends* of the truss. The other web members are more strained by a part of the load as it passes along the truss; so that if they be correctly proportioned for a full load, they will

* Except that, when the roadway is on the lower chord, the two web members resting upon the abutments are generally inclined struts.

be too weak for a partial one. If all be made as strong as the end ones, they will, it is true, be safe for a passing load; but this would require an expense of material that would be justified only in the case of moderate spans, especially of wood; in which the additional trouble and expense of getting out and fitting together pieces of many diff sizes, may more than counterbalance the saving in material.

Art. 5. Trusses with moving loads require calculation diff from that for uniform loads. We shall first treat of the latter only; and in so doing shall not employ the shortest methods, but such as will render the general principles clear to any one acquainted with the simple elements of "Composition and Resolution of Forces." The strains on trusses may be found with all the accuracy needed for practical purposes, by means of diagrams drawn to a scale. The same division of the scale that answers for a foot of length, may also represent a ton, 1000 lbs, or any other convenient wt, load, or strain, and may thus be used for measuring the lines which represent such.

The chords, verts, and obliques heretofore mentioned, constitute all the essential elements of a complete **truss:** but other pieces are necessary for a complete **bridge;** such as roof and floor beams; transverse bracing for connecting two parallel trusses with one another, so as better to resist lateral or sidewise motion from winds or lurchings of trains; bars for tying the truss to the piers and abutments in some cases, &c. The same may be said of the extension frequently made at the ends of either an upper or a lower chord of a bridge as shown at *v v* in the bottom chord of Fig 31. Here the *trusses* are perfect without the extensions; but the *bridge* requires them, to allow the load to reach and to leave it. They may be needed for the same purpose in an upper chord of a top-road bridge; or for extending a roof over an entire span, &c. The end vert posts *p u*, of the same Fig are not parts of the truss, but supports for upholding it; also, the posts *p* and *d*, Fig 28, are not essential to the *truss*.

Rem. Besides the forms of truss already mentioned, there are many others, in some of which arches are introduced either as principal members, or merely as auxiliaries; as **Town's Lattice,** Fig 33; **the Bow and String,** Fig 35; and **the Burr,** Fig 36, all of much merit. The Lattice and the Burr have both fallen into undeserved disrepute, from the fact that being the first trusses that were extensively introduced upon the railroads in this country, they were built too weak for the heavy engines and trains of the present day, and consequently failed.

Art. 6. Chords. When a beam *a b*, Fig 3, supported at both ends, breaks either under its own wt, or under the action of a load placed on top of it, or suspended from it below, it does so because the lower fibres, near its center *l*, are *pulled asunder;* and its upper ones at *u*, *crushed together* to such an extent as to offer no effective resistance. The fig shows this in a somewhat exaggerated manner. The extreme upper particles at *u*, and the extreme lower ones at *l*, being the most strained, give way first; and the strength of the beam being thereby diminished, the adjacent ones give way in rapid succession. The compressed particles of the beam are all *above* a certain point *n*; while the extended ones are *below* it. If we imagine an infinitely fine needle to be held perp to this page, and in that position to be stuck through the point *n*, passing *entirely through* the beam, or page, then the infinitely fine hole thus made will pass along what is called the **neutral axis** of the beam. It is so named because the fibres situated in that line, and which were cut in two by the needle, are neither compressed nor extended, until the strain becomes so great that on its removal the beam will not entirely recover itself; or, in other words, until the strain exceeds the *elastic limit* of the beam. Within the limits of elasticity, the neutral axis may be assumed to pass through the cen of grav of the cross-section of the beam. Thus, if the cross-section be of any of the forms shown in Fig 4, then so long as the beam is safe, or the load within the elastic limits, the line *n a* will pass along its cen of grav; which is at the same time its neutral axis. But the chords of a truss differ essentially in condition from the fibres of the beam, as will be seen by comparing pp 485 and 486 with Art. 2, p 528, where it is shown that while the resistance of a *closed* beam is in proportion to the *square* of its depth, that of a truss, or *open* beam, is proportional simply to its depth.

Fig. 3.

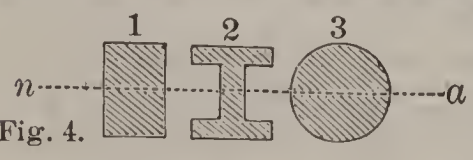

Fig. 4.

The same quantity of material that composes the beam *a b* Fig 3, will present far more resistance to bending or breaking if it be cut in two lengthwise along the hori-

zontal plane *a b*, and converted into top and bottom chords of a truss; because the *leverage* with which the resistance acts is thus greatly increased. Besides, the depths of the chords are so small compared with their distance from the neutral axis, that their fibres may be assumed to act *unitedly* and *equally*. Hence, practically, *all* the fibres in the upper chord must be crushed, or *all* those in the lower chord pulled apart, *at the same instant*, before the truss can give way; whereas, in the solid beam, *a b* Fig 3, the extreme upper or lower fibres yield first; then those next to them, and so on, one after the other.

Art. 7. In the designing of trusses, especially such as may have to bear unequal loads at different parts, as in a bridge, the point chiefly to be aimed at is to dispose its various parts so as to form a series of properly connected **triangles,** because in that shape they present more resistance to derangement of form, than in figs of a greater number of sides. Thus, in the three beams at *a*, Figs 4½, with a bolt at each junction or joint, the triangular form evidently cannot be changed by any but a force sufficient to either bend or break either the beams or the bolts. But in the 4-sided fig *b*, the form may readily be changed to that at *c*, by a force at *n* entirely too small to injure either the beams or the bolts. In *a* the bolts assist to prevent change of form; but in *b* they are merely pivots, around which great changes may easily take place.

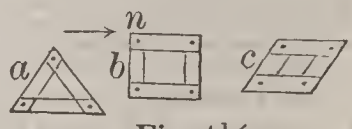

Fig. 4½.

Before the strains can be calculated, and the truss proportioned to those strains, ITS WEIGHT MUST BE KNOWN; for this tends to break it, as well as the extraneous load. But, on the other hand, we cannot learn its wt until we know the size of its diff members. In this dilemma we must *assume* for it an *approximate* wt, based upon our knowledge of somewhat similar trusses already built. This becomes the more necessary as the truss increases in size, so that its own wt becomes greater in proportion to that of the load. The table, p 605, gives *safe* assumed wts for bridge trusses; and p 580 will aid in the case of roofs. In very small spans, especially of bridges, the load is generally so much greater than the wt of the truss, that the latter might almost be neglected entirely.

Rem. For finding the strains on a paneled truss by means of a drawing, it is best to represent each member by a single line, as in Figs 1, 10, 14, 23, &c. Such is called a **skeleton drawing** or diagram of the truss. Each of the parts into which the panel-points divide either chord, or a rafter, is to be regarded as a separate member.

As will be shown farther on, a load consisting of some portion of the wt of the truss and its load, is assumed to be supported at each panel-point. All the forces which meet at any panel-point (namely, the aforesaid partial load, and the forces acting lengthwise of the members which meet there) hold each other in equilibrium.

The forces acting upon a truss (omitting wind) are the downward one of the wt of itself and load; and the upward one of the reaction of the abutments; and these two forces are equal. They produce all the strains along the members.

Fig 5 is the most simple form of a roof truss. It consists of two equal rafters *o a*, *o b*; and a hor tie-beam *a b*. Here, as in roofs generally, the entire weight of the truss, and of its load of roof-covering, snow, wind, &c, may be assumed to be uniformly distributed across the whole span. A roof consists of several trusses, placed usually from 8 to 12 ft apart; but sometimes much less, and at others much more. The trusses rest on longitudinal timbers, *p*, *p*, called *wall-plates*, stretching along the top of the wall; and serving to distribute the wt of the truss and its load over a greater area. On the rafters, and at intervals of a few ft, are fixed pieces of timber called **purlins,** of

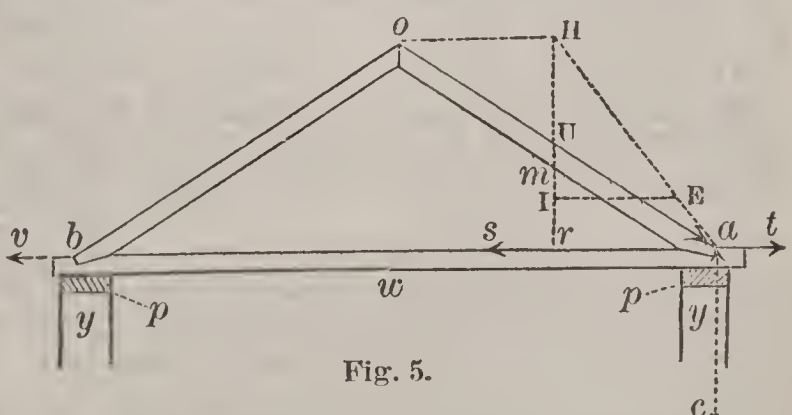

Fig. 5.

small scantling, running across from truss to truss; to which the laths or boards are nailed which support the shingles, tin, or slate, &c, which forms the roof-covering.

A truss plainly supports all the purlins, roof-covering, snow, &c, &c, which occupy the space halfway on each side of it to the next truss. Thus, suppose a span of 30 ft, and each rafter to be 16.8 ft long; and that the trusses are say 12 ft apart from center to center, and assume (as it is generally well to do,) that the wt of the truss, covering, snow, &c, may amount to 40 lbs per sq ft of area of roof. Then each truss has to sustain $33.6 \times 12 \times 40 = 16128$ lbs, including its own weight. Strictly the wt of the tie-beam should be omitted; because in Fig 5 no part of it is upheld by the rafters. It is very trifling however in comparison with the load.

To find the strains upon the different parts of a truss. Fig 5. First calculate in the manner just shown, the entire wt in lbs of a truss and its load. Through the center U of either rafter draw a vert line H *r*. From *o* draw a hor line *o* H. Join H *a*. Now on the vert line H *r*, lay off H I by any convenient scale to represent the entire uniformly distributed wt of *one rafter* and its load; and draw the hor line I E. Then will I E give by the same scale the hor force at the head of the rafter; and H E the amount and *direction* of the oblique force which presses *the foot* of the rafter. The *hor* force at the foot of the rafter will be equal to that at its head;* and equal also to the hor pull along the whole length of the tie-beam.†

Or, consider the force of gravity, G R, Fig 5½, (= H I) as resolved into two components; one, L R, in the direction of the length of the rafter; and the other, G L,

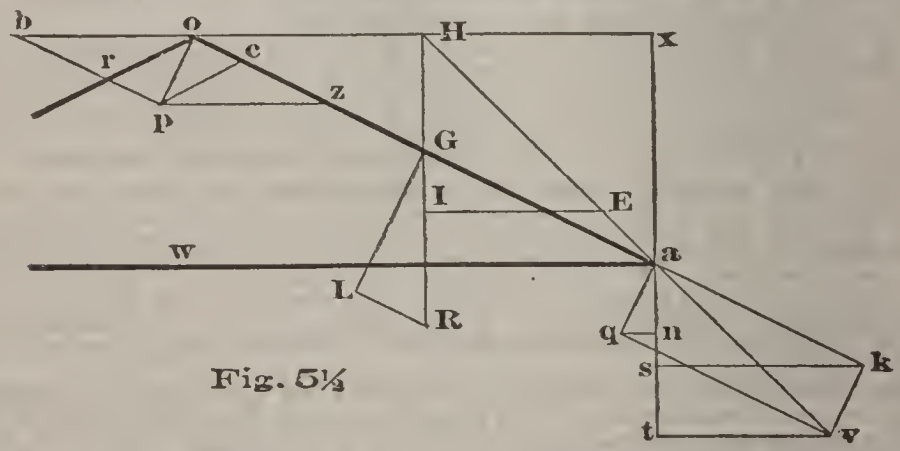

Fig. 5½

at right angles to it. The latter is the force which tends to break the rafter transversely, or like a beam. Since the rafter is uniformly loaded, the cen of grav G is at the center of its length. Hence, by the principle of the lever, Art. 54, p 339, *o p*, = ½ G L, is exerted against the top of the other rafter; and the other half *a q* at the abut *a*. At the top, *o p* causes, or is resolved into, two forces; first, the horizontal pressure *o b* (= I E) against the other rafter; and, second, a longitudinal thrust *o z* along *o a*.‡ *This* thrust (*o z*) is uniform from *o* to *a*. But at each point between *o* and *a* it is added to by a portion of the other longitudinal thrust L R which arises *directly* from the pres of the load. Since the load is uniformly distributed, this last thrust increases uniformly from nothing at the top *o*, to its full amount L R at the foot, *a*. At the top therefore, the *total* longitudinal thrust is *o z*. At G it is *o z* + *half* L R. At *a* it is *a k* = *o z* + L R; and combines with the transverse pres *a q* there, to form the *resultant* pressure *a v* of the foot of the rafter, which is of course the same as H E, the resultant formed by the load H I and the horizontal pres I E of the other rafter. It will be noticed that the *horizontal* components, *a q* and *s k*, of *a q* and *a k*, are in *opposite* directions. Their *difference* (= *t v*) is the pull on the tie beam, and is = I E = *o b*. But their *vert* components, *a n* and *a s*, are *both downward;* and their *sum* (= *a t*) is = H I = the load on *o a* = the upward reaction of the abut *a*.

The sizes in Fig 5 may be found in the following manner. Take, for example, a truss of white pine, of 30 ft span, and 7½ ft rise. The wt of the entire roof, snow, &c, &c, 40 lbs per sq ft of roof area. Trusses 12 ft apart from center to center; so that each truss will have to sustain a total load (including its own wt,) of $33.6 \times 40 \times 12 = 16128$ lbs, which we may call 16000, or each rafter 8000 lbs. We will calculate each part with a safety of 3; which we

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* The foot of each rafter tends to slide or push outward horizontally in the direction of the arrows *t* and *v*; each with a force equal to I E. But the tie-beam prevents them from so doing, and thus converts their *pushes* into *pulls* against each other; and thereby into a pulling strain along the whole tie-beam itself, to an amount equal to *one* of the forces; as two men pulling against each other at the two ends of a rope, each with a force of 10 lbs, only strain the rope 10 lbs. In other words, it requires *two* equal opposing *forces* of 10 lbs each, to produce *one strain* of 10 lbs.

† And there is a hor strain to the same degree generated at every point along the length of each rafter.

‡ It is immaterial whether we thus resolve *o p directly* into *o b* and *o z*, (as though the head of the rafter rested against a *vert wall* at *o*); or whether we first resolve it between the two rafters, into *o c* and *o r*. For in the latter case we must add to *o c a* thrust (= *o r* = *c z*) produced in *o a* by the transverse pres (similar to *o p*) of the head of the *other* rafter; and the sum of these two (*o c* and *o r*) is = *o z*.

think is abundantly sufficient, with the assumption of 40 lbs per sq ft. First prepare a diagram of the truss, on a scale of say ⅛ inch to a ft. This diagram will consist of but three lines. We will use the same scale of ⅛ inch to represent 1000 lbs of either wt or strain. Make H I by scale equal to 1 inch; that is to the 8000 lbs uniformly distributed wt of one rafter and its load. Also draw I E, and measure it. It will be equal in this case (accidentally) to H I, or 8000 lbs; and this is the amount of pull along the tie-beam.* Now we see by table, page 463, that average white pine breaks under a pull of 10000 lbs per sq inch; so that for a safety of 3, we must not subject it to more than $\frac{10000}{3} = 3333$ lbs pull per sq inch. The weakest part of the tie-beam is where it is cut into, near the ends, for footing the rafters; and even what is there left by the cut, is usually still farther reduced by the holes of the bolts or spikes driven into it through the feet of the rafters. Therefore, allowing for these things, we must give to the tie-beam *at that point* a transverse section of solid wood, equal at least to $\frac{8000}{3333} = 2.4$ sq ins. This would no doubt be sufficient to resist the pull; but there are other considerations, such as danger of sagging or breaking down if persons should get on it; or if a moderate load should chance to be laid upon it, &c, which cause the tie-beam (even when unloaded even by the wt of a plastered ceiling below,† as is here supposed to be the case,) to be made about as large as a rafter.

If, instead of a beam, we had used an iron rod to resist the 8000 lbs pull, we should have reqd one with a breaking strength of $8000 \times 3 = 24000$ lbs; and by the table of bolts, page 409, we see that a diam of full $1\frac{3}{16}$ inch would suffice if upset; or of full 1.04 inch if not upset. See Rem p 408.

Now, as to the rafters, each of them is an inclined beam, supported at both ends, and uniformly loaded with 8000 lbs; which is equal to a center load of 4000 lbs. But for a safety of 3 against 4000, we will find its dimensions for a *breaking* center load of 12000. Its length must here be taken as if measd *horizontally* between its end supports, or 15 ft.

We will assume for it some probable approximate depth; say 9 ins. Then by Art 20, p 497, we find that for a breaking center load of 12000 lbs, its breadth will be 4.94, say 5 ins. Therefore to be safe with 4000 lbs center load, each rafter must *at its head* be 5 ins broad, by 9 ins deep.‡

We may take G L (= 7200 lbs) instead of H I (= 8000 lbs) as the load; but then $o\,a$ (= 16.77 ft) must be taken as the span instead of $w\,a$ (= 15 ft). The result is the same in both cases.

As to the strength of the rafter against its longitudinal compressive strain, we find that G L Fig 5½ measures 7200 lbs; therefore $o\,p = 3600$ lbs. Drawing the parallelogram $o\,b\,p\,z$ at the top of the rafter Fig 5½ we find $o\,z = 7200$ lbs. The total longitudinal strain in the rafter is $= o\,z$ at the top; and $= a\,k = o\,z + L\,R = 7200 + 3600 = 10800$ lbs, at the foot. Thus, so far as the *longitudinal* strain is concerned, the rafter might be of less cross-section at top than at foot; but in practice the expense of cutting the timber to that shape would generally more than compensate for the slight difference of material, even assuming that it could be saved. It is uncertain how the transverse and longitudinal strains are distributed through the cross section; for the least sagging of the rafter would throw most of the longitudinal compressive strain on those fibres (on the upper side of the rafter) which are already under compression due to the transverse strain. For safety we will add to the cross-section required by the rafter as a beam, a sufficient area to be safe in itself against the greatest longitudinal compressive strain, or that ($a\,k$) at the foot, which we have just found to be = 10800 lbs; and will make the area uniform throughout. Now we find by table p 436 that average white pine or spruce crushes under a pressure of say 6000 lbs per sq inch. Therefore it will have a safety of 3 under 2000 lbs per sq inch; so that we must provide for each rafter $\frac{10800}{2000} =$ say 5½ sq ins of area of cross-section in addition to the 5×9 ins already found. These 5½ sq ins may be added either in the *breadth* of the rafter, thus making it say 5.6×9; or to its *depth*, making it say 5×10.2.

In the next three trusses we shall not enter into this detail of calculation; as we conceive that this example suffices to elucidate its principle.

Art. 8. Next to Fig 5, in point of simplicity, is Fig 6; which represents a truss for either a bridge or a roof of moderate span. It has two equal rafters, and a hor tie-beam $a\,b$ as before; but with the addition of a king-post, king-rod, or suspension-rod $o\,n$. Either the tie-beam, or the rafters, or both, may be uniformly loaded. It is immaterial whether the load on the former be equal to that on the latter or not. We shall here consider the truss only as that of a *roof*. Let $y\,y$ be points *halfway* between the king-rod and the abutments. Then will the king-rod sustain all the weight of the portion $y\,y$ of the tie-beam and its load. The portions of the tie-beam and its load between y, y, and the walls x, w, are sustained directly by the walls. The *entire* wt of the truss and its load, it is plain, is sustained ultimately by the abuts, or walls $x\,w$; but the wt of $y\,y$ and its load does not reach the walls until after having, as it were, first

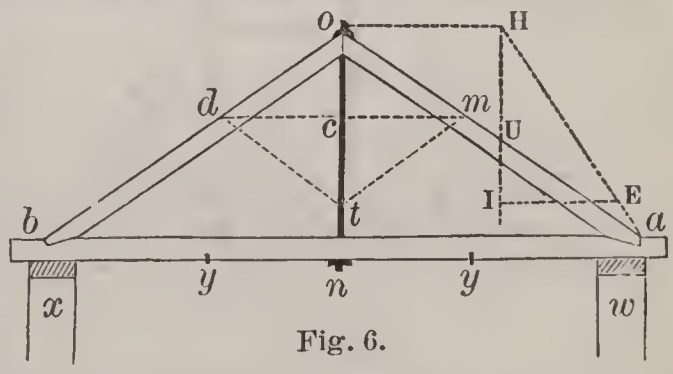

Fig. 6.

* The pull I E along the tie beam will thus be equal to the uniformly distributed load on the rafter only when the rise $o\,w$ is one fourth of the span. If $o\,w$ *exceeds* one fourth of the span, I E will be *less* than H I, and *vice versa*.

† **The weight of an ordinary lathed and plastered ceiling** is about 10 lbs per sq ft; and **that of an ordinary floor** of 1¼ inch boards, together with the usual 3 by 12 inch joists, 15 ins apart from center to center, is from 10 to 12 lbs per sq ft. In preliminary calculations it is well to take the two together at 25 lbs per sq ft.

‡ This is not a bad proportion of breadth to depth. If we had assumed say 15 ins for the depth, we should have got a rafter so thin as to be laterally weak. Frequently, two or three assumptions and calculations may have to be made before we hit upon a satisfactory proportion.

traveled up the king-rod to *o*, and from there down the rafters to *a* and *b*; or, *indirectly*, by a circuitous route. That the king-rod sustains all between *y* and *y*, will be evident when we reflect that a beam *a b*, when firmly suspended at its center *n*, may be regarded as two separate beams *n b*, *n a*. One-half of the beam *n b* and its load would, in that case, manifestly be borne by the wall *x*, and the other half by *n*; and so with *n a*. Therefore, *n* upholds one-half of the beam *a b* and its load; or, in other words, all between *y* and *y*. The king-rod transfers the wt of and on *y y*, to the heads of the rafters at o. *This* wt may, therefore, be considered precisely in the light of one resting upon *o*; and we may proceed to find the strains which it produces upon the rafters and tie-beam, by Art 33. of Force in Rigid Bodies. Namely, on o *n* make o *t*, by scale, equal to said total wt of *y y* and its load, and the wt of the king-rod itself. Complete the parallelogram of forces o *m t d*; and draw its hor diag *m d*. Then will *o m*, *o d* measure the strains produced *by said total weight only*, along their respective rafters; and c *m*, c *d* the pulling forces produced by the same wt only along the tie-beam *a b*; causing strain all along it equal to one of them.

The strain o *m* or o *d* must be added to the other longitudinal strains o *z* and L R (Fig 5½) in the rafter, found as already explained.

For light railroad bridges, a truss like Fig 6, of 30 ft span and 10 ft high, may have a chord of 15″ by 18″; rafters 10″ by 10″; one rod of 2⅜″ diam; or two rods of 1⅝″ diam, and several inches apart transversely of the bridge; which is far better than one.

The pull on the tie-beam will be I E added to *c m* or *c d*. Find the safe area by dividing their sum by 3333, which is the number of lbs per sq inch, giving a safety of 3. Then regarding *half* the length of the tie-beam supported at both ends, and loaded at its *center* with only *one-fourth* of the wt of and on the entire tie-beam, find its safe dimensions by the rules, p 497, or by table, page 499. The resulting area, added to the safe area for the pull just found, will be the entire section of the tie-beam, unless some addition be made to the depth, to allow for what is cut away for the feet of the rafters.* See Rem, p 500, also Rem, p 355.

As to the vertical king-rod, *n o*, it must be strong enough to bear safely a pull equal to its own weight, added to the weight of and upon *y y*. If the rod is of *good* bar iron, it should have one square inch for a safety of 3, of cross-section for each 20000 lbs of said weight; If of wood, it must, for a safety of 3, have at least one sq inch for about each 3333 lbs of said weight. A safety of 3 will be enough if the bar is not liable to vibration.

When the king-rod is of wood, it is improperly termed a king-*post*. Since a post is intended to sustain a load on its top, the term might lead to the inference that the upper ends of the rafters rested upon, or were upheld by the king-post; whereas, as we have seen, *they* actually uphold *it*.

We add the calculated approximate dimensions for a truss, Fig 6, of 30 ft span; and 7½ ft rise. Trusses 12 ft apart cen to cen. Wt of rafters and load on top of them, 40 lbs per sq ft of area of roof. Wt of and on the tie-beam, including floor, ceiling, load, and momentum, 100 lbs per sq ft. Timber white pine. Safety of each piece 3. Rafters 8¼ ins broad, by 11 deep. Tie-beam 8¼ broad, by 11 deep, without any allowance for cutting at feet of rafters. King-rod $1\frac{5}{16}$ inch diam if upset; or full 1⅛ if not upset.†

With no floor or loading on the tie-beam, except its own wt, say 500 lbs, we have, approximately enough, rafters 6½ ins broad by 9 ins deep. Tie-beam, say same as rafter, or 6½ × 9. King rod, ¼ inch diam; but it would be expedient to make it rather more. Trusses 12 ft apart center to center.

Art. 9. In Fig 7 we have a truss consisting of two rafters, *a b*, *a d*; a tie-beam, *b d*; a king-rod, *a c*; and two struts or braces, *e c*, *h c*. Either the rafters or the tie-beam, or both, may be supposed to be uniformly loaded.

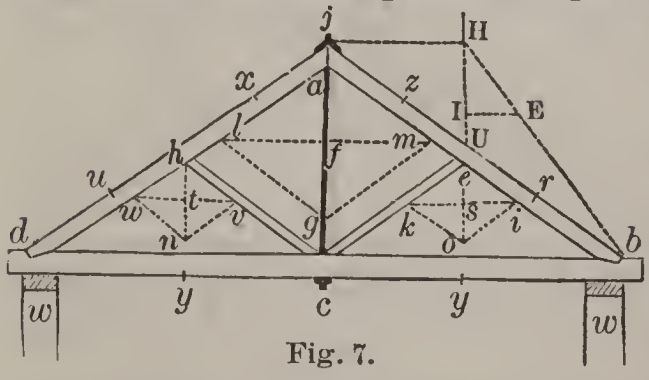

Fig. 7.

Here, as in Fig 6, **the king-rod *a c*, upholds** the weight of the portion *y y* of the tie-beam, and of any load of floor, ceiling, people, &c, that may be placed upon that portion; together with its own weight. But it also sustains, in addition to these, the weight of the two struts *e c*, *h c*; part of the weight of the portions *z r*, and *x u*, of the rafters; and part of the weight of the roof-covering, snow, &c, that may rest on said portions. That it upholds itself, *y y*, and the struts, is almost self-evident; but that it upolds part of *z r*, and *x u*, and their loads, is not at first sight so apparent. Such **struts are introduced into trusses** when the rafters become so long as to be in danger of bending too much, or of breaking under their loads; or else requiring the use of inconveniently large timbers to make them of. They act like posts in affording partial support to the rafters. They carry a part of the strain upon the rafters down to the foot c, of the king-rod; and the king-rod carries it from there up to the tops, *a*, of the rafters. From *a* it passes down through the entire length of the rafters to their feet. Thus, it is seen that the action of the struts consists in relieving the rafters from a *transverse*, or cross-strain which would endanger their safety; and in

* In cases where no appearance of sagging would be admissible, it is not always enough that the rafters and tie-beam be *safe*; for they may be perfectly safe, and yet sag too much for some purposes. When such is the case, refer to table, page 512.

† We have known country road bridges, Fig 6, of 30 ft span, and 7½ ft rise, of two trusses 18 ft apart, in which neither the timbers, nor the probable loads, were larger than in this example.

converting it into a *longitudinal* strain in the direction of their length, in which they can resist it with less danger. As we proceed with the subject of trusses for bridges as well as roofs, it will be seen that this is the grand duty of such struts and obliques generally. In roofs they thus assist the rafters; and in bridges the chords.

Where each rafter is a solid unbroken piece, as in our figures, it is uncertain what portion of the load $z\,r$ or $x\,u$ is actually borne by the strut $e\,c$ or $h\,c$. The introduction of the struts thus renders it impossible to calculate the strains with certainty by the above described method. For this reason, and for safety, we adopt another method, in which we begin with an assumption which is not strictly correct, but which simplifies the problem and enables us to calculate with exactness for each member a strain which is sufficiently near to the probable true one for most practical purposes. This assumption is that the rafter is in two parts, a U and U b; that these parts are connected by a perfectly flexible joint at U, so that all of the load of and on $z\,r$ rests upon the strut $e\,c$; and that the load of and on $x\,j$ and $j\,z$ rests directly upon j. Draw $e\,o$ and $h\,n$ vertically; and make each of them, by any convenient scale, equal to the weight in lbs of either $z\,r$ or $x\,u$, and its load. From o and n draw the dotted lines, $o\,i$, $n\,w$, parallel to the struts; and $o\,k$, $n\,v$, parallel to the rafters; thus completing the parallelograms of forces, $e\,k\,o\,i$, and $h\,w\,n\,v$. Draw the horizontal diagonals $i\,k$, and $v\,w$.*

Then by Composition and Resolution of Forces, either $e\,k$ or $h\,v$, measured by the same scale as before, will give the longitudinal strain in lbs upon each one of the struts. This strain presses the struts lengthwise from head to foot. They are also strained longitudinally and transversely by their own weight, as the rafters in Fig 5 were strained by *their* own weight and that of the roof; but in practice these strains in the struts, due to their own weight, are so trifling compared with that from the roof portions which they sustain, that they may be neglected.

Therefore, each strut may be regarded as if a vert pillar, bearing a load equal to $e\,k$ or $h\,v$. Now, the strain $e\,k$, along the strut $e\,c$, is compounded or composed of the vert strain $e\,s$, (which is equal to *half* of $e\,o$, or one-half of the wt of and on $z\,r$;) and of the hor strain $s\,k$. And the strain $h\,v$ along the strut $h\,c$, is compounded of the vert strain $h\,t$, (which is equal to *half* of $h\,n$, or one-half of the wt of and on $x\,u$);* and of the hor strain $t\,v$. These two hor strains $s\,k$ and $t\,v$ neutralize or counteract each other, by pressing against each other at the feet of the struts; and therefore only the vert ones $e\,s$ and $h\,t$ pull upon the king-rod; and they pull it to an extent equal to half the weights of and on $z\,r$ and $x\,u$.*

The king-rod, therefore, upholds in all, 1st, the weight of the two struts; 2d, the wt of and on $y\,y$: 3d, *half* the wt of and on $z\,r$ and $x\,u$;* and 4th, its own wt. It must, therefore, have sufficient sectional area to safely sustain a pull equal to the sum of these four. This area may be found by means of the table of bolts on p 409.

Make $a\,g$ by scale equal to the sum of these four wts, plus the weight of $x\,j$ and $j\,z$, which rests directly upon j. Draw $g\,m$, $g\,l$ parallel to the rafters; and $l\,m$ hor.

For the dimensions of the rafters, $a\,b$, $a\,d$, commencing with what they require as *beams*, supported at the ends, bear in mind that the introduction of the struts $e\,c$, $h\,c$ converts each rafter, as $a\,b$, into two shorter ones, $a\,e$, $e\,b$; each of which sustains, in the present case, only one-half the load of and on the whole rafter; or only ¼ of it as a *center* load. Find the safe dimensions for the short beam, with its smaller center load, by rules, p 497, or by table, p 499.

The compressive strain on a U is $a\,m$. That on U b (or the greatest comp strain) is $= a\,m + e\,r$. Divide this sum by 2000 (or by whatever other number of lbs may be considered the safe crushing strength of the timber). The quot is the safe area in sq ins reqd for that purpose. Add it to the area previously found for the rafter as a beam. The sum is the entire area required.

The tie-beam. The pull on the tie-beam is $f\,m + s\,i$. Divide it by 3333, the safe pull in lbs per sq inch. The quot will be the safe area reqd for *that* strain. Then consider *one-half* of the tie-beam to be a uniformly loaded beam supported at each end; and find the safe dimensions by rules, p 497, or by table, p 499. To these dimensions add the area just found for the hor pull; the sum is the entire area reqd for the tie-beam, unless some addition be made to compensate for the cutting away at the feet of the rafters.†

Below are the calculated dimensions for two trusses, Fig 7, of 40 ft span; 10 ft rise: and 12 ft apart from center to center. In the first of these the tie-beam with its floor, ceiling, and other load, are assumed at the rate of 100 lbs per sq ft of floor; while, in the second, no specific load is assumed for that member, for reasons before given. In both, the wt of the rafters, with their roof-coverings and load of snow, and wind, is taken at 40 lbs per sq ft of roof surface between the centers of two trusses. The safety of each separate part is taken at 3; except that the *unloaded* tie-beam is fixed by rule of thumb. Timbers white pine. The greatest dimension in each case is the depth. Dimensions in inches. **1st.** Rafters 8 × 10. Tie-beam 8 × 15. Each strut 4½ × 4½. King-rod 1¾ diam if upset; or 2 ins if not upset. In practice it is better to make the struts as broad as the rafters. **2d.** Rafters 6 × 8. The tie-beam requires, theoretically, only 16 sq ins area; we will make it 6 × 8, like the rafters. Each strut 4½ × 4½; (the same as in the other.) King-rod ¾ diam if upset; or scant 1 inch if not upset.

If a *tie-rod* were used instead of a tie-beam, its diam would be 1¼ inch if upset; or 1.6 if not.

Art. 10. Fig 9 is a truss with a tie-beam $a\,b$; two rafters $w\,a$, $z\,b$; two queen-rods,‡ or queens, $w\,t$, $z\,t$, and a hor straining beam d. It may represent a roof uniformly loaded along the rafters and straining beam; and having a uniform load along the tie-beam. Or only one of these loads may be supposed to exist, as in a bridge with a load along $a\,b$; or a roof with its load along $a\,w\,z\,b$. The queen $w\,t$ supports, besides its own weight, all the weight of and on the part $s\,y$

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* Each strut will thus bear *half* of the wt of and on $z\,r$, or $z\,u$, only when, as in Fig 7, the inclination of the strut is the same as that of the rafter. If the strut is steeper than the rafter, it will bear more than half; but if it is less steep than the rafter, it will bear less than half; the remainder being in every case borne by the rafter. The parallelogram of forces will of course show all this. When the inclinations of a rafter and strut are not equal, we cannot draw hor diags $i\,k$, $v\,w$; but from the points i,k,v,w, we must draw hor lines to the vert diags $e\,o$, and $h\,n$.

† WHEN A TIE-BEAM IS SO LONG THAT IT MUST BE SPLICED, allowance must be made for the weakening effect of the splice. For Splices, see p 611; and for other joints, p 613.

‡ The queens are frequently made of wood.

of the tie-beam; and the other one $z\,t$, that of and on $u\,y$; s and u each being halfway between a queen and an abut. These are the only strains on the queens; so that their proper diams can be found by table of bolts, p 409. The parts of the tie-beam from s and u to the abuts, or walls, as well as whatever loads those parts may bear, are sustained directly by the abuts.

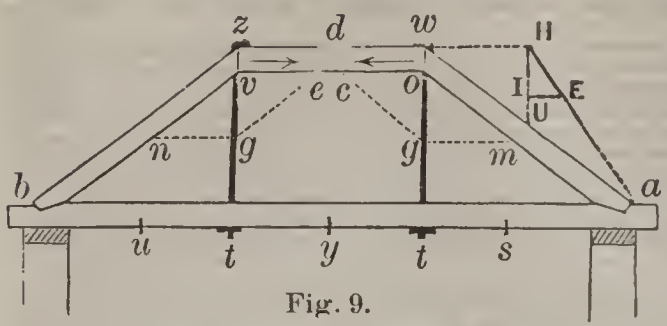

Fig. 9.

The queens transfer, as it were, the weights of themselves and of $s\,y$ and $u\,y$, with their loads, directly to w and z. To find the strains on the various parts of the truss, first from the center U of a rafter $a\,w$, draw a vert line U H; and from w draw a hor line w H to meet it. Join H a. Make H I by scale equal to the wt of only *one rafter* and its uniformly distributed load. Also draw o g vert, and equal, by the same scale, to the wt upheld by the queen-rod $w\,t$, added to *one-half* the wt of the straining-beam d. and its load; for it also presses vert at o. Draw $g\,m$ hor, or parallel to the straining-beam; and g e parallel to the rafter; thus completing the parallelogram $o\,c\,g\,m$ of forces.

The strains on the straining-beam d. The hor line I E and $o\,c$ together, give all the hor pres against the end w of the straining-beam d; and it is plain that a similar process on the other side of the truss, would give an equal pres against the end z. These two equal pressures reacting against each other, produce a strain, equal to one of them, throughout the entire length of the straining-beam; and therefore, the beam must be regarded as a pillar with a load equal to this strain, on its top; and the dimensions and area of section, for safely supporting it, may be found by the rule, p 458; or table, p 459.*

But beside this, the straining-beam, if loaded, must be regarded also as a beam supported at both ends; and the area necessary for this, as found by tables, page 499 or 512, must be added to that already found.

The strains on the rafters. *First*, consider a rafter $w\,a$ as an inclined loaded beam supported at both ends; and find the proper dimensions and area, by the rules on page 496; or by the tables, p 499, or 512.

Second, add together o m and the other longitudinal strains $o\,z$ and L R (found as in Fig 5½) in the rafter, and divide their sum by 2000 (or by whatever other number of lbs may be considered the safe crushing strength of the timber). The quot is the safe area in sq ins reqd for that purpose. Add it to that already found for the rafter as a beam. This last sum is the total area reqd.

The tie-beam. The hor strain, or pull on the tie-beam, will be equal to the push on the straining-beam; and is represented by I E and o e together. Find the safe area by table, page 463; or by dividing the hor strain by 3333, which is the pull in lbs per sq inch that ordinary building timber will bear with a safety of 3.

Then, since in this truss the queens divide the tie-beam into *three* lengths, each of these must be considered as a separate beam, (loaded or unloaded, as the case may be,) supported at each end. Its safe dimensions being found, add the area just found for resisting the pull. Add, if reqd, an allowance for the cutting away at the feet of the rafters.

Below are the calculated approximate dimensions for two trusses, Fig 9, of sixty ft span; 15 ft rise; and 12 ft apart from center to center. All the conditions the same as for the preceding example of Fig 7. **1st.** Rafters 12 ins broad, by 14 ins deep. Straining-beam 12 broad, by 12 deep. Tie-beam 12 broad, by 12 deep. Each queen rod 1½ ins diam if upset; 1⅝ if not.†

2d. Rafters 10 × 11¾. Straining-beam 10 × 11. Tie-beam, say 10 × 12. Each queen-rod ½ inch diam. Unloaded tie-rod, 1½.

The proper size for each piece, ***so that they shall all be suitable for the truss,*** cannot be determined at once. We must find any dimensions that will answer for each piece by itself; and afterwards adjust them by recalculation, perhaps 3 or 4 times. Great accuracy is not necessary in doing this. See Note, p 573.

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* **A strut or tie cannot be strained** along the direction of its length by a force acting at one end, unless there is at the other end an equal force acting in the same straight line but in the opposite direction, and which may be either one single force, or the resultant of two or more forces neither of which acts in that direction. Hence if in Fig 9 we place a load at Z only, a parallelogram $v\,e\,g\,n$ of forces will not give the hor strain v e along the beam Z W, because there is then no equal reacting hor force at the other end in the direction from H towards W. In that case a load at Z only, (represented by z c in Fig X) produces at z the two strains $z\,n$, $z\,e$; which last pressing towards a tends to make $z\,b$ revolve around b as a center, thus forcing z downwards, and the joint w upwards, thereby causing the distortions seen in Figs 9½, 9¾. The force $z\,e$ therefore evidently tends to break the joint w; and with a moment equal to the force $z\,e$ (in tons or lbs, &c) mult by its leverage $w\,o$ perp to $z\,a$. If the moment of resistance of the joint can withstand this the truss will remain unchanged; but a simple strut from z to a would remove all danger, by sustaining the whole of the force $z\,e$ effectively, and thus relieving the joint w entirely.

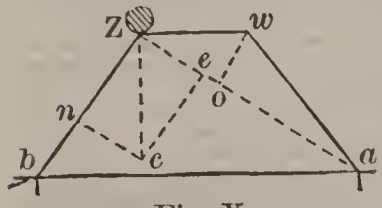

Fig. X.

† See Rem, p 408.

Rem. The truss in Figs 9 and 9¼ affords a good opportunity for alluding, in a general way, to the **principle of counterbracing,** and to the necessity (as stated in Art 7) of adhering to a triangular arrangement of the parts of a truss. So long as this truss is uniformly loaded throughout its length, it is well arranged for sustaining the resulting strains; because the strains on each side of the center are equal, and balance each other. But if a heavy load be placed along *a o* only (Fig 9¼) its tendency to depress *ae* will produce the derangement shown in Fig 9½; because the horizontal pressure from *s* toward *t*, Fig 9½, will then become greater than that from *t* toward *s*. The two triangular portions will still retain their original *fig;* but owing to the ease with which the 4-sided portion, *n m e e*, has been deranged, and changed to *s t c e*, their *position* becomes altered to the dangerous one in Fig 9½. The diag *e m* has been lengthened to *e t*; while the diag *e n* has been shortened to *e s*. Now, if there had been a strong bar of iron reaching from *e* to *m*, with its ends firmly attached to those points, it would have divided the whole truss into triangles; and then the diag *e m* could not have become lengthened to *e t* by any strain less than one sufficient to break this iron bar by *pulling* it apart; therefore the truss would have remained safe, and unchanged in figure; for the bar, while preventing *e m* from lengthening to *e t*, would, as a consequence, prevent *e n* from shortening to *e s*. Or, omitting the iron bar at *e m*, suppose a stiff, unbending inclined post to be inserted between *e* and *n*. This also will divide the whole truss into triangles; and it is then plain that *e n* could not be shortened to *e s* by any strain less than one sufficient to break the post by *crushing* it. Therefore, in this case also, the truss would have remained safe, and unchanged in figure; for the post, while preventing *e n* from shortening to *e s*, would, as a consequence, prevent *e m* from lengthening to *c t*. Either the bar or the post would be a counterbrace against the effect of unequal loading. With a uniform load it is not needed. Neither are additional counterbracing pieces needed in bridge trusses of the forms Figs 10, 11, 12, 13, provided each web member is so constructed as to bear alternately compression and extension.

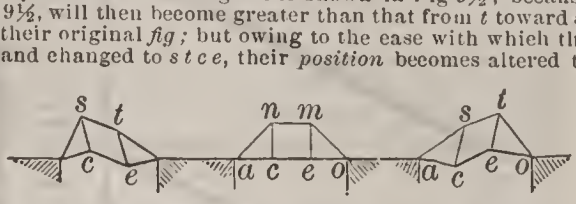

Fig. 9¾. Fig. 9¼. Fig. 9½.

The next Fig 9 *b*, shows the bad effect produced in a truss longer than Fig 9¼, when the web members are not so constructed. In the Burr bridge, Fig 36, and in some others, although the truss is divided into triangles, yet the inclined braces, *i* c, &c, are often improperly adapted to bear compression only; their ends not being firmly attached to the chords. Consequently, with a heavy load at *a*, the derangement shown in Fig 9 *b* (analogous to that in Fig 9½) takes place. To prevent it, counterbracing must be resorted to, either by inserting struts or ties along the dotted diagonals; or by making the braces capable of resisting tension as well as compression. The last method shows that counterbracing can be performed without the addition of pieces specially called counterbraces, and denoted by the dotted diagonals. All that is required in the principle of counterbracing, is to so arrange and connect the several web members, that the strain produced by *unequal* loading at any point, as *a*, between the abuts; or along any portion of that distance, shall be properly transferred by them to *both abutments*.

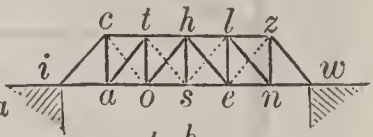

Fig. 9*a*

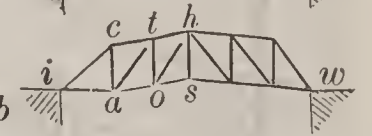

Fig. 9*b*

Art. 11. The strains in such trusses as Figs 10 and 11, may be found by three very simple processes when the truss and its load are uniform from the center each way.* When this is the case it is usual and safe to assume that the half load *e p*, Fig 10 or 11, on the right hand of the center *e*, rests on the right hand support *p;* and that the half load *e a* on the left hand of the center *e*, rests on the left hand support *a*.† It is often assumed also, for simplifying the calculations, that the entire weight of the truss and its load is distributed along one chord only. This is plainly incorrect; but inasmuch as the *extraneous* load (such as the covering of slate, snow, etc., on a roof, and the travelling load on a bridge) in many cases actually does rest on one chord only, and is great in comparison with the weight of the truss alone, the error arising from the assumption *in such cases* is not of practical importance.

But in bridges of great span the weight of the truss may bear a large proportion to that of its load; or there may be an upper and a lower roadway, one resting on each chord; and a roof truss may have to bear not only the covering, snow, &c., on its upper chord or rafters: but a floor with a plastered ceiling beneath it, and all the load incident to any ordinary room, on its lower chord. In such cases the entire weight of the truss and load must be properly distributed along both chords before we can correctly find the strains. But this will in no way affect the principle of the three processes which we are about to explain, and as we proceed we shall give directions for both cases.

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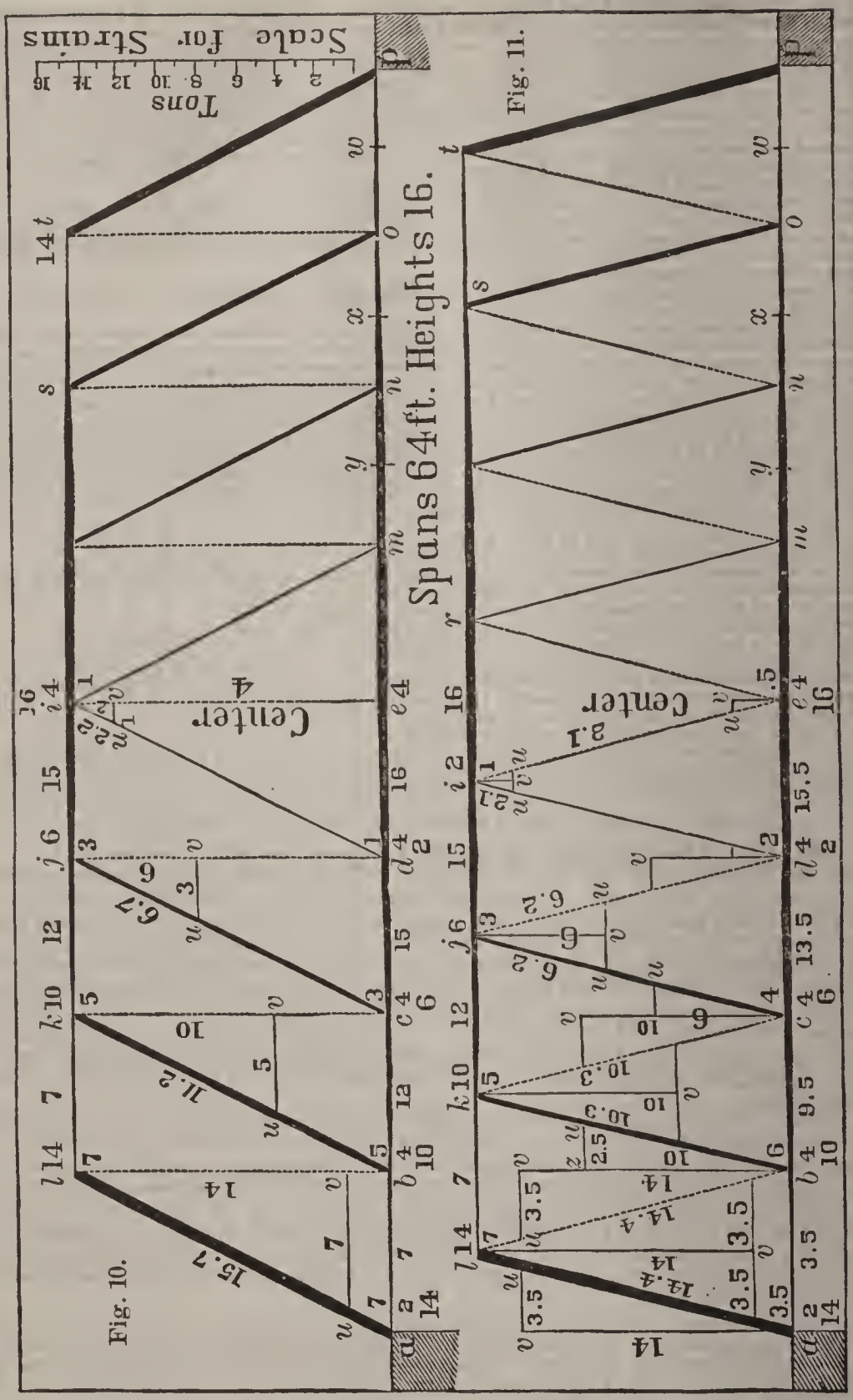
* It is not necessary that the entire load should *in itself* be uniform; but merely uniform *each way from the center*. Thus at *e* may be say 1 ton; at *d* and *m* each say 5 tons; at *c* and *n* each 2 tons, &c.

† **This assumption is untrue,** and opposed to the unvarying law that every individual portion of the entire weight rests partly on each support. Thus, one portion of the load at *o* rests partly on *p* and partly on *a;* and so with every other portion; and on this fact depends the difference in the methods of calculating the strains from uniform, and ununiform or moving loads. When, however, the weight of the truss and load is uniform each way from the center we obtain correct results, and more readily by adopting the erroneous assumption.


Spans 64ft. Heights 16.
Scale for Strains
Tons
Center
Fig. 10.
Fig. 11.

Beginning then with uniformly distributed weights of truss and load, and assuming all of said weights to rest on the long chord *a p*, prepare a correct skeleton diagram of the truss (or at least of one-half of it), such as Figs 10 and 11, in which the height or depth *e i*, Fig 10, is the vert distance between the centers of the depths of the two horizontal chords. A scale of from $\frac{1}{8}$ to $\frac{1}{4}$ of an inch to a foot will generally be large enough.

Then the **first process** is the very easy one of ascertaining how much of the total uniform weight is to be considered as sustained at each point of support along either the top or the bottom chord, as the case may be; remembering that one *half* of each end panel is sustained directly by the abut nearest to it, as in the preceding cases.

In order more fully to illustrate the following Articles, we shall assume each of the trusses, pp 558, 570, 571, 572, 574, 585 and 586, to be 64 ft long, and 16 ft high, and to be divided into 8 equal panels. Total uniform wt of one truss and its load, 32 tons; or 4 tons to a panel. Consequently there will be 9 points of support to each truss. Thus, in Figs 14, 15, and 16, in which the load is supposed to rest on top of the truss, and in Figs 10 and 11, in which it rests upon the bottom, the points of support are at *a*, *b*, *c*, *d*, *e*, *m*, *n*, *o*, *p*. Some of these are not shown in the first three Figs. If both chords are loaded, there will be points of support in the short one also. Thus, in Fig 10 there will be 7, and in Fig 11 there will be 8 of them. Now, in Figs 10 and 11, *w*, *x*, *y*, etc., being midway between the points of support, it is plain that (assuming *all* the weight to be on the lower chord) the point *o* must sustain that portion of it comprised between *w* and *x*; *n* all between *y* and *x*; while the abut *p* sustains *directly* the portion from *w* to *p*. The same principle applies to all the other trusses; and equally so whether the panels be of the same width or not; each point of support is assumed to sustain all the uniform wt of truss and load between itself and the two points midway to the adjacent points of support, however unequal the two distances may be. In our Figs 10 to 16, the strong dotted lines of the web members represent ties; the full lines, struts. The dots intimate that *chains* may serve as *ties*. When the panels are of equal length, *p o*, *o n*, etc., the distance from *p* to *w* will be but half a panel; so that each abut will *directly* sustain but half as much wt as each other point. Therefore, to find the amount of wt sustained at each of the nine points of support, we have only to div the total wt (32 tons) by a number less by 1 than the number of points. The quot $\frac{32}{8} = 4$ tons, will be a *full panel-load*, to be at each point, except the two end ones, *a* and *p*, at the abuts; at each of which it will be but half of one of the full panel-loads, or two tons.* The amounts of these panel and half-panel loads should at once be figured on the sketch at their proper points, as is done in our Figs; a 2 being placed at each end of the truss; and a 4 at the other points. Each of these panel-loads of course causes a vert strain equal to itself where it rests. As the strains on one half of the truss are the same as those on the other half, the numbers need only be written on one of them; indeed, the sketch, as a general rule, need show but one-half of the truss.

If there is a load on the other chord also, it must be in the same way divided among the points of support of that chord, and be figured as before.

The second process. All the panel-loads are of course eventually transmitted through the truss to the abuts; as is manifest from the fact that each abut sustains half the total load. But each panel-load, while travelling, as it were, up and down alternate web members from its original point of support, to the nearest abut, places, so to speak, an additional load, or more correctly produces an additional vert strain equal to itself, at every intervening point of support in each chord.† Our second process consists in finding the amount of this additional vert *strain* at each point of support.

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* This of course is only when the end panels are of the same length as the others. When not so, the loads at the points of support and on the abutments will plainly vary from the above.

† In trusses like Figs 10 and 11, with two horizontal chords, the panel loads are transferred directly from their points of support via the web members to the abuts. In such trusses the strains in the web members are least at the cen of the span, and greatest at its ends; while those in the chords are greatest at the cen and least at the ends. This is indicated in the Figs by the diff thicknesses of the lines representing these members. But in Figs 14, 15 and 16, the panel loads divide at their respective panel points, as explained in Rem p 573; a portion of each panel load going directly to the nearest abut via the sloping rafter; the remainder going first to the cen *e* via the web members, whence it finally reaches the abuts via the rafters Figs 14 and 16,

In Figs 10 and 11, with *parallel horizontal upper and lower chords*, the vert strains are very easily found, thus: Remembering that only *half* of the *center* panel-load strain at *e* goes to *each* abut, begin with the 4 tons at *e*.

In Fig. 10 these 4 tons first go up the tie *e i* to *i*, where they produce a vert strain of 4 tons, which figure as in the diagram. But at *i* these 4 tons separate; 2 of them going to the abut *p*, and the other 2 to the abut *a*. The last 2 first pass down *i d* to *d*, where also they produce a vert strain of 2 tons, which also figure, as must be done with all that follow. At *d* these 2 tons unite with, or as it were take up and carry along with them the 4 tons already there; and the entire 6 tons go up the tie *d j* to *j*, where they produce a vert strain of 6 tons. From *j* these 6 tons go down the strut *j c* to *c*, where they also produce a vert strain of 6 tons. At *c* these 6 tons take up the 4 tons already there, and the entire 10 tons go up *c k* to *k*; and thus the process continues until 14 tons find their way to the abut *a*, where they meet the 2 tons of load already there; thus making 16 tons, or *one-half the wt of the truss and its load*; which is a proof that our work is correct so far.

In Fig 11, the 4 tons at the center *e* separate *there*; 2 of them going up *e i* to *i*, and thence to the abut *a* as before.

In either Fig **if there is a uniform load on each chord** there is no difference in the second process; for after having by the "first process" divided each load among the points of support of its own chord, the portion at each point must be taken up as it occurs, and carried on with the others to the abutment as before.

The third and last process consists in completing our sketch, or diagram, in such a manner as to enable us to measure by scale the strains produced along every member of the truss, by these vert strains thus accumulated at the diff points of support, *a*, *b*, *c*, *l*, *k*, etc.

To do this in Fig 10 (that is, whenever the web members are alternately vertical and oblique) from each point of support of one chord only, beginning at the center apex *i*, draw a vert line as *i v*, *j v*, etc., to represent by any convenient scale, the vert strain figured at said point; except at the center one *i*, where the vert line must represent only half the vert strain, inasmuch as that is all that goes to each abut. Draw also the hor lines *v u*, *v u*, etc. Then will each oblique line *i u*, *j u*, etc., give by the same scale the strain (2.2, 6.7, 11.2, 15.7) along its own oblique web member, as figd. The hor lines give the hor strains exerted on the chords by each oblique at both its ends. We have figured all these strains (7, 5, 3, 1) at the head and foot of each oblique. Each of these hor strains extends from the ends of the oblique, to the center of the chord; therefore the end stretches of the chords bear 7 tons hor strain; the next ones $7 + 5 = 12$ tons; the next ones $7 + 5 + 3 = 15$ tons; and the center one $7 + 5 + 3 + 1 = 16$ tons; all of which are figured along the chords.*

We have said that the vert, hor, and oblique sides of the triangles give the **strains,** but it would be more correct to say that each of them gives a **force,** which being balanced by the other two, thereby causes a **strain** equal to itself, instead of **motion.**

The hor strains at the centers of the two chords will be equal in both Figs 10 and 11, whether one or both chords be uniformly loaded; or if the truss be inverted; with only the exception in the foot note.*

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or via the two inclined ties in Fig 15, of which *i a* is one. In such cases the strains on the web members are greatest at the cen of the span, and least at its ends; while those on the tie beam and rafters, Figs 14 and 16, and on the tie rods and hor chord Fig 15, are greatest at the ends of the span and least at its cen. This also is indicated in Figs 14, 15 and 16 by diffs in the thicknesses of the lines, and is the exact reverse of the case of Figs 10 and 11.

* When at the *central apex i*, Fig 10, the two ends of the obliques *i d*, *i m*, which meet there, are so arranged as to butt tight *against each other*, then the center hor strain of 1 ton at *that point* is not borne by the chords, but by the obliques themselves; so that there will then be that much less strain at the center point of that chord than along the center stretch *d m* of the other chord. But if instead of this, they abut *against the chord*, at some little distance from each other, then the chord also receives the strain; so that the hor strains at the centers of the two chords become equal, as we assume to be the case in all our Figs of uniform trusses uniformly loaded each way from the center. The same remark applies to the hor strain of .5 of a ton at the center apex *e* of Fig 11.

The strain along the center vert tie *e i* of Fig 10, will be equal to the 4 tons at *e; and when the entire wt is assumed to be on the long chord*, the vert lines at the other points of support will give the vert pulling strains on the other verts, as 6, 10, 14.

But with loads upon both chords this last will not be the case; but the strain on each vert tie will then be equal to the vert strain at its foot.*

If the loaded truss is inverted, the verts become struts or posts, and the obliques ties; also the strain on each vert is then the one figured *at its top;* but the *amount* of strain on each part of the entire truss will remain as before.

In Fig 10 all the uniform wt is on the long chord, and the resulting strains are all figured on the diagram. We add the strains that would occur in case there were an *additional* uniform load of 6 tons on the upper chord from *l* to *t*. This would give 1 ton at each point of support along that chord, except the two end ones *l* and *t*, at each of which it would be but .5 of a ton. All these must be figured on the short chord of the diagram, as were 4, 4, &c, on the long one. The student may then work out the case for himself. We repeat that uniform trusses and loads require no counter-bracing.

For Fig 10, but for a load on each chord.

$e\,i = 4.$ tons.	$i\,d = 2.8$ tons.	$a\,b$ or $l\,k = 8.5$ tons.
$d\,j = 6.5$ "	$j\,c = 8.39$ "	$b\,c$ or $k\,j = 14.75$ "
$c\,k = 11.5$ "	$k\,b = 13.98$ "	$c\,d$ or $j\,i = 18.50$ "
$b\,l = 16.5$ "	$l\,a = 19.01$ "	$d\,e$ or at $i = 19.75$ "

For the "third process" in Fig 11 (or when all the web members are oblique, whether equally so or not) after having found and figured the loads and vert strains at each point of support precisely as directed for Fig 10, then from every such point in both chords draw a vert line as *e v*, *i v*, *d v*, &c; and on it lay off separately by scale both the vert strain that *comes to* that point through a web member from towards the center of the truss; and the one that *goes from* it through another web member towards the abut; except that at the very starting point *e*, Fig 11, there is but one vert strain (the one of 2 tons going from it); and at the very end *a* also there is but one, namely that of 14 tons coming to it along the oblique *l a;* for the 2 tons at *a* are not to be included, because they do not reach *a* by means of a web member. Therefore both at *e* and at *a* only a single vert strain is to be laid off.†

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* **It is so in both cases,** for in any of these trusses under stationary loads the strain along a web tie whether vert or oblique may be considered to commence at its lower end, that being the end at which the panel loads first act on their route to the abut, and up which they as it were work their way. But under moving loads the same member may have to act both as a tie and a strut; hence the remark will not apply to such. Referring to what is said above, when the entire wt is on only one chord, the vert strains at the two ends of any tie in Fig 10 are equal. Hence a line drawn to represent the upper one, may be assumed to represent also the lower one. But when both chords are loaded, the vert strains figured at the two ends of any tie are unequal, and we must then have regard to the true principle. **If the verts should be struts or posts** (as if Fig 10 should be inverted) then any strain along them must be received from their **tops,** or the reverse of the case with ties. It will aid the student very much in what follows to familiarize himself with the idea that **strains pass only down the struts, and up the ties.**

† When all the wt of truss and load is assumed to be on the long chord as in our Fig 11, then the vert strain that comes to any point in the short chord by one web member is plainly the same in amount as that which goes from it by the other web member; and hence only one vert measurement need be laid off for it, as is seen at *i v*, *j v*, *k v*, *l v*, in the Fig. But at the long chord (or at both chords when there is a load on both) the vert strain that comes to any point is less than the one that goes from it towards the abut, and is evidently the one last figured at that point, as the 2, 6, 10, &c, tons at *d*, *c*, *b*, &c, in Fig 11; while the one that goes from that point towards the abut is as evidently equal to the sum of the two strains figured at that point, as the 6, 10, 14, &c, tons at *d*, *c*, *b*, &c. When both chords are loaded there will be two vert strains to be figured at each point of support in both chords, except at the very starting point, and at each abut, where will be but one in any case.

Draw also the hor lines, thus forming a series of triangles (as $i\,v\,u$, $i\,v\,u$, $j\,v\,u$, $j\,v\,u$, &c, of the upper chord; and $a\,v\,u$, $b\,v\,u$, $b\,z\,u$, &c, of the lower chord), each with one vert, one hor, and one oblique side. Then the *combined* **hor strain** exerted upon either chord, *by the obliques* at any point of support and *by the part load* (if any) supported there; will be measured by the *sum* of the two hor lines *directly opposite* to said point, except at the center e, and at the end a, at each of which it will be measured by the *single* hor line $v\,u$, or $v\,u$, opposite each.* These hor strains (7, 5, 3, 1 tons on the upper chord; and 3.5, 6, 4, 2, .5 on the lower chord) are figured close to the points of support at which they occur; and the *total* hor strains on the several stretches of the chords are figured midway of said stretches.†

The strain along any oblique as $j\,c$, will be measured by the oblique side $j\,u$, or $c\,u$, of either one of the two triangles on either side of it; and this will be the case whether one or both chords be loaded; or if the truss and load be inverted. All the strains in Fig 11 (loaded on the long chord only) being figured on the diagram, we give below the strains that would occur in case there were an *additional* uniform load of 7 tons on the short chord from l to t; which would give 1 ton at each point of support along that chord, except the two end ones l and t, at each of which it would be but .5 of a ton. All these must first be figured on the short chord of the diagram, as were 4, 4, &c, on the long one. The student may then work out the case for himself.

For Fig 11, but for a load on each chord.

$e\,i = 2.06$ tons.	$c\,k = 12.36$ tons.	$a\,b = 4.38$ tons.	$l\,k = 8.63$ tons.
$i\,d = 3.09$ "	$k\,b = 13.40$ "	$b\,c = 11.88$ "	$k\,j = 14.88$ "
$d\,j = 7.22$ "	$b\,l = 17.53$ "	$c\,d = 16.88$ "	$j\,i = 18.63$ "
$j\,c = 8.25$ "	$l\,a = 18.04$ "	$d\,e = 19.38$ "	i to center $= 19.88$ "
		at $e = 19.88$ "	

Art 12. The strains in Figs 10 and 11 may readily be calculated (after having by the "first and second processes" found the vert strains at all the points of support) whether one or both chords be loaded, or if the truss and load be inverted. Thus, divide the hor stretch of an oblique by its vert stretch; the quotient will be the **natural tangent** (.5 for Fig 10, and .25 for Fig 11) of the angle (26° 34′ in Fig 10, and 14° 2′ in Fig 11) which the oblique forms with a vert line. Divide the actual length of an oblique by its vert stretch; the quotient will be the **nat secant** (1.12 for Fig 10, and 1.03 for Fig 11) of the same angle. Then **the strain along any oblique** in Fig 10 or 11, is found by multiplying the vert strain that travels towards the abutment along said oblique, by the nat secant.

The hor strain on either chord, caused by either end of any oblique, is in Fig 10 equal to the vert strain that travels along said oblique towards the abutment, mult by nat tangent. And in Fig 11, it is equal to the vert strain that *comes to*, added to that which *goes from* any end of an oblique, mult by nat tang; except at the center and end, where the *single* vert strain must be mult by nat tang.

All this is simply because that if we assume the vert side of any one of the triangles to be radius or 1, then the hor side becomes by that same scale the nat tang of the angle which it subtends; while the oblique side becomes the nat secant of the same angle.

The principle of the mode of finding the strains in Figs 10 and 11 is this. We know that if three forces are in equilibrium with each other at any point, the lines which represent them will form a triangle. Now at every point of

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* Each of the upper hor lines $u\,u$, $u\,u$, &c, in Fig 11 is to be considered as composed of two separate ones $v\,u$, $v\,u$, &c; the right hand one of which measures the hor strain caused by the vert strain that *comes to* i, j, &c; while the left hand one measures the hor strain caused by the vert one that *goes from* i, j, &c, towards the abutment. Such lines as $u\,u$, $u\,u$, &c, occur only when all the wt is on one chord; for when both chords are loaded, the vert strain that comes to, and that which goes from any point of support differ, therefore requiring two unequal vert measurements, and two unequal hor lines at each point of support of both chords, except at the center, and at the ends; at each of which will be but one.

† **The common rule** $\dfrac{\textbf{Total uniform wt} \times \textbf{span}}{\textbf{8 times the height}} =$ hor strain at center of either hor chord, is not strictly correct except when both chords extend the full length of the span, and are both loaded throughout their entire length; or in the impossible case of the entire wt being on the long chord. Still in ordinary cases it is a sufficiently close approximation. On this subject see Art 19.

support in Fig 10 we have one set of three such forces; and in Fig 11, two sets. In Fig 10 it was not necessary to show those at the long chord. Now each set, or each triangle represents a vert force, a hor one, and an oblique one, keeping each other in equilibrium at the point of support. It is true that there are other forces acting at the same point, but as they hold *each other* in equilibrium, they do not interfere with the first ones. Thus, both the 7 and the 12 tons hor forces along the chord at *k* are balanced or held in equilibrium by the equal ones from *t* and *s*, on the other half of the truss; without disturbing the forces represented by the sides of the triangles. Hence by measuring those sides we obtain the forces and strains themselves.

The same principle either by diagram or by calculation applies to Figs 12 to 13½, when uniform and uniformly loaded. In those Figs all the weight is here assumed to be on the long chord; (but after what we have said, no difficulty can arise when placing loads on the other chord also.) All said wt is first to be properly distributed among the points of support on said long chord, and there figured, as shown by the upper 4s and 2s along that chord in the Figs. This being done, we have figured all the other vert strains, thus providing the data for drawing the vert sides of the triangles; and these in turn give us the hor and oblique sides which measure the corresponding strains, and all of which are drawn on the Figs.

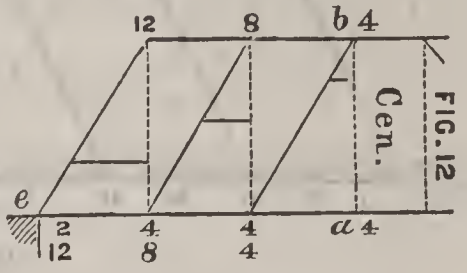

FIG. 12

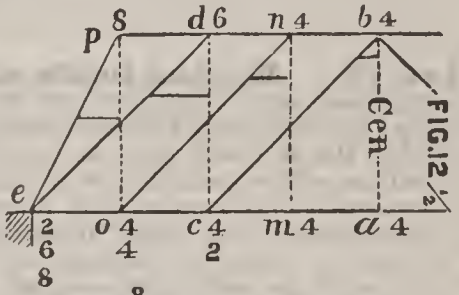

FIG. 12½

All these trusses being uniform and uniformly loaded from the center each way require no counter bracing. Bear in mind that the vert strains that accumulate at an abut must equal half the wt of the truss and its load.

In Fig 12, with no oblique at the center, the 4 tons at *a* having no oblique in contact on either side of them, go to *b*; and on their way to *b* strain *ab* 4 tons. From *b* all 4 go along the web members to the nearest abut *e* as figured.

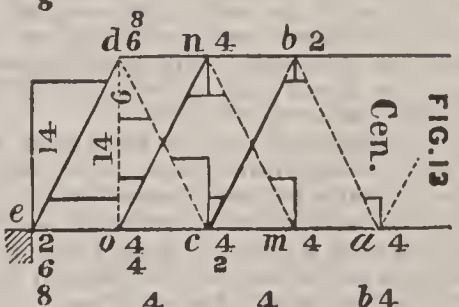

FIG. 13

In Figs 12½ and 13 the web members of each are to be regarded in some degree as belonging to two separate trusses, namely *a b c d e* and *m n o p e* in Fig 12½; and *a b c d e* and *m n o d e* in Fig 13; and the vert strains at their ends are to be found on that assumption, as figured. In Fig 13, *o d* is a vertical tie.

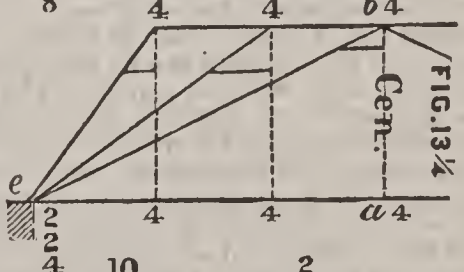

FIG. 13¼

In Fig 13¼ there being at none of the 4 ton loads on the long chord an oblique in contact with them on either side, they (like that at *e* Fig 10, or at *a* Fig 12) pass each by itself vertically to the upper chord, where figure them. Of those at *b*, 2 go to the abut *e* by way of the oblique *b e*; but the other two 4s *all* go to *e*, each by its own oblique. Each of the three hor lines gives the strain exerted upon the upper chord at the respective panel point; but the hor strain along the lower chord is uniform from end to end, because all the forces that produce it act at its ends only. It is equal to the sum of the three hor lines.

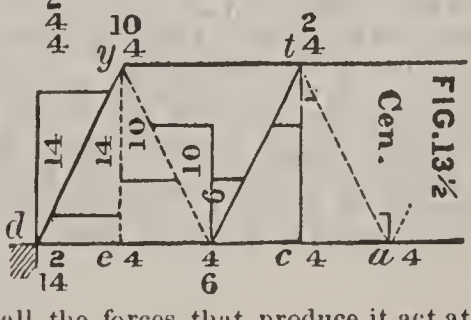

FIG. 13½

In Fig 13½, at the 4 ton loads at *c* and *e*, there is no oblique in contact with them on either side; therefore they pass at once vertically to *t* and *y*, where figure them. Then begin with 2 of the 4 tons at *a*.

All loads that have no oblique in contact **on either side,** whether sustained by vert ties or by vert posts, are to be thus transferred **at once** to the opposite chord in order to meet an oblique along which to travel to the nearest abut; and said vert ties or posts will be strained to the amounts of said loads.

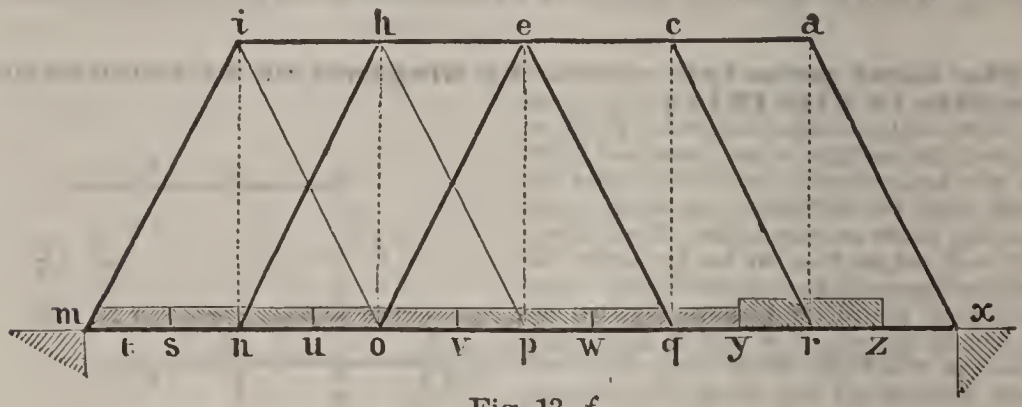

Fig. 13. *f*

Art. 12½. Moving loads and counterbracing. The foregoing investigations refer to cases where the load is always in the same position, and of the same amount; and where, consequently, the strain on any given member is constant. But in many cases, as in railroad bridges, the amount, and manner of distribution, of the load, change greatly from time to time, so that the strain on a given member may vary greatly in amount, and may even change from tension to compression or *vice versa*. In the present article we investigate these changes in the strains, caused by changes in the position of the load.

To simplify the subject, we regard the weight of the locomotive as being *uniformly distributed* over its length, as shown in Fig 13 *f*; and this suffices to illustrate the *principle*. But specifications frequently call for calculations based upon the actual load *on each pair of wheels* of an actual or assumed engine and tender, as shown on p 546.

For simplicity, also, we here seek only the *maximum* strain on each member, in order to determine *where counterbraces are required*. In practice it is necessary to note not only the maximum but also the *minimum* strain on each member; for when the strain on a member is subject to great and frequent variations, especially where it changes from tension to compression or *vice versa*, the ultimate strength of the member is reduced to an important extent. See p 435.

We will calculate the maximum strains in a bridge Fig 13 *f* of 120 ft span *mx*, divided into 6 panels, each 20 ft long and 30 ft high. For convenience of calculation we will suppose the fig to represent the *two* trusses of such a bridge *combined into one*. We first calculate the max strain in each member of this supposed *double* truss, and then divide each strain by 2 for the *actual* strain in the corresponding member of each *actual* truss.

The weight of this double truss is 48 tons; or 8 tons per panel; or .4 ton per ft run. The floor, and its several timbers, such as cross girders, hor bracing, &c, which, although not portions of the *truss proper*, are essential to the *bridge*; and to be considered as so much permanent load, equally distributed along the truss, are assumed to weigh an additional 24 tons; or 4 tons per panel; or .2 ton per ft run of the double truss. Therefore, the truss and its accompaniments together, or in other words, the bridge superstructure, weighs 72 tons; or 12 tons per panel; or .6 of a ton per ft run. Since each panel is 20 ft long, and 30 ft high, each oblique is $\sqrt{20^2 + 30^2} = 36$ ft long: and the secant of the angle which it forms with a vert, (or of the *brace angle*,) is = length of oblique ÷ its vert spread = 36 ÷ 30 = 1.2: and the nat tang of the same angle is = hor spread ÷ vert spread = 20 ÷ 30 = .6667.

The moving load is assumed to consist of an engine, $y\,z$, weighing 36 tons: all of which is supposed to be so concentrated on its drivers, as to stand upon the length of one panel. The engine is supposed to be followed by a tender and cars, weighing 21 tons per panel; or 1.05 ton per ft run.

The greater danger from engines, arises from the fact that in many of them, especially in heavy freight engines, all the wt is concentrated upon their driving wheels; which are so close together that a 30 or 40 ton engine on 8 drivers, will often have its entire wt sustained upon only 12 to 15 feet length of the truss; thus bringing a great strain upon each individual web-member, as it crosses the bridge. This point is one of the greatest importance in arranging the preliminary data on which to form the basis for calculating the strains on a bridge truss; and we should allow liberally for the greatest load that can possibly be brought upon one panel at a time. When the panels are quite short, each one will have of course to sustain only a smaller portion of the wt of engine; it may, indeed, have to be divided among 2, 3, or 4 of them.

The fact that the engine produces upon each panel in succession, a greater strain than an equal length of loaded cars can do, causes a modification of the calculations; as will be seen as we proceed.

Such data must be prepared before beginning the calculations Our assumptions in the case before us have been made entirely with reference to ease of calculating our example with but few figures. Our truss, together with a load of cars extending from end to end, will weigh 198 tons; or 1.65 tons per ft run; or 33 tons per panel.

Having prepared a diagram, begin by finding the strains caused in only the verts and obliques, by the bridge itself, half the wt of the truss alone being considered to be on the top chord, while all of the weight (24 tons) of floor rests of course directly on the lower chord.

By our "first process," p 559, there will then be 6 tons each at h, e, and c; 3 tons each at i and a; 8 tons each at n, o, p, q, and r; and 4 tons each at m and x. By our second and third processes, the strains on the web members will be as follows:

On $e\,p$	8	tons.
" $h\,o$ and $c\,q$, each	15	"
" $i\,n$ and $a\,r$ "	29	"
" $e\,o$ and $e\,q$ "	8.4	"
" $h\,n$ and $c\,r$ "	25.2	"
" $i\,m$ and $a\,x$ "	38.4	"

Write these strains at the *feet* of the respective members.

We now have the strains on the web members, resulting from the truss itself, and from its uniformly distributed permanent load of floor, &c; and are prepared to begin with the additional strains resulting from the moving load. We find those on the counters, on one-half of the diagram; and those on the main obliques and verticals, on the other half. First suppose the engine to be in the position $y\,z$, on the half diagram $p\,x$; with the train reaching to the farthest end m of the truss. In this position of the moving load, the vertical $a\,r$, and the end oblique $a\,x$, will be more strained by it than in any other; and their strains will be produced by that portion of the load that may be regarded as being first concentrated at r; and as passing thence up $r\,a$ to a; and from a, down $a\,x$ to the abutment x. To find this portion, the truss may be considered as a simple beam or lever, marked off into six equal divisions at r, q, p, o, n. In that case we know (from the principle of the lever) that if any load, as for instance our engine of 36 tons, be placed uniformly along $y\,z$, its whole weight may be assumed to be concentrated at the middle point r; and that five-sixths of it will be borne by the nearest abutment x; and only one-sixth by the farthest abutment m. So also with the panel length $y\,w$ of cars; its weight being supposed concentrated at q, four-sixths of it must be borne by x; and two-sixths of it by m. Of the cars $w\,v$, three-sixths are borne by x; and three-sixths by m. Of those along $v\,u$, two-sixths are borne by x; and four-sixths by m; of those along $u\,s$, one-sixth by x; and five-sixths by m. The half panel of cars, $s\,m$, rests directly upon x, and produces no strains in the truss.

By this process, then, we find that five-sixths of the engine, or 30 tons; four-sixths of the cars $y\,w$, or 14 tons; three-sixths of the cars $w\,v$, or 10.5 tons; two-sixths of $v\,u$, or 7 tons; and one-sixth of $u\,s$, or 3.5 tons; making 65 tons in all, are borne by the abutment x, when the moving load is in the position shown in our fig. This

load of 65 tons passes from r to a; and from a to x.* The calculation is made as shown under our fig below the heading "With engine at r."

We next suppose the engine to back into the position $y\,w$; with the train reaching to m; and with no load between y and x. In this position the strains on $q\,c$, and $c\,r$, are greater than in any other; and by a process precisely as before, and as shown under our fig, below the heading "Engine at q," we find that 45 tons of the moving load go to the abut x; passing from q to c; and from c to r, on their way to it. Then placing the engine at $w\,v$, the cars reaching to m, and no load between w and x, the strains on $p\,e$ and $e\,q$, are the maximum that they can be subjected to by an engine and train; and are found like the others, as shown under "Engine at p." We have now reached the center of the truss, and have obtained data for the greatest strains that can occur on the verts and main obliques on one-half of it. So far as regards the corresponding members on the other half, we stop here; for it is manifest that an engine and train crossing the bridge in the opposite direction, must produce the same maximum strains upon them as we have already found for the others. That is, $i\,m$ will be strained the same as $a\,x$; $n\,i$ the same as $r\,a$; $e\,o$ the same as $e\,q$, and so on, by a returning train. Write these strains near the *tops* of the verticals.

But as yet, we have not sufficient data for determining where *counters* will be required. To obtain it, we draw the two lines $h\,p$, and $i\,o$, parallel to the obliques on the opposite side of the truss; and taking it for granted for the present, that these two lines are counters, we back the engine to $v\,u$, and then to $u\,s$; performing each time, calculations precisely similar to the former ones: and as shown under the headings "Engine at o," &c. We have stated in a former Art, that a counter is most strained when the moving load extends from it to the *nearest* abut. Therefore, a counter $h\,p$ would be most strained with the engine on $v\,u$, and the train reaching to m; and $i\,o$, by the engine on $u\,s$, with a train to m.

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* With the engine at r, as in the Fig, the counters $i\,o$ and $h\,p$ are not required. Consider them removed, and it will be seen that the several sixths of the panel loads reach their respective abutments by the following routes:

Of $u\,s$, one-sixth to x, via $n\,h\,o\,e\,q\,c\,r\,a\,x$; five-sixths to m, via $n\,i\,m$. Of $v\,u$, two-sixths to x, via $o\,e\,q\,c\,r\,a\,x$; four-sixths to m, via $o\,h\,n\,i\,m$. Of $w\,v$, three-sixths to x, via $p\,e\,q\,c\,r\,a\,x$; three-sixths to m, via $p\,e\,o\,h\,n\,i\,m$. Of $y\,w$, four-sixths to x, via $q\,c\,r\,a\,x$; two-sixths to m, via $q\,e\,o\,h\,n\,i\,m$. Of the engine $z\,y$, five-sixths to x, via $r\,a\,x$; one-sixth to m, via $r\,c\,q\,e\,o\,h\,n\,i\,m$.

The one sixth of engine going to m, in passing from e to m, and the five-sixths going to x, plainly bring upon each member the *same* kind of strain as that brought upon it by the weight of truss and floor; *i. e.*, *tension* upon the ties and *compression* upon the struts; and thus *increase* their strains; but in passing from r to e, the one-sixth going to m *pulls* the *struts* $c\,r$ and $e\,q$, and *presses* the *tie* $c\,q$, thus *diminishing* the strains upon those members.

And, similarly, in any position of the engine and train, each panel length of moving load, except $w\,v$, divides at its point of support; one portion passing up the vertical tie (and *increasing* the *tension* upon it) on its way to one abutment; and the other portion passing up the inclined strut (and *diminishing* the *thrust* upon it) on its way to the other abutment.

Caution. When a web member is subjected to its *maximum* strain, no such relieving strain can thus come to it from the moving load. Thus $c\,q$ has its maximum strain (tension) when the engine is at q, with train reaching to m; and there is then no load at r to relieve $c\,q$ by sending a *compressive* strain through it.

The *final* or *resultant* strain (which may or may not be the *maximum* strain) upon any member, for any given position of the train, is the *difference* between the tension and compression upon it at that moment; and is of course a pull if the tension is greater than the compression, and *vice versa*.

Thus, with engine at r, as in the figure, the final strain on the tie $a\,r$ is =

Tension.		Tension.		Compression.		Tension.
29 tons from bridge and floor, as per p 565	+	65 tons of train going to x, as above	—	0	=	94 tons,

and this is also the *maximum* strain on $a\,r$. But on $c\,q$, the *final* strain (with engine still at r) is =

Tension.		Tension.		Compression.
15 tons from bridge and floor	+ (65 tons going to x	— Five-sixths of engine which do not pass through $c\,q$)	—	$\frac{1}{6}$ of engine going to m, which *presses* $c\ q$, and thus relieves its tension.

$$= \underset{\text{T}}{15} + \underset{\text{T}}{35} - \underset{\text{C}}{6} = \underset{\text{T}}{44}\text{ tons.}$$

But this is not the *maximum* strain on $c\,q$; for this takes place, as already remarked, with engine at q, and amounts to 45 tons.

ERRATA.

For Trautwine's Civil Engineer's Pocket Book.

9th to 15th Editions (1885 to 1891) inclusive.

To face lower part of page 567.

P. 565. Line 5 from foot: For upon x read upon m.

Pp. 567, 568. Under **Verticals:** Strain on Vertical $e\ p$ with engine at p.

If the counter $h\ p$ be made to act as a *tie*, the strain on the middle vertical $e\ p$ is equal to the 28.5 tons upward reaction of the abutment x, as stated; but otherwise it is equal to the 36 tons weight of engine at p.

In either case we have 36 tons engine at p, *minus* 28.5 tons upward reaction of x,=7.5* tons, tending to *shorten* the panel $e\ h\ o\ p$ along $e\ o$ and to *lengthen* it along $h\ p$.

Now if the ends of $h\ p$ be secured, it may be sufficiently tightened to act as a *tie*, relieving the main brace $e\ o$ of all compressive strain. We may then suppose $e\ o$ removed. In this case the 28.5 tons upward reaction of x would go via $x\ a\ r\ c\ q\ e\ p$ to p, causing a live-load strain of 28.5 tons in $e\ p$, the same as in $a\ r$ and $c\ q$. Half of the engine load (or 18 tons) would then go via $h\ p$ to m.

But the proper office of $h\ p$ is to act as a counter *strut* when the engine is at o, and it should not be depended upon to act as a *tie*. Hence the true maximum live-load strain in the middle vertical, $e\ p$, is the 36 tons engine at p; and the maximum *total* strain in $e\ p$ is:

live-load (entire weight of engine at p), -	36 tons.
dead-load (see page 565) - - -	8 "
Total,	44 tons.

instead of 36.5 tons as stated on page 568.

*=18 tons (half weight of engine) going to m, *minus* 10.5 tons (two sixths of o + one-sixth of n) going to x.

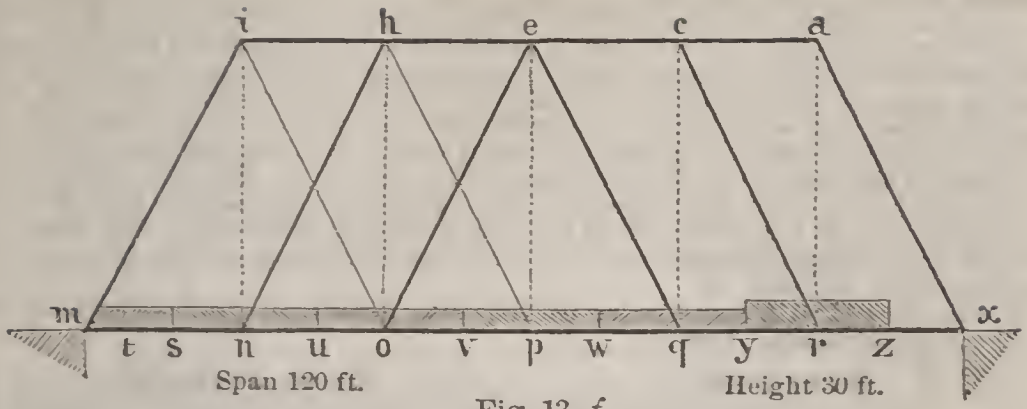


Fig. 13. *f*

MAIN BRACES.

With Engine at *p*,
there go to x.

Tons.
18 = 3-6 *engine.*
7 = 2-6 v u.
3.5 = 1-6 u s.

28.50 *go to* x.
1.2 *nat sec.*

34.2 *on* e q
from load only.

With Engine at *q*,
there go to x.

Tons.
24 = 4-6 *engine.*
10.5 = 3-6 w v.
7 = 2-6 v u.
3.5 = 1-6 u s.

45. *go to* x.
1.2 *nat sec.*

54. *on* c r
from load only.

With Engine at *r*,
there go to x.

Tons.
30 = 5-6 *engine.*
14 = 4-6 y w.
10.5 = 3-6 w v.
7 = 2-6 v u.
3.5 = 1-6 u s.

65. *go to* x.
1.2 *nat sec.*

78. *on* a x
from load only.

VERTICALS.

Vertical e p.

28.5 *go to* x.

Total on e p *from load only.*

Vertical c q.

45 *go to* x.

Total on c q *from load only.*

Vertical a r.

65 *go to* x.

Total on a r *from load only.*

COUNTERS.

Counter i o.

Engine at *n*.

6 = 1-6 *engine go to* x.
1.2 *nat sec.*

7.2 *on* i o
from load only.

Counter h p.

Engine at *o*.

12 = 2-6 *engine.*
3.5 = 1-6 u s.

15.5 *go to* x.
1.2 *nat sec.*

18.6 *on* h p
from load only.

Supposing the calculations to have been made as above, we have thereby found the several loads, say 65; 45; 28.5; 15.5; and 6; which go from train to *x*, from the

several points r, q, p, o, n, when the engine is at each of those points in succession; with a train reaching in each case to m. Each of these loads, in travelling, as it were, from its particular point, to the abut x, first ascends to the top of its own vert; and then descends along the adjacent oblique; producing in said oblique a strain as much greater than the load itself, as the length of the oblique is greater than its vert spread. Now, each of our obliques, as before stated, is 36 ft long, or 1.2 times as long as its vert spread 30 ft, (or the height of truss;) which 1.2 is therefore the nat sec of the angle which any of our obliques forms with a vert line. Therefore, to find the strain which each of our moving loads produces on the oblique next to it, on its way to the abut x, mult each said load by 1.2, as in the above calculations. We thus obtain for these strains, 78 tons on $a x$; 54 on $c r$; and 34.2 on $e q$; all which write near the tops of the obliques.

For the **verticals** and **main obliques;** *Total* max strain = truss strain + moving-load strain. Thus:

Verticals		**Main obliques**	
$e p$	$8 + 28.5 = 36.5$ tons	$e o$, $e q$, each	$8.4 + 34.2 = 42.6$ tons
$h o$, $c q$, each	$15 + 45 = 60$ "	$h n$, $c r$, "	$25.2 + 54 = 79.2$ "
$i u$, $a r$, "	$29 + 65 = 94$ "	$i m$, $a x$, "	$38.4 + 78 = 116.4$ "

For the counters, we go to the other side, $p m$, of the truss. Beginning with the panel $e h o p$, we examine its two diags, $h p$, $e o$. and see that with the engine at $v u$, and the cars reaching to m, there is produced in the counter $h p$ its maximum strain of 18.6 tons; which tends to cause in the truss the kind of derangement shown in Figs 9½. 9¾, and 9 *b*. Now, to resist this derangement, there is nothing but the 8.4 tons produced by the truss, floor, &c. upon the opposite diag $e o$ of the same panel. Since, therefore, the deranging effect of the load is greater than the preventive effect of the weight of the truss, there must be a counter at $h p$, able to bear a pres equal at least to the difference between the two, or to $18.6 - 8.4 = 10.2$ tons; or else the strut $e o$ must be made so that it can also act as a *tie*, capable of sustaining safely a *pull* of 10.2 tons. This last method relieves $h o$ and $e p$ from 10.2 tons each, but this relief comes when they have not their maximum load.

We now go to the next panel, $h i n o$. But here we find that the deranging effect of the load on the counter $i o$, with the engine at $u s$, is but 7.2 tons; while the preventive effect of the wt of the truss, exerted through the opposite diag $h n$, is 25.2 tons. Hence, the moving load can produce no derangement of the truss; and consequently the counter $i o$ may be omitted. On this same principle each panel on one side of the truss must be examined, when there are many of them; and the insertion or omission of counterbraces be determined upon. When we thus arrive at a panel at which no counter is reqd, none will be needed between it and the nearest end of the bridge. Similar counters will, of course, be needed on the other side of the truss. In practice it is better to retain the first apparently unnecessary counterbrace; counting from the center of the truss. Thus, although calculation shows $i o$ to be unnecessary, it is well to retain it. The lighter a bridge is, in proportion to its moving load, the greater will be the number of panels requiring counters.

The strains on the chords are greatest when the truss is loaded from end to end; and for Fig 13 *f*, as well as for Fig 13 *g*, may readily be calculated by Art 12; or found by a diagram with a max load. Or sufficiently close for most purposes, the hor strain along either chord at the center of the truss where the strain is greatest, will be equal to

$$\frac{\text{Total weight of truss and load} \times \text{span.}}{\text{8 times the depth or height of truss.}}$$

$$\text{which here is} = \frac{198 \times 120}{8 \times 30} = \frac{23760}{240} = 99 \text{ tons.}$$

Finally, each of all the strains in our double truss must be div by 2; for proportioning them among the two actual trusses, which we have all along supposed (for convenience) to be combined into one.

Art 12¾. The Warren or triangular truss. Fig. 13 *g*.

Here the dotted web-members which supplant the ties in Fig 13 *f*, are not vert; but inclined to the same extent as the struts or braces. Hence the hor stretch of each oblique will be but *half* the length of a panel, or 10 ft; or only half as great as in Fig 13 *f*. Consequently, the length of each oblique will be $\sqrt{10^2 + 30^2} = 31.6$ ft; and the nat sec of the brace angle will be $\frac{31.6}{30} = 1.05$; and the nat tang of the same angle will be $\frac{10}{30} = .333$. All the other data being the same as in the foregoing ex-

ample, prepare a diagram, and from it find the strains on the obliques from the wt of the *bridge* alone; one-half of the wt of the

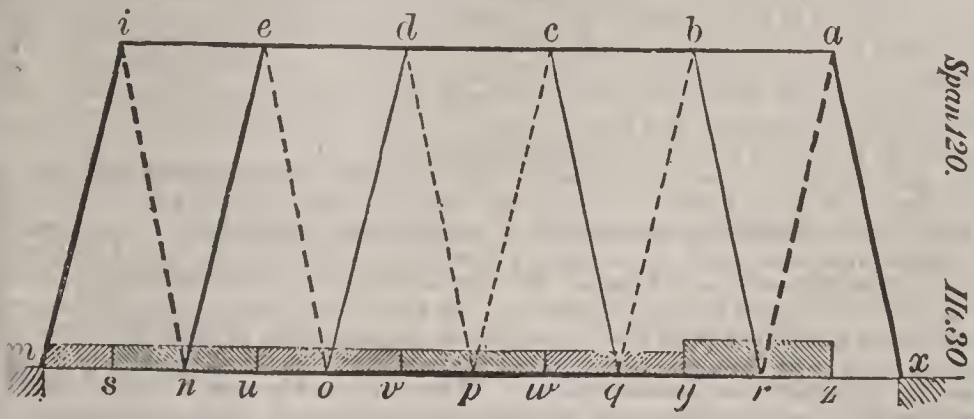


Fig. 13. g

truss only being supposed to be on the top chord, making, by our first process, p 559, 4.8 tons each at *e*, *d*, *c*, and *b*; 2.4 tons each at *i* and *a*; 8 tons each at *n*, *o*, *p*, *q*, and *r*; and 4 tons each at *m* and *x*. By our second and third processes, the strains on the ob members will be as follows:

On *p d* and *p c*, each..... 4.2 tons.	On *e n* and *b r*, each.....22.68 tons.
" *d o* and *c q*, " ..... 9.24 "	" *n i* and *r a*, " .....31.08 "
" *o e* and *q b*, " .....17.64 "	" *i m* and *a x*, " .....33.6 "

Write these strains at the *feet* of the respective members, as before.

Having now the correct strains arising from the weight of the truss and floor: next find, precisely as in the preceding example, how much of the moving load will go to *x* when the engine is at *r*, as in the fig, with the train reaching to *m*; and afterward with the engine at *q*, *p*, *o*, and *n*, in succession. These loads will of course be the same as in the former example, namely:

Engine at n.	*Engine at* o.	*Engine at* p.	*Engine at* q.	*Engine at* r.
6 *go to* x.	15.5 *go to* x.	28.5 *go to* x.	45 *go to* x.	65 *go to* x.

Multiplying each of these by the nat sec 1.05, we get the compressing strains which they produce on the end oblique strut *a x*; and on the other obliques that *are parallel to it*; namely:

On e o.	*On* d p.	*On* c q.	*On* b r.	*On* a x.
6.3	16.275	29.925	47.25	68.25

which write near the *tops* of said obliques. But they also produce precisely the same amount of strains, in the shape of tensions or pulls, on the respective dotted ties which carry them to the struts between *x* and *p*; and on the struts which carry them to the dotted ties between *m* and *p*. Write them all near the heads of said obliques also.*

Now, if on the half truss *x p*, we add together the strains written at the head and foot of each oblique separately, the sums will be the total or maximum strain (compressive on the struts, and tensile on the ties) which said obliques will be subjected to on the passage of the train. They will of course be the same on the other half of the truss when the train crosses in the opposite direction.

Finally, as to **counterbracing,** we go to the other half, *m p*, of the truss. Beginning with the oblique *d p*, we see that with the engine at *o*, and

---

* The remarks in foot-note p 566 apply also, in principle, to Fig 13g. Thus, with the engine at *r*, as in the fig, the five-sixths of engine, going to *x*, increase the strains on *r a* and *a x*. The one-sixth, going to *m*, diminishes the strains in *r b*, *b q*, *q c*, and *c p* (but these members have not now their maximum strains), and increases those in *p d*, *d o*, *o e*, *e n*, *n i*, and *i m*.

The final strains, also, are found in the same way as in Fig 13f. Thus, with engine at *q*, train reaching to *m*, we have final strain on *q b* (maximum)

TENSION. TENSION. COMPRESSION. TENSION.
= 17.64 from truss and floor + 45 going to $x \times 1.05 = 47.25$ — 0 = 64.89;

and on *q c* (not maximum)

COMPRESSION. COMPRESSION. TENSION.
= 9.24 from truss and floor + [(45 to x — $\frac{1}{6}$ engine) $\times$ 1.05] — ($\frac{5}{6}$ engine $\times$ 1.05)

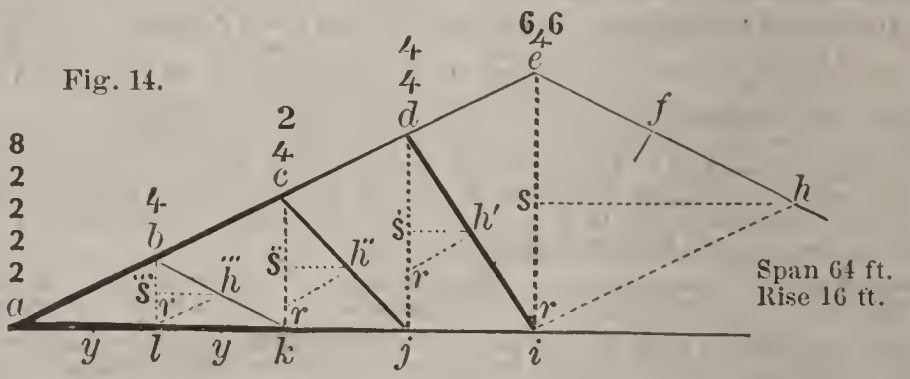
C C T C
= 9.24 + 22.05 — 12.60 = 18.69 tons.

the train reaching to m, the deranging compressive strain, 16.3 tons, of the moving load, is greater than the preservative tensile strain, 4.2 tons, of the truss and floor, acting on it at the same time. Therefore, $d\ p$, although a tie, the same as $c\ p$, is liable *at times* to be compressed rather than pulled. Therefore, it must be so arranged as to act also as a strut; at least so far as to bear a pressure equal to the difference between the 16.3 tons of pressure from the load and the 4.2 tons of tension from the weight of the truss and floor; or to 12.1 tons. The same end would be accomplished by inserting a counter *tie* reaching from o to c.

On the next oblique $o\ d$, which is a strut, the same as $c\ q$, the moving load on $v\ m$ produces a pull of 16.3; while the truss and floor produce on it a pres of only 9.2 at the same time. Therefore, although it is a strut, it is liable *at times* to be pulled rather than compressed; and consequently it must be made able to bear a pull also, equal at least to $16.3 - 9.2 = 7.1$ tons. The introduction of a counter *strut* reaching from e to p would answer the same purpose. On the tie $e\ o$, the deranging compressive load strain 6.3 is less than the preservative tensile strain 17.6 of the truss and floor acting upon it at the same time. Therefore, it may remain as a tie only; or in other words, it requires no counterbracing. When this is the case, no other oblique between it and the nearest abutment m needs counterbracing. It is almost needless to remark, that the half $x\ p$ of the truss requires the same as the half $m\ p$, when the engine crosses in the opposite direction.

The strains on the chords are found as directed on p 568.

Fig. 14.



Art. 13. In Fig 14, as in Figs 10 and 11, the truss is of 64 ft span and 16 ft rise, with an extraneous load of 32 tons; of which our **first process** gives, as before, 2 tons each at a and at the other end of the span, and 4 tons at each other panel point b, c, d, e, f etc. The wt of the truss itself is neglected, as before.

Second and third processes.* Each panel load divides at its point of support, as explained more fully in Rem, p 573, one portion, $r\ s'''$ etc, going directly down the rafter to the nearest abut; the other, $b\ s'''$ etc, by way of the web members to the peak e of the truss, and thence via the rafters to the abuts.

Thus, beginning at the panel load b, nearest the abut, lay off by scale the vert $b\ r$ = that panel load, 4 tons. Draw $r\ h'''$ parallel to the rafter, and $h'''\ s'''$ horizontal or parallel to the tie beam. Measure $s'''\ r$, which represents the portion (2 tons) of b going directly to a via the rafter, and write its amount (2) at a. Also measure $b\ s'''$, the portion (2 tons) of b going to e via $b\ k\ c\ j\ d\ i\ e$, and write its amount (2) at c, over the original panel-load (4) of c; thus making the final load at $c = 6$ tons. Make $c\ r$ = this load, draw $r\ h''$ parallel to the rafter, and $h''\ s''$ hor. Then $s''\ r$ (= 2 tons) goes from c to a via the rafter; and $c\ s''$ (= 4 tons) goes to e via $c\ j\ d\ i\ e$. Write these amounts at a and d respectively. The final load at d thus becomes 8 tons. Therefore make $d\ r = 8$ tons; draw $r\ h'$ parallel to the rafter, and $h'\ s'$ hor. Then $s'\ r$ (2 tons) goes from d to a via the rafter; and $d\ s'$ (6 tons) to e via $d\ i\ e$. Write these amounts

---

* When wishing to know merely the *amounts* of the several *final* panel loads in a truss like Fig 14, *uniformly loaded and divided into panels of equal length*; without caring to trace the process of their accumulation from the original panel loads; our **second process may be shortened, thus:**

$$\textit{original} \text{ panel load at abut } a = \frac{\frac{1}{2} \text{ the entire wt of truss and load}}{\text{entire number of points of support } a\ b\ c\ d\ e\ f \text{ etc } \textit{minus } 1}$$

Then, calling this *original* panel load at a "a", the *final* panel loads are as follows: at $b = 2\,a$; at $c = 3\,a$; at $d = 4\,a$; and so on with any number of points of support along a rafter, except at the apex e, where the final load is twice the final load at the nearest panel point (d in our Fig) or = ½ the wt of the entire truss and its load. The *final* load at a = half the weight of entire truss and load.

at a and e respectively. Now it is plain that the same process, on the other half of the truss, brings another 6 tons to e. Write this at e also. Thus we get for the final load at e, $4 + 6 + 6 = 16$ tons, or ½ the wt of the entire truss and its load.

The 16 tons thus concentrated at e divide there, half going down each rafter to an abut. Draw $e\,r$ vert, and = these 16 tons. Draw $r\,h$ parallel to $a\,e$, and $h\,s$ hor. Then $e\,s$ and $s\,r$ are the 2 halves (8 tons each) of $e\,i$, passing from e to the abuts via the rafters. Therefore write 8 at a.

It is by mere accident that the two vertical lines $b\,l$ and $e\,r$, representing the loads at b and e, happen just to extend to the tie $a\,i$ in our Fig.

We thus find, for Fig 14,

Strains along the verticals.

Along $b\,l$ = nothing, except weight of tie-bar from y to y.

" $c\,k$, $= b\,\overset{\cdots}{s} = 2$ tons.

" $d\,j = c\,\overset{\cdot\cdot}{s} = 4$.

Strains along the obliques.

Along $b\,k = b\,\overset{'''}{h} = 4.47$ tons.

" $c\,j = c\,\overset{''}{h} = 5.66$.

" $d\,i = d\,\overset{'}{h} = 7.21$.

$e\,i = 2\,d\,s = 2 \times 6 = 12$; for while each other vertical tie bears only the vert strain brought upon it by the oblique strut next nearer the abut; the *center* tie $e\,i$, of course bears those from 2 obliques; one on each side of it.

Strains along the horizontal tie-bar $a\,i$.

$$\text{At } i = s\,h = 16 \text{ tons; also} = \frac{\text{½ wt of truss and load} \times \text{¼ span}}{\text{height of truss.}}$$

From j to $i = s\,h + \overset{'}{s}\,\overset{'}{h} = 16 + 4 = 20$.

" k to $j = s\,h + \overset{'}{s}\,\overset{'}{h} + \overset{''}{s}\,\overset{''}{h} = 16 + 4 + 4 = 24$.

" a to $k = s\,h + \overset{'}{s}\,\overset{'}{h} + \overset{''}{s}\,\overset{''}{h} + \overset{'''}{s}\,\overset{'''}{h} = 16 + 4 + 4 + 4 = 28$.

Strains along the rafter $e\,a$.

From e to $d = h\,r = 17.9$ tons.

" d to $c = h\,r + \overset{'}{h}\,r = 17.9 + 4.47 = 22.4$.

" c to $b = h\,r + \overset{'}{h}\,r + \overset{''}{h}\,r = 17.9 + 4.47 + 4.47 = 26.8$.

" b to $a = h\,r + \overset{'}{h}\,r + \overset{''}{h}\,r = \overset{'''}{h}\,r = 17.9 + 4.47 + 4.47 + 4.47 = 31.3$.

It will be observed that the hor components $h\,s$, except the center one, have equal lengths; also those marked $h\,r$, parallel to the rafter; while the oblique ones have not.

For a span of 100 feet, rise 20 ft, or ⅕ of the span; trusses 10 feet apart from center to center; loaded on top only; the following dimensions will be amply sufficient for a covering of slate. Rafters and tie-beam, each 10″ × 12″ deep. The rafters may, if preferred, be reduced to 9 × 12 at top. The verts of round bar-iron of good quality, ½ inch, ¾ inch, 1 inch, and 1¾ inches diameter. The obliques or braces, 5 × 10, 6 × 10, 8 × 10; thus making the truss-thickness uniformly 10″. See Table 2, p. 579. **For shorter spans,** see NOTE, p 573.

Art. 14. In Fig 15, the process is the same as in Fig 14, except that the vert lines representing the strains at the points of support a, b, c, d, e, are to be drawn *upward* from l, k, j, i; and the strains $l\,s'''$, $k\,s''$, $j\,s'$, are to be carried forward to the next

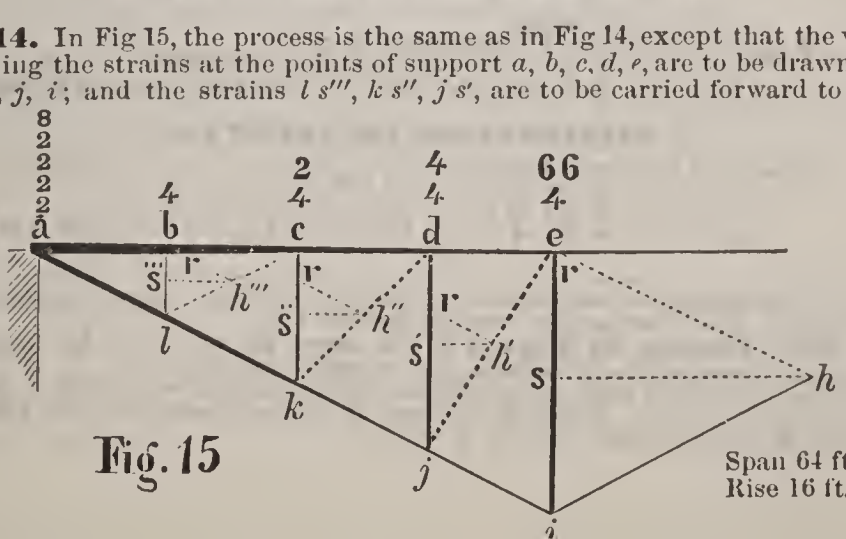


Fig. 15

Span 64 ft.
Rise 16 ft.

panel-load. Fig 15 is simply Fig 14 inverted, and those members which resisted *pressure* in Fig 14, resist *pull* in Fig 15, and *vice versa*. In other words, the struts become ties, and the ties, struts. All the strains are *equal* to those in Fig 14, except those on the *verts*, each of which is 4 tons *greater* than the corresponding one in Fig 14, because the original panel loads of 4 tons each, instead of being applied directly to the ends of the obliques, as in Fig 14, have first to pass through the vert struts *b l*, *c k*, *d j*, *e i*; the total loads on which will be, respectively, 4, 6, 8, and 16 tons.

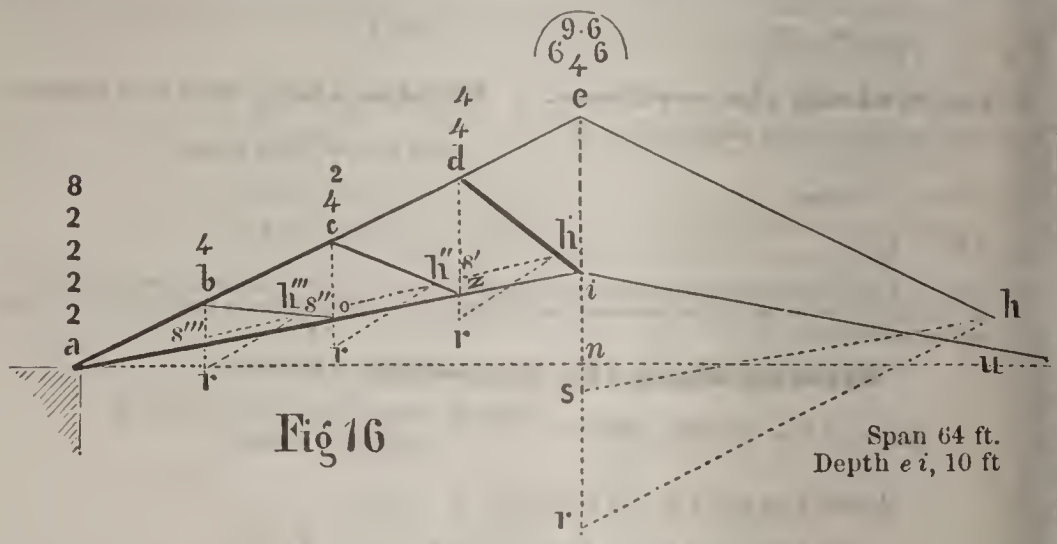


Fig 16

Span 64 ft.
Depth *e i*, 10 ft

Art. 15. In Fig 16, the process is the same as in Fig 14, except that the lines *h s*, &c, must be drawn and measured parallel to the inclined tie *a i*; instead of being hor. As in Fig 14 *b s'''* is carried forward to *c*; *c s''* to *d*; *d s'* to *e*. In this way, we find, as before, the vert strain of 6 + 4 + 6 = 16 tons at *e*. But we must now add to these 16 tons, another strain generated by the obliquity of the tie-rod *a i*. This strain is found by mult the one at *e*, (16 tons, or half the wt of the entire truss and its load,) by the vert dist *n i*, (6 ft,) which the center *i* of the tie-rod is raised above the horizontal *a u*, and div the prod by the dist *i e*, (10 ft.)

That is, $\frac{16 \times 6}{10} = 9.6$ tons; which also write down as in the Fig; making the total vert strain at *e* 25.6 tons, instead of the 16 tons of Fig 14.*

Now, make *e r* by scale, = 25.6 tons; draw *r h* parallel to the rafter *e a*, and meeting the other rafter; also draw *h s*, parallel to the raised tie-bar *i a*. Then the strains along the members of the truss will be as follows, taken from a Fig on a larger scale.

Strains along the verticals.

Along the one at *b* = nothing, except weight of tie-bar for the width of one panel (8 ft.)
Along *c o* = *b s'''* = 2 tons.
" *d z* = *c s''* = 4 "
" *e i* = 2 *d s'* + 9.6 = 21.6.

Strains along the obliques.

Along *b o* = *b h'''* = 6.43.
" *c z* = *c h''* = 6.96.
" *d i* = *d h'* = 8.04.

Strains along the raised tie-bar *a i*.

At *i* = *h s* = 26 tons.
From *z* to *i* = *h s* + *h' s'* = 26 + 6.5 = 32.5.
" *o* to *z* = *h s* + *h' s'* + *h'' s''* = 26 + 6.5 + 6.5 = 39.
" *a* to *o* = *h s* + *h' s'* + *h'' s''* + *h''' s'''* = 26 + 6.5 + 6.5 + 6.5 = 45.5.

Strains along the rafter *e a*.

From *d* to *e* = *h r* = 28.5 tons.
" *c* to *d* = *h r* + *h' r* = 28.5 + 7.13 = 35.63.
" *b* to *c* = *h r* + *h' r* + *h'' r* = 28.5 + 7.13 + 7.13 = 42.76.
" *a* to *b* = *h r* + *h' r* + *h'' r* + *h''' r* = 28.5 + 7.13 + 7.13 + 7.13 = 49.9.

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* It is probable that the tie rod is sometimes raised in this manner by persons ignorant of the fact that they thereby greatly increase the strains on the rafters, &c.

All the strains in Figs 14, 15, and 16 may also be found by precisely the same process as that for bowstring and crescent trusses, in Art 19.

The tension in the tie-rod which brings the 9.6 tons additional load to *e*, causes at the same time an equal *upward* pull at *a*. Hence the final pres of the truss upon each abut remains 8 tons (= half the weight of truss and load) as in Fig 14.

Rem. The reason for measuring only *parts* of the vert lines which, in Figs 14, 15, 16, represent the whole panel-loads, is that the rafter *a e*, Figs 14 and 16; or the tie *a i* of Fig 15, being *inclined*, also bear a part of each panel-load; and since that part does not go forward to the next point of support, but goes backward, along said inclined member, to the abut at *a*, it must be omitted in the *second process*. Thus, in Fig 17, if *b a* be an *inclined* rafter resting on an abut *a*; *b g* a strut; and *b r* a vert line representing the load sustained at *b* by *b a* and *b g*: if we complete the parallelogram *b m r n* of forces, then will *b m* give by scale the strain along the strut; and *b n* that along the rafter. The strain along the strut is made up or composed of the portion *b s* of vert force; and the hor force *s m*. The vert portion *b s* alone goes to the next point of support; while *s m* strains the tie *a g* hor. So also the strain *b n* is made up of the other portion (*b o* or *s r*) of the vert force *b r*; and of the hor force *o n*; which, when the strain *b n* reaches *a*, become again resolved into two; one of which, *b o*, presses vert upon the abut; or, in other words, transfers to the abut the portion *b o* of the load resting on *b*; while the portion *o n*, which is equal to *s m*, strains the tie *a g* hor.

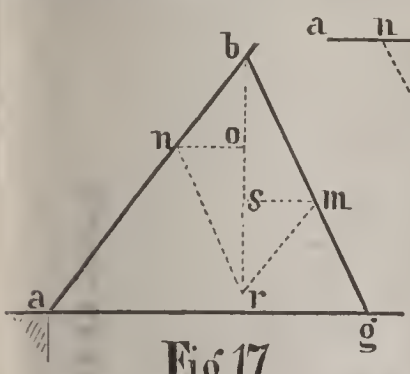

Fig 17

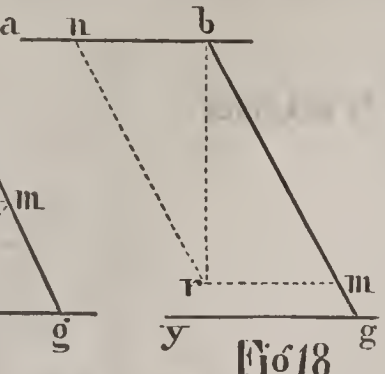

Fig 18

But in Fig 18, where *b r* also represents a load resting on *b*, and supported by a strut *b g*, and by a *hor* chord *b a*, if we complete the parallelogram *b m r n*, we have the strain *b m* along the strut, composed of *all* the vert force *b r*, and the hor force *r m*. The *whole* of *b r* is transferred to the next point of support; while *r m* and *b n* produce only *hor* strains along *b a* and *g y*.

Art. 16. The roof truss, Figs 19, 20, and 21, consists of two complete Fink trusses, *a n e* and *e k l*, Fig 21. It is supposed to be of the same span (64 ft) and ht

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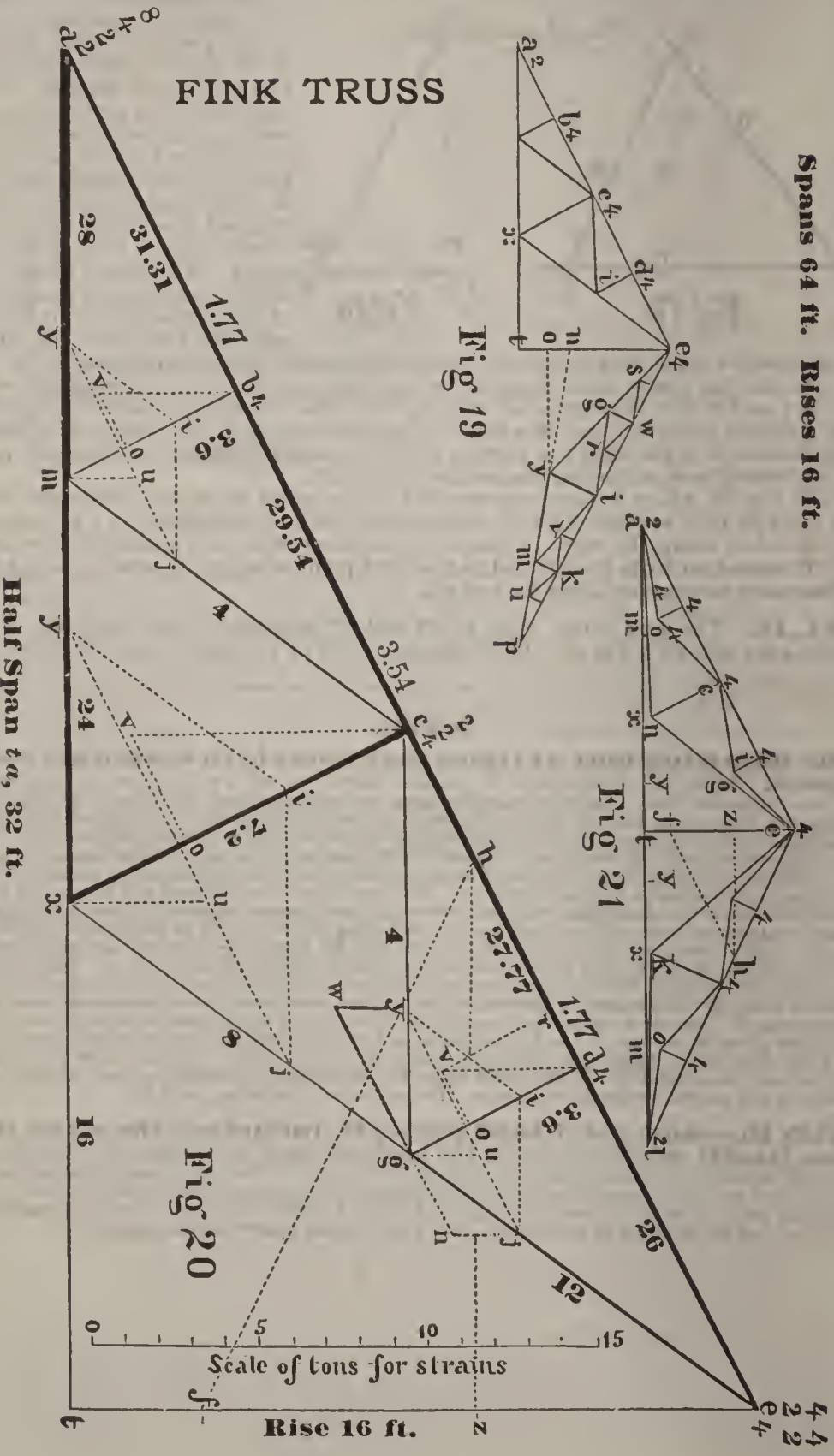
NOTE.

The following may at times save trouble in designing roof trusses. After the dimensions of all the members of a roof truss of any span have been calculated, then those of any *smaller* span similarly arranged, and having the same rise in proportion to its span, and the same extraneous load per sq ft; but with the trusses at the same dist apart as in the large span; may be found *safely;* and often near enough for practice, thus:

Find the area of cross section of each member of the large truss, in sq ins. Then make the area of cross section of each member of the small truss less than that of the corresponding member of the large truss in proportion as its span is smaller. The small truss thus obtained will be stronger than the large one under the same extraneous load per sq ft. For instance, suppose a truss of 175 ft span has been designed to carry safely a *total* load (*i e including* its own wt) of 40 lbs per sq ft of ground covered. Such a truss, by table, p 580, will weigh 8.05 lbs per sq ft of ground covered, leaving $40 - 8.05 = 31.95$ lbs per sq ft as its safe extraneous load of purlins, slate, snow, &c. Now a truss of one-fifth this span, or 35 ft, proportioned by the above rule, would sustain safely one-fifth the same *total* load; or (with trusses at same dist apart in both cases) would sustain the *same* total load (40 lbs) *per sq ft.* But the weight of the small truss itself, per sq ft of ground covered, is only one-fifth of that of the larger one, or 1.61 lbs, leaving $40 - 1.61 = 38.39$ lbs per sq ft as its safe *extraneous* load, while that of the larger one, as shown above, is only 31.95 lbs. Reductions will, however, rarely be made to as small as one-fifth; and where the short span is not less than half the long one, the method will answer very well in practice. For examples of reducing, see p 581.

With the same total load per sq ft (including the wt of the truss itself) and with trusses at the same dist apart in all cases, the strains on the several members of trusses proportioned by the above rule, will be in the same proportion as the spans; as will also the areas of cross section and weights per ft run, of each member, the wt of the truss alone, *per ft of span* and *per sq ft of ground covered*, and the total load on a truss, including its own wt; *but the total wts of the trusses alone* will be as the *squares* of the spans.

(16 ft) as Figs 10, 11, 14, 15, 16; and to have the same number (9) of points of sup port for the weight (32 tons,) supposed to be uniformly distributed along its top Consequently from **our first process** there will, as before, be 2 tons of panel load at each of the end supports; and 4 tons at each of the others. Write thes



down as in Fig 20.* The part truss $e\,x\,a$ may be regarded as being composed of three separate trusses $e\,x\,a$, $e\,g\,c$, $c\,m\,a$; as will appear more plainly from $e\,n\,a$, $e\,i\,c$, and $c\,o\,a$, in Fig 21. These may be called *first* and *second secondary trusses*. In Fig 19, the half $e\,y\,p$ exhibits a truss on the same principle, but having a greater number of points of support for the uniform wt. That half truss consists of first, second, and *third* secondaries, as shown by $e\,y\,p$, $e\,g\,i$, and $e\,s\,w$. However far this subdivision may be carried, if the struts occur at equal dists, each of the smallest divisions, as $c\,o\,a$, Fig 21, is to be regarded as a complete truss in itself, loaded at its center only. One-half, therefore, of its wt must rest upon each of its supports a and c.

Thus, with our **second process** at one of the shortest struts, as $b\,m$, Fig 20, 2 of its 4 tons go to c at the next longer strut, $c\,x$; and 2 of them to the end a of the rafter, as written on the Fig. Then, at the other of the shortest struts, $d\,g$, 2 tons go to c; and 2 to the end e of the rafter.

We will suppose, for the present, that the end e, and the corresponding end of the other half truss $e\,k\,l$, Fig 21, rest upon an *abutment* at e, as a rests upon *its* abut.

We thus have 4 tons at b; 4 at d; and 8 at c.

In like manner we now regard the first secondary truss $a\,w\,e$ (see $a\,n\,e$, Fig 21) as loaded at its center, c, only, with 8 tons as just explained. Of these 8 tons, 4 are of course supported by the abut at a, and 4 by the similar abut supposed, for the present, to be at e; both of these 4 tons are therefore set down as at a and e. When there are more points of support, as along the rafter $e\,p$, Fig 19, the process is precisely the same: we first adjust the strains of the four third secondaries, $e\,s\,w$, $w\,r\,i$, $i\,v\,k$, $k\,u\,p$; placing them at e, w, i, k, and p: then we transfer those thus accumulated at w and k, to e, i, and p; and finally transfer them from i to e and p, at the ends of the rafter. Now, returning to Fig 20, we see that in addition to the original panel-load of 4 tons at e, we have accumulated 6 tons of vert strain from the other panel-loads; and it is plain that the same process, performed along the other half truss $e\,k\,l$, Fig 21, would bring $2 + 4 = 6$ tons more to e, as written in Fig 20. Thus it appears that we have 16 tons in all at e;† resting upon our supposed abut there. But as this abut has no existence, the 16 tons really come upon the rafters themselves at e, and of course half of this wt (or 8 tons) is transferred by a rafter to each abut. Write down the 8 tons at a as in Fig 20. We thus have for the total vert pressure at a, $2 + 2 + 4 + 8 = 16$ tons = half the total wt of truss and load, and this is a further proof of the correctness of the operation.

Having thus finished our second process of finding the additional strains at the several points of support produced by the original panel-loads on their way to the abutments, we have only by our **third process,** to complete the drawing, so that we may measure by scale the strains along all the members of the truss. To do this, from the tops of the struts draw vert lines $b\,v$, $c\,v$, $d\,v$ to represent the *total* vert strains accumulated at those respective points; namely, 4 tons at b, 4 at d, 8 at c. Draw $v\,o$, $v\,o$, $v\,o$ parallel to the rafter $a\,e$. Then $b\,o$, $c\,o$, $d\,o$ will give the strains along the struts; 3.6 tons on $b\,m$ or $d\,g$; and 7.2 tons on $c\,x$; and $v\,o$, $v\,o$, $v\,o$ will give those produced directly by the final panel-loads from their points of support, upon the lower halves of the rafters of their respective secondary trusses. Thus the 4 tons at d exert a pres there of 1.77 tons (as shown by the upper $v\,o$) upon the lower half $d\,c$ of $e\,c$. The 4 tons at b exert an equal pres upon $b\,a$, and the 8 tons at c strain $c\,a$ 3.54 tons. These strains $v\,o$ of course become proportionally greater, and $b\,o$, $c\,o$, $d\,o$ proportionally less, as the rise of the truss increases in proportion to the span. They will be referred to further on.

Lay off $m\,i$, $x\,i$, $g\,i$ respectively equal to $b\,o$, $c\,o$, $d\,o$; draw $i\,j$, $i\,j$, $i\,j$; and $i\,y$, $i\,y$, $i\,y$, parallel to the ties; thus completing the parallelograms of forces $i\,j\,m\,y$, $i\,j\,x\,y$, $i\,j\,g\,y$. Draw the diags $y\,j$, $y\,j$, $y\,j$; and the vert lines $m\,u$, $x\,u$, $g\,u$. Lay off the vert dist $e\,f$ equal to the total vert strain (16 tons) at e; make $e\,z =$ to half of $e\,f$; draw $z\,h$ hor; and $h\,f$.

Now $m\,y$ and $m\,j$ give the strains (4 tons each) along the ties $m\,a$, $m\,c$, caused by the 4 tons at b; which strains extend from m to a and c.‡ In like manner $g\,y$ and

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* When merely wishing to ascertain the strains along the members of such a truss, without caring to trace their progress, we may omit part of the following; and, after having made a correct diagram of the truss, we may at once write down the vert strains at the points of support, thus: At e (the apex or peak of the truss) write one-*half* of the entire wt of a truss and its load, (for which, *per sq ft*, see Table 4, p 581) at the foot a of the rafter, one *half*; at the center strut c one *fourth*; at b and d, one *eighth* at each; and when there are four intermediate subdivisions of the same kind, as along the rafter $c\,p$, Fig 19, one *sixteenth* of said entire weight at each of such additional points, &c. Thus the total load at any longer strut will be twice as great as that at any next shorter one. Then begin at "Having thus finished our second process,"

† This is precisely half the wt (32 tons) of the entire truss and its load; and as this will always be the case in trusses on this principle, it proves our work to be correct thus far.

‡ When the main tie $a\,t$ is hor, as in Fig 20, these strains along the ties will be equal to those at the points of support, only where the height of the truss is equal to ¼ of its span; as in the case before us. When the height is less than ¼, the strains on the ties will be greater than those at the points of support; and vice versa.

$g\,j$ give the strains (4 tons each) extending from g to c and e. In Fig 21, the short ties, $o\,a$, $o\,c$, $i\,c$, $i\,e$, show this more distinctly.*

Next $x\,y$ and $x\,j$, Fig 20, give the strains (8 tons each) produced along $x\,a$ and $x\,e$ by the 8 tons at c. This also is shown more plainly in Fig 21, by the ties $n\,a$ and $n\,e$. The hor line $h\,z$ gives the strain (16 tons) produced along the entire hor tie $a\,l$, Fig 21, by the 16 tons at e. Fig 20 may be considered one-half of Fig 21.

We have for **the total strains on the ties** as follows:

Along c m and c g, strain $= m\,j$ or $g\,y = 4$ tons.
Along $x\,e$, from x to g, strain $= x\,j = 8$ tons.
Along $x\,e$, from g to e, strain $= x\,j + g\,j = 8 + 4 = 12$ tons.
Along $t\,a$, from t to x, strain $= h\,z = 16$ tons.
Along $t\,a$, from x to m, strain $= h\,z + x\,y = 16 + 8 = 24$ tons.
Along $t\,a$, from m to a, strain $= h\,z + x\,y + m\,y = 16 + 8 + 4 = 28$ tons.

The line $f\,h$ or $e\,h$, Figs 20 and 21, gives the longitudinal pres (17.9 tons) brought upon each rafter by the 16 tons vert load which our second process brought to e. *This* pres strains the *entire* rafter uniformly from end to end; but the several portions of the rafter sustain, *in addition*, pressures arising more directly from the panel loads. For instance, the 4 ton load at d, produces at d, as already explained, a pres $v\,o = 1.77$ tons along $d\,c$, and one, $d\,o = 3.6$ tons, along the strut $d\,g$; and this last exerts pulls, $y\,g$ and $g\,j$, $= 4$ tons each, along $g\,c$ and $g\,e$. Now if on $g\,j$ we draw the parallelogram $g\,n\,j\,u$, making $j\,n$ and $u\,g$ vert, and $n\,j$ and $g\,n$ parallel to the rafter, then $j\,n$ or $u\,g$ will give the vert load of 2 tons which travels to e from the 4 tons at d; while $j\,u$ or $g\,n$ will give the strain, $= 2.7$ tons, exerted along the second secondary rafter $e\,c$ by the tie $g\,e$. Similarly the parallelogram $y\,u\,w\,g$ gives the vert load $y\,w$ or $u\,g$, $= 2$ tons, which travels from d to c, and the strain $y\,u$ or $w\,g$, $= 4.47$ tons, exerted upon $c\,e$ at c by the tie $c\,g$. The 4 tons at d therefore produce a strain $y\,u = 4.47$ tons along $c\,d$, and one, $u\,j = 2.7$ tons along $d\,e$: because $u\,j = 2.7$ tons exerted at e, and $v\,o = 1.77$ tons exerted at d ($= 4.47$ tons in all) *both* press the *lower* part $d\,c$, and strain it against the equal pres of $y\,u$ at c; whereas this upward pres, $= 4.47$ tons, of $y\,u$, is *diminished* at d by the *downward* pres, 1.77 tons, of $v\,o$, leaving an upward pres of 2.7 tons to strain $d\,e$ against the equal downward pres $u\,j$ at e. In the same way $y\,u$ and $u\,j$ of the lower parallelogram show the strains (4.47 and 2.7 tons) brought upon $a\,b$ and $b\,c$ respectively by the 4 tons load at b; and $y\,u$, $u\,j$, of the middle parallelogram give the corresponding strains (8.94 and 5.4 tons) brought upon $a\,c$ and $c\,e$ respectively, by the 8 tons at c.

The *total* strain upon any portion of the rafter is found by adding together the uniform strain $e\,h$ or $f\,h$, common to all parts, and the strains peculiar to such portion, as shown by the lines $y\,u$ and $u\,j$. Taking the part $c\,d$ for instance; as the lower half of $c\,e$ it sustains $y\,u$ of the upper parallelogram, $= 4.47$ tons; as part of the upper half of $a\,e$ it sustains $u\,j$ of the middle parallelogram, $= 5.4$ tons; and as part of the entire rafter it sustains $e\,h = 17.9$ tons; or, in all, $4.47 + 5.4 + 17.9 = 27.77$ tons.

Thus we have, **for the total strains along a rafter,**

From e to d, strain $= u\,j$ of upper parallelogram $+ u\,j$ of middle parallelogram $+ e\,h = 2.7 + 5.4 + 17.9 = 26$ tons.
From d to c, strain $=$ upper $y\,u +$ middle $u\,j + e\,h = 4.47 + 5.4 + 17.9 = 27.77$ tons.
" c to b, strain $=$ lower $u\,j +$ middle $y\,u + e\,h = 2.7 + 8.94 + 17.9 = 29.54$ tons.
" b to a, strain $=$ lower $y\,u +$ middle $y\,u + e\,h = 4.47 + 8.94 + 17.9 = 31.31$ tons.

The center vert $e\,t$ may be omitted in short spans *where the tie bar is hor*, as $a\,l$ Fig 21; since it then sustains nothing but the wt of the half ($y\,y$) of the central spread $x\,x$ of the hor tie $a\,l$.

Art 16 A. If the main or primary tie is raised at its center, as $p\,n$, Fig 19, proceed as at Fig 16, and, after having found the vert strains at all the points of support, as before, add to that (16 tons) at e, an amount

$$= \frac{\text{Said 16 tons} \times \text{the vert ht } t\,n\text{, Fig 19, to which the tie } p\,n \text{ is raised above the hor } p\,t.}{\text{the remaining ht, } n\,e\text{, of the truss.}}$$

This additional amount is the strain on the cen vert rod $e\,n$, which is indispensable in such cases. Then, as in Art 15, lay off the vert $e\,f$, Fig 20, equal to the *total* vert strain at e, thus found; and after drawing $f\,h$ parallel to the rafter, draw

* In practice, the ties $a\,o$, $a\,n$, &c, Fig 21, of the secondaries, are not always made distinct from that ($a\,l$) of the primary truss $a\,e\,l$; but they are so represented in Fig 21, merely to show more plainly that the central portion $x\,x$ of the primary tie $a\,l$ needs only such dimensions as will enable it to sustain the thrust produced by the 16 ton strain at e: whereas, along its portions $x\,m$, $x\,m$ it must be stout enough to bear, *in addition*, the pull along the first secondary ties, $n\,a$, $k\,l$; while at its ends $m\,a$, $m\,l$ it must resist not only the two preceding forces, but also those along the second secondary ties $o\,a$, $o\,l$. Likewise it is plain that the portion $g\,e$ of the first secondary tie $n\,c$, must be stouter than the portion $n\,g$: because $g\,e$ has to bear also the pull along the second secondary tie $i\,e$. In Fig 20, those portions of the ties which are most strained are shown by stouter lines.

h z parallel to the inclined tie, instead of hor. *f h* gives the pres throughout the rafter, due to the *total* vert load at *e*; and *h z* gives the pull throughout the raised tie-rod, due to the same load. Both *f h* and *h z* are of course greater than the corresponding strains *e h*, *h z*, Fig 20. Like them, they are to be used in finding the *total* strains in the several parts of the rafter and tie-rod.

If we omit the *third* secondary trusses *e s w*, &c, Fig 19, the strains on the *struts*, *w g*, *i y*, *k m*, will be the same as those on the corresponding struts, *d g*, *c x*, *b m*, Fig 20; but the strains on the *rafter*, corresponding to *y u*, *u j*, Fig 20, and those on all the ties, corresponding to *m y*, *m j*, &c, Fig 20, although found by the same process as in Fig 20, will be *greater* than in that case; in addition to which the *uniform* strains, *f h*, *h z*, exerted throughout the *entire* rafter and tie-rod respectively by the final vert load at *e*, will also be greater, as explained above.

Art 16 B. If the main tie is raised only part way, as *p y*, Fig 19, and then continued hor, as *y o*; draw it as if it extended to *n*, as in Art 16 A, and use the same lits *t n*, *n e*, for finding the additional vert pres at *e*. Find *f h* and *h z* as in Art 16 A. On *p n* lay off by scale the pull *h z* on *p y* found as above; and on this as a diag draw a parallelogram with sides parallel respectively to *y e* and *y o*. The latter (hor) give the total strain on *y o*, and the former give an additional strain on *y e*, to be added to those found for each part of that member as directed in Art 16 A.

In this case, as in Fig 20, the cen vert *e o* sustains only the wt of half the hor stretch of the tie bar, and may be omitted in short spans.

Rem. 1. It is not necessary actually to draw all three of the parallelograms, as in Fig 20. The large or center one alone will suffice; for we need only div the several strains measured along the strut *c n*, Fig 21; and along the ties *n a*, *n e*, by 2, to get those along the struts 4 *o* and 4 *i*; and along the ties *a o*, *c o*, *c i*, *e i*. And these, in turn, div by 2, will give those along the smaller subdivisions shown between *e* and *p*, Fig 19, if there are such; and so on with any number of still smaller ones.

Art. 16C. Remarks on king and queen; and on Fink trusses, for roofs. The following comparison is founded upon total spans, or lengths of truss, of 154 ft. Rise 30.8 ft; or $\frac{1}{5}$ of the total span. Trusses 7 ft apart from center to center. Each rafter 83 ft long. Total load, including the truss itself. 40 lbs per sq ft of roof; or 20.8 tons to each truss. There are seventeen points of support in each truss; consequently a full panel-load (Art 11) is $\frac{20.8}{16} = 1.3$ tons. Trusses as shown, half of each, in Figs 47A and 48, p 606. The strain in tons (calculated as if all the weight of truss and load were on the rafters) is marked on each member. The assumed coefficient of safety for ties is 3. Iron is supposed to be used that will not break with a less pull than 20 tons per sq inch; the assumed safe allowable pull being therefore here taken at $\frac{20}{3} = 6\frac{2}{3}$ tons per sq inch. The safe pressure *along* the rafters is taken at $3\frac{1}{2}$ tons per sq inch. The struts are assumed to be wrought cylindrical tubes, with an outer diam equal to $\frac{1}{40}$ of their length; and of such thickness as will give them a metal area of 1 sq inch for each 2 tons of strain. The rafters are in the present case supposed to be 9-inch rolled Phœnix beams; $7\frac{1}{2}$ sq ins transverse area; weighing 25 lbs per foot rnn. The ends of all ties are supposed to be enlarged, or *upset*; so that the cutting of the screw-threads shall not diminish their effective area. The purlins are supposed to be at or near the "points of support," so as to produce no cross-strains on the rafters.

Table 1. Weight of the Fink truss, of which Fig 47A shows one-half.

Length 154 ft. Rise $\frac{1}{5}$ length. Trusses 7 ft apart. Load 40 lbs per sq ft of roof; including truss.

(Original.)

Name of part.	Number of parts.	Area of each part. sq ins.	Lbs. per foot run of each part.	Total weight of all the parts. Lbs.	Lbs.
Rafter	2	7.50	25.	4150	4150
Main tie, *n*	2	1.95	6.5	430	1446
Main tie, *o*	2	2.93	9.77	450	
Main tie, *p*	2	3.41	11.37	272	
Main tie, *r*	2	3.66	12.22	294	
Secondary ties, *u*	4	0.50	1.67	78	676
Secondary ties, *t*	4	0.73	2.44	118	
Secondary ties, *m*	2	0.98	3.27	150	
Secondary ties, *l*	2	1.47	4.89	118	
Secondary ties, *k*	2	1.71	5.70	136	
Secondary ties, *s*	8	0.25	0.83	76	
Struts, *j*	2	2.40	8.0	272	476
Struts, *w*	4	1.20	4.0	136	
Struts, *i*	8	0.60	2.0	68	
Center vertical *y* ..... say	1	.....	.....	40	40
Joint and splicing-pieces, nuts, &c, &c... say	.....	.....	.....	400	400
Shoes at ends of rafters, say	.....	.....	.....	400	400
Wt of purlins not included. Total wt of truss	.....	.....	.....	=7588	7588

With the same total load per sq ft, including the trusses, (with trusses 7 ft apart; rise $\frac{1}{5}$ span) the *area* of each part, its wt *per ft run*, and the strain upon it, are *as the spans;* but the *total* wts of the *trusses alone* are as the *squares* of the spans, Hence, it is easy to deduce from the table the areas reqd for smaller spans. The rafters for small spans are frequently made of round iron rods from $1\frac{1}{4}$ to $1\frac{3}{4}$ ins diam; or of ordinary flat bars. Tubes with the same area of metal, would be better. For trusses also of different spans, and rise of $\frac{1}{5}$ the span, 7 ft apart, in which the rafters and struts are of wood, with ties of iron, the strains may be deduced quite closely from those in Figs 47A and 48. They will, however, be somewhat greater, because wooden struts, not being hollow like our assumed iron ones, must be heavier than the latter to prevent bending. The weight of the load, however, is generally so much greater than that of the truss, that this consideration of the strut is not very material; so that a roof partly of wood may be assumed in practice to weigh, *together with its load*, but little more than an iron one; and the strains on the several parts will be nearly the same in both cases.

Table 2. Weight of the king and queen truss, of which Fig 48 shows one-half.

Length 154 ft. Rise $\frac{1}{5}$ length. Trusses 7 ft apart. Load 40 lbs per sq ft of roof; including truss.

(Original.)

Name of part.		Number of parts.	Area of each part. sq. ins.	Lbs per foot run of each part.	Total weight of all the parts. Lbs.	
Rafter		2	7.5	25	4150	4150.00
Main tie	H	2	2.2	7.33	146 70	
	G	2	2.44	8.14	162.80	
	F	2	2.68	8.94	178.80	
	E	2	2.92	9.74	194.80	1606.30
	D	2	3.16	10.54	210.80	
	C	2	3.41	11.34	226.80	
	B	2	3.65	12.14	242.80	
	A	2	3.65	12.14	242.80	
Verticals	*i*	2	.0	.0	.0	
	j	2	.1	.33	5.34	
	k	2	.2	.67	16.00	
	l	2	.3	1.00	32.00	298.66
	m	2	.4	1.33	53.33	
	n	2	.5	1.67	80.00	
	o	2	.6	2.00	112.00	
Center vertical	*p*	1	1.4	4.67	150.00	150.00
Struts	*q*	2	.88	2.92	64.20	
	r	2	1.05	3.50	91.00	
	s	2	1.28	4.25	136.00	
	t	2	1.55	5.17	196.37	1587.50
	u	2	1.82	6.08	267.63	
	v	2	2.12	7.08	368.30	
	w	2	2.40	8.00	464.00	
Joint and splicing-pieces, nuts, &c, &c .. say		.....	.....	.....	400.00	400.00
Shoes at ends of rafters..... say		.....	.....	.....	400 00	400.00
Total weight of truss =		.....	.....	.....	8592.46	8592.46

Wt of purlins not included.

Hence, the wt of the king and queen truss in this instance is equal to $\frac{8592.46}{7588}$ = 1.132 times (say $1\frac{1}{7}$ times) that of the equally strong Fink; or the Fink is about $\frac{1}{8}$ part lighter than the K and Q. Theoretically the diff would be less, because the rafters of the K and Q truss being so much less strained at top than at foot, may be diminished toward their upper ends, instead of being proportioned throughout with reference to the max strain at their feet. If the theoretical diminution toward the tops of the rafters, were made in both cases, the wts of the two forms of truss would be nearly equal. But in practice, on the score of inconvenience, this is rarely done in roofs of moderate span: say less than about 100 ft. No such diminution, or but very slight, would be admissible even theoretically, when the purlins are not placed at the points of support only. **With same total load per sq ft, including trusses themselves at same dist apart, the total wts of trusses are as the squares of their spans; but their wts per ft of span, as well as the cross areas, wts per ft run, and strains along**

individual members, are directly as the spans.

When the dist apart of the trusses is 7 ft from center to center; the rise $\frac{1}{5}$ of the span; assumed load, including the wt of the trusses themselves, 40 lbs per sq ft of roof covering; and the various parts proportioned for the several strains per sq inch assumed in Tables 1 and 2; the weight of a properly constructed Fink truss will be approximately as follows:

$$\textbf{Total wt in lbs of a Fink roof-truss} = \frac{\textit{square of span in ft}}{3.1};$$

and the wt in lbs **per ft of span** $= \frac{\textit{span in feet}}{3.1}$

A total K and Q truss, will be about $\frac{1}{7}$ part more; or $\frac{\textit{square of span in ft.}}{2.7}$

Or per foot of span, $= \frac{\textit{span in feet}}{2.7.}$

These rules give $\frac{23716}{3.1} = 7650$ lbs, for the foregoing Fink; and $\frac{23716}{2.7} = 8784$ lbs, for the K and Q truss. From these rules we have drawn up the following

Table 3. Approximate weights of roof-trusses of the Fink system. (Original.)

Rise $\frac{1}{5}$ span. Trusses 7 ft apart. Load 40 lbs per sq ft of roof, including truss.

Total Span.	Total wt of a Truss.	Wt per ft. of Span.	Wt per sq ft of ground covered.	Total Span.	Total wt of a Truss.	Wt per ft. of Span.	Wt per sq ft of ground covered.
Feet.	Lbs.	Lbs.	Lbs.	Feet.	Lbs.	Lbs.	Lbs.
20	129	6.46	.92	100	3228	32.3	4.60
25	202	8 08	1.15	105	3557	33.9	4.83
30	290	9.67	1.38	110	3904	35.5	5.06
35	396	11.3	1 61	115	4267	37.1	5.29
40	516	12.9	1.84	120	4640	38.7	5 52
45	654	14.5	2.07	125	5041	40.4	5.75
50	807	16.1	2.30	130	5452	42.0	5.98
55	976	17.8	2.33	135	5880	43.6	6.21
60	1160	19.4	2.76	140	6336	45.2	6.44
65	1363	21.0	2 99	145	6782	46.8	6.67
70	1584	22 6	3.22	150	7260	48.4	6.90
75	1815	24.2	3.45	155	7750	50.0	7.13
80	2064	25.8	3.68	160	8256	51.6	7.36
85	2331	27.5	3.91	165	8782	53.3	7.59
90	2616	29.1	4.14	170	9324	54.9	7.82
95	2912	30.7	4.37	175	9879	56.5	8.05

For king and queen trusses add $\frac{1}{7}$ part to the tabular wts; when the rafters are as usual of the same size throughout.

The wts in the 4th column will remain nearly the same, whatever may be the dist apart. For if this be increased say to 14 ft, each truss will sustain twice as many sq ft of roof; and must itself be at least twice as strong and heavy, in order to do so. We say "*at least*," because if the dist apart is increased, the wt of the *purlins* will generally increase more rapidly than said dist. Thus, if the dist be doubled, the purlins will not only be doubled in length, which alone would double their wt; but they must also be deeper. In practice, however, long purlins are usually prevented from becoming very heavy, by trussing them, as at 7, Figs 21½.

The cost, at shop, of trusses alone for iron roofs and bridges, generally varies between 2 and 2½ times the cost of ordinary "refined" bar iron. The putting up alone from ¼ to ½ the cost of the iron. With roof trusses 7 ft apart, iron purlins will weigh about 2 lbs per sq ft of ground covered by the roof. Therefore to any wt in the 4th col add 2 lbs. Add for covering with tin, slate, or corrugated iron. See pp 404, 418, 429.

Rem. 1. **As to the proper total weight, or load, per sq ft** *of roof, (including snow and wind,) that should* be assumed to be sustained by the trusses, engineers differ considerably. The French appear to consider 30 lbs as sufficient; while the English use 40. Since roofs are not subject to violent vibrations like bridges, they do not require so high a coefficient of safety; this should not, however, in our opinion, be taken at *less* than 3; and this we consider sufficient. The load is evidently influenced by the character of the roof-covering. Within ordinary limits, for spans not exceeding about 75 ft, and with trusses 7 ft apart, the total load per sq ft, including the truss itself, purlins, &c, complete, may be safely taken as follows:

Table 4.

Span 75 ft or less.

		Wind and Snow.*	Total.
Roof covered with corrugated iron, unboarded,†	8 lbs.	20 lbs.	28 lbs.
If plastered below the rafters,	18 "	20 "	38 "
" " " corrugated iron, on boards,	11 "	20 "	31 "
If plastered below the rafters,	21 "	20 "	41 "
" " " slate, unboarded, or on laths,	13 "	20 "	33 "
" " " " on boards, 1¼ ins thick.	16 "	20 "	35 "
" " " " if plastered below the rafters,	26 "	20 "	46 "
" " " shingles on laths.	10 "	20 "	30 "
If plastered below rafters or below tie beam.	20 "	20 "	40 "

For spans from 75 to 150 ft, it will suffice to add 4 lbs to each of these totals.

Example of use of foregoing tables. A Fink roof 60 feet span; rise $\frac{1}{5}$; trusses 14 ft apart; and to be covered with slate, on boards 1¼ inch thick. Here we see at once from Table 3 that at 7 ft apart, its wt would be about 1160 lbs; therefore, at 14 ft apart, it would be 2320 lbs. But our table is for 40 lbs per sq ft of roof: while, for slate on boards, 35 lbs, or ⅛ part less, is sufficient. Therefore, we may reduce the weight of the truss ⅛ part, making it only 2030 lbs.

Ex. 2. Roof as before, 60 span; *trusses only 7 ft apart.* Turn to Table 1, where the areas are given for a total length or span of 154 ft. But 60 ft is the $\frac{60}{154}$ = say the .4 part of 154 ft; therefore, the areas, and the wts *per foot run* of each member of the 60 ft span, will be .4 of those of the 154 ft one. Thus, the area of a rafter will be $7.5 \times .4 = 3$. sq ins; which corresponds with a rolled T iron of $3 \times 3\frac{1}{2}$ ins, and ½ inch average thickness. Its wt per foot run will be $25 \times .4 = 10$ lbs. The area of the part *n* of the main tie will be $1.95 \times .4 = .78$ sq inch, which we see at once from a table of circular areas, is equal to a round rod very nearly 1 inch diam. Its wt per ft run $= 6.5 \times .4 = 2.6$ lbs; and so with all the other members. But the *total* wts will be as the *squares* of the span. The square of 154 is 23716; and that of 60 is 3600. And $\frac{3600}{23716} = .152$; therefore, the total wts will be .152 of those in Table 1. Thus, the two rafters will weigh $4150 \times .152 = 631$ lbs. The main tie, $1446 \times .152 = 220$ lbs, &c. Lastly, if for 35 lbs per sq ft, reduce each area and wt ⅛ part.

Since the rafters are generally made of T or I iron, a pattern precisely adapted to the calculated strains, will not always be procurable; and in that case we may either take the nearest one in excess; or change the dist apart of the truss to suit the pattern on hand. Owing to the variety of modes of arranging the details of the junctions, &c, an exact coincidence between the calculated and the actual wts, is not to be expected; but we suspect that in properly proportioned roofs, the discrepancy will rarely be found to vary more than about 5 per ct from the results of our rules.

It might be supposed that with iron of a tensile quality considerably higher than our assumption of 20 tons per sq inch; as say of 25 to 30 tons, the truss might be made *much* lighter. But this is not the case; because the superiority would affect the ties only; inasmuch as the compressive strength of iron does not increase with its tensile strength; but to a certain extent rather the reverse. Now, by Table 1, it appears that the ties in a Fink roof-truss, constitute less than $\frac{3}{10}$ of its entire wt. Therefore, iron of even 30 tons, would reduce the weight of the truss less than ⅓ part of $\frac{3}{10}$ part; or $\frac{1}{10}$; and 25 ton iron, about $\frac{1}{20}$ part.

Short spans need not have as many subdivisions, or "points of support," as a large one; and this will lessen the number of parts of the truss; but inasmuch as the remaining parts will require to be proportionally stronger, this consideration will not materially affect the wts. While on this subject, we will remark that too few points of support are probably used at times; owing to either an undervaluation, or an ignorance of the effect of the transverse strains produced by the load on the parts of the rafter between said points. These parts must be regarded as so many separate *beams* supported at both ends; or rather, as *firmly fixed* at both ends, when the pieces composing a rafter are, as usual, strongly connected together; in which case the beam is about twice as strong as when merely supported. If the separate parts be trussed, like the purlin at 7, page 583, to neutralize this transverse action, it must be remembered that additional *compression* will be thereby produced lengthwise along the rafter. The best practice is, as far as practicable, to increase the number of points of support, so that the purlins may rest upon them alone, or near them; and thus relieve the rafters entirely, or in part, from transverse strain.

Rem. 2. **As to the effect produced on the weight of a truss, by changing its rise,** no short correct rule can be laid down. Although as a roof becomes flatter, its area becomes less, so that each truss has less total wt of roof-covering, snow, and wind, to sustain, still the strains on most of its members become greater; requiring greater wt of truss. To find this increase with accuracy, it is

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* See Snow and Wind, p 216, 221.

† The corrugated iron itself will weigh from 1½ to 2 lbs per sq ft on the roof. If not plastered underneath, the condensed moisture of the air, especially from crowded rooms, will fall from the iron into the rooms below. Mere boarding will not prevent this, even if tongued and grooved, unless the circulation of air against the under side of the iron is effectually cut off.

necessary to make a diagram, and perform all the calculations. The strains on a Fink rafter become more nearly uniform throughout its length, as the pitch of the roof becomes less; while, with a rise of ⅓ span, the strain at its foot is about $1\frac{4}{10}$ times that at its head. On the contrary, the strains on its struts remain *nearly the same in amount* for all ordinary rises.

In the king and queen truss the strains at the heads and feet of the rafters retain the same *proportions* to each other, at all rises; the strains on the verticals become *less* as the roof becomes flatter; while those on the obliques vary according to their several obliquities. Under these irregularities, which affect the K and Q, much more than the Fink, we can do nothing more than say that when it is merely wished at the moment to form a rough idea of the effect of changing the rise, we may assume the weight of a *Fink* truss to increase about in the *same proportion* as we diminish the rise; or to diminish as we increase the rise. Thus, if we increase the rise of the roof in Table I, one-fourth part, so as to make it equal to .25 or ¼ of the span, instead of .2 or $\frac{1}{5}$ of the span, we may diminish its wt ¼ part; making it about 6000 lbs, instead of 8000. Or if we reduce the rise from $\frac{1}{5}$ to $\frac{1}{10}$, making it only half as great, we shall double its weight, making it 16000 lbs; as rude approximations.

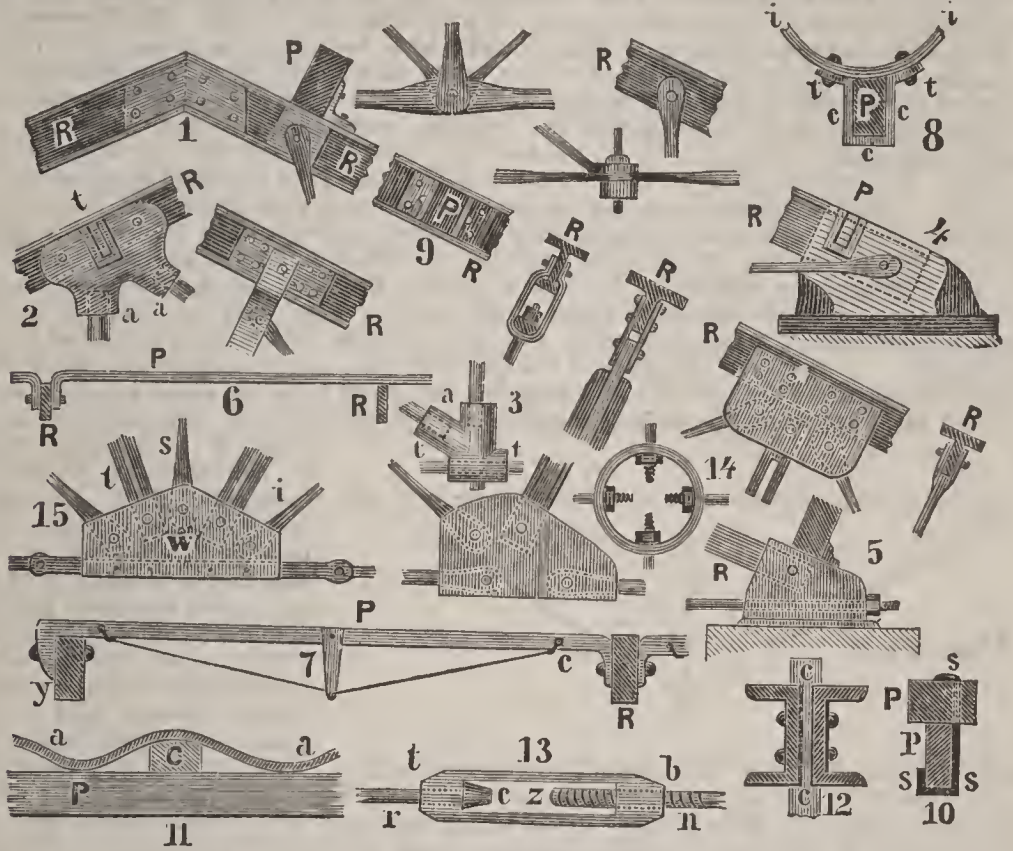
Figs 21½ show a few of the many forms of the details of iron roofs. Every maker has his own modifications of them. Most of the figs explain themselves. They will serve as hints.

R and P stand for rafter and purlin. In small roofs, with the trusses only 3 or 4 ft apart, the purlins may, as at 6, be simple ½ inch or ⅝ round rods, about 9 ins apart; and the slates may rest immediately on them, being tied to them by iron wire. They may be bent down at their ends, and riveted to the rafters. As the dist between the trusses increases, these purlins may be made of flat iron, from 1 to 3 ins deep, and ⅛ inch thick; or of light T iron, &c; and may be trussed, as at 7, so as to admit of being placed several feet apart. When, however, they have to bear great weight, the mode at *c*, Fig 7, of confining their ends to the rafters, will be too weak. Sometimes they may be arranged as at *y*. Or the purlins, of either iron or wood, may rest on top of the rafters, as at 1 and 5; or their ends may rest in a kind of stirrup, as at *t*, Fig 2; and at P, Fig 4; in castings placed at the "points of support" of the truss; or they may be confined to the sides of the rafters by two angle-irons, as at P, Fig 9. **Purlins should, when practicable, be supported only at or near the "points of support"** of the truss: and as a general rule, it will be expedient to arrange the number of these points with reference to this particular. The rafters are then relieved from transverse strains; and may be proportioned with regard only to the compressive strain in the direction of their length. Too little attention is sometimes given to this point, and the transverse strain is overlooked, to the serious injury of the roof. It is well, however, to bear in mind that thin deep rafters are liable to yield by buckling *sideways;* and that this tendency is diminished by purlins well secured to them *between* the "points of support." Sometimes castings similar to 2, are used at the heads; and 3, at the feet, of the struts and vert ties; which last have their ends cut into right and left hand screws, for insertion into corresponding female screws cut in the castings. At 3, *t t* is the main tie passing loosely through the lower opening through the casting. Below it, is seen the head of a small set-screw, for tightening together the casting and the tie; to prevent the former from slipping out of place. There must be different patterns of these castings, to suit the obliquities of the several obliques; or, in small roofs, the parts *a a* may be made with hinges, for the same purpose.

At 4 and 5 are cast-iron shoes for supporting the ends of the trusses upon the walls. With the exception, perhaps, of these shoes, it is better that the details generally should be of wrought iron.

At 8 is a mode of confining thin metal roof-covering *i*, **to the purlins** P, by means of short (about an inch) U-shaped pieces (*c c t t* is one of them) of the same metal; to which *i* is riveted by an ⅛ inch rivet through each flange *t t*. This may be adopted with corrugated iron covering, which, by its strength, allows the purlins to be placed several feet apart. See Corrugated Iron. Flat sheets require boards beneath them.

At 10 is a mode of confining a wooden purlin P on top of an iron one *p*, by means of a crooked spike *s s s*; which, after being driven from below, is clinched or bent on top. Wooden purlins are sometimes thus required, for nailing slates or plain sheet metal. At 11, *c*, is a stick of timber inserted between an iron



Figs 21½

purlin P and the corrugated roof-covering *a a*. To such sticks plastering-laths may be nailed, when the roof is to be plastered beneath, to avoid condensed moisture. There is room for much ingenuity in all these details. Fig 12 is a rafter made of two channel-bars riveted together; with a web member *c c* between them. Two angle-bars are often thus riveted together for a rafter.

Fig 13 shows a **turnbuckle** or **arm swivel** *tb*, for shortening a tie-bar made in two lengths. If *tb* is made of a pipe or solid bar it is called a **double nut** or **pipe swivel,** and, at least for a part of its length, it is made square or hexagonal, so that it can be turned with a wrench. In either case a female screw is tapped in each end of the nut, right and left hand respectively; and corresponding screws are cut on the ends of the rods. When the swivel is revolved, the two ends of the rods are drawn nearer together. In the *arm* swivel, *one* rod-end may be left plain and round, as in fig, and furnished with a head *c*.

Fig 14 is a mode of tightening four lengths of tie-bars crossing each other, by means of a ring. The ends inside of the ring are cut into screws, and provided with tightening nuts, as in the fig. The rings are usually ¾ to 1¼ inch thick; 3 to 5 deep; and 7 to 10 diam.

Horizontal Fink Truss, uniformly loaded.

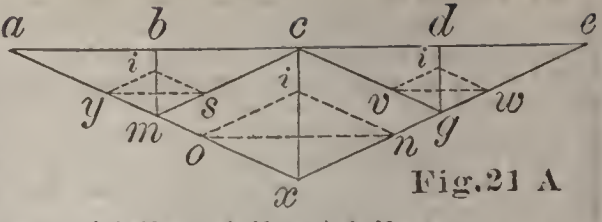


Fig. 21 A

Strains on posts = *final* panel loads: at $c = x\,i =$ half the total wt of truss and load; at $b = m\,i$, and at $d = g\,i$, each = half $x\,i$. **Strains on ties:** on $m\,c$ and $g\,c$, each $= m\,s = g\,v$; on $x\,m$ and $x\,g$, each $= x\,o = x\,n$; on $m\,a$ and $g\,e$, each $= x\,o + m\,y = x\,n + g\,w$. **Strain on chord** $a\,e$ (uniform throughout) = half $o\,n$ + half $y\,s$ = half $o\,n$ + half $v\,w$.

In the Fink truss, the effects of a moving load may be calculated as for a full uniform maximum load from end to end. Thus assuming at first, as in the preceding cases, that everything is borne by one truss only; then, when the load is upon the *top* chord of the truss, each vert post may in practice be regarded as upholding one-half of that portion of the entire wt of bridge and distributed load which is between the two extreme ends of the two obliques which uphold said post. Thus, in Fig 46, the half-way post $d\,c$, bears half of all between a and b. The post $m\,g$, half of all between a and d; the post $h\,o$, half of all between m and d. This is equivalent to saying that the half-way post bears half the entire wt of the bridge and load; each quarter-way post, one-quarter; each eighth-way post, one-eighth, &c, &c, of this same entire wt of bridge and load; and these constitute theoretically the strains on the several posts. But after having got thus far, it is necessary to examine whether some of the smaller ones may not have to be increased, for the following plain reason: Suppose we have assumed our max load to be a string of engines, weighing 1 ton per foot run; or, including the wt of the bridge itself, say 1.4 ton per ft; and suppose our posts to be as close together as 5 ft; then the least loaded posts would each bear $5 \times 1.4 = 7$ tons. But we know that from 16 to 20 tons may be concentrated within a length of 5 ft, on four drivers of an engine; and half of it will have to be supported by each post in succession as a train passes. When we thus find by trial, which posts will be more strained by an engine than by our assumed max per ft run of the whole truss, we must increase the load first found, correspondingly. In the Fink, and Bollman trusses, the verts are always struts or posts. Having fixed upon the load for each post, as $p\,o$, Fig 21 *h*, then for the strain which said load will produce upon each of the obliques, or ties, $p\,c$, $p\,h$, upholding said post, take any dist $p\,d$ on the post, to represent the load by scale; and draw $d\,w$, $d\,n$, parallel to the ties; then $p\,w$, $p\,n$, measured by the same scale, will respectively give the strains on each; whether they be equally inclined as usual, or not. The two hor lines $n\,a$, $w\,a$, by the same scale, give the two hor forces which the load at the top o of the post, acting through the ties, produces upon the chord at c and h; which two equal and opposing *forces* produce along the intermediate stretch $c\,h$ of chord, a *strain* equal to one of them. In other words, either $n\,a$, or $w\,a$, gives the hor strain produced along $c\,h$, by the load at o only. See "Strain," Art 2, of Force in Rigid Bodies.

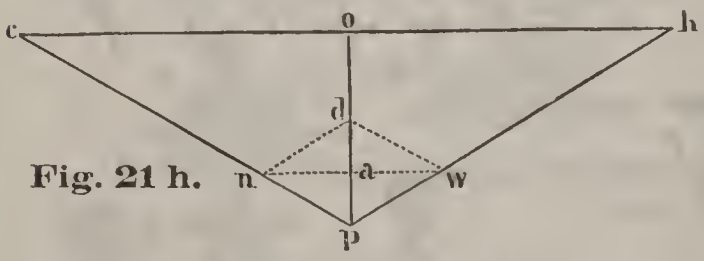
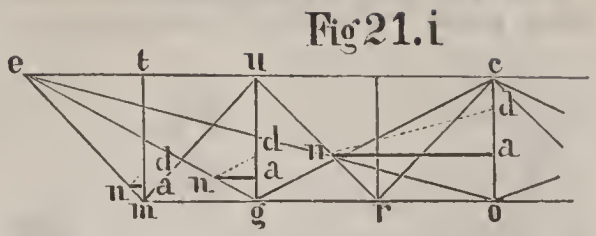


Fig. 21 h.

Strain on the chord. This, from a uniformly distributed load, is the same throughout the entire length of a Fink chord. To find it, observe which obliques, (as $m\,e$, $g\,e$, $o\,e$, Fig 21 *i*,) of one-half of the truss, *terminate at one end*, e, of the chord. Then, having previously found the loads on the posts, $c\,o$, $u\,g$, $t\,m$, which pertain to

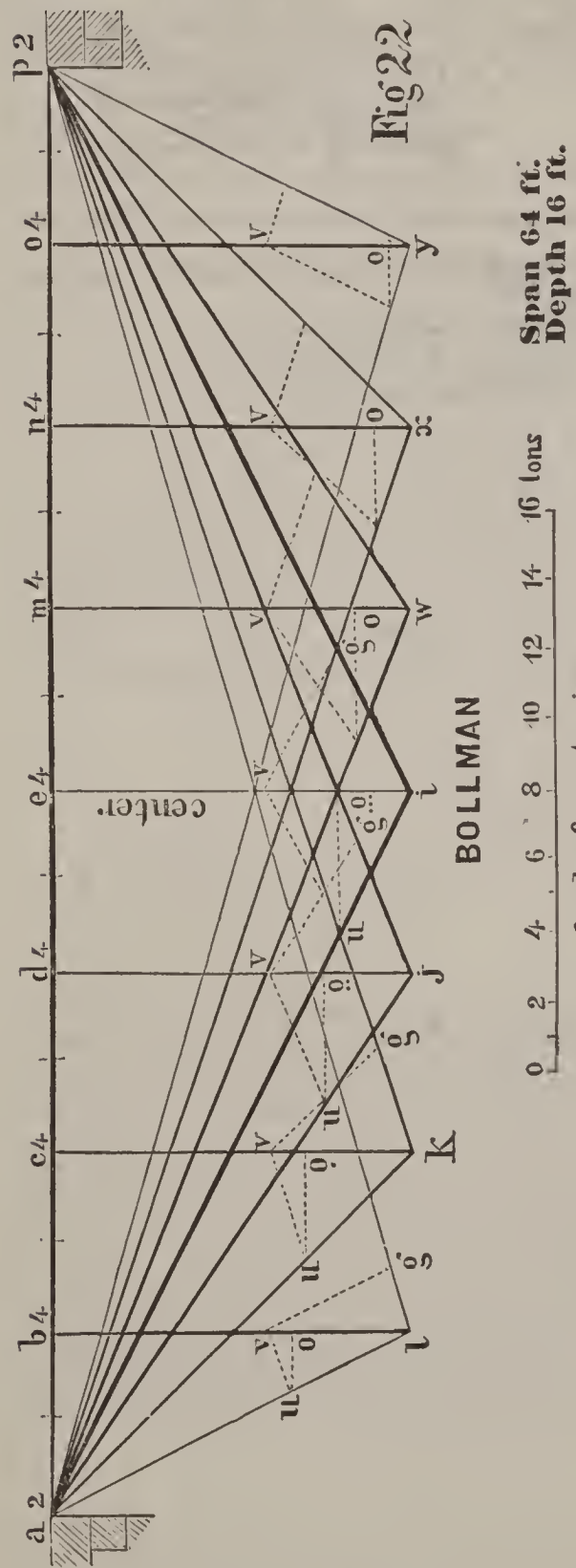


those obliques, ascertain by the process in Fig 21 *h*, the hor force *n a*, (in both figs,) which each of those loads produces on one oblique. Add together these forces *n a*, (there will be but three of them in Fig 21 *i*, as marked by the dark lines;) their sum will be the strain along the entire chord. The obliques *m u*, *u r*, *r c*, *g c*, do not terminate at *e*; and are, therefore, omitted in finding the chord strain. The process is the same whether the verts are all of the same length or not.

Or the hor chord-strain produced by each of the loads on the posts *c o*, *u g*, *t m*, may be calculated thus, and added together.

$$\textit{Horizontal strain} = \frac{\textit{entire load on post} \times \textit{hor dist from post to end e of chord}}{\textit{twice the length of the post.}}$$

Art. 17. Fig 22 represents a suspension truss on the Bollman plan;* the whole weight supposed to be along the top *a p*.



In this, the strain from each panel-load, as for instance that at *d*, passes down to the foot of its supporting post *d j*; and from there is transferred to the two ends *a* and *p* of the chord, by means of two ties, as *j a*, *j p*, upon which the post stands. In this manner the vert strain from each panel-load is separately sustained; and transferred directly as a hor strain to the ends of the chords, by its own post and pair of ties; without producing, as in the foregoing cases, an additional vert strain at the points of support of the other panel-loads. So **omit our 2nd process**; and having divided the uniform wt of the truss and its load, among the several points of support *a*, *b*, *c*, *d*, *e*, &c, as before, we proceed at once to draw the parallelograms of forces *v u l g*, *v u k g*, &c, for measuring the strains. To do this we have only to set up the equal vert dists *l v*, *k v*, *j v*, &c, each to represent by scale the 4 ton panel-loads on top of the respective posts; then complete each parallelogram by drawing *v u*, *v g* parallel to the two ties which support each post. Then the lines *l u*, *l g*; *k u*, *k g*, &c. give by scale the strains along the respective ties. The end *a* of the hor chord is pressed hor by the seven hor forces $u\,o$, $u\,o_{,}$, $u\,o_{,,}$, $u\,o_{,,,}$, &c., equal to $1.75 + 3 + 3.75 + 4 + 3.75 + 3 + 1.75 = 21$ tons; and the other end *p* is in like manner pressed by the seven corresponding forces not shown; and these two sets of equal opposing forces produce a strain equal to one of them; or to 21 tons, *uniform throughout the entire chord*. The tie *l a* carries to *a* so much of the *weight* of the 4 tons at *b* as is represented by *l o*, or 3.5 tons; *k a* carries to *a* a *weight* equal to $k\,o_{,}$, or 3 tons; *j a* carries $j\,o_{,,}$, = 2.5 tons; *i a* carries $i\,o_{,,,} = 2$ tons; *w a*, $w\,o = 1.5$; *x a*, $x\,o =$ 1; and *y a*, $y\,o = .5$ ton. All these amount to 14 tons; which, with the 2 tons of half panel-load at *a*, give 16 tons; or half the entire weight (32 tons) of the truss and its load. This is a proof that the strains have been drawn and measured correctly. The other half weight of truss and load is carried in the same way to the other end *p*, by means of the ties *y p*, *x p*, *l p*, &c. These wts cause the following strains:

* Invented by Mr. Wendel Bollman, C. E.

The strain $l\,u = 3.91$ tons.	The strain $l\,g = 1.82$ tons.
" $k\,u = 4.25$ "	" $k\,g = 3.17$ "
" $j\,u = 4.52$ "	" $j\,g = 4.05$ "
" $i\,u = 4.47$ "	" $i\,g = 4.47$ "

Each post or vert is of course strained to the amount of a full panel-load, when the whole wt is supposed to be on top of the truss.

In the Bollman, for a moving load, having first prepared the working diagram, determine the max weight that can come upon a post. This will be the same for each post. If the moving load is on top of the truss, this load on each post will consist of the greatest wt of engine that can stand upon one panel-length of truss; together with (approximately enough for practice) one panel-length of floor: and the half of a panel-length of truss. If the load is at the bottom of the truss, the posts bear no part of either the moving load, or of the floor; but each of them will be strained to the amount of the wt of half a panel of truss.

The loads on the posts may then be written upon the diagram.

The obliques or ties, however, when the load is at the bottom, bear (as in the Fink) the same amount of strain from the moving load and floor, as when it is on top. Therefore, when it is at the bottom, each pair of ties sustains not only the load resting on the post which they uphold; but the wt of one panel-length of floor, and the max panel-weight of engine. In other words, *whether the load be on top, or at bottom*, the two ties at the foot of each post, sustain a wt equal to a full panel-length of truss and floor; together with the max panel-wt of engine. Having added these wts together, lay off their sum by scale at each post, as shown at $l\,v$. $k\,v$, $j\,v$, $i\,v$. Fig 22; complete $l\,g\,v\,u$, $k\,g\,v\,u$, &c; and measure the strains $l\,u$, $l\,g$, $k\,u$, $k\,g$, &c, along the ties.

The strains on any pair of ties, may also be calculated thus; having the load they sustain.

$$\textit{Strain on short tie} = \frac{\textit{load} \times \textit{hor dist from post to farthest end of chord}}{\textit{total length of truss}} \times \frac{\textit{length of short tie}}{\textit{length of post.}}$$

$$\textit{Strain on long tie} = \frac{\textit{load} \times \textit{hor dist from post to nearest end of chord}}{\textit{total length of truss}} \times \frac{\textit{length of long tie}}{\textit{length of post.}}$$

The hor strain on the chord will be uniform throughout, as in the Fink truss; and will depend upon the max uniform load that can cover the whole bridge; and not, as in the case of the ties, upon the greatest load which each pair of ties may have to sustain *in succession;* unless we assume our max uniform load to be a string of engines which may bring a max panel-wt of engine upon *every pair of ties at once.* In that case we have only to measure upon one-half of our working diagram, the several hor lines corresponding to $u\,o$, &c, in Fig 22; and their sum will be the reqd hor strain on the chord. But if we take our max uniform load on the whole truss, to be a string of *cars*, we must diminish the chord-strain thus found, in this manner: Add together a full panel-weight of truss, floor, and *cars;* then, as the full panel-wt of truss, floor, and *engine*, (which we before assumed as the straining load of each pair of ties,) is to the panel-wt of truss, floor, and cars, just found, so is the hor chord-strain before found, to the one reqd.

We will repeat, that chords must be strong enough to bear not only the hor pull or push to which they are exposed; but also to sustain safely, *as beams*, the transverse strains from the floor, and from the moving load, when these rest upon them

BOWSTRING, AND CRESCENT TRUSSES, UNIFORMLY LOADED.

Art. 18. Before attempting to find the strains on either a uniformly loaded bowstring or a crescent truss, Figs 23, 23 *b*, 23 c, by means of a diagram, the student should familiarize himself with the following remarks:

Rem. 1. **The basis of the entire process is** that at every point of support, beginning at an abut as the first one, we have acting one or more *known* forces, balanced or held in equilibrium by either one or two *unknown* ones; and the object at each point is first, by means of the parallelogram of forces, to find the resultant of the known ones; and second, by the same principle, to resolve this resultaut into two components in the directions of the unknown oues. **This is all that is required in either the bowstring or the crescent truss.***

Rem. 2. **While more than two unknown forces exist** at any point of support, their amounts cannot be found. **If one force is known, and two unknown,** the three balancing each other, draw a line by scale to represent the amount and direction of the known one; and, considering it as one side of a triangle, from its two euds draw lines parallel to the two unknown ones, to meet each other, thus completing the triangle. Theu these last two will by the scale give the two unknowu ones; because when three forces meeting at one point, balance each other, three liues representing them both iu amount and in direction, will form a triangle.†

If there are two or more known forces, first find the single resultant of them all and, taking it as one side of the triangle, find the other two sides (that is, the two unknown forces) as before. After a little practice the student will find it unnecessary to draw more than half the sides of the parallelogram of forces.

Rem. 3. **The bow is to be considered straight from apex to apex.** If actually curved it will be much weakened.

Rem. 4. At each point of support, or apex, consider every member that meets there, to be a force either **pushing towards** said point, if along **a strut;** or **pulling from it,** if along **a tie.** This is shown by the arrows in Figs 23, 23 *a*, &c.; thus in Fig 23, at *o*, the forces along the struts *c o*, *s o*, and *q o*, *push towards o; q o* also pushes towards *q;* while the force along the tie *r o pulls from o*, as also from *r*. **All loads** on the bow are forces pushing vert downwards.

Rem. 5. **If the known forces are not all alike** at any point, (that is, neither all pulls nor all pushes,) then while constructing the parallelograms of forces, one or more of the forces must be changed, and be regarded as acting at the opposite side of said point, but in the same direction as before; so as to make them all alike; otherwise the parallelogram will not give the correct resultant. For instance, in Fig 23 *a*, we have (as will be seen hereafter) three known forces acting at c, namely *e c* pushing towards c: a load (not shown) on the bow at c, pushing vert downwards towards c; and *p c* pulling from c. In this case we must, while drawing the sides of the parallelograms, either consider the pull *p c* to be changed to a push in the direction

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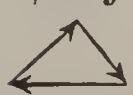
* **The same process applies equally to all our figs from 10 to 16 also;** whether uniformly loaded on one chord or on both; also to the inverted bowstring, and to Fig 23 *d*. In this last, as in the others, the lower member (*n n n*) is a tie which prevents the truss from spreadiug, and thus causes it to exert only vert pres against the abuts. This is a distinctive character of all so-called **truss girders,** including all our figs back to Fig 5. But when *n n n* is converted into an **arch for sustaining compression,** it ceases to be a tie, and the truss then exerts hor as well as vert force against the abuts; and becomes what is called a **braced arch.** The process requires a slight modification before it can be used for such, as shown in Art 20. Although strictly speaking the process does not apply to the Fink truss, Figs 19, 20, 21, because we there encounter *three* unkuown forces at any point where three web members as *c m*, *c x*, *c g*, Fig 20, meet at a rafter as at c, still as we can readily determine the strain on one of these, *c x*, by means of the eutire vert strain at such point, (which strain can be found in a Fink truss by a mere mental calculatiou) we thus reducc the unknown forces to two, and therefore may employ the process even for such trusses. The student would do well to test Figs 10 to 16 by this process.

† **Any one of the three forces** is then the **anti-resultant,** or balancer, or opposer, of the other two; and if **three arrow-heads** showing their directions be added to the sides of the triangle as in this Fig, they will, as it were, **chase each other** aromnd the triangle; that is, the head of any one of them will touch the butt end of the one next to it; or no two arrow-heads will meet. **But this is not so** when three sides of a triangle represent two forces and their **resultant,** or equivalent in effect; for the arrow-head of the resultant will then meet that of one of the other forces.



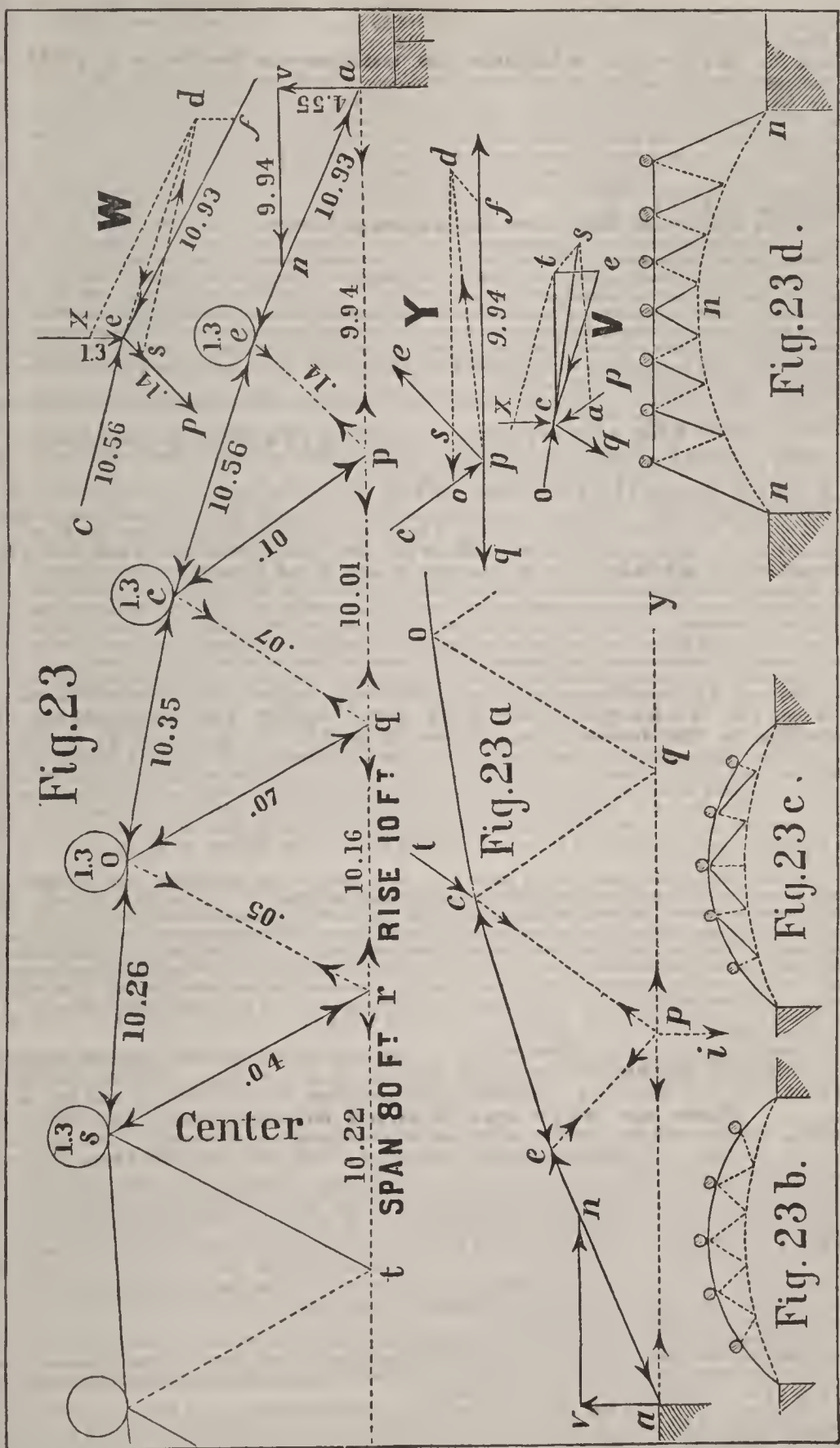


Fig.23

Fig.23 a

Fig.23 b.

Fig.23 c.

Fig.23 d.

from *t* towards c; or the other two to be similarly changed to pulls. The first of course is the easier. Accordiug as the known forces (after one or more are so changed, if necessary) are pulls or pushes, the diagonal or resultant will be the same.

Rem. 6. **To decide whether an unknown force is a pull or a push;** that is whether the member along which said force acts is a tie or a strut. Having found the resultant of the known forces, add to it an arrow-head to show its direction. Then having on this resultant completed the triangle by means of the two unknown forces, add arrow-heads to them also, placiug them so that the three arrows shall chase each other around the triangle. Theu imagine each arrow of the unknown forces to be placed one at a time without changing its direction, upon the line representing its respective web member. If, in this position, the arrow *pushes toward* the given apex, the member is a *strut*. If the arrow *pulls away from* the apex, the member is a *tie*.

When there is but one unknown force, and it is found to form a straight line with the resultant of the known ones, then its arrow must point in the opposite direction from that of the resultant. We may add that of the two unknown forces at any apex, one is always along either the bow or the string; and must plainly be a strut in the first case, and a tie in the last one. For the present we will call it the chord-force. The other unknown force will be along a web member. Now it can always be seen at once that the resultant and the chord-force together tend to displace the apex at which they act, by moving it either outwards or inwards, from or towards the truss; and if we simply consider that it is the duty of the web-member to counteract this displacing tendency, we shall have no trouble in deciding whether it must for that purpose pull or push at the apex, or in other words be a tie or a strut.

Rem. 7. After having found the resultant of the known forces, said known forces themselves must be considered as no longer existing.

Rem. 8. **To find the strains correctly requires great care** and attention. Plain as the foregoing remarks are, the student will in his first attempts probably commit many errors. A little practice however will rectify this.

A good metallic parallel ruler on rollers, and about 18 inches long, is almost indispensable. Paper ruled in squares facilitates the work. The lead-pencil must be kept sharp, and the lines should be drawn lightly. A scale of from ¼ to ½ of an inch to a foot or ton will generally be found convenient. **It will be difficult to find all the strains to within the nearest .1 or .2 of a ton,** or even more: because, as the work progresses errors which are inappreciable at the start may insensibly enlarge themselves. It will be seen from our table of bowstrings of 80 ft span, p 592, that the strains on some of the web members are less than .1 of a ton; so that the diagram may even with great care mislead us to the extent of 100 per cent, or more, in these small strains. Fortunately this is a matter of little importance, for these members are so small that a liberal allowance for errors involves but a trifling waste of material. In the larger strains errors of .1 or .2 of a ton are of no consequence. Never consider the work of a diagram complete, however, until after testing it by some of the proofs in Art 19.

Art. 19. Example. We will now apply the foregoing remarks to the bowstring truss, Fig 23. Its span is 80 ft; its rise 10 ft. The bow is divided into 8 equal parts; and the lower apices are horizontally half-way between the upper ones. The trusses are assumed to be 7 ft apart from center to center. The total wt of the truss and its load is supposed to be equally distributed along the bow only, and to amount to 10.4 tons, which corresponds to 40 lbs per square ft of roof covering. This gives 1.3 tons for a full panel load; and half as much, or .65 of a ton resting *directly* (or without passing along the web members) on each abut; and the finding of these, and figuring them on the diagram as shown, constitute our "first process," Art 11.*

This done, draw at the abut *a* a vert line *a v* equal by scale to half the entire wt of the truss and load, minus the part panel load which rests directly upon one abut; or to 5.2 — .65 = 4.55 tons. This line represents the vert upward reaction of the abut against that portion of the wt of the half truss and load that causes the strains which we are about to seek.† This reacting force, which is a known one, balances the two unknown forces, *a e* along the bow, and *a p* along the string. To find these we have only (Rem 2) to consider *a v* as one side of a triangle, and from its two ends to draw two meeting lines *a n* and *v n* parallel to, or in the same directions as, the unknown forces. Then will *v n* give by the same scale 9.94 tons strain along the hor string; and *a n* 10.93 tons, along the bow. The arrow on *a n pushes toward a*. Hence *a e* is a *strut* (Rem 6). But the arrow on *v n*, if placed on *a p*, *pulls away from a*. Heuce *a p* is a *tie*.‡

Now let us go to the apex *p*. There we find that we have one known force, (the 9.94 tons along *a p*) balancing **three** unknown ones, namely *p q*, *p c*, and *p e*. Hence (Rem 2) we cannot now find these last; therefore we leave them for the present, and try at the apex *e*. **Here we have two known forces,** namely the 10.93 tons pushing along *a e*, and the 1.3 tons of load which of course push vert downwards. Both these being pushes, neither of them requires to be reversed; and they balance **two** unknown forces, uamely *e c* along the bow; and *e p* along the oblique. Hence we have only to draw *f e* (see Fig W) to represent *a e*; and *x e* to represent the 1.3 ton load, and completing the parallelogram *f e x d*, draw its diagonal *d e*, which is the resultant of *f e* and *x e*; or it alone would balance the two unknown forces. By measuring *d e* by scale it becomes a known force. Therefore takiug it as one side of a triangle, from its two ends draw *d s* parallel to the bow *e c*, and *e s* parallel to the oblique

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* Remember that in a truss of any form it is only when the stretches along which a load is uniformly distributed are equal, that the panel loads are also equal; or that the portion which rests *directly* on an abut is just *half* a panel load.

† Each abut of course reacts vert upwards against the *entire* half wt of the truss and its load; but inasmuch as that portion of said wt which rests *directly* on an abut, does not reach the abut by way of the web members, and therefore has nothing to do with their strains, it is omitted from the pressure set up at the abut for ascertaining said strains.

‡ When, as in Fig 5½, p 552, the load is uniformly distributed along the rafter, and the latter is supported only at its ends, it will not do to assume that the load is concentrated at said ends. Such a rafter presses, at its foot, in the direction H *a*, and not in the direction of its length *o a*. Therefore, if *a x* in that Fig be drawn to represent the upward reaction of the abut, we must draw the triangle *a* H *x* (not *a o x*); and H *x* (not *o x*) gives the pull on the tie beam. But in such simple trusses the strains are more readily found by the method given in connection with Figs 5, 5½ and 6.

ep, thus completing the triangle des. Then ds gives by scale the strain 10.56 along ec; and es gives .14 of a ton along ep.*

Now going again to the apex p, we find that we have two known forces pa, pe, both pulls, balancing two unknown forces pq, pc. Therefore as at Fig Y. take pf and ps to represent the two known forces; complete the parallelogram $fpsd$; draw its diagonal pd; and taking it as one side of a triangle, draw do parallel to pq, and po parallel to pc. Then is do by scale 10.01 tons strain along pq; and po is .10 of a ton along pc. Going to c, **we have three known forces,** ce, cp, and the 1.3 ton panel load, all of them pushes, so that none of them need be reversed; and balancing co, cq, both unknown. In this case, as shown at Fig V, we must draw two parallelograms, beginning with any two of the known forces. Say we begin with cx representing the 1.3 ton load, and ce, representing the 10.56 tons. On these two draw the parallelogram $cetx$, and its diagonal ct. Then draw ca to represent the third known force cp of .10 of a ton; and on it and the diagonal ct draw the second parallelogram $ctsa$, and its diagonal cs. This last diagonal is the resultant or single force which would balance the two unknown forces co, cq; therefore take it as one side of a triangle, and from its two ends draw two meeting lines parallel to said unknown forces, and measure them by scale to obtain the amounts of those forces. On account of the smallness of our scale we have not shown these two lines. In practice, in order to prevent confusion, it is well to rub out the sides of the parallelogram after having found the first diagonal. In this manner proceed until the final strains on os, sr, and rt complete the whole.

If the entire uniform wt of truss and load is assumed to be on the string or lower chord, as in Fig 23 a, then all the web members become ties; but the process remains unchanged. Therefore first distribute the entire wt among the lower apices and abuts by our "first process." Then, as in Fig 23, draw the vert line av (= half the entire wt, minus what rests directly on one abut) and from it find the strains on ae and ap. Then going to e we have one known force ae balancing the two unknown ones ec and ep. After finding these go to p. Here we have three known forces, pe, pa, and the panel load; the first two of which are pulls, while the third (resting on top of the string) is a push vert downwards. Therefore we will reverse this last, and consider it as being a vert pull pi; and draw the first parallelogram on pi and pa. After finding the resultant of the three pulls, and by it the two forces pq, pc, we go to c. Here of the two known forces, ec, being a push; and pc a pull, we must reverse one of them, say pc; representing it by an arrow tc; and complete the parallelogram on tc and ce.

We have now had an instance of all the cases that occur, and have shown how to manage them.

Proofs of accuracy of the work. The resultant of the strains on those members (os and rs), Fig 23, of the half truss that meet at the center s, of the bow, and of *half* the panel load, s, at the center, should come out to be a hor line, and equal to the hor strain along the center stretch, rt, of the string. With all the care, however, that can be taken, it will be very difficult to make the coincidence *exact*. In some trusses the half truss will have *three* members meeting at the center of the bow; one of them (a center vert one) belonging partly to each half of the truss. Then only *half* the strain on this one, as well as half the center panel load, is to be used in the proof. All this applies to any form of truss however loaded.

Again, if all the uniform wt of the truss and its load is assumed to be on the *hor string*, **and if the string is divided into equal, or nearly equal, parts** by the web members, the hor strain at the center should be equal, or about equal, to

$$\frac{\text{Wt of half truss and load} \times .25 \text{ of the span}}{\text{Depth of truss.}}$$

But with very unequal divisions of the string, such as will rarely occur, this formula is not even roughly approx.

With all the wt uniformly on the bow, unequal divisions of the string have no effect on the center hor strain; and if the rise does not exceed about one tenth to one eighth of the span, and the bow is about evenly divided, the above formula will be nearly as approx as when the load is on the string. For greater rises, however, multiply the span by the following multipliers (instead of by the .25 of the above formula), when the load is on the bow; and the half bow about equally divided into at least two parts.

Rise, in Parts of the Span. (Original.)									
.1 or less	.15	.2	.25	.3	.333	.35	.4	.45	.5
Multipliers.									
.246	.243	.239	.233	.226	.222	.219	.211	.202	.192

Rem. The multipliers for intermediate rises may be taken in simple proportion. When multiplied by the span they give the hor dist from the abut to the center of gravity of one half the loaded bow, (as the .25 of the formula gives that of one half the loaded string,) assuming the load to be concentrated at the points of support on the abut and at the apices. **It is this center of gravity of half load so concentrated that must be used for finding the strains in the truss.** and not that of half load as actually evenly distributed; for these two centers of gravity under these two aspects may differ greatly from each other, not only in the evenly loaded bow, but in the string, if divided into very unequal parts by the web members. Even the *number* of divisions of the loaded bow makes some difference in this respect, but not so great but that the multipliers in the above table will be correct to within about three per ct at most in any case in which the entire bow has at least four nearly equal divisions.

* It is usual, and far more convenient, to draw all such lines at the apices of the diagram itself; but on account of the smallness of the scale of Figs 23, and 23 a, we have drawn them at W, Y, and V, to prevent confusion. Also our lines are not drawn to scale because some of the forces are too small to be appreciable with so small a scale.

If both the bow and the string are uniformly loaded, it is plain that the multiplier for any given rise must be somewhere between the .25 of the formula, and the decimal for that rise in our table; and this furnishes an easy method of finding, approx enough for practice, the hor dist from the abut to the cen of grav of a half truss thus loaded. Thus after allotting to the bow and string their respective proportions of the entire wt of the truss and load, find the hor dist for each of the two separately; and then combine them.

Table of approximate strains in tons on Bowstring trusses of 80 ft span. Trusses 7 ft apart from center to center. Load (including wt of trusses) 40 lbs per square ft of roof covering; all assumed to be uniformly distributed on the bow. Bow divided into 8 equal parts; like Fig 23; and straight from apex to apex. Lower apices half-way hor between the upper ones. Each column of the table commences near an abut, or end of truss. The first or end web member in the columns is a tie, the next one a strut, and so on alternately towards the center, as in Fig 23. Below the table is given the wt of each entire truss and its load.

Rise 20 ft, or ¼ Span.			Rise 13⅓ ft, or ⅙ Span.			Rise 10 ft, or ⅛ Span.			Rise 8 ft, or $\frac{1}{10}$ Span.		
Bow. Tons.	Tie. Tons.	Web. Tons.	Bow. Tons.	Tie. Tons.	Web. Tons.	Bow. Tons.	Tie. Tons.	Web. Tons.	Bow. Tons.	Tie. Tons.	Web. Tons.
7.01	4.85	0.38	8.80	7.50	0.22	10.9	9.94	0.14	13.3	12.5	0.07
6.02	5.18	0.35	8.30	7 72	0.18	10.6	10.01	0.10	13.0	12.6	0.05
5.56	5.34	0.25	8.03	7.78	0.15	10.4	10.16	0.07	12.8	12.7	0.07
5.34	5.38	0.25	7.90	7.88	0.15	10.3	10.22	0.07	12.7	12.7	0.07
		0.10			0.10			0.05			0.03
		0.10			0.10			0.04			0.03
Total wt = 11.6 tons.			Total wt = 10.75 tons.			Total wt = 10.375 tons.			Total wt = 10.25 tons.		

Art. 20. The braced arch, Fig 23 W. Find the center of gravity of

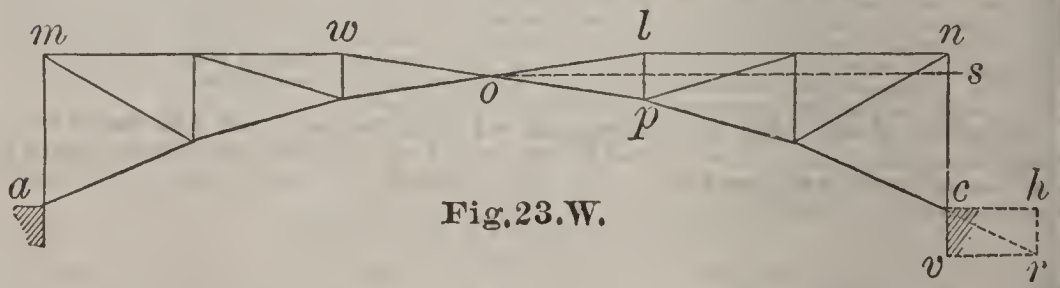


Fig. 23. W.

either half, as *o n c*, of the truss and its load. Then

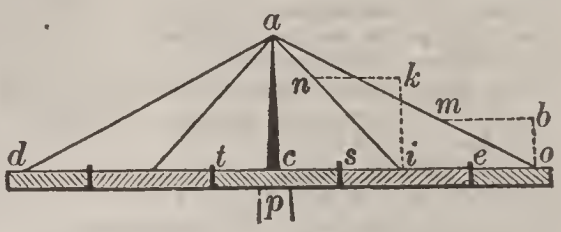
$$\frac{\text{Weight of said half truss and load, in tons} \times \text{Hor dist of said cen of grav from the nearest abut}}{\text{Vert dist } c\ s \text{ from abut } c \text{ to cen } o \text{ of truss}} = \text{Hor pres at the cen } o \text{ of the truss.}$$

Having thus found the hor strain *h c* at the center, and knowing the wt *v c* of the half truss and load minus the part panel-load that rests directly on the abut, draw them as shown, and find their resultant *r c*. Then use this resultant precisely as the vert line *a v*, Figs 23 and 23*a*, is used; namely, by drawing from its ends two lines for finding the first two unknown forces; then proceed precisely as in those figs.

Proof of accuracy of the work. If all has been done correctly, the last resultant near the center of the truss will be in a straight line with the last member; the strain along which it represents. Also, the resultant of those members of the half truss which meet at the center of the truss, and of *half* the center panel load, should come out a hor line equal to the calculated hor strain. But as in the bowstring, &c., it is almost impossible to secure a perfect agreement. It would be well to test the strains near the center, especially the very small ones, by the principle indicated by the following remark.

Rem. **It is not necessary to begin at an abut in order to work out the strains;** for after having disposed the load properly among the apices, and calculated the hor strain at the center of a truss, we may employ said strain, and one *half* of the panel load at the center, as two known forces acting against the half truss at the center; find their resultant; and resolve it along the two members of said half truss that meet there; and thus work on down to an abut. The line representing the hor strain must evidently be drawn as if *pushing against* the half truss whose strains are sought. **This remark applies also to bowstring trusses, and others,** as Figs 10 and 11, &c., in which we have two or more known forces acting against our half truss at the center; and not more than two unknown forces of our half truss meeting there also. In an actual bridge there would be a hor member *w l*, supporting the flooring, and a short vert one extending upward from *o*, and helping to support *w l*. But these do not form part of the truss proper.

Cantilevers. Suppose the half *o*, *n*, *c*, of the braced arch, Fig 23 W, to be a trussed cantilever, with *n* and c firmly built into, or attached to a wall; and loaded either along the top *o n*, or otherwise. Then to calculate the strains, begin with only the load concentrated at the outer end or apex *o*, as a given known force, and resolve said force along *o l* and *o p*; and so on to *n* and *c*, taking in on the way the loads at the other apices as before. **The upper chord will be in tension;** the lower in compression. **A revolving truss drawbridge,** when open, assimilates to two cantilevers joined back to back.



Art. 21. This fig represents an **opened swing-bridge** supported on rollers on the pier *p*, and by tie-rods *a o*, *a i*, &c. All the wt of *o e* is upheld by *a o*; that of *e s* by *a i*, &c; and that of *s t* directly by the rollers. Draw *o b* and *i k* vertical to represent the wts of *o e* and *e s*; and draw *b m*, *k n* horizontal. Then will *o m* and *i n* give the strains along *a o* and *a i*. Also *b m* will give a hor compressive strain reaching from *o* to *c*: and *k n* one reaching from *i* to *c*.

Art. 22. Fig 28, shows the general arrangement of a small wooden Howe bridge-truss; Fig 29, some of its details; and Fig 30, those of an iron truss. High trusses are sometimes made as in Fig 1. The top and bottom chords of the wooden one are each made up of three or more parallel timbers *c c c*, placed a small dist apart, to let the vert tie-rods *r r* pass between them. The main braces, *o o*, are in pairs or in threes. The pieces composing them, abut at top and bottom, against triangular **angle blocks,** *s*; which if of hard wood, are solid; and if of cast-iron, hollow; as shown at T, Figs 30; strengthened by inner ribs. These extend entirely across the three or more chord-pieces. Against their centers, abut also the counterbraces *e*. These are single pieces in small bridges; or in pairs, in large ones; and pass between the pieces which compose a main brace. Where the wooden braces and counters cross each other, they are bolted together. For wooden chords, the angle-blocks are cast,* as at T. The dotted lines show the strengthening ribs; and *x* serves to keep the block in place. The vert tie-rods *r r*, of iron, are in pairs, threes, or fours, &c, according to size of bridge; with a screw and nut at each end. The heads and feet of the braces and counters, butt square against the angle-blocks: and are kept in place only by the tightening of the screws of the vert ties. When the floor is below, as in Fig 28, the end posts *p d*; and the ends *g i* and *w b*, of the upper chord, *may* be omitted; also *i c* and *b y*; but it is seldom done.

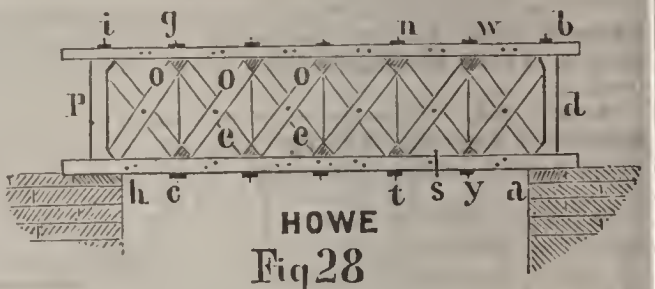


Fig 28

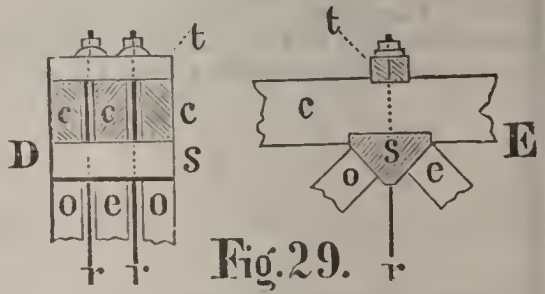
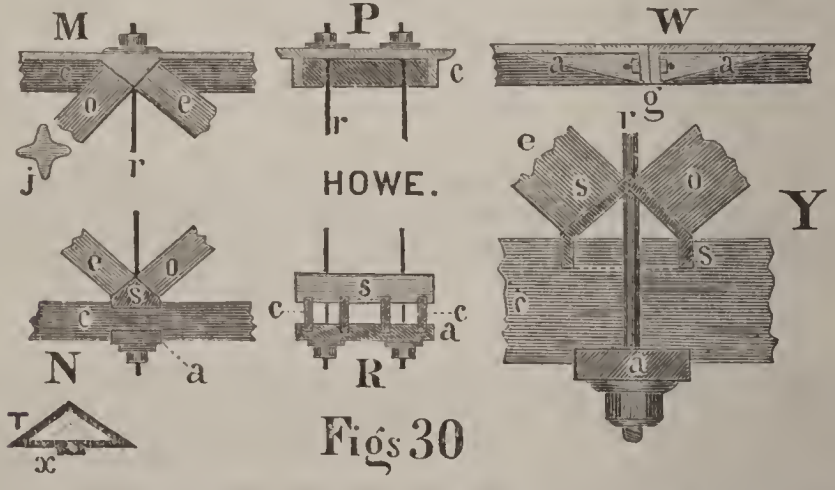


Fig. 29.



Figs 30

In Figs 30, of an iron Howe truss, the top chord P, M, and W, is cast in one piece transversely, as at P. Its separate lengths are connected together by flanges and bolts, somewhat as shown at W; where *a a* are cast longitudinal flanges for strengthening the transverse bolting-flanges *g*. Instead of separate angle-blocks at the upper chord, solid ones may be cast in the same piece with the chord itself, as shown at M. The lower chord usually consists, as in other iron bridges, of four or more flat bars of rolled iron, *c*, placed on edge: and some dist apart, as at R. On top of them rest the lower angle-blocks *s*, which have shallow channels below, for receiving the chord pieces; and thus securing them from lateral motion. A cast washer, *a*, below the chords, is provided with similar channels on top, for the same purpose. The braces

* In **large spans,** to prevent the pressure of the heads and feet of the obliques from crushing the chords, the angle-blocks are cast with deep projecting flanges under their bases; and which, passing between the pieces which compose a chord, extend to the opposite face of the chord. There the flanges bear upon broad washers at the ends of the vert rods. By this means the vert components of the strains along the obliques are transferred directly to the verts, without at all affecting the chords. Angle blocks of course have openings for the passage of the vert rods.

and counters, *o*, *e*, in moderate spans are usually cast in a star-shape, as at *j*.* The following table gives dimensions sufficient for a strong Howe bridge; although in wooden bridges it is customary to add arches when the span exceeds about 150 feet.

Dimensions for each of two trusses of a Howe bridge for a single-track railway. Timber not to be strained more than 800 lbs per sq inch; nor iron more than 5 tons per sq inch. Iron supposed to be of rather superior quality, requiring from 25 to 27 tons (60480 lbs) per sq inch to break it. The rods to be *upset* at their screw-ends. To each of the two sides of each lower chord is supposed to be added, and firmly connected, a piece at least half as thick as one of the chord-pieces; and as long as three panels; at the center of the span.

Clear Span. Feet.	Rise. Feet.	No. of Panels.	An upper Chord.		A lower Chord.		An End Brace.		A Center Brace.		A Counter.		End Rod.		Center Rod.	
			No. of Pieces.	Size.	No. of Pieces.	Size.	No. of Pieces.	Size.	No. of Pieces.	Size.	No. of Pieces.	Size.	No. of Rods.	Diam.	No. of Rods.	Diam.
				Ins.		Ins.		Ins.		Ins.		Ins.		Ins.		
25	6	8	3	5×6	3	5×12	2	5×8	2	5×6	1	5×6	2	1½	2	1
50	9	9	3	6×9	3	6×14	2	6×9	2	5×8	1	5×8	2	1⅞	2	1¼
75	12	10	3	6×12	3	6×14	2	6×11	2	6×8	1	6×8	2	2¼	2	1⅜
100	15	11	3	6×14	3	6×16	2	8×12	2	6×10	1	6×10	2	2½	2	1½
125	18	12	4	6×14	4	6×16	2	9×14	2	6×12	1	6×12	2	3	2	1⅞
150	21	13	4	8×14	4	8×18	3	8×14	3	6×10	2	6×10	3	2¾	3	1⅜
175	24	14	4	10×16	4	10×20	3	8×15	3	8×10	2	8×10	3	3	3	1½
200	27	15	4	12×16	4	12×20	3	9×16	3	8×14	2	8×14	3	3½	3	1⅞

The same dimensions will serve for a *double* road for common travel.

For bridges of iron, assuming the safe strain for iron to be 5 tons per sq inch, or 14 times as great as the 800 lbs assumed for wood; the areas of the cross-sections of the individual members will as a general rude approximation, be about one-fourteenth part as great as those of wooden ones. **Equally strong wooden, and iron bridges** of the same span, will not differ very materially in weight.

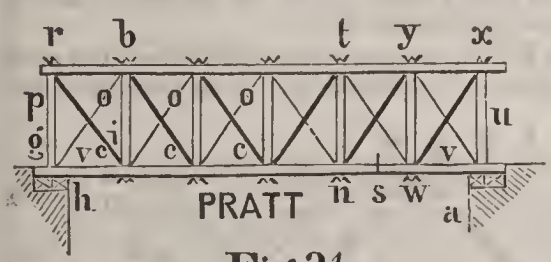
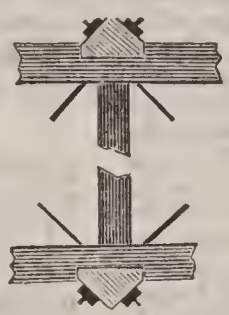
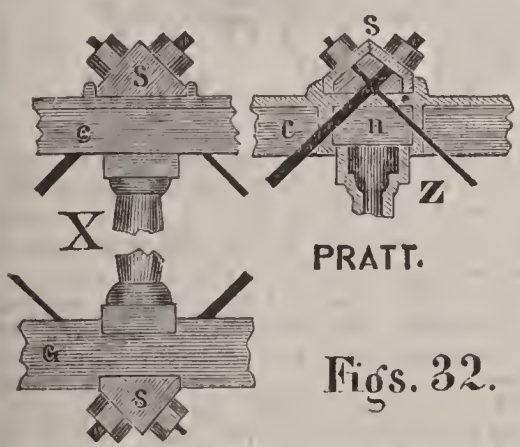


Fig 31





Figs. 32.

Art. 23. Fig. 31, shows in like manner, a **wooden Pratt truss:** and Fig 32, some details of a small iron one.* After the foregoing, they do not need much explanation. Since the angle-blocks support tension rods instead of struts, they are placed above the top chord, and below the bottom one. The main obliques are in pairs: and the smaller single counters pass between them, as in the Howe. In large bridges they are in threes, fours, &c. The vertical posts, which, when of iron, are hollow,*are retained in their positions both by the strains on the obliques, which terminate above and below their ends; and by being let into the chords. In large spans, the details generally vary more or less from those in the Figs. In Fig 31, *ccc* are the main braces; and *ooo* the counters.

* Cast iron is now rarely used in trusses, except for short blocks sustaining compression, such as angle-blocks etc. Rolled iron and rolled steel have taken its place.

When the roadway is below, as in Fig 31, the ends, $r\,b$, $y\,x$, of the upper chord; the end verticals p and u; and the two tension obliques in each end panel, may be omitted; and two diagonal *struts* from b and y must then be substituted, extending to the abutments, for upholding the upper chord, &c. **In the Pratt** the chords may be of the same dimensions as in the foregoing table for Howe's. The posts may have about $\frac{1}{5}$th less area than the main braces of the Howe. The main brace rods, and others, (of the same number as the main brace pieces of the Howe,) may have the following diams in ins; allowing the safe strain to be five tons per sq inch.

For each Truss of a Pratt Bridge.

Spans.							
25 Ft.	50 Ft.	75 Ft.	100 Ft.	125 Ft.	150 Ft.	175 Ft.	200 Ft.
End Main-brace Rods.							
Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
2 of 1⅜	2 of 2¼	2 of 2½	2 of 3	2 of 3	3 of 3	3 of 3¼	3 of 3¾
Center Main-brace Rods.							
Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
2 of 1¼	2 of 1⅜	2 of 1½	2 of 1⅝	2 of 1¾	3 of 1½	3 of 1⅜	3 of 1¾
Counter Rods at Center.							
Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
1 of 1⅝	1 of 1⅞	1 of 2½	1 of 2½	1 of 2½	2 of 1⅜	2 of 2	2 of 2¼

In Pratt's truss the directions of the main braces and counters are respectively the reverse of the Howe. Many of the remarks in the preceding Art, apply equally in this. Neither the Howe nor the Pratt possesses any special advantage over the other as regards ease of adjustment, &c. In both trusses, arches are frequently added in wooden railroad bridges when the span exceeds about 150 ft.

Art. 24. Town's lattice truss, Fig 33, as originally introduced, and very extensively employed, was of extremely simple construction; being composed entirely of planks from 2 to 3 ins thick; and from 9 to 12 wide; depending on the span. Two sets of these were placed crossing each other at angles of about 90°; and were connected together at their intersections by either 2 or 4 treenails of locust, or other hard wood, about 2 ins diam. At the top and bottom, similar planks, $a\ a$, $c\ c$, were treenailed hor, to form the chords; or in large spans, (many exceeded 150 ft,) there were two upper and two lower chords, (sometimes of timber 6 ins thick,) as shown in the fig, by $n\ n$, $o\ o$. The transverse section A shows the two upper chords on a larger scale; each chord consisting of two planks; one on each side of the lattices. Two trusses of this kind, with a depth equal to $\frac{1}{8}$ or $\frac{1}{9}$ of any clear span not exceeding about 175 ft; planks of 3×12 white pine; the open squares $2\frac{1}{2}$ ft on a side, in the clear, were considered sufficient for a common-road bridge 20 ft wide. Many of these bridges warped sideways very badly; and when applied to railroad purposes, failed entirely. In some cases the better mode of three lattices was employed; two of them running in one direction; and the third in the other direction, passing between them. A fundamental defect was that the parts were of equal size throughout the span; whereas the chords should be stoutest at the center; and the lattices, near the ends. These defects caused the lattice to fall into unmerited neglect, in the United States; whereas

Fig 33

in Europe it is, when properly proportioned, highly esteemed; especially for iron bridges; some of which, on this principle, have been constructed of more than 300 feet span. The tendency to warp, owing to the thinness of the trusses, is obviated in the large bridges referred to, by placing a *double* truss, Fig 34, (instead of a single one,) at each side of the bridge. The trusses T, D, composing a double one, are placed a foot or more apart; and are connected together at proper intervals, by short pieces riveted to each one, for stiffening them. At T and D are seen the three lattices or lattice-bars, of each truss; two of which, on the outside, constitute the main braces; while the center one is the counterbrace. Such a double truss bears some resemblance to the Fairbairn box-girder; the diff being chiefly that in the former the sides are composed of lattice-bars; and in the latter of solid plates.

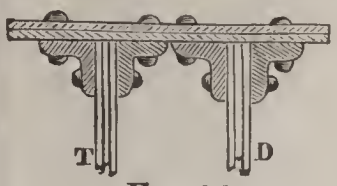


Fig 34

The Bowstring truss, Fig 35, is an excellent one as regards strength, and economy of construction. It has, however, the disadvantage, in large spans, of a difficulty in connecting together overhead the two trusses of a span, so as to be as free from lateral vibration as a bridge with parallel upper and lower chords. In the latter, this connection can be made from end to end of the span; but in the Bowstring it can be done only for some distance each way from the center; from want of headway near the ends. In short spans with low trusses, this defect is not felt. For a graphical method of finding the strains in bowstring and similar trusses, see pp 588 to 592.

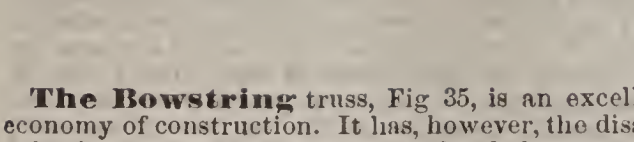
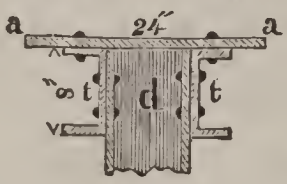


Fig 35

BOWSTRING



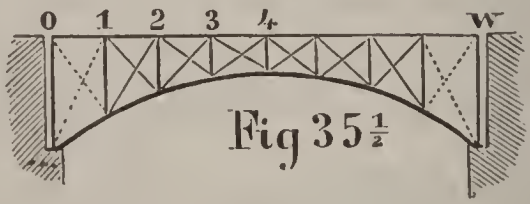


Fig 35½

The Lock Ken viaduct, England, of 130 ft clear span, and 18 ft high, has two trusses, 13 ft 8 ins apart clear, for single-track railroad, on the Bowstring principle, Fig 35, omitting only the verts; which, however, affects the strains on the obliques. For convenience of construction, it was built chiefly of rolled channel-iron, (see *t t*,) of 8 ins by 4, by 4, by ½ inch. At Fig 35, *d* shows a transverse section of the arch or bow; which is uniform throughout. That of the uniform chord or string is of the same fig and size as the arch. Each has an area of 33 sq ins in each truss. The top strip, *a, a*, of rolled iron, is 24 ins by ¾; and is riveted to the upper flanges of the two channel-irons *t t*; which are 8 ins apart, so as just to allow the passage between them of the obliques *d*; which also are of the same channel-iron. The panels are about 12 ft long near the center of a truss; and 8 ft at its ends. The wt of the two trusses alone, without the roadway, is very nearly 50 tons. The wt was increased beyond the theoretical requirements, to save the trouble and expense of preparing and fitting together many pieces of diff dimensions; yet, although the bridge is a strong one, the trusses alone, weigh together but .37 of a ton per foot run. Where two obliques cross, one is in two pieces, riveted to the other by straps.

In the State of New York are many much-used **bridges for common travel, by Mr. Whipple,** of 100 feet span, and 12½ ft rise; with two trusses 19 ft apart from center to center; for two roadways; and having two outside footways, each 6 ft wide. Each truss has 9 panels, braced altogether by vert and oblique tie-rods (no struts) arranged as in Fig 35. The verts next the center of *each* truss, consist each of two rods of 1⅝ ins diam, welded together at top; and straddling 2 ft at bottom. The other verts are single, and 2 ins diam. The obliques are all single, and 1⅛ ins diam. The arches are of cast-iron. The transverse area of metal of each arch is 18 sq ins at the crown; and 21 sq ins at the spring. The shape of arch transversely resembles a channel-iron with its back upward; the total depth of flange 7 ins; the width of arch on top, 11 ins at center of span; but increasing uniformly by means of wide open-work on top, to 3 ft at springs. Each consists of 9 straight segments, held together at their butting flanges, by the verts themselves; which pass through them, and have screws and nuts at their ends. The screw-ends are not upset. The thickness of metal in the arches nowhere varies much from ¾ inch. Under the floor, and between the trusses, are horizontal diagonal braces of rods ¾ inch diam; two of them to each panel; each of them with a tightening swivel. The chord of each arch consists of 4 rods of 2 ins diam. In the same State, are also many similar common road bridges of 72 ft span. Rise 9 ft; two trusses, 19 ft apart from center to center; * and two outside footways of 6 ft each in addition. Each truss has 7 panels, with vert and oblique ties, as in Fig 35. Each cast-iron arch is in 7 straight segments, of the same shape as the foregoing; with a cross-area of metal of about 12 and 15 sq ins. Its width at center of truss 10 ins; at springs, 30 ins. The two verts next the center of each truss, consist each of 2 rods of 1½ diam; the other verts are single, each 1¾ diam. The obliques are all single, 1 inch diam. The chord or string of each arch, is 4 rods of 1⅝ inch diam. Horizontal diag bracing of ¾ inch rods under the floor, as in the foregoing.

Some cast-iron bridges of the Severn Valley Railroad, England, of 200 ft clear span, consist of arches rising 20 ft, and supporting the railroad *on a level* with the tops of the arches, instead of *above* them as in Fig 35½. There are no diags between the arches and the roadway, as in that fig; but cast-iron verts only, placed 4 ft apart. The railroad is double track; and there are four arches, one under each line of rails. The transverse section of an arch is I; each flange is 15½ ins wide, by 2 ins deep; the web is 2 ins thick. Total depth at center of span, 4 ft; and at the skewbacks, 4 ft 9 ins Transverse area of each rib at crown, 150 sq ins. Each arch is cast in 9 segments of eqnal length.

The cast-iron bridge across the Schuylkill at Chestnut St, Phila, Strickland Kneass, Esq, Engineer, roadway on top, has two arches of 185 ft clear span each, and 20 ft rise. Clear width, 42 ft. Each arch has 6 ribs, about 8 ft apart in the clear; and of the uniform depth of 4 feet, including a hor top rib 8 ins wide; and a similar one at the bottom. Thickness everywhere 2½ ins; thus giving to each rib a transverse area of 147½ sq ins. The standards are vert, with ornamentation. It is a city street bridge. The roadway consists of cast-iron plates, which support a pavement of cubical blocks of granite, laid in gravel. The arches are cast in segments 12 ft 10 ins long; each with end flanges 12 ins wide, for bolting them together with four 1¾ inch diam screw-bolts at each end. For a change of temperature from 12° to 99° Fah, the crowns of the arches rise $2\frac{5}{16}$ ins. Under a uniform extraneous load of 100 lbs per sq foot, the greatest pres on the arches is but 3600 lbs

* With only two trusses, the width between them, in the clear, should not be *less* than 16 ft, to allow two ordinary vehicles to pass each other readily; but 18 or 20 ft is still better; more would be unnecessary when there are outside footways. The headway should not be *less* than 13 ft.

per sq inch of their cross-section; or not more than $\frac{1}{2\,8}$ of the ultimate crushing strength of average cast iron, in short blocks.

The Moseley Bridge. Figs 35¾, by Thos. W. H. Moseley, of Kentucky, is essentially a wrought-iron Bowstring, with a hollow plate-iron arch of triangular cross-section, apex up; and formed of three plates riveted together; the two side-plates, *a b*, *a c*, having their top and bottom edges bent to form flanges for this purpose. The chords *o e*; the verts; and the two counter-arches *t t*; are also of iron. These counter-arches are *intended* as a substitute for the obliques of Fig 35. Each of them consists of two angle-irons, back to back, riveted together, and to the verticals, which pass between them. Each of them has a sectional area equal to half that of the main arch. The verticals are placed about 2 ft apart. They are flat (not square) bars, for convenience of riveting. They pass through holes in the bottom-plate of the main arch, (see dotted line of top Fig,) and are fastened at *a* by the same rivets which connect the upper flanges of the two side-plates, *a b*, *a c*.

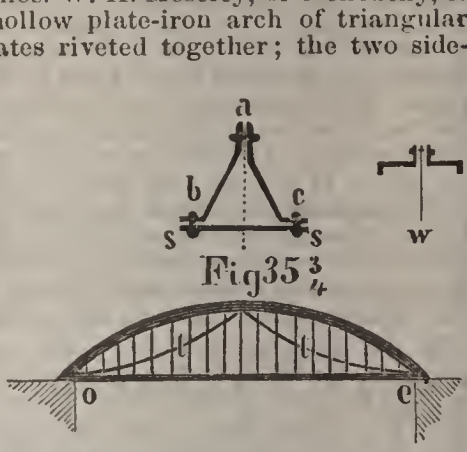


Fig 35 ¾

The chords, *o e*, are also flat bars; and have a transverse area half as great as that of the main arches. At their ends they are attached to strong wrought-iron shoes upon which the feet of the main arches rest and abut. The rise of the main arches, measured to the bottom of the arch, is $\frac{1}{9}$ or $\frac{1}{1\,0}$ of the clear span.

The following are the principal dimensions of a single-track bridge of 93 ft clear span, (97 ft from out to out of arch,) carrying the "Iron Railroad" at Ironton, Ohio. It was built in 1860, and is traversed by heavy engines with trains of pig iron, coal, &c. Rise to bottom of arch 10⅓ ft; to middle of arch 11 ft. Bottom-plate *s s* of arch, 16½ ins, by .44 inch. Side-plates *a b*, *a c*, in clear of flanges, 14½ ins, by .29 inch. Top flanges at *a*, each 3 ins. Bottom flanges *b* and *c*, each 1.88 ins. Vert rods, 3 ins, by ⅜ inch; and 2 ft apart from center to center. The chord of each arch consists of two flat bars, each of 4½ sq ins of cross-section. The bridge was tested for three weeks by a dead load of ½ a ton; and a rolling load of 1 ton at the same time, per foot run; and deflected only ⅝ inch. With a load of 1 ton per foot, the pressure at the center of the arches would be about 5½ tons per sq inch of metal; and a trifle more at the feet. The sectional area of metal in each main arch is 18½ sq ins. These bridges are of easy construction, and consequently cheap. For long spans, vert diagonal bracing (see Fig 35) would probably be essential for preserving the form of the arches under heavy moving loads.*

An iron arch roof in Philadelphia, clear span 80 ft, rise 16 ft, consists of a uniform arch of single 7-inch Phœnix beam of 6 sq ins sectional area; weighing 20 lbs per foot. This rests on cast-iron shoes on the walls. The hor chord or tie at the feet, is of two rods of 1⅜ diam. At the center of this tie is an arrangement similar to No. 15, of page 583, from which diverge upward, to the arch, a central vert rod 1 inch diam; two struts of 6-inch Phœnix beam, 20 ft apart at the arch; and two ties each 1⅛ diam, reaching the arch at half-way between the struts and the feet of the arch. There are 10 such trusses, each of which by itself weighs about 3900 lbs; they are placed 16 ft apart; and by means of purlins resting upon them, support the entire weight of the roof, which is of inch boards, covered by thin sheet-iron.

The iron roof of a rolling-mill near Boston, Mass. of about 80 ft span, and 16 ft rise, has arches of the Moseley section *a*, *b*, *c*, Fig 35¾; but without counter arches. The trusses are 12 ft apart. Sides of the arches, clear of the flanges, 7 ins; upper flanges, 2 ins; lower ones, 1 inch; total of each side, 10 ins, by .19 inch thick. Bottom plate 8½ inch by ¼ inch thick. Total sectional area of an arch, 5.925 sq ins. There are besides, a chord; and 24 vert suspending rods; but no obliques. The roof is covered with corrugated iron, on purlins. When required, the heavy iron rolls of the mill are lifted by tackle supported by a roof-truss.

Figs 36, represent the Burr truss; which was formerly more used

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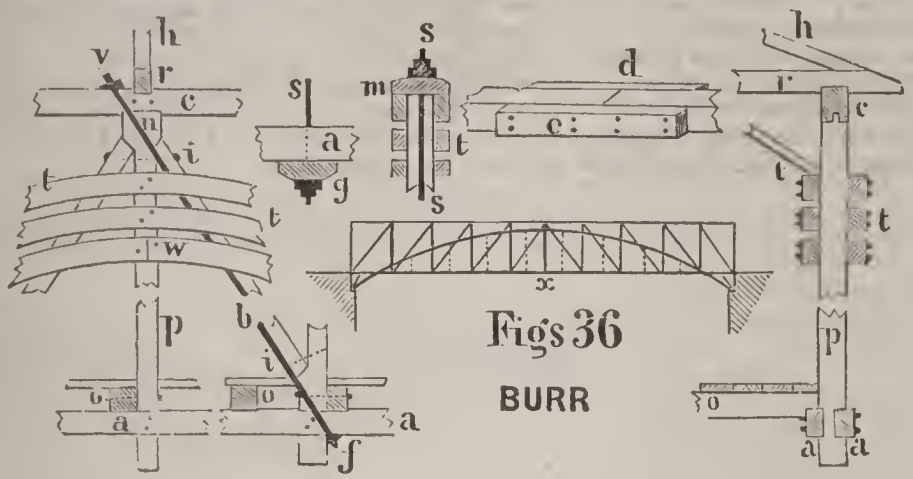
* Although this bridge seems to have stood very well for several years, the writer would prefer not to exceed two tons of compressive strain per sq inch *on plate-iron* in such structures. In several bridges, General Moseley has used a *continuous web* of ⅛ inch iron, instead of the vert suspenders. But in such cases the triangular tube is not applicable for the arch; and he substitutes two Z bars, riveted together, and to the web, *w*, which passes between them, as in the foregoing fig. The counter arches being here unnecessary, are omitted. This web would be objectionable in large spans, especially of draw-bridges, on account of the wind. More recently still, he has also introduced latticed, instead of vert bars, in some of his bridges; together with many innovations on the arrangement first described. At 1 ton per ft run, the pull on the chord above, = 11 tons per sq in.

than any other in the United States. It is at present regarded with disfavor by some, because many early ones failed under railroad traffic, in consequence of bad proportions, and the absence (as in our Fig 36) of counterbracing. When properly constructed it makes an excellent bridge. The common objection to it, and not without reason, is that a truss and an arch cannot be so combined as to act entirely in concert; yet, as soon as any ordinary truss begins to fail, the almost invariable remedy is to add an arch. When, however, the two are to be united, it is better to so proportion the arch as to be capable by itself of safely sustaining the max load *at rest;*



and to confine the duty of the truss to preventing the arch from changing its form under a *moving* load. Counterbracing may be effected by strapping the heads and feet of the braces to the chords; or by iron rods parallel to the braces; two to a brace; with screws and nuts, as at *v f*. Or by similar rods across the other diags of the panels. **The following dimensions** answer for a single-track R R bridge of about 150 ft span. Rise from out to out of chords one-eighth of the span; about fourteen or sixteen panels. Width in clear of arches, 14 ft. Six arch-pieces *t*, of 10″ × 13″ each, to each truss. Upper chord *c*, 14″ × 16″. Lower chord *a a*, two pieces each 10″ × 15″. Posts *p*, 14″ transversely of the bridge, as in the right-hand fig; by 10″; except at the heads and feet, where enlarged to receive the ends of the braces. Braces 10″ deep, by 13″ wide. Floor girders *o*, 10″ × 16″; and 2½ to 3 ft apart from center to center. Suspending rods (shown at *s*; and dotted in *x*) 1⅝ diam. Counterbrace rods in pairs parallel to braces, about 1½″ diam. Bolts for arches, lower chords, &c, 1½″ diam. Theoretically, the posts, braces, and arches should gradually diminish from the ends, toward the center of the truss; while the chords should increase; but in practice, the additional labor of getting out and fitting pieces of diff sizes, frequently makes it more economical to use uniform sizes.* The same amount of arch would answer also, if trussed, as in Fig 35; and the arch by itself with a full max load, would be strained less than 800 lbs per sq inch, at its feet. For a span of 200 ft, with the same proportion of rise, the transverse areas of the several arch and truss pieces should be increased 33 per cent; and for one of 100 ft, they may be diminished the same. None of these dimensions are the result of close calculation. The dimensions just given will answer for *common travel*, for a span of 200 ft; with a depth from out to out of chords, of ⅛ the span; panels 10 to 12 ft long. Many such spans have been built with timbers of about ⅕ less transverse section; and without counterbracing. The heads of the posts are notched about 2″ to 3″ into the bottom of the upper chords; and are moreover tenoned into it some ins further; with two wooden pins through the tenon; see *n*, Figs 36. Their feet are notched both *into* and *upon* the lower chords, so as to leave the two chord-pieces *a a* only about 2″ apart. Through these and the post pass two bolts of about 1″ to 1¼ diam.

Since the upper chord resists compression only, its pieces may come together with a plain butt joint, *d*. To this may be added fishes *e d*, of stout plank, on the sides of the chord, bolted through by 4 or 8 bolts.

The lower chords resist pull; and the pieces composing each lower chord must therefore be joined together These pieces should be as

* It will be borne in mind that our examples are not intended to illustrate perfectly proportioned structures. None of them would endure strict criticism. There is more waste of timber in an arch built with a uniform transverse section throughout, than in the straight upper chord of a Howe or Pratt similarly built; for both must be proportioned to the greatest strain. This is at the center of the two last; but while that at the center of the arch is as great as in these, that at its feet is much greater. See Example 2, Art 38, of Force in Rigid Bodies.

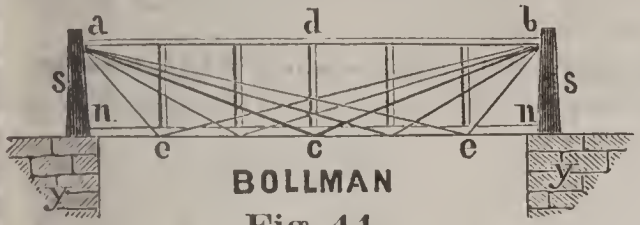
long as possible, and should never be jointed opposite to each other, but one opposite the middle of the other.

The braces are merely cut to fit to the heads and feet of the posts, after these last have been fixed in their places; and usually have no other connection to them than one or two spikes at each end, for small bridges; or screw-bolts for large ones.

The ends of the timbers composing the arches, butt full square against each other; and may also have a wooden dowel. The joints should occur at the posts, as shown at W. The arches are screw-bolted to the posts, as shown in the figs, by bolts of 1 to 1¼ ins diam. Where the arches pass the lower chord, both are notched, and well bolted together. The feet of the arches abut against cast-iron plates.

When suspension rods (dotted in Fig 36) are used for assisting to support the roadway, they are placed as shown at *s s*; *m* being a strong block of wood slightly notched on top of the upper arch-pieces. The rods are suspended by a washer and nut on top of the block; and after passing down between the arches and chord, have a similar arrangement, but inverted, below the last; as shown at *g*.

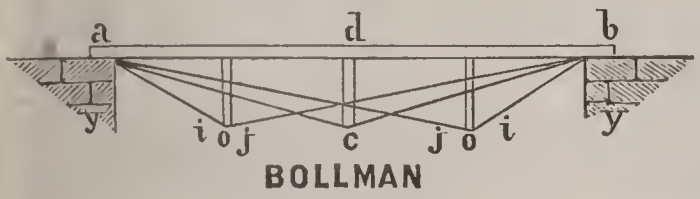
If a long beam, *a b*, Figs 45 and 47, **requires to be strengthened,** this may be done by adding a vert post *d c*; and 2 inclined tie-rods *c a*, *c b*. And if, after this, the two halves, *d a*, *d b*, of the beam still are found to be too weak, additional intermediate posts, *o o*, may be introduced; with other ties, *i j*, to sustain them.



BOLLMAN

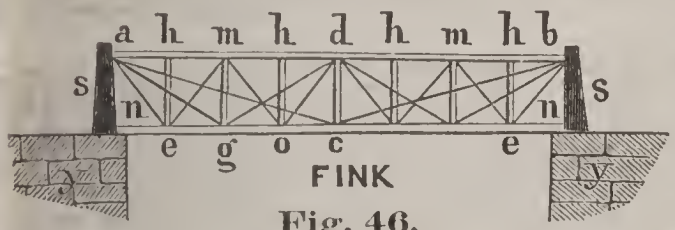
Fig. 44.

In Figs 45 and 47, the roadway is at the chord *a b*; no parallel lower chord being necessary, it may be omitted. The inclined ties act as substitutes for it. But if the bridge is so near the water as not to allow the posts and ties to be placed *beneath* the roadway *a b*, we may raise the entire truss upon two posts or piers *s s*, Figs 44, 46; and place the roadway *n n*, at the lower ends of the posts and ties; instead of letting it rest on top of the chord, as in Figs 45 and 47. In Figs 44 and 46, the truss and its load do not then rest *directly* upon the abuts *y y*, but upon the tops of the posts *s*, *s*; and the only part that does rest directly on the abuts, is *one-half* of that small portion of the roadway comprised at each end, between *e* and *n*; in other words, only one half the wt of the roadway of the end panels; the other half being sustained by the inclined ties which meet at *e*.



BOLLMAN

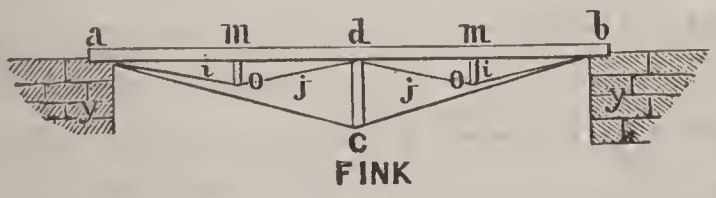
Fig. 45.



FINK

Fig. 46.

When the tie-rods *all* pass from the feet of the posts to the ends of the chords, as in Figs 44 and 45, we have the **Bollman truss.** And when, as in Figs 46 and 47, only those which sustain the center post *d c*, *both* pass to the ends of the chord, while the others are disposed as in said figs, the **Fink truss** is the result.



FINK

Fig. 47.

The following dimensions for single-track Fink bridges with chords and posts of wood; and iron suspension bars; are on the assumption that all the bars deflect 1/8 of the span; that the road is on top; that the bars shall not be strained more than 10000 lbs, or 4½ tons per sq inch, under a weight of bridge and load, amounting in all to two tons per running foot. Assumed wt on each driving-wheel of engine, 5 tons. Screw ends upset.

Dimensions for one truss only, of a single-track Fink bridge.

The spans are in feet; the other dimensions are square inches of cross-section of each member.

Areas in sq ins.						Areas in sq ins.						
Span. Ft.	Chord.	1st Rod.	2d Rod.	3d Rod.	1st Post.	Span. Ft.	Chord.	1st Rod.	2d Rod.	3d Rod.	4th Rod.	t Pc
35	275	8¼	2½	...	56	90	470	21	5¾	1¾	...	146
40	300	9½	2½	...	64	100	505	23⅜	6¼	2	...	169
45	322	10½	2⅞	...	72	125	580	29¼	8	2½	1¼	225
50	340	11¾	3¼	...	81	150	650	35	9½	3	1½	280
60	375	14	3¾	1⅝	95	175	730	41	11	3½	1½	335
70	410	16½	4⅓	1⅝	110	200	800	46¾	12½	4	1¾	39.
80	440	18¾	5	1⅝	126							

We have given the area of the 1st post only. For that of the 2d, we may take ⅔ of the first; for the 3d, ⅔ of the second; and for the 4th, ⅔ of the third; without pretending to any great accuracy. The iron rods may be flat, square, or round, so that the proper area be maintained. It will usually be best to have them flat.

A width of 18 feet is necessary for allowing two ordinary vehicles to pass each other *readily*. It should never be less than 16 ft; and nothing is gained by exceeding 20 ft.

It will of course be understood that each member, especially in large spans, will consist of two or more pieces, side by side. Thus, in a span of 200 ft, the 800 sq inches of each chord, will probably consist of four beams of about 10″ × 20″, or 12″ × 16″, placed side by side; but with sufficient intervals between them to allow the several oblique bars to pass. Or it may consist of six beams of smaller size. So also, the 1st, or main rod, of 46¾ sq ins, will probably be made up of from 4 to 8 bars, of 11 or 5.85 sq ins each, placed side by side; occasionally some inches apart. And so with the others. The feet of the opposite posts of the two trusses of a Fink span, are connected together by ties of wood or iron, to prevent lateral motion; and for the same purpose, diagonals are carried from each upper chord to the foot of the opposite post; as is usually done in top-road bridges of any kind. These had better be tie-struts.

The weights of bridges of the same span, designed by different persons, vary considerably, from several causes; such as the form of truss; quality of iron; coefficients adopted for safety, and for strength of materials; whether the roadway is on the top chord, or on the bottom one, &c, &c.

For a mere approximate weight to be assumed as a preliminary in calculating the strength and proportioning the parts of a bridge, or for forming some rude idea of the quantity of iron required, we suggest the following purely empirical formulæ. They give the **weight** in pounds per foot run of span, of only the two trusses or main girders together and their lateral bracing, for a single-track railroad bridge of standard gauge (4 ft 8½ ins). The weight of cross-beams, flooring, rails, etc, is not included.

For spans not exceeding 75 feet.

(Plate girders.)

Weight per foot run (see above) $= 5 \times$ span in feet $+ 50 \times \sqrt{\text{span in feet.}}$

For spans of from 75 to 250 feet.

(Pratt, Whipple, or Warren trusses.)

Weight per foot run (see above) $= 4.5 \times$ span in feet $+ 22 \times \sqrt{\text{span in feet.}}$

For spans exceeding 250 feet.*

(Trusses of various designs.)

$$\textbf{Weight} \text{ per foot run (see above)} = \frac{\text{span}^2 \text{ in feet}}{60} + 500.$$

* In spans longer than about 300 feet, the differences in truss designs are so great that the actual weights may differ widely from those obtained by simple practical formulæ of any kind.

Table of approximate weights of single-track iron railroad bridges of standard (4 ft 8 1-2 inches) gauge.

Span.	Weight of the two trusses, or main girders, and their lateral bracing.		
Feet.	Lbs per foot run of span.	Lbs total.	
20	325	6500	Plate girders.
30	425	13000	
40	520	21000	
50	600	30000	
60	690	41000	
70	770	54000	
80	550	44000	Pratt, Whipple, or Warren trusses.
100	670	67000	
150	940	141000	
200	1200	240000	
250	1500	375000	
300	2000	600000	Trusses of various designs.
400*	3200	1280000	
500*	4700	2350000	
600*	6500	3900000	

For **double track** bridges add 80 per cent to the weight of single-track ones as obtained by the formulæ.

For **narrow gauge** bridges take 75 per cent of the weights for standard gauge ones as given by the formulæ.

Iron floor systems, with two stringers under each track and adapted for heavy loads, such as A p 546, may be taken as weighing approximately as follows:

Span, in feet.	Weight of iron floor system, in pounds per foot run.	
	Single track.	Double track.
20 to 100	200 to 275	550 to 700
100 to 250	250 to 350	700 to 850
250 to 300	325 to 400	750 to 900
300 to 400	375 to 450	"
400 to 500	425 to 500	"
500 to 600	475 to 575	800 to 1000

Two **iron safety stringers** will together weigh about 150 lbs per foot run.

For **wooden floor systems,** we may, as a rude average, set down for spans not exceeding about 200 feet, cross floor girders of about 7″ × 15″, and about 2½ to 3 feet apart, from center to center; clear span 14 feet; together with substantial string-pieces of about 10″ × 12″, for supporting the rails; the rails themselves; and a plank pathway between the rails, all complete, at about .14 ton, or 314 lbs, per foot of span; or with a full floor of 3″ plank, 14 feet wide, about .2 ton, or 448 lbs. For greater spans, with the trusses farther apart, increase this to .25 ton, up to 300 feet; .3 ton, to 400 feet; .35 ton, to 500 feet; and .4 ton, to 600 feet.

Two **safety stringers** will together weigh about 100 to 150 lbs per foot run.

When a bridge is to be roofed and weather-boarded, an addition must, of course, be made to our weights.

Wooden bridges weigh about the same as iron ones of equal strength.

In the Northern Pacific R R bridge over the Missouri River, at Bismarck, Dak, built 1881–2, the two inverted bowstring trusses of the 115-foot approach spans, weigh (together) .38 ton per foot run; two Pratt trusses of the 400-foot channel spans, 1.09 tons.

Two trusses of the Newark Dyke bridge, England, Warren girder, 240½ foot span, weigh 1.02 tons. The single-track tubes of the Victoria bridge at Montreal, 244 feet span, 1.14 tons; the 330 foot span, 2 tons. The single-track Britannia tube, Eng, 460 foot span, 3.43 tons. Two trusses alone, of the Penna R R, at Philadelphia, 180 foot span (Pratt's system,) by Mr Linville, .52 ton. The fine 320 foot span across the Ohio at Steubenville, (Pratt,) also by Mr Linville, 1.6 tons per foot. All these are of iron; single track.

* See foot-note (*), page 604.

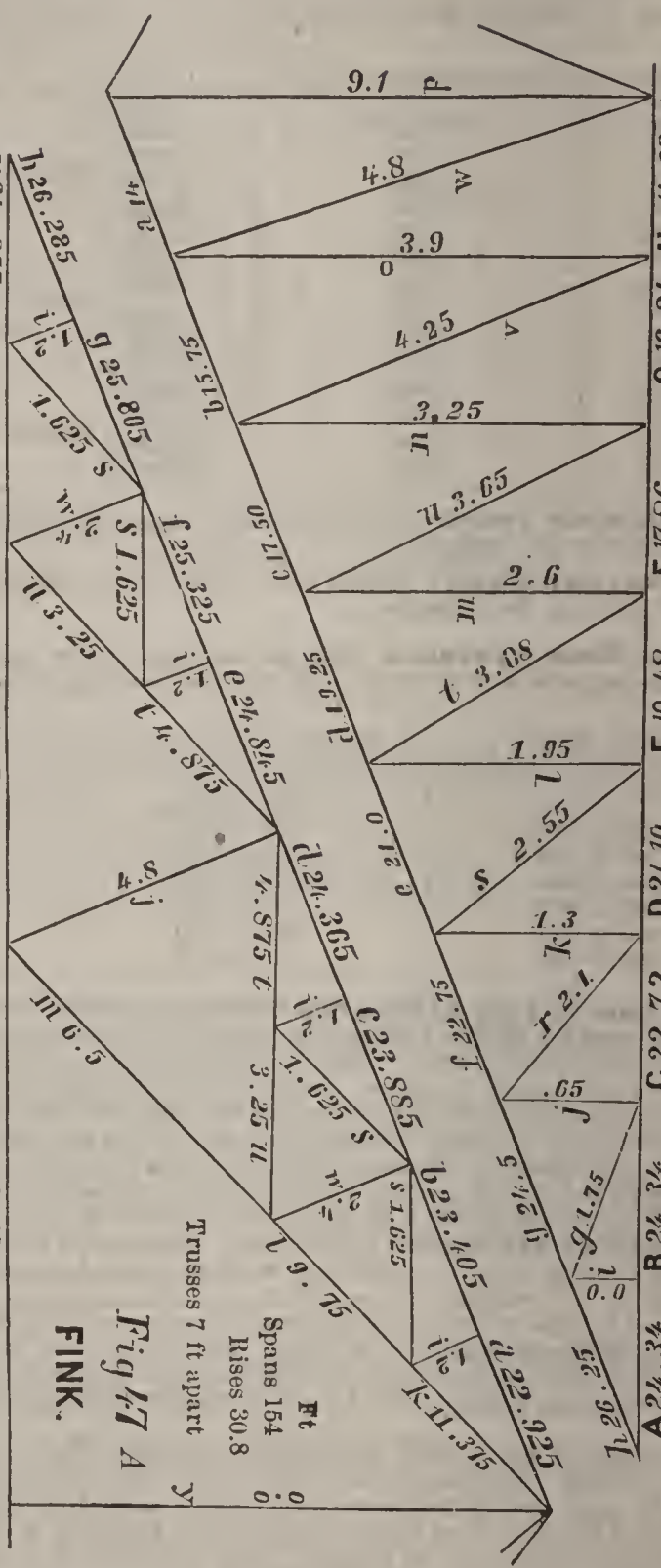
The greatest load that can come upon a bridge,

If for a single-track railroad, can scarcely exceed that of a string of heavy locomotives coupled together, without their tenders. Such engines will weigh from one to two tons per foot of their extreme length but a long string of such, without their tenders, is hardly probable.

Our table on page 605 makes sufficient allowance for the greater loads per ft run that may probably come upon small spans. The diffs between our 4th and 6th cols will give our assumed max loads.

On very small spans the loads cannot be assumed to be equally distributed. On bridges for turnpikes and common roads, no probable contingency could crowd people upon them to such an extent as to weigh more than 80 lbs per sq ft of floor. The French standard indeed is but half of this, or 42 lbs per sq ft; and is sufficient for *probability*, but not for *possibility*. The latter *may* increase it to 80 lbs; and this may safely be taken as the maximum load on spans of 20 or more feet. To compensate, however, for momentum, we recommend to adopt 100 lbs, or .045 of a ton, as the limit for crowds. † A bridge for a single-track carriage-way; with room between the trusses for a footway also, should not be *less* than 12 ft wide in the clear; in which case its greatest load at 100 lbs per sq ft, would be ½ a ton per ft run; or if 24 ft wide, with but two trusses, the load would be full 1 ton per ft run.

But in a common bridge also, the greatest load per ft run, on a very short span, will be greater than in a long one; as in the case of two wheels of a truck hauling a large block of stone, &c; and this must be taken into consideration in building such.



† The engineers of the Chelsea bridge, London, *packed picked* men upon the platform of a weigh-bridge; with a result of 84 lbs per sq ft. Mr Nash, architect of Buckingham Palace, experimenting with reference to fire-proof floors for that building, *wedged* men together as closely as they could possibly stand upon an area of 20 ft diam; the last man being lowered down from above, among the others. Result, 120 lbs per sq ft. See foot-note p 623

It must also be remembered that each transverse floor-girder must bear *at least* all the weight resting upon two wheels; no matter how close together the girders may be placed. If they are farther apart than the dist between two axles of a vehicle, they will have to bear more than the load on one pair of wheels.

The allowance for safety in a truss bridge.

As the result of a long-continued series of deflections applied to an experimental plate-iron girder of 20 ft span, Mr Fairbairn concludes that a bridge subject to 100 deflections per day, each equal to that produced by ½ of its extraneous breaking load, would probably break down in about 8 years; while, with 100 daily deflections equal to that arising from but ¼ of its breaking load, it would last fully 300 years. We are of the opinion that a bridge should not have a safety of *less* than 4 for its max extraneous load, and **its own weight,** combined; nor do we see any use in exceeding 6. From 4 to 5 may be used in temporary structures, or in those rarely exposed to maximum strains; and 6 in more important ones frequently so exposed. The last will (roughly speaking) generally give a safety of about 2 against reaching the **elastic** strength, which is the true guide in such matters. But 4, 6, &c, usually refer to the **ultimate** or breaking-down strength; so that a truss with **such** a safety of 2 would in fact be very unsafe.

On the camber of truss bridges. In practice, the upper and lower chords of bridges are not made perfectly straight, but are curved slightly upward; and this curve is called the **camber** of the truss or bridge. Its object is to prevent the truss from bending down below a hor line when heavily loaded. A cambered chord is of course longer than a straight line uniting its ends; but in practice the camber is so small that this diff is inappreciable, and may be entirely neglected. But when the chords are cambered, (see *y s* and *c d*, Fig 51,) they become concentric arcs of two large circles, of which the center is at *t*; and the upper one plainly becomes longer than the lower, to an extent which, although much exaggerated in our fig, cannot be overlooked in practice. The verticals, instead of remaining truly vert, become portions of radii of the aforesaid large circles; and although their lengths remain the same, yet their tops become a little farther apart than their feet; and this renders it necessary to lengthen the obliques or diags a trifle. Therefore, we must find how great is this increase of length of the upper chord beyond the lower one; and divide it equally among all the panels, along said chord; otherwise the several parts of the truss will not fit accurately together.

1st. To find the amount of camber of the lower chord. Divide the span in feet, (measured from center to center of the outer panel-points,) by 50. The quot will be a sufficient camber, in inches; as shown in the following

Table of cambers for bridge trusses.

Span. Feet.	Camber. Ins.	Span. Feet.	Camber. Ins.	Span. Feet.	Camber. Ins.
25	0.5	100	2.0	250	5.0
50	1.0	150	3.0	300	6.0
75	1.5	200	4.0	350	7.0

Rem. 1. It is by no means necessary to adhere strictly to this rule; and the camber by experienced builders of **iron bridges** is often but one-half the above, or 1 inch per 100 ft of span.

Rem. 2. A well built bridge of good design should not, under its greatest load, **deflect** more than about 1 inch for each 100 feet of its span. The deflection is frequently much less than this.

2d. To find the increase of length in the upper chord, beyond the lower one, having the span; the depth of truss; and the camber; (all in feet, or all in ins.)

With any camber not exceeding $\frac{1}{50}$ of the span; (which, however, is about 7 times as great as is usually given to trusses;) mult together the depth of truss, the camber, and the number 8; div the prod by the span. The quot will be the increase, in ft, or in ins, as the case may be. Or as a formula,

$$\underset{\text{or } in\ ins,}{Increase\ in\ ft,} = \frac{depth \times camber \times 8}{span,} \quad \underset{all\ in\ inches.}{all\ in\ feet;\ or}$$

This rule may be considered practically perfect with any camber not exceeding $\frac{1}{50}$ of the span. Based upon this principle, we have prepared the following table, which may be used instead of making the above calculations.

Table for finding increase of length of upper chord beyond lower one.

Depth of Truss.	Mult Camber by	Depth of Truss.	Mult Camber by	Depth of Truss.	Mult Camber by	Depth of Truss.	Mult Camber by
¼ span.	2.00	⅛ span.	1.00	1-12 span.	.666	1-16 span.	.500
1-5 "	1.60	1-9 "	.888	1-13 "	.614	1 17 "	.470
⅙ "	1.33	1-10 "	.800	1-14 "	.571	1-18 "	.444
1-7 "	1.15	1-11 "	.727	1-15 "	.533	1-20 "	.400

Ex. How much longer is the upper chord than the lower one, when the depth of the truss is $\frac{1}{7}$ of the span; and the camber 5 ins? Here in the table, and opposite $\frac{1}{7}$ span, we find the multiplier 1.15. Therefore, 5 ins $\times$ 1.15 = 5.75 ins, Ans. If the truss has say 8 panels, then $\frac{5.75}{8} = .72$ inch of this increase must be given to each panel, along the upper chord.

The length of a diag. or oblique, *b c*, Fig 50, may readily be found thus: Let *a s n c* in this fig represent a panel when there is no camber; then *o b n c* will represent a panel when there is a camber; and *o a* and *s b* together are the portion of the increased length of upper chord given to each panel; but to an exaggerated scale. Now, to find *b c*, we have the right-angled triangle *a b c*, in which we know the side *a c*, (the depth of truss;) and the side *a b*, (equal to the panel width *c n* on the lower chord; added to *s b*, or *half* the portion of the increased length of upper chord given to one panel.) Hence, we have only to square each of those two sides; add the two squares together; and take the sq rt of the sum. This sq rt is *b c*.

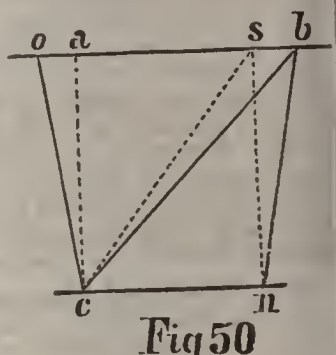


Fig 50

Example. Span 200 ft. Height (*a c*) of truss ⅛ of the span, or 25 ft, or 300 ins. Camber 5 ins; 10 panels each 20 ft, or 240 ins, (*c n*,) measured on the lower chord or span. Now, the height being ⅛ of the span, the increase of length of upper chord will be equal to the camber, 5 ins; and this divided among 10 panels, will be $\frac{5}{10} = .5$ inch to each panel; or *o b* will be .5 inch longer than *c n*; and *o a* and *s b* will each be .25 inch. Hence, in the right-angled triangle *a b c*, we have *a b* = 240.25 ins; and *a c* = 300 ins. Hence,

$$b\,c = \sqrt{a\,b^2 + a\,c^2} = \sqrt{57720.0625 + 90000} = \sqrt{147720.0625} = 384.344 \text{ ins};$$

or 32 ft, .344 ins. Without any camber, *b c* would be in the position *s c*, which is 32 ft, .187 ins long; or .157 of an inch (about ⅙ inch) shorter than *b c*.

An error to this extent would prove seriously inconvenient if the oblique were a cast-iron strut with carefully planed ends, intended to fit closely between planed bearings at the chords; or a bar with a drilled hole at each end for fitting over pins whose position was fixed and unalterable. In many cases, as when the obliques are merely rods with screw-ends, it is only necessary to be sure that they are *long enough;* because their exact length can then be adjusted when put into place, by means of the nuts on their ends. So also when the obliques or other pieces are flat bars intended to be bolted or riveted to the sides of the chords; for the final rivet-holes may be made when the pieces come to be finally fitted in place. In the case of wooden obliques, &c, if too long, they can readily be reduced by the saw or chisel.

When the panels are all of one size, as is generally the case, it is usual for builders to draw one of them full size on a board platform or floor, to guide in fitting the parts together.

In raising a truss, or in other words, when putting its parts together in their proper position on the abutments and piers, a scaffold or **false-works,** must first be erected for sustaining the parts until they are joined together so as to form the complete self-sustaining truss. Upon the false works the bottom chords are first laid as nearly level as may be; and the top chords are then raised upon temporary supports which foot upon the one that carries the lower chord. The upper chords are at first placed a few inches higher than their final position, or than the true height of the truss, in order that the obliques and verts may be readily slipped into place. After this is done, the top chords are gradually let down until all rests upon the lower chords. The screws are then gradually tightened to bring all the surfaces of the joints into their proper contact; and by this operation (the upper chord being supposed to have the increased length given by the foregoing rule) the

camber, as it were, forms itself; and lifts the lower chords clear off from their falseworks; leaving the truss resting only upon the abuts or piers, as the case may be.

As a support for the falseworks themselves on soft bottoms piles may be driven, to which the uprights of the falseworks may be notched and bolted or banded. In some cases, as of rock bottom in a strong current, it may become expedient to sink cribs filled with stone, as a support for the falseworks.

The falseworks should be well protected by fender-piles or otherwise from passing boats, ice and other floating bodies, especially in positions liable to sudden floods; and numerous accidents have shown the expediency of guarding the unfinished truss itself against **high winds.** This last remark applies as well to roofs as to bridges; and is too frequently neglected.

To prevent an overturning tendency in a whole truss when it is not high enough to admit of being horizontally braced overhead, we may introduce wooden knees, or short straight struts or ties, of either wood or iron; which may foot upon the cross-girders of the floor; and head against either some of the web members, or the upper chord. These braces or ties may be placed either between the two trusses of a span; or outside of them; or both. When outside, some of the floor-girders may be lengthened out a few feet beyond the lower chord, for receiving the feet of the braces or ties.

The clear **distance apart of the trusses** in railroad bridges for 4 ft 8½ inch gauge, is generally made not less than 14 or 15 ft, and 26 or 28 ft, respectively, for single and double track, in *through* bridges; and 11 or 12 ft, and 16 ft, respectively, in *deck* bridges. For lateral stability, **in long spans,** the distance between centers of trusses is generally made not less than one-twentieth of the span.

A headway of 18 to 20 ft should be allowed for clearing smoke-stacks, &c.

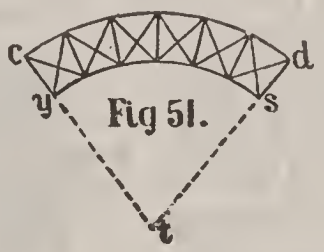


Fig 51.

Floor-girders not exceeding 14 ft clear span, may be 8 ins. by 15 ins deep; and placed not more than about 2½ ft apart from center to center. Upon them should be notched and spiked stout string-pieces, say 12″ wide, by 9″ deep, to carry the rails; and to distribute the pressure of the load.

Hor diag bracing for diminishing lateral motion, can be used only under the floors of low bridges; but in high ones it is introduced also at the top of the trusses. When of timber, these braces are about 4 to 6 inches thick; by 6 to 9 deep; and form a hor cross between each two opposite panels of the two trusses. If the bridge is roofed, and has girders *r*, Figs 36, 56½, upon and well secured to the upper chords *c c*, the upper lateral bracing may consist simply of 4 iron rods *n n*, passing through the chords about midway of their depth; and having heads and washers on their outer sides. At the center of the cross the rods terminate in an adjusting-ring; see No 14, of page 583. In a bridge of 150 ft span, these rods need not exceed ¾ inch diam at the center panel, and 1½ at the end ones. If the bridge is high, and not roofed, but open at top, then cross-struts *r r*, Figs 56½, must be inserted purposely, when this rod-bracing is used. If it is also used at the lower chords, the floor-girders perform the duty of these struts. Iron-bracing is not liable to catch fire from the locomotives.

A favorite mode of lateral bracing. W, resembles a Howe truss laid flat on its side. In it the diags of the cross are struts of timber; and the pieces *r r* are round rods. One of the struts is whole, with the exception of a slight mortice on each vert side, at its center, for receiving tenons cut on the inner ends of the two pieces which compose the other diag. At the sides of the chords, the ends of the diags rest upon a ledge, (shown by the dotted line *i i*,) about 1½ ins wide, cast at the bottom of the cast-iron angle-block. The tie-rod *r r*, passing through the chords of both trusses, being tightened by means of the nut *s*, holds the diags firmly in place; and in case of their shrinking a little in time, can be again tightened up by the same means.

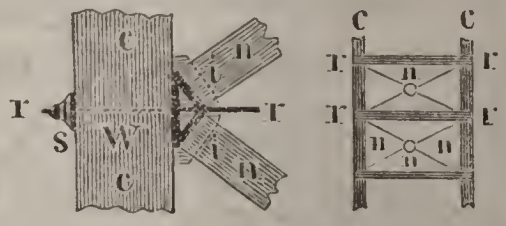


Fig. 56½.

Various modifications of these methods are in use; but we cannot afford them space here. The cast angle-block is as deep as a brace; its thickness need not exceed ½ inch, in a large bridge. The dark triangle is a top view of it. It has holes for the passage of the rod *r r*.

Art. 25. Lengthening-scarfs, splices, or joints. The lower chords of bridges, being exposed to great pulling strains, require much care in connecting together the ends of the several pieces of which they are composed. There is much uncertainty regarding the strength of the joint-fastenings in common use for this purpose. Experiments on the subject are much needed. When only two pieces, as *t* and *y*, Fig 57, or 58, are joined by any of the ordinary methods, it is probably not safe to depend on their possessing more than ⅓ of the tensile strength of a single solid beam of equal cross-section. When the chord, as in Fig 59, is composed of two parallel parts *a a*, *n n*, made up of long pieces, breaking joint with each other, as at *j j j*, each of the two parts may be made somewhat stronger than either one of them would be by itself. This is owing to the opportunity afforded of connecting

them also by bolts *b b*, and packing-blocks, *c c*, of wood or iron, intermediate of the joints *j j*, &c. By this means the strength of the entire chord may probably be practically rendered equal to one-half of what it would be if solid. If the chord consists of 3 or 4 parallel parts, of long pieces, breaking joint, and connected in the same way, it will probably have about ⅔ of the strength of the solid. Care must of course be taken that the serviceable area of the pieces shall not be reduced at any intermediate point, to less than it is at the joints.

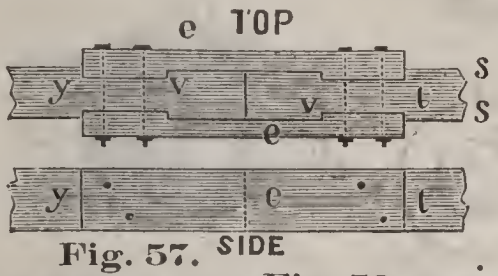


Fig. 57.

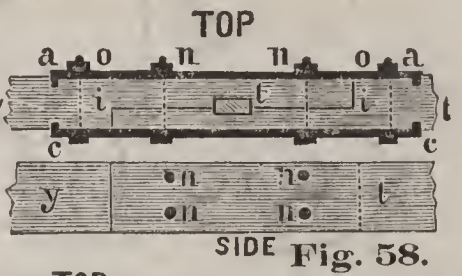


Fig. 58.

Fig. 59.

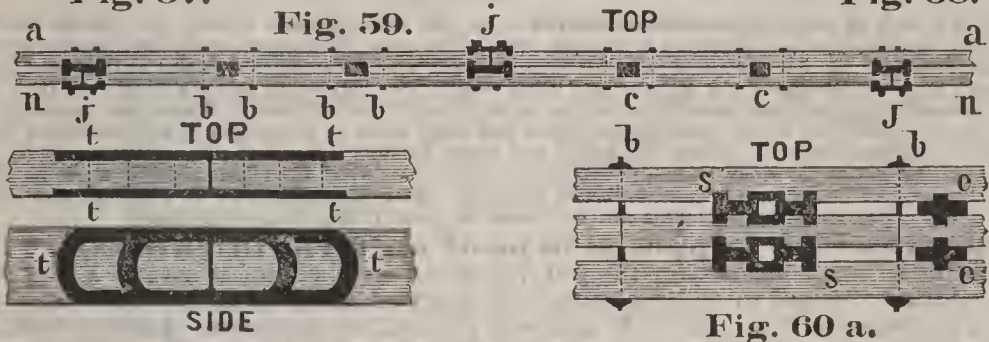


Fig. 60.

Fig. 60 a.

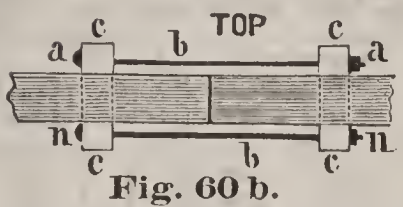


Fig. 60 b.

Fig 58 is a simple and efficient form of scarf. Its length *i i* may be about 3 to 4 times the greatest transverse dimension of the beam. At the center is a block *t* of hard wood, with a thickness equal to ⅓ that of the beam; a width of 2 or 3 times its thickness; and a length just sufficient to reach entirely through the beam. The beams are connected by 4 screw-bolts *n n*; or by 8 of them, if the length requires it. Plates of stout rolled iron, *a a*, *c c*, with their ends bent down into the beams, are occasionally added. These require bolts *o o*, beyond the ends *i i* of the scarf. These bolts are not shown in the side view.

Fig 57 is another excellent joint with *splicing-blocks e e*, instead of the block *t* of Fig 58. The indentations, *v v*, may each be about ⅙ as deep as the beam is thick. The length of each splice-block, about 6 times *s s*. From 4 to 8 screw-bolts, as the case may require. Length of each indent about ¼ that of the block itself.

Fig 60 is a joint formed by two flat iron links or rings, *t t*, let flush into the timbers, and retained in place by spikes. The iron may vary from ¼ to 1 inch in thickness; from 1 to 4 or 5 ins in width; and 2 to 6 ft in length, as occasion may require.

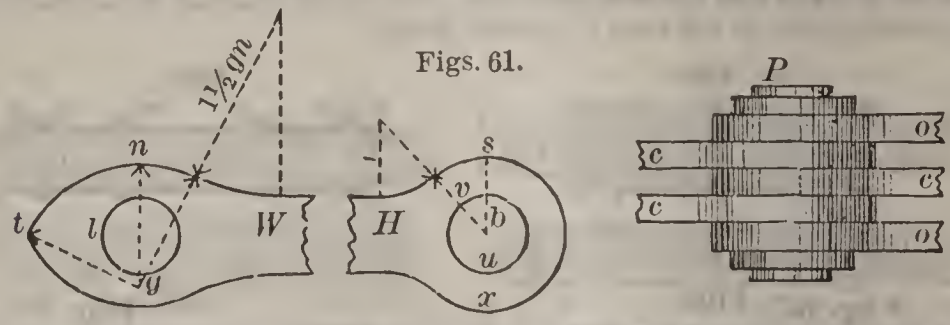
Fig 60 *b*, is a joint formed by two blocks, *c c*, of hard wood, passing through the timbers; and connected by bolts. *a a*, *n n*.

In Fig 60 *a*, *s s* are cast-iron packing-blocks, sometimes used instead of plain wooden ones, at points *b b*, Fig 59, intermediate of the joints *j j*. The openings in the centers of the blocks are needed only when vertical truss-rods have to pass through those points. At *e e*, of the same fig, is shown another form, much used in chords composed of two or more parallel strings. Both these are as deep as the chord; and their cross-sections, or end views shown in the fig, may be from 4 to 10 ins long: 2 to 4 ins wide; and from ½ to 1½ ins thick; according to size of bridge, &c.

Rem. In selecting hard wood for splicing-blocks, treenails, or for any part of a bridge, it is well to remember that the oaks when in contact with the pines, expedite the decay of the latter; therefore, it is generally better to employ the best southern yellow pine heart wood for such blocks, &c, or interpose sheet iron.*

* The tendency of some kinds of timber to produce rapid decay when brought into close contact

Eye-Bars and Pins. The lower chords of iron bridges usually cousist of flat links or bars *c* and o, W and H, Figs 61 on edge aud connected by tight-fitting wrought-irou pius *b* and P. After deciding on the size of the body W or H of the bars to bear safely the pull upon tbem, the proper proportioning of their heads or eyes and pius is an abstruse aud difficult poiut upon which much has been written. It was formerly supposed that the diam of the pin should be governed by its resistance to sbearing, but experience has shown tbat this was entirely insufficient.



Figs. 61.

We give a table of practical conclusions arrived at by that accomplished expert, Chs. Shaler Smith, from 111 experiments by himself on a working scale. Tbe table shows some irregularities, for as Mr. Smith remarks "the bars declined to break by formula." The pin is more strained at the outer links o o than at tbe inner ones c c c, so that the latter would not require so large a diam, but that this must be uniform throughout in order to secure tight fitting for all of them. When web members as well as chords are held by the same pin the diam and bead must he proportioned for that bar of them all which is most strained. When the heads are made by pressure in one piece with the body, the metal *v s* and *u x* at the sides of tbe pin *b* must be wider tban when the beads are first made in separate pieces by bammering and tben welded to the body. But tbe welded one W requires more iron *back* of tbe pin as shown at *l t*. This width *l t* must be equal to tbe diam of the pin.*

The links are supposed to be of uniform thickness. Having drawn a circle *b* for the pin, lay off on each side of it as at *v s*, *u x*, *half* the width of metal in the table for the head of W or H as the case may be. Then for forming the head of H use only the rad *b s* as shown. For the head of W lay off also *l t* = diam of pin. Find by trial the rad *g n* or *g t* and use it, except for uniting the head to the body, where use a rad = 1.5 *g n* as shown.

Width of bar.	Thicks. of bar.	Diam. of pin.	Metal in head across pin. W.	Metal in head across pin. H.	Width of bar.	Thicks. of bar.	Diam. of pin.	Metal in head across pin. W.	Metal in head across pin. H.
1.	.2	.67	1.33	1.50	1.	.55	1.28	1.50	1.60
1.	.25	.77	1.33	1.50	1.	.60	1.36	1.55	1.72
1.	.30	.86	1.40	1.50	1.	.65	1.43	1.60	1.76
1.	.35	.95	1.50	1.50	1.	.70	1.50	1.67	1.85
1.	.40	1.04	1.50	1.50	1.	.80	1.64	1.67	1.95
1.	.45	1.12	1.50	1.53	1.	.90	1 77	1.70	2.05
1.	.50	1.20	1.50	1.56	1.	1.00	1.90	1.76	2.21

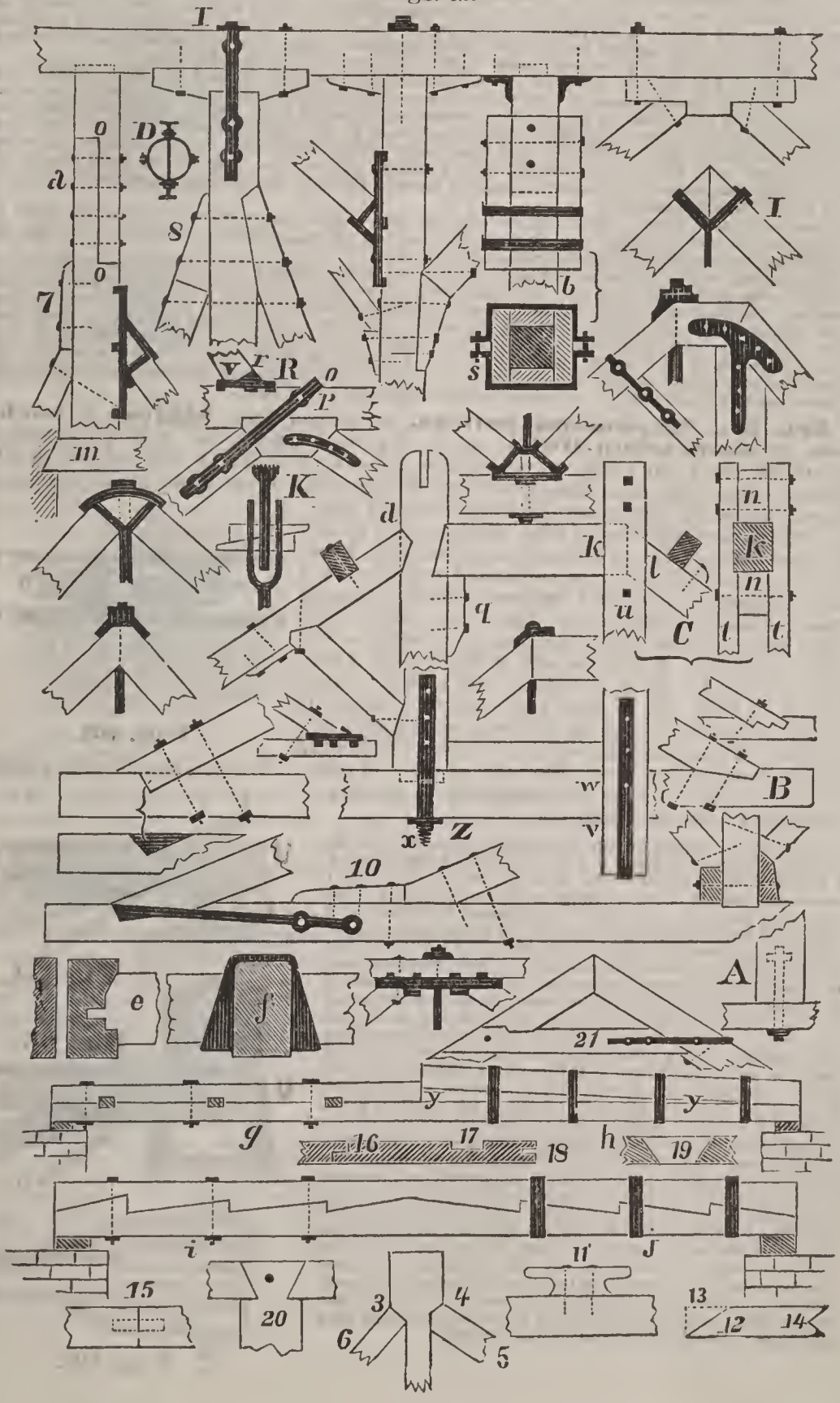
Art. 26. Figs 62 exhibit joints adapted to most of the cases that occur in practice with wooden beams, &c. They need bnt little explanation. Fig *a* is a good mode of splicing a post; in doing whieh the line *o o* should never be inclined or sloped, bnt be made vert; otherwise, in case of shrinkage, or of great pressnre, the parts on eaeh side of it tend to slide along each other, and thus bring a great strain upon the bolts. When greater strength is reqd, iron hoops may be used, as at *b*, *h*, and *j*, instead of bolts. Fig *b*, a post spliced by 4 fishing pieees; whieh may be fastened either by bolts, as in the npper part; or by hoops, as in the lower. The hoops may be tightened by flanges and screws, as at *s*; or thin iron wedges may be driven between them and the timbers, if necessary. Fig C shows a good strong arrangement for nniting a straining-beam *k*, a rafter *l*, and a queen-post *u*; by letting *k* and *l* abut against eaeh other, and confining them between a double queen-post *t t*; *n n* are two blocks through whieh the bolts pass. A similar arrangement is equally good for uniting the tie-beam *w*, with the foot *v*, of the queens; with the addition of a strap, as in the fig. Fig *e* is a method of framing one beam into another, at right angles to it. **An iron stirrup,** as at *f*, may be used for the same purpose; and is stronger. Figs *g h*, *i j* are **built beams.** When a beam or girder of great depth is required, if we obtain it by merely laying one beam flat

with other kinds, is a subject of great practical importance; but one which hitherto has received but little attention. Black walnut and cypress are said to cause mutual rot within a year or two. Observed cases of this kind should be reported to the leading professional journals.

* The strength of a given hinged-end pillar (see p 439) is increased to an important extent by enlarging the diameter of the pin.

upon another, we secure only as much strength as the two beams would have if separate. But if we prevent them from *sliding* on one another, by inserting transverse blocks or keys, as at *g*; or by indenting them into one another, as at *i j*; and

Figs. 62.



then bolt or strap them firmly together to create friction; we obtain nearly the strength of a solid beam of the total depth; which strength is as the *square* of the depth.

The strength of a built beam is increased by increasing its depth at its center, where it is most strained; as in the upper chords of a bridge. This may be done by adding the triangular strip *y y* between the two beams.

Tredgold directs that the combined thicknesses of all the **keys** be not less than 1.4 times the entire depth of the girder; or when indents are used, as in *i j*, that their combined depths be at least ⅗ that of the girder. If the girder *i j* be inverted, it will lose much of its strength.

A piece of plate-iron may be placed at the joints of timbers when there is a great pressure; which is thus more equalized over the entire area of the joint; or cast iron may be used.

Frequently a simple strap will not suffice, when it is necessary to draw the two timbers very tightly together. In such cases, one end of each strap may, as at *x*, terminate as a screw; and after passing through a cross-bar Z, all may be tightened up by a nut at *x*. Or the principle of the DOUBLE KEY, shown at K, may be applied. Sometimes, as at A, the hole for the bolt is first bored; then a hole is cut in one side of the timber, and reaching to the bolt-hole, large enough to allow the screw nut to be inserted. This being done, the hole is refilled by a wooden plug, which holds the nut in place. Then the screw-bolt is inserted, passing through the nut. By turning the screw the timbers may then be tightened together.

When the ends of beams, joists, &c, are inserted into walls in the usual square manner, there is danger that in case of being burnt in two, they may, in falling, overturn the wall. This may be avoided by cutting the ends into the shape shown at *m*.

When a strap *o*, Fig R, has to bear a strain so great as to endanger its crushing the timber *p*, on which it rests, a casting like *v* may be used under it. The strap will pass around the hack *r* of the casting. The small projections in the bottom being notched into the timber, will prevent the casting from sliding under the oblique strain of the strap. The same may be used for oblique bolts, and below a timber as well as above it. When below, it may become necessary to bolt or spike the casting to the under side of the timber. When the pull on a strap is at right angles to the timber, if there is much strain, a piece of plate-iron, instead of a casting, may be inserted between the strap and the timber, to prevent the latter from being crushed or crippled; see I and I.

Art. 27. Expansion rollers, or planed iron **Slides,** or **rockers,** or **suspension-links,** must be provided when an iron span exceeds about 80 feet; in order to allow the trusses to contract and expand freely under changes of temperature, without undue strain upon some of its members. Figure 63 shows the general arrangement of rollers; which are cylinders of cast iron or steel, from 3 to 6 ins diam; and 1 to 4 ft long; planed smooth. From 4 to 8 or more of these are connected together by a kind of framing, *n n*; and one such frame is placed under at least *one end* of the truss. The rollers rest upon a strong planed cast bed-plate *o o*; bolted to the masonry below. Under the end of the truss is a similar plate *s s*, by which it rests on the rollers. Since a truss of even 200 ft span will scarcely change its length as much as 3 ins by extremes of temperature, the play of the rollers is but small. They are kept in line by flanges cast along the side of the bed-plate. Flanges should also project downward from *s s*, so as completely to protect the rollers from dust, rain, &c.

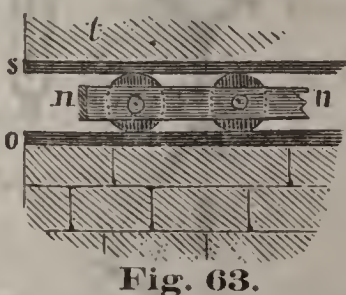


Fig. 63.

In Fig 64, *r r* gives a general idea of a ROCKER; and Fig 65, *s s*, of a SUSPENSION-LINK. U U in each fig is a side view of a cast-iron Fink upper chord, through each end of which passes a round pin o, which sustains the entire weight of the truss and its load; and which is sustained by from 4 to 6 rockers or links, as the case may be. In a railroad bridge of 205 ft span, across the Monongahela, the links are 3½ ft long; and the pins 5 ins diam; and in others of the same size, over Barren and other rivers, the rockers are a foot wide from *r* to *r*; and about 5 ins wide transversely on the curved tread or rim. For the accommodation of these several links and rockers; as well as of the various bars *b b*, which constitute the oblique ties of a Fink truss; the ends of the octagonal cast-iron upper chords are widened out, as shown by Fig 66; which is a top view of a longitudinal section of such an end. The rockers, or links, and bars, *b*, occupy the spaces *n n*, &c, between the several partitions of the chord; and the pin o o passes through them all, except when it is expedient to attach some of the bars to the sides, or to the top of the chord, as at *t*. These figs are intended merely to illustrate the general principle, without regard to detail of construction.

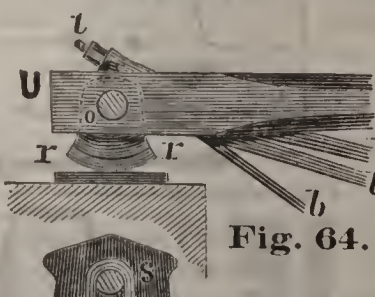


Fig. 64.

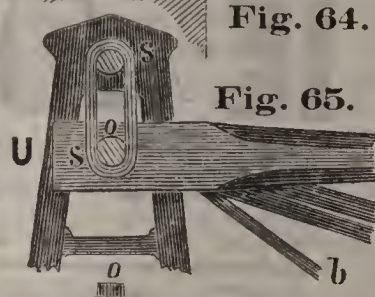


Fig. 65.

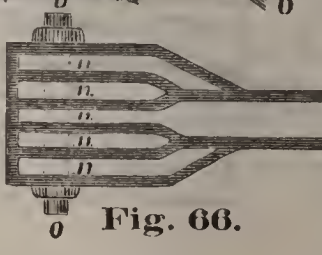


Fig. 66.

In some English bridges of considerable size, such as the Crumlin Viaduct, of 150 ft spans; and the Newark Dyke bridge, of 240 span; (both of them Warren girders,) and sustained like the foregoing, by the ends of the upper chords; no further precaution is taken with regard to expansion and contraction, than merely to rest the ends of said chords upon smoothly planed iron plates, upon which they may *slide*. So also several American bridges.

SUSPENSION BRIDGES.

Art. 1. Table of data required for calculating the main chains or cables of suspension bridges. Original.

Deflection in parts of the Chord.	Deflection in Decimals of the Chord.	Length of Main Chains between Suspension Piers, in parts of the Chord.	Tension on all the Main Chains at either Suspension Pier, in parts of the entire Suspended Wt. of the Bridge, and its Load.	Tension at the Center of all the Main Chains; in parts of the entire Suspended Wt. of the Bridge, and its Load.	Angle of Direction of the Chains at the Piers.	Natural Sine of the Angle of Direction of the Chains, at the Piers.	Natural Cosine of the Angle of Direction of the Chains at the Piers.
					Deg. Min.		
1-40	.025	1.002	5.03	5.00	5 43	.0995	.9950
1-35	.0286	1.002	4.40	4.37	6 31	.1135	.9935
1-30	.0333	1.003	3.78	3.75	7 36	.1322	.9912
1-25	.04	1.004	3.16	3.12	9 6	.1580	.9874
1-20	.05	1.006	2.55	2.51	11 19	.1961	.9806
1-19	.0526	1.007	2.43	2.38	11 53	.2060	.9786
1-18	.0555	1.008	2.30	2.25	12 32	.2169	.9762
1-17	.0588	1.009	2.18	2.12	13 14	.2290	.9734
1-16	.0625	1.010	2.06	2.00	14 2	.2425	.9701
1-15	.0667	1.012	1.94	1.87	14 55	.2573	.9633
1-14	.0714	1.013	1.82	1.74	15 57	.2747	.9615
1-13	.0769	1.016	1.70	1.62	17 6	.2941	.9558
1-12	.0833	1.018	1.57	1.49	18 33	.3180	.9480
1-11	.0919	1.022	1.46	1.37	19 59	.3418	.9398
1-10	.1	1.026	1.35	1.25	21 48	.3714	.9285
1-9	.1111	1.033	1.23	1.12	23 58	.4062	.9138
⅛	.125	1.041	1.12	1.00	26 33	.4471	.8915
1-7	.1429	1.053	1.01	.881	29 45	.4961	.8726
3-20	.15	1.058	.972	.833	30 58	.5145	.8574
⅙	.1667	1.070	.901	.750	33 41	.5547	.8320
1-5	.2	1.098	.800	.625	38 40	.6247	.7808
	.225	1.122	.747	.555	42 0	.6690	.7433
¼	.25	1.149	.707	.500	45 00	.7071	.7071
.3	.3	1.205	.651	.417	50 12	.7682	.6401
⅓	.3333	1.247	.625	.375	53 8	.8000	.6000
.4	.4	1.332	.589	.312	58 2	.8483	.5294
9-20	.45	1.403	.572	.278	60 57	.8742	.4855
½	.5	1.480	.559	.250	63 26	.8944	.4472

These calculations are based on the assumption that the curve formed by the main chains is a parabola; which is not strictly correct. In a finished bridge, the curve is between a parabola and a catenary; and is not susceptible of a rigorous determination. **It may save some trouble in making the drawings** of a suspension bridge, to remember that when the deflection does not exceed about $\frac{1}{10}$ of the span, a segment of a circle may be used instead of the true curve; inasmuch as the two then coincide very closely; and the more so as the deflection becomes less than $\frac{1}{10}$. The dimensions taken from the drawing of a segment will answer all the purposes of estimating the quantities of materials.

For some particulars respecting wire for cables, see pages 412 and 413.

The deflection usually adopted by engineers for great spans is about $\frac{1}{12}$ to $\frac{1}{15}$ the span. As much as $\frac{1}{10}$ is generally confined to small spans. The bridge will be stronger, or will require less area of cable, if the deflection is greater; but it then undulates more readily; and as undulations tend to destroy the bridge by loosening the joints, and by increasing the momentum, they must be specially guarded against as much as possible. The usual mode of doing this is by trussing the hand-railing; which with this view may be made higher, and of stouter timbers than would otherwise be necessary. In large spans, indeed, it may be supplanted by regular bridge-trusses, sufficiently high to be braced together overhead, as in the Niagara Railroad bridge, where the trusses are 18 ft high; supporting a single-track railroad on top; and a common roadway of 19 ft clear width, below.*

* The writer believes himself to have been the first person to suggest the addition of very deep trusses braced together transversely, for large suspension bridges. Early in 1851, he designed such a bridge, with four spans of 1000 ft each: and two of 500; with wire cables; and trusses 20 ft high. It was intended for crossing the Delaware at Market Street, Philada. It was publicly exhibited for several months at the Franklin Institute, and at the Merchants' Exchange; and was finally stolen from the hall of the latter. Mr Roebling's Niagara bridge, of 800 ft span, with trusses 18 ft high, was not commenced until the latter part of 1852; or about 18 months after mine had been publicly exhibited.

Another very important aid is found in deep longitudinal floor timbers, firmly united where their ends meet each other. These assist by distributing among several suspender-rods, and by that means along a considerable length of main cable, the weight of heavy passing loads; and thus prevent the undue undulation that would take place if the load were concentrated upon only two opposite suspenders. With this view, the wooden stringers under the rails on the Niagara bridge are made virtually 4 ft deep. The same principle is evidently good for ordinary trussed bridges.

Another mode of relieving the main cables is by means of *cable-stays;* which are bars of iron, or wire ropes, extending like c *y*, Fig 1, from the saddles at the points of suspension c, *d*, obliquely down to the floor, or to some part of the truss. In the Niagara bridge are 64 such stays, of wire ropes of 1⅜ inch diam; the longest of which reach more than quarter way across the span from each tower. They transfer much of the strain of the wt of the bridge and its load directly to the saddles at the top of the towers: thereby relieving every part of the main cable, and diminishing undulation. They end at c and *d*, where they are attached, not to the cables, but to the saddles. They of course do not relieve the *back stays*.

The greatest danger arises from the action of strong winds striking below the floor, and either lifting the whole platform, and letting it fall suddenly; or imparting to it violent wavelike undulations. The bridge of 1010 ft span across the Ohio at Wheeling, by Charles Ellet, Jr, was destroyed in this manner. It is said to have undulated 20 ft vertically before giving way. It had no effective guards against undulation; for although its hand-railing was trussed, it was too low and slight to be of much service in so great a span. Many other bridges have been either destroyed or injured in the same way. When the height of the roadway above the water admits of it, the precaution may be adopted of tie-rods, or anchor rods, under the floor at different points along the span, and carried from thence, inclining downward, to the abutments, to which they should be very strongly confined. In the Niagara Railroad bridge 56 such ties, made of wire ropes 1¼ inch diam, extend diagonally from the bottom of the bridge, to the rocks below. They, however, detract greatly from the dignity of a structure.

Mr Brunel, in some cases, for checking undulations from violent winds striking *beneath* the platform, used also *inverted* or *up-curving* cables under the floor. Their ends were strongly confined to the abuts several ft below the platform; and the cables were connected at intervals, with the platform, so as to hold it down.

Art. 2. The angle *a d g*, or *a c i*, Fig 1, which a tang *d g* or *c i* to the curve at either point of suspension c or *d*, forms with the *hor* line c *d* or chord, is called **the angle of direction of the main chains,** or cables, at those points. Frequently the ends *c h*, and *d r*, of the chains, called the **backstays.** are carried away from the suspension piers in straight lines; in which case the angles *l d r*, *e c h*, formed between the hor line *e l* and the chain itself, become the angles of direction of the backstays.

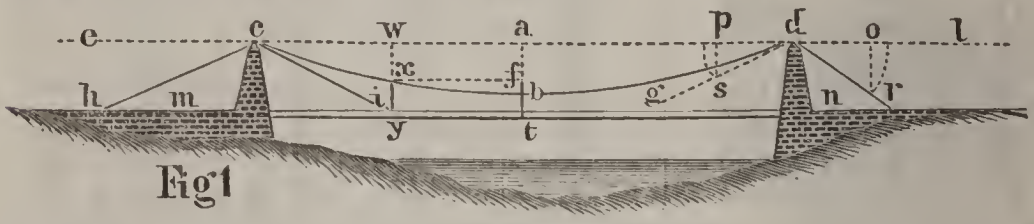


Fig 1

$$\textbf{Sine of angle of direction } a\,d\,g = \frac{\text{Twice the deflection } a\,b}{\sqrt{(\text{twice the deflection})^2 + (\text{Half the chord})^2}}$$

Note 1. The direction of the tang *d g* or c *i*, can be laid down on a drawing, thus: Continue the line *a b*, making it twice as long as *a b*; then lines drawn from *d* and c to its lower end, will be tangs to the parabolic curve at the points of suspension.

Note 2. **If the chord *c d* be not hor,** as sometimes is the case, the angle must be measured from a hor line drawn for the purpose at each point of suspension; as the two angles will in that case be unequal, the piers being of unequal heights.

$$\textbf{Tension on all the main chains or cables, together, at either one of the piers, } c \textbf{ or } d\textbf{, Fig 1.} = \frac{\text{Half the entire suspended weight of the clear span and its load}}{\text{Sine of angle of direction } a\,d\,g}$$

$$\textbf{or} = \frac{\sqrt{(\frac{1}{2}\text{ Span})^2 + (2\text{ Defl})^2}}{2\text{ Deflection}} \times \text{Half the entire suspended weight of the clear span and its load.}$$

$$\textbf{Tension on all the main chains or cables, together, at the middle, } b\textbf{, of the span, Fig 1.} = \frac{\text{Half the entire suspended weight of the clear span and its load} \times \text{Cosine of angle of direction } a\,d\,g}{\text{Sine of angle of direction } a\,d\,g}$$

$$\textbf{or} = \frac{\text{Half the entire suspended weight of the clear span and its load} \times \text{Half the span}}{\text{Twice the deflection}}$$

The diff between the tensions at the middle, and at the points of suspension, is so trifling with the proportion of chord and deflection commonly adopted in practice, viz, from about $\frac{1}{10}$ to $\frac{1}{16}$, that it is usually neglected; inasmuch as the saving in the weight of metal would be fully compensated for by the increased labor of manufacture in gradually reducing the dimensions of the chains from the points of suspension toward the middle; and in preparing fittings for parts of many different sizes. The reduction has, however, been made in some large bridges with wrought-iron main chains; but in none with wire cables.

Art. 2A. As it is sometimes convenient to form a rough idea at the moment, of the size of cables required for a bridge, we suggest the following rule for finding approximately the area in sq ins of *solid* iron in the wire required to sustain, with a safety of 3,* the weight of the bridge itself, together with an extraneous load of 1.205 tons per foot run of span; which corresponds to 100 lbs per sq ft of platform of 27 ft *clear available* width. This suffices for a double carriage-way, and two footways. The deflection is assumed at $\frac{1}{12}$ of the span; and the wire to have an ultimate strength of 36 tons per solid square inch, as per table, page 412; and which can be procured without difficulty. **For spans of 100 ft or more,**

Rule. Mult the span in feet, by the square root of the span. Divide the prod by 100. To the quot add the sq rt of the span. Or, as a formula,

$$\begin{matrix}\textit{Area of solid metal of all}\\ \textit{the cables; in square ins;}\\ \textit{for spans over 100 feet}\end{matrix} = \frac{\textit{span} \times \textit{sq rt of span}}{100} + \textit{sq rt of span.}$$

For spans less than 100 feet, proportion the area to that at 100 ft.

If a defl of $\frac{1}{10}$ is adopted instead of $\frac{1}{12}$, the area of the cables may be reduced very nearly $\frac{1}{7}$ part.

The following table is drawn up from this rule. The 3d col gives the united areas of all the actual wire cables, when made up, including voids. (Original.)

Span in Feet.	Solid Iron in all the Cables.	Areas of all the Finished Cables.	Span in Feet.	Solid Iron in all the Cables.	Areas of all the Finished Cables.	Span in Feet.	Solid Iron in all the Cables.	Areas of all the Finished Cables.
	Sq. Ins.	Sq. Ins.		Sq. Ins.	Sq. Ins.		Sq. Ins.	Sq. Ins.
1000	348	446	400	100	128	150	30.6	39.2
900	300	385	350	84	108	125	25.2	32.3
800	254	326	300	69	89	100	20	25.6
700	212	272	250	55	71	75	15	19.2
600	171	219	200	42	54	50	10	12.8
500	134	172	175	36.4	46.7	25	5	6.4

Having the areas of all the actual cables, we can readily find their diam. Thus, suppose with a span of 500 ft, we intend to use four cables. Then the area of each of them will be $\frac{172}{4} = 43$ sq ins; and from the table of circles, we see that the corresponding diam is full 7⅜ ins.

The above areas are supposed to allow for the increased wt of a depth of truss, and other additions necessary to secure the bridge from violent winds, and from undue vibrations from passing loads.

When these considerations are neglected, and a less maximum load assumed, the following descriptions of the Wheeling and Freyburg bridges show what reductions are practicable. Weight, sufficiently provided for, is of great service in reducing undulation.

* The writer must not be understood to advocate a safety of 3 against 100 lbs per sq ft, in addition to the weight of the bridge, in all cases. He believes that limit to be about a sufficient one for a properly designed wire suspension bridge for ordinary travel; but for an important railroad bridge, he would (according to position, exposure, &c) adopt a safety of at least from 4 to 6 against the greatest possible load, added to the wt of the bridge. A train of cars opposes a great surface to the action of side winds; and trains must run during violent storms, as well as during calms; but a large open bridge for common travel is not likely to be densely crowded with people during a severe storm.

Art. 3. Tension on the back-stays, *c h* and *d r*, Fig 1. and strains on the piers, or towers, or pillars. If the angle of direction *a d g* and the angle *l d r*, between the back-stays and the horizontal, are equal to each other, the tension on the back-stays will be equal to that on the main cables at the tops of the piers; and the pressure on the piers will be vertical; but if the two angles are unequal, then these tensions and pressures will depend, to a very important extent, upon the manner in which the chains or cables are fixed to, or laid upon, the tops of the piers.

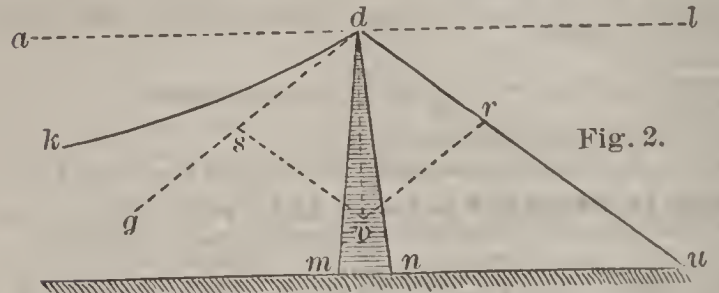


Fig. 2.

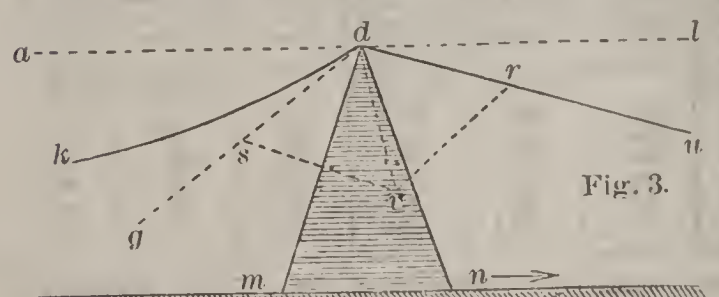


Fig. 3.

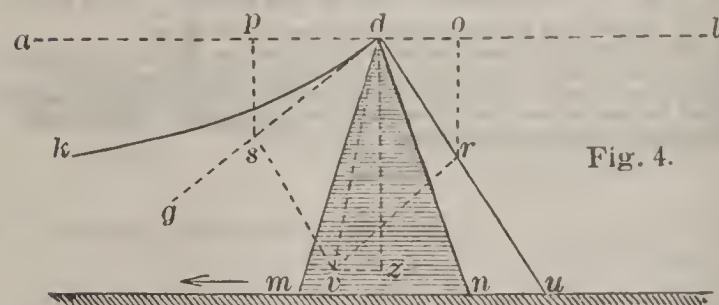


Fig. 4.

Art. 4. In Figs 2, 3 and 4, the piers *d n m* are supposed to be immovable; and the cables *k d u*, passing over them, rest immediately upon horizontal rollers, *which have no other motion than that of revolving about their horizontal axes; the frame to which they are attached being bolted to the top of the pier.* On these rollers the cables slide, when changes of loading or of temperature produce changes in their directions.

In this case the **tension on the back-stays** is equal to that on the main cable. See Funicular Machine.

To find the direction and amount of the **pressure on the pier;** from *d*, Fig 2, 3 or 4, lay off *d s* and *d r*, each equal, by scale, to the tension, in tons, on the main chain at *d*; and from *s* and *r* lay it off to *v*. In other words, draw the parallelogram *d s v r*, and its diagonal *d v*. Then will *d v* give the *direction* and *amount* of the pressure upon the pier.

When, as in Fig 2, the angles *a d g* and *l d u* are *equal*, the pressure *d v* will be vertical, and equal to the *entire* weight of the clear span and its load.

When, as in Figs 3 and 4, the angles *a d g* and *l d u* are *unequal*, the pressure *d v* will not be vertical, but will incline from *d* toward the smaller angle.

When, as in Fig 3, *a d g* exceeds *l d u*, the pressure *d v* will be *less* than the entire weight of the clear span and its load.

When, as in Fig 4, *l d u* exceeds *a d g*, the pressure *d v* will be *greater* than the entire weight of the clear span and its load.

If we suppose symmetrical piers, *d n m*, to be used in each case, the base *m n* of that in Fig 2, may be much narrower than in the other two figs; because, the direction of *d v* being vertical, the pressure has no tendency to overturn the pier. In Fig 2, the masonry of the pier should be laid in the usual horizontal courses, in order that its bed joints may be at right angles to the pressure upon them.

But, in Figs 3 and 4, if the bases were made as narrow as in Fig 2, the lines of direction *d v*, of the pressure, would fall outside of them; and the piers would consequently be in danger of overturning. Also, the stones of the masonry, if laid in horizontal courses, would have a tendency to slide on each other. To prevent this, the beds should be at right angles to *d v*.

In Fig 3, the obliquity of the pressure would tend to slide the base of the pier *outward* as shown by the arrow; but in Fig 4, *inward*. This tendency is produced by the *horizontal* component of the force *d v*. The amount of this may be found thus, in either fig: From *d* downward draw a vert line as in Fig 4; and from *v* a hor one, meeting it in *z*, then *v z*, measured by the same scale of tons as before, will give this horizontal force, and *d z* will give the vertical component of the pressure *d v*. The effect upon the pier, of the one pressure *d v* is precisely the same as would be produced upon it by one vertical force equal to *d z* and a horizontal one equal to *v z* acting at the same time, as explained under Composition and Resolution of Forces.

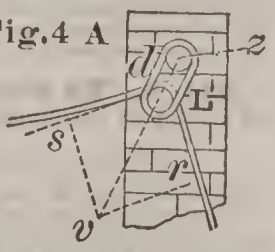
If, in either fig, we draw the vert lines *s p* and *r o*, see Fig 4, then *d o*, measd by the foregoing scale, will give the tons of horizontal pull, and *r o* the vertical pressure, produced on the pier by the back-stay; and *p d* and *p s* will, in like manner, give the corresponding forces produced by the main chain. If we add together *r o* and *p s* they will be found to be equal to *d z*, and if we subtract *d o* from *p d*, their difference will equal *v z*. It is this difference only that tends to slide, or to upset, the pier; the other portions of *d o* and *p d* neutralizing each other in that respect.

The foregoing strains may all be *calculated*, thus:

Horizontal pull inward by the main chain = Tension × Cosine of *a d g*
" " **outward by the back-stay** = Tension × Cosine of *l d u*.
Vertical pressure by main chain = Tension × Sine of *a d g*.
" " " **back-stay** = Tension × Sine of *l d u*.

Art. 5. If the cables pass freely over a loose pin, d, Fig 4 A, supported by a link L, hanging from the fixed pin z, and capable of moving freely about both of its pins; the tension in the back-stay will, as before, be equal to that in the main cable; and the direction and amount of the strain on the piers will be found in the same way as for Figs 2, 3 and 4; namely: lay off $d\,s$ and $d\,r$, each equal to the tension, and draw the parallelogram $d\,s\,v\,r$. Then will $d\,v$ give the amount and direction of the strain on the piers. This last will, of course, be transmitted through the pins and the link. The amount of tension on the link will be given by the length of $d\,v$; and the link (being free to move) will be in line with this tension. The shearing strain on each pin is also given by $d\,v$.

Fig.4 A



Art. 6. But if the ends of the cable and back-stay, Figs 4 B, 4 C and 4 D, at the top of the pier, be *made fast* to a truck or wagon which is supported by rollers on a smooth platform on top of the pier, the axles of the rollers being fixed in the truck; then the strain on the back-stay will not be the same as that on the cable, unless the angles $a\,d\,g$ and $l\,d\,u$ are equal, as in Fig 4 B.

If $a\,d\,g$ exceeds $l\,d'\,u$, as in Fig 4 C, the strain on the back-stay will be less than that on the cable, and vice versa (Fig 4 D).

But, in either case, if the top of the pier is horizontal, as is usually the case, the horizontal components of the strains on the cables and on the back-stays, will be equal, and will thus counteract each other, and there will consequently be no horizontal or oblique strain on the pier. That is, the strain on the pier will be vertical.

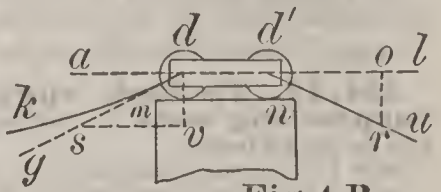


Fig.4 B

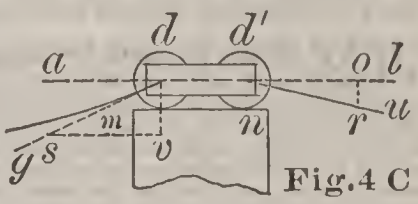


Fig.4 C

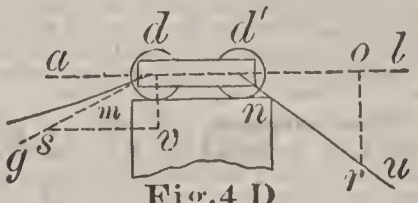


Fig.4 D

To find the amount of the tension on the back-stay, and of the **pressure on the pier;** on $d\,g$ in either Fig. 4 B, 4 C or 4 D, lay off $d\,s$, equal, by scale, to the tension on the cable at d. Draw $d\,v$ perpendicular to the surface $m\,n$ on which the rollers rest. We assume that $m\,n$ is horizontal, as is generally, *but not necessarily*, the case; and $d\,v$, therefore, vertical. Draw $s\,v$ horizontal, or parallel to $m\,n$.*

Then $s\,v$ will give the horizontal pull of the main cable on the wagon, and $d\,v$ will give the vertical pressure of the wheel d on the tower (to which that of the wheel d' has yet to be added). From d' lay off $d'\,o$ horizontal, and equal to $s\,v$; and draw $r\,o$ vertically. Then $d'\,r$ will give the amount of the pull on the back-stay; and $r\,o$ will give the vertical pressure of the wheel d' on the pier; which must be added to $d\,v$ for the *total* vertical pressure.

Or the various strains may be *calculated*, thus:

Horizontal pull $s\,v$ or $d'\,o$ at the top of the pier } = Horizontal pull at middle of span = Tension $d\,s$ in cable at d × Cosine of $a\,d\,g$.

Strain $d'\,r$ in back-stay } = Horizontal pull $s\,v$ or $d'\,o$ at top of pier, or at middle of span ÷ Cosine of $l\,d'\,u$.

Pres on pier, perp to surf on which the rollers rest } $= d\,v + r\,o = \left(\begin{matrix}\text{Tension } d\,s \\ \text{on main} \\ \text{cable at } d\end{matrix} \times \begin{matrix}\text{Sine of} \\ a\,d\,g\end{matrix}\right) + \left(\begin{matrix}\text{Tension } d'\,r \\ \text{on back-stay}\end{matrix} \times \begin{matrix}\text{Sine of} \\ l\,d'\,u\end{matrix}\right)$

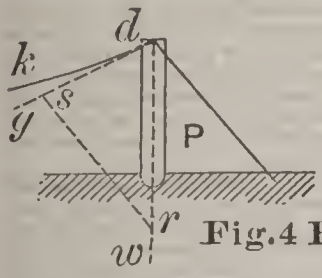


Fig.4 E

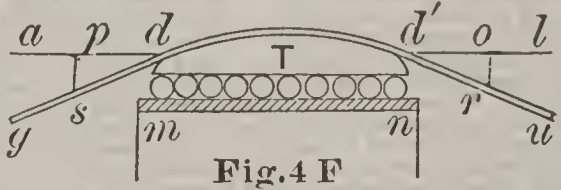


Fig.4 F

Art. 7. When, as is sometimes the case in light bridges, the piers are posts, P Fig 4 E, of wood or iron, hinged at the bottom, and having the cables and back-stays firmly fixed to their tops; from d draw $d\,s$, equal, by scale, to the tension on the main cable at d; and $d\,w$ toward the foot of the post. From s draw $s\,r$ parallel to the back-stay, and meeting $d\,w$ in r. Then will $s\,r$ give the strain in the back-stay, and $d\,r$ will give the amount and direction of the pressure upon the post.

Art. 8. As in the Niagara bridge, the cables often merely rest upon movable trucks, or saddles, T Fig 4 F, curved on top to avoid sudden bends in the cables, and resting upon loose rollers which lie upon a thick horizontal iron plate bolted to the top of the pier, and are free to move horizontally. In such cases the angles $a\,d\,g$ and $l\,d'\,u$ are made equal; so that the pulls

* The lines $a\,l$ and $s\,v$ must be drawn *parallel to the surface in or on which the wagon rests*, whether said surface be horizontal or inclined.

$d\ s$ and $d'\ r$ are equal, as are also their horizontal components $p\ d$ and $d'\ o$; and the pressures on the pier are vertical; and if changes of temperature or of loading produce slight changes in the angles $a\ d\ g$ and $l\ d'\ u$, the truck will (by reason of the inequality thus brought about between the horizontal components) move far enough to restore the equality between the angles, and between the horizontal components, and consequently the pressure upon the pier will at all times be vertical.

Art. 9. To find, approximately, the length of a main chain $c\ b\ d$, Fig. 1; having the span $c\ d$, and the middle defl $a\ b$. See preceding table, Art 1.

$$\textit{Half length of main chain} = \sqrt{1\tfrac{1}{3}\ (\text{defl}^2) + (\tfrac{1}{2}\ \text{chord})^2}.$$

In Menal bridge the chord $c\ d$ is 579.874 ft; and the defl is 43 ft.

According to the above formula, the entire length is 588.3 feet. By actual measurement the chain is precisely 590 feet. The approximate rule below gives 589.764 ft.

Note. The lengths obtained by this rule are only approximate, because the calculation is based upon the supposition that the chains form a parabolic curve; whereas, in fact, the curve of a finished bridge is neither precisely a parabola, nor a catenary, but intermediate of the two.

The following simple rule by the writer is quite as approximate as the foregoing tedious one, when, as is generally the case, the defl is not greater than $\frac{1}{12}$ of the chord, or span.

Length of main chain when defl does not exceed one-twelfth of the span = chord + .23 defl.

Art. 10. To find, approximately, the length of the vert suspending rods $x\ y$, &c, Fig 1; **assuming the curve to be a parabola.**

Let x, Fig 1, be any point whatever in the curve; and let $x\ w$ be drawn perp to the chord $c\ d$: and $x\ f$ perp to $a\ b$; then in any parabola, as $a\ c^2 : a\ w^2 :: a\ b : b\ f$. And $b\ f$ thus found, added to $b\ t$, (which is supposed to be already known, being the length decided on for the middle suspending rod,) gives $x\ y$, the length of rod reqd at the point x; and so at any other point.

If $b\ f$ thus found be taken from the middle deflection $a\ b$, it leaves $w\ x$: and thus any deflection $w\ x$ of the main chain or cable, may be found when we know its hor dist, $a\ w$, from the center, a, of the span.

In the foregoing rule, the floor of the bridge is supposed to be straight; but generally it is raised toward the center; and in that case, the rods must first be calculated as if the floor were straight, and the requisite deductions be made afterward. When it rises in two straight lines meeting in the center, the method of doing this is obvious. When an arc of a circle is used, its ordinates may be calculated by the rule given on page 141, and deducted from the lengths obtained by this rule. Or, having drawn the curve by the rule for drawing a parabola, the dimensions can be approximated to by a scale. The adjustments to the *precise* lengths must be made during the actual construction of the bridge, by means of nuts on their lower screw-ends. The rods require, therefore, only to be made *long enough* at first.

The towers, piers, or pillars, which uphold the chains or cables, admit of an endless variety in design. According to circumstances, they may consist each of a single vertical piece of timber, or a pillar of cast or wrought iron; or of two or more such, placed obliquely, either with or without connecting pieces; like the bents of a trestle. Or they may be made (with any degree of ornamentation) of cast-iron plates; as in iron house-fronts. Or they may be of masonry, brick, or concrete; or of any of these combined.

Each of the suspending-rods, through which the floor of the bridge is upheld by the main chains, requires merely strength sufficient to support safely the greatest load that can come upon the interval between it and half-way to the nearest rod on each side of it; including the wt of the platform, &c, along the same interval.

In anchoring the backstays into the ground, it is necessary to secure for them a sufficiently safe resistance against a pull equal to the strain upon the backstay.

As to the anchorage of the cables below the surface of the ground, natural rock of firm character is the most favorable material that can present itself. When it is not present, serious expense in masonry must be incurred in large spans, in order to secure the necessary weight to resist the pull of the cables. Our Figs 4½ give ideas of the modes most frequently adopted. For a very small bridge, such as a short foot-bridge, for instance, the backstays may simply be anchored to large stones, t, Fig A, buried to a sufficient depth. Or, if the pull is too great for so simple a precaution, the block of masonry, $m\ m$, may be added, enclosing the backstay. A close covering of the mortar or cement of the masonry has a protecting effect upon the iron.

To avoid the necessity for extending the backstays to so great a dist under ground, they are usually curved near where they descend below the surface, as shown at B, D, and E; so as sooner to reach the reqd depth. This curving, however, gives rise to a new strain, in the direction shown by the arrows in Figs B and D. The nature of this strain, and the mode of finding its amount, (knowing the pull on the backstay,) are very simple; and fully explained under the head of Funicular Machine. The masonry must be disposed with reference to resisting this strain, as well as that of the direct pull of the backstay. With this view, the blocks of stone on which the bend rests should be laid in the position shown in Fig D; or by the single block in Fig B. Sometimes the bend is made over a cast-iron chair or standard, as at x, Fig F, firmly bolted to the masonry.

Fig E shows the arrangement at the Niagara railway bridge of 821½ ft span. The *wire* backstays end at $c\ c$; and from there down to their anchors, they consist of heavy chains; each link of which is composed of (alternately) 7 or 8 parallel bars of flat iron, with eye ends, through which pass bolts *. Each of the 7 bars of each link is 1.4 ins thick, by 7 ins wide, near the

* When chains of iron bars are used instead of wire cables, they are usually made as at p 612. Since bar iron has but about half the tensile strength of wire, the chains must have a sectional area twice as great as that of a cable.

lowest part of the chain; but they gradually increase from thence upward, until at c, c, where they unite with the wire cable, the sectional area of each *link* is 93 sq ins. These chain backstays pass in a curve through the massive approach walls, (28 ft high,) and descend vertically down shafts *s*, *s*, 25 ft deep in the solid rock. Here they pass through the cast-iron anchor-plates, to which they are confined below by a bolt 3½ ins diam. The anchor-plates are 6½ feet square, and 2½ ins thick; except for a space of about 20 ins by 26 ins, at the center where the chains pass through, where they are 1

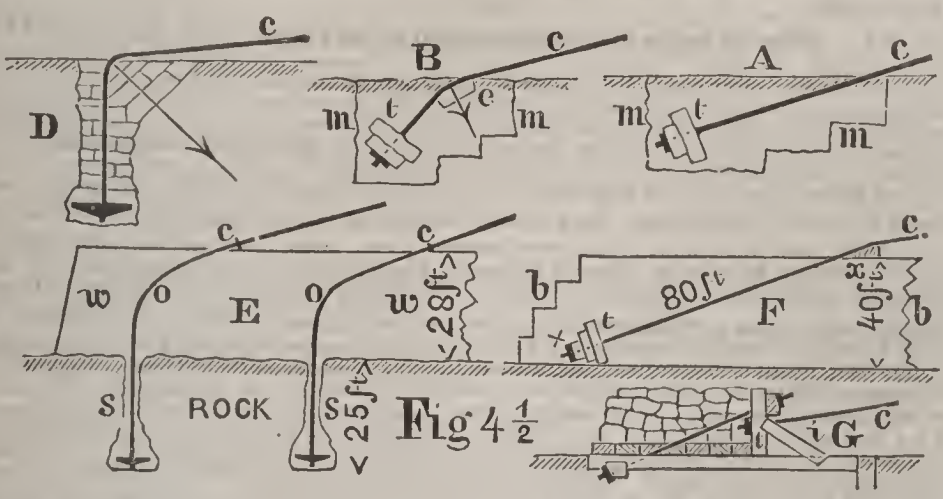


Fig 4 ½

foot thick. Through this thick part is a separate opening for each bar composing the lowest link. From this part also radiate to the outer edges of the lower face of the plate, eight ribs, 2½ ins thick. The shafts *s*, *s*, have rough sides, as they were blasted; and average 3 ft by 7 ft across; except at the bottom, where they are 8 ft square. They are completely filled with cement masonry, with dressed beds, well in contact with the sides of the shafts; and thoroughly grouted, thus tightly envelopiug the chaius at every point; as does also the masonry of the approach wall *w w*; which extends 28 ft above ground; and is 6 ft thick at top, aud 10½ ft thick at its base on the natural rock.

D, Figs 4½, shows a mode that may be used in most cases, for bridges of any span. The depth and the area of transverse section of the shaft, and consequently the quantity of masonry in it, will depend chiefly upon whether it is sunk through rock, or through earth. If through firm rock, theu if its sides be made irregular, and the masonry made to fit securely into the irregularities, much reliance may be placed upon it to assist the weight of the masonry in resisting the pull on the backstays. Earth also assists materially in this respect.

F is the arrangement in the Chelsea bridge of 333 feet span, across the Thames, at London; Thos. Page, eng. The space from one wall *b b*, to the opposite one, is 45 feet; and is built up solid with brickwork and concrete; except a passage-way 4 ft wide, and 5 ft high, along the backstay; and a small chamber behind the anchor-plates. It rests chiefly on piles.

The arrangement by Mr Brunel, in the Charing Cross bridge, London, is very similar. In it also the entire abutment rests on piles; and is 40 ft high, 30 ft thick, and solid, except a narrow passage-way along the chains. The backstays extend into it 60 ft. Span 676 feet. Dell 50 feet.

G is intended merely as a general hint, which, variously modified, may find its application in the case of a small temporary, or even permanent bridge; for the number of pieces, *i*, *t*, &c, may be increased to any necessary extent; and they may be made of iron or stone, instead of wood.

In estimating the action of the backstays upon the masonry, &c, to which they are anchored, it is safest to consider the tension aloug them to continue undiminished to their very ends; although, when they are embedded in masonry, friction causes it to diminish; especially when they are curved, as in E, Figs 4½, in which case the friction is greatly increased, and the tension thereby materially reduced as the ends are approached. Frequently, however, they are not so embedded; for, although embedding preserves the iron, many engineers prefer to leave an open space around the entire length of the anchor-chains; as well as arouud the anchor-plates; in order that they may be examined from time to time. To this end, the masses *m m* of masonry in Figs 4½, may be made not solid, but to consist of two parallel walls, between which the backstay may pass; and the anchor-stones, or anchor-plates, will extend across the space between the walls, and have their bearing against the ends of the walls. In F, the cable may be supposed either to be tightly surrounded by the masonry, and grouted to it; or else to be surrounded by a cylindrical passage-way like a culvert, so as to be at all times accessible. And the same with regard to the cable in the vertical shafts at D or E.

Art 35, Art 49, &c, of Force in Rigid Bodies, will assist in calculating the resistance which the masses *m m* of anchorage masonry oppose to the pull of the backstays. Soft friable stone must be carefully excluded from such parts of these masses as are most directly opposed to this pull.

If blocks of stone large enough for securing good bond are not procurable, heavy bars of iron, or I beams, may be advantageously introduced for that purpose.

The masses must be founded at such a depth as not to slide by the yielding of the earth in front of them.

Experience shows that with due attention to periodical painting, and renewal of woodwork, a properly designed suspension bridge will be very durable. The transverse floor joists should be of wrought iron; to prevent interruption to travel while putting in new wooden ones.

Particular care should be bestowed upon the strength of the joints of the side parapets; for the undulations and lateral motions of the bridge expose them to violent deranging forces in every direction. The parapets should be high and stout; and not restricted to mere service as hand-rails, or guards.

As a rule of thumb, one-half the sq rt of the span will be about a good height for them in ordinary cases, provided it is not *less* than a hand-rail requires.

We do not think that diagonal horizontal bracing should, as is usual, be omitted under the floor. It may readily be effected by iron rods.

All the cables need not be at the *sides* of the bridge. One or more of them may be over its axis; especially in a wide bridge. One wide footpath in the center may be used, instead of two narrow ones at the sides.

The platform or roadway should be slightly cambered, or curved upward, to the extent say of about $\frac{1}{200}$ of the span.

Art. 11. The Niagara suspension bridge, built in 1852–3,*John A Roebling, engineer, consists of a single span of 821½ ft measured straight from center to center of towers; and 800 ft of clear suspended length of roadway. It has two floors or roadways: the upper one, for a single-track railway, is 25½ ft; and the lower one, for common travel, 24½ ft wide, out to out of everything. The lower one is 19 ft wide in the clear of everything. They are 17 ft apart vertically. The trusses are 18 ft total height, throughout. They are on the Pratt arrangement; with verticals 5 ft apart from cen to cen; and single oblique iron rods, 1 inch square, running in each direction across four of the 5 ft panels. Where these rods pass each other, they are tied together by 10 or 12 turns of $\frac{1}{16}$ inch wire. Each vertical consists of two pieces of 4¼ by 6¼ timber, placed 4½ ins apart, to allow the oblique rods to pass between them. Both upper and lower floor girders are in two pieces, of 4 by 16 ins each. Pairs 5 ft apart. The tops and bottoms of the verticals pass between the two pieces which form each floor girder. No tenons or mortises are used in the framing.

There are four cables of iron wire; two on each side of the bridge. Each cable is 10 ins diam. The wire is scant No. 9 of the Birmingham wire gauge, or scant .148 inch diam. Sixty wires have a united transverse section equal to one square inch of *solid* iron. Each of the four cables contains 3640 wires, with a united cross-section of 60.4 sq ins of solid metal. Therefore, the area of solid metal in a section of all the four cables together is 241.6 sq ins, or 1.678 sq ft; weighing 814 lbs per ft of span. The wires of each cable are first made up, in place, into 7 small strands; and these are firmly bound together throughout by a continuous close wrapping of wire. The strength of each individual wire is 1640 lbs, or .73214 of a ton. This is equal to 98400 lbs, or 43.93 tons per sq inch of solid metal; or to 5943360 lbs, or 2653.3 tons per cable; or to 10613.2 tons ultimate strength of the four cables together. One cable on each side of the bridge deflects 54 ft; and the other 64 ft; average deflection 59 ft, or about $\frac{1}{14}$ of the span. With this av defl the tension on the cables at the tops of the towers averages 1.82 times the total suspended wt of the span and its load. See table, Art 1. The wt of the suspended span itself is about 900 tons; and if the greatest extraneous load on the two floors together be taken at 1¼ tons per ft run, we have the total suspended wt $900 + (800 \times 1\frac{1}{4}) = 1900$ tons. And $1900 \times 1.82 = 3458$ tons tension at towers; or very nearly ½ of the ultimate strength of the cables, without any allowance for momentum, or wind. But such loads, although possible, are not permitted to come upon the bridge.

The wires were perfectly oiled before being made into strands; and when the strands were being bound together to form a cable, the whole was again thoroughly saturated with oil and paint.

The cables do not hang vertically; but the two upper ones are about 37 ft apart from center to center, where they rest upon the towers, (where all four are on the same level;) and are drawn to within 13 ft of each other at the center of the span; and at the level of the railway track on top of the bridge; while the two lower ones are about 39 ft apart at the towers, and 25 ft at the center of the span, and at the level of about halfway between the two floors.

This drawing-in of the cables contributes much to lateral stability; as do also the upper and lower floor of stout plank. There is no horizontal diagonal floor bracing.

There are 624 suspenders of wire rope, 1⅜ ins diam, and 5 ft apart, or corresponding with the floor girders, which they uphold; and with the wooden verticals of the trusses. They do not hang vertically; but incline inward.

The masonry towers are all founded on rock. They are 78½ ft high above the bottom of the bridge; and 60½ ft above the upper floor. The two at each end of the span are 39 ft apart from center to center. At the level of the lower floor they are 19×20 ft; and 21 ft apart in the clear. At the level of the upper floor they are 15 ft square; and 24 ft apart in the clear. From there they taper regularly to the top, where they are 8 ft square. They are built of limestone, in heavy dressed hor courses; laid in cement; vertical joints grouted. The upper courses are dowelled. On top of each tower is a cast-iron plate, 8 ft sq, and 2½ ins thick, bedded in cement. Part of the top of this plate is planed, as upon it move the rollers which support the cast-iron saddles on which the cables rest. At each tower, each cable has its separate saddle and rollers. Each saddle rests on 10 cast-iron rollers 25½ ins long, and 5 ins diam, carefully planed. See Fig 4 F. They lie loosely, and close together; and are kept in place by side flanges on the bed-plate.

The cast saddles are each 5 ft long, by 25½ ins wide. Their bottoms, which rest on the rollers, are flat, and planed. Their tops are curved to a rad of 6¼ ft; to suit the bend of the cables over the piers; and each saddle has a longitudinal groove, in which the cable lies. The passage of the heaviest trains produces less than ½ an inch of movement in a saddle.

The floors have a camber of 5 feet.

A change of 100° Fah of temperature causes an average variation of about 2½ ft in the deflection of the cables, or in the camber of the roadways; and one of 150°, (about the extreme to which the bridge is exposed,) about 3¾ ft. The passage of a train weighing 291 tons, and covering the entire length of the span, caused a deflection of 10 ins; and an ordinary train deflects it only from 3 to 5 inches.

This bridge has, since the year 1853, demonstrated the applicability of the suspension principle to large span railway bridges. Its entire cost was not quite $400,000.

Art. 12. The wire suspension bridge near Freyburg, Switzerland, finished in 1834, Mr. Chaley, engineer, and still in full service, is of very simple construction, and has served as the prototype for several in this country. It is for common travel only; and is narrow: its entire width of platform being but 21¼ ft; and its clear available width but 19 ft. The dist from cen to cen of its towers is 889 feet; and its clear span between abutments 800 ft; or the same as the Niagara. There are 4 cables, each 5 ins diam. Each of them consists of 1056 wires of No. 10, or full ⅛ inch diam, (or 71 wires to the sq inch of *solid* metal;) arranged in 20 strands of about 53 wires each. The four cables, therefore, have a united area of but 60 sq ins of solid metal; weighing 202 lbs or .09 of a ton, per ft run of span. All its suspenders are

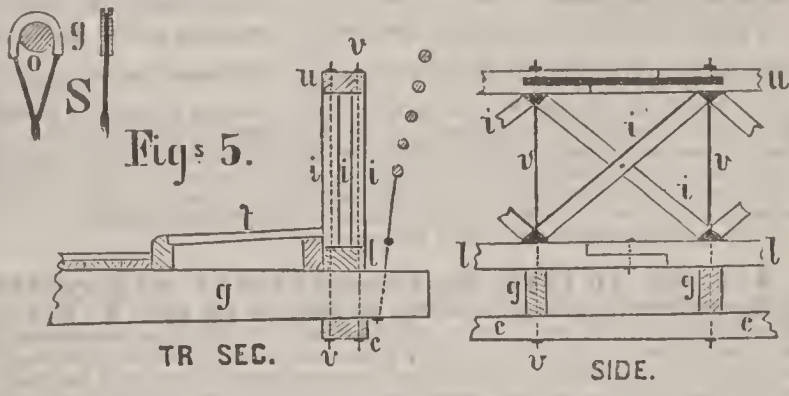
* In 1886 the wooden piers and trusses here described were replaced by iron ones.

vertical; about 5 ft apart; and each upholds one end of a transverse floor girder. It has no side trussing except the slight one of the wooden hand-railiug, which is about 6 feet high; and consequently, with its great span it is quite flexible. The deflection of the cables is $\frac{1}{14}$ of the span; hence the strain upon them at the top of the towers at either end, is 1.82 times (see table p 615) the wt of the suspended span itself, and its extraneous load; and supposing the wire to be as good as that of the Niagara, the *breaking* strain of the four cables would be $60 \times 44 = 2640$ tons; and their safe strain cannot be taken at *more* than ⅓ as much, or 880 tons. The suspended weight reqd to produce this safe strain would of course be $\frac{880}{1.82} = 484$ tons. The suspended weight of the span itself cannot well be less than .3 of a ton per ft run; or 240 tons in all;* thus, leaving $484 - 240 = 244$ tons for the maximum safe extraneous load. This amounts to .305 of a ton per ft run of span; or 36 lbs per sq ft of its platform, 19 ft wide *in the clear*. The French allowance is 41 lbs per sq ft;† and since no allowance is here made for momentum or wind, it is plain that this celebrated bridge, on account of its slight cables, and its flexibility, is by no means a strong one. In that respect, as well as steadiness, it is much inferior to the one next spoken of. It is said, however, to have withstood very severe tempests; and also to have been occasionally completely covered by crowds of people. If so, their lives were not very secure.

Art. 13. The wire suspension bridge across the Schuylkill at Philada, finished in 1842,‡ Chas Ellet, Jr, engineer, is somewhat similar in character, and in the dimensions of its details, to the preceding; but being of much less span, is much stronger. Its span from cen to cen of towers is 358 ft; suspended platform between abuts 342 ft. It has ten cables of 3 ins diam; five on each side. Their united sections present 55 sq ins of solid iron; or nearly as much as the preceding bridge of 800 ft clear span. The five cables on either side have different deflections, ranging between the $\frac{1}{12}$ and the $\frac{1}{13}$ of the span from tower to tower. The dist from cen to cen of towers at either end of the span is 35½ ft; and on top of each tower the cables (considerably flattened at that point) lie side by side on a single roller about 30 ins long, and 6 ins smallest diam, which has 5 grooves, for their reception. Each cable is drawn in about 3½ ft at the center of the span. At intervals of 20 ins the parallel wires of the cable have a close wrapping of finer wire for a distance of 3 ins.

The suspenders are of wire; and are ¾ inch diam; and 4 ft apart. On any one cable they are 20 ft apart. They all incline slightly inward.

The width of the platform from out to out is 27 ft; and in clear of hand-rails 25 ft. Inside of the hand-rail is a footway, 4 ft 4 ins wide, on each side of the bridge. The remaining 16 ft 4 ins serves for a double carriage way, or double-track street railway. Figs 5 show the arrangement of the woodwork, on a scale of ⅛ inch to a ft. The trussing of the parapets is on the Howe system, which does not appear to be as well adapted as the Pratt, to suspension bridges. The diagonals in the Fairmount bridge work themselves out of place laterally, by the vibrations of the bridge; and we have occasionally seen several of them almost on the point of falling out entirely. Being under municipal charge, it is of course neglected. The upper chords *u*, are 12 ins wide by 6 ins deep; the lower ones *l*, and the stringer c, below them, are each 12 wide, by 7 deep. The diagonals *i* are all 4 ins wide, by 5 deep. The angle-blocks at their ends are of cast iron, hollow, and about ½ inch thick. The vert iron rods *v*, (in pairs,) are ⅞ inch diam near the center of the span; and 1⅛ at its ends. The top chords are spliced on each vert face by an iron bar, of 5 ft by 3 ins, by ½ inch; with 4 bolts passing through them. The splice of the bottom chord has merely 2 bolts, side by side; (see Figs 5;) which (except S) are to a scale of ⅛ inch to a ft. The floor girders *g*, 4 ft apart from cen to cen, are 6 by 14 ins at their ends; and 6 by 16 at center.



The floor is of two thicknesses of 2-inch plank; except the footpaths, which are single thickness.

The wires were well oiled when the cables were made; and afterward painted.

At S is shown the mode of uniting a suspender with a cable, *o*, by means of a small cast-iron yoke *g*, which straddles the cable; and on the back of which is a groove ¾ of an inch wide, in which the suspender rests. The metal of the yoke is about ¼ inch thick. Since the lower ends of the wires which compose a suspender cannot themselves be formed into a screw-bolt, for upholding the floor girders, they are passed through the eye of a screw-bolt of bar iron; then doubled on themselves, and held by a wrapping of wire. It is well to introduce a yoke here also, to prevent the wear of the wires by friction. The small fig on the right of S is an edge view of the yoke *g*.

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* This is probably nearly its actual weight, as obtained by comparing it with the Fairmount bridge‡ which, by a careful estimate by the writer, weighs .375 of a ton per ft run; but is considerably wider than the Freyburg; and carries four lines of light street-rails. But if the Freyburg has longitudinal joists, it will weigh about .03 ton more per ft run.

† The greatest load that can come upon an ordinary bridge, is a dense crowd of people; and this the French engineers estimate at 41 lbs per sq ft of platform. This is certainly as great as can well occur under ordinary circumstances; but it *may* be considerably exceeded. The French estimate, moreover, includes no allowance for wind, or for the crowd being in motion. Including these, the writer thinks that no suspension bridge should have a *less* safety than 3, against 100 lbs per sq ft; added to the weight of the bridge itself. A less coeff of safety is admissible in a wire bridge than in an iron trussed one, on account of the greater reliability of the material. See foot-note, p 606.

‡ Removed, 1873, and replaced by a truss bridge of 348 ft span, by Keystone Bridge Co.

There is no transverse bracing under the floor; nor are there longitudinal floor joists resting on the girders. Owiug to the want of the distributing effect of these; and to the use of so many small cables instead of but 2 or 4 larger ones; as well as to the inefficient trussing of the haud-railiug or parapets, the bridge is much less steady thau it would otberwise be.

With wire of the same quality as the Niagara, (or 44 tons per sq inch breaking strength,) the Fairmount bridge would, with a safety of 3, (omitting momentum and wind,) sustain an extraneous load of 346 tons; which is equal to 1.01 ton per ft ruu of span; or 90 lbs per sq ft of its clear platform. This last is 2.5 times as great as the streugth of the Freyburg, with the same quality of wire. The Fairmouut is, however, we believe, built with wire of but 36 tons per sq inch ultimate strength. If so, its greatest extraneous load becomes reduced to 260 tons; or .76 ton per ft run; or 68 lbs per sq ft of platform, or nearly twice that of the Freyburg.

The towers are of cut granite, iu heavy courses. They are 8½ ft square at the ground linc, or level of the floor; about 5 ft sq at the top; and about 30 ft high. The backstays have the same augle of direction as the main cables.

Art. 14. The Wheeling bridge across the Ohio at Wheeling, Virginia, also by Mr Ellet, had a span of 1010 ft between the towers; and 960 feet clear span betweeu tho abuts; and was 26 ft wide from out to out. Its mode of construction was much the same even in detail as that of the Fairmount bridge; except in having 12 cables instead of 10. The 12 cables consisted of 6600 wires of No. 10 Birmingham gauge, presenting a sectional area of 93 sq ins of solid metal, weighing 313 lbs, or .14 of a ton, per foot of span. The weight of the woodwork was about the same per foot run of span as in the Fairmount. Although its clear span was 2.8 times as great as the Fairmount, yet its cables had but 1.7 times as great area of solid metal. Tbe entire suspeuded wt betwcen towers, is stated at but 440 tons; tberefore, with an average deflection of $\frac{1}{13}$ of the span, for a safety of 3 against 100 lbs per sq ft of platform of 24 ft clear width; or 1.07 tons per ft run of span, the arca of solid metal in the cables should have been 175 sq ins, with 44 ton wire like that of the Niagara: or 214 sq ins, with 36 ton wire, which we believe was the quality actually used.

Art. 15. The suspension canal aqueduct at Pittsburg, Penn, built in 1845, John A Roebling, Esq, engineer, has seveu spans of 160 ft each. Deflection 14½ ft; or about $\frac{1}{11}$ of the span. It has but two cables, cach 7 ins diam. The two together contain 3800 No. 10 wires, making 53 sq ins of *solid* metal section. Ultimate strength of each wire 1100 lbs; equal to 35.2 tons per sq inch of solid metal; and making the ultimate strength of the two cables together 1866 tons.

The prism of water in the wooden aqueduct is 4 ft deep; by 14¾ ft average width; and weighs 265 tons per span. The wt of one span of the structure itself is about 111 tons; making the total suspended wt at each span 376 tons. Thc tension on the two cables at either end of a span, witb a defl of $\frac{1}{11}$, is 1.46 times the total suspended weight; see table, p 615. Hence it is in this case $376 \times 1.40 = 549$ tons; and the strength of the cables is $\frac{1866}{549} = 3.4$ times the *constant* strain upon them.

On one side of the water is a towpath for horses; and on the other a footpath; each 7 ft clear width. With these occupied by horses and people, the foregoing safety would be reduced to about 3. The loaded boats do not add materially to the weight, inasmuch as they displace a bulk of water equal to their own wt; and but little of the displaced water remains on a span at the same time with the boat.

The great wt of the water prevents undulations; and the aqueduct is therefore very steady. On this account a less coeff of safety is admissible than on a common bridge.

The aqueduct leaked badly along its lower corners.

Art. 16. In 1796, Mr James Finley, of Fayette County, Penn, introduced suspension bridges in the U. S.; and built several with spans of 200 feet and less. Many of them were very primitive structurcs; but answered sufficiently well for the times. They had usually either two or four chains, composed of links from 7 to 10 feet long, formed by bending about 1½-inch square bars of iron, and welding their ends togcther. At each link-end, was a vertical suspender rod of 2 ins by ½ inch iron; which, at its lower end, was bent and welded into a stirrup for upholding one end of a transverse floor beam. On these beams rested longitudinal joists supporting the floor plank. Finley used defiections as great as $\frac{1}{7}$, or even $\frac{1}{6}$ of the span; and his piers were frequently single wooden posts; the two at each end being braced together at top. Such were used in a span of 151½ ft clear, across Will's Creek, Alleghany Co, Penn. It had two chains. The defl was $\frac{1}{6}$ of the span. Thc double links of 1⅜ inch sq irou, were 10 feet long. The center link was borizontal, and at the level of the floor; and at its ends were stirruped the two central transverse girders. From thc ends of this central link, the chains were carried *in straight lines* to the tops of the single posts, 25 ft high, which served as piers or towers. The backstays were carried away straight, at the same angle as the cables: and each end was confined to four buried stones of about ½ a cub yard each. The floor was only wide enough for a single line of vehicles. All the transverse girders were 10 ft apart; and supported longitudinal joists, to which tho floor was spiked. There were no restrictions as to travel; but lines of carts and wagons in close succession, and heavily loaded with coal, stone, iron, &c, crossed it almost daily: together with droves of cattle in full run. The slight hand-railing of iron was *hinged, so as not to be bent by the undulations of the bridge.* Six-horse wagons were frequently driven across in a rapid trot. It was built in 1820; and an observant engineer friend, who in 1838 took the sketch and measurements upon which this description is based, informed the writer that the iron was as perfect, and as sharp on all its edges, as on the day it was built. The iron was the old-fashioned charcoal, of full 30 tons per sq inch ultimate strength. The united cross-section of the two double links was 7.56 sq ins; which, at 30 tons per sq inch, gives 227 tons for their ultimate strength; or say 76 tons, with a safety of 3. Now, with a defl of $\frac{1}{6}$ span, the tension on the cables (see table) is but .9 of the suspended total wt. The two chains in place would therefore sustain, with a safety of 3, a quiet suspended load of $76 \times .9 = 68$ tons; and as the span itself did not weigh *more* than 15 tons, we have 53 tons for the safe extraneous weight, omitting all consideration of wind and momentum. This is equal to .35 ton per ft of span; equal to .7 ton for a bridge wide enough for two vehicles to pass. **This primitive**

bridge would therefore safely sustain a greater load per foot run of span, than the Freyburg.

These old bridges frequently failed by the rotting of the end posts; or were carried away by freshets; but we have never heard of a failure from the breaking of the chains. Many of them were built in a much more perfect style than the one just described; and on the most used roads in the Union.

Pierce's Well-borer, made by the Pierce Well-Excavator Co, of New York City and Long Island City, N. Y., is an excellent tool **for boring into soils, clay, sand,** or **gravel,** even when quite indurated, or when frozen. It will not bore through hard rock, or through large boulders. It consists of two sheet-iron cylindrical segments S S, called "pods," having their lower or cutting edges shod with steel. These edges project (as shown in Fig 1) beyond the sides of the anger, and thus make the hole larger than it, so that it cannot bind or stick. The two cutting edges are equidistant from the vert cen line of the tool, and this insures a straight and vert hole. At *a* the auger is attached to the lower end of a vert boring rod composed of a number of 1½-inch square iron bars, or 2½-inch iron tubes, about 10 to 15 ft long, jointed together at their ends by means of square socket-joints. At the top of this boring-rod is a swivel-hook, by means of which the entire apparatus is hung to the end of a rope, which passes over a pulley at the top of a derrick or tripod, and down to a drum worked by a windlass and gearing. By means of this drum and rope, the anger and boring-rod (which at first consists of only one bar) are lifted, and suspended over the intended hole. The anger is then lowered, and rotated hor by two men or one horse, working at the ends of levers which grip the boring-rod a few ft above the ground. The swivel at the top of the boring-rod permits this rotation to take place without twisting the rope. The shape of the auger is such that its rotation feeds or screws it into the ground; and the man at the windlass has, during the boring, merely to keep the rope tight, so as to prevent the auger from boring too fast, and becoming clogged. In about 8 revolutions the anger fills with earth. By means of the windlass it is then raised to about 2 ft above the ground; and by unkeying and removing the band *b* the anger is opened like a pair of tongs, and the earth emptied into a wooden box which has in the meantime been placed over the hole. The box is then removed and emptied, and the boring proceeds as before. When the boring has reached a depth of about 10 ft, a second bar must be added to the top of the rod. For this purpose the rod and anger are raised a few inches; a slight frame-work of boards is placed on the ground, close to the boring-rod and surrounding it; and a flange is clasped tightly to the rod just above, and close to, the framework. The framework and flange now support the rod and anger; the swivel-hook and rope are removed, and attached to the upper end of the second bar, which is then raised, and its lower end is fastened into the socket-joint upon the top of the first one. The rope is then drawn tight; the flange removed; the auger lowered to the bottom of the hole; and the boring resumed. Additional lengths of boring-rod are attached in the same way from time to time, as required by the descent of the anger.

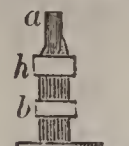


Fig 1.

Fig 2.

The borers are made from 6 to 18 ins diam, or larger to special order. If desired, the boring may be made from 24 to 36 ins diam by attaching a reamer to the anger. This anger will bore to a depth of 100 ft or more at the rate of from 5 to 20 ft per hour. It removes stones as large as half the diam of the hole. In dry soils a bucketful of water is poured into the hole each time the auger is raised.

The Pierce well-borer may be advantageously used in boring the holes for **sand-piles,** p 650, and at times, **instead of driving wooden piles,** it may be better to *plant* them (butt down if preferred) in holes bored by this anger; ramming the earth well around them afterwards. This will save adjacent buildings from the jarring and injury done by a pile driver.

If sand, mud, or loose gravel is reached in boring with this tool, the hole is reamed out 4 ins larger, and a **tubing** of inch boards is inserted into the hole, and driven into and through the sand or gravel, which is then removed from within the tubing by means of the **sand-pump** furnished with each machine. This consists of a hollow iron cylinder, about 5 ins diam × 30 ins long, with a valve at its foot, opening upward. It is lowered to the bottom of the hole; covered with water to a depth of 2 to 4 ft, and churned quickly up and down 4 to 6 ins, by hand, 20 or 30 times, during which the sand fills the pump, which is then drawn up and emptied. From 10 to 20 ft in depth of sand, mud, &c, per hour can thus be taken from a 6 to 18-inch hole. This pump is also used for removing broken earth, &c, from a hole bored in compact earth by the Pierce borer first described.

The cost of a Pierce anger, with derrick, boring-rods, rope, sand-pump, &c, &c, complete, is (1886) about $150. **The auger weighs** from 150 to 200 lbs, according to size. Boring-rod 1½ ins sq, 3½ lbs per ft. Derrick, 150 lbs.

Pierce's Sand-borer, Figs 3 and 4, like the sand-pump just described, is used inside of tubing, and for the same purpose. The hollow iron cylinder C, 10 ins diam × 30 ins long, slides vertically on the rod, but the screw is fast to the rod. While boring, the sand below and around the cyl keeps it in the position shown in

Fig 3. Six revolutions of the rod and screw fill the cyl with sand. The rod is then lifted. This first draws the screw up into the cyl, as in Fig 4; and a valve at the foot of the screw closes the bottom of the cyl, and prevents the sand from falling out when the borer is lifted from the hole. The rod is hollow, and open at top and bottom. This allows passage of the air, and thus prevents resistance from suction in withdrawing the borer. This tool is rotated and withdrawn in the same way as the earth borer first described. Price (1886), $32.

FIGS 3, 4.

The Pierce Co also furnish a **steel prospecting auger,** from 2 to 4 ins diam, and 2 ft long, for boring holes from 2½ to 6 ins diam, and to depths of 10 to 50 ft, into **clay, sand,** or **fine gravel,** of all of which it brings up samples. It is turned by wrenches, and by man or horse power, as is the well-borer; but requires no derrick, as it can be withdrawn by hand. Price (1886), of auger alone, $10 to $25; or, with boring-rods for a depth of 50 ft, levers, &c, complete, $50 to $75.

The boring tool shown in vert section by Fig 6, and in hor cross section by Fig 5, is very useful **for boring shallow holes by hand through surface soils,** clay, and gravel, and bringing up samples. The borer proper consists of a cylinder of spring steel, 3 or 4 ins diam, and 4 or 5 ins high, with sides ¼ inch thick, having a vert slit (see cross section) throughout its height, and beveled to a cutting edge all around its foot, as shown in the vert section. At its top it is riveted, as shown, or welded, to the inverted-*v*-shaped forging, which, by means of the socket at its top, is screwed to a length of gas-pipe which serves as a handle, and to which other pieces are joined by sockets as boring proceeds.

The boring is done by two men, who grasp the handle, and, holding the tool vert, drive it into the ground by repeatedly lifting it and forcibly bringing it down upon the same spot. As the tool strikes the ground, the beveled shape of its cutting edge causes it to open slightly, and when the downward pres is relieved in lifting it, it springs back and grasps the earth which has entered it. It soon fills; and the men, finding that it ceases to penetrate readily, lift it to the surface and empty it. The character of its contents from different depths, measured along the handle, is noted from time to time.

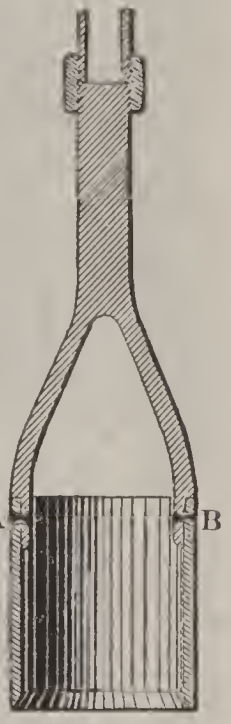


FIG 6.

In six days of 8 hours each, three men (one resting at intervals) using one such auger between them, bored 20 holes, averaging 9½ ft each, in loam, gravel, clay, and decomposed mica schist, at a cost of 22 cts per foot. Wages of each man, $2 per day.

FIG 5.

For work in loam, clay, or non-running sand, **an effective screw-auger** can be made by any good blacksmith, by merely forming a one-inch sq bar of iron or steel into corkscrew shape about 2 ft long with 6 complete turns 6 ins in diam; its lower end sharpened to form a vertical cutting edge, which should project say .5 of an inch beyond the spiral of the screw, in order to diminish friction. It will bring up full samples. Requires a derrick, or some other simple mode of lifting, when the screw is full.

Artesian Well Drilling. Deep vert holes in earth and rock, 6 and 8 ins in diam, such as are reqd for artesian wells for water and oil, and for mining explorations, are drilled by repeatedly lifting and dropping, in the same vert line, a heavy iron bit, Fig 1, p 629, with a steel cutting-edge. The bit is partly revolved horizontally after each blow, to insure roundness of hole. The length of the cutting-edge of the bit is a little greater than the diam of the bit, and the hole is thus made sufficiently large to prevent the bit from binding in it. See also diamond drill, p 652.

The bit is the lowest one of a series of iron and steel bars, &c, Figs p 629, screwed together at their ends, and called a "**string of tools.**" The string of tools varies in length from 25 to 60 ft, according to the size and depth of the hole, and the hardness of the rock; and its diam throughout (above the cutting-edge) is an inch or two less than that of the hole. Its weight is from 800 to 4000 lbs. Its upper member is always a "rope-socket," Fig 4 (*without* a swivel), to which the lower end of the supporting rope **cable** is attached. This cable passes up out of the hole to a **hor lever,** which, by means of a horse-power or steam-engine, is kept constantly moving up and down with a see-saw motion. The string of tools, with the cutting-edge of the bit at its lower end, is thus alternately lifted from 2 to 4 ft, and

let fall, from 30 to 50 times per minute, and so drills its way into the rock or earth. From 4 to 10 ft in depth of water are kept in the hole, to facilitate the drilling and the removal of debris. After water is reached, the drilling may be continued, even if the hole is full of water; but a great depth of water of course diminishes the force of the blows of the bit. A suitable arrangement must be provided for **paying out the rope** as the boring tool descends. A clamp is attached to the cable; and the man in charge, by turning the clamp, twists the rope, and thus **turns the bit horizontally** about one-fifth of a revolution after each stroke, until six or eight complete revolutions have been made in one direction. He then reverses the motion, and makes an equal number of turns, at the same rate, in the opposite direction.

After drilling a few feet, the string of tools is lifted out of the hole by means of the cable, to allow the **removal of the debris** which has accumulated in the hole. This is done by means of a **sand-pump,** which is a sheet-iron cylinder, say 4 ins diam, and 4 to 6 ft long, provided, at its foot, with a valve opening upward. The pump is lowered to the bottom of the hole, and filled with the mixed water and debris by churning it up and down a number of times. Sometimes, in addition to the valve, the pump is fitted with a plunger, which is at the foot of the pump when the latter is let down to the bottom of the hole. The plunger is then drawn up into the pump, and the debris follows it. In either case, the pump, when filled, is lifted out of the hole and emptied; the string of tools is again lowered into the hole, and the drilling resumed. The debris must be removed after every 3 to 5 ft of drilling. Otherwise it would interfere too greatly with the action of the bit.

Wells are usually drilled from 6 to 8 ins diam. For diams less than 6 ins, the tools are so slender that they are liable to be broken in a deep hole.

The same apparatus is used for **drilling through the earth above the rock,** before the latter is reached. This is called "spudding." In this case the sides of the hole must be prevented from caving in. For this purpose a wrought-iron pipe of such diam as to fit the hole closely, and ¼ inch thick, is inserted into the hole, and is driven down from time to time as the drilling proceeds. The pipe is driven by means of a heavy maul of oak, or other hard wood, 14 to 18 ins square, and 10 to 16 ft long. This maul is attached, by one end, to the lower end of the same cable which, during drilling, supports the string of tools. It is thus repeatedly lifted, and dropped upon the head of the tube, which is protected by a cast-iron "driving-cap." The foot of the tube is shod with a steel cutting-edge ring, or "steel shoe." When the tube has been driven as far as it will readily go, the maul is removed from the end of the rope; the string of tools substituted; and the drilling resumed within the pipe.

The pipe is put together in lengths of from 8 to 18 ft, and the drilling and pipe-driving proceed alternately until the rock is reached, and the foot of the pipe forced into it to a depth of a few ins, or far enough to shut off quicksand or surface water.

If quicksand is encountered, the string of tools is removed, and the sand-pump is used inside of the pipe.

For reaming out, or **enlarging, holes,** or for **straightening** crooked ones, &c, special tools, such as reamers, &c, are substituted in place of the boring bit.

Special care must be taken to have **all the rubbing surfaces thoroughly lubricated.** The pulley in the mast-head, and the pinion-wheels of the horse power (if such be used) should be well oiled every two or three hours.

In very **cold or wet weather,** a **shed of rough boards,** or a covering of canvas, about 8 ft high, should be erected, to protect the men; and, if steam is used, 2 or 3 boards should be used as a covering for the belt, which will slip if wet.

The following description is based upon the improved machines made by the **Pierce Well Excavator Co,** of New York City, and Long Island City, N Y, who make a specialty of artesian-well machinery, and of steam-engines and horse-powers for its operation. **They also sink wells** to any reqd depth, in any country.

For holes from 200 to 1000 ft deep, this Co furnish **portable drilling machines,** to be worked by horse or steam power. In these machines, the drill-rope, extending from the string of tools up out of the hole, passes over a sheave at the top of a wooden mast; down to, and around, a pulley fast to the working lever; and thence, by way of a pulley fixed at the foot of the mast, to a drum upon which it is wound. To this drum a friction and ratchet wheel is attached, for paying out the cable as the tools descend.

The mast is hinged six feet above its foot, so that its upper part may be laid hor when the machine is to be moved. When at work, it is held in position by two timber struts or braces, bolted to it near its top, and having their lower ends fastened to the **"drill-jack,"** which is a light and strong framework, 9 ft long, 3 ft wide, and 4 ft high, at the foot of the mast, containing the working lever which

raises the rope and lets it fall, the drum on which the rope is wound, the shaft and cam which work the lever, &c. The operator stands at the foot of the mast, and, by means of foot- and hand-levers within his reach, regulates all the movements of the machine. One of these governs the pawl and ratchet wheel regulating the paying out of the cable. By letting the ratchet-wheel of the drum move one notch, the bit is fed down quarter of an inch.

The operator, by moving a slide with his foot, holds the working lever down, out of reach of the cam, thus stopping the up-and-down motion of the rope and tools. By means of another lever he can now put the rope-drum in gear with the main driving-shaft, so that the rope is wound up on the drum, and the tools drawn up out of the hole. Another lever controls the separate reel on which the light rope, carrying the sand-pump, is wound. All these operations are performed by the same power (horse or steam, as the case may be), which works on without stopping; the various changes being made by merely throwing the different parts into, or out of, gear with the main driving-shaft.

One of these portable machines requires two horses or a small steam-engine, a man to attend the same, and another man to operate the machine, empty the sand-pump, change the tools, &c. It can be transported on a farm wagon over any common road. Two men can unload it, set it up, and commence drilling, in two hours; and, unless steam is preferred, the two horses used for its transportation furnish the motive power. The machine can be taken down and reloaded in the wagon in two hours.

Figs 1 to 4 show the **tools used with these machines.** For the different sizes of machine they differ chiefly in their dimensions and weights. Fig 1 is the **drilling bit,** called a "Z" bit from the shape of its cutting-edge. This edge is 6 ins long. The bit is 30 to 36 ins long, and weighs about 100 lbs. Its top is screwed into the foot of the **"auger-stem,"** Fig 2, which is of 3-inch round iron, 12 ft long, and weighs 350 lbs. Its use is that of a weight, giving additional force to the blows of the bit. Its top is screwed into the foot of the **"drill-jars,"** Fig 3; and to the top of these is screwed the **"rope-socket,"** Fig 4, to which the drilling cable is attached. **If the bit, or auger-stem, becomes wedged in the hole by any means,** the operator stops the churning motion of the tools, and the rope is let out about 12 ins. This permits the upper link U of the drill-jars, Fig 3, to slide down about 12 ins in the slot S in their lower link. The churning motion is then started again, and the upward jerk of the link U against the upper end of the slot loosens the tools.

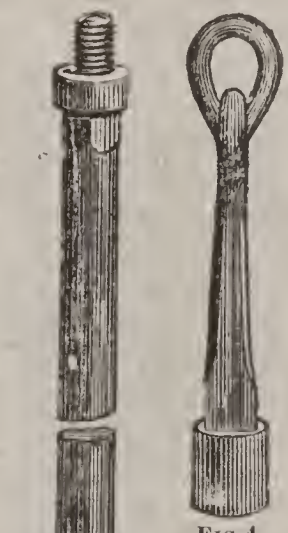


FIG 4.

FIG 2.

FIG 3.

FIG 1.

Four sizes of these machines are furnished, as follows:

Depth of hole, ft.	Letter.	Ht of mast, ft.	Length of string of tools, ft.	Wt of string of tools, lbs.	Total wt of machine, mast, tools, rope, &c. but exclusive of power, lbs.	Horses reqd.	Horse power of engine reqd.	Wt of horse power, lbs.	Wt of steam-engine, lbs.	Cost (1886) of machine complete, but exclusive of horse power * or steam-engine, $.
200	A	25	23	800	1800 to 2500	2	3	800	1600	800
350	B	25	23	800	1850 to 2500	2	3	800	1600	875
600	C	35	30	1200	2800 to 3500	Steam	5		2200	1000
1000	D	40	35	1800	3800 to 4500		10		3600	2200

* The cost of a horse power (1886) is $75.

For wells from 1000 to 3000 ft deep, a stationary machine, with a walking-beam, is used, similar to those employed in the oil regions of Penna. A square pyramidal derrick is erected,

74 ft high, 20 ft sq at base, 4 ft sq at top. Each of its 4 corner legs is of 2 in × 8 in and 2 in × 10 in planks, spiked together so as to form a 10 in × 10 in angle-piece 2 ins thick. The legs are braced together by hor and diag timbers. The walking beam is of timber, 26 ft long, 12 ins wide, and 26 ins deep at the middle of its length where it is pivoted to the top of a wooden post 18 ins sq and 12 ft high, called a "Samson post." This post, at its foot, is dovetailed into the main sill of the machine, which is 18 ins wide × 24 ins deep.

The motive power is a 15-hp steam-engine, which, by means of a belt and pulley, crank and pitman, working at one end of the walking-beam, gives to the latter its see-saw motion. To the other end of the beam, and immediately over the well, is suspended, by means of a hook, a "temper-screw." This last is composed of two bars of iron, about ⅝ × 2 ins, 5 ft long, hung 2 ins apart, fastened together at their top ends, at which point there is an eye, which is suspended on the walking-beam hook. At the bottom of the two bars there is a sleeve-nut, and between the two bars and passing through the nut, is a screw 5 ft long, at the bottom of which there is a head, which carries a swivel, set-screw, and a pair of clamps. These grasp the cable, 2 or 2¼ ins diam, which carries at its lower end the **string of tools.** This, for a 2000-ft hole, consists of a steel bit, 3 or 4 ft long, weighing 200 to 400 lbs; an anger-stem of 4 or 5-inch round iron, from 24 to 30 ft long, and weighing from 1200 to 2100 lbs; steel-lined drill-jars 8 ft long, weighing 600 to 700 lbs; a sinker-bar of round iron of same diam as the anger-stem, 12 to 15 ft long, and weighing from 600 to 1100 lbs; and a rope-socket, 2½ ft long, weighing 200 lbs. Total length of string of tools, 50 to 60 ft, total weight, 3000 lbs; or, for an 8-inch hole in the hardest rock, 4000 lbs. **The sinker-bar** is added to give additional wt, and thus to assist in pulling the cable down through the water, either in lowering the string of tools or in working the drill-jars. The shapes of the other tools are given by Figs 1 to 4. **Special tools** are used for recovering articles that may be accidentally dropped into the hole.

The drilling cable is wound on a drum, called a bull-wheel shaft, at the foot of, and inside of, the derrick. While drilling is going on, it passes from the bull-wheel shaft loosely over the sheave at the top of the derrick, and down to the clamps at the lower end of the temper-screw on the end of the walking-beam. As the drilling progresses, the temper screw is turned or fed out by the man in charge, who also, by means of a clamp, twists the rope, so as to change the position of the bit after each stroke.

When the tools are to be lifted out of the hole, the cable is disengaged from the clamps on the temper-screw, and is wound upon the bull-wheel shaft, which, for this purpose, is thrown into gear with the steam-engine; the pitman being at the same time removed from the crank-pin, so that the walking-beam is at rest. As in the portable machines, the sand-pump is also raised by the same power which does the drilling.

About 10000 ft b m of rough lumber are reqd for the derrick, walking-beam, sills, &c, and about 3000 ft more for sheds over the boiler, engine, and belt.

In ordinary hard limestone rock, such a **machine will drill** about 1½ ft per hour under the most favorable circumstances. **Two men are required;** one to attend to the boiler, sharpen the bits, &c, and one to operate the machine. **The cost of the apparatus,** in 1886, is from $1800 to $2500. **Lumber** for the derrick, $350 to $400 more. **The cost of drilling,** in 1886, in limestone, is about $6 per ft run for 6-inch holes; $8 for 8-inch. In granite and trap rock, $10 and $12.

For **quantity of masonry in walls of wells,** see p 158.

COST OF DREDGING.

DREDGING is generally done by skilled contractors, who own the requisite machines, scows or lighters, &c; and who make it a specialty. It is necessary to specify whether the dredged material is to be measured in place before it is loosened; or after being deposited in the scow: because it occupies more bulk after being dredged. It was found, in the extensive dredgings for deepening the River St Lawrence through the Lake of St Peter, that on an average a cub yd of tolerably stiff mud in place, makes 1.4 yds in the scow; or 1 in the scow, makes .715 in place. Also stipulate whether the removal of bowlders, sunken trees, &c, is to constitute an extra. These often require sawing and blasting under water. The cost per cub yd for dredging varies much with the depth of water; the quantity and character of the material; the dist to which it has to be removed; whether it can be at once discharged from the machine by means of projecting side-shoots or slides; or must be discharged into scows, to be removed to a short dist by poling, or to a greater dist by steam tugs; whether it can be *dropped* or dumped into deep water by means of flap or trap doors in the bottom of the hoppers of the scows; or must be *shovelled* from the scows *into shallow water*, (at say 4 to 8 cts per yd;) or *upon land*, (at say from 6 to 10 or 20 cts for the shovelling alone, or shovelling and wheeling, as the case may be;) whether much time must be consumed in moving the machine forward frequently, as when the excavation is narrow, and of but little depth; as in deepening a canal, &c; whether many bowlders and sunken trees are to be lifted; whether interruptions may occur from waves in storms; whether fuel can be readily obtained, &c, &c. These considerations may make the cost per cub yd in one case from 2 to 4 times as great as in another. The *actual cost* of deepening a ship-channel through Lake St Peter, to 18 ft, from its original depth of 11 ft, for several miles through moderately stiff mud, was 14 cts per cub yd in place, or 10 cts in the scows; including removing the material by steam tugs to a dist of about ½ a mile, and dropping it into deep water. This includes repairs of plant of all kinds, but no profit. It was a favorable case. When the buckets work in deep water they do not become so well filled as when the water is shallower, because they have a more vertical movement, and, therefore, do not scrape along as great a distance of the bottom. Hence one reason why deep dredging costs more per yard; in addition to having to be lifted through a greater height. Perhaps the following table is tolerably approximate for large works in ordinary mud, sand, or gravel; assuming the plant to have been paid for by the company; and that common labor costs $1 per day.

Table of actual cost of dredging on a large scale; including dropping the material into scows, alongside; or into side-shoots, on board. Common labor $1 per day. Repairs of plant are included: but no profit to contractor. (Original.

Depth in Ft.	Cts per Yard, in place.	Cts per Yard, in scow.	Depth in Ft.	Cts per Yard, in place.	Cts per Yard in scow.
Less than 10	8.4	6	25 to 30	18.2	13
10 to 15	9.8	7	30 to 35	25.2	18
15 to 20	11.2	8	35 to 40	35.0	25
20 to 25	14.0	10			

For towing of the scows by steam tugs to a dist of ¼ mile, and dropping the mud into deep water, add 4 cts per yard in the scow; for ½ mile, 6 cts; for ¾ mile, 8 cts; for 1 mile, 10 cts Add profit to contractor. On a small scale work is done to a less advantage; and a corresponding increase must be made in these prices. Also, if the contractor himself furnishes the dredgers and plant, a still further addition must be made. It is evident that the subject admits of no great precision. Small jobs, even in favorable material, but in inconvenient positions, may readily cost two or three times as much per yd as the above; and in very hard material, as in cemented gravel and clay, four or five times as much for the *dredging*. The cost of *towing*, however, will remain as before, if wages are the same.

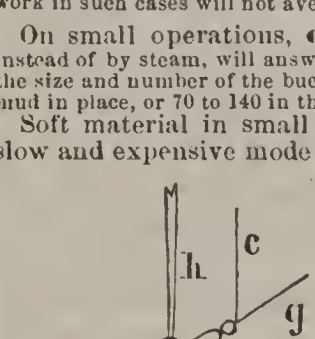
The cost of dredgers, tugs, &c, will vary of course with their capabilities, strength of construction, style of finish, whether having accommodations for the men to live on board or not, &c. When for use in salt water, the bottoms of both dredgers and scows should be coppered, to protect them from sea-worms; and if occasionally exposed to high waves, both should be extra strong. The most powerful machines on the St Lawrence cost about $45000 each; and removed in 10 working hours on an average about 1800 cub yds in place, or 2520 in the scows. Good machines, capable, under similar circumstances, of doing as much, may, however, be built for about $25000 to $30000. To remove this quantity to a dist of ½ to 1 mile, would require two steam tugs, costing about $6000 to $10000 each; and 4 to 6 scows, (some to be loading while others are away,) holding from 30 to 60 cub yds each; and costing from $800 to $1500 each at the shop. Scows with two hoppers are best. Such a dredger would require at least 8 or 10 men, including captain, engineer, fireman, and cook. Each tug 4 or 5 men; and each scow 2 men. The engineer should be a blacksmith; or a blacksmith should be added. In certain cases a physician, clerk, assistant engineer, &c, may be needed.

Dredgers are often built on the principle of the Yankee Excavator, with but a single bucket or dipper, of from 1 to 2 cub yds capacity. Hull about 25 by 60 ft. Draft 3 ft. Cylinder about 7 or 8 ins diam; 15 to 18 inch stroke; ordinary working pressure 50 to 80 lbs per sq inch, according to hardness of material. Cost $8000 to $12000. Will raise as an average days' work (10 hours) from 200 to 500 yds in place, or 280 to 700 in the scow, according to the depth, nature of the material, &c. Require 5 or 7 men in all aboard, including cook. Burn ½ to 1 ton of coal daily. Tolerably large bowlders, and sunken logs, can be raised by the dipper.*

When the material is hard and compacted, the buckets of dredgers should be armed with strong steel teeth projecting from their cutting edge. On arriving at such material, every alternate bucket is sometimes unshipped. By arranging the buckets so as to dredge a few feet in advance of the hull, low tongues of dry land may be cut away; the machine thus digging its own channel. The daily work in such cases will not average half as much as in wet soil.

On small operations, **dredgers worked by two or more horses,** instead of by steam, will answer very well in soft material; or even in moderately hard, by reducing the size and number of the buckets. A two-horse machine will raise from 50 to 100 yards of ordinary mud in place, or 70 to 140 in the scow, per day, at from 12 to 15 ft depth.

Soft material in small quantity, and at moderate depth, may be removed by the slow and expensive mode of the **bag-scoop,** or **bag-spoon.**



This is simply a bag B, made of canvas or leather, and having its mouth surrounded by an oval iron ring, the lower part of which is sharpened to form a cutting edge. It has a fixed handle *h*, and a swivel handle *i*. One man pushes the bag down into the mud by *h*, while another pulls it along by the rope *g*; and when filled, another raises it by the rope *c*, and empties it. If the bag is large, a windlass may be used for raising it. The men may work from a scow or raft properly anchored. Or a long-handled metal spoon, shaped like a deeply-dished hoe, may be used by only one man; or a larger spoon may be guided by a man, and dragged forward and backward by a horse walking in a circle on the scow, &c, &c.

The weight of a cub yd of wet dredged mud, pure sand, or gravel, averages about 1⅓ tons; say 111 lbs per cub ft; muddy gravel, full 1½ tons; say 125 lbs per cub ft. Pure sand or gravel dredges easily; also beds of shells. Wet dredged clay will slide down a shoot inclined at from 5 to 1, to 3 to 1, according to its freedom from sand, &c; but wet sand or gravel will not slide down even 3 to 1, without a free flow of water to aid it; otherwise it requires much pushing.

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* The writer has seen cases in which a **circular saw** for logs in deep water, would have been a very useful addition to a dredger. It should be worked by steam; and be adjustable to different depths. It would cost but about $500.

The American Dredging Co, No. 234 Walnut St, Philada, make dredgers of many patterns; and contract for dredging on any scale.

FOUNDATIONS.

A VOLUME might be occupied by this important subject alone. We have space for only a few general hints; leaving it to the student to determine how far they may be applicable in any given case. In ordinary cases, as in culverts, retaining walls, &c, if excavations, or wells, &c, in the vicinity, have not already proved that the soil is reliable to a considerable depth, it will usually be a sufficient precaution, after having dug and levelled off the foundation pits or trenches to a depth of 3 to 5 ft, to test it by an iron rod, or a pump-auger; or to sink holes, in a few spots, to the depth of 4 to 8 ft farther; (depending upon the weight of the intended structure;) to ascertain if the soil continues firm to that distance. If it does, there will rarely be any risk in proceeding at once with the masonry; because a stratum of firm soil, from 4 to 8 ft thick, will be safe for almost any ordinary structure; even though it should be underlaid by a much softer stratum. If, however, the firm upper stratum is exposed to running water, as in the case of a bridge-pier in a river, care must be taken to preserve it from gradually washing away; or from becoming loosened and broken up by violent freshets; especially if they bring down heavy masses of ice, trees, and other floating matter. These are sometimes arrested by piers, and accumulate so as to form dams extending to the bottom of the stream; thus creating an increase of velocity, and of scouring action, that is very dangerous to the stability both of the bottom and of the structure. When the testing has to be made to a considerable depth, it may be necessary to drive down a tube of either wrought or cast iron, to prevent the soil from falling into the unfinished hole. If necessary, this tube may be in short lengths, connected by screw joints, for convenience of driving; and the earth inside of it may be removed by a small scoop with a long handle.†

Borings in common soils or clay may be made 100 ft deep in a day or two by a common wood auger 1½ ins diam, turned by two to four men with 3 ft levers. This will bring up samples. For this and other earth-boring tools see p 626.

In starting the masonry, the largest stones should of course be placed at the bottom of the pit, so as to equalize the pressure as much as possible; and care should be taken to bed them solidly in the soil, so as to have no rocking tendency. The next few courses at least should be of large stones, so laid as to break joint thoroughly with those below. The trenches should be refilled with earth as soon as the masonry will permit; so as to exclude rain, which would injure the mortar, and soften the foundation. It is well to ram or tread the earth to some extent as it is being deposited.

If the tests show that the soil (not exposed to running water) is too soft to support the masonry, then the pits should be made considerably wider and deeper: and afterward be filled to their entire width, and to a depth of from 3 to 6 or more ft, (depending on the weight to be sustained,) with rammed or rolled layers of sand, gravel, or stone broken to turnpike size; or with concrete in which there is a good proportion of cement. On this deposit the masonry may be started. The common practice in such cases, of laying planks or wooden platforms in the foundations, for building

† Subterranean caverns in limestone regions are a frequent source of trouble, against which it is difficult to adopt precautions.

upon, is a very bad one. For if the planks are not constantly kept thoroughly wet, they will decay in a few years; causing cracks and settlements in the masonry.

Some portions of the circular brick aqueduct for supplying Boston with water, gave a great deal of trouble where its trenches passed through running quicksands, and other treacherous soils. Concrete was tried, but the wet quicksand mixed itself with it, and *killed* it. Wooden cradles, &c, also failed; and the difficulty was finally overcome by simply depositing in the trenches about two feet in depth of strong gravel.* Sand or gravel, *when prevented from spreading sideways*, forms one of the best of foundations. To prevent this spreading, the area to be built on may be surrounded by a wall; or by squared piles driven so close as to touch each other; or in less important cases, by short sheet piles only. But generally it is sufficient simply to give the trenches a good width; and to ram the sand or gravel (which are all the better if wet) in layers; taking care to compact it well against the *sides* of the trench also. Under heavy loads, some settlement will of course take place, as is the case in all foundations except rock. If very heavy, adopt piling, &c. See GRILLAGE, p 641.

When an unreliable soil overlies a firm one, but at such a depth that the excavation of the trenches (which then must evidently be made wider, as well as deeper,) becomes too troublesome, and expensive; especially when (as generally happens in that case) water percolates rapidly into the trenches from the adjacent strata, we may resort to piles. See p 641.

When making deep foundation pits in **damp clay,** we must remember that this material, being soft, has, to a certain degree, a tendency to press in every direction, like water. This causes it to bulge inward at the sides; and upward at the bottom. The excavations for tunnels, or for vertical shafts, often close in all around, and become much contracted thereby before they can be lined; therefore they should be dug larger than would otherwise be necessary. The bottoms of canal and railroad excavations in moist clay are frequently pressed upward by the weight of the sides.

Dry clay rapidly absorbs moisture from the air, and swells, producing effects similar to the foregoing. Its expansion is attended by great pressure: so that retaining-walls backed with dry rammed clay will be in danger of bulging if the clay should become wet. It is a treacherous material to work in. **For concrete foundations,** see ps 680 &c.

As to the greatest load that may safely be trusted on an earth foundation, Rankine advises not to exceed 1 to 1.5 tons per sq ft. But experience proves that on good compact gravel, sand, or loam, at a depth beyond atmospheric influences, 2 to 3 tons are safe, or even 4 to 6 tons if a few ins of settlement may be allowed, as is often the case in isolated structures without tremors. Years may elapse before this settlement ceases entirely. Pure clay, especially if damp, is more compressible, and should not be trusted with more than 1 to 2.5 tons, according to the case. All earth foundations must yield *somewhat*. **Equality of pressure** is a main point to aim at. **Tremor** increases settlements, and causes them to continue for a longer period, especially in weak soils, great care must be taken not to overload in such cases, even if piled. **Foundations in silty soils** will probably settle, in years, at the rate of from 3 to 12 ins per ton (up to 2 tons) per sq ft of **quiet** load, if not on piles.

Fig 2 shows an easy mode of obtaining a foundation in certain cases. It is the **"pierre perdue"** (lost stone) of the French; in English, **"random stone."** or **rip-rap.**

It is merely a deposit of rough angular quarry stone thrown into the water: the largest ones being at the outside, to resist disturbance from freshets, ice, floating trees, &c. A part of the interior may be of small quarry chips, with some gravel, sand, clay, &c. When the bottom is irregular rock, this process saves the expense of levelling it off to receive the masonry. For 2 or 3 feet below the surface of the water, the stones may generally be disposed by hand so as to lie close and firmly. Small spawls packed between the larger ones will make the work smoother, and less liable to be displaced by violence. Cramps or chains may at times be useful for connecting several of the large stones together for greater stability. **Rip-rap, however, is apt to settle.**

If the bottom is so yielding as to be liable to wash away in freshets, it may, in addition, be protected, as in Fig 2, by a covering of the same kind of stones, as at *c*: extending all around the structure. Or the main pile of stones may be extended as per dotted line at *d*; so that if the bottom should wash away, as per dotted line at *o*, the stones *d* will fall into the cavity, and thus prevent further damage. Sheet-piles, *s s*, may be driven as an additional precaution. For greater security, the bed of the river may be dredged or scooped under the entire space to be covered by the main deposit, as per dotted lines in Fig 3, to as great a depth as any scouring would be apt to reach;

Fig 2

* Smeaton mentions a stone bridge built upon a natural bed of gravel only about 2 ft thick, overlying deep mud so soft that an iron bar 40 ft long sank to the head by its own weight. One of the piers, however, sank while the arches were being turned; and was restored by Smeaton. Although a wretched precedent for bridge building, this example illustrates the bearing power of a thick layer of well-compacted gravel.

this excavation also to be filled with stone. Such foundations are evidently best adapted to quiet water. The masonry should rest on a strong platform.

Large deposits of stone, as in these two figs, greatly increase the velocity, and the scouring action of the stream around them, especially in freshets; unless the bottom on each side from the deposit be dredged out to such an extent that the original area of water shall not be reduced. If the bottom is treacherous, this should be done before depositing the covering stones c, Fig 2. Judgment and experience are necessary in such matters, as in all others connected with engineering. Mere study will not guard against constant failures. Theory and practice must guide each other.

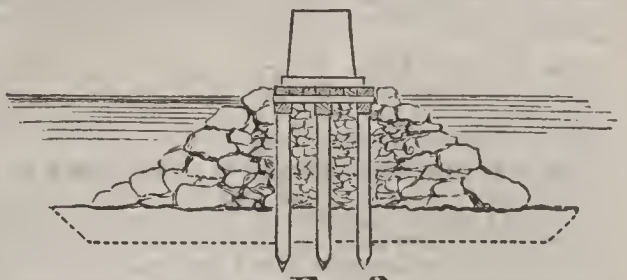
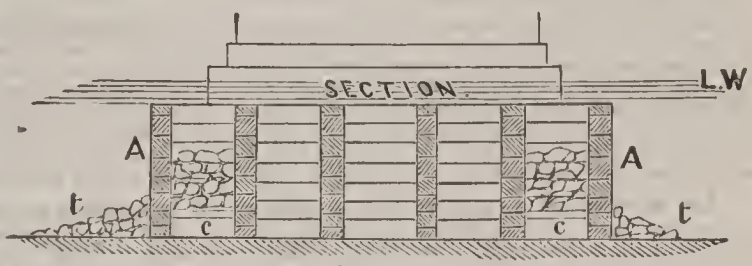


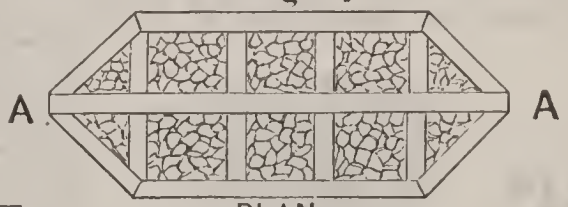
Fig 3

Fig 3 is another simple method; and when it does not create too great an obstruction to the navigation of the stream, or to the escape of its waters in time of high freshets, is a very effective one. Here the piles are first driven into the river bottom, for the support of the pier; then the deposit of stone is thrown in, for the support and protection of the piles; preventing them from bending under their loads; and shielding them from blows from floating bodies. The tops of the piles being cut off to a level, a strong platform of timber is laid on top of them, as a base for the masonry. The top of the platform should not be less than about 12 or 18 ins below ordinary low water, to prevent decay. Mitchell's iron screw pile; or hollow piles of cast iron, may be used instead of wooden ones. See pp 641 to 646.

Figs 4 represent a convenient method of establishing a foundation in water, by means of a **timber crib, A A, without a bottom.** It should be built of squared timbers, notched together at their crossings, as shown at Fig 5; each notch being ¼ of the depth of the stick. By this means each timber is supported throughout its entire length by the one below it; and resists pulling in both directions. Bolts also are driven at the intersections; at least in the sides of the crib, to prevent one portion from being floated off from the other. The crib is thus divided into square or rectangular cells, from 2 to 4 or 5 ft on a side, according to the requirements of the case. The partitions between the cells are put together in the same manner as those at the sides of the cribs; and consequently, like the latter, form solid wooden walls.



Figs 4



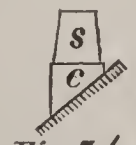
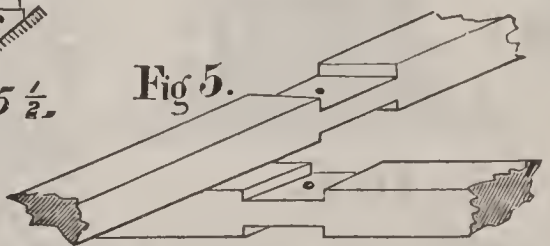


Fig. 5 ½.

Fig 5.



The crib may be framed afloat, at any convenient spot; and when finished, may be towed to its final place, where it is carefully moored in position, and then sunk by throwing stone into a few cells provided with platforms, as at e e, for that purpose. These platforms should be placed a little above the lower edge of the cells, so as not to prevent the crib from settling slightly into the soil, and thus coming to a full bearing upon the bottom. After it has been sunk, all the cells are filled with rough stone. A stout top platform may be added or not, as the case may be; also, a protection, *t t*, of random stone, to prevent undermining by the current. If the sides are exposed to abrasion from ice, &c, they may be covered in whole or in part with plank, or plate iron; and the angles strengthened by iron straps, &c. In deep water, a foundation may be made partly of random stone, as in Figs 2 and 3; and on top of this may be sunk a crib, with its top about 2 ft under low water, as a base for the masonry. This is much safer than random stone alone.

On uneven rock bottom it may be necessary to scribe the bottom of the crib to fit the rock; or the crib may first be sunk by means of a loaded platform on its top, or by filling some of its cells, until its lowest timbers are within a short distance above the bottom. Being there kept in a horizontal position, small stones may be thrown into the cells, and allowed to find their way under the timbers of the crib, thus forming a level support for it. The cells may then be filled; and rip-rap deposited outside around the crib to prevent the small stones from being displaced.

A crib with only an outside row of cells for sinking it may be built; and the interior chamber may be filled with concrete under water. The masonry may then rest on the concrete alone. If the crib rests upon a foundation of broken stone, the upper interstices of this stone should first be levelled off by small stone or coarse gravel to receive the concrete of the inner chamber.

Or a crib like Fig. 4 may be sunk, and piles be driven in the cells, which may afterward be filled with broken stone or concrete. The masonry may then rest on the piles only, which in turn will be defended by the crib. If the bottom is liable to scour, place sheet-piles or rip-rap around the base of the crib.

By all means avoid a crib like *e*, Fig 5½, much higher at one part than at another, *if the superstructure s is to rest on the timber of the crib* instead of on piles, or on concrete independent of the timber; for the high part of the crib will compress more under its load than the low part, and will thus cause the superstructure to lean or to crack.

A crib either straight sided or circular, with only an outer row of cells for **puddling may be used as a cofferdam** (see cofferdams, p 637). The joints between the outer timbers should be well caulked; and care be taken, by means of outside pile-planks, gravel, &c, to prevent water from entering beneath it.

The cast-iron Bridge across the Schuylkill at Chestnut St, Phila, Mr. Strickland Kneass, Engineer, affords a striking example of crib foundation. The center pier stands on a crib, an oblong octagon in plan; 31 by 87 feet at base; 24 by 80 ft at top; and (with its platform) 29 ft high. Its timbers are of yellow pine, hewn 12 ins square: and framed as at Fig 5. The lower timbers were carefully cut or scribed to conform to the irregularities of the tolerably level rock upon which it rests. These were ascertained (after the 8 ft depth of gravel had been dredged off) in the usual manner of mooring above the site a large floating wooden platform, composed of timbers corresponding in position with all those of the lower course of the intended crib, both longitudinal and transverse. Soundings were then taken close together along all these lines of timber. Most of the cells are about 3 by 4 ft on a side, in the clear. A few of them had platforms at the level of the second course from the bottom, for receiving stone for sinking the crib; the others are open to the bottom.

The crib was built in the water; and was kept floating, during its construction, with its unfinished top continually just above water, by gradually loading it with more stone as new timbers were added. The stone required for this purpose alone was 309 tons. When the crib was towed into position, and moored, 150 tons more were added for sinking it. All the cells were afterward filled with rough dry stone, and coarse gravel screenings; making a total of 1666 tons. A platform of 12 by 12 inch squared timber covered the whole; its top being 2½ ft below low water. The pier alone, which stands on this crib, weighs 3255 tons; and during its construction it compressed the crib 6½ ins. The weight of superstructure resting on the pier, may be roughly taken at 1000 tons more.

An ordinary caisson is merely a strong scow, or a box without a lid; and with sides which may at pleasure be readily detached from its bottom. It is built on land, and then launched. The masonry may first be built in it, either in whole or in part, while afloat; and the whole being then towed into place, and moored, may be sunk to the bottom of the river, to rest upon a foundation previously prepared for it, either by piling, if necessary; or by merely levelling off the natural surface, &c. The bottom of the caisson constitutes a strong timber platform, upon which the masonry rests; and is so arranged, that after it is sunk, the sides may be detached from it, and removed to be rebottomed for use at another pier, if needed. This detaching may be effected by some such contrivance as that shown in Fig 6, where P P *w* is the bottom of the caisson, to which are firmly attached at intervals strong iron eyes *t*; which are taken hold of by hooks *d*, at the lower end of long bolts E *n*, reaching to the top timbers S of the crib, where they are confined by screw nuts *n*. By loosening the nuts *n*, the hooks *d* can be detached from the eyes *t*; and the sides can then be removed from the bottom; there being no other connection between the two. These hooks and eyes are usually placed outside of the caisson; the screw nuts *n* being sustained by the projecting ends of cross pieces, as *t t*, Fig 9. The improper position given them in our Fig was merely for convenience of illustrating the principle. It will sometimes be necessary to have one side detachable from the others, in order to float the caisson away clear from the finished pier; unless it be floated away before the masonry has been built so high as to render the precaution useless. Fig 6 shows one of many ways of constructing a caisson; with sides consisting of upright corner-posts, 1; cap pieces S, on top; and sills *g* at bottom, resting on the bottom platform P P *w*; intermediate uprights T, framed into the caps and sills; the whole being covered outside by one or two thicknesses of planking B, which, as well as the platform, should be well calked, to prevent leaking. Tarpaulin also may be nailed outside to assist in this. The greatest trouble from leaking is where the sides join the platform. On top of the platform is firmly spiked a timber o o, extending all around it just inside of the inner lower edge of the sides of the caisson. Its use is to prevent the sides from being forced inward by the pressure of the water outside. The details of construction will of course vary with the requirements of the case. In deep caissons, inside cross-braces or struts from side to side, as at c c, Fig 7, will be required to prevent the sides from being forced inward by the pressure of the water, as the vessel gradually sinks while the masonry is being built within it. As the masonry is carried up, the struts are removed; and short ones, extending from the sides of the caisson to the masonry, are inserted in their place. When the caisson is shallow, only the upper course of braces will be required, they also support a platform for the workmen and their materials. In deep caissons, in order not to be in the way of the masons, the outer planking of the sides may, in part, be gradually built up as the masonry progresses. It may sometimes be expedient to build the masonry hollow at first, with thin transverse walls inside to stiffen it if necessary; and to com-

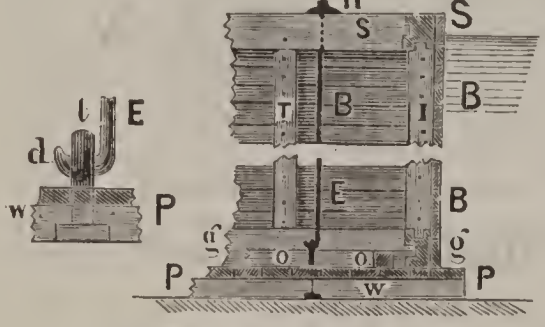


Fig 6.

plete the interior after sinking the caisson. Indeed, masonry or brickwork, in cement, may thus be built hollow at first, resting on the platform; the masonry itself forming the sides of the caisson. Or the sides may consist of a water-tight casing of iron, or wood, of the shape of the intended pier, &c. This casing being confined to the platform, becomes, in fact, a mould, in which the pier may be formed, and sunk at the same time by filling it with hydraulic concrete. **For concrete foundations,** see p 680 &c.

On rock bottom the under timbers of the platform may be cut to suit the irregularities as already stated under "Cribs." Or the bottom may be levelled up by first depositing large stones around the area upon which the caisson is to rest; and then filling between these with smaller stones and gravel; testing the depth by sounding. Or a level bed of cement concrete may, with care, be deposited in the water. If there are deep narrow crevices in the rock, through which the concrete may escape, they may be first covered with tarpaulin. Diving bells may often be used to advantage, in all such operations. But in the case of very irregular rock, it will often be better to resort to coffer-dams. The draft of a caisson (the depth of water which it draws) whether empty or loaded, can be found by page 236. Valves for the admission of water for sinking the caisson are usually introduced. If, after sinking, it should be necessary to again raise the whole, it is only necessary to close the valves, and pump out the water. Guide piles may be driven and braced alongside of the caisson, to insure its sinking vertically, and at the proper spot. Or it may be lowered by screws supported by strong temporary framework.

Assuming the uprights I, T, &c. Fig 6, to be sufficiently braced, as at c c, Fig 7, the following table will show the thickness of planking necessary for different distances apart of the uprights, (in the clear,) to insure a safety of six against the pressure of the water at different depths; and at the same time not to bend inward under said pressure, more than $\frac{1}{480}$ part of the distance to which they stretch from upright to upright; or at the rate of ¼ inch in 10 ft stretch; ⅛ inch in 5 ft, &c. Such a table may be of use in other matters.

Table of thickness of white pine plank required not to bend more than $\frac{1}{480}$ part of its clear horizontal stretch, under different heads of water. (Original.)

Stretch in Ft.	HEADS IN FEET. 40	30	20	10	5
	Thickness in Inches.				
3	3⅛	3	2¾	2¼	1¾
4	4⅛	4	3½	2¾	2¼
6	6¾	6	5¼	4¼	3¼
8	9	8	7	5½	4½
10	11¼	10	8¾	7	5½
12	13½	12¼	10¾	8½	6¾
15	16¾	15	13	10½	8½
20	22¼	20	17½	14	11

Coffer-dams are enclosures from which the water may be pumped out, so as to allow the work to be done in the open air. Their construction of course varies greatly. In still shallow water, a mere well-built bank of clay and gravel; or of bags partly filled with those materials when there is much current, will answer every purpose; or (depending on the depth) a single or double row of sheet-piles; or of squared piles of larger dimensions, driven touching each other; their lower ends a few feet in the soil; and their upper ones a little above high water, and protected outside by heaps of gravelly soil or puddle, (as at P in Fig 7,) to prevent leaking. The sheet-piles may be of wood; or of cast iron, of a strong form.

The sufficiency of a mere bank of well-packed earth in still water, is shown by the embankments or levees, thrown up in all countries, to prevent rivers from overflowing adjacent low lands. The general average of the levees along 700 miles of the Mississippi, is about 6 ft high; only 3 ft wide on top; side-slopes 1½ to 1. In floods the river rises to within a foot or less of their tops; and frequently bursts through them, doing immense damage. They are entirely too slight.

The method of a single row of 12 by 12 inch squared piles, driven in contact with each other, (*close piles,*) and simply backed by an outer deposit of impervious soil, is very effective; and with the addition of interior cross-braces or struts, like c c, Fig 7, to prevent crushing inward by the outside pressure of the water and puddle when pumped out, has been successfully employed in from 20 to 25 ft depth of water, in which there was not sufficient current to wash away the puddle. The cross-braces are inserted successively, as the water is being pumped out; beginning, of course, with the upper ones. The ends of these braces may abut on longitudinal timbers, bolted to the piles for the purpose. Another method is **a strong crib,** composed of uprights framed into caps and sills; and covered outside with squared timbers or plank, laid touching each other, and well calked; as in the caisson, Fig 6; but without a bottom. Between the opposite pairs of uprights are strong interior struts, as c c, Fig 7, reaching from side to side, to prevent crushing inward. The

upper series of these usually supports a platform for the workmen, windlasses, &c. The crib having been built on land, is launched, taken to its final place, and sunk by piling stones on a temporary platform resting on the cross-struts; the bottom of the stream having been previously levelled off, if necessary, for its reception.

To prevent leaking under the bottom of the crib, sheet-piles may be driven around it, their heads extending a few feet above its bottom; or a small deposit of outside puddle may be placed around it, as shown at the stone deposits *t t*, Fig 4. Or a broad flap of tarpaulin may be closely nailed around and a little above the lower edge of the crib; so arranged that it may be spread out loosely on the river bottom, to a width of a few feet all around the outside of the crib: and the puddle may be placed upon it. Such a tarpaulin is also very useful in case the river bottom is somewhat irregular, and cannot be levelled off without too great expense; in which case the crib cannot come to a full bearing upon it; and consequently the water would leak or flow beneath freely. It is especially adapted to uneven rock; where sheet-piles cannot be driven. An artificial stratum of impervious soil may, however, be deposited on bare rock; in which case the sinking of the crib, and the subsequent operations will be the same as on a natural stratum. These expedients are evidently more or less applicable in other cases, where, to avoid repetition, they are not specially mentioned.

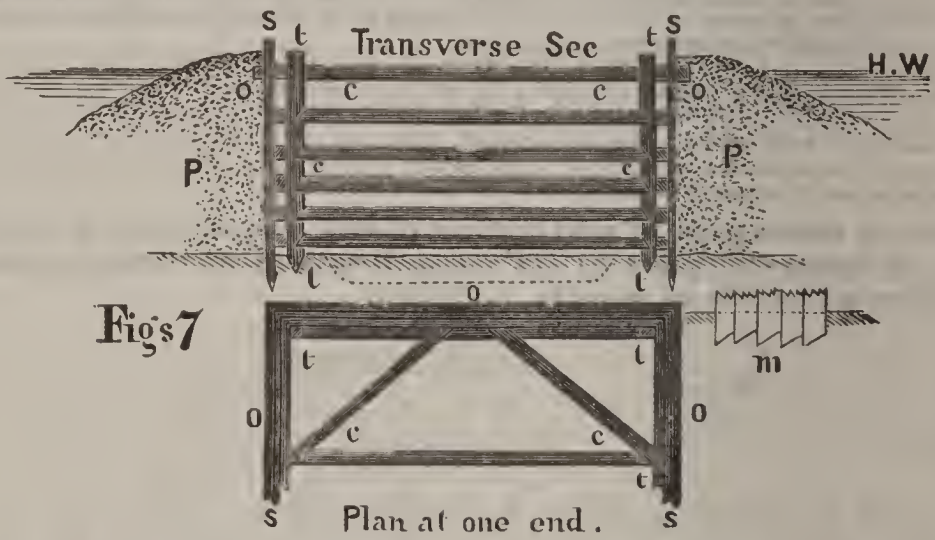
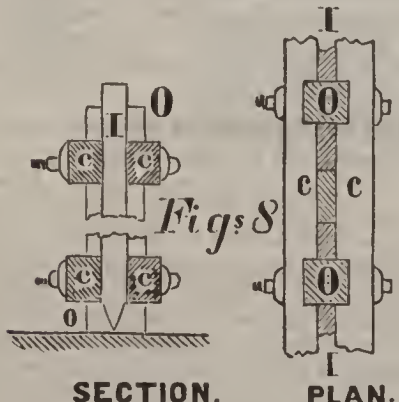


Fig 7

Fig 7 is another crib coffer-dam: in which the sides, instead of being planked longitudinally, as in the last instance, are sheathed with vertical sheet-piles *s*, driven after the crib is sunk. It is much inferior to the last, owing to its greater liability to leak. In one of this description, Fig 7, successfully used in 16 ft water, the dimensions of the crib were 34 ft by 80 ft. Along each long side were 7 uprights *t, t*, 19 ft long, 12 ins square, 12¾ ft apart. Into each opposite pair of these were notched, and held by dog-irons, 6 cross-braces *c c*, of 12 ins square. The distance between the two upper ones was 3 ft in the clear; gradually diminishing to 18 ins between the two lower ones, on account of the increased pressure of the water in descending. On the outside of the uprights, and opposite the ends of the braces, were bolted longi-



Figs 8

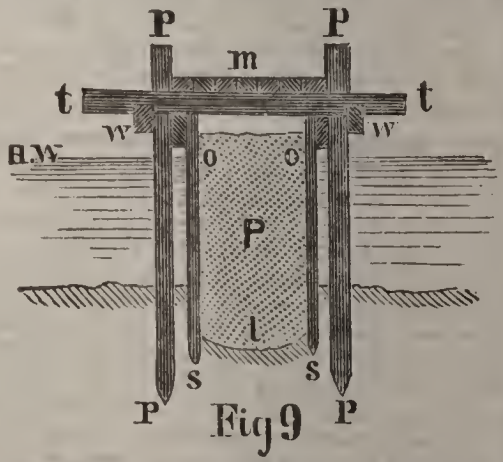


Fig 9

tudinal timbers to support the outside pressure against the 3-inch sheet-piling *s s*. Other longitudinal pieces *o o*, confine the heads of the sheet-piles to the top of the crib after they are driven. The feet of the sheet-piles were cut to an angle, as at *m*; to make them draw close to each other at bottom in driving.

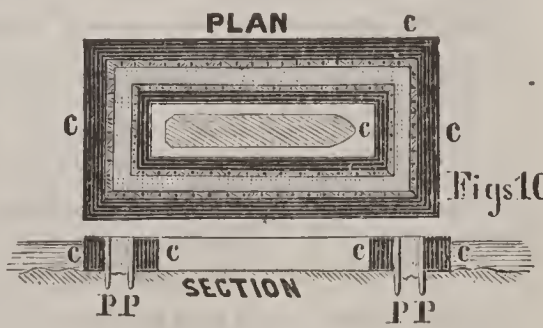
The sheet-piles will drive in a far more regular and satisfactory manner, with the arrangement shown in Figs 8. Here *o o* are the uprights; *c c* are pairs of longitudinal

pieces, notched and bolted to the uprights, near both their tops and their feet; and at as many intermediate points as may be desired. The sheet-piles I, are inserted between these; and of course are guided during their descent much more perfectly than in Fig 7. **The crib at top of p 636** may be used as a cofferdam.

When the current is too strong to permit the use of outside puddle, P, Fig 7, the principle of coffer-dam shown in Fig 9, is generally used; in which both sides of the puddle are protected from washing away. The space to be enclosed by the dam is surrounded by two rows of firmly-driven main piles *p p*, on which the strength chiefly depends. They may be round. In deciding upon their number, it must be remembered that they may have to resist floating ice, or accidental blows from vessels, &c. With reference to this, extra *fender*-piles may be driven. A little below the tops of the main piles are bolted two outside longitudinal pieces *w w*, called *wales;* and opposite to them two inner ones, as in the fig. The outer ones serve to support cross-timbers *t t*, which unite each pair of opposite piles, and steady them; and prevent their spreading apart by the pressure of the puddle P. The inner ones act as guides for the sheet-piles *s s*, while being driven; after which the heads of the sheet-piles are spiked to them. In deep water these sheet-piles must be very stout, say 12 ins square; to resist the pressure of the compacted puddle.

A gangway m, is often laid on top of the cross-pieces *t t*, for the use of the workmen in wheeling materials, &c. The puddle P is deposited in the water in the space, or boxing, between the sheet-piles. It should be put in in layers, and compacted as well as can be done without causing the sheet-piles to bulge, and thus open their joints. The bottom of the puddle-ditch should be deepened, as in the fig, in case it consists, as it often does, of loose porous material which would allow water to leak in beneath it and the sheet-piles. This leaking *under* the dam is frequently a source of much trouble and expense. Water will find its way readily through almost any depth and distance of clean coarse gravelly and pebbly bottom, unmixed with earth. Sand is also troublesome; and if a stratum of either should present itself extending to a great depth, it will generally be expedient to resort to either simple cribs, Fig 4; or to caissons; with or without piles in either case, according to circumstances. But if such open gravel, or any other permeable or shifting material, as soft mud, quicksand, &c, is present in a stratum but a few feet in thickness, and underlaid by stiff clay, or other safe material, leaking may be prevented, or at least much reduced, by driving the sheeting-piles 2 or 3 ft into this last; and by deepening the puddle-trench to the same extent. It may sometimes be better, and more convenient, to dredge away the bad material entirely from all the space to be enclosed by the dam, and for a short distance beyond, before commencing the construction of the latter. If the dam, Fig 9, is (as it should be) well provided with cross-braces, like *c c*, Fig 7, extending across the enclosed area, the thickness or width *o o* of the puddle, need not be more than 4 or 5 feet for shallow depths; or than 5 to 10 ft for great ones: because its use is then merely to prevent *leaking*. But if there are no braces, it must be made wider, so as to resist *upsetting* bodily; and then, with good puddle, *o o* may, as a rule of thumb, be ¾ of the vertical depth *o l* below high water; except when this gives less than 4 ft: in which case make it 4 ft; unless more should be required for the use of the workmen, for depositing materials, &c. Or if the excavation for the masonry is sunk deeper than the puddle, the dam must be wider; else it may be upset into the excavated pit.

The excavated soil may be raised in buckets by windlasses, or by hand, in successive stages. The pumps may be worked by hand, or by steam, as the case may require; as also the windlasses generally needed for lowering mortar, stone, &c. More or less leaking may always be anticipated, notwithstanding every precaution.



Figs 10

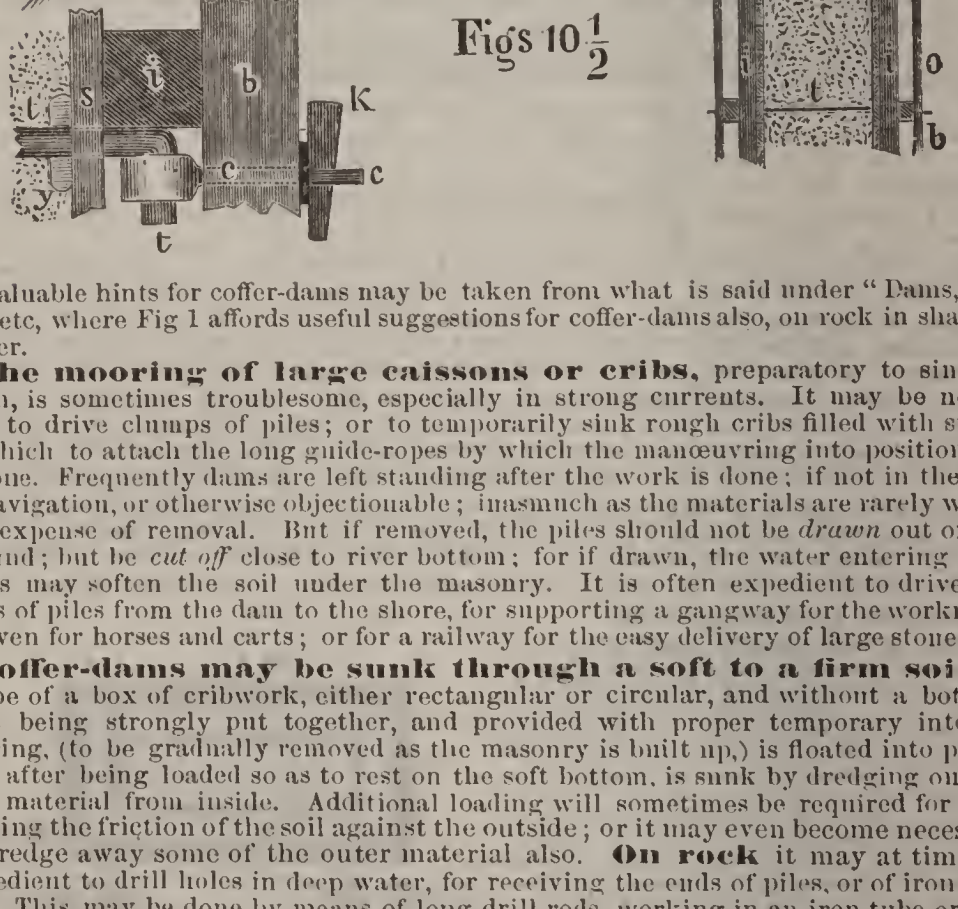
Where a coffer-dam is exposed to a violent current, and great danger from ice, &c, the expensive mode shown in Figs 10 may become necessary. The two black rectangles c c, represent two lines of rough cribs filled with stone, and sunk in position; one row being enclosed by the other; with a space several feet wide between them. Sheet-piles *p p* are then driven around the opposite faces of the two rows of cribs; and the puddle is deposited within the boxing thus provided for it, as shown in the fig.

Where the current is not strong enough to wash away gravel backing, we may, on rock especially, enclose the space to be built on, by a single quadrangle of cribs sunk by stone; and after adopting precautions to prevent the gravel from being pressed in beneath the cribs, apply the backing.*

Figs 10½ show the plan, outside view, and transverse section, to a scale of 20 ft to an inch, of a coffer-dam on rock, in 8 to 9 ft water, used successfully on the Schuylkill Navigation.

* A pure clean coarse gravel is entirely unfit for such purposes. A considerable proportion of earth is essential for preventing leaks.

Coffer-dam on rock. Uprights *b*, about 1 ft square, and 10 ft apart from center to center along the sides of the dam; and 10 ft in the clear, transversely of the dam, support two lines of horizontal striugers, *i i*; inside of which are the two lines of sheeting-piles, *s s*, enclosing between them a width of 7 ft of gravel puddle. Two flat iron bars (*t t*, of the transverse section) tie together each pair of uprights *b b*. These bars are ½ inch thick, by 2½ ius deep, and 9 ft long. Their hooked ends fit into eye-bolts c, which pass through the uprights *b*; outside of which they are fastened by keys, *k*, (see detail sketch.) Between the keys and *b*, were washers. At the corners of the dam (see plan) were additional tie-bars, as shown. A small band of straw, as seen at *y*, wrapped around the tie-bars just inside of the sheet-piles; and kept in place by the puddle; effectually prevented the leaking which generally proves so troublesome in such cases. The stout oblique braces, *o o*, were merely spiked to the outside faces of the uprights *b*. They are not shown in the transverse section. This dam was built on shore; in sections 30 to 40 ft long. These were floated into place, and weighted down, sheet-piled, and puddled with gravel. The dam had sluices by which water was admitted when necessary for preventing the outside head from exceeding 9 ft. The lengths of the uprights *b b* were first found by careful soundings.



Figs 10 ½

Valuable hints for coffer-dams may be taken from what is said under "Dams," pp 282 etc, where Fig 1 affords useful suggestions for coffer-dams also, on rock in shallow water.

The mooring of large caissons or cribs, preparatory to sinking them, is sometimes troublesome, especially in strong currents. It may be necessary to drive clumps of piles; or to temporarily sink rough cribs filled with stone, to which to attach the long guide-ropes by which the manœuvring into position, &c, is done. Frequently dams are left standing after the work is done; if not in the way of navigation, or otherwise objectionable; inasmuch as the materials are rarely worth the expense of removal. But if removed, the piles should not be *drawn* out of the ground; but be *cut off* close to river bottom; for if drawn, the water entering their holes may soften the soil under the masonry. It is often expedient to drive two rows of piles from the dam to the shore, for supporting a gangway for the workmen; or even for horses and carts; or for a railway for the easy delivery of large stones, &c.

Coffer-dams may be sunk through a soft to a firm soil, in shape of a box of cribwork, either rectangular or circular, and without a bottom. This being strongly put together, and provided with proper temporary internal bracing, (to be gradually removed as the masonry is built up,) is floated into place; and after being loaded so as to rest on the soft bottom, is sunk by dredging out the soft material from inside. Additional loading will sometimes be required for overcoming the friction of the soil against the outside; or it may even become necessary to dredge away some of the outer material also. **On rock** it may at times be expedient to drill holes in deep water, for receiving the ends of piles, or of iron rods, &c. This may be done by means of long drill-rods, working in an iron tube or pipe sunk as a guide to the rod; with its lower end over the spot to be bored. Or a diving-bell may be used. Or **a cylinder** of staves 4 to 12 inches thick, long enough to reach above the surface, and having a broad tarpaulin flap or apron around its lower edge, to be covered with gravel to prevent leaking: may be sunk, and the water pumped out, to allow a workman to descend, and work in the open air.

Piles. When driven in close contact, as in Fig 11, for preventing leakage; for confining puddle in a coffer-dam; or for enclosing a piece of soft or sandy ground, to prevent its spreading when loaded; or if the outside soil should wash away from

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Cost of piles delivered at wharf, Philada, 1886. Hemlock, 5 to 6 cts per foot lineal, Bay yellow pine, up to 35 ft, 6 cts, Southern yellow pine, 10 cts.

around them, &c, they are called **sheet-piles.** Generally these are thinner than they are wide; but frequently they are square; and as large as bearing piles; and are then called **close piles.** To make them drive tight together at foot, they are cut obliquely as at *f*. Occasionally, when driven down to rock through soft soil, their feet are in addition cut to an edge, as at *i*, so as to become somewhat bruised when they reach the rock, and thus fit closer to its surface. Their heads are kept in line while driving, by means of either one or two longitudinal pieces *a* and *o*, called **wales** or *stringers*. These wales are supported by *gauge-piles*, or *guide-piles*, previously driven in the required line of the work, and several ft apart, for this purpose. See Figs 8.

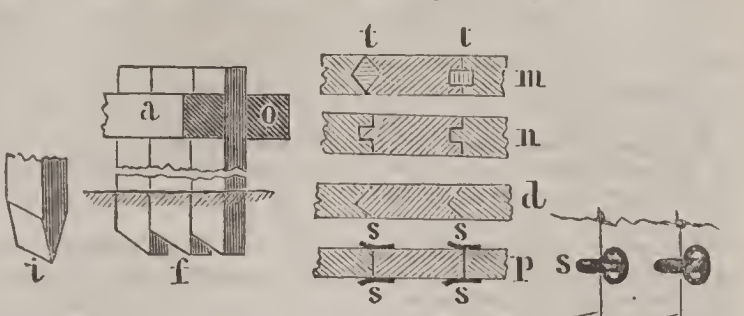


Fig 11

A dog-iron d, of round iron, may also be used for keeping the edges of the piles close at top to those previously driven, both during and after the driving. Its sharp ends, c c, being driven into the tops of the wales *w w*, (shown in plan,) it holds the descending pile *o* firmly in place. At *n*, *d*, *p*, Fig 11, are other modes occasionally used for keeping the piles in proper line. At *p*, the letters *s s* denote small pieces of iron well screwed to the piles, a little above their feet, to act as guides; very rarely used. At *m* are shown wooden tongues *t t*, sometimes driven down between the piles after they themselves have been driven; to assist in preventing leaks. In some cases sheet-piles are employed without being driven. A trench is first dug to their full depth for receiving them; and the piles are simply placed in these, which are then refilled. Closer joints can be secured in this manner than by driving.

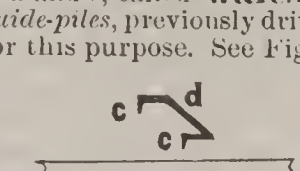
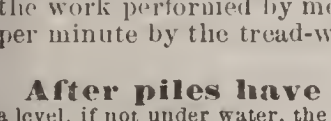


Fig 12

When piles are intended to sustain loads on their tops, whether driven all their length into the ground, or only partly so, as in Fig 3, they are called **bearing piles.** They are generally round; from 9 to 18 ins diam at top; and should be straight, but the bark need not be removed. White pine, spruce, or even hemlock, answer very well in soft soils; good yellow pine for firmer ones; and hard oaks, elm, beach, &c, for the more compact ones. They are usually driven from about 2½ to 4 ft apart each way, from center to center, depending on the character of the soil, and the weight to be sustained. **A tread-wheel** is more economical than the winch for raising the hammer, when this is done by men. Morin found that the work performed by men working 8 hours per day, was 3900 foot-pounds per man, per minute by the tread-wheel; and only 2600 by a winch.

After piles have been driven, and their heads carefully sawed off to a level, if not under water, the spaces between them are in important cases filled up level with their tops with well rammed gravel, stone spawls, or concrete, in order to impart some sustaining power to the soil between the piles. Two courses of stout timbers, (from 8 to 12 ins square, according to the weight to be carried) are then bolted or treenailed to the tops of the piles and to each other, as shown in the Fig, forming what is called a **grillage.** On top of these is bolted a floor or platform of thick plank for the support of the masonry; or the timbers of the upper course of the grillage may be laid close together to form the floor. The space below the floor should also, in important cases, be well packed with gravel, spawls, or concrete.



If under water, the piles are sawed off by a diver, or by a circular saw driven by the engine of the pile-driver, and the grillage is omitted. Instead of it the masonry or concrete may be built in the open air in a caisson, which gradually sinks as it becomes filled; or on a strong platform which is lowered upon the piles by screws as the work progresses. Or a strong caisson may first be sunk entirely under water, and then be filled with concrete, up to near low water; the caisson being allowed to remain. Or the caisson may form a cofferdam, to be first sunk, and then pumped out. If the ground is liable to wash away from around the piles, as in the case of bridge piers, &c, defend it by sheet-piles, or rip-rap, or both.

The cost of a floating steam pile driver, in Philada, scow 24 ft by 50 ft, draft 18 ins, with one engine for driving, and one (to save time) for getting another pile ready; with one ton hammer, is about $6000; and $500 more will add a circular saw, &c, for sawing off piles at any reqd depth. Requires engineman, cook, and 4 or 5 others. Will burn about half a ton of coal per day. Driving 20 ft into gravel, and sawing off, will average from 15 to 20 piles per day of 10 hours. In mud about twice as many. On land about half as many as in water.

The gunpowder pile driver invented by Mr. Thomas Shaw, the well-known mechanical engineer of Philada, is a very meritorious machine. The hammer is worked by small cartridges of powder, placed one by one in a receptacle on top of the pile; and exploded by the hammer itself. It can readily make 30 to 40 blows of 5 to 10 ft, per minute; and, since the hammer does not come into actual contact with the piles, it does not injure their heads at all; thus dispensing with iron hoops, &c, for preserving them. When only a slight blow is required, a smaller cartridge

is used. To drive a pile 20 ft into mud averages about one-third of a pound of powder; into gravel, 4 times as much. This machine does not assist in raising the pile, and placing it in position, as is done by ordinary steam pile drivers; the latter, however, average but from 6 to 14 blows per minute.

Piles have been driven by exploding small charges of **dynamite** laid upon their heads, which are protected by iron plates.

Steam-hammer pile drivers, operating on the principle of that devised by **Nasmyth** about 1850, are economical in driving to great depths in difficult soils where there are say 200 or more piles in clusters or rows, so that the machine can readily be moved from pile to pile.

The steam **cylinder** is upright, and is confined between the upper ends of two vertical and parallel I or channel **beams** about 6 to 12 ft long and 18 ins apart, the lower ends of which confine between them a hollow conical **"bonnet casting,"** which fits over the head of the pile. This casting is open at top, and through it the hammer, which is fastened to the foot of the piston-rod,* strikes the head of the pile. Each of the vertical beams encloses one of the two upright guide-timbers, or "leaders," of the pile driver, between which the driving apparatus, above described, is free to slide up or down as a whole.

When a pile has been placed in position, ready for driving, the bonnet casting is placed upon its head, thus bringing the weight of the beams, cylinder, hammer, and casting upon the pile. This weight rests upon the pile throughout the driving, the apparatus sliding down between the leaders as the pile descends.

The steam is conveyed from the boiler to the cyl by a flexible pipe. When it is admitted to the cyl, the hammer is lifted about 30 or 40 ins, and upon its escape the hammer falls, striking the head of the pile. About 60 blows are delivered per minute. The hammer is provided with a trip-piece which automatically admits steam to the cylinder after each blow, and opens a valve for its escape at the end of the up-stroke. By altering the adjustment of this trip-piece, the length of stroke (and thus the force of the blows) can be increased or diminished. The admission and escape of steam, to and from the cyl, can also be controlled directly by the attendant. The number of blows per minute is increased or diminished by regulating the supply of steam.

In making the up-stroke, the steam, pressing against the lower cyl head, of course presses downward on the pile and aids its descent.

The chief advantage of these machines lies in the great rapidity with which the blows follow one another, allowing no time for the disturbed earth, sand, &c, to recompact itself around the sides, and under the foot, of the pile. This enables the machines to do work which cannot be done with ordinary pile drivers. They have driven Norway pine piles 42 ft into sand. They are less liable than others to split and broom the pile, so that these may be of softer and cheaper wood. The bonnet casting keeps the head of the pile constantly in place, so that the piles do not "dodge" or get out of line. Their heads have, in some cases, been set on fire by the rapidly succeeding blows.

These machines consume from 1 to 2 tons of **coal** in 10 hours, and **require a crew** of 5 men. They work with a **boiler pressure** of from 50 to 75 lbs per sq inch.

They are made by R. J. & A. B. Cram, 80 Griswold St, Detroit, Mich, and by Vulcan Iron Works, 86 N. Clinton St, Chicago, Ill.

The largest (No. 1) Vulcan machine has a **hammer weighing** about 3640 lbs; **stroke,** 36 ins; **diam of cyl,** 12 ins; space between leaders, 19½ ins. **Cost** (1886) including hose, fittings, &c, but exclusive of hoisting engines and boiler, about $1075 on cars at works.

The Vulcan Works make a railroad **car, furnished with a pile-driving machine,** and 2 hor engines, with boiler, for raising the hammers. Either the Nasmyth, or the ordinary, hammer may be used. The car can be made self-propelling when desired. It is larger than an ordinary platform car, and is provided with lateral supports to enable it to drive piles to the right or left of the road-bed. The leaders are hinged, so that they can be laid horizontally upon the car when not in use.

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*In the Cram machine the hammer is fastened to the lower end of the *cylinder*, instead of to that of the piston-rod. The cylinder rises and falls with the hammer, and its weight is thus utilized in increasing the force of the blows. Steam is supplied to the cylinder through the piston-rod, which is made hollow for this purpose. The piston-rod is fixed in place between the vertical beams by means of a cross-head at its top. The piston is fastened to the foot of the piston-rod.

Rules for the Sustaining Power of Piles.

They differ very much. No rule can apply correctly to all conditions. The ground itself between the piles, in most cases, supports a part of the load; although the whole of it is usually assigned to the piles. Again, in very clayey soils, there is greater liability to sink somewhat with the lapse of time, in consequence of the admission of water between the pile and the clay; thus diminishing the friction between them. The less firm the soil, the more will the piles be affected by tremors; which also tend in time to cause sinking. In some cases this sinking will not be that of the piles settling deeper into the earth *around* them; but that of the entire compacted mass of piles and earth into which they were driven, settling down into the less dense mass *below* them. Piles are sometimes blamed for settlements which are really due to the crushing (flatways) of the timbers which rest immediately upon their heads. See p 436.

In the fine **London bridge** across the Thames, each pile under some of the piers sustains the very heavy load of 80 tons. They are driven but 20 feet into the stiff, blue London clay; and are placed nearly 4 ft apart from center to center; which is too much for such piers and arches. At 3 ft apart scant, they would have had but 45 tons to sustain. They are 1 ft in diam at the middle of their length. Ugly settlements, some of them to the extent of about a ft, have occurred under these piers. **Blackfriars bridge,** in the same vicinity, exhibits the same defect. By some this is ascribed in both cases to the gradual admission of water between the clay and the piles, perhaps by capillary action of the piles themselves; or perhaps by direct leaking. It may, however, be owing in part to the crushing of the platforms on top of the piles; or to a bodily settlement of the entire mass of piled clay, into the unpiled clay beneath, under the immense load that rests upon it. This here amounts to 5½ tons per sq foot of area covered by a pier; and is probably too much to trust upon damp clay, when even the slightest sinking is prejudicial.

Maj J. Sanders, U. S. Engs, experimented largely at Fort Delaware in river mud; and gave the following in the Jour. Franklin Inst, Nov 1851. For the **safe** load for a common wooden pile, driven until it sinks through only small and nearly equal distances, under successive blows, divide the height of the fall in ins, by the small sinking at each blow in ins. Mult the quot by the weight of the hammer, ram, or monkey, in tons or pounds, as the case may be. Divide the prod by 8. He does not state any specific coefficient of safety.

Example. At the **Chestnut St Bridge,** Philada, the greatest weight on any pile is 18 tons. Mr Kneass had the piles driven until they sank ¾, or .75 of an inch under each blow from a 1200 lb hammer, falling 20 ft. Was he safe in doing so? Here we have the fall in ins $= 20 \times 12 = 240$. And $\frac{240}{.75} = 320$; and $320 \times 1200 = 384000$ lbs; and $\frac{384000}{8} = 48000$ lbs, $= 21.4$ tons safe load by Maj Sanders' rule. The soil was river mud.

Our own rule is as follows. Mult together the cube rt of the fall in ft; the wt of hammer in lbs; and the decimal .023. Divide the prod by the last sinking in ins. + 1. The quotient will be the **extreme load** that will be just at the point of causing more sinking. For the safe load take from one twelfth to one half of this, according to circumstances. Or, as a formula,

$$\underset{\text{in tons}}{\textbf{Extreme load}} = \frac{\text{Cube rt of fall in feet} \times \text{Wt of hammer in pounds} \times .023.}{\text{Last sinking in inches} + 1}$$

Example. The same as the foregoing at Chestnut St Bridge. Here the cube rt of 20 ft fall is 2.714 ft. Hence we have

$$\underset{\text{in tons}}{\textbf{Extreme load}} = \frac{2.714 \times 1200 \times .023}{.75 + 1} = \frac{74.9}{1.75} = 42.8 \text{ tons.}$$

Or say half of this, or 21.4 tons, the load for a safety of 2. Major Sanders' rule makes the **safe** load 21.4. The actual one is 18 tons.

A safety of 2 is not enough for river mud. See "Proper load for safety." p 644.

But although Major Sanders' rule and our own agree very well in this instance *if a safety of 2 be taken for each,* they differ widely in some others. Thus at **Neuilly Bridge,** France, Perronet's heaviest hammer weighed 2000 lbs, fall 5 ft, sinkage .25 of an inch in the last 16 blows; or say .016 inch per blow. The piles sustain 47 tons each. Our rule gives 38.8 tons for a safety of 2; while Sanders' rule gives 515 tons safe load! If, as we think probable, there was no actual sinking at the last blow, then our rule gives 39.3 tons for a safety of 2; while Sanders' gives infinity.

At the **Hull Docks,** England, piles 10 ins square, driven 16 ft into alluvial mud, by a 1500 lb hammer, falling 24 ft, sank 2 ins per blow at the end of the driving. They sustain at least 20 tons each, or according to some statements 25 tons. Our rule gives 33.2 tons for the extreme load; or 16.6 for a safety of only 2. Sanders gives for *safety* 12.06 tons. As before remarked, 2 is not safety enough for mud. In mud, it is not primarily the piles, but the piled **soil** that settles, bodily, for years.

At the **Royal Border Bridge,** England, piles were very firmly driven from 30 to 40 ft in sand and gravel, in some cases wet. Pine was first tried, but it split and broomed so badly under the hard driving, that American elm was substituted, with success. They were driven until they sank but .05 inch per blow, under a 1700 lb monkey, falling 16 ft. They support 70 tons each. Our rule gives 47 tons for a safety of 2; while Sanders gives 364 tons safe load!

It is the writer's opinion, however, that the piles did not actually sink, as was (and always is, in such cases) taken for granted by the observers; but that they were merely compressed or partially crushed by overdriving. Most of the piles were driven until they sank (?) only an inch under 150 blows; but we doubt whether they were any safer, or farther in the ground, than when they had received only one of them; and consider such extreme precaution worse than useless.

In some experiments (1873) at Philada, a trial pile was driven 15 ft into soft river mud, by a 1600 lb hammer; its last sinking being 18 ins under a fall of 36 ft. Only 5 hours after it was driven it was loaded with 6 5 tons; which caused a sinking of but a very small fraction of an inch. Our rule

gives 6.4 tons as the extreme load. Under 9 tons it sank .75 of an inch; and under 15 tons, 5 ft. By Maj Sanders' rule its safe load would be 2.14 tons.

A U. S. Govt trial pile, about 12 ins sq, driven 29 ft through layers of silt, sand, and clay, hammer 910 lbs, fall 5 ft, last sinking .375 of an inch, bore 26.6 tons; but sank slowly under 27.9 tons. Our rule gives 26 tons extreme load.

French engineers consider a pile safe for a load of 25 tons, when it is driven to the refusal of 1344 lbs, falling 4 ft; our rule gives 24.2 tons for safety 2. They estimate the refusal by its not sinking more than .4 of an inch under 30 blows. In many important bridges &c they drive until there is no sinking under an 800 lb hammer, falling 5 ft. Our rule here gives 31.5 tons extreme load; or 15.7 for safety 2.

As to the proper load for safety, we think that not *more* than one-half the extreme load given by our rule should be taken for piles *thoroughly* driven in *firm* soils: nor more than one-sixth when in river mud or marsh; assuming, as we have hitherto done, that their feet do not rest upon rock.

If liable to tremors, take only **half** these loads.

Piles may be made of any required size as regards either length or cross section, by bolting and fishing together sidewise and lengthwise, a number of squared timbers.

Piles with blunt ends. At South Street Bridge, Phila, 1200 stout piles of Nova Scotia spruce with blunt ends were driven 15 to 35 ft, partly in strong gravel, by a common steam pile driver, at a total cost (piles and driving) of $7 to $8 each. At **Wilmington Harbor,** Cal, Mr. C. B. Sears, U. S. Army, (Jour. Am. Soc. C. E., Dec 1876) found that in firm compact wet sand, after the first few blows the piles would not penetrate more than .5 to 1.5 ins at a blow, no matter how far the 2400 lb hammer fell. The unpointed ones of which there were many thousands, drove quite as readily to average depths of 15 ft in this sand as the pointed ones, and with much less tendency to cant. As a high fall had no farther effect than to batter the heads he reduced it to 10 ft, which drove an average of about .72 inch to a blow. To insure straight driving, the ends must be at right angles to the length. **Instead of driving** piles to moderate depths it may at times be better to merely **plant** them butt down in holes bored by an auger like **Pierce's Well Borer.** See p 626. This will avoid shaking adjacent buildings. See "In Mobile Bay," p 646.

The ultimate friction of piles even with the bark on, and driven about 3 ft apart from cen to cen probably never much exceeds about 1 ton per sq ft even when well driven into dense moist sand or loamy gravel; nor more than .5 to .75 of a ton in common soils and clays; or than .1 to .2 of a ton in silt or wet river mud depending on the depth and density.

The friction of cast iron cylinders seems to be about .3 that of piles.

There is a great difference in the penetrability of different sands. Thus, in the Lary bridge, no special difficulty was found in driving piles 35 ft into deep wet sand; while, in other wet localities, piles of very tough wood, well shod with iron, cannot be driven 6 ft into sand, without being battered to pieces. The same difference has been found in the case of screw-piles. At the Brandywine light-house these could not be forced more than 10 ft into the clean wet sand. Stiff wet clay (and clean gravels) also differ very much in this respect. Generally they are penetrable to any required depth with comparative ease; but we have seen stout hemlock piles battered to pieces in driving 6 ft through wet gravel; and Mr. Rendel found that at Plymouth he "could not by any force drive screw-piles more than about 5 ft into the clay, which is not as stiff as the Loudon clay," on which the forementioned new London and Blackfriars bridges were founded; and into which even ordinary woodeu piles were driven 20 ft without special difficulty.

A mixture of mud with the sand or gravel facilitates driving very much; but before beginning an extensive system of piling, a few experimental ones should be driven, to remove doubt as to the trouble and expense that may be anticipated. Mere boring will often be but a poor substitute for this.

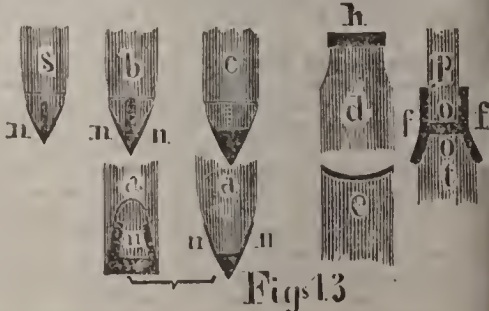
As a general rule, a heavy hammer with a low fall, drives more pleasantly than a light one with a high fall. Where a hammer of ⅔ ton (1500 lbs) falling 25 ft, in a very strong ground, shattered the piles; one of 2 tons, (4500 lbs.) with 7 ft fall, drove them satisfactorily. More blows can be made in the same time with a low fall; and this gives less time for the soil to compact itself around the piles between the blows. At times a pile may resist the hammer after sinking some distance; but start again after a short rest; or it may refuse a heavy hammer, and start under a lighter one. It may drive slowly at first, and more rapidly afterward, from causes that may be difficult to discover. The driving of one sometimes causes adjacent ones previously driven, to spring upward several feet. A pile is in the most favorable position when its foot rests upon rock, after its entire length has been driven through a firm soil, which affords perfect protection against its bending like an overloaded column; and at the same time creates great friction against its sides; thus assisting much in sustaining the load, and thereby relieving the pressure upon the foot. A pile may rest upon rock, and yet be very weak; for if driven through very soft soil, all the pressure is borne by the sharp point; and the pile becomes merely a column in a worse condition than a pillar with one rounded end. See Fig 1, page 439, Strength of Iron Pillars. In such soils the piles need very little sharpening; indeed, had better be driven without any; or even butt end down.

The driving of a pile in soft ground or mud will generally cause an adjacent one previously driven, to lean outwards unless means be taken to prevent it.

In piling an area of firm soil it is best to begin at its center and work outwards; otherwise the soil may become so consolidated that the central ones can scarcely be driven at all.

Elastic reaction of the soil has been known to cause entire piled areas to rise, together with the piles, before they were built upon.

In very firm soil, especially if stony; or even in soft soil, if the piles are pointed, and are to be driven to rock; their feet should be protected by **shoes** of either wrought iron, as at *a*, *s*, and *b*, Figs 13; spiked to the pile by means of the iron straps *n*, forged to them; or of cast iron, as at *c*, where the shoe is a solid inverted cone, the wide flat base of which affords a good bearing for the flat bottom of the pile-point. The dotted line is a stout wrought-iron spike, well secured in the cone, which is cast around it; this holds the shoe to the pile. Regular



Figs 13

wrought-iron shoes will generally weigh 18 to 30 lbs; but sheet iron may be used when the soil is but moderately compact; plate iron when more so; and solid iron or steel points, from 2 to 4 ins square at the butt, and 4 to 8 ins long, when very compact and stony. **Holes may be drilled in rock** for receiving the points of piles, and thus preventing them from slipping; by first driving down a tube, as a guide to the drill, after the earth is cleaned out of the tube. **To preserve the heads** to some extent from splitting under the blows of the hammer, they are usually surrounded by a hoop *h*, Fig *d*; from ½ to 1 inch thick; and 1½ to 3 ins wide. These are, however, sometimes but imperfect aids; for in hard driving the head will crush, split, and bulge out on all sides, frequently for many feet below the hoop: moreover, the hoops often split open. The heads, therefore, often have to be sawed, or pared off several times before the pile is completely driven; and allowance must be made for this loss in ordering piles for any given work; especially in hard soil. Capt Turnbull, U S Top Eng, states that at the Potomac aqueduct, his pileheads were preserved from injury by the simple expedient of dishing them out to a depth of about an inch, and covering them by a loose plate of sheet iron; as shown in section at *e*, Figs 13. A very slight degree of brooming or crushing of the head, materially diminishes the force of the ram. Piles may be driven through small loose rubble without much labor. Shaw's driver does not injure the heads. Piles which foot on sloping rock may slide when loaded.

To drive a pile head below water a wooden punch, or follower, as at *p*, Figs 13, may be used. The foot of this punch fits into the upper part of a casting *f f*, round or square, according to the shape of the pile; and having a transverse partition *o o*. The lower part of the casting is fitted to the head of the pile *t*; and the hammer falls on top of the punch. When driving piles vertically in very soft soil, to support retaining-walls, or other structures exposed to horizontal or inclined forces, care must be taken that these forces do not push over the piles themselves; for in such soils piles are adapted to resist vertical forces only, unless they be driven at an inclination corresponding to the oblique force.

A broken pile may be drawn out, or at least be started, if not very firmly driven, by attaching scows to it at low water, depending on the rising tide to loosen it. Or a long timber may be used as a lever, with the head of an adjacent pile for its fulcrum. Or a crab worked by the engine of the pile driver. In very difficult cases the method devised by Mr J. Monroe, C E, may be used. A 4 inch gas pipe 15 ft long, shod with a solid steel point, and having an outer shoulder for sustaining a circular punch, was thereby driven close to and 2 or 3 ft deeper than two piles driven 12 ft, in 37 ft water, and broken off by ice. Four pounds of powder were then deposited in the lower end of the pipe, and exploded, lifting the piles completely out of place. It will often be best to let a broken pile remain, and to drive another close to it. May be drawn by hydraulic press.

Ice adheres to piles with a force of about 30 to 40 lbs per sq inch, and in rising water may lift them out of place if not sufficiently driven.

Iron piles and cylinders. Cast iron in various shapes has been much used in Europe for sheet piles: especially when intended to remain as a facing for the protection of concrete work, filled in behind and against them.* Cast-iron cylinders, open at both ends, may be used as bearing piles; and may be cleaned out, and filled with concrete, if required. The friction in driving is greater than in solid piles, inasmuch as it takes place along both the inner and the outer surfaces. This may be diminished by gradually extracting the inside soil as they go down. They require much care, and a lighter hammer, or less fall than wooden ones, to prevent breaking: to which end a piece of wood should be interposed between the hammer and the pile; or the ram may be of wood. But it is better to use them in the shape of **screw cylinders,** which, moreover, gives them the advantage of a broad base, as in the following.

Brunel's process. He experimented with an open cast-iron cylinder, 3 ft outer diam; 1½ ins thick: in lengths of 10 ft, connected together by internal socket and joggle joints, secured by pins, and run with lead. It had a sharp-edged hoop or cutter at bottom: and a little above this, one turn of a screw, with a pitch of 7 ins, and projecting one foot all around the outside of the cylinder. By means of capstan bars and winches, he screwed this down through stiff clay and sand, 58 feet to rock, on the bank of a river. In descending this distance the cylinder made 142 revolutions; sinking on an average about 5 ins at each. The time occupied in actually screwing was 48¼ hours; or about $1\frac{2}{10}$ ft per hour. There were, however, many long intervals of rest for cleaning away the soil in the inside. After resting, there was no great difficulty in restarting. The next fig will give an idea of the arrangement of the screw.

The screw-pile of Alex. Mitchell, Belfast, consists usually of a rolled iron shaft A, Figs 14, from 3 to 8 ins diam; and having at its foot a cast-iron screw S S S, with a blade of from 18 ins to 5 ft diam. The screws used for light-houses, exposed to moderate seas, or heavy ice-fields, are ordinarily about 3 ft diam, have 1½ turns or threads, and weigh about 600 lbs. The round rolled shafts are from 5 to 8 ins diam. They are screwed down from 10 to 20 ft into clay, sand, or coral, by about 30 to 40 men, pushing with 6 to 8 capstan bars, the ends of which describe a circle of about 30 to 40 ft diam. For this purpose a platform on piles has frequently to be prepared. In quiet water, this may be supported on scows; or a raft well moored may be used when the driving is easy; or the deck of a large scow with a well-hole in the center for the pile to pass through. Roughly made temporary cribs, filled with stone and sunk, might support a platform in some positions. The platform must evidently be able to resist revolving horizontally under the great pushing force of the men at the capstan bars; and on this account it is difficult to drive screws to a sufficient depth, in clean compact sand, by means of a floating platform. The feet of the piles must be firmly secured to the screws, to prevent

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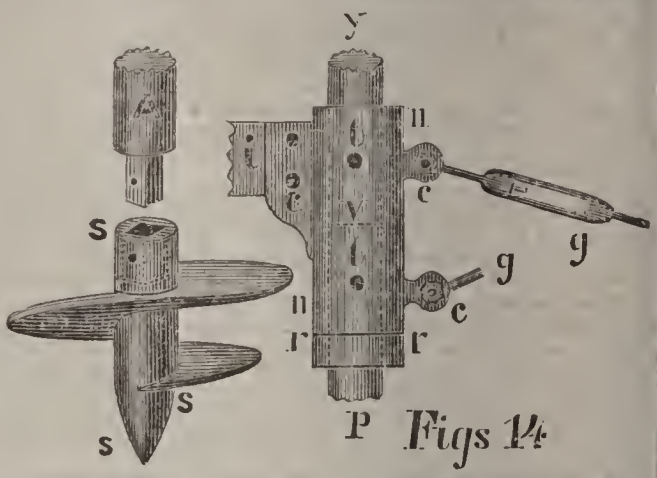
* **Cast iron, intended to resist sea-water,** should be close-grained, hard, white metal. In such, the small quantity of contained carbon is chemically combined with the metal; but in the darker or motled irons it is mechanically combined, and such iron soon becomes soft, (somewhat like plumbago,) when exposed to sea-water. Hard white iron has been proved to resist for at least 40 years without any deterioration; whether constantly under water, or alternately wet and dry. Copper and bronze are but slightly and superficially affected by sea-water; but destructive galvanic action takes place if diff metals are in contact. Soe p 218.

their being lifted out of them by the upward force of waves against the superstructure. At *y p*, Figs 14, is shown a mode of splicing or uniting the different lengths or sections of a pile. The point of junction is at *v*; *r r* is a stout iron ring forged on to the lower pile *p*, about a foot or 18 ins below its top *r*. A strong cylindrical casting *n n*, enclosing the ends of the sections, rests on this ring, and is pinned through the piles, as at *t t*. On this casting are also cast projections *c c c*, for attaching rods *g g*, and beams *i*, &c, necessary for bracing the structure from pile to pile. The time actually required for driving a screw is from 2 to 10 hours, in favorable circumstances.



Figs 14

At the Brandywine lighthouse, on a sand-bank of very pure sand, covered 6 or 8 ft at low water, and from 11 to 13 ft at high, they could not be forced down, from a fixed platform, for more than 10 ft. At other places 20 ft in sand is reached without much trouble, where the sand contains a good deal of mud, but its **bearing power** is then less. This (ultimate) ranges between about 1 and 6 tons per sq ft according to purity, depth, compactness, &c, of the sand. In important cases the bearing power should be tested.

Mitchell's piles have been screwed about 40 feet into a mixture of clay and sand, with screws 4 ft diam. They pass through small broken stone and coral rock without much difficulty; and will push aside bowlders of moderate size. Ordinarily, clay or sand will present no great obstruction; but occasionally either of them will do so. Perfectly pure clean sand, as a general rule, gives most difficulty. At the Brandywine shoal the driving was aided by a spur and pinion placed as low as the water permitted; and the levers were worked by 30 men. The danger of twisting off the shaft is the limit for screwing them. They are much used for the anchoring of chains for mooring buoys, &c. On land, small screws, with short hollow shafts, make good durable supports for depot pillars, cranes, wooden telegraph poles, station signals in marine surveying, &c, &c. They can readily be unscrewed for removal. Horses or oxen may be used in driving large screws. The Brandywine light-house stands on 9 screw-piles, which are surrounded by 30 others of 5 ins diam, as fenders. They have to resist not only moderate seas, but immense fields of floating ice, miles in extent. An unfinished structure was destroyed by ice, which at times injures the bracing of the standing one.

Test borings should be made to ensure that the screws do not stop just above a very weak stratum which may endanger their bearing power. So with *any* piles.

By means of a jet of water forcibly impelled through a tube by a force pump, the most obstinate sands (but not stiff clay or cemented gravel) will be loosened, and the sinking of screw-piles, or wooden ones, or even the largest cylinders, be greatly facilitated. In a government **pier at Cape Henlopen** in very compact sand, in which 6 out of 7 screws previously broke before reaching 10 ft, the use of the jet was found to remove more than three-fourths of the resistance.* The pile *p* to be sunk having first been placed in position as in Fig 15, the lower open ends *t t* of a bent iron tube *t s t* of one and a quarter ins bore were stood upon the upper face of the screw disk, and there held firmly by 3 or 4 men while the pile was being screwed down by the capstan c, which was worked by a leading rope *r*. From the bend *s* of the pipe, a hose *h*, 2 ins diam, led to the force pump, the cylinder of which was 5 ins bore, and 9 ins stroke, and worked about 80 full strokes per minute, by a mule walking on a tread wheel on a floating platform *f*. There was now no trouble in screwing the piles to any required depth. Previous trials by playing the jet *beneath* the disk gave unsatisfactory results.

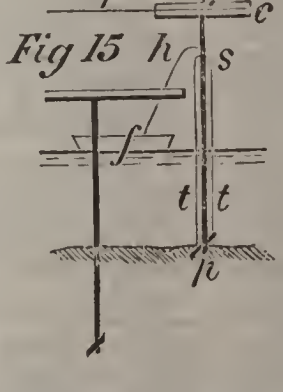


Fig 15

In Mobile Bay several thousands of wooden piles, from 18 to 48 ins diam, were sunk from 10 to 20 ft into obstinate sand, at the average sinking rate of about 1 ft per second, entirely by means of jets. The jet was propelled by a city steam fire engine, on a steamboat, through its own hose, with a one and a quarter inch nozzle. During the descent the nozzle *n n* was held loosely in its place near the foot of the pile, by two staples *s s* and by a string *t* reaching to the surface. The piles were suspended by their heads from shears, by the tackle of which their descent was regulated. The sand settled firmly around the piles in a few minutes after they were sunk.†

At Tensas River, Alabama, for iron cylinders 6 ft diam (enclosing piles, see p 651), in deep light shifting sand, the jet was forced by a small rotary pump of 200 to 300 revolutions per minute, through a canvas hose 3 ins diam, into a central conical cast iron vessel 10 ins diam, from which radiated 12 gas pipes 1 inch diam, and about 30 ins long. At the outer end of each of these radii was an elbow to which was attached a long vertical pipe reaching down into the cylinder, and made in 10 ft lengths with screw ends for prolonging them as the cylinder went down. This apparatus was raised and lowered by a light block and line; and by it alone each cylinder was sunk about 16 ft into the light sand in a few hours.‡

* Report Sec of War 1872. † John W. Glenn, C E, Van Nostrand, June 1874.
‡ Gabriel Jordan, C E; Trans Am Soc C E, Feb 1874.

At the Levan Viaduct, Mr James Brunlee, England, in a light sandy marl of great depth, sunk hollow cast iron cylinders of 10 ins outer diam, to a depth of 20 ft, by means of a jet pipe 2 ins diam passing down inside of the cylinder, and through a hole in its base, which was a cast iron disk 30 ins diam, and 1 inch thick, strengthened by outside flanges. The connecting flanges of the cylinder sections are *outside*, thus impeding the descent, as did also the broad bottom disk; still 3 or 4 hours usually sufficed for the sinking of each, to 20 ft depth. Actual trial showed that their safe sustaining power was about 5 tons per sq ft of bottom disk.

At Lock Ken viaduct each pier consists of two cylinders, open at both ends; of cast iron, 8 ft in diam; $1\frac{1}{8}$ ins thick; in lengths of 6 ft, weighing 4 tons each; and bolted together by inside flanges, with iron cement between them. The cylinders stand 8 ft apart in the clear; and are in 36 ft water. "A strong staging was erected; and 4 guide-piles driven for each cylinder. The several lengths being previously bolted together, these were lowered into their places. Each cylinder sank by its own weight one or two ft through the top mud, and then settled upon the sand and gravel which form the substratum for a great depth. Into this last they were sunk about 8 or 9 ft farther, by excavating the inside earth under water, by means of an inverted conical **screw-pan,** or dredger, of $\frac{1}{4}$ inch plate iron. This was 2 ft greatest diam, and 1 ft deep; and to its bottom was attached a screw about 1 ft long, for assisting in screwing it down into the soil. Its sides had openings for the entrance of the soil; and leather flaps, opening inward, to prevent its escape. From opposite sides of the pan, 3 rods of $\frac{3}{4}$ inch diam projected upward 4 feet, and were there forged together, and connected by an eye-and-bolt joint to a long rod or shaft, at the upper end of which was a four-armed cross-handle, by which the pan was screwed down by 4 men on the staging."

"When a pan was full, a slide which passed over the joint at the bottom was lifted; and the pan was raised by a tackle. This pan raised about 1 cub ft at a time. A smaller one of only 1 ft diam, and 1 ft deep, raising about $\frac{1}{4}$ cub ft, was used when the material was very hard. By this means the cylinders were sunk at the rate of from 2 to 18 ins per day. The slow rate of 2 ins was caused by stones, some of them of 50 lbs. These were first loosened by a screw-pick, which was a bar of iron 3 ft long, with circular arms 12 ins long projecting from the sides. After being loosened by this, the stones were raised by the pan. The expense of all this apparatus was very trifling; and the excavation was done easily and cheaply. After the excavation was finished, and the cylinder sunk, before pumping out the water, concrete (gravel 2, hydraulic cement 1 measure) was filled in to the depth of 12 feet, by means of a large pan with a movable bottom; and about 12 days were left it to harden. The water was then pumped out, and the masonry built in open air. In some of the cylinders, however, the water rose so fast, notwithstanding the 12 ft of concrete, that the pumps could not keep them clear; and 6 ft more of concrete had to be added in those. Finally random-stone, or rough dry rubble, was thrown in around the outsides of the cylinders, to preserve them from blows and undermining."* The masonry extends 20 ft above the cylinders, and above water.

The vacuum and plenum processes. We can barely allude to the general principles of these two modes of sinking large hollow iron cylinders. In the **vacuum process** of Dr. Lawrence Holker Potts, of London, the cylinder *c*, Fig 16, while being sunk, is closed air-tight at top, by a trap door, opening upward. A flexible pipe *p*, of India-rubber, long enough to adapt itself to the sinking of the cylinder, and provided with a stopcock *s*, leads from the cylinder to a vessel *v*; which may be placed on a raft, or a scow, or on land, as may suit circumstances. The cylinder being first stood up in position, as in the fig, the water is pumped out, and the interior soil removed if the cylinder has sunk some distance by its own weight. The cock *s* is then closed, and the air is drawn out from the vessel *v* by an air pump. The cock is then opened, and most of the air in the cylinder rushes into the void vessel *v*; thus leaving the cylinder comparatively empty, and therefore less capable of resisting the downward pressure of the external air upon its top. This pressure, as is well known, amounts to nearly 15 lbs on every sq inch; or nearly 1 ton per sq ft of area of the top. Consequently the cylinder is forced downward in the bed of the river, by this amount of pressure, in addition to its own weight. At the same time, the pressure of the air upon the surface of the water is transmitted through the water to the soil around the open foot of the cylinder; so that if this soil be soft or semi-fluid, it will be pressed up into the nearly void cylinder, in which is no downward pressure to resist it. The descent varies from a few inches, to 4 or 5 ft each time. The process is then repeated, by admitting air again into the cylinder, opening the trap-door, removing the water and soil, as before, &c. Additional lengths of cylinder may be bolted on, by means of interior flanges.

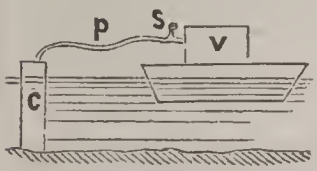


Fig 16

It is **adapted only to soft soils,** and to wet sandy ones; but is not sufficiently powerful in very compact ones; nor does it answer where obstructions from bowlders, logs, &c, occur;

*__Hollow Iron Piles__ either cast or wrought with solid pointed feet, to be driven by the hammer falling *inside* of them and striking against the top of the solid foot, are a recent device of great use in many cases. They are made in sections of which enough can be gradually united to reach any required depth. They avoid the danger of bending which attends striking the top. The iron feet are swelled outwardly a little to diminish earth-friction against the pile above them.

the removal of which requires men to enter the cylinder to its foot; which they cannot do in the rarefied air. The pipe *p* should be of sufficient diam to allow the air to leave the cylinder rapidly, so that the outer pressure may act upon the top as suddenly as possible.

At the Goodwin Sands light-house, England, hollow cylinders 2½ ft in diam, were sunk 34 ft into sand by this process, in about 6 hours; where a steel bar could be driven only 8 ft by a sledge-hammer. Others, 12 ins in diam, have been sunk 16 ft into sand within less than an hour. In this last instance the air-pump had two barrels, 4½ ins diam, 16 inch stroke, worked by 4 men. The pipe *p* was of lead, and only ½ inch diam.

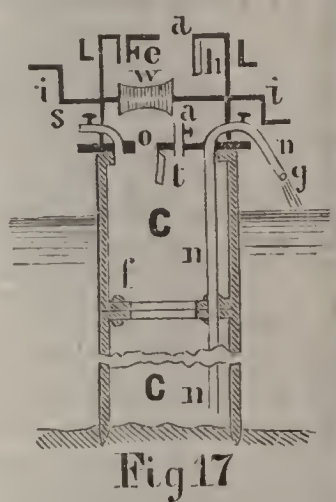


Fig 17

The plenum process, invented by Mr Triger, of France, consists in forcing air into the cylinder C C, Fig 17, to such an extent as to force out the water, compelling it to escape beneath the open foot, into the surrounding water. The interior of the cylinder being thus left dry to the bottom, men pass down it to loosen and remove the soil at and below its base. When this is done, they leave; the compressed air is allowed to escape; and the cylinder, being no longer sustained by the upward pressure of the compressed air beneath its top, sinks into the cavity, or the loosened material at its foot. Fig 17 shows the simple arrangement by which workmen are enabled to enter or leave the cylinder, without allowing the compressed air to escape; as well as the general principle of the entire process.

L L is a separate small chamber, **the air-lock,** which is removed when a new length of pipe is to be added; and afterward replaced and firmly bolted on. This chamber has a small air-tight door *d*, by which it can be entered from without; and another, o, opening into the cylinder. The flaps, *t*, *h*, of both doors, open inward, or toward the cylinder. This chamber also has two stopcocks; one, *a*, in its floor, communicating with the cylinder; and one *e*, above, communicating with the open air. At *s* is a bent tube, also with a cock, which passes air-tight through the side and the bottom of the air-lock. Through it the compressed air is forced into the cylinder, by an air force pump or condenser: and through it the same air is allowed to escape at a later period. A siphon is shown at *n n n*. A drum *w* is used for hoisting the excavated material from the bottom, to the air-lock: its axle *i i* passes air-tight through stuffing-boxes in the sides of the lock; the hoisting being done by men outside. This is the general arrangement employed by Mr W. J. McAlpine, C E, of New York, at Harlem bridge; and from his description of it, ours has been condensed. The cylinders were there 6 ft diam, 1¼ ins thick, and in lengths of 9 ft, bolted together through inside flanges *f*, as the sinking went on. The air-lock is 6 ft diam, by nearly 6 ft high; with sides of boiler iron; and top and bottom of cast iron.

Now suppose the cylinder C C to be let down, and steadied in position, as in the fig; and the air-lock L L to be adjusted on top of it. The next process is to force in air through the curved tube *s*; the flap *t* of the lower door o, and the cock *a*, being previously closed. As the compressed air accumulates in the cylinder, it forces out the water; which escapes partly beneath the bottom of the cylinder, and partly by rising through the siphon *n n*, and flowing out at *q*. The door *o* being already closed, and that at *d* open, the air in the air-lock is in the same condition as that outside; so that workmen can enter it readily. Having done so, they close the door *d*, and the cock *e*; and open the cock *a*, through which condensed air from the cylinder rushes upward, soon filling the air-lock. When this is done, the flap *t* is opened, and the men descend through the door *o* by a ladder, or by a bucket lowered by the drum *w*, to the bottom. Here they loosen and excavate the material as deep as they can; and, filling it into a bucket or bag, they signal to those outside, who raise it to the air-lock. When done, they ascend to the air-lock, close the door *o*, and the cock *a*; and open the cock *e*, through which the condensed air in the lock soon escapes, leaving the internal air the same as that outside. The door *d* is then opened, the buckets of earth are removed, and the men go out. Finally the cock at *s* is opened, the condensed air in the cylinder escapes through it to the outside air, and the cylinder sinks by its own weight into the cavity and loosened soil prepared for it at its base, and which is now forced up into the cylinder by the rush of the returning water. The process is then repeated. The sinking will often vary from 0 to 10 or more feet at one operation. Until depths of 40 or 50 ft, most men can endure the pressure of the condensed air; but as the depth increases this becomes more difficult, and positively dangerous to life. Cast-iron cylinders 15 ft diam; and great caissons, Fig 18, have been thus sunk; but at times at great expense and trouble.

The cylinder should be guided in its descent by a strong frame, which may be supported by piles. Otherwise it will be apt to tilt, and thus give great trouble to settle it upon its exact place. Have been sunk in deep water by divers undermining inside.

The plenum process as applied at the South St bridge, Philada, by Mr. John W. Murphy, contracting engineer, differs materially from that described above; and moreover deserves notice on account of the great simplicity and efficacy of his plant. This consisted partly of two canal boats, decked, each 100 ft long, by 17½ ft wide, and 8 ft depth of hold. They were anchored parallel to each other, 15 ft apart. Supported by the boats, and over the space between them, was a strong four-legged shears about 50 ft high; at the top of which was attached tackle for handling the cast iron cylinders. In the hold of one of the boats was a **Burleigh Compressor** having two pistons of 10 ins diam, and 9 ins stroke; together with its boiler. On the deck of the same boat stood a vertical **air-tank or regulator,** 22 ft long, by 2 ft diam, made of quarter inch boiler iron. This served to maintain a supply of compressed air in the submerged cylinder in case of an accidental stopping of the compressor; which otherwise would probably be fatal to the laborers in the cylinder. The condensed air flowed from this air-tank to the air-lock of the cylinder through a hose 4 ins diam, made of gum elastic and canvas, and so long, and so placed, as to extend itself as the cylinder went down, thus maintaining the communication at all times. Entirely across both boats, and across the interval between them, extended two heavy wooden **clamps,** each 3 ft wide by 18 ins high; each composed

of three pieces of 12 × 18 inch timber strongly bolted together. At the centers of these clamps the two inner vertical sides which faced each other were hollowed out to the depth of a foot by concavities corresponding to the curve of the cylinders. The distance apart of the clamps was regulated by two strong iron rods, having screws and nuts at their ends for that purpose. Thus when a section of a cylinder was hoisted by means of the shears into its position over the space between the two boats, the two concavities of the clamps were brought into contact with it, and the nuts being then screwed up, the cylinder was firmly held in place by the clamps. The shears could then be used to raise another section of the cylinder to its place upon the first one, that the two might be bolted together. By repeating this process the height of the cylinder would soon become too great to allow the shears to place another section upon it; in which case the nuts of the screws were slightly loosened, and the cylinder was allowed to slip down slowly into the water until its top was but a little above the surface. The screws were then again tightened, and the cylinder again held fast until other sections were added and bolted to it. When there was danger that the upward pressure of the condensed air might lift a cylinder, the clamps were raised by the shears clear of the boats; then tightened to the cylinder, and a platform of planks laid upon them, and loaded with stone.

The **air-lock** was so arranged as not to require to be removed when a new section was to be bolted on. This was effected as follows. Sections of the cylinder were bolted together in the manner just described, until its foot rested on the bottom, with its top a few feet above high water. A heavy cast iron **diaphragm** 1¼ inches thick, to form the floor of the air-lock, was then placed on top. Then was added another 10 ft high section of the cylinder, to form the chamber of the air-lock. These were bolted together; and then another diaphragm was added at top to form the roof of the air-lock. These diaphragms were furnished with openings, and with doors and valves corresponding with those shown in Fig 17. and remained permanently in the cylinders when the work was finished. If the depth of soil to be passed through before reaching rock is so great as to require other sections of cylinder to be bolted on above the top of the air-lock, this may be done to any extent, inasmuch as it is immaterial whether the air-lock is under water or not. **To keep the cylinder both air- and water-tight** the faces of the flanges before being bolted together were smeared with a mixture of red and white lead and cotton fiber.

At the South St bridge the cylinders were 4, 6, and 8 ft diam; in lengths or sections 10 ft long. They were all 1¼ inch thick. Inside flanges 2¾ ins wide, 1¼ thick, with bolt-holes 1¼ inch diam, by 5 ins apart from center to center. The bottom edge has no flange. A 10 ft section of an 8 ft cylinder weighs 14600 lbs; of a 6 ft one, 10800; of a 4 ft one, 6800. An 8 ft diaphragm, 2800 lbs; 6 ft, 1600; 4 ft, 783. The rock under the soil was quite uneven in places; but was levelled off as the cylinders went down. These were then bolted to it by cast iron brackets. **The work went on, day and night,** summer and winter: with no interruption from the tides, floods, or floating ice; and the thirteen columns were sunk, filled with concrete, and completed in 11 months; much of which was consumed in levelling off the rock, and bolting the brackets. The want of guides caused much tilting, trouble, and delay.

Rise and fall of tide about 7 ft. Greatest depth of soil, gravel, &c, passed through, 30 ft: least, 6 ft. Depth of water about 25 ft. The work was under charge of John Anderson, a very skillful and energetic superintendent of such matters. **The entire neat cost** of the cylinders in place, and filled with hydraulic concrete, was probably not far from $120 per ft of total length for the 8 ft ones; $80 for the 6 ft; and $50 for the 4 ft diams. There were three gangs of workmen; and each gang worked 4 hours at a time. See a full and very instructive description with engravings, by D. M. Stauffer, Superintending Engineer for the city, in the Journal of the Franklin Inst, Nov., 1872. From it the above few items are taken. Mr. Anderson's firm (Anderson & Barr, Tribune Building, N. Y.) have since successfully sunk a number of such cylinders, including (1884-5) four of wrought-iron, 8 ft diam, 66 ft long, at an angle of 45° with the hor, intended as struts to prevent the movement of one of the abut piers of Chestnut St bridge, Phila.

Cast iron cylinders **have cracked through,** around their entire circumference, in many parts of the U. S. in very cold weather; owing to the diff of contraction of the iron, and of the concrete filling. Ignorant use of them may be attended by great danger.

The shaded part of Fig 18 shows a transverse section of the **caisson of yellow-pine** timber and cement, for the **Brooklyn tower of East River** (N Y) suspension bridge, of 1600 ft clear span. It is 168 ft long at bottom, and 102 ft wide. A longitudinal section resembles the transverse one, except in being longer, and in showing more shafts J. Of these there are 6, arranged in pairs, for expedition and as a precaution against accident. Namely, two water-shafts J, each 7 ft by 6½ ft across, for removing by buckets and hoisting apparatus, the material excavated beneath the caisson; together with such water as may accumulate at *o o*; two air-shafts of 21 ins diam, through which air is forced from above, to expel the water from the chamber C S S D below the caisson, so as to allow the laborers to work there at undermining; the expelled water escaping under the foot C D of the caisson, into the river; and two supply shafts of 42 ins diam, for admitting laborers, tools, &c. The several shafts of course have air-chambers on top, on the same principle as Fig 17, to prevent the escape of the compressed air in *s s*.

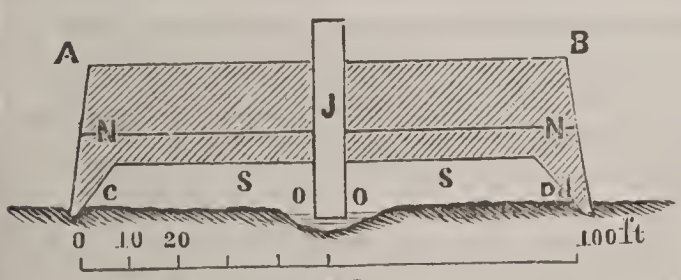


Fig 18

The shafts are of ¼ inch boiler iron. The foot C D, nine timbers high, is continuous, extending entirely around the caisson; its bottom is shod with cast iron; its four corners are strengthened by wooden knees 20 ft long.

From the bottom, up to the line N, N, 14 ft, the caisson is built of horizontal layers of timbers one foot square; the layers crossing each other at right angles; and the timbers of each layer touching each other well forced and bolted together; and all the joints filled with pitch. To aid in preventing leakage, the nuts and heads of the screws have India-rubber washers; also all outside seams, as well as all the seams of the layer of timbers N, N, are thoroughly calked; and a layer of tin, enclosed between two layers of felt, is placed outside of each outer joint; and over the entire top of the layer next below N, N.

When the caisson was built up to N, N, on land, it was launched, floated into position, and anchored; after which were added for sinking it, fifteen courses of timbers one ft square; and laid one ft apart in the clear; with the intervals filled with concrete. The top course A B is of solid timber, to serve as a floor for supporting machinery, &c. It was sunk some feet below the very bottom of the river, in order to avoid the teredo.

Cribs are sunk outside of the caisson, to form temporary wharves for boats carrying away excavated material; and for vessels bringing stone, &c.

When the caisson was sunk, and the water forced out from the chamber or space C S S D, workmen began to excavate uniformly the enclosed area of river bottom, so as to allow the caisson to descend slowly until it reached a firm substratum. The space C S S D, as well as the shafts, was then filled up solid with concrete masonry. A coffer-dam was built on top of the caisson; and in it the regular masonry of the tower was started. The total height of this tower including the caisson, is about 300 ft. For full details see report, 1873, of W. A. Roebling the chief engineer.

Hollow cylinders, or other forms of brickwork or masonry, with a strong curb or open ring of timber or iron beneath them, may be gradually sunk by undermining and excavating from the inside; and form very stable foundations. Under water this may be done by properly shaped scoops, with or without the aid of the diving-bell, according to the depth, &c. On land it will often be the most economical and satisfactory mode, especially in firm soils. The descent may be assisted by loading them, if, as sometimes happens, the friction of their sides against the earth outside prevents their sinking by their own weight. A brick cylinder, 46 ft outer diam, walls 3 ft thick, has been sunk 40 ft in dry sand and gravel, without any difficulty. It was built 18 ft high, (on a wooden curb 21 ins thick,) and weighed 300 tons before the sinking was begun. The interior earth was excavated slowly, so that the sinking was about 1 ft per day; the walls being built up as it sank. Tunnel shafts are at times so sunk.

On the Rhine for a coal shaft, a brick cylinder 25½ feet diam was first thus sunk by its own weight 76 ft through sand and gravel; then an interior one, 15 ft diam, was sunk in the same way to the depth of 256 ft below the surface; of which depth all the 180 ft below the first cylinder was a running quicksand. At 256 ft friction rendered the cylinder immovable. The quick sand was removed by boring; no pumping was done; but the water was permitted to keep the cyl full.

The entire foundation for a large pier of masonry has been sunk in this manner, in a single mass; a sufficient number of vertical openings being left in it for the workmen to descend, or for tools to be inserted for undermining. This is generally a very slow and tedious operation, especially under water. It may often be expedited by diving-bells or by diving-dresses. It will generally be better to make the mass wider at bottom than above it, so as to diminish friction against the outside earth. On land, water may at times be used for softening the bottom earth. By keeping the interior of such hollow masonry dry, it may even be *built downward* from the surface; by undermining only a portion of its circumference at a time, filling said portion with masonry, and then removing and filling the other portion; and so on in successive stages of 2 or 3 ft downward at a time. This mode may be adopted also when friction has stopped the sinking of a mass by its own weight when undermined.

The sand pump as used at the St Louis bridge will often be of service in raising sand from cylinders while being sunk in water. With a pump pipe of 3.5 ins bore, and a water jet under a pressure of 150 lbs per sq inch, 20 cub yds of sand per hour were raised 125 feet. A jet of air has also been successfully used in the same way, as at the East River, N Y, suspension bridge, &c.

Fascines. On marshy or wet quicksand bottoms, foundations may be laid by first depositing large areas of layers of fascines, or stout twigs and small branches, strongly tied together in bundles from 6 to 12 ft long, and from 6 ins to 2 ft in diam.

The layers or strata of bundles should cross each other. A kind of floating raft or large mattress is first made of these, and then sunk to the bottom by being loaded with earth, gravel, stones, &c. In this manner the abutments and piers of the great suspension bridge at Kieff, in Russia, with spans of 440 ft, were founded in 1852, on a shifting quicksand. There the fascine mattresses extend 100 ft beyond the bases of the masonry which rests upon them.

Fascines may be used in the same way for sustaining railway embankments, &c, over marshy ground, but they will settle considerably.

Sand-piles. We have already alluded to the use of sand well rammed in layers into trenches or foundation pits; but it may also be used in soft soils, in the shape of piles. A short stout wooden pile is first driven 5 to 10 feet or more, according to the case. It is then drawn out, and the hole is filled with wet sand well rammed. The pile is then again driven in another place, and the process repeated. The intervals may be from 1 to 3 ft in the clear. Platforms may be used on these piles as on wooden ones. If the sand is not put in wet, it will be in danger of afterward sinking from rain or spring water. In this case, as with fascines, it is well to test the foundation by means of trial loads. Some settlement must inevitably take place until all the parts come to a full bearing: but it will be *comparatively* trifling. The same occurs in every large work to some extent; as in a roof or arch of great span, whether of wood, iron, or masonry; so also with all tall piers, walls, &c, &c. Sandy foundations under water should be surrounded by stout well-driven sheet-piling, to prevent the enclosed sand from running out in case the outer sand is washed away; and should also be defended by a deposit of random-stone. See Sand-piles, p 626.

On bad bottoms under water, **small artificial islands** of good soil have been deposited; and the masonry founded upon them. Canal locks and other structures may at times be advantageously founded in this way in marshy soils. If necessary, a depth of several feet of the bad soil may be dredged out before the firmer soil is deposited; and the latter may be weighted by a trial load to test its stability.

The mode of laying a foundation under water, by building the masonry upon a timber platform above water, upheld by **strong screws**, and lowered into the water as the work is finished in the open air, a course or two at a time, has of late been much employed with entire success, in large bridge-piers in deep water. It however is not new. It was suggested more than 100 years ago by Belidor.

Piles are driven 6 to 10 ft apart around the space to be occupied by the pier; having their tops connected by heavy timber cap-pieces. These last uphold the screws, which work through them. The whole is braced against lateral motion.

A CLUMP OF PILES WELL DRIVEN; and then enclosed by an iron cylinder sunk to a firm bearing, and filled with concrete, is an excellent foundation. The piles may extend to the top of the cylinder, and thus be enclosed in the concrete. Such an arrangement has been patented by S. B. Cushing, C. E., Providence, R. I. The cylinder and concrete serve to protect the piles from sea-worms, and from decay above low water; and are not intended to support the load above them.

Cost of a diving outfit, with two dresses, air-pump and tubes, about $750, or $690 with cheaper pump. Alfred Hale & Co, 30 School St, Boston, Mass.

Two men can work the air-pump to 50 ft depth.

STONEWORK.

Where work is done on a large scale, blasting can sometimes be done at from 10 to 20 per cent less cost per cubic yard by means of **machine drills and dynamite**, than by **hand drills and gunpowder.** Ordinarily, however, **the cost is about the same**, and the advantage of the newer methods consists rather in economy of time, convenience, and having the work more entirely under control. In ordinary railroad work in average hard rock, and when common labor costs $1 per day of ten hours, the cost per cubic yard, for loosening, will ordinarily range between 30 and 60 cts, including tools, drilling, powder, &c; average 45 cts.

Holes for blasting, drilled by hand, are generally from 2½ to 4 ft deep; and from 1½ to 2 ins diam. **Churn-drilling** is much more expeditious and economical than that by *jumping*, mentioned below. The churn-drill is merely a round iron bar, usually about 1¼ ins diam, and 6 to 8 ft long; with a steel cutting edge, or bit, (weighing about a lb, and a little wider than the diam of the bar,) welded to its lower end. A man lifts it a few inches; or rather catches it as it rebounds, turns it partially around; and lets it fall again. By this means he drills from 5 to 15 feet of hole, nearly 2 ins diam, in a day of 10 working hours, depending on the character of the rock. From 7 to 8 ft of holes 1¾ ins diam, is about a fair day's work in hard gneiss, granite, or compact siliceous limestone; 5 to 7 ft in tough compact hornblende; 3 to 5 in solid quartz; 8 to 9 in ordinary marble or limestone; 9 to 10 in sandstone; which, however, may vary within all these limits. When the hole is more than about 4 ft deep, two men are put to the drill. Artesian, and oil wells, in rock, are bored on the principle of the churn-drill. See also diamond drill, p 652.

The **jumper**, as now used, is much shorter than the churn-drill. One man (the *holder*) sitting down, lifts it slightly, and turns it partly around, during the intervals between the blows from about 8 to 12 lb hammers, wielded by two other laborers, the *strikers*. It can be used for holes of smaller diameters than can be made by the churn-drill; because the holder can more readily keep the cutting end at the exact spot required to be drilled. It is also better in conglomerate rock; the hard siliceous pebbles of which deflect the churn-drill from its vertical direction, so that the hole becomes crooked, and the tool becomes bound in it. The coal conglomerates are by no means hard to drill with a jumper. The jumper was formerly used for large deep holes also, before the superiority of the churn-drill became established.

Either tool requires resharpening at about each 6 to 18 inches depth of hole; and the wear of the steel edge requires a new one to be put on every 2 to 4 days. With iron jumpers, the top also becomes battered away rapidly. As the hole becomes deeper, longer drills are frequently used than at the beginning. The smaller the diameter of the hole, the greater depth can be drilled in a given time; and the depth will be greater in proportion than the decrease of diam. Under similar circumstances, three laborers with a jumper will about average as much depth as one with a churn-drill.

The **hand-drill**, in which the same man uses both the hammer and the short drill, is chiefly used for shallow holes of small diam. With it a fair workman will drill about as many feet of hole from 6 to 12 ins deep, and about ¾ inch diam, as one with a churn-drill can do in holes about 3 ft deep, and 2 ins diam, in the same time. Only the jumper or the hand-drill can be used for boring holes which are horizontal, or much inclined.

MACHINE ROCK-DRILLS.

Art. 1. Machine Rock-drills bore much more rapidly than hand drills; and more economically, provided the work is so great as to justify the preliminary outfit. They drill in any direction, and can often be used in boring holes so located that they could not be bored by hand. They are worked either by steam directly; or by air, compressed by steam or water power into a tank called a "receiver," and thence led to the drills through iron pipes. The air is best for tunnels and shafts, because, after leaving the drills, it aids ventilation.

Art. 2. Such drills are of two kinds: rotating drills and **percussion drills.** In the former, the drill-rod is a long tube, revolving about its axis. The end of this tube, hardened so as to form an annular cutting-edge, is kept in contact with the rock, and, by its rotation, cuts in it a cylindrical hole, generally with a solid core in the center. The core occupies the core-barrel, Art 8. The drill-rod is fed forward, or into the hole, as the drilling proceeds. The debris is removed from the hole by a constant stream of water, which is led to the bottom of the hole through the hollow drill-rod, and which carries the debris up through the narrow space between the outside of the drill-rod and the sides of the hole.

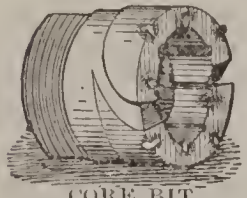
In **percussion drills,** the drill-rod is solid, and its action is that of the churn drill, p 651.

Art. 3. In the **Brandt** (European) **rotary drill,** the cutting-edge at the end of the tubular drill-rod is armed with hardened steel teeth. It is pressed against the rock under enormous hydraulic pressure, and makes but from 5 to 8 revolutions per minute.

Art. 4. The Diamond drill is the only form of rotary rock-drill extensively used in America. In it, the boring-rod consists of a number of tubes jointed rigidly together at their ends by hollow interior sleeves.

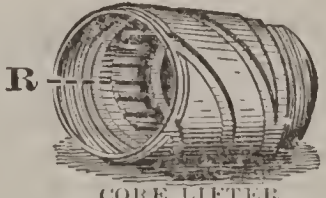
Art. 5. The boring-bit, Fig 1, is called a "core-bit." Its cutting-edge has imbedded in it a number of **diamonds** as shown. These are so arranged as to project slightly from both its inner and outer edges. Annular spaces are thus left between core and core-barrel, and between the latter and the walls of the hole. These spaces permit the ingress and egress of the water used in removing the debris from the hole, and, at the same time, prevent the core from binding in the barrel, or the latter in the hole.

Fig. 1



CORE BIT.

Fig. 2



CORE LIFTER.

Fig. 3

BORING HEAD.

Art. 6. Just above the "core-bit," the **"core-lifter,"** Fig 2, is screwed to the barrel. This is a tube about 8 ins long and of the same outer diam as the barrel. Inside it is slightly coned, with the base of the cone upward, and furnished with a loose split-ring, R, toothed inside, and similarly coned. While the drilling is going on, this ring encircles the core closely, and remains loose from the outer cylinder; but when the drilling is stopped, and the drill-rod begins to be raised, the ring is caught and raised by the outer cylinder: and, by reason of its beveled shape, is pressed hard against the core of rock, which is pulled apart close to its foot by the power which lifts the drill-rod.

Art. 7. This power is supplied by a **rope-drum,** fastened to the top of the frame which supports the drill and worked by the same engine which rotates the drill-rod. The rope from the drum passes up to a pulley at the top of a **derrick,** and thence down to the upper end of the drill-rod. The considerable height of the derrick enables from 40 to 50 feet of the drill-rod to be removed in one piece.

Art. 8. Above the "core-lifter" is the **"core-barrel."** This is a wrought-iron tube from 8 to 16 ft long. It is spirally grooved outside, to permit the ascent of the water and debris from the hole; and is sometimes set with diamonds on its outer surface, to prevent wear. The bit, lifter, and barrel are of uniform outer diam, a little less than the diam of the hole. The outer diam of the drill-rod varies from about 1⅞ ins for 2-inch barrel to 5½ ins for 12-inch barrel.

Art. 9. Where it is not desired to preserve the core intact, a "**boring-head,**" Fig 3, may be used instead of the "core-bit," Fig 1. This is a solid bit (except that it is perforated with holes which allow the water to pass out from the drill-rod), and is armed with diamonds, some of which project beyond its circumference.

Art. 10. The drill-rod **revolves at a speed** of from 200 to 400 revolutions per minute. **The engine,** by which it is rotated, consists usually of two cylinders, either fixed or oscillating, operated by steam or compressed air, and working at right angles to each other. By means of cranks they turn a shaft, which communicates its motion, through bevel gearing, to the drill-rod. The latter is **fed down,** as the hole progresses, either by other **bevel gearing** driven by the same engine; or by being attached to a cross-head which connects the piston rods of 2 **hydraulic cylinders,** the piston rods being parallel with the drill rod.

Art. 11. The diamond drill bores perfectly **circular holes, in straight lines and in any direction, to great depths;** from 300 to 1500 feet being not uncommon. This, with the fact that **it brings up unbroken cores,** from 8 to 16 ft long, which show the precise nature and stratification of the rock penetrated, renders it very valuable in test-boring, prospecting of mines, &c. They are also furnished of sufficient size to bore holes from 6 to 15 ins diam, for **artesian wells.** The roundness of the holes bored enables the use of casing of nearly as great diam as that of the hole; and their straightness is advantageous in case a pump has to be used.

Art. 12. In soft rock a bit may drill through 200 ft or more without resetting. On the other hand, in very hard rocks, similar drills will wear out in 10 ft or less. In 1883–4, a diamond drill by the Am'n Diamond Rock Boring Co, weighing complete about 1400 lbs, and costing about $2800, bored, in 1428 hours of actual boring, 53 holes of 2 ins diam, and aggregating 9141 lineal ft. Average length of hole 172.5 ft. **Average rate,** 6.4 lin ft per hour; greatest, 12.8. **Average total cost,** about 96 cts per lin ft. The rock was principally limestone, with some quartz and sandstone. The holes were bored at angles varying from 0° to 45° with the vertical.

As a rough average we may say that **in ordinary rocks,** as granite, limestone, and hard sandstone, these drills will bore deep holes, 2 to 3 ins diam, at **from 1 to 2 ft per hour,** and at a cost of from **$1 to $2 per ft.**

Art. 13. These drills are made of many widely **different sizes,** and with **different mountings,** depending upon the nature of the work to be done.

They are made by the Penna Diamond Drill Co, Pottsville, Pa; American Diamond Rock-Boring Co, office 15 Cortlandt St, New York; M. C. Bullock Mfg Co, Chicago, Ill, and others. These companies usually contract to do the drilling themselves. They also sell the machines, generally under restrictions as to the location and extent of the territory in which they are to be used. **The prices** depend, to a great extent, upon the nature of these restrictions. Roughly, the card prices for some of the leading sizes, are, in 1886, as follows:

Diam of hole.	Greatest length of hole.	Weight of machine.	Card price, 1886.
ins.	ft.	lbs.	$
1½	250	400	1500
2	500	1000	2000
2	2000	3500	4000
4	2000	6200	6000

Art. 14. In **percussion drilling machines,** the drill-bar is driven forcibly against the rock by the **pressure of steam or of compressed air,** acting upon a piston, P, Fig 4, moving in a cylinder, C C, Figs 4 and 5; and makes about 300 strokes per minute. The rotation of the drill-bar is accomplished automatically, as explained in Art 27.

Art. 15. The cylinder, C C, is free to slide longitudinally in the fixed frame or shell, S S, Fig 5, to which it is attached, and which, in turn, is fixed to the tripod or other stand (see Arts 18 and 19) upon which the machine is supported.

Art. 16. The drill-rod, R, corresponding to the churn drill, p 651, is fastened, by an appropriate chuck, K, to the end of the piston-rod, O. The drilling is begun with a short drill-rod, and with the cylinder as far from the hole as the length of the shell, S, will permit. As the bit penetrates the rock, the cylinder is fed forward,* either automatically or by hand (see Art 28), as far as the length of

* By **forward,** or **downward,** we mean *toward* the hole which is being drilled. By **backward,** or **upward,** *from* the hole.

the shell permits. The drilling is then stopped, by shutting off the steam,* and the cylinder is run back, by reversing the motion of the feeding apparatus. The short drill-bar is then removed, and, if the drilling is to be continued, a longer one is substituted in its place, and the process repeated.

Art. 17. Inasmuch as the act of drilling wears the edges of the bit, thus reducing its diam somewhat, **the hole will of course be tapering,** or of slightly less diam at bottom than at top. The second bit must therefore be of slightly less diam than the first; say from $\frac{1}{16}$ to $\frac{1}{8}$ inch less; the third must be less than the second, and so on. On the other hand, in long holes, the drill-bar will seldom be in a perfectly straight line, so that the bit, instead of striking always in the same spot, will describe a circle, and thus enlarge the hole.

Art. 18. The shell, S, in which the cylinder slides, is provided with an arrangement by which it may be clamped, either to a **tripod,** as in Fig 5, or to a long **bar** or **column,** along which it may slide. The column, if hor, may rest upon two pairs of legs; or it may be braced, in any position, against the opposite sides of a narrow cut, or against the floor and ceiling of a tunnel-heading, &c, in which case one of its ends is provided with a screw which is run out so as to cause the two ends of the col to press firmly against the opposite rock walls; or rather against wooden blocks which are always placed between each end of the col and the rock. In any case, the supports of the drill are so jointed that it can bore in any direction.

Art. 19. Frequently the drill is **clamped to a short arm,** which, in turn, is clamped to the column, and projects at right angles from it. The arm may be slid lengthwise of the column, and may be revolved around it, and thus may be placed in any desired position, and there clamped. This gives the drill a greater range of motion, and enables it to bore holes over a greater space than would otherwise be possible without moving the column.

Art. 20. In tunnels, one or more drills may be mounted upon a **drill-carriage,** travelling upon a railroad track running longitudinally of the tunnel. Upon this track the carriage is moved up to the work, or run back from it when a blast is to be fired. The gauge of the track may be made wide enough to admit of a second track, of narrower gauge, running underneath the drill-carriage. Upon said narrower track the cars are run which carry away the debris. Drill-carriages are less commonly used in this country than in Europe.

Art. 21. The pressure used in the cylinders of percussion drills is usually from about 60 to 70 lbs per sq inch. **In an hour, one will drill** a hole from 1 to 2 ins diam, and from 3 to 10 ft deep, depending on the character of the rock and the size of the machine at from 10 to 25 **cts per lin ft** with labor at $1 per day. **A bit requires sharpening** at about every 2 to 4 ft depth of hole. One blacksmith and helper can sharpen drills for 5 or 6 machines.

Art. 22. The bits are of many different shapes, varying with the nature of the work to be done. For uniform hard rock, the bit has two cutting-edges, forming a cross with equal arms at right angles to each other. For seamy rock, the arms of the cross are equal, but form two acute and two obtuse angles with each other, as in the letter X. For soft rock, the cutting-edge sometimes has the shape of the letter Z.

Art. 23. Each drill requires one man to operate it. Two or three men are required for moving the heavier sizes from place to place. One man can attend to a small air-compressor and its boiler.

Art. 24. Figs 4 and 5 represent the **"Eclipse"** percussion drill of the Ingersoll Rock-Drill Co, No 10 Park Place, New York. Fig 5 shows the drill, mounted (as is most frequently the case) upon a tripod. Fig 4 is a longitudinal section through the cylinder, valve-chest, and piston.

Art. 25. The cylinder, C, is provided at each end with a **rubber cushion,** N, for deadening the blows of the piston, which, in all percussion drills, is liable, at times, to strike either cylinder-head. The side of each cushion nearest the piston is protected by a thin iron plate. The cushions have to be renewed from time to time.

Art. 26. The valve, V, is shaped somewhat like a spool. The bolt, B, passes loosely through its center and guides it. Steam is admitted from the boiler to the steam-chest, and occupies all of the space between the two end flanges of the valve, except *a*. It drives the valve alternately from one end of the valve-chest to the other, and back, according as one end or the other is relieved from opposing pressure by being put into communication with the exhaust, *E*, by way of the passages, D D′ and F F′. D and D′ communicate with the ends of the steam-chest through passages not shown; while F and F′ communicate, through similar passages, with the exhaust, E. The piston has an annular channel, L L′, encircling it. Whatever the position of the piston, one of the passages, D or D′, is always, by means of this channel, in communication with its corresponding passage, F or F′, leading

* To avoid repetitions, we will use the word **steam** to signify either *steam* or *compressed air*, whichever happens to be used.

to the exhaust. Thus, one or the other end of the valve-chest is always in communication with the open air; and to that end the valve is driven by the pres of the steam surrounding it, admitting steam to the cyl, C, from the other end.

Art. 27. The rotation of the piston, and, with it, that of the drill-bar, is effected thus: The spirally-grooved, cylindrical steel bar, A, called a **rifle-bar,** passes through and works in, the **rifle-nut,** H, which is firmly fixed in the end of the piston, and has spiral grooves corresponding with those on the rifle-bar. Said bar is fixed, at its upper end, to the ratchet-wheel, J, the pawls of which are so arranged that, on the *down* stroke of the piston, the rifle-nut, H, acting upon the grooves on the rifle-bar, causes it, and, with it, the ratchet-wheel, to revolve about their common axis. The weight and momentum of the piston, &c, are such that it thus readily turns the ratchet-wheel without itself turning. Thus the bit is prevented from rotating while delivering its blow. But, on the *up* stroke, the tendency of the rifle-nut is to turn the rifle-bar and ratchet-wheel in the *opposite* direction; and as this is prevented by the pawls, the *rifle-bar* remains *stationary*, while the *piston*, *piston-rod*, and *drill* are *made to revolve* about their common axis.

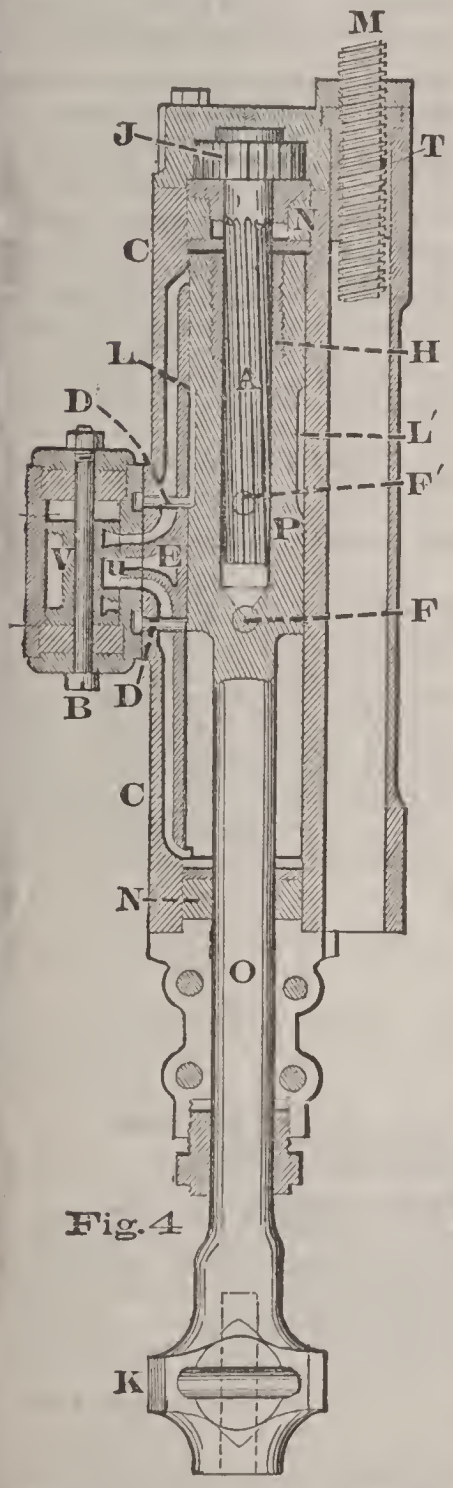


Fig. 4.

Art. 28. The feed-screw, M, is collared, at its upper end, to the fixed frame, Q. It is thus prevented from moving longitudinally when revolved by means of the crank fixed to its top. Its lower end works in a nut, T, fixed to the cylinder, which last is thus moved longitudinally backward or forward as the crank is turned.

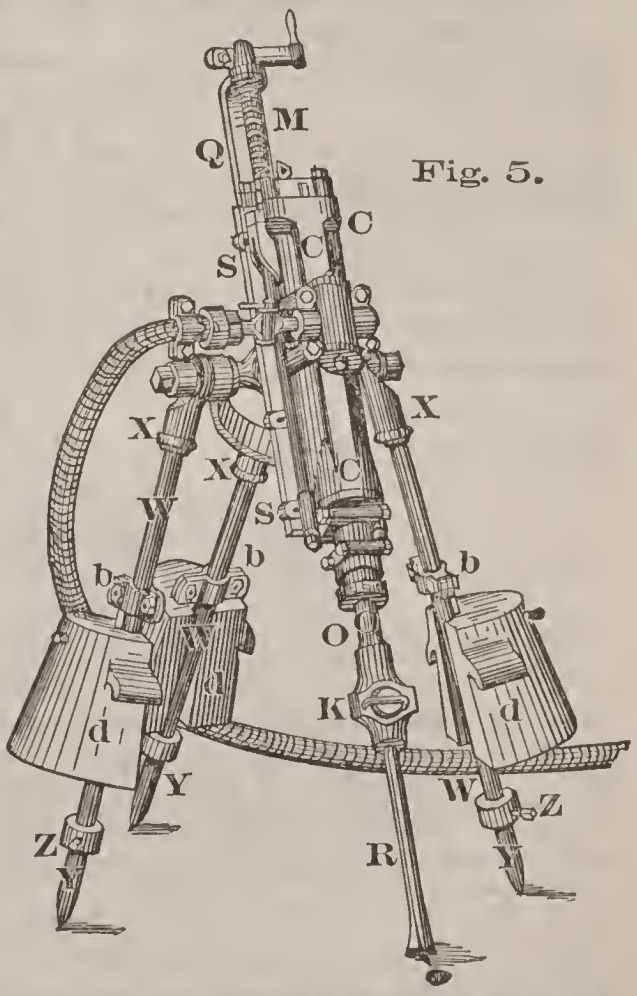


Fig. 5.

Large drills are frequently furnished with an ***automatic*** **feeding arrangement** in addition to the hand-crank. In this arrangement, when the cylinder requires feeding forward, and when, consequently, the piston is running nearly to

the forward limit of its stroke, the piston presses against a cam projecting into cyl near the forward end, and presenting an inclined plane to it. The motion this cam, by means of an exterior axle, running alongside of the cyl and furnish at its top with a dog, turns a ratchet-wheel fixed to the feed-screw. When desir the automatic feed may be thrown out of gear, and the feed moved by hand.

Art. 29. The tripod legs consist of wrought-iron tubes, W W. These screwed at their upper ends into sockets, X X. At their lower ends, they rece the pointed and tapering steel bars, Y Y, about 2 or 3 ft long. The legs may lengthened or shortened by turning the set-screws, Z Z, thus regulating the distan to which the bars, Y Y, can enter the legs. The clamps, *b b*, have L-shaped hoo of ½ inch to 1 inch round iron forged to them. On these hooks **the weight** *d d*. are hung, which hold the machine down against the upward reaction of blows.

Art. 30. The following **table** gives the principal **dimensions** of the drills, with the **diams and lengths of holes** to which each is adapted.

For prices, apply to the Co at the above address. We give the card prices 1886. These may be taken as giving, approximately, the present range of prices percussion drills of any first-class make. Size H is used for submarine work, hea tunneling, and deep rock cutting. G and F for tunneling, street grading, quarryi and sewer work. E, D, and C for general mining purposes. B is adapted only very light work. In asking for estimates on drills and compressors, give the full possible description (accompanied by a sketch) of the work to be done, stating present and proposed extent. State whether the work is on the surface or und ground. State how far the steam or compressed air will be carried. Give depth holes to be drilled, nature of rock, &c. Percussion drills are sold without restricti as to the purpose or extent to which they are to be used.

LIST OF INGERSOLL "ECLIPSE" PERCUSSION ROCK DRILLING MACHINES.

		Letter designating the size of the machine.						
	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**
Inner diam of cylinder.....ins.	1¾	2½	2¾	3	3⅛	3½	4¼	5
Length of full stroke..... "	3	4	5	6	6	6½	7	7
" feed..... ... "	12	20	24	24	24	26	34	34
" machine*..... "	36	34	36	40	42	53	60	60
Wt of machine, unmounted....lbs.	80	155	195	230	250	345	605	670
" tripod, without the wts. "	‡	125	125	125	125	150	275	275
" three wts for tripod legs. "	‡	250	250	250	250	350	400	400
" column, arm and clamp. "	‡	200	280	280	280	420	420	420
Diam of hole drilled.....ins.	½ to ⅞	¾ to 1¼	1 to 2	1 to 2	1 to 2	1½ to 2½	2 to 4	3 to 5
Maximum depth of vert† hole..... ft.	‡	4	8	10	12	16	30	40
Prices.		$	$	$	$	$	$	$
Machine, unmount'd, with't drills.	‡	230	255	280	300	330	375	425
Set of drills for above depth of hole.	‡	8	18	24	31	69	139	400
Tripod, with weights.....	‡	45	45	45	45	45	55	55
		Column 4½ ins diam.				Column 6 ins diam.		
Column, 8 ft long, with arm & clamp..	‡	80	80	80	80	110	110	110

* From top of handle of feed-crank to lower end of piston at the end of the down stroke.
† For greatest advisable length of *hor* holes, deduct one-fourth from these depths.
‡ Machine **A** is mounted on a small frame. Price, so mounted, $150. Hole 18 ins deep. Drills $

Art. 31. The drills of different makers differ chiefly in th methods by which the valve is operated. In some this is done, as in the Ingerso "Eclipse" drill, Art 26, by the **pres of steam.** In others, the valve is moved b a **lever or tappet,** which projects into the cylinder so as to come into contac with, and be moved by, the piston at each stroke. As these strokes are made wit great force some 300 or more times per minute, such valve-gear is necessarily subjec to great wear.

Art. 32. In the **"Little Giant Drill,"** made by the **Rand Drill Co** office 23 Park Place, New York, the valve, V, Fig 6, is slid backward and forward in the same direction in which the piston is moving, by the tappet, T, which i pivoted at *p*. The inclined lower corners of this tappet ride up as they come, alter nately, in contact with the shoulders, *s s*, of the piston.

Art. 33. The same Co have recently, 1884, brought out **two ne drills,** the **"Economizer"** and the **"Slugger";** in each of which t valve, as in the Ingersoll "Eclipse" drill, is moved by steam, but upon a quite

rent principle. In these two drills, there is no steam cushion for the piston strike against on the down stroke, the force of which is thus more completely spended upon the rock. The cushion behind or above the piston, on the return roke, is formed by exhaust steam. Both of these drills cut off steam before e completion of either stroke, thus using the steam expansively. On the down oke, the "Economizer" cuts off earlier than the "Slugger." Hence its name. both machines the point of cut-f is fixed when the machine is nade.

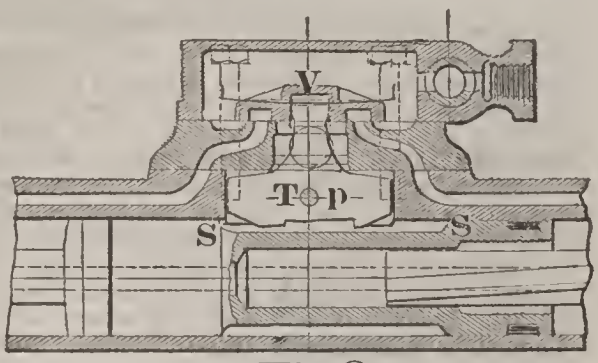


Fig.6

Art. 34. In the improved **Burleigh** drill, the valve, V, Fig 7, is moved by **two** tappets, T T', which are alternately struck by the ends of the piston, P. **Burleigh Rock-Drill Co,** mfrs, tchburg, Mass; office 115 Liberty New York.

Art. 35. In the "Dynamic" k-drill, invented by **Prof De olson Wood,** and developed d manufactured by the **Grayton & Denton Mfg Co,** 15 Cortlandt St, New York, **the valve** is attached to a valve-piston, V, Fig 8, which s moved backward and forward by steam, which is admitted so as to act alternately upon its two ends. The admission of this steam is controlled by a small auxiliary valve, *a*. A hub on the back of the auxiliary valve fits in the spiral groove shown on the plug, *n*. This plug is constantly pressed downward (as the Fig stands) by steam pressing upon its upper shoulder, but it is lifted at each forward stroke by the conical surface of the piston, P, pressing against its foot. It thus moves constantly up and down, carrying the valve, *a*, with it. By turning the plug, *n*, by means of the adjusting-stem, *s*, the hub of the valve is made to occupy a higher or lower point in the spiral groove, and thus the stroke of the piston may be varied, or may be confined to any part of the cylinder.

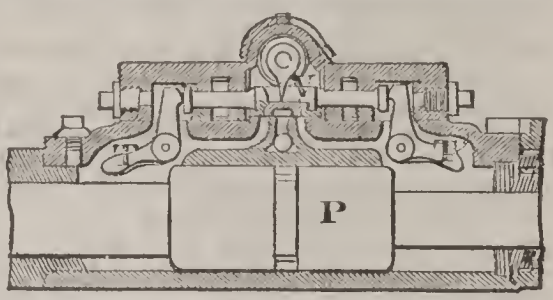


Fig.7

In this drill, unlike the Ingersoll, Art 27, the **piston rotates** while making

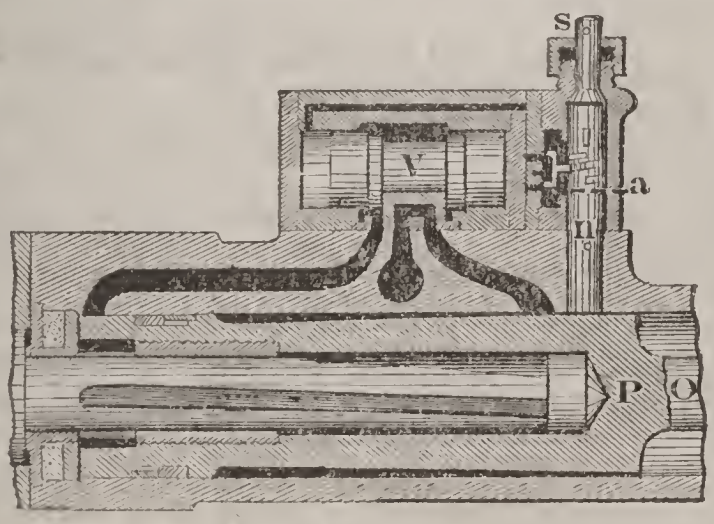


Fig.8

the *downward* stroke. **The piston-rod,** *o*, is **made lighter than in other drills.** This gives a greater surface under the piston for the pressure of steam on the up stroke, and, consequently, greater lifting power. This is useful when the drill sticks in the hole.

The **tripod legs** are of bar iron. Their length is adjustable.

Art. 36. The **hand rock-drilling machine** of the **Pierce Well Excavator Co,** New York, and Long Island City, N Y, is a percussion drill. It is worked by a crank which turns a disc about 2 ft in diam. The disc has a semi-circular slot, in which works the arm which raises the drill-rod. This arm, in rising, compresses a coil-spring, which, on the down stroke, drives the drill against the rock. An iron ball, weighing 30 lbs or more, is furnished with each machine. This ball may be screwed to the top of the drill-rod, for giving greater force to the blows of the drill. The ball may be used without the spring, by disengaging the latter.

The drill makes about 40 strokes of 10 or 12 ins per minute; and bores holes from ¾ to 2½ ins diam. It can be arranged to drill to depths of 30 ft and over. For sharpening the bits, it has an emery wheel attached, which is turned by the crank. The latter, at such times, is thrown out of gear with the disc.

The drill is mounted on a rectangular two-legged frame, about 5 ft high by 2 ft wide, made of iron tubes. To the top of this frame a third leg is attached, by adjusting which the angle of the drill-rod with the vert may be changed. Like other percussion drills worked by hand-power, this one ceases to work to advantage when said angle exceeds about 45°. These drills **cost,** ready for work, in 1886, $225 each. They **weigh** from 200 to 400 lbs. They are moved from place to place like a wheelbarrow, the disc serving as a wheel.

Art. 37. Channeling consists in making long, deep, and narrow cuts in the rock. In this way large blocks can be gotten out without blasting and the consequent danger of fracture. This is ordinarily done by boring a row of holes about an inch apart in the clear, and then breaking down the intermediate spaces by means of a blunt tool, called a **broach.** This is called **broach channeling.** For this purpose a steam drilling machine is mounted upon a hor bar resting upon two pairs of legs. The hor bar is placed over the intended row of holes, and the drill is slid along upon it from one hole to the next. In using the *broach*, the rotating apparatus is thrown out of gear, so that the edge of the broach maintains its position in line with the row of holes.

Art. 38. The Saunders patent channeling machine, of the Ingersoll Co, consists of a rock-drilling machine, having, in place of the usual drilling-bit, a gang of tools consisting of a number of chisels, clamped together side by side, and thus forming a cutting tool about 7 ins long by ¾ inch wide. This tool has as many cutting-edges (each as long as the tool is wide) as there are chisels. The machine is supported upon a carriage, moving on a track parallel with the channel to be cut. The tool is of course not rotated; but the rifle-bar, A, Fig 4, is employed to move the carriage along the track about an inch after each blow. The carriage remains stationary while a blow is being struck. Under favorable circumstances **this machine has cut** from 80 to 100 sq ft of channel per day of ten hours. Its **weight,** including carriage, is about 5000 lbs. **Cost,** 1886, about $2300.

A valve is provided, by which, if desired, the **steam may be shut off** from the piston on the down stroke, so that said stroke may be made with only the *weight* of the piston, rod, and drill.

Art. 39. The Ingersoll Co have a special appliance, designed by Mr. W. L. Saunders, C E, for **drilling and blasting rocks under water,** even when they are covered by a considerable depth of mud.

Art. 40. Air compressors for rock-drills, as made and used in this country, are mostly hor, direct-acting engines. That is, the axes of the steam- and air-cylinders are hor; and the piston-rod passes directly from the steam-cylinder into the air-cylinder. A fly-wheel is attached, by a crank and connecting-rod, to the piston-rod. Sometimes the steam-engine is separate from the compressor, and the power is conveyed to the latter by belts or gearing; or water-power may be used in the same way. The air is forced into a receiver, which is generally a plate-iron cylinder, 3 or 4 ft in diam, and 5 to 12 ft long.

If the air- or pumping-cylinder of the compressor is so arranged as to take in air on one stroke only, and force it out into the receiver upon the return stroke, it is **"single-acting."** If, at each stroke, it both takes in and forces out air, it is **"double-acting."** If the compressor has only one air-cylinder, it is **"single."** If it has two, and thus practically consists of two single compressors, it is **"duplex."**

The **valves** may be either **"poppet"** valves, held in place by springs, and operated by the pressure of the air itself; or **slide** valves, operated by eccentrics and rods, as in steam-engines.

The compression of the air develops **heat.** This is removed either by causing cold water to circulate through the air-piston, and through jackets surrounding the air-cylinder; or by injecting it into the air-cylinder in the form of spray. Or both methods may be used together.

Art. 41. Compressors are furnished by the Ingersoll Rock-Drill Co, 10 Park Place, New York; Rand Drill Co, 23 Park Place, New York; Burleigh

Rock-Drill Co, 115 Liberty St, New York; Graydon & Denton Mfg Co, 15 Cortlandt St, New York; Clayton Steam-Pump Works, office, 43 Dey St, New York, and others.

The following partial list of Clayton compressors, compiled from data given by the makers, shows the **dimensions** and **performance** of each. We give also a list of their **receivers.** For **prices,** apply to the Co as above. We give the card prices of 1886 as an approximation.

CLAYTON DOUBLE-ACTING AIR-COMPRESSORS. Partial List.

	Number, designating the size of the machine.			
	1	**2½**	**4**	**7**
Duplex Direct-acting* Compressors.				
Diam of steam-cylinders.....ins.	8	10	14	18
" air " .....ins.	8	10	14	18
Length of stroke.....ins.	12	13	15	24
Number of revolutions per minute.....	120 to 140	100 to 130	100 to 120	80 to 90
Cub ft of free air compressed per minute .....Actual.	136	210	438	900
Approximate wt of compressor .....lbs.	3000	7000	15000	25000
Approx number of rock-drills† with 3-inch cyls supplied with air at 60 lbs per sq inch	2	4	8	18
Card price, 1886.*.....$	1320	1540	3025	6050
Single Direct-acting* Compressors.				
Diam of steam-cylinder.....ins.	8	10	14	18
" air " .....ins.	8	10	14	18
Length of stroke.....ins.	12	13	15	24
Number of revolutions per minute	120 to 140	100 to 130	100 to 120	80 to 90
Cub ft of free air compressed per minute.....Actual.	68	105	219	450
Approx wt of compressor.....	1650	3850	8250	13750
Approx number of rock-drills† with 3-inch cyls supplied with air at 60 lbs per sq inch.....	1	2	4	9
Card price, 1886.*.....$	725	850	1650	3300

* The price of a compressor alone, to be worked by a separate steam-engine or water-power, is of course less than that of the above compressor and engine combined.

† Some makes of rock-drills require much more air than others.

Air-Receivers; vertical or horizontal.

Diam, Ins.	L'gth, Ft.	Approx wt, lbs.	Card Price, 1886.	Diam, Ins.	L'gth, Ft.	Approx wt, lbs.	Card Price, 1886.
33	5	700	$100	40	8	1675	$168
30	7	890	120	40	10	1900	200
36	8	1560	150	40	11	2000	212
40	6	1600	160	40	12	2100	225

The above prices include brass-face pressure-gauge, glass water-gauge, safety-valve, blow-off valve, try-cocks, flanges, and connections to automatic feed on compressor.

GUNPOWDER.

The explosive force of powder is about 40000 lbs, or 18 tons, per square inch. Its **weight** averages about the same as that of water, or 62½ lbs per cubic foot; hence, 1 lb = about 28 cubic inches. In ordinary quarrying, a cubic yard of solid rock in place, (or about 1.9 cubic yards piled up after being quarried,) requires from ¼ to ¾ lb. In very refractory rock, lying badly for quarrying, a solid yard may require from 1 to 2 lbs. In some of the most successful great blasts for stone for the Holyhead Breakwater, England, (where several thousands of lbs of powder were usually exploded by electricity at a single blast,) from 2 to 4 cubic yards solid were *loosened* per lb; but in many instances not more than 1 to 1½ yards. Tunnels and shafts require 2 to 6 lbs per solid yard; usually 3 to 5 lbs. Soft, partially decomposed rock frequently requires more than harder ones. Usually sold in kegs of 25 lbs.*

Weight of powder in one foot depth of hole.

Diameter of hole in inches.

1	1¼	1½	2	2½	3	3½	4	4½	5	5½	6
Weight of powder in pounds and ounces avoir.											
0″5	0″8	0″11	1″4	2	2″13	3″14	5″0	6″6	7″14	9″8	11 ″5

* **Price,** 1886, in Atlantic cities, about $2.50 per 25 lb keg.

MODERN EXPLOSIVES.

Art. 1. Most of the explosives, which, of late years, have been taking the place of gunpowder (p 660), **consist of** a powdered substance, partly saturated with nitro-glycerine, a fluid produced by mixing glycerine with nitric and sulphuric acids.

Art. 2. Pure **nitro-glycerine,** at 60° Fah, has a sp grav of 1.6. It is odorless, nearly or quite colorless, and has a sweetish, burning taste. It is poisonous, even in very small quantities. Handling it is apt to cause headaches. It is insoluble in water. At about 306° Fah it takes fire, and, if unconfined, burns harmlessly, unless it is in such quantity that a part of it, before coming in contact with air, becomes heated to the exploding point, which is about 380° Fah.

N-G, and the powders containing it, are always **exploded by means of** sharp percussion. See Arts 36, &c. After N-G is made, great care is required to wash it completely from the **surplus acids remaining in it** from the process of manufacture. Their presence, either in the liquid N-G, or in the powders containing it, renders the N-G liable to spontaneous decomposition, which, by raising the temperature, increases the danger of explosion.

Art. 3. N-G freezes at about 45° Fah. It is then very **difficult of explosion,** and must be thawed *gradually*, as by leaving it for a sufficient length of time in a comfortably warm room, or by placing the vessel containing it in a second vessel containing hot water, not over 100° Fah; but never by exposing it to intense heat, as in placing it before a fire, or setting it on a stove or boiler. Extra strong caps are made for exploding N-G and its powders when frozen.

Art. 4. N-G, owing to its incompressibility, is **liable to explosion through accidental percussion.** This, and its **liability to leakage,** render it inconvenient to transport and handle. Hence it is rarely used in the liquid state in ordinary quarrying and other blasting. In the oil regions of Penna, it is largely used in **oil wells,** in order to increase the flow. For this purpose it is confined in cylindrical tin casings, from 1 to 5 inches diam, called torpedo-shells. These are suspended from, and lowered into the well by means of, a cord or wire wound on a reel; and are destroyed when the charge is exploded. They are about 1 inch less in diam than the well, and contain usually from one to twenty quarts = 3 lbs, 5⅛ oz to 66 lbs, 6¼ oz of N-G. They are pointed at their lower ends, in order to facilitate their passage through the oil or water which may be in the well. When a greater charge than about 66½ lbs is required, two or more of these shells are placed in the well, one on top of another, the conical point on the lower end of each one fitting into the top of the one next below. In this case, the N-G is fired by means of a cap or series of caps placed in the top of the charge before it is lowered. When the charge is in place, the caps are exploded by electricity led to them by conducting wires, as in Art 37, or (as in the method more commonly practised) by letting a weight fall on them.

When a well has been repeatedly torpedoed, and a cavity has thus been formed in it so large that the space surrounding a torpedo would interfere too greatly with the effect of the explosion of the N-G on the walls of the well, the latter is placed directly in the well, by lowering a tin cylinder, filled with it, and provided with an automatic arrangement which allows the N-G to escape when at the bottom of the well. The N-G is then fired by a torpedo suspended on a line, and having caps placed in its top. These caps are exploded by a leaden or iron weight sliding down the line, or by electricity. When the rock is seamy, the N-G is confined in short cylindrical tin shells, lowered into the cavity, and fired by a torpedo. N-G and torpedoes of N-G, and of "Atlas" and "Hercules" powders, are furnished by The Torpedo Co of Delaware, office Warren, Pa. **Price** of N-G, 1886, about 75 cts per lb. It is also used for increasing the flow of springs of *water*. It of course cannot be used in **hor or upward** holes, such as often occur in tunneling, &c.

Art. 5. N-G explodes so suddenly that **very little tamping is required.** Moist sand or earth, or even water, is sufficient. This, with the fact that N-G is unaffected by immersion in water, and is heavier than water, render it particularly **suitable for sub-aqueous work,** or for holes containing water, provided the rock has no seams which would permit the N-G to escape. **If the rock is seamy,** the N-G must be confined in a water-tight casing. Such casings, however, necessarily leave some spaces between the rock and the explosive, and these diminish considerably the effect of the latter.

Art. 6. The great **explosive force of N-G is due** partly to the very large volume of gas into which a small quantity of it is converted by explosion, and

partly to the *suddenness* with which this conversion takes place, the gases being liberated almost instantaneously,* while with gunpowder their liberation requires a longer time. The suddenness of the explosion increases its effect, not only by applying all of its force practically at one instant, but also by greatly *heating* the gases produced, and thus still further increasing their *volume*.

Art. 7. The liquid condition of N-G is useful in causing it to **fill the drill-hole completely,** so that there are no vacant spaces in it to waste the force of the explosion. On the other hand, the liquid form is a disadvantage, because, when thus used without a containing vessel in seamy rock, portions of the N-G leak away and remain unexploded and unsuspected, and may cause accidental explosion at a future time.

Art. 8. N-G is stored in tin cans or earthenware jars. If properly washed from acid it does not injure tin. For transportation, these cans or jars are packed in boxes with sawdust, or in padded boxes, and loaded in *wagons*. The R R companies do not receive it.

Art. 9. When N-G and its compounds are *completely* exploded, **the gases given out** are not troublesome, but those resulting from *incomplete* explosion, such as generally takes place, or from combustion, are very offensive.

Art. 10. For convenience, we apply the name **"dynamite"** to any explosive which contains nitro-glycerine mixed with a granular absorbent; **"true dynamite"** to those in which the absorbent of the N-G is "Kieselguhr,"† or some other *inert* powder which takes no part in the explosion; and **"false dynamite"** to those in which the absorbent itself contains explosive substances other than N-G.

Art. 11. The absorbent, by its granular and compressible condition, **acts as a cushion to the N-G,** and protects it from percussion, and from the consequent danger of accidental explosion.

N-G undergoes no change in composition by being absorbed; and it then freezes, burns, explodes, &c, under the same conditions as to pressure, temperature, &c, as when in the liquid form. The cushioning effect of the absorbent merely renders it more difficult to bring about sufficient percussive pressure to cause explosion. The absorption of the N-G in dyn enables the latter to be used in *hor* holes, or in holes drilled *upward*.

Art. 12. N-G and dyn **explode much more readily when rigidly confined,** as by a *metallic* vessel, or by the walls of a hole drilled in rock, than when confined by a *yielding* substance, as wood. Therefore the fact that dyn, not being liquid, can be packed in *wooden* boxes, renders it safer than N-G which has to be kept in stone or metal vessels.

Art. 13. True dynamites must contain at least about 50 per cent of N-G. Otherwise the latter will be too completely cushioned by the absorbent, and the powder will be too difficult to explode. False dynamites, on the contrary, may contain as small a percentage of N-G as may be desired; some containing as little as 15 per cent. The added explosive substances in the false dynamites generally contain large quantities of oxygen, which are liberated upon explosion, and aid in effecting the complete combustion of any noxious gases arising from the N-G.

Art. 14. Dynamites which contain large percentages of N-G explode (like the liquid N-G. Art 6) with great suddenness, tending to *shatter* the rock in their vicinity into small fragments. They are most useful in *very hard* rock. In such rock, **No 1 dynamite,** or that containing 75 per cent of N-G, is roughly estimated to have **about 6 times the force of an equal wt of gunpowder.**

For soft or decomposed rocks, sand, and earth, the **lower grades** of dynamite, or those containing a smaller percentage of N-G, are more suitable. They explode with less suddenness, and their tendency is rather to upheave large masses of rock, &c, than to splinter small masses of it. They thus more nearly resemble gunpowder in their action.

Judgment must be exercised as to the **grade and quantity of explosive** to be used in any given case. Where it is not objectionable to break the rock into small pieces, or where it is desired to do so for convenience of removal, the higher, *shattering* grades are useful. Where it is desired to get the rock out in large masses, as in quarrying, the lower grades are preferable.

For very difficult work in hard rock, and for submarine blasting, the highest grades, containing 70 to 75 per cent of N-G, are used. A small charge of these does the same execution as a larger charge of lower grade, and of course does not require

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* Such sudden liberation of gas is called **"detonation."**

† Kieselguhr is an earthy, silicious limestone, composed of the fossil remains of small shells. Each shell acts as a minute receptacle for nitro-glycerine. Kieselguhr is found in Hanover, Germany, and in New Jersey.

the drilling of so large a hole. In submarine work their sharp explosion is not deadened by the water.

For general railroad work, ordinary tunneling, mining of ores, &c, the **average grade,** containing 40 per cent of N-G, is used; **for quarrying,** 35 per cent; **for blasting stumps,** trees, piles, &c, 30 per cent; **for sand and earth,** 15 per cent.

Art. 15. Dynamite, like N-G, **can be readily exploded under water,** provided it is so immersed as not to be *scattered;* but **long exposure to water is injurious** to it. In the higher grades, the water, by its greater affinity for the absorbent, drives out the N-G. In the lower grades it is apt to wash away the salts used as additional explosives.

Art. 16. In dyns containing a large percentage of N-G, the latter is **liable to exude** in liquid form, or to "leak," especially in warm weather, and then to explode through accidental percussion. The same danger exists, even though the percentage of N-G be small, if the absorbent has but small absorbing power, and is, consequently, easily saturated.

Art. 17. True dyn resembles moist brown sugar. **Its properties** are generally those of the N-G contained in it. Thus, it takes fire at about 350° F, and burns freely. It freezes at 45° F, and is then difficult to explode. It is not exploded by friction, or by ordinary percussion, but requires, for general purposes, a strong cap, or exploder, containing fulminating powder, see Arts 36, 38, &c. It may, however, be exploded by a priming of gunpowder, tightly tamped, and fired by an ordinary safety-fuse.

Art. 18. The charge should fill the cross section of the hole as completely as possible. If water is not standing in the hole, the cartridge should be cut open before insertion, so that the powder may escape from it and fill the hole; or the powder may be simply emptied from the cartridge into the hole.

Art. 19. For **blasting ice** in place, holes are cut in it, and a number of dyn cartridges (one of which must contain an exploding cap) are tied together and lowered from 1 to 5 ft into the water. They are fired as soon as possible after immersion, to avoid the danger of freezing. Electrical exploders (Arts 37, &c,) are best for sub-aqueous work.

Art. 20. Dyn is useful for **breaking up pieces of metal,** such as old cannon, condemned machinery, "salamanders" (masses of hardened slag) in blast furnaces, &c. In cannon, the dyn is of course exploded in the bore. In other pieces, small holes are generally drilled to receive it; but plates, even of considerable thickness, may be broken by merely exploding dyn upon their surface.

Art. 21. For **blasting trees or stumps,** one or more cartridges are fired in a hole bored in the trunk or roots, or under the latter. This shatters both trunk and roots. **A tree may be felled neatly** by boring a number of small radial holes into it, at equal short dists in a hor line around its circumf, and, by means of an electric battery (Arts 37, &c), exploding simultaneously a small charge of dyn in each. Or a single long cartridge may be tied around the trunk of a small tree, and fired.

Art. 22. Piles may be blasted in the same way as trees; or a hole may be bored for the cartridge in the axis of the pile; or the cartridge may be simply tied to the side of the pile at any desired ht.

Art. 23. The higher grades of dyn, like N-G, **require but little tamping.** Use a *wooden* **tamping-bar,** *never* a *metallic* one, for any explosive. If a charge of dyn **"hangs fire,"** it is dangerous to attempt to remove it. Remove the *tamping*, all but a few ins in depth, on top of which insert another cartridge, containing an exploder, and try again. See electrical exploders, Arts 37, &c. Dyn, like N-G, if frozen, must be thawed *gradually*, by leaving it in a warm room, far from the fire; or by placing it in a metallic vessel, which is then placed in another vessel containing hot water. The water should not be hotter than can be borne by the hand. Otherwise the N-G is liable to separate from the absorbent. The N-G in dyn may freeze without cementing together the particles of the absorbent; in which case the powder of course is still soft to the touch. **An overcharge** of N-G, or of dyn, is liable to be burned, and thus wasted, giving off offensive gases.

Art. 24. Dyn is sold in cylindrical, paper-covered cartridges, from ⅞ to 2 ins in diam, and 6 to 8 ins long, or longer. They are furnished to order of any required size, and are packed in boxes containing 25 lbs or 50 lbs each. The layers of cartridges are separated by sawdust.

Art. 25. Some of the **R R companies** decline to carry dyn or N-G in any shape. Others carry dyn under certain restrictions, based upon State laws; providing that it must be dry (*i e*, that no N-G shall be exuding from it); that boxes and cars containing it shall be plainly marked with some cautionary words, as "explosive," "dangerous," &c; that the cartridges shall be so packed in the boxes, and the boxes so loaded in the cars, that both shall lie upon their *sides*, and the boxes

be in no danger of falling to the floor; that caps, &c, shall not be loaded in the same car with dyn, &c, &c.

Art. 26. A great many varieties of dyn are made. They differ (generally but slightly) in the composition of the absorbent, and in the method of manufacture. Each maker usually makes a number of grades, containing different percentages of N-G, &c, and gives to his powders some fanciful name.

Art. 27. The following **table of explosives** made by the Repauno Chemical Co, at Thompson's Point, N J, office Wilmington, Del, and known as **"Atlas" powders,** gives the percentage of N-G in each, and the **approx card price** for 1886. It gives a general idea of the range of American dyns.

Brand.	Percentage of N-G.	Card price, 1886, cts per lb.	Brand.	Percentage of N-G.	Card price, 1886, cts per lb.
A	75	36	D+	33	21
B+	60	32			
B	50	28	E+	27	19
C+	45	26	E	20	16
C	40	24			

The absorbents contain: in "A" brand, 18 per cent wood pulp and 7 per cent carbonate of magnesia; in "C" brand (the average grade), 46 per cent nitrate of soda (soda saltpetre), 11 per cent wood pulp, and 3 per cent carbonate of magnesia; in "E" brand, 62 per cent nitrate of soda, 16 per cent wood pulp, &c, and 2 per cent carbonate of magnesia.

Art. 28. "Miner's Friend" powder, made by the Hecla Powder Co, office 239 Broadway, New York, contains nitrate of soda, wood pulp, resin, and carbonate of magnesia. It freezes at 42°, and is then, like other dyn, difficult to explode. When used under water, the cartridges should not be broken, because the powder is injured by direct contact with water. Their **"Hecla" powder** is a lower grade. It is in granulated form, like ordinary blasting powder, but is said to be much stronger. It is intended as a substitute for it.

Art. 29. "Giant" powder is made by Atlantic Dynamite Co, office 245 Broadway, New York. No 1 is dyn proper, containing 75 per cent N-G, and 25 per cent Kieselguhr obtained near their works in New Jersey. Their lowest grade, branded "M," contains 20 per cent N-G. The name "giant powder" was originally applied to dynamite in general.

Art. 30. Other brands are **"Hercules"** powder, Hercules Powder Co, 40 Prospect St, Cleveland, O; and **"Judson R R P powder,"** Atlantic Dyn Co., New York, a substitute for ordinary blasting powder. It is put up in water-proof paper bags, of 6¼, 12½, and 25 lbs each, and these are packed in wooden boxes holding 50 lbs each. The same Co furnish also **"Judson F F F dynamite,"** a higher grade, in cartridges of the usual shape, packed in 50-lb boxes.

Art. 31. "Rackarock" cartridges, furnished by Rendrock Powder Co, 23 Park Place, New York, are said to contain no N-G, and to be entirely inexplosive until immersed, for a few seconds, in an inexplosive liquid furnished by the same Co. They are then allowed to stand for 15 mins, after which they may be used at any time. They are fired in the same way as dyn, and can be used under water. The mfrs claim that they "approximate N-G in strength, and are stronger than dyn."

Art. 32. The following explosives are made and used in Europe, but have not yet been regularly imported into the U S.

Compressed gun-cotton, made at Stowmarket, Eng, is cotton dipped in a mixture of nitric and sulphuric acids, then reduced to a fine pulp, and made into discs 1 to 2 ins thick, and ⅞ to 2 ins diam, or larger. It is generally used wet, for the sake of greater safety. It then requires extra strong caps or primers. Roughly speaking, it is about as strong as dyn No 1, but is less *shattering* in its effect. Being lighter than dyn, it requires larger holes; and, owing to its rigidity, is less easily inserted, and does not fit the hole so completely. When dry, it is very inflammable, but, if not confined, it burns harmlessly. It contains no liquid, to freeze or to exude; and is safe to handle.

Art. 33. Tonite consists of finely divided gun-cotton mixed with nitrate of baryta. It is made by the Cotton Powder Co, Limited, at Faversham, Eng. It is compressed into candle-shaped cartridges having, at one end, a recess for the reception of an exploder containing fulminate of mercury. The cartridges weigh about the same as dyn. They are generally made waterproof.

Art. 34. Forcite, Lithofracteur, and **Dualin** are foreign makes of nitro-glycerine explosives. In Dualin the absorbent is sawdust. It has greater bulk than dyn for a given wt, and requires larger holes.

Art. 35. Explosive gelatine is made by the Nobels Explosives Co, Lim (office Glasgow, Scotland), at their several works in England. It is a transparent, pale yellow, elastic substance, and is composed of 90 per cent N-G and 10 per cent gun-cotton. It is less sensitive than dyn to percussion, friction, or pressure, and is not affected by water. Its specific gravity is 1.6. It burns in the open air. For complete detonation a special primer is required. The addition of a small proportion of camphor renders it still less sensitive, and increases its explosive force. The camphor evaporates to some extent.

In some experiments on the power of different explosives to increase the contents of a small cavity in a leaden block, explosive gelatine caused an increase 50 per cent greater than that caused by dyn No 1. In hard rock the diff would probably have been greater. The increase was 10 per cent less than that caused by N-G.

Art. 36. The cap or exploder, used with ordinary safety fuse for exploding N-G and dyn, is a hollow copper cylinder, abont ¼ inch diam, and an inch or two in length. It contains from 15 to 20 per cent, or more, of fulminate of mercury, mixed with other ingredients into a cement, which fills the closed end of the cap. The cap is called "single-force," "triple-force," &c, according to the quantity of explosive it contains.

The end of the fuse, cut off square, is inserted into the open end of this cap, far enough to touch the fulminating mixture in it. In doing this, care must be taken not to roughly scratch the latter. The neck of the cap is then pinched, near its open end, so as to hold the fuse securely. The cap, with the fuse thus attached, is then inserted into the charge of N-G or dyn, care being taken not to let the fuse come into contact with the explosive, which would then be burned and wasted. If a dyn cartridge is used, the fuse, with cap, is first inserted into it. The neck of the cartridge is then tied around the fuse with a string, and the cartridge is then ready to be placed in the hole and fired.

Art. 37. The Siemens magneto-electric blasting apparatus, now in general use, consists of a wooden box abont as large as a transit-box. Outside it has two metallic binding-posts with screws, for attaching the two wires leading to the exploder. From the top of the box projects a handle at the end of a vert bar. This bar, which is abont as long as the box is high, is made so as to slide up and down in it, and is toothed, and gears with a small pinion inside the box. When a blast is to be fired, the bar is drawn up, by means of the handle, as far as it will come. It is then pressed quickly down to the bottom of the box. In its descent it puts into operation, by means of the pinion, a magneto-electric machine inside the box. This generates a current of electricity, which increases in force with the downward motion of the bar, but which is confined to a short circuit of wire *within the box*, until the foot of the bar strikes a spring near the bottom of the box, breaking the short circuit and forcing the electricity to travel through the two longer "leading wires," which lead it from the two binding-posts on the outside of the box to the cap or exploder placed in the charge.

Art. 38. The cap used with this machine is similar to that used with safety fuse (Art 36), except that its mouth is closed with a cork of sulphur cement, through which pass the two wires leading from the electric machine. The ends of these wires project into the fulminating mixture in the cap. They are ⅛ inch apart, but are connected by a platinum wire, which is so fine as to be heated to redness by the current from the battery. Its heat ignites the fulminate and thus explodes the cap.

These exploders, called **platinum caps.** or (improperly) platinum *fuses*, **cost,** 1886, abont 3 to 10 cts each, depending upon the length (from 1 to 16 ft) of the two cotton-covered wires attached to them. With gutta-percha-covered wires, 20 to 40 cts. The outer end of each of these short wires is connected with the electrical machine by a cotton-covered **"leading wire"** costing 1 cent per ft.

Art. 39. Where a number of holes are to be fired simultaneously (thus increasing their effect), each hole has a platinum cap inserted into its charge, and one of the short wires attached to each cap is joined to one of those of the next cap, so that at each end of the series of caps there is one free end of a short wire. Each of these two ends is fastened to the end of one of the leading wires, placing the whole series "in one circuit." Where the holes are too far apart for the caps to be thus joined by the short wires attached to them, the ends of the latter are connected by cotton-covered **"connecting wires,"** costing, 1886, 30 cts per *pound*.

Art. 40. The magneto-electrical **machine weighs** about 16 lbs, an **costs,** 1886, size No 3, $25. It can fire about 12 caps at once. A larger size, No costs $50; and a still larger one, said to be capable of firing over 50 holes at onc $100.

Frictional electric blasting machines, costing about $75 each, ar now (1884) nearly obsolete.

Caps for ordinary fuse and for electrical firing, fuses, wires, electrical machine &c, are made by Laflin & Rand Powder Co, 29 Murray St, New York, and are sold b most of the makers of, and dealers in, explosives, rock-drilling machines, &c.

Art. 41. Simultaneous firing of a number of holes can be conveniently accon plished only by electricity. Electric blasting apparatus is specially useful fo blasting under water, where ordinary fuses are apt, especially at great depths, t become saturated and useless.

If an electrical machine fails to fire a charge, it is known that the charge canno explode until the attempt is repeated. Therefore no time need be lost, and no risk run, on account of "hanging fire."

Cost of quarrying stone. After the preliminary expenses of purchasing the site of a good quarry; cleaning off the surface earth and disintegrated top rock; and providing the necessary tools, trucks, cranes, &c; the total neat expenses for *getting out* the rough stone for masonry, per cub yard, ready for delivery, may be roughly approximated thus: Stones of such sizes as two men can readily lift, measured *in piles*, will cost about as much as from ¼ to ½ the daily wages of a quarry laborer. Large stones, ranging from ½ to 1 cub yd each, got out by blasting, from 1 to 2 daily wages per cub yd. Large stones, ranging from 1 to 1½ cub yds each, in which most of the work must be done by wedges, in order that the individual stones shall come out in tolerably regular shape, and conform to stipulated dimensions; from 2 to 4 daily wages per cub yard. The smaller prices are low for sandstone, while the higher ones are high for granite. Under ordinary circumstances, about 1⅓ cub yds of good sandstone can be quarried at the same cost as 1 of granite; or, in other words, calling the cost of granite 1, that of sandstone will be ¾: so that the means of the foregoing limits may be regarded as rather full prices for sandstone; rather scant ones for granite: and about fair for limestone or marble.

Cost of dressing stone. In the first place, a liberal allowance should be made for **waste.** Even when the stone wedges out handsomely on all sides from the quarry, in large blocks of nearly the required shape and size, from ⅙ to ¼ of the rough block will generally not more than cover waste when well dressed. In moderate-sized blocks, (say averaging about ½ a cub yard each,) and got out by blasting, from ¼ to ⅓ will not be too much for stone of medium character as to straight splitting. About the last allowance should also be made for well-scabbled rubble. The smaller the stones, the greater must be the allowance for waste in dressing. In large operations, it becomes expedient to have the stones dressed, as far as possible, at the quarry; in order to diminish the cost of transportation, which, when the distance is great, constitutes an important item — especially when by land, and on common roads.

A stonecutter will first take out of wind; and then fairly patent-hammer dress, about 8 to 10 sq ft of plain face in hard granite, in a day of 8 working hours; or twice as much of such inferior dressing as is usually bestowed on the beds and joints; and generally on the faces also of bridge masonry, &c, when a very fine finish is not required. In good sandstone, or marble, he can do about ¼ more than in granite. Of *finest* hammer finish, *granite*, 4 to 5 sq ft.

Cost of masonry. Every item composing the total cost is liable to much variation; therefore, we can merely give an example to show *the general principle* upon which an approximate estimate may be made; assuming the **wages of a laborer** to be $2.00 per day of 8 working hours; and $3.50 for a mason. **The monopoly of quarries** affects prices very much.*

Cost of ashlar facing masonry. Average size of the stones, say 5 ft long, 2 ft wide, and 1.4 thick; or two such stones to a cub yd. Then, supposing the stone to be granite or gneiss, the cost per cub yd of masonry **at such wages** will be,

Getting out the stone from the quarry by blasting, allowing ¼ for waste in dressing; 1⅓ cub yds, at $3.00 per yard	$4.00
Dressing 14 sq ft of face at 35 cts	4.90
" 52 " beds and joints, at 18 cts	9.36
Neat cost of the dressed stone at the quarry	18.26
Hauling, say 1 mile; loading and unloading	1.20
Mortar, say	.40
Laying, including scaffold, hoisting machinery, superintendence, &c	2.00
Neat cost	21.86
Profit to contractor, say 15 per ct	3.28
Total cost	25.14

Dressing will cost more if the faces are to be rounded, or moulded. If the stones are smaller than we have assumed, there will be more sq ft per cub yd to be dressed, &c.

If in the foregoing case, the stones be *perfectly* well dressed on all sides, including the back, the cost per cub yd would be increased about $10; and if some of the sides be curved, as in arch stones, say $12 or $14; and if the blocks be carefully wedged out to given dimensions, $16 or $18; thus making the neat cost of the dressed stone *at the quarry* say $28, $31, or $35 per cub yd.

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* The blocks of granite for **Bunker Hill monument** averaging 2 cub yds each, were quarried by wedging, and delivered at the site of the monument, at a neat actual cost of $5.40 per cub yd; by the Monument Association; from a quarry opened by themselves for the purpose. The Association received no profit; their services being voluntary. The average contract offers for the same, were $24.30! The actual cost of getting out the rough blocks at the quarry was $2.70. Loading upon trucks at quarry, about 15 cts. Transportation 8 miles by railway and common road, $2.55. Total, $5.40. In 1825 to 1845; common unskilled labor averaging $1 per day.

In 1886, Granite blocks about a cub yd each, with dressed beds and joints, but with only a 2 inch draft around the showing-face, (which is left rough,) are del'd on the wharf at Philada, from Port Deposit, Md, by McClenahan & Bros, at $16 each.

The item of laying will be much increased if the stone has to be raised to great heights; or if it h to be much handled; as when carried in scows, to be deposited in water-piers, &c. Almost eve large work presents certain modifying peculiarities, which must be left to the judgment of the en neer and contractor. The percentage of contractors' profit will usually be less on large works th on small ones.

Cost of ashlar facing masonry. If the stone be **sandston** with good natural beds, the gettlug out may be put at $3.00 per cubic yard. Face dressing at 26 per sq ft: say $3.64 per cubic yd. Beds and joints 13 cts per sq ft; say $6.76 per cub yd. The ne cost, laid, $17.00.

And **the total cost of large well scabbled range sandstone masonry in mortar,** may be taken at about $10 per cub y

Cost of large scabbled granite rubble, such as is generally used backing for the foregoing ashlar; stones averaging about ½ cub yd each:

Labor at $1 per day.	Cost per cub yd of masonry.
Getting out the stone from the quarry by blasting, allowing ⅛ for waste in scabbling; 1⅛ cub yds at $3.00	$3.43
Hauling 1 mile, loading and unloading	1.20
Mortar; (2 cub ft, or 1.6 struck bushels quicklime, either in lump or ground; and 10 cub ft, or 8 struck bushels of sand, or gravel; and mixing)	1.50
Scabbling; laying, including scaffold, hoisting machinery, &c	2.50
Neat cost	8.63
Profit to contractor, say 15 per ct	1.30
Total cost	9.93

Common rubble of small stones, the average size being such as tw men can handle, costs, to get it out of the quarry, about 80 cts per yard of pile or to allow for waste, say $1.00. Hauling 1 mile, $1.00. It can be roughly scabbled and laid, for $1.20 more; mortar as foregoing, $1.50. Total neat cost, $4.70; or, wit 15 per ct profit, $5.40, *at the above wages for labor.*

With smaller stones, such as one man can handle, we may say, stone 70 cts; hauling $1 laying and scaffold, tools &c, $1; mortar $1.50 Making the neat cost $4.20; or with 15 per ct profit, $4.8 Neat scabbled irregular range-work costs from $2 to $3 more per yd than rubble; according to the charac ter of the stone &c. The laying of thin walls costs more than that of thick ones, such as abutments &c.

The cost of plain 8 inch thick ashlar facings for dwellings &c i Philada, in 1886, is about as follows per square foot showing, put up, including everything. Sand stone, $1.50 to $2.25. Pennsylvania marble, $2.50. New England marble, $2.75 to $3.25. Granite $2.25 to $2.75. If 6 ins thick, deduct one-eighth part. **First class artificial stone** could be made and put up at one-third the price. See p 681. **North River blue stone flags,** 3 ins thick, for footwalks, put down, including gravel &c, 70 cts per sq foot. **Belgian street pavement,** with gravel, complete, $3.50 per sq yard in Eastern cities.

When dressed ashlar facing is backed by rubble, the expense per cub yard of th entire mass will of course vary according to the proportions of the two. Thus, i ashlar at $12 per yd, is backed by an equal thickness of rubble at $5, the mean cos will be ($12 + $5) ÷ 2 = $8.50; or if the rubble is twice as thick as the ashlar the ($12 + $5 + $5) ÷ 3 = $7.33, &c. **Such compound walls are weak** an apt to separate in time, as also walls of cut stone backed by concrete, or by brick from unequal settlement of the two parts.

At times the contractor must be **allowed extra** in opening new quarries; in formin short roads to his work; in digging foundations; or for pumping or otherwise draining them, whe springs are unexpectedly met with; for the centers for arches, &c; unless these items are expressl included in the contract per cub yd.

For quantity of masonry in walls of wells, see p 158.

Approximate cost of buildings per cubic foot, at Philada price In 1873; including every cub ft of space from roof to cellar floor. Plain brick dwellings, such as most of those in Philada, 12 to 15 cts. Better class, highly finished throughout, 15 to 18 cts. First class, with cut stone fronts, 20 to 30 cts. Plain brick churches, public schools, court-houses, theaters, &c, 12 to 16 cts. Ornate Gothic churches with much cut stone facing, 30 to 45 cts, exclusive of spires. Large plain brick or rubble R R shops, depots, station-houses, &c, 9 to 12 cts; or with ornamental finish and best materials, 15 to 20 cts. First class city stores, marble fronts, high stories, fire-proof, (so called,) throughout to roof; best materials and workmanship, 18 to 25 cts.

Small buildings cost more per cub ft than large ones of the same finish. Also isolated or corner buildings, cost more than those which have two party-walls.

In Philada dwellings of brick, the carpentry and lumber usually cost each about one-fourth of the entire building. **Memorial Hall** of the Centen nial Buildings cost 68 cts per cub ft, exclusive of iron dome.

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* In Philadelphia, in 1886, cellar and other walls of rough rubble, $3 to $4 per perch of 22 *cub ft of wall.* Outside walls with a facing of broken range rock-work of sandstone, (as common in Gothic churches,) $5 to $7 per 22 cub ft, including everything.

MORTAR, BRICKS, &c.

Art. 1. Mortar. The proportion of 1 measure of quicklime, either in irregular lumps, or ground,* and 5 measures of sand, is about the average used for common mortar, by good builders in our principal Atlantic cities; and if both materials are good, and well mixed (or *tempered*) with clean water, the mortar is certainly as good as can be desired for such ordinary purposes as require no addition of hydraulic cement. The bulk of the mixed mortar will usually exceed that of the dry loose sand alone about $\frac{1}{8}$ part.

Quantity required. 20 cub ft, or 16 struck bushels of sand, and 4 cub ft, or 3.2 struck bushels of quicklime, the measures slightly shaken in both cases, will make abt 22½ cub ft of mortar; sufficient to lay 1000 bricks of the ordinary average size of 8¼ by 4 by 2 ins, with the coarse mortar joints usual in interior house-walls, varying say from ⅜ to ½ inch. With such joints, 1000 such bricks make 2 cubic yards of massive work. Nearly one-third of the mass is mortar. For outside or showing joints, where a whiter and neater looking mortar is required, house-builders increase the proportion of lime to 1 in 4, or 1 in 3. For mortar of fine screened gravel, for cellar-walls of stone rubble, or coarse brickwork, 1 measure of lime to 6 or 8 of gravel, is usual; and the mortar is good. In average rough massive rubble, as in the foregoing brickwork, about one-third the mass is mortar: consequently a cubic yard will require about as much as 500 such bricks; or 10 cubic feet, (8 struck bushels) of sand; and 2 cub ft, or 1.6 bushels of quicklime. Superior, well-scabbled rubble, carefully laid, will contain but about $\frac{1}{5}$ of its bulk of mortar; or 5½ cub ft sand, and 1.1 cub ft lime, per cub yard.

For public engineering works, especially in massive ones, or where exposed to *dampness*, an addition should be made in either of the foregoing mortars, of a quantity of good **hydraulic cement,** equal to about ⅓ of the lime; or still better, ⅓ of the lime should be omitted, and an equal measure of cement be substituted for it. **If exposed to water while quite new,** use little or no lime outside.

With bricks of 8¼ by 4 by 2 ins, the following are the **quantities of mortar and of bricks for a cubic yard of massive work.**

Thickness of Joints.	Proportion of Mortar in the whole mass.	No. of Bricks per cub yard.	No. of Bricks per cub foot.
$\frac{1}{8}$ inch	about $\frac{1}{9}$	638	23.63
$\frac{1}{4}$ "	" $\frac{1}{4}$	574	21.26
$\frac{3}{8}$ "	" $\frac{3}{10}$	522	19.33
$\frac{1}{2}$ "	" $\frac{1}{3}$	475	17.60
$\frac{5}{8}$ "	" $\frac{4}{10}$	433	16.04

In estimating for bricks in massive work, **allow 2 or 3 per ct for waste;** and in common buildings, 5 per ct, or more. Much of the waste is incurred in cutting bricks to fit angles, &c. In Philadelphia a barrel of lump lime is allowed for 1000 bricks; or for 2 perches (25 cub ft each) of rough cellar-wall rubble. Somewhat less mortar per 1000 is contained in thin walls, than in massive engineering structures; because the former have proportionally more outside face, which does not require to be covered with mortar; but thin walls involve more waste while building; so that both require about the same quantity of materials to be provided. Careful experiments show that mortar becomes harder, and more adhesive to brick or stone, if the proportion of lime is increased. Hence, on our public works the proportion of one measure of quicklime to 3 of sand, is usually specified, but probably never used.

Lime is usually sold in lump, by the barrel,* of about 230 lbs net, or 250 lbs gross. **A heaped bushel** of lump lime averages about 75 lbs. **Ground quicklime,** loose, averages about 70 lbs per struck bushel; and 3 bushels loose just fill a common flour barrel; but from 3.5 to 3.75 bushels, or 245 to 280 lbs can readily be compacted into a barrel.

General remarks on mortar and lime. On too great a proportion of our public works, the common lime mortar may be seen to be rotten and useless, where it has been exposed to moisture; which will be carried by the capillary action of earth to several feet above the natural surface: or as far below the artificial surface of embankments deposited behind abutments, retaining-walls, &c. The same will frequently be seen in the soffits of arches under embankments. *Common lime mortar, thus exposed to constant moisture, will never harden properly.* Even when very old and hard, it absorbs water freely. *Cement also does so, but hardens.*

Brickdust, or burnt clay, improves common mortar; and makes it hydraulic. In localities where sand cannot be obtained, burnt clay, ground, may be substituted; and will generally give a better mortar.

Protection of quicklime from moisture, even that of the air, is absolutely essential, otherwise it undergoes the process of **air-slacking,** or

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* **Price of quicklime** in lump in Philadelphia, about 25 cts per bushel. Bar **sand** $1.40 per cubic yard. J. Rex Allen, 1319 Washington Ave.

spontaneous slacking, by which it becomes reduced to powder as when slacked by water as usual but without heating, and with but little swelling. As this air slacking requires from a few months t a year or more, depending on quality and exposure, it gives the lime time to absorb sufficient carboni acid from the air to injure or destroy its efficacy. But **quicklime will keep good for a long time** if first ground, and then well packed in air-tigh barrels. The grinding also breaks down refractory particles found in all limes, and which injure th mortar by not slacking until it has been made and used. For the same reason it is better that lim should not be made into mortar as soon as it is slacked, but be allowed to remain slacked for a day o two (or even several) protected from rain, sun, and dust.

Lime slacked in great bulk may char or even set fire to wood.

Lime paste and mortar will keep for years, and improve, if wel buried in the earth. Also for months if merely covered in heaps under shelter, with a thick layer o sand. The paste shrinks and cracks in drying; but the sand in mortar prevents this.

As approximate averages varying much according to the character and degree of burning of the limestone; and to the fineness or coarseness of the sand, one measure o good quicklime, either in lump, or ground; if wet with about ¼ a measure of water, will within less than an hour, slack to about 2 measures of dry powder. And if to this powder there be added about ¾ more measures of water, and 3 measures of dry sand, and the whole thoroughly mixed, the resul will be about 3⅓ measures of mortar. Or the same slacked dry powder, with about 1 measure of water, and 5 measures of sand, will make about 5⅔ measures of mortar. In both cases the bulk of the mortar will be about ⅙ part greater than that of the dry sand alone. If ¾ of a measure of water be used for slacking, the result, instead of a dry powder, will be about 1⅙ measures of stiff paste; or with 1 whole measure of water for slacking, the result will be about 1¼ measures of thin paste, of about the proper consistence for mixing with the sand. Very pure, *fat* limes, slack quickly, and make about from 2 to 3 measures of powder; while poor, *meagre* ones, require more time, and swell less. Slow slacking, and small swelling, in case the lime has been properly burnt, are not in general bad properties; but on the contrary, usually indicate that it is to some extent hydraulic. In this case it makes a better mortar; especially for works exposed to moisture, or to the weather. Very pure limes are the worst of all for such exposures; or are bad *weather-limes;* and in important works, should never be used without cement.

Shell lime appears to be about the same as that from the purest limestones; but that from chalk is still more inferior, and will not bear more than about 1½ measures of sand; its mortar never becomes very hard. Madrepores (commonly called coral) appear to furnish a lime intermediate between those of chalk and limestone. They require to be but moderately burnt.

The average weight of common hardened mortar is about 105 to 115 lbs per cub ft.

Grout is merely common mortar made so thin as to flow almost like cream. It is intended to fill interstices left in the mortar-joints of rough masonry; but unless it contains a large amount of cement, it is probably entirely worthless; since the great quantity of water injures the properties of lime; and moreover, its ingredients separate from each other; the sand settling below the lime. Besides this, it will never harden thoroughly in the interior of thick masses of masonry; indeed, the same may probably be said of any common lime mortar. In such positions, it has been found to be perfectly soft, after the lapse of many years.

Both the sand and the water for lime mortar, should be **free from clay and salt.** The clay may be removed by thorough washing; but it is extremely difficult to get rid of the salt from seashore sand, even by repeated washings. Enough will generally remain to keep the work damp, and to produce efflorescences of nitre on the surface; whether with lime, or with cement mortar. Slacking by salt water gives less paste than fresh.

Mortar should not be mixed upon the surface of clayey ground; but a rough board, brick, or stone platform should be interposed. Pit sand sifted from decomposed gneiss, and other allied rocks, is excellent for mortar; its sharp angles making with the lime a more coherent mass than the rounded grains of river or sea sand. Mortar should be applied wetter in hot than in cold weather; especially in brickwork; otherwise the water is too much absorbed by the masonry, and the mortar is thereby injured.

The tenacity, or cohesive strength, that is, the resistance to a pull of good common lime mortar of the usual proportions of lime and sand, and 6 months old, is about from 15 to 30 lbs per sq inch; or .96 to 1.9 tons per sq ft. With less sand, or with greater age, it will be stronger.

The crushing strength of good common mortar 6 months old is from 150 to 300 lbs per sq inch, or 9.7 to 19.3 tons per sq foot.

The sliding resistance, or that which common mortar opposes to any force tending to make one course of masonry slide upon another, is stated by Roudelet, to be but 5 lbs per sq inch; or about one third of a ton per sq ft, in mortar 6 months old.

Transverse strength of good common mortar 6 months old. A bar 1 inch square and 12 ins clear span, breaks with a center load of 4 to 8 lbs.

The lime in mortar decays wood rapidly, especially in close, damp situations. Still the soaking of timber for a week or two in a solution of quicklime in water appears to act as a preservative. **Iron,** so completely embedded in mortar as to exclude air and moisture, has been found perfect after 1400 years; but if the mortar admits moisture the iron decays. So, probably, with other metals.

The adhesion to common bricks, or to rough rubble at any age will average about ¾ of the cohesive strength at the same age; or say 12 to 24 lbs per sq inch, or .75 to 1.5 ton per sq ft at 6 months old. If care be taken to exclude dust entirely, by dipping each brick into water before laying it, or by sprinkling the stone by a hose, &c, the adhesion will be increased. On the other hand, much dust may almost prevent any adhesion at all. The precaution of wetting is especially necessary in very hot weather, to prevent the warm bricks or stone from **killing the mortar** by the rapid absorption and evaporation of its water. **The adhesion to very smooth hard pressed bricks,** or to smoothly dressed or sawed stone is considerably less.

Art. 2. Bricks, size, wt, &c.* A common size in our eastern cities is about 8.25 × 4 × 2 ins; which is equal to 66 cub ins; or 26.2 bricks to a cub ft; or 707 bricks to a cub yard. For the number required with mortar, see table, p 669.

In ordering a large number a miuimum limit of dimeusion should be specified in order to prevent fraud. A brick ¼ inch less each way than the above, contains but 52.5 cub ius; thus requiring full 25 per cent more bricks to do the same work; in addition to 25 per ct more cost for laying, which is geuerally paid for by the 1000.

The weight of a good common brick of 8.25 × 4 × 2 ins, will average about 4.5 lbs; or 118 lbs per cub ft = 3186 lbs or 1.42 tons per cub yard; or **2.01 tons per 1000.** **A good pressed brick** of the same size will average about 5 lbs, = 131 lbs per cub ft = 3537 lbs or 1.58 tons per cub yd; or **2.23 tons per 1000.** **Immersed in water,** either of them will in a few minutes **absorb** from ½ to ¾ lb of water; the last being about $\frac{1}{7}$ of the weight of a hand moulded one; or $\frac{1}{3}$ of its own bulk. Siuce the weight of hardened mortar averages but little less than that of good common brick, we may for ordinary calculations assume the weight of such brickwork at 1.4 tons per cub yard; 1.3 tons per perch of 25 cub ft; or 116 lbs per cub ft; or for machine-moulded, at 1.56 tons per cub yd; 1.44 tons per perch; or 129 lbs per cub ft.

Allowing for the usnal waste in cutting bricks to fit corners, jambs, &c, the average **number of 8¼ × 4 × 2, required per sq foot of wall** is as follows:

Thickness of Wall.	No. of Bricks.
8¼ ins, or 1 brick	14
12¾ " or 1½ "	21
17 " or 2 "	28
21½ " or 2½ "	35
25¾ " or 3 "	42

Laying per day, a bricklayer, with a laborer to keep him supplied with materials, will in common house walls, lay on an average about 1500 bricks per day of 10 working hours. In the neater outer faces of back buildings, from 1000 to 1200; in good ordinary street fronts, 800 to 1000; or of the very fiuest lower story faces used in street fronts, from 150 to 300, dependiug on the number of angles, &c. In plain massive engineering work, he should average about 2000 per day, or 4 cub yds; aud iu large arches, about 1500, or 3 cub yds.†

Since bricks shrink about $\frac{1}{13}$ part of each dimension in drying and burning, the moulds should be about $\frac{1}{12}$ part larger every way than the burnt brick is intended to be.

Good well-burnt bricks will ring when two are struck together.

At the brick-yards about Philadelphia, a brick-moulder's work is 2333 bricks per day; or 14000 per week. He is assisted by two boys, one of whom supplies the prepared clay, moulding sand, aud water; while the other carries away the bricks as they are moulded. A fourth person arranges them in rows for drying. About ¾ of a cord, or 96 cub ft of wood, is allowed per 1000 for burning. Where coal is used, the kilns are fired up with anthracite, and the finishing is done with bituminous. One ton of coal, in all, makes 4500 bricks.

Paving with brick. In our cities this is done over a 6-inch layer of gravel, which should be free from clay, and well consolidated. With bricks of 8¼×4×2 ins, with joiuts of from ⅛ to ¼ inch wide, a sq yard requires, flatwise, as is usual in streets, 38 bricks; edgewise, 73; endwise, 149. An average workman, with a laborer to supply the bricks and gravel, will in 10 hours pave about 2000 bricks; or 53 sq yds flat, 27 edgewise, 13 endwise. When done, saud is brushed iuto the joints.

Art. 3. The crushing strength of bricks of course varies greatly. A rather soft one will crush under from 450 to 600 lbs per sq inch; or about 30 to 40 tons per sq ft: while a first-rate machine-pressed one will require about 200 to 400 tons per square foot. This last is about the crushing limit of the best sandstone; ⅔ as much as the best marbles or limestones; aud ½ as much as the best granites, or roofing slates. But masses of brickwork crush under much smaller loads than single bricks. In some English experiments, small cubical masses only 9 inches on each edge, laid in cement, crushed under 27 to 40 tons per sq ft. Otbers, with piers 9 ins square, and 2 ft 3 ins high, in cement, only two days after being built, required 44 to 62 tons per sq ft to crush them. Another, of pressed brick, in best Portland cement, is said to have withstood 202 tons per sq ft; and with common lime mortar only ¼ as much. See page 436.

It must, however, be remembered, that cracking and splitting usually commence under about one-half the crushing loads. To be safe, the load should not exceed ⅛ or $\frac{1}{10}$ of the crushing one; and so with stone. Moreover, these experiments were made upon low masses; but the strength decreases with the proportion of the height to the thickness.

The pressure at the base of a brick shot-tower in Baltimore, 246 feet high, is estimated at 6½ tons

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The Peerless Brick Co, office No. 1003 Walnut St, Philada, make superb smooth (not shining) bricks of various shapes and colors (as white, black, gray, buff, brown, red, &c) for ornamental architectural purposes. Their standard size is 8⅜ × 4⅛ × 2⅜ = 82 cub ins, or ¼ larger than the above 8¼ × 4 × 2 ins. The color extends throughout the body of the brick. With a few of theso judiciously distributed among common bricks, beautiful architectural effects may be produced, both indoors and out, at far less cost than in stone. For prices and illustrated catalogue, address as above.

The same Co will, if a sufficient order is given, furnish voussoir bricks for specified radii, but of the quality and finish of ordinary good hard brick, at from 25 to 50 per ct advance on prevailing market rates of common plain ones.

* **Prices** in Philada in 1886. **Bricks alone;** Salmon, or soft, $5 per 1000. Hard brick, $8. Back stretchers (generally used for the facings of back buildings, &c), $11. Paviug brick, $11. Pressed, (for lower stories of first-class fronts,) $22.

† **Bricklaying;** including mortar; averaging an entire dwelling. $7 per 1000. Best pressed bricks in first-class fronts, $15. "Tuck" fronts, laid with steel wire, special rates.

per sq ft; and in a brick chimney at Glasgow, Scotland, 468 feet high, at 9 tons. Professor Rankin calculates that in heavy gales this is increased to 15 tons, on the leeward side. The walls of both ar of course much thicker at bottom than at top. With walls 100 feet high, of uniform thickness, th pressure at base would be 5.4 tons per sq ft.

With our present imperfect knowledge on this subject, it cannot be considered safe to expose eve first-class pressed brickwork, *in cement*, to more than 12 or 15 tons per sq ft; or good hand-moulded to more than two-thirds as much.

Tensile strength of brick, 40 to 400 lbs per sq inch; or 2.6 to 26 tons per sq ft

The English rod of brickwork is 306 cub feet, or 11⅓ cub yards; and requires about 4500 bricks of the English standard size; with about 75 cub ft of mortar. The English *hundred of lime*, is a cub yd.

Frozen mortar. There is risk in using common mortar in cold weather. If the cold should continue long enough to allow the frozen mortar to set well, the work may remain safe, but if a warm day should occur between the freezing and the setting of the mortar, the sun shining on one side of the wall may melt the mortar on that side, while that on the other side may remain frozen hard. In that case, the wall will be apt to fall; or if it does not, it will at least always be weak; for mortar that has partially set while frozen, if then melted, will never regain its strength. By the writer's own trials hydraulic cements seemed not to be injured by freezing.

Experiments for rendering brick masonry impervious to water. Abstract of a paper read before the American Society of Civil Engineers, May 4, 1870, by William L. Dearborn, Civil Engineer, member of the Society.

The face walls of the Back Bays of the Gate-houses of the new Croton reservoir, located north of Eighty-sixth Street, in Central Park, were built of the best quality of hard-burnt brick; laid in mortar composed of hydraulic cement of New York, and sand mixed in the proportion of one measure of cement to two of sand. The space between the walls is 4 ft; and was filled with concrete. The face walls were laid up with great care, and every precaution was taken to have the joints well filled and insure good work. They are 12 ins thick, and 40 ft high; and the Bays when full generally have 36 ft of water in them.

When the reservoir was first filled, and the water was let into the Gate-houses, it was found to filter through these walls to a considerable amount. As soon as this was discovered, the water was drawn out of the Bays, with the intention of attempting to remedy or prevent this infiltration. After carefully considering several modes of accomplishing the object desired, I came to the conclusion to try "Sylvester's Process for Repelling Moisture from External Walls."

The process consists in using two washes or solutions for covering the surface of brick walls; one composed of Castile soap and water; and one of alum and water. The proportions are: three-quarters of a pound of soap to one gallon of water; and half a pound of alum to four gallons of water both substances to be perfectly dissolved in the water before being used.

The walls should be perfectly clean and dry; and the temperature of the air should not be below 50 degrees Fahrenheit, when the compositions are applied.

The first, or soap wash, should be laid on when at boiling heat, with a flat brush, taking care not to form a froth on the brickwork. This wash should remain twenty-four hours; so as to become dry and hard before the second or alum wash is applied; which should be done in the same manner as the first. The temperature of this wash when applied may be 60° or 70°; and it should also remain twenty-four hours before a second coat of the soap wash is put on; and these coats are to be repeated alternately until the walls are made impervious to water.

The alum and soap thus combined form an insoluble compound, filling the pores of the masonry, and entirely preventing the water from penetrating the walls.

Before applying these compositions to the walls of the Bays, some experiments were made to test the absorption of water by bricks under pressure after being covered with these washes, in order to determine how many coats the wall would require to render them impervious to water.

To do this, a strong wooden box was made, put together with screws, large enough to hold 2 bricks; and on the top was inserted an inch pipe forty feet long.

In this box were placed two bricks after being made perfectly dry, and then covered with a coat of each of the washes, as before directed, and weighed. They were then subjected to the pressure of a column of water 40 feet high; and, after remaining a sufficient length of time, they were taken out and weighed again, to ascertain the amount of water they had absorbed.

The bricks were then dried, and again coated with the washes and weighed, and subjected to pressure as before; and this operation was repeated until the bricks were found not to absorb any water. Four coatings rendered the bricks impenetrable under the pressure of 40 ft head.

The mean weight of the bricks (dry) before being coated, was 3⅛ lbs; the mean absorption was one-half pound of water. An hydrometer was used in testing the solutions.

As this experiment was made in the fall and winter, (1863,) after the temporary roofs were put on to the Gate-house, artificial heat had to be resorted to, to dry the walls and keep the air at a proper temperature. The cost was 10.06 cts per sq ft. As soon as the last coat had become hard, the water was let into the Bays, and the walls were found to be perfectly impervious to water, and they still remain so in 1870, after about 6½ years.

Brick arch (footway of High Bridge). The brick arch of the footway of High Bridge is the arc of a circle 29 ft 6 in radius; and is 12 in thick; the width on top is 17 ft; and the length covered was 1381 ft.

The first two courses of the brick of the arch are composed of the best hard-burnt brick, laid edgewise in mortar composed of one part, by measure, of hydraulic cement of New York, and two parts of sand. The top of these bricks, and the inside of the granite coping against which the two top courses of brick rest was, when they were perfectly dry, covered with a coat of asphalt one-half an inch thick, laid on when the asphalt was heated to a temperature of from 360° to 518° Fahrenheit.

On top of this was laid a course of brick flatwise, dipped in asphalt, and laid when the asphalt was hot; and the joints were run full of hot asphalt.

On top of this a course of pressed brick was laid flatwise in hydraulic cement mortar, forming the paving and floor of the bridge. This asphalt was the Trinidad variety; and was mixed with 10 per cent, by measure, of coal tar; and 25 per cent of sand. A few experiments for testing the strength of this asphalt, when used to cement bricks together, were made, and two of them are given below.

Six bricks, pressed together flatwise, with asphalt joints, were, after lying six months, broken. The distance between the supports was 12 ins; breaking weight, 900 lbs; area of single joint, 28½ sq ins. The asphalt adhered so strongly to the brick as to tear away the surface in many places.

Two bricks pressed together end to end, cemented with asphalt, were, after lying 6 months, broken. The distance between the supports was 10 ins; area of joint, 8½ sq ins; breaking weight, 150 lbs.

The area of the bridge covered with asphalted brick, was 23065 sq ft. There was used 94200 lbs of asphalt, 33 barrels of coal tar, 10 cub yds of sand, 93800 bricks.

The time occupied was 109 days of masons, and 148 days of laborers. Two masons and two laborers will melt and spread, of the first coat, 1650 sq ft per day. The total cost of this coat was 5.25 cents per sq ft, exclusive of duty on asphalt. There were three grooves, 2 ins wide by 4 ins deep, made entirely across the brick arch, and immediately under the first coat of asphalt, dividing the arch into four equal parts. These grooves were filled with elastic paint cement.

This arrangement was intended to guard against the evil effects of the contraction of the arch in winter; as it was expected to yield slightly at these points, and at no other point; and then the elastic cement would prevent any leakage there.

The entire experiment has proved a very successful one, and the arch has remained perfectly tight.

In proposing the above plan for working the asphalt with the brickwork, the object was to avoid depending on a large continued surface of asphalt, as is usual in covering arches, which very frequently cracks from the greater contraction of the asphalt than that of the masonry with which it is in contact; the extent of the asphalt on this work being only about one-quarter of an inch to each brick. This is deemed to be an essential element in the success of the impervious covering."

A cheap and effective process for preventing the percolation of water through the arches of aqueducts, and even of bridges, is a great desideratum. Many expensive trials with resinous compounds have proved failures. Hydraulic cement appears to merely diminish the evil. Much of the trouble is probably due to cracks produced by changes of temperature.

The **white efflorescence** so common on walls, especially on those of brick, is due to the presence of soluble salts in the bricks and mortar. These are dissolved, and carried to the face of the wall, by rain and other moisture. Sulphate of magnesia (Epsom Salt) appears to be the most frequent cause of the disfiguration. In many places mortar lime is made from dolomite, or *magnesian* limestone, which often contains 30 per cent or more of magnesia; which also occurs frequently in brick clay. Coal generally contains sulphur, most frequently in combination with iron, forming the well-known "iron pyrites". The combustion of the coal, as in burning the limestone or clay, in manufactures, in cooking etc, converts the sulphur into sulphurous acid gas, which, when in contact with magnesia and air, as in the lime or brick kiln, or in the finished wall or chimney, becomes sulphuric acid and unites with the magnesia, forming the soluble sulphate. We are not aware of any remedy that will prevent its appearance under such circumstances; but the formation of the sulphate may be prevented by the use of limestone and brick-clay free from magnesia. See also p 678.

Art. 4. Hydraulic cements.* Certain limestones, when burnt, will not slack with water; but when the burnt stone is finely ground, and made into a paste, it possesses the property of hardening under water; and is therefore called *hydraulic cement*. So long as the proportion of those ingredients which impart hydraulicity, is so small that the burnt stone will slack; but still make a paste or a mortar, which will harden under water, it is called *hydraulic lime*. This does not harden so promptly, or to so great a degree, as the cements. Hydraulic limes slack more slowly, and swell less, in proportion to their hydraulicity; some requiring many hours. Artificial hydraulic limes and cements, of excellent quality, may be made by mixing lime and clay thoroughly together; then moulding the mixture into blocks like bricks; which are first dried, then burnt, and finely ground. The celebrated artificial English Portland cement, is made by grinding together in water chalk and clay. The fine particles are floated away to other vessels, and allowed to settle as a paste; which is then collected, moulded, dried, burnt, and ground. **Natural Portland** is that made from limestone, or other material of very rare occurrence, which combines *naturally* that proportion of lime and clay which gives the above artificial Portlands their pre-eminence. This alone constitutes its difference from our common natural hyd cements.

The weight of the Rosendale, and other American cements, together with some foreign ones, will be found on page 382. **Saylor's Portland** weighs about 120 lbs per struck bushel.

The writer found by 10 years' trial that if, after setting, dampness is absolutely excluded, **Cements preserve iron,** lead, zinc, copper, and brass; and that **Plaster of Paris** preserves all except iron, which it rusts somewhat unless galvanized. **Lime-mortar** probably preserves all of them, if kept free from damp.

Protection from moisture, even that of the air, is very essential for the preservation of cements, as well as of quicklime. On this account the barrels are generally lined with stout paper. With this precaution, aided by keeping the barrels stored in a dry place, raised above the ground, the cement, although it may require more time to set, will not otherwise very

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* **Prices of hyd cements** in Philada, 1886, by the large importing firm of Samuel H. French & Co, corner of York Avenue and Callowhill St. **English Portland** cement, $2.60 to $3.25 per barrel of about 400 lbs gross; according to quality and quantity. **German Portland,** $2.40 to $3.00 per barrel of about 400 lbs gross. **Saylor's American Portland,** $2.35 to $2.65 per barrel of 400 lbs gross. **Rosendale,** per barrel of about 300 lbs net, $1.15 to $1.40. **Other U. S. cements,** per barrel of about 300 lbs net, $1.00 to $1.30. **Ground calcined plaster of Paris: selected,** barrel of 300 lbs net, $2.00 to $2.25. **Commercial,** barrel of about 200 lbs net, $1.20 to $1.80.

The **American Improved Cements Co, Egypt Pa,** office 216 S. 3d St, Philadelphia, make

"Giant" Portland. Price per barrel of 400 lbs net, $2.40 in 1886
"Improved Union" " " " 300 " $1.40 "

appreciably deteriorate for six months; but after 14 or 16 months, Gillmore says it is unfit for use in important works. But in lumps, kept dry, it will remain good for 2 or 3 years; and may be groun as required for use.

Good Portland cement is stated by good authority rather to improve by free exposure to the ai under cover; but whether this is correct or not, we cannot say.

Restoration by reburning may be effected. If the injured ground ce ment is spread in a thin layer, on a red-hot iron plate, for about 15 minutes, its good qualities will b in a great measure restored. The time should be ascertained by trial. If it has been actually we and lumpy, or cemented into a mass, it should first be broken into small pieces, and then ground. O these pieces may be first kiln-burnt at a bright red-heat for about 1½ hours; and then ground.

Art. 5. For roughcasting, or stuccoing the outside of walls, very few hydraulic cements are fit. Mr. Downing, in his work on "Country Houses," excepts that from Berlin Connecticut, as the only one within his extended knowledge, that is suitable. Portland cement i said to be good for that purpose. A wall with a northern exposure in Philada was coated with it i 1860; and appears to be in perfect condition in 1880.

Quantity required. A barrel of cement, 300 lbs; and 2 barrels of sand, (bushels, or 7½ cub ft;) mixed with about ½ a barrel of water, will make about 8 cub ft of mortar sufficient for

192 sq ft of mortar-joint	½	inch thick	=	21⅓	sq yards.					
288 " " " "	⅜	"	=	32	"	"				
384 " " " "	¼	"	=	42⅔	"	"				
768 " " " "	⅛	"	=	85½	"	"				

Or, to lay 1 cubic yard, or 522 bricks of 8¼ by 4, by 2 ins, with joints ⅜ inch thick; or a cubic yar of roughly scrabbled rubble stonework. The quantity of sand may be increased, however, to 3 or measures for ordinary work.

Pointing mortar. Gen Gillmore recommends "1 part by *weight* of good cement powder, to 3 or 3½ parts of sand. To be mixed under shelter, and in quantities of only 2 o 3 pints at a time, using very little water, so that the mortar, when ready for use, shall appear rathe incoherent, and quite deficient in plasticity. The joints being previously scraped out to a depth o at least ½ an inch, the mortar is put in by the trowel; a straight-edge being held just below the joint if straight, as an auxiliary. The mortar is then to be well calked into the joint by a calking iron and hammer; then more mortar is put in, and calked, until the joint is full. It is then rubbe and polished under as great pressure as the mason can exert. If the joints are very fine they shoul be enlarged by a stonecutter, to about $\frac{3}{16}$ inch, to receive the pointing. The wall should be well we before the pointing is put in, and kept in such condition as neither to give water to, nor take it from th mortar. In hot weather, the pointing should be kept sheltered for some days from the sun, so as no to dry too quickly." Why not finish joints at once, without subsequent pointing? Author.

Art. 6.* Color is no indication of strength in hyd cements. **The finer they are ground the better.** At least 90 per ct should pass through a sieve of 50 meshes per lineal inch, of Wire No 35 Amer wire gauge (.0056 inch thick); or 2500 meshes per sq inch. **Weight** is a good indication when equally well ground. A flat cake of good cement paste placed in water as soon as it admits of so doing safely, and left in it for a week, should show no cracks. **New cement** is not as good as when a few weeks old. **The term Setting** does not imply that the cement has hardened to any great extent, but merely that it has ceased to be *pasty* and has become *brittle*. Quick setting cements may do this sufficiently to allow small experimental samples to be lifted and handled carefully within five to thirty minutes; while others may require from one to eight or more hours. **Slow setting does not indicate inferiority,** for many of the very best are the slowest setting. A layer of very quick setting cement may partially set, especially in warm weather, before the masonry is properly lowered and adjusted upon it, and **any disturbance after setting** has commenced is prejudicial. Such are to be regarded with suspicion, and submitted to longer tests than slow ones. Still, quick setting ones are best in certain cases, as when exposed to running water, &c. They may be rendered slower by adding a bulk of lime paste equal to 5 or 15 per ct of the cement paste, without weakening them seriously. As a general rule cements **set and harden better in water than in air,** especially in warm weather. If, however, the temp for the first few days does not exceed 55° to 65° Fah, there seems to be no appreciable difference in this respect; but in warm air cement *dries* instead of *setting*, and thus loses most of its strength. In hot weather every precaution should be used against this. **The time reqd to attain the greatest hardness** is many years, but after about a year the increase is usually very small and slow, especially with neat cement. Moreover, any subsequent increase is a matter of little importance, because generally by that time, and often much sooner, the work is completed and exposed to its maximum strains. **Sand retards setting,** and weakens the cement paste. But although with sand the strength of the mortar may never attain to that of the neat paste, yet it increases with age in a greater proportion; so that a neat paste which at the end of a year would be but twice as strong as in 7 days, may with sand yield a mortar which at the end of a year will be 3, 4, or 5 times as strong as *it* was in 7 days. Good Portlands neat usually have at the end of a year from 1.5 to 2 times their strength at the end of 7 days; and the American natural cements, Rosendale, Louisville, Cumberland, &c, from 2.5 to 3.5 times; but inasmuch as Portlands average (roughly speaking) about 5 or 6 times the strength of the others in 7 days, they still average about 2.5 to 3 times as strong in a year or longer. Cements of the same class differ much in their **rapidity of hardening.** One may at the end of a month gain nearly one-half, and another not more than one-sixth of its increase at the end of a year, at which time both may have about the same strength. Hence, tests for 1 week or 1 month are by no means conclusive as to their final comparative merits.

There seems to be a period occurring from a few weeks to several months after having been laid, at which cement and its mortars for a short time not only cease from hardening, but **actually lose strength.** They then recover, and the hardening goes on as before. It has been suggested that this opinion has originated in some oversight of the experimenters, but the writer believes it to be founded on fact. In his expts with various hyd cements of the consistence of mortar, even without sand, the writer detected **no change of bulk in setting.**

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* See "**Mr. Eliot C. Clarke**", p 678.

Art. 7.* Mr. Wm. W. Maclay, C. E. (see his very instructive paper in Trans. Am. Soc. C. E., Dec, 1877), found that in **the testing of cements** the temperature of the air and water had far more influence than had before been suspected. Thus neat Portland moulded in air at 30° Fah. and kept 6 days in water at 40°, had a tensile strength of but 156 lbs per sq inch, while that kept in water of 70° had 299 lbs, or nearly twice as much. Other bars moulded in air at 60°, after 6 days in water of 40°, broke with 113 lbs tensile per sq inch, while those in water at 70° required 254 lbs, or about 2.25 times as much. But at the end of only 20 days the strengths of these last were as 212 to 336 lbs, or as 1 to 1.6; the weaker one having in that time gained rapidly on the stronger. As longer time would of course bring them still nearer to an equality, the *ultimate* effects of temperature within certain limits are fortunately not so important in actual practice as the first expts might lead us to infer. Work must go on notwithstanding changes of temperature, but we must take care that our mortar shall at all times be *strong enough* even under their most injurious influences. Cements in open air are certainly more or less injured by *drying* instead of *setting*, as the temp exceeds about 65° to 70°. But if mixed only in small quantities at a time, and quickly laid in masonry of dampened stone, so as to be sheltered from the air, the injury is much reduced. The sand and stone should both be *damp*, not wet, in hot weather, and a *little* more water may be used in the cement paste; also if possible not only the mortar while being mixed, but the masonry also should then be shaded Mr Maclay found that 6 day specimens of neat Portland broken *direct from the water* were much stronger than if first left 24 hours to dry in the shade at tolerably high temps. But the reverse occurs with such U. S. natural cements as Rosendale, &c, the strength of some being largely *increased* by such drying. Experiments in Europe with Portlands kept 3 months in water, seemed to show the weakest period for such to be at 2 days' exposure, when the strength was but half as great as when first taken from the water. But on the 4th day they were even stronger than at first; and the strength then increased with time as if there had been no interruption.

The effects of cold, although it retards the setting, do not appear to be serious otherwise. If the cement mortar even *freezes* almost as quickly as the masonry is laid with it, it does not seem to depreciate appreciably. The writer has found this to be the case also with lime mortar, even when a few hours after freezing the temp became so high as to soften the frozen mortar again. But although the *mortar* of either lime or cement may not be thereby injured, the *work*, especially in thin brick walls, may be ruined and overthrown. Thus if soon after the mortar through the entire thickness of such a wall be frozen, the sun shines on one face of it so as to soften the mortar of that face, while the mortar behind it remains hard, it is plain that the wall will be liable to settle at the heated face, and at least bend outward if it does not fall. The writer has observed that coatings of cement applied to the backs of arches on the approach of winter, and left unprotected, were entirely broken up and worthless on resuming work the next spring. **The heating of sand and cement** in freezing weather seems to be a bad practice, especially if to be placed in cold water. But for use out of water Mr Maclay says they may be heated to 50° or 60°. **Cold water for mixing** is probably no farther injurious than that it retards the setting. All cements when mixed with sand to a proper consistence for mortar will **fall to pieces** if placed in water before setting has commenced. Portlands do so even without sand; but U. S. natural cements of good quality do not.

Art. 8.* Strength of cements. The strength as before stated is much affected by the temp of the air and water, as also by the degree of force with which the cement is pressed into the moulds; by the extent of setting before being put into the water, and of drying when taken out; and still more by the consideration of whether or not it sets while under the influence of pressure, which increases the strength materially. On this account cements in actual masonry may under ordinary circumstances give better results than in-door expts. These causes, together with the degree of thoroughness of the **mixing or gauging,** the proportion of water used, and other considerations may easily affect the results 100 per ct, or even much more. Hence, the discrepancies in the reports of different experimenters.

Rem.* Portlands require more water than the common U. S. cements, and shrink less in mixing. See next Art. Also, mortars require more than concrete, especially when the last is to be **well rammed,** in which case it should be merely *moist*, so as barely to cohere when pressed into a ball by hand. If more water is present, the consolidation by ramming is proportionally imperfect. **To assure himself that the quality** of cement furnished is equal to that contracted for, the engineer should reserve the right to bore with a long auger into any part of each barrel, and to reject every barrel of which the sample drawn out does not satisfy the stipulations. On works using large quantities, there should be one person specially detailed to this duty. One advantage of very strong cements is their economy, even at a higher cost, in allowing the use of a larger proportion of the cheaper ingredients, sand, gravel, and broken stone. Almost any common U. S. cement, *if of good quality*, will with 1.5 or 2 measures of sand give a mortar strong enough for most engineering purposes; but a good Portland will give one equally strong with 3 or 4 meas of sand; and will, therefore, be equally cheap at twice the price; beside requiring the handling, storing, and testing of only half the number of barrels.

After what has been said it is plain that great latitude must be allowed in attempting to prepare a table of approximate average strengths. The writer can pretend to nothing more than the following, which is deduced from reliable reports, aided by a few experiments of his own on transverse strengths, a summary of which last forms the last column.

If one measure of cement slightly shaken be mixed to a paste with about .35 meas of water if a common U. S. cement, or about .40 meas if Portland, in the shade, and in a temp of from 60° to 90°, this paste will occupy about .7 meas if common, and about .86 if Portland, when well pressed into wooden moulds by the fingers (protected from corrosion by gloves of rubber or buckskin). If then allowed from 30 minutes to some hours (according to its setting properties) to set; then removed from the moulds, and at the end of 24 hours total, placed in water of the above limits of temp for 7 days, and broken at once when taken from the water, the samples will generally exhibit about the following strengths. Those for compression are supposed to be cubes; and those for transverse strength in the table were beams 1 inch square, and 12 ins clear span, loaded at the center.

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* See "**Mr. Eliot C. Clarke**", p 678.

Table A. Average Ultimate Strengths of neat Cements after 6 days in water, and broken directly from the water.

	Tensile. lbs per sq in.	Compres. lbs per sq in.	Compres tons per sq ft.	Transv. 1" > 1" × 12". lbs
Portlands, artificial, either foreign, or the "National" of Kingston, N.Y.	200 to 350	1400 to 2400	90 to 154	25 to 45
" Saylor's natural, Coplay, Penn..	170 to 370	1100 to 1700	71 to 109	26
U. S. common hydraulic cements.....	40 to 70	250 to 450	16 to 29	3 to 7

All below the lowest of these should be rejected; the average of the table may be considered fair and all above the highest superior. **After only 24 hours in water** the strength of the common U. S. cements averages about half that for 6 days, but with considerable variations both ways. In like manner **at the end of a year** neat Portlands average from 1.5 to 2 times as strong as in 6 days; and our common cements from 2.5 to 3.5 times. **The London board of works** require that Portlands after 7 days in water shall have at least 35 lbs transverse, and 350 lbs tensile strength. Some have reqd 500 or more tensile to break them. **For Portlands the writer found** the transverse strengths of several well known English brands moulded as before described, to be 26 to 40 lbs after 7 days in water; National Portland of Kingston, N. Y., 40 and 46; Saylor's Portland (only 2 trials) 26 lbs. **Toepffer, Grawitz, & Co,** of Stettin, Germany, warrant all their Portland (known as the "Steru" brand) equal to 569 lbs tensile after 7 days in water. Some of it has borne 760 lbs.

Mr. J. Herbert Shedd, as Engineer of the Water Works and Sewers of Providence, R. I., rejected all Rosendale which when mixed to a stiff paste, and allowed 30 min in air to set, and then put into water for only 24 hours, broke with less tension than 70 lbs. At first he found some that broke with 10 to 15 lbs; some that would not set at all in water; and but little that bore 30 lbs. Now samples frequently bear 100 lbs or more; but that usually sold still rarely exceeds 40 to 50, and frequently scarce half as much. **The Sewer Department of St. Louis,** Missouri, requires all Louisville, Kentucky, cement to bear at least 40 lbs tensile after 24 hours in water. Some of it now shows as high as 100 or more; and 60 or 70 would have been adopted as the minimum, but for the fear that it would have encouraged the making of *too quick setting* cement. Most of that sold will probably not exceed 30 lbs.

Art. 9. Cement mortar is cement mixed with water and sand only. The writer found that for making cement *pastes* of about equal consistency and fit for mortar by themselves, the English Portlands, slightly shaken in the measure, required an average of about .4 of their own bulk of water; and the U. S. common cements about .35. The Portland pastes when thoroughly mixed and slightly pressed by hand into a box shrank about one-eighth of their bulk as dry shaken cement; and the others about one-fourth; or in other words the common U. S. cements shrink about twice as much as the Portlands; and these are about the proper proportions to assume in estimating the quantity of cement for *theoretically* filling the voids in sand.

But when sand is added, more water is reqd. It is impossible to lay down rules for all cases, but as a very rough average, mortar will require an addition equal to about .2 of the bulk of dry sand; varying of course with the weather, &c. Trial on the work in hand is better than rules.

Any addition of sand weakens cement, especially as regards tension; as it does also lime mortar. But economy requires its use. Sand also retards the setting, so that cement which by itself would set in half an hour, may not do so for some days if mixed with a large proportion of sand. This weakening effect will of course vary with different cements, and with many circumstances inferable from Art. 7, &c. **As a rough average** the following is perhaps not far from the truth as regards either tensile or transverse strength when not rammed. See p 682.

Sand.	0	½	1	1½	2	3	4	5	6	7	8
Strength.	1	⅔	½	.4	⅓	.3	¼	⅕	⅙	⅐	⅛

Tensile Strength of Cement Mortars,*

of medium coarse sea-beach sand, and good Rosendale, and English Portland cements; being averages of about 25000 experiments in the years 1878 to 1882. The area of breaking section was 2.25 sq ins. The proportions of sand and cement were by measure. The mortar was rammed into the moulds, and the specimens were immersed in water as soon as they would bear handling, and so remained for 1 day or 1 week, or for 1, 6, or 12 months. The strengths are in **lbs per sq inch.**

* From experiments by Mr. Eliot C. Clarke, C. E; of Boston. See also p 678.

Rosendale.

Neat.					Cement 1. Sand 1.				Cement 1. Sand 1.5.			
1 D.	1 W.	1 M.	6 M.	1 Y.	1 W.	1 M.	6 M.	1 Y.	1 W.	1 M.	6 M.	1 Y.
71	92	145	282	290	56	116	180	236	41	90	135	210
Cement 1. Sand 2.					Cement 1. Sand 3.				Cement 1. Sand 5.			
	22	49	105	169	12	25	65	121		10	36	80

Portland.

Neat.					Cement 1. Sand 1.				Cement 1. Sand 1.5.			
102	303	412	468	494	160	225	347	387				
Cement 1. Sand 2.					Cement 1. Sand 3.				Cement 1. Sand 5.			
	126	163	279	323	95	130	198	257	55	78	116	145

The crushing strength. For each proportion of sand we may take the trength preceding it in the table, p 676. Moreover the crushing streugth with sand increases with ge much more rapidly than the tensile; and the more so the greater the proportion of sand.

As a general rule with cements of good quality we shall have mortars fit for most engineering purposes if we do not exceed from 1 to 1.5 measures of dry sand to 1 of the common cements; or from 2 o 3 of sand to 1 of Portland.

The shearing strength of neat cements averages about one-fourth of the ensile.

The adhesion of cements to bricks or rough rubble, at different ages, and whether neat or with sand, may probably be taken at an average of about three-ourths of the cohesive or tensile strength of the cement or mortar at the same age. If the bricks and tone are moist and entirely free from dust when laid, the adhesion is increased; whereas if very dry nd dusty, especially in hot weather, it may be reduced almost to 0. The adhesion to very hard, mooth bricks, or to finely dressed or sawed masonry is less.

The voids in sand of pure quartz like that found on most of our seahores, when perfectly dry and loose, occupy from .303 of the mass in sand weighing 115 lbs per cub ft, o .515 in that weighing 80 lbs. Usually, however, such dry sand weighs say from 105 lbs with voids of 364: to 95 lbs. with volds .424; the mean being 100 lbs, with voids .394.* But the wet sand in mortar ccupies about from 5 to 7 per ceut less space than when dry; the shrinkage averaging say 6 per ct or $\frac{1}{17}$ part; thus making the voids .304 of the 105 lb sand when wet; and .364 of the 95 lb; the mean of whioh is .334. But to allow for imperfect mixing, &c, it is better to assume the voids at .4 of the dry sand. Moreover, since the cements, as before stated, shrink more or less when mixed with water, and worked up into mortar, it would be as well to assume that to make suffioient paste to thoroughly fill the voids, we should not use a less volume of dry common cement, slightly shaken, than half the bulk of the dry sand; or than .45 of the bulk if Portland. **The bulk of the mixed mortar** will then be about equal to or a trifle less than that of the dry sand alone.

The best sand is that with grains of very uneven sizes, and sharp. The more uneven the sizes the smaller are the voids, and the heavier is the sand. It should be well washed if it contaius clay or mud, for these are very injurious to mortar or concrete.

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* **If greater accuracy is desired** pour into a graduated cylindrical measuring-glass 100 measures of dry sand. Pour this out, and fill the glass up to 60 measures with water. Into this **sprinkle slowly** the same 100 measures of dry sand. These will now be found to fill the glass only to say 94 measures, having shrunk say 6 per ct; while the water will reach to say 121 measures: of which 121—94 = 27 measures will be above the sand: leaving 60—27 = 33 measures filling the voids lu 94 measures of wet sand; showing the volds in the wet sand to be $\frac{3}{9}\frac{3}{4}$ = .351 of the wet mass. **If the sand is poured into the** water hastily, alr is carried in with it, the voids will not be filled, and the result will be quite different.

Since a cubic foot of pure quartz weighs 165 lbs, it follows that if we welgh a cubic foot of purc dry sand either loose or rammed, then as 165 is to the wt found, so is 1 to the solid part of the sand. And if thls solid part be subtracted from 1, the remainder will be the voids, as below.

Wt in lbs per cub ft dry	80	85	90	95	100	105	110	115
Proportiou of solid	.485	.515	.546	.576	.606	.636	.667	.697
Proportion of voids	.515	.485	.454	.424	.394	.364	.333	.303

The common (not Portland) cements, when used as mortar for brickwork, often disfigure it, especially near sea coasts, and in damp climates, by **white efflorescences** which sometimes spread over the entire exposed face of the work, and also injure the bricks. This also occurs in stone masonry, but to a much less extent, and is confined to the mortar joints, and injures only porous stone. It is usually a hydrous carbonate of soda or of potash often containing other salts. Gen'l Gillmore recommends as a preventive to add to every 300 lbs (1 barrel) of the cement powder 100 lbs of quicklime, and from 8 to 12 lbs of any cheap animal fat. The fat to be well incorporated with the quicklime before slacking it preparatory to adding it to the cement. This addition will retard the setting, and somewhat diminish the strength of the cement. It is also said by others that linseed oil at the rate of 2 gallons to 300 lbs of dry cement, either with or without lime, will in all exposures prevent efflorescence, but like the fat it greatly retards setting, and weakens. See also § 673.

Mr. Eliot C. Clarke, to whom we are already indebted for tables on pp 677 and 682, has published, in Trans Am Soc C E April 1885, the results of a series of experiments made for the Boston Main Drainage Works. From his paper we condense as follows, by permission.

Variations in **shade** *in a given kind of cement* may indicate diffs in the character of the rock or degree of burning. Thus, with Rosendale, a light color generally indicated an inferior or under-burned rock. A coarse-ground cement, light in color and weight, would be viewed with suspicion.

Finer sieves than No 50 (about 50 meshes to the lineal inch) should be used. No 120 (120 meshes) was used in the experiments.

The highest strength was obtained by the use of just **enough water** to thoroughly dampen the cement. An excess of water retards setting. American cement needed more water than Portland; fine-ground more than coarse; quick-setting more than slow. Neat Rosendale, a year old, was strongest with 35 per cent water. Neat Portland, same age, with 20 per cent.

The finer the sand, the less is the strength.

Salt, either in the water used for mixing, or in that in which the cement is laid, retards setting somewhat, but has no important effect upon the strength.

Adding **clay** gives a much more dense, plastic, water-tight paste, useful for plaster or for stopping leaks. Half a part of clay did not seem to weaken mortar materially, except in the case of sample blocks exposed to the weather for 2½ years after a week's hardening in water.

A year's saturation in fresh or salt water, and in contact with **oak, hard pine, white pine, spruce or ash,** did not affect the mortars.

With sand, **fine-ground** cements make the strongest mortar; but when tested neat, **coarse-ground** cements are strongest. This is especially the case with Portlands.

Good results were obtained from **mixing cements.** A mortar of half a part each of Rosendale and Portland, and two parts sand, was stronger, at 1 wk, 1 mo, 6 mos and 1 year, than the average of two mortars, one of 1 part Rosendale and one of 1 part Portland; each with 2 parts sand. Mixtures of Roman (quick setting) and Portland (slow) set about as quickly as Roman alone, and were much stronger.

Portland **resisted abrasion** best when mixed with 2 parts sand; Rosendale with 1 part. A little more or less sand rapidly reduced the resistance in both cases.

Cements **expand in setting;** but not more than 1 part in 1000 of any given dimension.

Art. 10. Cement concrete or Beton, is the foregoing cement mortar mixed with gravel or broken stone, brick, oyster shells, &c, or with all together. In concrete as in mortar, it is advisable on the score of strength that all the voids be filled or more than filled. Those of broken stone of tolerably uniform size and shape are about .5 of the mass; with more irregularity of size and shape they may decrease to .4. Those of gravel vary like those of sand, and had like better be taken at .5 when estimating the dry cement. We shall then have as follows.

For 1 cub yd of concrete of stone, gravel, and sand, without voids.

1 cub yd broken stone	with .5 of its bulk voids, requiring					.5 cub yd gravel.
0.5 cub yd gravel	" .5	"	"	"		.25 cub yd sand.
0.25 cub yd sand	" .5	"	"	"		.125 (or ⅛) cub yd dry cement.

It is probable that mistakes have occurred from inadvertently assuming that because the voids in a broken mass, constitute a certain proportion of the bulk of said mass; therefore, the original solid has swelled in only that same proportion. Thus, if a solid cubic yard of stone be broken into small irregular pieces, which have among themselves about the same proportions of large and small ones, as usually occurs in quarrying, or in railroad rock-cuttings; and if these be loosely thrown into a heap the .47 of this heap, or rather less than half of it, will be voids. But it does not follow, therefore, that the original solid cub yd has swelled only .47, or nearly one-half, or makes only 1½ cub yds of broken stone; although many young engineers would probably consider this a very full allowance; and would suppose that they were quite just to the company, if they counted for the contractor one solid yard of excavation for every 1½ yds of fragments. Now, it is plain that if .47 of the broken heap are voids the remaining .53 must be stone. But these .53 constituted the original solid cubic yard; and the still remain equal to it in actual solidity. Hence we must say as follows: If .53 of the broken mass occupies one cub yd of actual space, how much space will the whole mass occupy; or,

Of the broken mass.	Cub yd of space.	Entire broken mass.	Cub yds of space.
			$\frac{1 \times 1}{}$

Hence, we see that a solid cub yd of stone, when so broken, swells to 1.9, or nearly 2 cub yds; and ence a proper allowance to a contractor, would be 1 cub yd solid, for every 1.9 cub yds of pieces; or he yds of pieces must be divided by 1.9 for the solid yards.

If we know that a cubic yard of any stone, breaks to, say 1.9 yds, then to find the proportions of oids, and solid, in the broken mass, proceed thus: The solid part of the broken mass must occupy 1 ub yd of space; and the question is what part of 1.9 yds does this 1 yd constitute. The answer is $\frac{1}{1.9} = .53$; therefore, 53 hundredths of the broken mass is solid; and of course the remaining 47 hundredths are voids.

If a cubic foot solid weighs N lbs; but when broken up, or ground, only *n* lbs *per cub ft*, then *n* ivided by N, will be the proportion of solid in the broken mass.

If the broken stone is loosely piled up, it will occupy a little less space, say about 1.8 cub yds; in hich case the voids will be .44; and the solid, .56 of the mass. We will here venture to express our ouhts whether hard rock when blasted and made into embankment, settles to *less* than 1⅘ yds for very solid yd. Mr Ellwood Morris gives as the result of certain embankment of hard sandstone, ade under his supervision, an increase of bulk of $\frac{5}{12}$; or in other words, that 1 cub yd of rock in lace, made $1\frac{5}{12}$, or 1.417 yds of embankment. This corresponds to very nearly .7 solid; and .3 oids; while 1⅘ yds to 1 solid, corresponds to .6 solid; and .4 voids. The rough sides of rock excavaons, make it difficult to measure them with accuracy; and we cannot but suspect that something f this kind has interfered with the results obtained by Mr Morris. He, however, may be right, and e wrong.

By some careful experiments of our own, an ordinary pure sand from the sea shore, perfectly dry, nd loose, weighed 97 lbs per cub ft; and its voids were .41, and the solid .59 of the mass. By thorough aking, and jarring, it could be settled the .1333 part, (halfway between $\frac{1}{7}$ and $\frac{1}{8}$,) and no more. s then weighed 112 lbs per cub ft; and its voids were then .32; and the solid, .68 of the mass.

Another pure quartz sand, of much finer grain, perfectly dry and loose, weighed but 88 lbs per cub .; the voids were .466; and the solid .534 of the mass. By thorough shaking and jarring it could e reduced; like the former, only the .1333 part: it then weighed 101.6 lbs per cub ft; and its voids ere .384; and the solid .616. Another, consisting of the finest sifted grains, of the last, weighed 82 s per cub ft; so that its voids, and solid, each were very nearly .5 of the mass. This could be comacted about ⅛ part; and then weighed 98½ lbs per cub ft.

The first, or coarsest of these sands, when quite moist, but not wet, perfectly loose, weighed but 86 s per cub ft; or 11 lbs less than when dry. It could be rammed in thin layers until it settled onefth part; and then weighed 107½ lbs per cub ft. Voids .348, solid .652.

The second sand, similarly moist and loose, weighed but 69 lbs per cub ft; or 19 lbs less than when ry. It could be rammed in thin layers, until it settled ⅓ part; and then weighed 103½ lbs per cub . Voids .373, solid .627.

None of these sands when dry, and loose, if poured gently into water to a depth of 15 inches, seted more than about one-fifteenth part; the coarsest one, considerably less.

Here the .125 cub yd of dry cement constitutes one-eighth of the single mass; or one-fifteenth of ll the dry ingredients as measured separately.

For 1 cub yd of concrete of broken stone and sand without voids.

cub yd broken stone, with .5 of its bulk voids requiring | .5 cub yd sand.
cub yard sand, " .5 " " " " | .25 cub yd dry cement.

The strength of concrete is affected by the quality of the broken stone, s well as by that of the cement, the degree of ramming, &c. Cubes of either of the above with Portnd, as well as one composed of 1 meas of good Portland to 5 of sand only, well made, and rammed, hould either in air or in water require to crush them at different ages, *not less* than about as follows.

Age in months.....	**1**	**3**	**6**	**9**	**12**
Tons per sq ft.....	15	40	65	85	100

Under favorable conditions of materials, workmanship and weather, the strengths may be from 50 100 per ct greater. For **transverse strength** as beams see p 682.

If not rammed the strength will average about one-third part less.

With common U. S. cements, if of good quality from .2 to .3 of the rength of Portland concrete may be had.

Slow setting cements are best for concrete, especially when to be ammed.

It may not be amiss to state here that **when masonry is backed by oncrete** the two are liable in time to crack apart from unequal settlement, specially if the ramming has not been thorough; also that in variable climates ast **iron cylinders filled with concrete** are frequently split horizonally by unequal expansion and contraction. In such structures it is safest to consider the cyls as ieie moulds for the concrete: and to depend upon the last only for sustaining the load.

The concrete for the New York City docks consists of 1 measure f either English or Saylor's Portland, 2 of sand, 5 of broken stone (hard trap). That made of Engsh Portland, after drying a few days, and then being immersed 6 weeks, required about 30 tons per q ft to crush it. Saylor's would probably require the same. **At the Mississippi Jetties,** (see "South Pass Jetties" by Max E. Schmidt, C. E., Trans Am oc C E, Aug 1879) Saylor's Portland 1; sand 2.76; gravel 1.46; broken stone 5.

In the foundations of the **Washington Monument** at Washington, D.C., 1880) English Portland (J. B. White & Bros) 1; sand 2; gravel 3; broken stone 4; and according to Govt. Report, has a crushing strength of 155 tons per sq ft when 7.5 months old.

At Croton Dam, N. Y., (1870) Rosendale 1; sand 2; broken stone 4.5. Some t the same work, and deposited under water, had 6 meas of stone; and at the end of a year had be ome so hard that it was found necessary to drill and blast a portion that had to be removed.

Lime with cement weakens all of them, but General Q. A. Gillmore, ou best authority, repeatedly states that even in important concrete work in either the air or water, (pro vided the water does not come into contact with it until setting takes place), from .25 to even .5 of th neat cement paste of the U. S. common cements may be replaced by lime paste without serious dim nution of either strength or hydraulicity; and with decided economy. It retards the setting whic is often of great advantage, especially with quick setting cements which at times cannot on that ac count be advantageously used without some lime.

Moulded blocks of Portland concrete of even 50 tons wt can generally b handled and removed to their places in from 1 to 2 weeks.

Ramming of concrete, when properly done, consolidates the mass abou 5 or 6 per ct, rendering it less porous, and very materially stronger. The rammers are like thos used in street paving, of wood, about 4 ft long, 6 to 8 ins diam at foot with a lifting handle, and sho with iron; weight about 35 lbs. They are let fall 6 or 8 ins. The men using them, if standin on the concrete, should wear india-rubber boots to preserve their feet from corrosion by the cement

Ramming cannot be done under water, except partially, when th concrete is enclosed in bags. A rake may, however, be used gently for levelling concrete under water

Blake's Stone Crusher (Co, New Haven, Connecticut), is useful fo breaking the stone more cheaply than by hand on a large work. The two sizes best adapted to thi purpose cost about $900 and $1200; break 6 to 7 cub yds per hour; and require steam-engines of abou 8 to 10 horse power to run them properly. According to Mr. J. J. R. Croes, C. E. (see "Constructio of Croton Dam," Trans. Am. Soc. C. E., Feb, 1875), a machine will require about as follows: engine man, 1 or 2 men to break the larger stones to a size that will enter the machine, 1 driver t horse-cart, 1 man to feed the stone into the machine, 2 to keep him supplied with stone, 1 at th screen, 2 wheeling away the broken stone to the stone-heap, 1 or 2 to receive it at the heap. Say 1 or 12 men in all. **The size of the broken stone for concrete** is gen erally specified not to exceed about 2 ins on any edge; but if it is well freed from dust by screening o washing, all sizes from .5 to 4 ins on any edge may be used, care being taken that the other ingredi ents completely fill the voids.

Concrete is good for bringing up an uneven foundation t a level before starting the masonry. By this means the number of horizonta joints in the masonry is equalized, and unequal settlement is thereby preveuted.

Concrete may readily be deposited under water in the usua way of lowering it, soon after it is mixed, in a V shaped box of wood or plate-iron, with a lid th may be closed while the box descends. The lid however is often omitted. This box is so arrange that on reaching bottom a pin may be drawn out by a string reaching to the surface, thus permittin one of the sloping sides to swing open below, and allow the concrete to fall out. The box is the raised to be refilled. In large works the box may contain a cubic yard or more, and should be sus pended from a travelling crane, by which it can readily be brought over any required spot in th work. The concrete may if necessary be gently levelled by a rake soon after it leaves the box. I consistency and strength will of course be impaired by falling through the water from the box: an moreover it cannot be rammed under water without still greater injury. Still, if good it will in du time become sufficiently strong for all engineering purposes. Concrete has been safely deposited in the above manner in depths of 50 ft.

The Tremie, sometimes used for depositing concrete under water, is a box of wood or of plate iron, round or square, and open at top and bottom; and of a length suited to the depth of water. It may be about 18 ins diam. Its top, which is always kept above water, is hopper shaped, for receiving the concrete more readily. It is moved laterally and vertically by a travelling crane or other device suited to the case. Its lower end rests on the river bottom, or on the deposited concrete. In commencing operations, its lower end resting on the river bottom, it is first entirely filled with concrete, which (to prevent its being washed to pieces by falling through the water in the tremie) is lowered in a cylindrical tub, with a bottom somewhat like the box before described, which can be opened when it arrives at its proper place. After being filled it is kept so by throwing fresh concrete into the hopper to supply the place of that which gradually falls out below, as the tremie is lifted a little to allow it to do so. The wt of the filled tremie compacts the concrete as it is deposited. A tremie had better widen out downwards, to allow the concrete to fall out more readily. See "Gillmore on Cements."

The area upon which it is deposited must previously be surrounded by some kind of enclosure, to prevent the concrete from spreading beyond its proper limits; and to serve as a mould to give it its intended shape. This enclosure must be so strong that its sides may not be bulged outwards by the weight of the concrete. It will usually be a close crib of timber o plate-iron without a bottom: and will remain after the work is done. If of timber it may require an outer row of cells, to be filled with stone or gravel for sinking it into place. Care must be taken to prevent the escape of the concrete through open spaces under the sides of the crib or enclosure. To this end the crib may be scribed to suit the inequalities of the bottom when the latter cannot readily be levelled off. Or inside sheet piles will be better in some cases; or an outer or inner broad flap of tarpaulin may be fastened all around the lower edge of the crib, and be weighted with stone or gravel to keep it in place on the bottom. Broken stone or gravel or even earth (the last two where there is no current) heaped up ontside of a weak crib will prevent the bulging outwards of its sides by the pressure of the concrete. After the concrete has been carried up to within some feet of low water and levelled off, the masonry may be started upon it by means of a caisson (page 636); or by men in diving dresses. Or if the concrete reaches very nearly to low water, a first deep course of stone may be laid and the work thus brought at once above low water without any such aids.

The concrete should extend out from 2 to 5 feet (according to the case) beyond the base of the masonry. **All soft mud should be removed** before depositing concrete. **Bags partly filled with concrete,** and merely thrown into the water may be useful in certain cases. If the texture of the bags is slightly open, a portion of the cement will ooze out, and bind the whole into a tolerably compact mass. Such bags, by the aid of divers, may be employed for stopping leaks, underpinning, and various other purposes, that may suggest themselves. Such bags may be rammed to some extent.

Tarpaulin may be spread over deep seams in rock to prevent the loss of concrete; and in some cases, to prevent it from being washed away by springs.

There is much room for judgment in the various applications of concrete; especially under water.

Concrete has been used in very large masses; as in the foundation of a graving dock at Toulon, France: where it was deposited to a thickness of 15 feet, over an area of 400 ft by 100 ft; forming, as it were, a single artificial stone of that size. It was deposited under water; an immense mould having first been prepared for its reception, by enclosing the area with close piling, lined inside with tarpaulin. On top of this foundation were similarly built, likewise under water, the sides of the dock; inside of great boxes, or enclosures of timber, conforming to the shape. The last deposits of concrete were then faced with masonry. Walls of buildings are also frequently built of cement concrete deposited between planks as a mould; and which are moved upward as the building goes on. Flues may be made in these walls by ramming concrete around a tube, which can afterwards be lifted out; and be used for the next course above. **The dome of the Pantheon,** at Rome, 142 ft diam, and now nearly 2000 years old, is of concrete. **The R. R. viaducts Pont Napoleon,** and Pont d'Alma, at Paris, have arches of 115 and 141 feet span, of concrete.

With regard to the mixing of concrete, Gen Gillmore gives the method pursued and described by Lieut Wright. The gravel and pebbles being first separated by screening, into different sizes, "the concrete was prepared by spreading out the gravel on a platform of rough boards, in a layer from 8 to 12 ins thick; the smaller pebbles at the bottom, and the larger ones on top. The mortar was then spread over the gravel as uniformly as possible. The materials were then mixed by 4 men: 2 with shovels, and 2 with hoes; the former facing each other, and always working from the outside of the heap, to the center. They then went back to the outside, and repeated this operation, until the whole mass was turned. The men with hoes worked each in conjunction with a shoveller, and were required *to rub each shovelful well into the mortar,* as it was turned and spread, or rather scattered on the platform by a jerking motion. The heap was turned over a second time, in the same manner, but in the opposite direction; and the ingredients were thus thoroughly incorporated; the surface of every pebble being well covered with mortar. Two turnings usually sufficed, and the concrete was then carried to the foundation in which it was to be used. The success of the operation depends, however, entirely upon the proper management of the hoe and shovel; and although this may be easily learned by the laborer, yet he seldom acquires it *without the particular attention* of the overseer." It is hard work.

Or simple machinery is sometimes employed for incorporating the ingredients of concrete, when large quantities are required. A machine that has been much used successfully in Germany, consists simply of a cylinder about 13 ft long, and 4 ft diam, open at both ends; and lined on the inside, which is perfectly smooth, with sheet iron. It is inclined 6 or 8 degrees with the horizon. This cylinder is made to revolve 15 or 20 times per min, by means of a simple leather strap or band around its outside; and to which motion is given by a locomotive, which at the same time worked a heavy mill for mixing the mortar. This simple machine easily turns out from 105 to 130 cub yds of concrete in 10 hours; and when worked in connection with a mortar mill, at a trifling expense."

"When concrete is deposited in water, especially in the sea, a pulpy gelatinous fluid exudes from the cement, and rises to the surface. This causes the water to assume a milky hue; hence the term **laitance,** which French engineers apply to this substance. As it sets very imperfectly, and with some varieties of cements scarcely at all, its interposition between the layers of concrete, even in moderate quantities, will have a tendency to lessen, more or less sensibly, the continuity and strength of the mass. It is usually removed from the enclosed space by pumps. Its proportion is greatly diminished by reducing the area of concrete exposed to the water, by using *large* boxes, say from 1 to 1½ cub yds capacity, for immersing the concrete."

Weight of good concrete 130 to 160 lbs per cub ft, dry.

Cost of concrete $5 to $9 per cub yard if roughly deposited; and $9 to $15 if first made into blocks; depending on size, cement, locality, wages, &c.

M. F. Coignet's beton. The artificial stone which bears this engineer's name has for several years been used in France with perfect success, not only for dwellings, depots, large city sewers, &c, but for the piers and arches of bridges, light-houses, &c. Bridge arches of 116 ft span, and of low rise, have been built of it. It is composed of 5 measures of sand, 1 of sifted dry-slaked lime, and from ¼ to ½ measure of ground Portland cement. Or of sand 6, cement 1, lime ½; &c.

These are first well mixed together dry, and then placed in a mixing-mill: at the same time sprinkling them with .3 to .4 measure of water, so as to moisten them slightly, without wetting them. They are then thoroughly incorporated by mixing, until they form a stiff pasty mass, slightly coherent. This is then placed in a mould, in successive thin layers, each of which is well compacted by blows of a 16 lb rammer. The top of each layer may be scored or cross-cut, to make the next one unite better with it. Owing to the small proportion of water, it sets soon; and may generally be taken from the mould in from a few hours to a few days, depending on the size of the block; and left to harden. River sand is the best, inasmuch as it requires less lime and cement than pit sand, to make equally good stone.(?) The cement should be a rather slow-setting one; and both it and the lime should be screened, to exclude lumps. About 1⅓ bushels, or 1⅔ cub ft of the dry materials, make 1 cubic foot of finished stone, weighing about 140 lbs; resisting 100 to 150 tons per square foot at 3 months old. 250 to 400 in 2 years. Arches of it are made no thicker than brick ones. An arch, pier, wall, foundation, &c. may be built of it, as one stone, instead of in separate blocks. In sewers the centers may be struck within 10 to 15 hours after the arch is finished; and the water may be admitted within a week or less. The distinctive features of Coignet's beton are: the very small proportion of water; the thorough incorporation of the ingredients; and the consolidation of the separate layers by ramming. It is difficult for a person who has never seen the process, to credit the rapidity, facility, and economy with which blocks of good stone can be made by it. Its cost, as compared with *perfectly plain* dressed granite, does not *exceed* one-half; while for ornamental work it compares even far more favorably. Hence the Coignet beton, or artificial stone, is nothing more than good, well-prepared mortar, *mixed with very little water;* and well rammed into moulds, in successive layers. A mixture of 1 measure of hydraulic cement, and 3 measures of sand, similarly treated, has been successfully used in the U. S., for some years, in buildings of all kinds. Ornamental work can be furnished at ¼ the price of stone; and will answer equally well. For full information, see Gillmore's "Coignet Beton."

Transverse Strength of Concrete Beams.*

Average results of 24 beams, 10 ins square, made of good Rosendale and Englis Portland cements, pit sand and screened pebbles, few exceeding 1 inch diam. Th beams were buried for 6 months, in a pit 4 ft deep, in gravelly soil, exposed to th rain, snow, &c. A first set of beams all broke on being taken from the moulds afte 7 or 8 days, although carefully handled. To avoid this, the bottom of the pit itsel was rammed to a smooth, hard surface: immediately upon which a new set wa made by ramming the concrete into 2 inch planed plank moulds without bottoms The moulds were removed after 24 hours, and when all were done the earth wa filled in over the undisturbed beams. Very little of the soil adhered to them. Thei wt in all cases when tested was about 150 lbs per cub ft, or 520 lbs wt of 5 ft clea span of beam; one half of which, or 260 lbs, must be deducted from the cen breakg loads of the 5 ft spans below; and 124 lbs from the 2 ft 4.5 ins ones. The coefficien or **Constant c** is the cen breakg load in lbs for a beam 1 inch square, and 1 f clear span, like those in table p 493; and like them is found by the formula at to of p. 492. Its use is shown by the formula at foot of p 492.

Proportions of materials by measure.			**Center Breaking load** in lbs, including half wt of beam.		**Constant c**
Cement.	Sand.	Pebbles.	Span 2 ft 4.5 ins.	Span 5 ft.	
Rosendale 1	2	5	1782	690	3.7
" 1	3	7	all broke in handling		
Portland 1	3	7	3926	1995	9.8
" 1	4	9	3648		8.1
" 1	6	11	2822	1190	6.2

* This useful table and that on p 677 were kindly furnished us by **Eliot C. Clarke**, Esq., then Principal Ass't in charge of the Improved Sewerage Works of Boston, Mass.; for which the experiments were made. See also "**Mr. Eliot C. Clarke**", p 678.

RETAINING-WALLS.

Art. 1. We here speak only of walls sustaining *earth*; for those sustaining *water*, see pp 229 to 232, and 236. **A retaining-wall** is one for sustaining the pres of earth, sand, or other *filling* or *backing*, deposited behind it after it is built; in distinction to a **face-wall,** which is a similar structure for preventing the fall of earth which is in its undisturbed natural position, but in which a vert or inclined face has been excavated. The earth is then in so consolidated a condition as to exert little or no lateral pres, and therefore the wall may generally be thinner than a retaining one.

This, however, will depend upon the nature and position of the strata in which the face is cut. If the strata are of rock, with interposed beds of clay, earth, or sand; and if they dip or incline toward the wall, it may require to be of far greater thickness than any ordinary retaining-wall; because when the thin seams of earth become softened by infiltrating rain, they act as lubrics, like soap, or tallow, to facilitate the sliding of the rock strata; and thus bring an enormous pres against the wall. Or the rock may be set in motion by the action of frost upon the clay seams; or, as sometimes occurs, by the tremor produced by passing trains. Even if there be no rock, still if the strata of *soil* dip toward the wall, there will always be danger of a similar result; and additional precautions must be adopted, especially when the strata reach to a much greater height than the wall. **A vertical wall** has both *c o* and *d s* vert.

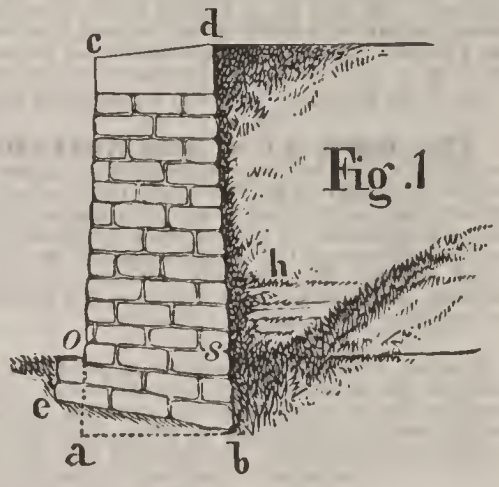


Fig. 1

Experience, rather than theory, must be our guide in the building of both kinds of wall. We recommend that the hor thickness *a b*, Fig 1, at the base of a vert or nearly vert retaining-wall *c d b a*, which sustains a backing of either sand, gravel, or earth, level with its top *c d*, as in the fig, should not be less than the following, in railroad practice, when the foundations are not more than about three feet deep.

When the backing is deposited loosely, as usual, *as when dumped from carts, cars, &c.*

Wall of cut-stone, or of first-class large ranged rubble, in mortar.....a.b.....	.35	*of its entire vert height*	d	b.	
" *good common scabbled mortar-rubble, or brick.*	.4	"	"	"	"
" *well-scabbled dry rubble.....*	.5	"	"	"	"

With good masonry, however, we may take the height *d s* instead of *d b*, and then the above proportions of *d s* will give a sufficient thickness at the ground-line *o s*. See Table, p 690.

When the backing is somewhat consolidated in hor layers, each of these thicknesses *may* be reduced, but no rule can be given for this.

The **offset** *o e*, in front of the wall, is not included in these thicknesses.

When, however, the backing is a pure clean sand, or gravel, we should use only the full dimensions; inasmuch as the tremor, caused by passing trains, would neutralize any supposed advantage from ramming materials so devoid of cohesion. Such sand may be rammed with much advantage for the purpose of compacting it in foundations; but a diff principle is involved in that case. When it is done even with cohesive earths, with a view of saving masonry in retaining-walls, it is probable that the expense will generally be found quite equal to that of the masonry saved. See Rem 4, p 691.

The base *a b* in Fig 1, is $\frac{4}{10}$ of the height *b d*. In the foregoing thicknesses at base, the back *d b* of the wall is supposed to be vert; and the face *c a* either vert, or battered (sloped or inclined backward) to an extent not exceeding about 1½ inches to a foot; which limit it is rarely advisable to exceed in practice, owing to the bad effect of rain, &c, upon the mortar when the batter is great. The base of a vert wall need not in fact be as thick as one with a battered face; but when the batter does not exceed 1.5 inches to a foot, the diff is very small. See Table, Art 7

Rem. 1. **A mixture of sand, or earth, with a large proportion** of round bowlders, paving pebbles, &c, will weigh considerably more than the materials ordinarily used for backing; and will exert a greater pres against the wall; the thickness of which should be increased, say about one-eighth to one-sixth part, when such backing has to be used.

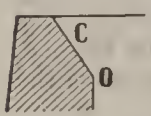
Rem. 2. The wall will be stronger if all the **courses of masonry be laid with an inclination inward,** as at *o e b*; especially if of dry masonry, or if time cannot be allowed (as it always should be, when practicable) for the mortar to set properly, before the backing is deposited behind it. The object of inclin-

ing the courses, is to place the joints more nearly at right angles to the direction *f* P, Figs 6, 7, and 8, of the pres against the back of the wall; and thus diminish the tendency of the stones to slide on one another, and cause the wall to bulge. See Art 19 of Force &c, p 314. When the courses are hor, there is nothing to prevent this sliding, except the friction of the stones, one upon the other, when of dry masonry; or friction and the mortar, when the last is used. But if, as is frequently the case, (especially in thick and hastily built walls,) this has not had time to harden properly, it will oppose but little resistance to sliding. But when the courses are inclined, they cannot *slide*, without at the same time being *lifted up* the inclined planes formed by themselves. In retaining-walls, as in the abuts of important arches, the engineer should place as little dependence as possible upon mortar; but should rely more upon the position of the joints, for stability.

An objection to this inclining of the joints in *dry* (without mortar) walls, is that rain-water, falling on the battered face, is thereby carried inward to the earth backing: which thus becomes soft, and settles. This may be in a great measure obviated by laying the outer or face-courses hor; or by using mortar for a depth of only about a foot from the face. The top of the wall should be protected by a coping c *d*, Fig 1, which had better project a few ins in front. After the masonry has been built up to the surface of the ground, the foundation pit should be filled up; and it is well to consolidate the filling by ramming, especially in front of the wall.

The back *d b* of the wall should be left rough. In brickwork it would be well to let every third or fourth course project an inch or two. This increases the friction of the earth against the back, and thus causes the resultant of the forces acting behind the wall to become more nearly vert; and to fall farther within the base, giving increased stability. It also conduces to strength not to make each course of uniform height throughout the thickness of the wall; but to have some of the stones (especially near the back) sufficiently high to reach up through two or three courses. By this means the whole masonry becomes more effectually interlocked or bonded together as one mass; and therefore less liable to bulge. Very thick walls may consist of a facing of masonry, and a backing of concrete.

Rem. 3. It is the pres itself of the earth against the back, that creates the friction, which in turn modifies the action of the pres; as the wt or pres of a body upon an inclined plane produces friction between the body and the plane, sufficient, perhaps, to prevent the body from sliding down it. A retaining-wall is *overthrown* by being made to revolve around its outer toe or edge *e*, Fig 1, as a fulcrum, or turning-point; but in order thus to revolve, its back must first plainly rise; and in doing so must rub against the backing, and thus encounter and overcome this friction. The friction exists the same, whether the wall stands firm or not; as in the case of the body on an inclined plane; the only diff is that in one case it *prevents* motion; and in the other only *retards* it.



Where deep freezing occurs the back of the wall should be sloped forwards for 3 or 4 ft below its top as at c o, which should be quite smooth so as to lessen the hold of the frost and prevent displacement.

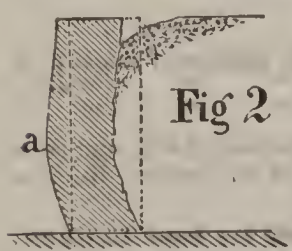


Fig 2

Rem. 4. When the wall is too thin, it will generally fail by **bulging** outward, at about ⅓ of its height above the ground, as at *a*, in Fig 2. A slight bulging in a new wall does not *necessarily* prove it to be actually unsafe. It is generally due to the newness of the mortar, and to the greater pres exerted by the fresh backing; and will often cease to increase after a few months. It need not excite apprehension if it does not exceed ¼ inch for each foot in thickness at *a*. See Remark 3, Art 7, p 691.

Art. 2. The young engineer need not in practice concern himself particularly about the PRECISE SP GRAV OF HIS BACKING, or about the ANGLE OF SLOPE at which it will stand; for the material which he deposits behind his wall one day, may be dry and incoherent, so as to slope at 1½ to 1; the next day rain may convert it into liquid mud, seeking its own level, like water; the next it may be ice, capable of sustaining a considerable load, as a vert pillar.

Moreover, he cannot foretell what may be the nature of his backing; for, as a general rule, this must consist of whatever the adjacent excavation may produce from time to time; sand to-day, rock to-morrow, &c. Retaining-walls are therefore usually built before the engineer knows the character of their backing; so that in practice, these theoretical considerations have comparatively but little weight. Theory, uncontrolled by observation and common sense, will lead to great errors in every department of engineering; but, on the other hand, no amount of experience alone will compensate for an ignorance of theory. The two must go hand-in-hand.

Again, **the settlement of the backing under its own wt,** aided by the tremors produced by heavy trains at high speed; its expansion by frost, or by the infiltration of rain; the hydrostatic pressure arising from the admission of the latter through cracks produced in the backing during long droughts; as well as its lubricating action upon it, (diminishing its friction, and giving it a tendency to slide,) &c, exert at times quite as powerful an overturning tendency as the legitimate theoretical pres does. The action of these agencies is gradual. Careful observation of retaining-walls year after year, will often show that their battered faces are becoming vertical. Then they will begin to incline outward; and eventually the wall will fail. Theory omits loads that may come on backing increasing its pres.

Assuming the theoretical views advanced by Professor Moseley to be correct as theories, the thicknesses which we have recommended in Art 1, for mortar walls, correspond to from 7 to 14 times; and for dry walls about 10 to 20 times, the pres assigned by him; and we do not consider ours greater than experience has shown to be necessary. See Table 3. Retaining-walls designed by good engineers, but in too close accordance with theory, (which assumes that a resistance equal to twice the theoretical pres is sufficient,) have failed; and the inference is fair that many of those which stand have too small a coefficient of safety.

The fact is, (or at least so it appears to us,) there must be defects in the theoretical assumptions of some of the most prominent writers who give practical rules on this subject. Thus Poncelet, who certainly is at their head, states that his tables, for practical use, give thicknesses of base for sustaining $1\frac{8}{10}$ times the theoretical pres; and this he considers amply safe. Yet, for a vert wall of cut granite, his base for sustaining dry sand level with the top, as in Fig 1, is .35 of the vert height; and for brick, .45. But the writer found that *when not subject to tremor*, a wooden model of a vert wall, weighing but 28 lbs per cub ft, and with a base of .35 of its height, balanced perfectly dry sand sloping at 1½ to 1, and weighing 89 lbs per cub ft.

NOW, THE RESISTANCE OF SIMILAR WALLS, OF THE SAME DIMENSIONS, VARIES AS THEIR SPECIFIC GRAVITIES; and, since granite weighs about 165 lbs per cub foot, or 6 times as much as our model, it follows, we conceive, that a wall of that material, with a base of .35 of its height, must have a resistance of 6 times any *true theoretical* pres, instead of only 1.8 times; and that his brick wall must have about 5 times the mere balancing resistance. Our experiments were made in an upper room of a strongly built dwelling; and we found that the tremor produced by passing vehicles in the street, by the shutting of doors, and walking about the room, sufficed to gradually produce leaning in walls of considerably more than twice the mere balancing stability while quiet: and it appears to us that the injurious effects of a heavy train would be comparatively quite as great upon an actual retaining-wall, supporting so incohesive a material as dry sand.

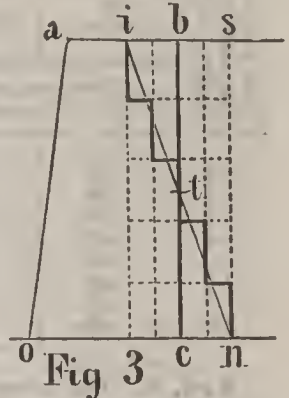


Fig 3

Since, therefore, Poncelet's wall is in this instance sufficiently stable for *practice*, it seems to us that his *theory*, which neglects the effect of tremors, &c, must be defective. He also gives $\frac{1}{5}$ of the height as a sufficiently safe thickness for a vert granite wall supporting *stiff earth;* but we suspect that very few engineers would be willing to trust to that proportion, when, as usual, the earth is dumped in from carts, or cars; especially during a rainy period. If deposited, and consolidated in layers, theory could scarcely assign any thickness for the wall; for the backing thus becomes, as it were, a mass of unburnt brick, exerting no hor thrust; and requiring nothing but protection from atmospheric influence, to insure its stability without any *retaining*-wall. It is with great diffidence, and distrust in our opinions, that we venture to express doubts respecting the assumptions of so profound an investigator and writer as Poncelet; and we do so only with the hope that the views of more competent persons than ourselves, may be thereby elicited. Our own have no better foundation than experiments with wooden and brick models, by ourselves; combined with observation of actual walls.

Art. 3. After a wall *a b c o*, Fig 3. with a vert back, has been proportioned by our rule in Art 1, it may be **converted into one with an offsetted back,** as *a i n o*. This will present greater resistance to overturning; and yet contain no more material. Thus, through the center *t* of the back, draw any line *i n*; from *n* draw *n s*, vert; divide *i s* into any *even* number of equal parts; (in the fig there are 4;) and divide *s n*, into *one more* equal parts; (in the fig there are 5.) From the points of division draw hor, and vert lines, for forming the offsets, as in the fig.

In the offsetted wall, the cen of grav is thrown farther back from the toe *o*, than in the other, thus giving it increased leverage and resistance; but within ordinary practical limits, the diff is very small; and since the triangle of supported earth is greater than when the back is vert, its pres is also greater; so that probably no appreciable advantage attends *that* consideration. **The increase of thickness near the base, diminishes, however, the leverage *v a*, Fig 8, of the pres *f* P, of the earth** against the back. The *center of pressure* of this pres is in both cases at ⅓ the vert height, measured from the bottom; and it is therefore plain that the farther back from the front it is applied, the shorter must *v a* become. Moreover, in the offsetted back, the *direction* of the pres becomes more nearly vert than when the back is upright. It is to these causes, rather than to the throwing back of the cent of gray, that the offsetted wall owes its increase of stability over one with a vert back.

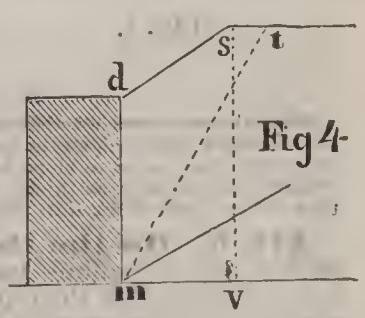


Fig 4

Art. 4. When, as in Fig 4, the backing is higher than the wall, and slopes away from its *inner* edge *d*, at the natural slope *d s*, of 1½ to 1, we are confident that the following thicknesses at base will at least be found *sufficient*

for vert walls with sand. They are deduced from the experiments just alluded to and are but rude approximations, with no scientific basis. We should not have inserted them, but for the fact that we know of no others for this case. See p 689.

The first column contains the vert height $s\,r$, of the earth, as compared with the vert height of the wall; which latter is assumed to be 1; so that the table begins with backing of the same height as the wall, as in Fig 1. These vert walls may be changed to others, with battered faces, by Art 8; or without any such proceeding their faces may be battered to any extent not exceeding 1½ inches to a foot, or 1 in 8, without sensibly affecting their stability, without increasing the base.

TABLE 1. (Original.)

Total height of the earth, compared with the height of the wall above ground.	Wall of Cut Stone, in Mortar.	Good Mortar Rubble, or Brick.	Wall of good dry Rubble.	Total height of the earth, compared with the height of the wall above ground.	Wall of Cut Stone in Mortar.	Good Mortar Rubble, or Brick.	Wall of good dry Rubble.
	Thickness at Base, in parts of the height.				Thickness at Base, in parts of the height.		
1.	.35	.40	.50	2.	.58	.63	.73
1.1	.42	.47	.57	2.5	.60	.65	.75
1.2	.46	.51	.61	3.	.62	.67	.77
1.3	.49	.54	.64	4.	.63	.68	.78
1.4	.51	.56	.66	6.	.64	.69	.79
1.5	.52	.57	.67	9.	.65	.70	.80
1.6	.54	.59	.69	14.	.66	.71	.81
1.7	.55	.60	.70	25.			
1.8	.56	.61	.71	or more	.68	.73	.83

Art. 5. But when the slope $n\,r$. Fig 5, of 1½ to 1, starts from the *outer* edge r of the wall, greater thickness is required. Poncelet gives the following for this case, for *dry sand*.

TABLE 2.

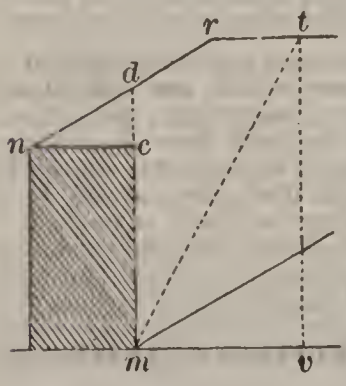


Fig. 5.

Total depth of earth, compared with height of wall.	Wall of Cut Stone in Mortar.	Wall of Brickwork.	Total depth of earth, compared with height of wall.	Wall of Cut Stone in Mortar.	Wall of Brickwork.
1	.35	.452	2.4	.762	1.02
1.1	.393	.498	3 0	.811	1.11
1.2	.439	.548	4.0	.852	1.18
1.3	.485	.604	6.0	.883	1.25
1.4	.532	.665	11.0	.909	1 28
1.5	.579	.726	21.0	.922	1.31
1.6	.617	.778	31.0	.926	1.32
1.7	.645	.824	Infinite.	.934	1.34
1.8	.668	.847			
1.9	.690	.903			
2.0	.707	.930			

When the earth reaches above the top of the wall, as in Figs 4 and 5, the wall is **surcharged**: and the earth that is *above* the top, is called the SURCHARGE. When the surcharge is carefully deposited above the wall, so as to slope back at a steeper angle than 1½ to 1, as say at 1 to 1, theory does not require the wall to be as thick. Notwithstanding Poncelet's high position, the writer cannot imagine that the base of a brick wall need be so great as 1⅓ times its height for any height of sand whatever.

Art. 6. On the theory of retaining-walls. Let $b\,c\,a\,m$, Fig 6, be such a wall, upholding backing or filling $c\,s\,m\,g$; the upper surf $c\,s$ of which is hor, and level with the top $b\,c$ of the wall; and let $m\,s$ represent the nat slope of the earth which composes the backing; $m\,g$ being hor.

Abundant experience on public works shows that this slope, whether for sand, gravel, or earth, when dry, may be practically taken at 1½ to 1; that is, 1½ hor. to 1 of vert measurement; which corresponds to an angle $s\,m\,g$ of 33° 41′ with the hor: which is also about the angle at which bricks and roughly dressed masonry begin to slide on each other. This angle, however, varies considera-

bly; being greatly influenced by the degree of dryness, or dampness, of the material; so that moderately damp sand or earth will stand at a slope of 1 to 1, or at an angle of 45°. Whatever it may be, it is called THE ANGLE OF NAT SLOPE of the material under consideration. In theoretical calculations for walls, it is safest to assume (as we have done throughout) that the backing is perfectly dry, since

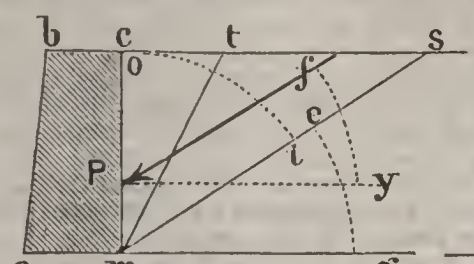


Fig. 6

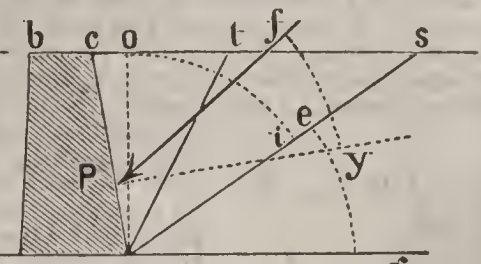


Fig. 7

ts pres is then greatest; unless it be supposed to be so wet as to possess some degree of fluidity. The triangle $c\,m\,s$ of earth above the nat slope $m\,s$, tends to slide down said slope, but is prevented from so doing by the wall.

It is assumed in all cases, that the wall is secured from *sliding* along its base, Art 9, p 692, that it is thick enough to prevent failure by *bulging*; and that it will fail only by *overturning*, by rotating around its toe, a, as a fulcrum. The thickness necessary to insure safety against the last will also be sufficient to prevent bulging. Now referring only to Fig 6 with a vert back, if the angle $o\,m\,s$, contained between the natural slope $m\,s$, and a vert line $m\,o$, drawn from the inner bottom edge m of the wall, be divided by a line $m\,t$, into two equal angles, $o\,m\,t$, $t\,m\,s$, then the angle $o\,m\,t$ is called THE ANGLE, and $m\,t$ THE SLOPE, OF MAXIMUM PRESSURE. The triangular prism of earth, of which $o\,m\,t$ is a section, or an end view, is called THE PRISM OF MAX PRES; because, if considered as a wedge acting against the back of the wall, it would produce a greater pres upon it than would the entire triangle $c\,m\,s$ of earth, considered as a single wedge. For although the last is the heaviest, yet it is more supported by the earth below it. Calculation shows that if we consider the earth $o\,m\,s$ to be thus div into wedges by *any* line $m\,t$, the wedge that will press most against the wall is that formed when $m\,t$ divides the angle $o\,m\,s$, or the arc $o\,i$, into two equal parts. But see Art 11.

Since $m\,g$ is hor, and $m\,o$ vert, the two form an angle of 90°: consequently the angle of max pres is plainly found by taking the angle $s\,m\,g$ of nat slope from 90°, and div the rem by 2. Thus a nat slope of 1½ to 1, or 33° 41′, taken from 90°, leaves 56° 18′; and $\frac{56°\ 18'}{2} = \mathbf{28°\ 9'}$, **the corresponding angle o m t of max pres.**

For case of calculation, only one foot of the *length* of the wall, and of its backing, is usually considered. The number of *cub* ft of wall, or of backing, is then equal to that of the *square* feet in their respective profiles, or cross-sections.

Now, according to Moseley, if we assume the particles of earth composing the backing to be perfectly dry, and devoid of cohesion, (or tendency to stick to each other,) which is very nearly the case in pure sand; and if we suppose the wall to be suddenly removed, then the triangle of earth $c\,m\,t$, comprised between the slope $m\,t$ of max pres, and the vert back $c\,m$ of the wall, Fig 6, would slide down, under the influence of a force which may be represented by y P, acting in a *direction* y P, at right angles to the face $c\,m$ of the triangle of earth; (or in other words, at right angles to the back of the vert wall,) its *center of force* being at P, distant ⅓ way between m and c, measured from the bottom; and its *amount* equal to either of the following:

No 1. $$\text{Perp pres } y\,P = \frac{\text{Wt of the triangle of earth } c\,m\,t \times o\,t}{\text{vert depth } o\,m};$$ or

No 2. $$\text{Perp pres } y\,P = \frac{\text{Wt of a single cub ft of the backing} \times \text{sq of } o\,t}{2.}$$

} See Art. 11, p 692.

In view of the great uncertainty involved in the matter of the actual pressure of earth against retaining-walls in practice (see Art 2.), and in order to furnish a simple rule which, although entirely unsupported by theory, is still (in the writer's opinion) sufficiently approximate for ordinary practical purposes, we shall assume that No 1 of the two foregoing formulas applies near enough to walls **with inclined backs** $c\,m$, also, as Figs 7 and 8, (precisely as they are lettered,) at least **until the back of the wall inclines forward as much as 6 ins hor, to 1 foot vert,** or at an angle $c\,m\,o$ of 26° 34′. **What follows on retaining-walls will involve this incorrect assumption, and must be regarded merely as giving safe approximation.**

Some appear to assume this perp pres to be the only one acting against the back of the wall; and hence arrive at erroneous practical conclusions. For when, in order to prevent this force from causing the triangle of earth to slide, we place a retaining-wall in front of it, then, instead of *motion*, the force will produce *pres* of the earth against the wall. But in producing pres, it

necessarily produces the new force of *friction*, between the pressed surfaces of the earth and wall. That is, if a wall were to begin to overturn around its toe a as a fulcrum, its back $c\,m$ must of course rise, and in so doing must rub against the earth filling in contact with it; and this rubbing would evidently act to *impede* the overturning. So long as the wall does not move, the same friction assists in *preventing* overturning. To ascertain the amount and effect of this friction, let y P, Fig 8, represent *by scale* the force perp to the back $c\,m$; and supposed to have been previously calculated by the foregoing formula No 1. Make the angle y P f equal to the angle of wall friction,* draw $y\,f$ at right angles to y P, or parallel to $m\,c$; make P x equal to $y\,f$, and complete the parallelogram P $y\,f\,x$. Then will x P represent by the same scale, **the amount of the friction against the back of the wall.** Since the friction acts in the direction of the back $c\,m$, (see end of Art 62, of Force, etc, p 354), it may be considered as acting at any point P, in that line: (see Art 18, of Force p 314). Hence we have acting at P, two forces; namely, the perp force y P, and the friction x P; consequently, by comp and res of force, the diag f P of the parallelogram P $y\,f\,x$, if measured by the same scale, will give us the amount of their resultant; **which is the approx single theoretical force, both in amount and in direction, which the wall has to resist,** including the wall friction.

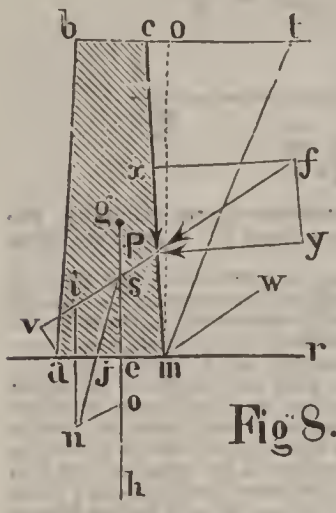


Fig 8.

But this force, f P, is also always equal to the perp force y P, mult by the nat sec of the angle y P f of the wall friction; (or divided by its nat cosine) and of course may be ascertained thus:

$$\text{Approx theoretical pres f P} = \frac{\text{wt of triangle c m t} \times \text{o t} \times \text{nat sec of angle y P f of wall friction}}{\text{vert depth o m}} = \frac{\text{wt of c m t} \times \text{o t}}{\cos \text{y P f} \times \text{o m}}$$

Or finally, if it is assumed, as we do throughout, that the earth is perfectly dry (inasmuch as its pressure is then the greatest) and that the angles of nat slope, and of wall friction are then each 33° 41′ or 1.5 to 1, then in Figs 6, 7 and 8, if the angle $c\,m\,o$ between the back $c\,m$ and the vert $o\,m$ does not exceed about 26° 34′ we may assume

$$\text{Approx theoretical pres f P} = \text{wt of triangle c m t} \times .643$$

which includes the action of the friction of the earth against the back of the wall.

Rem. 1. **When the back of the wall is offsetted or stepped,** as in Fig 3, instead of being simply battered, as in Figs 7 and 8, the direction of the pres of the earth will be the same as if the back had the batter $i\,n$, on the principle given in Art 34, Fig 17, of Force in Rigid Bodies, p 326.

Rem. 2. **Now to find both the overturning tendency of the earth,** and the resistance of the wall against being overturned around its toe a as a fulcrum, first find the cen of grav g of the wall (p 348), and through it draw a vert line $g\,h$. Prolong f P towards v and draw $a\,v$ perp to it. By any scale make $s\,o$ = wt of wall, and $s\,i$ = calculated pres f P. Complete the parallelogram $s\,i\,n\,o$, and draw its diagonal $s\,n$, which will be the resultant of the pres f P and of the wt of the wall; and should for safety be such that $a\,j$ be not *less* than about one-fifth of $a\,m$, *even with best masonry and unyielding soil.* Otherwise the great pressure so near the toe a may either fracture the wall or compress the soil near that point so that the wall will lean forward. In walls built by our rule, Art 1, or by table, p 690, $a\,j$ will be more than one-fifth of $a\,m$. The pres f P if mult by its leverage $a\,v$ will give the moment of the pres about a; and the wt of the wall mult by its leverage $e\,a$ will give that of the wall. The wall is safe from overturning in proportion as its moment exceeds that of the pres. It is assumed to be safe against *sliding*, *breaking*, or *settling* into the soil. See Art 13, p 231.

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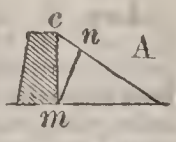
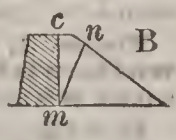
* **This angle of wall friction** is that at which a plane of masonry must be inclined to the horizontal so that dry sand or earth would slide down it. It is about the same as the nat slope, or 33° 41′, or 1.5 to 1; and its nat secant is 1.202, and its nat cos .832.



Rem. 4. If the earth slopes downward from C, as A or B, instead of being hor as in Figs 6, 7, 8, use the wt of the rth $c\ m\ n$ instead of $c\ m\ t$, $m\ n$ being the slope of max pressure. A the point of application will still be at P (at one-third of c) as in 6, 7, 8; but in B it will be a little higher as explained low for Fig 9.



Surcharged walls are those in which the earth backing tends above the tops of the walls.

According to theory, when as in Fig 9, there is a surcharge $c\ k$ of backing, sloping away from c at its natural slope $c\ v$, e max pres against the wall is tained when the earth reaches to e level of d, where the slope $m\ t\ d$ max pres intersects the face of the t slope $c\ v$; so that if afterward the rth is raised to v, or to any greater ight, no additional pres is thereby rown against the back of the wall. also if the earth slopes from b, or m between c and b, except that en the slope $m\ d$ of max pres must tend up to meet this other slope.

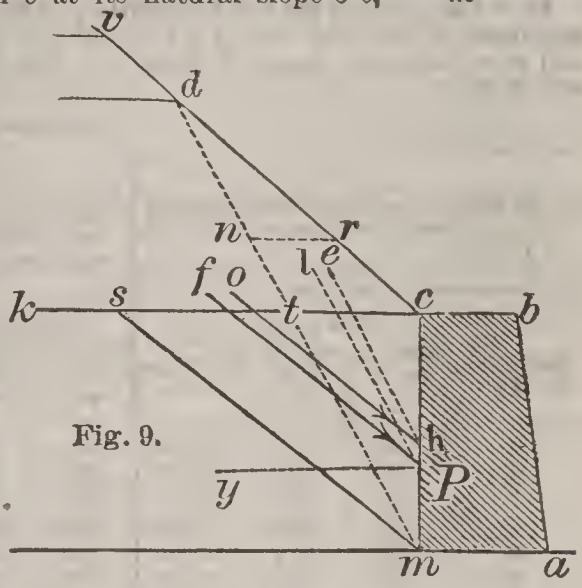


Fig. 9.

The approximate amount the oblique pres, when the wall is rcharged, (as in any of the Figs 4, 9,) may be found on the same prinple as when the earth is level with top; namely, instead of the *trian-* $c\ m\ t$ of earth, Figs 6, 7, 8, 9, find e wt of *all* the earth $d\ s\ m\ t$, Fig 4, $m\ t\ r$, Fig 5, or $c\ d\ m$, Fig 9 (if the rcharge reaches to d or v, or higher), tween the slope $m\ d$, Fig 9, $m\ t$, Figs 4 and 5, of max pres, the back of the wall, and e front slope; omitting any which, like $d\ c\ n$, Fig 5, rests on the top of the wall nd thus adds to its stability) when the slope starts in front of c. Having found is weight, then for dry backing the

$$\left.\begin{matrix}\textbf{Pressure}\\ \textbf{approximately}\end{matrix}\right\} = \textbf{Wt of the earth} \times .643,$$

cluding the action of the friction of the earth against the back of the wall; near ough (in the writer's opinion) for practical purposes in so uncertain a matter; it essentially empirical.

The direction of the pressure thus found will be the same as when the rth is level with the top $b\ c$; namely, as in Figs 6 and 7, first draw a line, as P y, rp to the back $c\ m$, whether vert or inclined. Then draw another line, as P f, aking the angle y P f = the angle of wall friction, which we all along assume to 33° 41′, or 1.5 to 1. Then P f will give the direction of the pressure. But its oint of application will not always be at P (one-third of the height of the wall ove m) as heretofore; for in all cases it will be at that point P, or at some **igher** one as h, where the back is cut by a line l P or $e\ h$, Fig 9, drawn from the n of grav of the sustained earth (omitting any that rests immediately on the top), and parallel to the slope $m\ d$ of max pres; and such a line will strike at one-ird the height of the wall only when the sustained earth $t\ c\ m$ or $d\ c\ m$ **forms a omplete triangle,** one of whose angles is at the inner top edge c of the wall. l all other cases said line for a surcharge will strike above P.

Art. 7. On page 683, Fig 1, we recommend that the base *o s* at the groun line of well built **vertical** walls should not be less than .35, or .4, or .5 of t height *d s* above said line, depending on the kind of masonry. But a wall with **battered** (inclined) front or face as found by Art 8, (by which the followi table was prepared), will be as strong, and at the same time contain less mason than a vert wall, although the battered one will have the thickest base *o s*.

Table 3, of thicknesses at base *o s*, Fig 1, and at top *c d*, walls with battered faces, so as to be as strong as vertic ones which contain more masonry.

For the cub yds of masonry above *o s* per foot run of wall, mult t **square** of the vert height *d s* by the number in the column of cub yds. Th add the foundation masonry below *o s*. See also Table, p 693. Also study Rems and 2, Art 8.

(Original.)

All the walls below have the same strength as a vert one whose base *o s*, fig 1=.35 of its ht *d s*.				All the walls below have the same strength as a vert one whose base *o s*, fig 1=.4 of its ht *d s*.			All the walls below have t same streugth as a v one whose base *o s*, fig 1 .5 of its ht *d s*.		
	Cut stone.			Mortar rubble.			Dry rubble.		
Batter, in ins to a ft.	Base, in pts of ht.	Top, in pts of ht.	C yds per ft run.	Base, in pts of ht.	Top, in pts of ht.	C yds per ft run.	Base, in pts of ht.	Top, in pts of ht.	C yds p ft run
0	.350	.350	.01296	.400	.400	.01482	.500	.500	.0185
½	.352	.310	.01226	.401	.359	.01407	.501	.459	.0177
1	.355	.270	.01158	.403	.320	.01339	.503	.420	.0170
1½	.359	.234	.01098	.408	.283	.01280	.506	.381	.0164
2	.364	.197	.01039	.413	.246	.01220	.510	.343	.0158
2½	.371	.163	.00989	.419	.210	.01165	.516	.308	.0152
3	.379	.129	.00941	.425	.175	.01111	.522	.272	.0147
3½	.389	.096	.00898	.435	.143	.01070	.528	.236	.0141
4	.400	.066	.00863	.445	.110	.01028	.537	.204	.0137
5	.425	.007	.00800	.468	.051	.00961	.555	.138	.0128
Triangle	.429	.000	.00794	.490	.000	.00907	.612	.000	.0113

Moseley and others quote Gadroy, for a DRY SAND SLOPING AT 21°. It would be better to cease fro circulating such evident mistakes. Dry sand will stand at no less angle for a savant than for an body else. For practical purposes, we may say that dry sand, gravel and earths, slope at 33° 41' 1½ to 1; as abundant experience on railroad embkts proves. Poncelet gives tables for walls to su port dry earth sloping at 1 to 1, or 45°; but as we do not believe in the existence of such earth, omit such tables. Sand, gravel, and earths may be moistened to diff degrees, so as to stand at a angle between hor and vert; and by moistening and ramming, the earths may be converted into co pact masses, exerting little or no pres; and may even so continue after they become dry; being the in fact, a kind of air-dried brick. It is sometimes difficult to know whether earth or sand is perfect dry or not; and an *exceedingly* small degree of moisture will cause them to stand at 1 to 1, in *sm heaps*, such as have probably been observed by the authorities on the subject. The writer found th fine sand from the sea-shore, and under cover, would stand at 1½ to 1 during warm dry weather, a at 1 to 1 when the air was damp. Yet no diff whatever in its degree of moisture was perceptible the feeling. Its susceptibility to dampness was of course owing to salt. A few handfuls of dry ear may perhaps be coquetted into standing at 1 to 1 on a table; but so far as our observation extend when it is dumped in large quantities from carts and wheelbarrows, its slope is about 1½ to 1; a this we consider the proper one to be used in practical calculations, where safety is the considerati of paramount importance.

The less the nat slope, the greater is the pres: and since t slope is least when the backing is perfectly dry, (omitting of course its conditi when so absolutely *wet* as to become partially fluid,) we have, on the score of safet confined our tables to dry backing. As stated in Art 1, we cannot recommend dime sions less than those there given, when we consider the rough treatment to whi masonry is exposed on public works.

In carrying a road along dangerous precipices, we shou rather be tempted at times to make thicker walls. We imagine, for instance, th the centrifugal force of a heavy train, whirling around a sharp curve, convex on t dangerous side, should not be overlooked in designing walls for such localities. Th force is hor; and is applied near the top of the wall: and, consequently, its levera may be considered as equal to the height; whereas the theoretical pres of the ear is oblique; and is applied at ⅓ of the height from the bottom; so that its levera about the toe of the wall is very short. Moreover, the simple *weight* of the train, pr duces pres against the wall; as well as that of the backing. All such consideratio are omitted by theorists. The dangerous pres caused by tremors, &c, cannot

sumed to be applied at ⅓ of the height from the bottom; nor indeed, can it be lculated at all.

Rem. 2. **Wharf walls** are an instance where the thickness should be increased, twithstanding that the pres of the water in front helps to sustain them. The earth hind such walls, is not only liable to be very heavily loaded when vessels are dis-arging; but is apt to become saturated with water, especially below low-water vel; and thus to exert a very great pres against the walls. Moreover, the water ts under the wall; and by its upward pressure virtually reduces its weight, and nsequently its stability. The same cause of course diminishes the friction of the ill upon its base. Such walls are, therefore, very liable to slide, if the foundation smooth, and horizontal; and have done so even when the foundation had a con-lerable inclination backward, as in Fig 1. See Art 9.

Rem. 3. A retaining-wall is usually in greater danger for a few months after its completion, than er time has been allowed for the mortar to harden perfectly; and for the backing to settle. When ere are suspicions of the safety of a new wall, it would be well to place strong temporary shores ainst it, at about ⅛ to ⅓ of its height above ground. In some cases, permanent buttresses of sonry may be built for the purpose. They should be well bonded into the wall.

Rem. 4. The pres of the earth backing will be much reduced, if the first few feet of its height be de up in thin hor layers, to be consolidated by being used by the masons instead of scaffolding; as own at *h*, Fig 1. Frequently this can be done without inconvenience; and at very trifling cost.

Art. 8. To change a vert retaining-wall, into one with a attered face, which shall present an equal resistance gainst overturning; although requiring less masonry. his is sometimes termed a **transformation of profile.** (Original.)

Let *a b o i*, Fig 10, be the vert wall. Mult its base ', by 1.225; (1.22475 is nearer;) the prod will be the se *o e*, of a triangular wall *b o e*, possessing the me stability; and yet not requiring much more an half the masonry of the vert one. See Rem 1. his being done, suppose a wall to be desired with a ce batter, of say 3 ins to a ft; or 1 in 4. From the oint *n*, where the face of the triangular wall inter-cts that of the vert one, step off vert any 4 short ual spaces; and from the upper one *m*, step off one ace hor, to *v*. Through *v* and *n* draw the dotted ne *s t*, which evidently will batter 1 in 4. Then is *s t o* approximately the reqd wall; but a little icker than necessary. To reduce it, from *t* draw e dotted line *t b*. Mark the point *c*, where this ne intersects the face *a i*, of the vert wall; and rough *c* draw *d l*, parallel to *s t*. Then is *b d l o* e reqd wall. Our fig is drawn in an exaggerated anner, so as to avoid confusion in the lines. The se *o e* of the triangular wall, would not in reality near so great as it is represented.

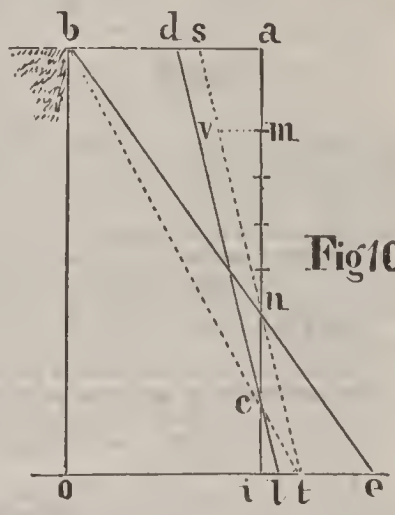


Fig 10

It will be observed that as the base increases, the quantity of masonry diminishes.

Rem. 1. **The battered wall will in fact be safer than the vert one.** The battered wall has the same moment of stability as the vert one; and the res of the earth against it also remains unchanged, but the *moment* or *tendency* of the pres to upset e wall has become less. For let *a b m n*, Fig 11, represent a vertical wall; and *f o* the *amount* and irection of pres behind it. (For ease of illustration, we have placed *o above* the true cen of pres of the arth filling, which would be at one-third of *a n* above *n*.) Now, the leverage with which this pres nds to overturn the wall around its toe *m*, is the dist *m s*, easured from the toe or fulcrum *m*, and at right angles to e direction *f o s c* of the pres; and this leverage mult by e force *f o*, gives the overturning tendency or moment of aid force. See "Moments and leverage." Again, t *a n y*, represent a triangular wall of the same stability s the other, as found by our rule. Here we still have the me amount *f o*, and direction *f o s c*, of pres force against e wall; but it now acts to overturn the wall *a n y* round the toe *y*; and therefore, with the reduced leverage *c*. Consequently, its overturning tendency is less than efore. Therefore, in ordinary language, we may say that e wall is stronger than before, although its moment of tability, or standing tendency, has in itself undergone no hange. If the pres *f o* against the vert back were hor, as a the case of water, then its leverage would evidently be ne same in both walls; and the proportion between the verturning moment of the pres, and the momeuts of tabillty of the two walls, would be constant. P 229.

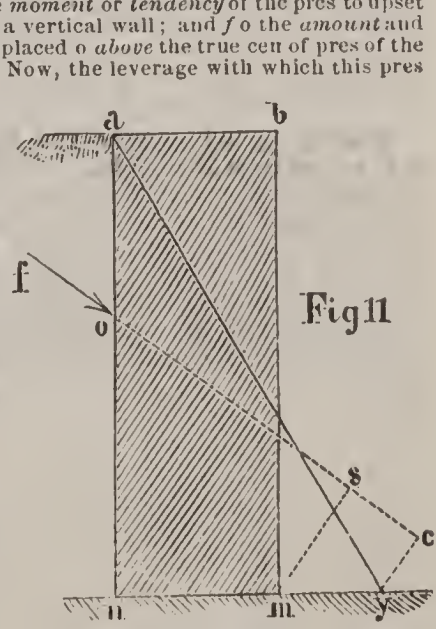


Fig 11

Rem. 2. In attempting to reduce the masonry by adopting a wall. *o b e*, Fig 10, of a triangular section: or of one early approaching a triangle, special attention should be iven to the quality of the masonry near the thin toe *e*; hich will otherwise be apt to crack, or fail under the pres.

Rem 3. Moreover, when common mortar is used without an admixture of cement, which it ne should be, in retaining-walls, where durability is an object, a great batter is obj tionable; inasmuch as the rain, combined with frost, &c, soon destroys the m tar. In such cases, therefore, the batter should not exceed 1 or 1½ ins to a ft; a even then, at least the pointing of the joints, and a few feet in height of b the upper and the lower courses of masonry, should be done with cement. cement-mortar. We have observed a most marked diff in the corrosion of the m tar, where, in the same walls, with the same exposure, one portion has been bu with a vert face; and another with a batter of but 1½ inch to a foot. Comm mortar will never set properly, and continue firm, when it is exposed to mo ture from the earth. This is very observable near the tops and bottoms abuts, retaining-walls, &c; the lime-mortar at those parts will generally found to be rendered entirely worthless. A profile somewhat like Fig 12, m at times prove serviceable, instead of the triangular. This is the form of t Gothic buttress; which probably had its origin in the cause just spoken of.

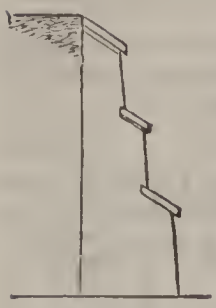


Fig 12

Art. 9. A retaining-wall may slide, without losing its verticality; and, indeed, without any dang of being overturned. This is very apt to occur if it is built up a hor wooden platform; or upon a level surf of rock, or cla without other means than mere friction to prevent sliding. This may be obviat by inclining the base, as in Fig 1; by founding the wall at such a depth as to pr vide a proper resistance from the soil in front; or in case of a platform, by securi one or more lines of strong beams to its upper surf, across the direction in whi sliding would take place. On **wet clay**, friction may be as low as from .2 ⅓ the weight of the wall; on dry earth, it is about ½ to ⅔; and on sand or grav about ⅔ to ¾. The friction of masonry on a wood platform, is about $\frac{6}{10}$ of the wt, if dry; and ¾ if we

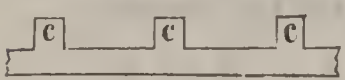


Fig 13

Counterforts, shown in plan at c c c, Fig 13, consist an increase of the thickness of the wall, *at its back*, at regular int vals of its length. We conceive them to be but little better than waste of masonry. When a wall of this kind fails, it almost i variably separates from its counterforts: to which it is connect merely by the adhesion of the mortar; and to a slight extent, by t bonding of the masonry. The table in Art 7 shows that a very small addition to the base of a wall, attended by a great increase of its strength; we therefore think that the masonry of counterfo would be much better, and more cheaply employed in giving the wall an additional thickness, alo its entire length: and for the lower third of its height. Counterforts are very generally used retaining-walls by European engineers; but rarely, if ever, by Americans.

Buttresses are like counterforts, except that they are placed *in front* of a wall instead of b hind it; and that their profile is generally triangular, or nearly so. They greatly increase its streng but, being unsightly, are seldom used, except as a remedy when a wall is seen to be failing.

Land-ties, or long rods of iron, have been employed as a makeshift for upholding weak r taining-walls. Extending through the wall from its face, the land ends are connected with ancho of masonry, cast-iron or wooden posts; the whole being at some dist below the surface.

Retaining walls with curved profiles are mentioned here merely to ca tion the young engineer against building them. Although sanctioned by the practice of some hi authorities, they really possess no merit sufficient to compensate for the additional expense and tro ble of their construction.

Art. 10. Among military men, a retaining-wall is called a **revetment.** When t earth is level with the top, a **scarp revetment**; when above it, a **counterscar** revetment, or a *demi-revetment*. When the face of the wall is battered, a *sloping*; and when the ba is battered, a *countersloping* revetment. The batter is called the **talus.**

Art. 11. The pres against a wall Fig 6, from sand etc level with its top, is n diminished by reducing the quantity of sand, until its top width *c s* becomes less tha that (*c t*) pertaining to the angle *c m t* of maximum pres. The pres then begins to d minish, but in practice *the diminution is not appreciable until the width is reduced to* abou one sixth of that (*c s*) pertaining to the angle *c m s* of natural slope, or about half o *c t*. The pres then begins to decrease rapidly as the width is further reduced.

Table 4, of contents in cub yards for each foot in length of retaining-walls, with a thickness at base equal to .4 of the vert heigh if the back is vert. If the back is stepped according to the rule in Art 3, p 685, th proportionate thickness at base will of course be increased. Face batter, 1½ inch to a foot; or ⅛th of the height. Back either vert, or stepped according to the ru in Art 3, Fig 3. The strength is very nearly equal to that of a vert wall with base of .4 its height. See table, p 690. Experience has proved that such wall when composed of well-scabbled mortar rubble, are safe under all ordinary circum stances for earth level with the top. Steps or offsets, *o e*, at foot, Fig 1, are not he included.

TABLE 4. (Original.)

Ht. Ft.	Cub. Yds.	Ht. Ft.	Cub. Yds.	Ht. Ft.	Cub. Yds.	Ht. Ft.	Cub. Yds.	Ht. Ft.	Cub. Yds.	Ht. Ft.	Cub. Yds.
1	.013	10½	1.38	20	5.00	29½	10.9	48	28.8	74	68.5
½	.028	11	1.51	½	5.25	30	11.3	49	30.0	76	72.2
2	.050	½	1.65	21	5.51	31	12.0	50	31.3	78	76.1
½	.078	12	1.80	½	5.78	32	12.8	51	32 5	80	80.0
3	.113	½	1.95	22	6.05	33	13.6	52	33.8	82	84.1
½	.153	13	2.11	½	6.33	34	14.5	53	35.1	84	88.4
4	.200	½	2.28	23	6.61	35	15.3	54	36 5	86	92.5
½	.253	14	2.45	½	6.90	36	16.2	55	37.8	88	96.8
5	.313	½	2.63	24	7.20	37	17.1	56	39.2	90	101.3
½	.378	15	2.81	½	7.50	38	18.1	57	40.6	92	105.8
6	.450	½	3.00	25	7.81	39	19 0	58	42.1	94	110.5
½	.528	16	3.20	½	8.13	40	20.0	59	43.5	96	115.2
7	.613	½	3.40	26	8.45	41	21.0	60	45.0	98	120.1
½	.703	17	3.61	½	8.78	42	22.1	62	48.1	100	125.0
8	.800	½	3.83	27	9.12	43	23.1	64	51.2	102	130.1
½	.903	18	4.05	½	9 45	44	24.2	66	54.5	104	135.2
9	1.01	½	4.28	28	9.80	45	25.3	68	57.8	106	140.5
½	1.13	19	4.51	½	10.2	46	26.5	70	61.3		
10	1.25	½	4.75	29	10.5	47	27.6	72	64 8		

STONE BRIDGES.

Art. 1. In an arch *s t s*, Fig 1, the dist *e o* is called its **span;** *i a* its **rise;** *t* its **crown;** its lower boundary line, *e a o*, its **soffit,** or **intrados;** the upper one, *r r*, its **back,** or **extrados.** The terms soffit and back are also applied to the entire lower and upper curved *surfaces* of the whole arch. The ends of an arch, or the showing areas comprised between its intrados and extrados, are its **faces;** thus the area *s t s a* is a face. The inclined surfaces or joints, *r e*, *r o*, upon which the *feet* of the arch rest, or from which the arch *springs*, are the **skewbacks.** Lines level with *e* and *o*, at right angles to the faces of the arch, and forming the lower edges of its feet, (see *n n*, Fig 2½,) are the **springing lines,** or **springs.** The blocks of which the arch itself is composed, are the **arch-stones,** or **voussoirs.** The center one, *t a*, is the **keystone;** and the lowest ones, *s s*, the **springers.** The term *archblock* might be substituted for voussoir, and like it would apply to brick or other material, as well as to stone. The parts *t r*, *t r*, are the **haunches;** and the spaces *t r l*, *t r b*, above these, are the **spandrels.** The material deposited in these spaces is the **spandrel filling;** it is sometimes earth, sometimes masonry; or partly of each, as in Fig 1.

In large arches, it often consists of several parallel SPANDREL-WALLS, *l l*, Fig 2½, running lengthwise the roadway, or astraddle of the arch. They are covered at top either by small arches from wall to wall, or by flat stones, for supporting the material of the roadway. They are also at times connected together by vert cross-walls at intervals, for steadying them laterally, as at *t t*, Fig 2½. The parts *p e n*, *g p o n*, Fig 1, are the ABUTMENTS of the arch; *e n*, *o n*, the *faces*; *g p*, *g p*, the backs; and *n*, *p n*, the *bases* of the abuts. The bases are usually widened by *feet*, *steps*, or *offsets*, *d d*, for distributing the wt of the bridge over a greater area of foundation; thus diminishing the danger of settlement. The distance *t a* in any arch-stone, is called its *depth.*

The only arches in common use for bridges, are the circular, (often called segmental); and the elliptic.

Art. 2. To find the depth of keystone for first-class cut-stone arches, whether circular or elliptic.*

Find the rad *c o*, Fig 1, which will touch the arch at *o*, *a*, and *e*. Add together this rad, and *half* the span *o e*. Take the sq rt of the sum. Div this sq rt by 4. To the quot add $\frac{2}{10}$ of a ft. Or by formula,

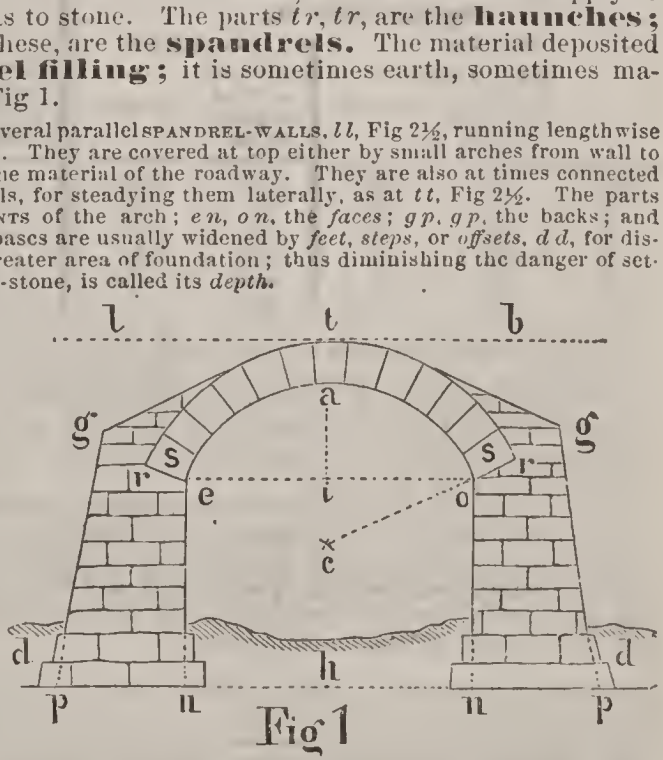


Fig 1

* Inasmuch as the rules which we give for arches and abuts are entirely original and novel, it may not be amiss to state that they are not altogether empirical; but are based upon accurate drawings

$$\text{Depth of key in feet} = \frac{\sqrt{Rad + \text{half span}}}{4} + .2 \text{ foot.}$$

For second-class work, this depth may be increased about ⅛th part; o **for brick or fair rubble,** about ⅓rd. See table of Keystones, p 697.

In large arches it is advisable to increase the depth of the archstones toward th springs; but when the span is as small as about 60 to 80 or 100 feet, this is not at al *necessary* if the stone is good; although the arch will be stronger if it is done. I practice this increase, even in the largest spans, does not exceed from ¼ to ½ th depth of the key; although theory would require much more in arches of great ris

Rem. **To find the rad *c o*,** whether the arch be circular or elliptic. Squar *half* the span *e o*. Square the whole rise *i a*. Add these squares together; div th sum by *twice* the rise *i a*. Or it may be found near enough for this purpose by th dividers, from a small arch drawn to a scale.

Amount of pressure sustained by archstones. In bridges of the same width of roadway; if all the other parts bore to each other the same propor tion as the spans, the *total* pres would increase as the squares of the spans, while th pressure *per square foot* would increase as the spans. But in practice the depth of th archstones increases much less rapidly than the span; while the thickness of th roadway material, and the extraneous load per sq ft, remain the same for all spans Hence the *total* pressures, at key and at spring, increase *less* rapidly than the *square* of the spans; but *more* rapidly than the simple *spans;* as do also the pressures *pe square foot.* Thus in two bridges of the same width, but with spans of 100 and 200 ft with depths of archstones taken from our table page 697, and uniform from key t spring; supposed to be filled up solid with masonry of 160 lbs per cub ft. to a level of about 15 inches above the crown, (including the stone paving of the roadway); wit an extraneous load of 100 lbs per sq ft; the pressures will be approximately as fol lows:

	Span 100 ft.				Span 200 ft.			
	AT KEY.		AT SPRING.		AT KEY.		AT SPRING.	
	For 1 ft in width of its entire depth.	Per sq ft.	For 1 ft in width of its entire depth.	Per sq ft.	For 1 ft in width of its entire depth.	Per sq ft.	For 1 ft in width of its entire depth.	Per sq ft
Rise.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.
⅙	42¼	13¾	58	18¾	126	29¼	179	42
⅕	36¾	12¼	57	19	112	27¼	181	44
¼	31¼	11	57½	20	97	24½	188	47½
⅓	25	9	61½	22½	80¼	21	207	54¼
½	18	6¾	67½	25	57½	15½	230	61½

It will be seen that with the same span, the pres at the key becomes less, while tha at the spring becomes greater, as the rise increases. Also that when the archstone are of uniform depth, the pres at either spring of a semicircular arch is about 4 time as great as at the key; whereas when the rise is but one-sixth of the span, the pres a spring averages but about one-third greater than at the key. These proportions var somewhat in different spans.

The greater pres per sq ft at the springs may be reduced by increasing the depth of the archstones towards the springs. This however is not *necessary* in moderate spans inasmuch as good stone will be safe even under this greater pres.

By using parallel spandrel walls, see Fig 2½, p 698, or by partly fill ing with earth instead of masonry, the pres on the archstones may be diminished say, as a rough average, about ⅕ part.

and calculations made by the writer, of lines of pres. &c. of arches from 1 to 300 ft span, and of every rise, from a semicircle to $\frac{1}{15}$ of the span. From these drawings he endeavored to find proportion which, although they might not endure the test of strict criticism, would still apply to all the case with an accuracy sufficient for ordinary practical purposes.

Table 1. Of some existing arches, with both their actual and their lculated depths (by our rule) of keystone. Where two depths are given in the column of keys the allest is for first class cut-stone, and the largest for good rubble, or brick. Those also which are t specified are of first-class cut-stone. C stands for circular, E for elliptic. For 2d class work, add out ⅛th part; and for brick, or fair rubble, about ¼th.

		Span.	Rise.	Radius.	Actual Key.	Key by our Rule.	Engineer.
		Ft.	Ft.	Ft.	Ft.	Ft.	
Cabin John, Washington Aqueduct	C	220	57.25	134.25	4 16	4.11	Meigs.
Grosvenor Bridge, Chester, England	C	200	42	140	4	4.07	Hartley.
London Bridge, England	E	152	29.5	112.6	4.75	3.62	Rennie.
Gloucester, across the Severn, England	E	150	35	98	4.50	3.49	Telford.
Dora Riparia, Turin, Italy	C	148	18	160.1	4.92	4.03	Mosca.
Pont-y-Pryddd, Wales. Rough rubble	C	140	35	87.5	1.5	3.3 to 4.2	Edwards.
Pont de Alma, Paris, France. Small rough rubble, in cement. Carries a railroad	E	141.4	28.2	102.8	4.92	3.5 to 4.44	Darcel.
Soupes, France. An experimental arch of cut-stone in Portland cement mortar*	C	124	6.92	281 1	2.67	4.82	Vaudray.
Waterloo. Across the Thames at London	E	120	32	72.2	4.5	3.07	Rennie.
Tongueland, England. Turnpike	C	118	38	64.8	3.5	3.0	Telford.
Pont Napoleon. R. R. viaduct, Paris. Small rough rubble in cement	C	115.5	14.75	120	4	3.54 to 4.48	Couche.
Holy Trinity, Florence. Very slightly pointed		95.5	15.1	83 9	2.75	3.07	Ammanati.
Dean Bridge, Scotland. Turnpike	C	90	30	48.9	3	2.62	Telford.
Dunkeld "	C	90	30	48.9	3.17	2.62	Telford.
Licking Aqueduct. Chesapeake & Ohio Canal	C	90	15	75	2.83	2.94	Fisk.
Royal Border Viaduct, England. Brick in cement	C	80	40	40	2.66	2.44 to 3.10	
Posen Viaduct, Germany. Brick in cement	C	80	16	58	4.66	2.68 to 3.40	
Allenton, England. Turnpike	C	75	11.5	67	2.5	2.75	Stevenson.
Falls Bridge, Philadelphia & Reading R R	C	78	25	43	3	2.46	
Staines, England. Turnpike	C	74	9.3	78	3	2.98	Rennie.
Brent R R Viaduct, England. Brick in cement	E	70	17.6	43.6	3	2.42 to 3.02	Brunel.
Bow Bridge. Turnpike. England	E	66	13.75	46.5	2.50	2.42	Walker.
Bewdly. "	C	60	20	33	2.2	2.18	Telford.
Chestnut Street Bridge, Philadelphia. Brick in cement	C	60	18	34	2.5	2.2 to 2.8	Kneass.
Monocacy Aqueduct, Chesapeake & Ohio Canal	E	54	9	45	2.5	2.32	Fisk.
Llanrwast, Wales. Turnpike	C	58	17	33	1.5	2.17	Inigo Jones.
Avon Viaduct, England. Brick in cement	E	50	15	28.3	2	2.08 to 2.67	Vignoles.
James River Aqueduct, Virginia	C	50	7	47	2.66	2.32	Ellet.
Tonoloway Culvert, under Ches & Ohio Canal. Rubble in cement	C	40	15	21	2	1.8 to 2.3	Fisk.
Philadelphia & Reading R R	C	44	8	34.3	2.5	2.08	Steele.
" " "	C	31.2	5	26.8	1.66	1.83	Steele.
Edinburg & Dalkeith R R. Scotland	E	24	4	20	.75	1.62	Jardine.

* Granite. Width 12 ft. The centers (not struck for 4 months) rested on sand in 1.32 inch sheet-iron cylinders 1 ft high, 1 ft diameter. Not injured by a distributed load of 360 tons, nor by a wt of 5 tons falling 18 ins on key. Total settlement about inch and a quarter. Depth of archstones 2.67 ft at key; 3.6 ft at springs. Rise about one-eighteenth of span. Mortar joints full quarter inch thick.

The arch on the Bourbonnais Railway, is probably the boldest;* and the Cabin John arch, by Capt, now Gen'l M. C. Meigs, U S Army, the grandest stone one in existence. Pont-y-Prydd, in Wales, is a common road bridge, of very rude construction; with a dangerously steep roadway. It was built entirely of rubble, in mortar, by a common country mason, in 1750: and is still in perfect condition. Only the outer, or *showing* arch-stones, are 2.5 ft deep; and that depth is made up of two stones. The inner arch-stones are but 1.5 ft deep; and but from 6 to 9 inches thick. The stone quarried with tolerably fair natural beds: and received little or no dressing in addition. The bridge is a fine example of that ignorance which often passes for boldness. Pont Napoleon carries a railroad across the Seine at Paris. The arches are of the uniform depth of 4 ft, from crown to spring. They are composed chiefly of *small rough quarry chips*, or *spalls;* well washed, to free them from dirt and dust; and then thoroughly bedded in good cement; and grouted with the same. It is in fact a arch of cement-concrete. The Pont de Alma, near it, *and built in the same way*, has elliptic arches of from 126 to 141 ft span; with rises of $\frac{1}{5}$ the span. Key 4.9 ft. These two bridges, considering the want of precedent in this kind of construction, on so large a scale, must be regarded as very bold and as reflecting the highest credit for practical science, upon their engineers, Darcel and Couche. Some trouble arose from the unequal contraction of the different thicknesses of cement. They show what may be readily accomplished in arches of moderate spans, by means of small stone, and *good hydraulic cement* when large stone fit for arches is not procurable. In Pont Napoleon the depth of arch is less than our rule gives for second class cut-stone.

Rem. **Our engineers are usually too sparing of cement.** It should be freely used, not only in the arches themselves, and in the masonry above them, as a protection from rain-soakage; but in abuts, wing-walls, retaining-walls, and all other important masonry exposed to dampness. The entire backs of important brick arches should be covered with a layer of good cement, about an inch thick. The want of it can be seen throughout most of our public works. The common mortar will be found to be decayed, and falling down from the soffits of arches; and from the joints of masonry generally, within from 3 to 6 ft of the surface of the ground. The moisture rises by capillary attraction, to that dist above the surf of the nat soil; or descends to it from the artificial surf of embankments, &c; therefore, cement-mortar should be employed in those portions at least. The mortar in the faces of battered walls, even when the batter is but 1 to 1½ inches per foot, is far more injured by rain and exposure, than in vert ones; and should therefore be of the best quality. See Mortar, &c.

We have, however, seen a quite free percolation of surface water through brick arches of nearly 3 ft in depth, even when cement was freely used. In aqueduct bridges, we believe that cement has not been found to prevent leaks, whether the arches were of brick, or even of cut-stone. May not this be the effect of cracks produced by settlement of the arch; or by contraction and expansion under atmospheric influence? Cement at any rate prevents the joints from crumbling.

Art. 3. The keystones for large elliptic arches by the best engineers, are generally made about ⅓ part deeper than our rule requires; or than is considered necessary for circular ones of the same span and rise; in order to keep the line of pres well within the joints; although the elliptic arch, with its spandrel filling, has slightly less wt; and that wt has a trifle less leverage than in a circular one; and consequently it exerts less pres both at the key, and at the skew-back. See London, Gloucester, and Waterloo bridges, in the preceding table.

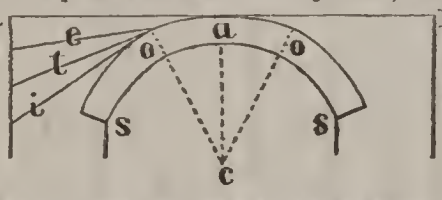


Fig. l½

Rem. Young engineers are apt to affect shallow arch-stones; but it would be far better to adopt the opposite course: for not only do deep ones make a more stable structure, but a thin arch is as unsightly an object as too slender a column. According to our own taste, arch-stones fully ⅓ deeper than our rule gives for first-class cut-stone, are greatly to be preferred when appearance is consulted. Especially when an arch is of rough rubble, which costs about the same whether it is built up as arch, or as spandrel filling, it is mere folly to make the arches shallow. Stability and durability should be the objects aimed at; and when they can be attained even to excess, without increased cost, it is best to do so.

* Built like that at Soupes in the preceding table.

Table 2. Depths of keystones for arches of first-class cut stone, by Art 2. For second class add full one-eighth part; and for superior brick one-fourth to one-third part, if the span exceeds about 15 or 20 ft. Original.

Span. Feet.	Rise, in parts of the span. $\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{10}$
	Key. Ft.	Key. Ft.	Key. Ft.	Key. Ft.	Key. Ft.	Key. Ft.	Key. Ft.
2	.55	.56	.58	.60	.61	.64	.68
4	.70	.72	.74	.76	.79	.83	.88
6	.81	.83	.86	.89	.92	.97	1.03
8	.91	.93	.96	1.00	1.03	1.09	1.16
10	.99	1.01	1.04	1.07	1.11	1.18	1.26
15	1.17	1.19	1.22	1.26	1.30	1.40	1.50
20	1.32	1.35	1.38	1.43	1.48	1.59	1.70
25	1.45	1.48	1.53	1.58	1.64	1.76	1.88
30	1.57	1.60	1.65	1.71	1.78	1.91	2.04
35	1.68	1.70	1.76	1.83	1.90	2.04	2.19
40	1.78	1.81	1.88	1.95	2.03	2.18	2.33
50	1.97	2 00	2.08	2.16	2.25	2.41	2.58
60	2.14	2.18	2.26	2.35	2.44	2.62	2.80
80	2.44	2.49	2.58	2.68	2.78	2.98	3.18
100	2.70	2.75	2.86	2.97	3.09	3.32	3.55
120	2.94	2.99	3.10	3.22	3.35	3 61	3.88
140	3.16	3.21	3.33	3.46	3.60	3.87	4.15
160	3.36	3.44	3.58	3.72	3.87	4.17	
180	3.56	3.63	3.75	3.90	4.06	4.38	
200	3.74	3.81	3.95	4.12	4.29		
220	3.91	4.00	4.13	4.30	4.48		
240	4.07	4.15	4.30	4.48			
260	4.23	4.31	4.47	4.66			
280	4.38	4.46	4.63				
300	4.53	4.62	4.80				

Art. 4. To proportion the abuts for an arch of stone or brick, whether circular or elliptic. (Original.)

The writer ventures to offer the following rule, in the belief that it will be found to combine the requirements of theory with those of economy and ease of application, to perhaps as great an extent as is attainable in an endeavor to reduce so complicated a subject, to **a simple and reliable working rule for practical bridge-builders.** This is all that he claims for it. Notwithstanding its simplicity, it is the result of much labor on his part. It applies equally to the smallest culvert, and to the largest bridge; whatever may be the proportions of span and rise; and to any height of abut whatever. It applies also to all the usual methods of filling above the arch; whether with solid masonry to the level *v f*, Fig 2, of the top of the arch; or entirely with earth; or partly with each, as represented in the fig: or with parallel spandrel-walls extending to the back of the abut, as in Fig 2½. Although the stability of an abut cannot remain precisely the same under all these conditions, yet the diff of thickness which would follow from a strict investigation of each particular case, is not sufficient to warrant us in embarrassing a rule intended for popular use, by a multitude of exceptions and modifications which would defeat the very object for which it was designed. We shall not touch upon the theory of arches, except in the way of incidental allusion to it. Theories for arches, and their abuts, omit all consideration of passing loads; and consequently are entirely inapplicable in practice when, as is frequently the case, (especially in railroad bridges of moderate spans,) the load bears a large ratio to the wt of the arch itself. Hence the theoretical line of thrust has no place in such cases. Our rule is intended for common practice: and we conceive that no error of practical importance will attend its application to any case whatever; whether the arch be circular or elliptic.

It gives a thickness of abut, which, without any backing of earth behind it, is safe in itself, and in all cases, against the pres. when the bridge is unloaded. Moreover, in very large arches, in which the greatest load likely to come upon them in practice is small in comparison with the wt of the arch itself, and the filling above it, our abuts would also be safe from the *loaded* bridge, without any dependence upon the earth behind them; but as the arches become less, and consequently the wt of the load becomes greater in proportion to that of the arch, and of the filling above it, we must depend more and more upon the resistance of the earth behind the abuts, in order to avoid the necessity of giving the latter an extravagant thickness. *It will therefore be understood throughout that, except when parallel spandrel walls are used, our rules suppose that after the bridge is finished, earth will be deposited behind the abuts, and to the height of the roadway, as usual.*

In small bridges and large culverts of first class railroads, subject to the jarring of heavy trains at high speeds, the comparative cheapness with which an excess of strength can thus be given to important structures, has led, in many cases, to the use of abutments from one-fourth to one-half thicker than by the following rule. **If of rough rubble** add 6 ins to insure full thickness in every part.

$$\left.\begin{array}{c}\textit{Thickness at spring in ft, when the height does not exceed } 1\frac{1}{2} \textit{ times the base}\end{array}\right\} = \frac{\textit{Rad in ft}}{5} + \frac{\textit{rise in ft}}{10} + 2\,\textit{ft}.$$

Mark the points n and y thus ascertained. Next, from the center i, of the span or chord $e\,o$, lay off $i\,h$, equal to $\frac{1}{24}$ part of the span. Join $a\,h$; and through n, and parallel to $a\,h$, draw the indefinite line $g\,n\,p$ of the abut. Do the same with the other abut. Make $y\,m$ and $n\,g$ each equal to half the entire height $i\,t$ of the arch; and from g draw a straight line $g\,x$, touching the back of the arch as high up as possible; or still better, as shown at $t\,m$, with a rad $d\,t$ or $d\,m$, (to be found by trial,) describe an arc $t\,m$. Then $g\,x$ or $t\,m$ will be the top of the masonry filling above the arch;* and this should be completed before striking the centers; before which, also, the embkt should be finished, at least up to $y\,n$.

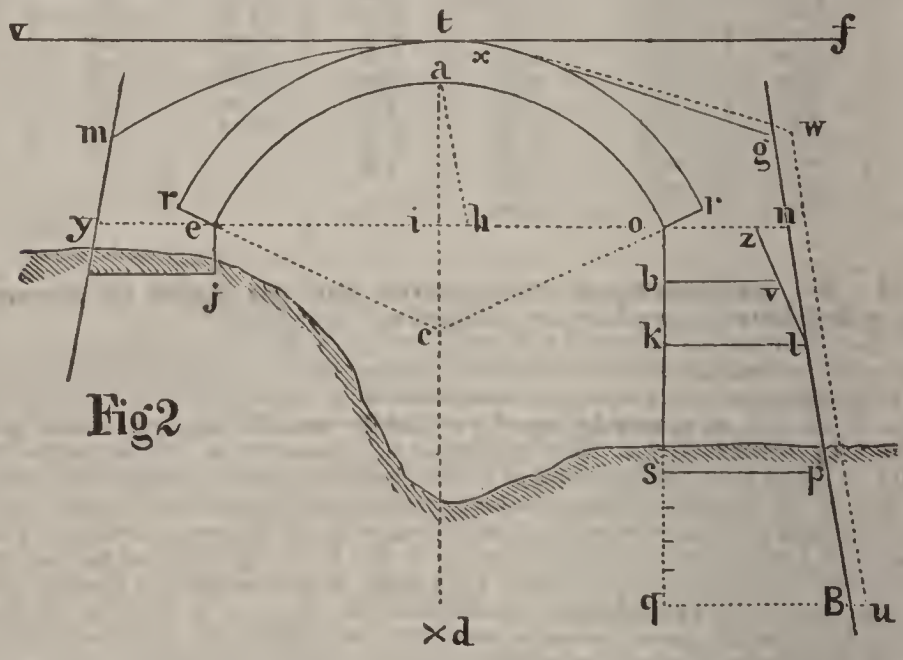


Fig 2

Now find by trial the point s, Fig 2, at which the thickness $s\,p$ is equal to two-

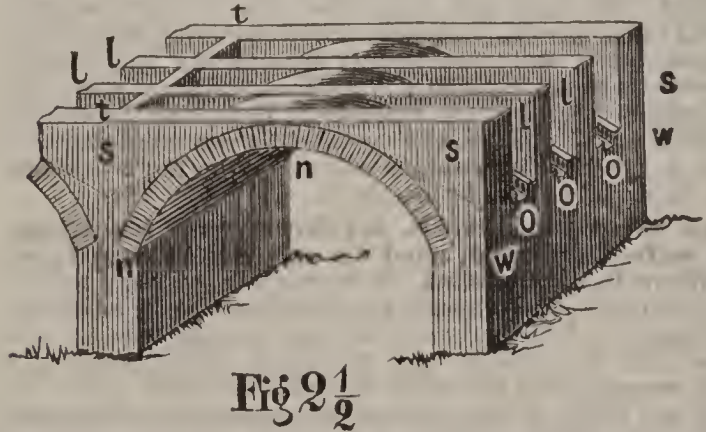


Fig 2½

* Except when the rise is but about $\frac{1}{5}$ of the span, or less; in which case carry the masonry up solid to the level $v\,t\,f$, of the top of the arch. Or if the arch is a large one, exceeding say about 60 ft span; and especially if its rise is greater than about $\frac{1}{5}$ of its span, it is better to economize masonry by the use of parallel interior spandrel-walls, $l\,l$, Fig 2½, carried up to $v\,t\,f$, Fig 2. Indeed, such interior walls may often be advantageously introduced in much smaller arches. When high, they are steadied by occasional cross-walls, as $t\,t$, Fig 2½. Their feet should be spread by offsets, as shown at $o\,o\,o$, so as to bear upon the whole surf of the back of the arch; thus equallizing the pres upon it. On top of the walls flagstones may be laid, or small arches may be turned from wall to wall, for supporting the ballast, &c, of the roadway. The spaces below are left hollow. In Fig 2½ the dark part $w\,w$ is supposed to be a section across an abutment; but omitting the second cross-wall, similar to $t\,t$. In R R bridges, put a spandrel-wall, $l\,l$, under each rail.

thirds of the corresponding vert height $o\,s$, and draw $s\,p$. Then will the *thickness* $o\,n$ or $e\,y$ be that at the springing line of the given circular or elliptic arch of any rise and span; and the line $g\,p$ will be the *back* of the abut; provided its height $o\,s$ does not exceed $1\frac{1}{2}$ times $s\,p$; or in other words, provided $s\,p$ is not less than $\frac{2}{3}$ of $o\,s$. In practice, $o\,s$ will rarely exceed this limit; and only in arches of considerable rise. But if it should, as for instance at $o\,q$, then make the base $q\,u$ equal to $s\,p$, added to one-fourth of the *additional* height $s\,q$; and draw the back $u\,w$, parallel to $g\,p$; and extending to the same height, &c, as in Fig 2. If, however, this addition of $\frac{1}{4}$ of $s\,q$ should in any case give a base $q\,u$ less than *one-half* the total height $o\,q$, (which will very rarely happen in practice,) then make $q\,u$ equal to half said total height; drawing the back parallel to $g\,p$, and extending it to the same height as before. The additional thicknesses thus found below $s\,p$, have reference rather to the pres of the earth behind the abut, than to the thrust of the arch. In a very high abut, the inner line $g\,p$ would give a thickness too slight to sustain this earth safely.

When the height $o\,b$, Fig 2, of the abut is less than the thickness $o\,n$ at spring, a small saving of masonry (not worth attending to, except in large flat arches) may be effected by reducing the thickness of the abut throughout, thus: Make $o\,k$ equal to $o\,n$, and draw $k\,l$. Make $o\,z$ equal to $\frac{3}{4}$ of $o\,n$, and draw $l\,z$. Then, for any height $o\,b$ of abut less than $o\,n$, draw $b\,v$, terminating in $l\,z$. This $b\,v$ will be sufficient base, if the foundations are firm. The back of the abut will be drawn upward from v, parallel to $g\,p$, and terminating at the same height as g or w.

Rem. 1. All the abuts thus found will (with the provisions in Art 6) be safe, without any dependence upon the wing-walls; no matter how high the embkt may extend above the top of the arch. If the bridge is narrow, and the inner faces of the wing-walls are consequently brought so near together as to afford material assistance to the abuts, the latter may be made thinner; but to what extent, must depend upon the judgment of the engineer.

We, however, caution the young practitioner to be careful how he adopts dimensions less than those given by our rule. There are certain practical considerations, such as carelessness of workmanship; newness of the mortar; danger of undue strains when removing the centers; liability of derangement during the process of depositing the earth behind the abuts, and over the arch; &c, which must not be overlooked; although it is impossible to reduce them to calculation.

Whenever it can be done, the centers should remain in place until the embkt is finished; and for some time afterward, to allow the mortar to set well. But for more on this see Rem 4, p 713.

Rem. 2. A good deal of liberty is sometimes taken, in reducing the quantity of masonry above the springing line of arches of considerable rise, and of moderate spans. When care is taken to leave the centers standing until the earth filling is completed above the arch, and behind its abuts, so that it may not be deranged by accident during that operation; and when good cement is used instead of common mortar, such experiments may be tried with comparative safety; especially with culvert arches, in which the depth of arch-stones is great in proportion to the span. They must, however, be left to the judgment of the engineer in charge; as no specific rules can be laid down for them. They can hardly be regarded as legitimate practice, and we cannot recommend them. We have known nearly semicircular arches, of 30 to 40 ft span, to be thus built successfully, with scarcely a particle of masonry above the springs to back them. Such arches, however, are apt to fall, if at any future period the earth filling is removed, without taking the precaution to first build a center or some other support for them. Even when the embkt can be finished before the centers are removed, we cannot recommend (and that only in small spans) to do *less* than to make $n\,g$, Fig 2, equal to $\frac{1}{8}$ of the total height $i\,t$ of the arch; and from g so found, to draw a straight line touching the back of the arch as high up as possible.

Rem. 3. We have said nothing about **battering the faces of the abuts,** because in the crossing of streams, the batter either diminishes the water-way; or requires a greater span of arch. Such a batter, however, to the extent of from $\frac{1}{2}$ to $1\frac{1}{2}$ ins to a ft, is useful, like the offsets, for distributing the wt of the structure, and its embkt, over a greater area of foundation; especially when the last is not naturally very firm; or when the embkt extends to a considerable height above the arch. In our tables, Nos 3 and 5, of approximate quantities of masonry in semicircular bridges of from 2 to 50 ft span, the faces are supposed to be vert.

Art. 5. Abutment-piers. When a bridge consists of several arches, sustained by piers of only the usual thickness, if one arch should by accident of flood, or otherwise, be destroyed, the adjacent ones would overturn the piers; and arch after arch would then fall. To prevent this, it is usual in important bridges to make some of the piers sufficiently thick to resist the pres of the adjacent arches, in case of such an accident; and thus preserve at least a portion of the bridge from ruin. Such are called abutment-piers.

Our formula of $\frac{\text{Rad}}{5} + \frac{\text{Rise}}{10} + 2$ ft, for the thickness at spring; with the back battering as before, at the rate of $\frac{1}{2\frac{1}{4}}$ of the span to the rise; face vert.; will of itself (*without any modification for great heights*) give a perfectly safe abut-pier, for any unloaded bridge; and to any height whatever; due regard being had, however, to the consideration alluded to in the next Art. Thus, for an abut-pier as high as $o\,q$, Fig 2; or of any greater height; it is only necessary first to find the thickness $o\,n$ at spring as before; and then draw the battered back $g\,n\,p$: extending it down to the base at B; without adding $\frac{1}{4}$ of the additional height $s\,q$. This addition is made in the case of abuts, that they

may be secure from the pres of the *earth* behind them; as well as from the pres of the arch; a consideration which does not apply to abut-piers; in which only the pres of the arch is to be resisted.

But although the abut-pier thus found by our formula, would be abundantly safe, yet its shape *a b c o*, Fig 3, is inadmissible. In practice it would be changed to one somewhat like that shown by the dotted lines; having an equal degree of batter on both faces. This of course requires more masonry, with but little increase of stability; but that cannot be avoided.

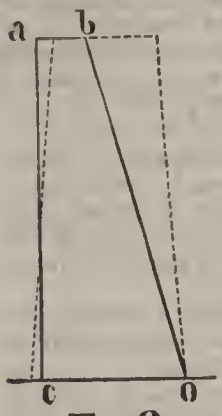


Fig 3

When an abut-pier is built in deep water, or in a shallow stream subject to high freshets, care must be taken that water cannot find its way *under* the pier, and thus produce an upward pres, which will either diminish, or entirely counteract its efficiency as an abut. See Remark 2, Art 4, of Hydrostatics.

Art. 6. Inclination of the courses of masonry below the springs of an arch. Although our foregoing rule gives a thickness of abut which cannot be *overturned*, or *upset*, by the pres of the arch, yet if the arch be of large span, and small rise, its great hor thrust may produce a *sliding* outward of the masonry near the level of the springs, if the stones are laid in *hor* courses; especially if the mortar has not set well.

This danger, it is true, could be avoided by confining the courses together by iron bolts and cramps; or by increasing considerably the thickness of the abuts; but the expense of doing either of these, leads to the cheaper expedient of inclining the masonry, as shown between o and *n*, Fig 4; the courses near o being steeper; and gradually becoming less steep near *n*. By this process the arch is virtually prolonged into the body of the abut, so far that when the inclination of the lower masonry ceases, as at *n*, the direction of the theoretical *line of thrust*, or *of pres* of the arch, (rudely represented by the dotted curved line *o n*) is nearly at right angles to the joints of the hor masonry below *n*; and consequently, said thrust is unable to produce sliding at that point. Between o and *n*, the line of pres is everywhere so nearly at right angles to the variously inclined joints, as to preclude the possibility of sliding in that interval also. See Art 63 of Force in Rigid Bodies.* The abut being thus safe throughout from both overturning and sliding, can fail only from defective foundations; or from the inferiority of the stone of which it is built; and which, if soft, may be crushed.

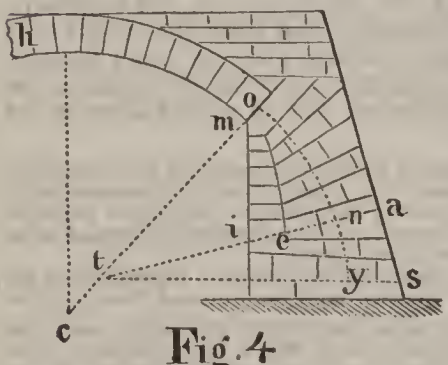


Fig. 4

This inclination of the masonry is as necessary in an elliptic arch, Fig 4½, as in a circular one.

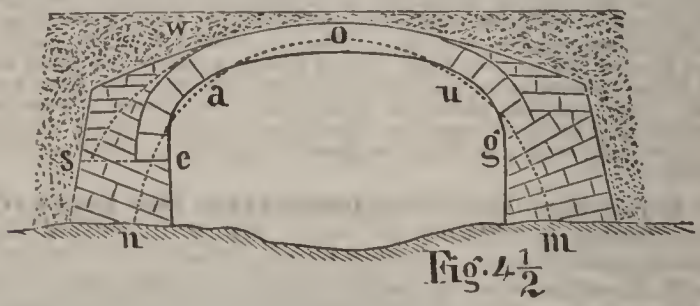


Fig. 4½

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* This curved line of pressures is found in the manner directed at Rem 1, p 360, and at Fig 25, p 231. Rankine, Moseley, and others call it **the line of resistance,** and apply *line of pressures* to another line which need not be introduced in a practical consideration of abutments, walls, dams, &c. They however call any given point in our line a *center of pressure*, because at any part of the height of the abut such point shows where all the pressure or thrust may for many purposes be assumed to be concentrated. The perversion of common technical terms is reprehensible. Said other line had better been called the *line of resultants*. We, both here and elsewhere, call the one to which we refer **the line of pressure, or of thrust,** simply because bridge masons have no idea of any other line curving through an abut, and inasmuch as the pressure or thrust is greatest in that line *they* very properly so term it. Again, we have said above and elsewhere that the bed-joints should be nearly perp to this line of thrust. Theory properly requires them to be **at right angles to the resultants which cut the bed-**joints at this line of thrust. Still we know that on account of the friction of masonry we may with perfect safety vary as much as about 30° from a right angle to these resultants, without depending at all on the strength of the mortar; and in using our rule of thumb on the next page for drawing inclined bed-joints, we shall always be far within the limit of 30°; and therefore fully safe from sliding. Our rules do not call for this line, nor for anything more than the span, rise, and radius of the arch.

The elliptic form is plainly unfavorable for uniting the arch-stones with the inclined masonry near the springs, so as to receive the thrust properly; or about at right angles to its resultant. In ordinary cases this difficulty may be overcome by making the joints of only the outside or showing arch-stones to conform to the elliptic curve; as between e and a; while the joints of the inner or hidden ones, may have the directions shown between g and u, nearly at right angles to the line of thrust. It will rarely happen, however, that the young engineer will have to construct elliptic arches of sufficient magnitude to require either this, or any equivalent expedient. For spans less than 50 ft, with rises not less than about $\frac{1}{5}$ of the span, nothing of the kind is actually necessary, if the mortar is good, and has time to harden.†

In order to incline the masonry of any abut with sufficient accuracy, it would be necessary first to trace the curved line of pres of the given arch, as directed in Art 72 of Force in Rigid Bodies, so as to arrange the bed joints about at right angles to it at every point of its course; but we offer the following process as sufficing for all ordinary practical purposes; while its simplicity places it within the reach of the common mason. In actual bridges the direction of the actual thrust changes as the load is passing; therefore, in practice no given degree of inclination of the abut masonry can conform to it precisely during the entire passage. Consequently, any excess of refinement in this particular, becomes simply ridiculous; especially in small spans.

Rule for inclining the beds of the masonry in the abuts. Add together the rad $c\,m$, Fig 4; and the span of the arch. Div the sum by 5. To the quot add 3 ft. Make $o\,t$, on the rad, equal to the last sum. Then is t a central point, toward which to draw the directions of the beds, as in the fig. Draw $t\,s$ hor, and from t as a center, describe the arc $o\,y$; o being the center of the depth of the springers. From y lay off on the arc the dist $y\,n$, equal to one-sixth part of $t\,y$: draw $t\,n\,a$. It will never be *necessary* to incline the masonry below this $t\,n\,a$. Neither *need* the inclination extend entirely to the face $m\,i$ of the abut; but may stop at e, about half-way between i and n. From e upward, the inclination may extend forward to the line $e\,m$.

† The feet of both elliptic and semicircular arches are always made hor; but it is plain from Fig 4½, that this practice is at variance with correct principles of stability in the case of the ellipse. It is the same in the semicircle. In ordinary bridges of the latter form, the vert pres, or weight resting on each skewback, is (roughly speaking) usually about from 3½ to 4 times the hor pres on the same; and the total pres is about 4 times as great as the pres on the keystone. Therefore, theoretically, the skewback should usually be about 4 times as deep as the keystone; and its bed, instead of being hor, should be inclined at the rate of about 1 vert to 4 hor.

When the arch is flat, this inclination may become so steep, especially in the upper parts, that struts, or shores of some kind, must be used for preventing the masonry from sliding down, until the completion of the arch secures it from doing so. The hor courses between the face *m i*, and the line *o c*, will aid somewhat in this respect.

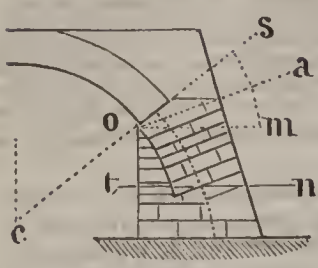


Fig 5

This method should be applied to *all very large arches* whose rise is one-third, or less, of the span. As before remarked, it is not actually necessary in arches not exceeding about 50 ft span, and not flatter than $\frac{1}{5}$ of the span. Indeed, if the earth filling can be deposited before the centers are removed, these limits may be considerably extended without danger. Still, since a certain degree of inclination is attended with very little trouble or expense, we would recommend for even such arches, a process somewhat like the following: From *half* the span take the rise. Div the rem by 3. Make o *t*, Fig 5, equal to the quot. Draw *t n*, and o *m*, hor. Div the angle *s o m* into two equal parts, by the line *o a*. Incline the masonry so as to be parallel to *o a*, as far down as *t n*. The inclined courses may extend out to the face o *t*, or not, at pleasure.

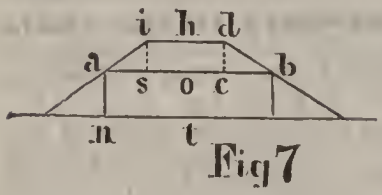


Fig 7

Rem. 1. **To find the length** (*a b*, Fig 7) **from face to face of a culvert.** From the height *h t* of the embkt, take the *above ground* height *n a* of the culvert; the rem will be the height *h o* of the embkt *above* the culvert. Then the reqd length *a b* is plainly equal to the top width *i d* of the embkt, added to the two dists *a s*, *c b*, which correspond to its steepness of side-slopes. Thus, if the side-slope is, as usual, 1½ to 1, then *a s* and *e b* will each be equal to 1½ times *o h*; or the two together will be 3 times *o h*. So that if the width *i d* is 14 ft, and *h o* 5 ft, the length *a b* will be

$$14 + (5 \times 3) = 14 + 15 = 29 \text{ ft.}$$

Art. 7. The following tables, 3, 4, and 5, of quantities, will be found useful for expediting preliminary estimates; for which purpose chiefly they are intended; hence no pains have been taken to make them scrupulously correct, but rather a little in excess of the truth. The first column of Table 3 contains the total vert height *o c*, Fig 6, from the crown *o* of a semicircular arch, to the foundation or base *g m* of its abut. The other columns give approximately the number of cub yds contained in each running foot, or foot in length of the culvert or bridge, measured from end to end (face to face) of the arch proper; and including only the arch and its abuts, as shown in Fig 1; or in the half section *o p m g y* in Fig 6; including footings to the abuts, but omitting the wing-walls (*w n*), and the spandrel-walls (*s*), Figs 6 and 2½. At the foot of each column is the approximate content in cub yds of the two spandrel-walls by themselves; one over each face of the arch.

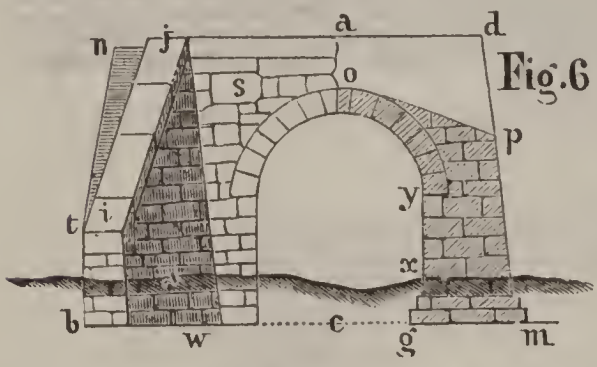


Fig.6

These spandrel-walls are calculated on the supposition that their thickness at base, at their junction with the wing-walls, where their height is greatest, is equal to $\frac{4}{10}$ of their height at that point: except where that proportion gives a less thickness at top than 2½ ft; and that they extend 2 ft (*o a*) above the top o of the arch. At the top of the arch, they are all supposed to be 2½ ft thick at top; that being assumed to be about the least thickness admissible in a rubble wall in such a position. Both the back and the face are supposed to be vert. The contents of these spandrel-walls will vary somewhat, however, even in the same span, with the height of the abut and the arrangement of the wings. They, however, constitute so small a proportion of the entire contents given in Table 5, that this consideration may be neglected in preliminary estimates. They are so firmly bonded into the masonry of the wings at their highest points, and so strongly connected by mortar with the backing of the arch at their bases, that they require no greater thickness however high the emb may be.

The contents of the four wing-walls, of which *n j w b*, Fig 6, is one, will be found in a table (No. 4) immediately following that for the body of the culvert. We have also added a table (No. 5) for complete semicircular culverts of various lengths, including their spandrel and wing walls.

Rem. 1. Although the thickness of wing-walls increases in all parts with their height, they are not made to *show* thicker at nj than at ti, Fig 6; but (as seen in the fig) are offsetted at their back tn, a little below their slanting upper surf ij, so as to give a uniform width for the steps or flagstones, as the case may be, with which they are covered. In the fig the covering is supposed to be of flagstones; but steps are preferable, being less liable to derangement. To prevent the flagstones from sliding down the inclined plane ji, the lower stone i should be deep and large, and laid with a hor bed. The flags are sometimes cramped together with iron, and bolted down to the wall. Steps require nothing of that kind, as seen at s, Fig 11.

Rem. 2. **The tables show the inexpediency of too much contracting the width of water-way,** with a view to economy, by adopting a small span of arch, when a culvert of greater span can be made, *of the same total height.*

For the wings must be the same, whether the span be great or small, provided the total height is the same in both cases; and since the wings constitute a large proportion of the entire quantity of masonry, in culverts of ordinary length, the span itself, within moderate limits, has comparatively little effect upon it. Thus, the total masonry in a semicircular culvert of 3 ft span, 8 ft total height, and 60 ft long between the faces of the arch, is, by Table 5, 151½ cub yds; while that of a 5 ft span, of the same height and length, is 152.4. A semicircular bridge of 25 ft span, 24 ft total height, and 40 ft between the faces of the arch, contains 1031 cub yds; while one of 35 ft span, of the same height and length, contains 1134 yds; so that in this case we may add nearly 50 per cent to the water-way, by increasing the masonry of the bridge but $\frac{1}{10}$th part.

Rem. 3. Partly for the same reason, and partly because **the culverts for a double-track road are not twice as long as those for a single-track one,** the quantity of culvert masonry for the former will not average more than about from ¼ to ⅙ part more than that for the latter; so that it frequently becomes expedient to *finish* the culverts at once to the full length required for a double track, although the embkts may at first be made wide enough for only a single one, with the intention of increasing them at a future time for a double one.

Thus, the average size of culverts for a single track may be roughly taken at 6 ft span, 30 ft long from face to face, and 10 ft total height; and such a one contains, by Table 5, 140 cub yds. For a double track, it would require to be about 12 feet longer; and we see by Table 3 that this will add $2.67 \times 12 = 32$ cub yds; making a total of 172 yds instead of 140; thus adding rather less than ¼ part. When the culverts are under very high embkts, and consequently much longer, the addition for a double track becomes comparatively quite trifling.

Table 3, of approximate numbers of cub yds of masonry per foot run, contained in the arches and abutments only, as shown in Fig 1 (omitting wings, and the spandrel-walls over the faces of the arches) of semicircular culverts and bridges, of from 2 to 50 ft span, and of different total heights, ht, Fig 1, or vc, Fig 6. It will be seen that in many cases, a bridge of larger span contains less masonry than one of smaller span, *when their total heights are the same.* There is a liberal allowance for footings or offsets at the bases of the abuts.

TABLE 3. (Original.)

Total Height.	Span 2 ft.	Span 3 ft.	Span 4 ft.	Span 5 ft.	Span 6 ft.	Span 8 ft.	Span 10 ft.	Span 12 ft.	Span 15 ft.
Feet.	Cub. y.	Cub. y.	Cub. y.	Cub. y.	Cub. y.	Cub. y.	Cub. y.	Cub. y.	Cub. y.
2	.42	.....	.....	.....	.....	.....	.....	.....	.....
3	.60	.63	.67	.....	.....	.....	.....	.....	.....
4	.79	.83	.87	.92	.97	.....	.....	.....	.....
5	.99	1.04	1.08	1.15	1.21	.....	.....	.....	.....
6	1.28	1.28	1.28	1.37	1.46	1.58	1.69	.....	.....
7	1.62	1.59	1.55	1.64	1.72	1 85	1.97	2.12	.....
8	2.01	1.96	1.91	1.95	1.99	2.13	2.26	2.38	.....
9	2.45	2.38	2.31	2.29	2.27	2.42	2.56	2.65	3.02
10	2.94	2.85	2.76	2.72	2.67	2.77	2.87	2.93	3.34
11	.....	3.38	3.26	3.19	3.12	3.16	3.19	3.23	3.67
12	.....	3.98	3.82	3.72	3.62	3.57	3.52	3.55	4.01
13	.....	.....	4.42	4.29	4.17	4.10	4.02	3.86	4.36
14	.....	.....	5.08	4.90	4.77	4.67	4.57	4.41	4.72
15	.....	.....	.....	5.57	5.42	5.30	5.17	5.01	5.09
16	.....	.....	.....	6 30	6.12	5.97	5.82	5.56	5.69
17	.....	.....	.....	.....	6.87	6.70	6.52	6.26	6.34
18	.....	.....	.....	.....	7.69	7.48	7.27	7.01	7.04
19	.....	.....	.....	.....	.....	8.32	8.07	7.71	7.69
20	.....	.....	.....	.....	.....	9.20	8.92	8.56	8.49
21	.....	.....	.....	.....	.....	.....	9.82	9.46	9.34
22	.....	.....	.....	.....	.....	.....	10.8	10.3	10.2
23	.....	.....	.....	.....	.....	.....	.....	11.3	11.1
24	.....	.....	.....	.....	.....	.....	.....	12.3	12.1
25	.....	.....	.....	.....	.....	.....	.....	.....	13.2
26	.....	.....	.....	.....	.....	.....	.....	.....	14.2
Contents of the two spandrel-walls, over the two ends of the arch, in cub yds.	2.9	3.7	4 4	5.2	5.8	7.9	9.8	12.	16.

TABLE 3. (Continued.)

Total Height.	Span 20 ft.	Span 25 ft.	Total Height.	Span 35 ft.	Total Height.	Span 50 ft.
Feet.	Cub. y.	Cub. y.	Feet.	Cub. y.	Feet.	Cub. y.
12	4 60	.....	20	10.5	27	18.0
13	4.98	.....	21	11.0	28	18.7
14	5.37	6.10	22	11.6	29	19.4
15	5.77	6.41	23	12.2	30	20.1
16	6 18	6.76	24	12.7	31	20.9
17	6.60	7.16	25	13.3	32	21.6
18	7.03	7.61	26	13.8	33	22.4
19	7.47	8.10	27	14.5	34	23.1
20	8.12	8.60	28	15.1	35	23.9
21	8.82	9.02	29	15.7	36	24.7
22	9.57	9.72	30	16.3	37	25.5
23	10.4	10.4	31	17.0	38	26.3
24	11.3	11.2	32	18 1	39	27.1
25	12.2	12.1	33	19.2	40	28.0
26	13.1	13.0	34	20.4	41	28.8
27	14.1	14.0	35	21.7	42	30.0
28	15.2	15.0	36	23.0	43	31.5
29	16.3	16.1	37	24 3	44	33.0
30	17.4	17.2	38	25.7	45	34.6
31	18.6	18.4	39	27.2	46	36.3
32	19.9	19.6	40	28.7	47	38.1
33	21.2	20.9	41	30.2	48	39.8
34	22.6	22.2	42	31.8	49	41.6
35	24.0	23.6	43	33.5	50	43.6
36	25.4	25.0	44	35.2	51	45.5
37	26.9	26.5	45	36.9	52	47.4
38	28 5	28.0	46	38.7	53	49.4
39	30.1	29.5	47	40.6	54	51.6
40	31.7	31.2	48	42.5	55	53.7
41	.....	32 8	49	44.4	56	55.9
42	.....	34.5	50	46.4	57	58.1
43	.....	36.3			58	60.4
44	.....	38.1			59	62.7
45	.....	40.0			60	65.1
Contents of the two spandrel-walls, over the two ends of the arch, in cub yds.						
	28.	42.		85.		195.

Art. 8. The **following table** of contents of wing-walls, or wings, will, like the preceding one, be useful in making preliminary estimates. The wings *n o*, *n o*, shown in plan at Fig 8, are supposed to form an angle *a o c*, of 120°, with the face, or end *o o* of the culvert. Their outer or small ends *n n*, are all assumed to be of the dimensions shown on a larger scale at E. Thickness at base at every part equal to $\frac{4}{10}$ of the height of the wall at said part: except when that proportion becomes too small to allow the width or thickness at top to be 2.5 ft; in which case it is enlarged at such parts sufficiently for that purpose. See Remark 2. This happens only

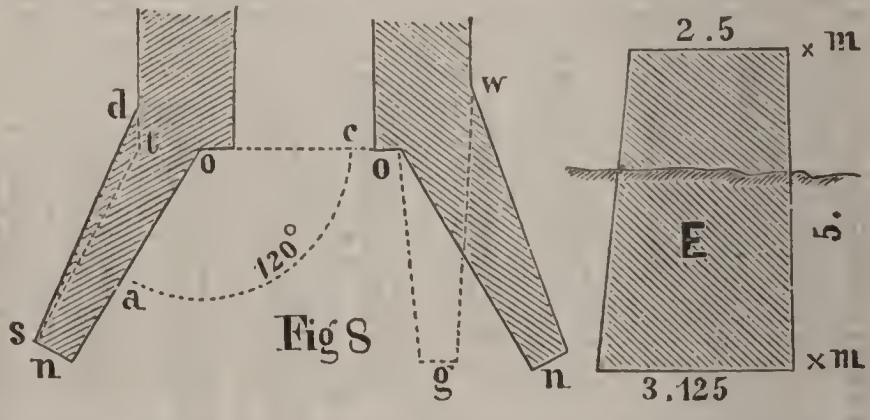


Fig 8

when the height *m m*, Fig E, of the wing, becomes less than 9 ft. Batter of face, 1½ ins to a ft; or 1 in 8. Back vert; but offsetted, if necessary, for a short dist below the top, so as to give a uniform *showing* top thickness of 2½ ft. The masonry is supposed to be good well-scabbled mortar rubble. The height given in the first column is the greatest one; or that at *o o*, (or *w j*, Fig 6,) where the wing joins the face of the culvert. In the table no allowance is made for footings (offsets or steps) at the base of the wings; as these are frequently omitted in wings on good founda-

tions. In taking out quantities from the table, bear in mind that the height of the wings is usually a little greater than that of the culvert itself.

Table 4. of approximate contents, in cub yds, of the four wing-walls of a culvert, or bridge. (Original.)

The heights are taken where greatest; as at *j w*, Fig 6

Height of wing.	Length of one wing.	Cub. yds. in 4 wings.	Height of wing.	Length of one wing.	Cub. yds. in 4 wings.
Feet.	Feet.		Feet.	Feet.	
6	1.73	4.04	30	43.3	818
7	3.46	8.85	32	46.8	997
8	5.20	14.6	34	50.3	1192
9	6.93	21.5	36	53.7	1414
10	8.66	30.2	38	57.2	1661
11	10.4	40.9	40	60.7	1928
12	12.1	53.7	42	64.2	2220
14	15.6	85.2	44	67.6	2552
16	19.1	128	46	71.1	2912
18	22.5	183	48	74.6	3306
20	26.0	247	50	78.0	3741
22	29.5	329	55	86.7	4942
24	32.9	426	60	95.3	6404
26	36.4	541	65	104	8131
28	39.8	672	70	113	10155

To reduce cub yds to perches of 25 cub ft, mult by 1.080.
To reduce perches to cub yds, mult by .926, or div by 1.08.

The contents for heights intermediate of those in the table may be found approximately by simple proportion.

Rem. 1. It is not recommended to actually prolong all wings until their dimensions become as small as shown at E, in Fig 8. In large ones it will generally be more economical to increase their end height *m m*, a few feet. The contents, however, may be readily found by the table in that case also. Thus suppose the height of the wings at one end to be 30 ft, and at the other end 8 ft: we have only to subtract the tabular content for 8 ft high, from that for 30 ft high. Thus, 818 — 14.6 = 803.4 cub yds required content.

Rem. 2. It might be supposed that inasmuch as the wings of arches often have to sustain the pressure from embankments reaching far above their tops, they should, like ordinary retaining-walls, be made much thicker in that case. But the fact that they derive great additional stability from being united at their high ends to the body of the bridge or culvert, renders such increase unnecessary when proportioned by our rule; no matter how far the earth may extend above them; as shown by abundant experience.

Relying upon this aid, we may indeed, when the earth does not extend above the top, reduce the base at *o* to one-third of the ht, as shown at *o t*; and by dotted line *t s*. Experience shows that we may also do the same even when the earth reaches to a great height above the top; provided that the wings, instead of being splayed or flared out, as at *o n*, *o n*, merely form straight prolongations of the abutments of the arch, as shown by the dotted lines at *o g w*. In this case the pressure of the earth against the wings is less than when they are splayed. We have known the thickness at *o* to be reduced in such cases to less than one-third the height, when the wings were 15 ft high, and the height of the embankment *above their tops* 16 feet in one case, and 36 ft in another. In another instance, similar wings 25½ ft high, and with 29 ft of embankment above their top, had their bases at *o* rather less than $\frac{1}{5}$ of the height. In all these cases, the uniform thickness at top was 2.5 feet; backs vertical. We mention them because this particular subject does not seem to be reducible to any practical rule. The last wall appears to us to be too thin; especially if the earth is not deposited in layers; and after allowing the mortar full time to set. The labor, however, required in compacting the earth carefully in layers, may cost more than is thereby saved in the masonry. The young practitioner must bear this in mind when he wishes to economize masonry by such means; and also that the thin wall may bulge, or fail entirely, if the earth backing is deposited while the mortar is imperfectly set.

Table 5. Approximate contents in cubic yards, of complete semicircular culverts and bridges of from 2 to 50 feet span; including the 2 spandrel walls; and the 4 wings; all proportioned by the foregoing directions; and taken from the two preceding tables. The height in the second column, is from the top of the *keystone* to the bottom of the foundation. The wings are calculated as being 2 ft higher than this, including the thickness of the coping. The wings are frequently carried only to the height of the top of the arch, thus saving a good deal of masonry. Table 4, of wings alone, will serve to make the proper deduction in this case.

The several lengths are from end to end, or from face to face, of the *arch proper*. The contents for intermediate lengths may be found exactly; and those for intermediate heights, quite approximately, by simple proportion. In this table, as in No. 3, it will be observed that *when the heights are the same in both cases*, a large span frequently contains less masonry than a smaller one. A semicircular culvert or bridge contains less masonry than a flatter one, when the total height is the same in both cases; therefore, the first is the most economical as regards cost; but it does not afford as much area of water-way; or width of headway.

(Original.)

Span.	Height.	Length, 15 Ft.	Length, 20 Ft.	Length, 30 Ft.	Length, 40 Ft.	Length, 60 Ft.	Length, 80 Ft.	Length, 100 Ft.	Length, 120 Ft.	Length, 140 Ft.	Length, 160 Ft.	Length, 180 Ft.	Length, 200 Ft.
Ft.	Ft.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.
2	5	27	32	42	52	72	92	112	132	152	172	192	212
	6	37	43	56	69	94	120	146	171	197	222	248	274
	7	49	57	73	89	122	154	187	219	251	284	316	349
	8	63	73	93	113	153	193	233	273	313	353	393	433
	10	101	116	145	175	234	291	351	410	469	527	586	645
3	5	28	34	44	54	75	96	117	138	158	179	200	221
	6	38	44	57	70	95	121	146	172	198	223	249	275
	7	49	57	73	89	121	153	184	216	247	280	312	343
	8	63	73	93	112	152	191	230	269	308	348	387	426
	10	101	115	143	172	229	286	343	400	457	514	571	628
	12	149	169	208	248	328	407	487	567	646	726	806	885
4	5	30	35	46	57	78	100	122	143	165	186	208	229
	6	38	45	58	70	96	122	147	173	198	224	250	275
	7	49	57	73	88	119	150	181	212	243	274	305	336
	8	63	73	92	111	149	188	226	264	302	340	379	417
	10	100	114	141	169	224	279	335	390	445	500	555	611
	12	147	166	204	243	319	395	472	548	625	701	777	854
	14	209	234	285	336	437	539	641	742	844	945	1047	1149
5	6	41	47	61	75	102	130	157	184	212	239	267	294
	7	52	60	76	93	125	158	191	224	257	289	322	355
	8	65	75	94	114	153	192	231	270	309	348	387	426
	10	100	114	141	168	223	277	331	386	440	495	549	603
	12	146	165	202	239	314	388	463	537	611	686	760	835
	14	207	231	280	329	427	525	623	721	819	917	1015	1113
6	7	53	62	79	96	131	165	200	234	268	303	337	372
	8	66	76	96	116	156	196	236	276	316	356	396	436
	10	100	113	140	167	220	274	327	380	434	487	541	594
	12	146	164	200	236	308	381	453	526	598	670	743	815
	14	206	219	277	325	420	516	611	706	802	897	993	1088
	16	281	311	373	434	556	679	801	923	1046	1168	1291	1413
8	7	57	67	85	104	141	178	215	252	289	326	363	400
	8	70	81	102	124	166	209	251	294	337	379	422	464
	10	104	118	145	173	228	284	339	395	450	505	561	616
	12	147	165	200	236	308	379	450	522	593	664	736	807
	14	206	230	276	323	416	510	603	696	790	883	977	1070
	16	281	310	370	430	549	669	788	908	1027	1146	1266	1385
	18	367	405	480	554	704	854	1003	1153	1302	1452	1602	1751
10	8	74	85	108	131	176	221	266	311	357	402	447	492
	10	107	121	150	179	236	294	351	408	466	523	581	638
	12	148	166	201	236	306	377	447	518	588	658	729	799
	14	207	229	275	321	412	504	595	686	778	869	961	1052
	16	280	309	368	426	542	659	775	891	1008	1124	1241	1357
	18	366	402	475	548	693	839	984	1129	1275	1420	1565	1711
12	10	110	125	154	183	242	301	359	418	476	535	594	652
	12	151	168	204	239	310	381	452	523	594	665	736	807
	14	206	228	272	317	405	493	581	669	758	846	934	1022
	16	279	306	362	418	529	640	751	862	974	1085	1196	1307
	18	364	399	469	540	680	820	960	1100	1241	1381	1521	1661
	20	470	512	598	684	855	1026	1197	1368	1540	1711	1882	2053

Table 5—(Continued.) (Original.)

Span.	Height.	Length. 15 Ft.	Length. 20 Ft.	Length. 30 Ft.	Length. 40 Ft.	Length. 60 Ft.	Length. 80 Ft.	Length. 100 Ft.	Length. 120 Ft.	Length. 140 Ft.	Length. 160 Ft.	Length. 180 Ft.	Length. 200 Ft.
Ft.	Ft.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.
15	12	162	182	222	262	342	422	502	583	663	743	823	903
	14	215	239	286	333	427	522	616	711	805	899	994	1088
	16	285	313	370	427	541	654	768	882	996	1110	1223	1337
	18	369	404	474	545	686	826	967	1108	1249	1390	1530	1671
	20	473	515	600	685	855	1024	1194	1364	1534	1704	1873	2043
	22	595	646	748	850	1054	1258	1462	1666	1870	2074	2278	2482
20	14	237	264	317	371	478	586	693	801	908	1015	1123	1230
	16	304	335	397	458	582	706	829	953	1076	1200	1324	1447
	18	381	416	486	556	697	838	978	1119	1259	1400	1541	1681
	20	479	520	601	682	844	1007	1169	1332	1494	1656	1819	1981
	22	598	646	741	837	1028	1220	1411	1603	1794	1985	2177	2368
	24	739	795	908	1021	1247	1473	1699	1925	2151	2377	2603	2829
25	16	327	360	428	496	631	766	901	1036	1172	1307	1442	1577
	18	403	441	517	594	746	898	1050	1202	1355	1507	1659	1811
	20	500	543	629	715	887	1059	1231	1403	1575	1747	1919	2091
	22	614	663	760	857	1051	1246	1440	1635	1829	2023	2218	2412
	24	751	807	919	1031	1255	1479	1703	1927	2151	2375	2599	2823
	26	909	974	1104	1234	1494	1754	2014	2274	2534	2794	3054	3314
	28	1085	1160	1310	1460	1760	2060	2360	2660	2960	3260	3560	3860
35	22	685	743	859	975	1207	1439	1671	1903	2135	2367	2599	2831
	24	817	880	1007	1134	1388	1642	1896	2150	2404	2658	2912	3166
	26	969	1033	1181	1309	1585	1861	2137	2413	2689	2965	3241	3517
	28	1130	1205	1356	1507	1809	2111	2413	2715	3017	3319	3621	3923
	30	1327	1408	1571	1734	2060	2386	2712	3038	3364	3690	4016	4342
	32	1549	1639	1820	2001	2363	2725	3087	3449	3811	4173	4535	4897
	35	1946	2054	2271	2488	2922	3356	3790	4224	4658	5092	5526	5960
50	30	1494	1594	1795	1996	2398	2800	3202	3604	4006	4408	4810	5212
	32	1711	1819	2035	2251	2683	3115	3547	3979	4411	4843	5275	5707
	34	1956	2071	2302	2533	2995	3457	3919	4381	4843	5305	5767	6229
	36	2228	2350	2597	2844	3338	3832	4326	4820	5314	5808	6302	6796
	38	2519	2650	2913	3176	3702	4228	4754	5280	5806	6332	6858	7384
	40	2835	2975	3255	3535	4095	4655	5215	5775	6335	6895	7455	8015
	42	3197	3347	3647	3947	4547	5147	5747	6347	6947	7547	8147	8747
	45	3818	3991	4337	4683	5375	6067	6759	7451	8143	8835	9527	10219
	50	5063	5281	5717	6153	7025	7897	8769	9641	10513	11385	12257	13129

Art. 9. Especial pains should be taken to **secure an unyielding foundation for culverts and drains under high embkts;** otherwise the superincumbent weight, especially under the middle of the embkt, may squeeze them into the soil below, if soft or marshy; and thus diminish the area of waterway, or at least cause an ugly settlement at the midlength of the culvert. Also, in soft ground, the embkt may press the side walls closer together, narrowing the channel. This may be prevented by an inverted arch, or a bed of masonry, between the walls. A stratum from 3 to 6 ft thick, of gravel, sand, or stone broken to turnpike size, will generally give a sufficient foundation for culverts in treacherous marshy ground; or quicksand, with but a moderate height of embkt. It should extend a few feet beyond the masonry in every direction, and should be rammed; the sand or gravel being thoroughly wet, if possible, to assist the consolidation. Piling will sometimes be necessary. If the masonry is built upon timber platforms, or a smooth surface of rock, care must be taken to prevent it from sliding, from the pres of the earth behind it. This same pres may even overthrow the piles, if they are not properly secured against it.

Art 10. Drains. Drains of the dimensions in Fig 11, contain 1 perch, of 25 cub ft; or .926 of a cub yd, per ft run.

They are frequently built of dry scabbled rubble, and paved with spawls. When there is much wash through them, with a considerable slope, it is better to continue the foundation

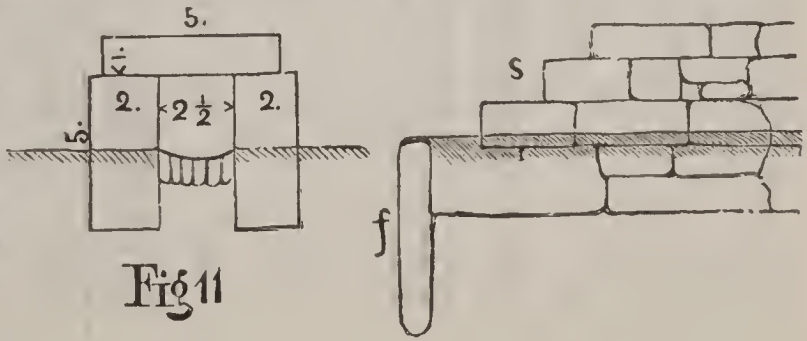


Fig 11

solid clear across. This is often done without those causes, inasmuch as the additional masonry is a mere trifle; and the excavation of a single broad foundation-pit is less troublesome than that of two narrow ones. A deep flag-stone *f* at the entrance, and others at short dists of the length, may be introduced in both drains and culverts, to protect from undermining.

These drains extend under the entire width of the embkt, from toe to toe; and may terminate in steps, as in the side view at S. They are of course better when built with mortar, with an admixture of cement to prevent the water when full from leaking into and softening the embankment.

Sometimes two or three such drains may be placed parallel to each other, instead of a culvert. When two are so placed, they contain only 1½ times the masonry of one; still their use will generally involve no saving of masonry over a culvert. A man can crawl through Fig 11 to clean it.

Art. 11. The drainage of the roadways of stone bridges of several arches, is generally effected by means of open gutters, which descend slightly from the crowns of the arches, each way, until they reach to near the ends of the respective spans.

There they discharge into vertical iron pipes built into the masonry. The upper ends of the pipes should be covered by gratings. When inconvenience would result from the water falling upon persons passing under the arches, these pipes may be carried down the entire height of the piers; but when such is not the case, they may extend only to the soffit, or under face of the arch; allowing the water to fall freely through the air from that height.

Table 6, of approximate contents, in cub yds, of a solid pier of masonry, 6 ft by 22 ft on top; and battering 1 inch to a ft on each of its 4 faces. The contents of masonry of such forms must be calculated by the prismoidal formula; and not by taking the length and breadth of the pier at half its height as an average length and breadth, as is sometimes done. This incorrect method would give only 6492 cub yds as the content of the pier 200 ft high; instead 7178 yds, its true content. High piers may for economy be built hollow, with or without interior cross-walls for strengthening them, as the case may require; and the batter is generally reduced to ½ inch or less to a foot. Hollow piers require good well-bedded masonry.

(Original.)

Ht. Ft.	Lgth at base.	Bdth at base.	Cubic yards	Ht. Ft.	Lgth at base.	Bdth at base.	Cubic yards	Ht. Ft.	Lgth at base.	Bdth at base.	Cubic yards.
6	23.	7.	32.5	52	30.67	14.67	537	128	43.33	27.33	2759
7	.17	.17	38.6	54	31.	15.	570	130	.67	.67	2848
8	.33	.33	44.9	56	.33	.33	605	132	44.	28.	2940
9	23.5	7.5	51.3	58	.67	.67	641	134	.33	.33	3032
10	.67	.67	58.	60	32.	16.	679	136	.67	.67	3126
11	.83	.83	64.8	62	.33	.33	717	138	45.	29.	3222
12	24.	8.	71.7	64	.67	.67	757	140	.33	.33	3320
13	.17	.17	79.	66	33.	17.	798	142	.67	.67	3420
14	.33	.33	86.4	68	.33	.33	840	144	46.	30.	3521
15	24.5	8.5	94.	70	.67	.67	884	146	.33	.33	3623
16	.67	.67	102	72	34.	18.	928	148	.67	.67	3728
17	.83	.83	110	74	.33	.33	973	150	47.	31.	3835
18	25.	9.	118	76	.67	.67	1021	152	.33	.33	3944
19	.17	.17	127	78	35.	19.	1070	154	.67	.67	4056
20	.33	.33	135	80	.33	.33	1120	156	48.	32.	4168
21	25.5	9.5	144	82	.67	.67	1171	158	.33	.33	4284
22	.67	.67	153	84	36.	20.	1224	160	.67	.67	4402
23	.83	.83	163	86	.33	.33	1278	162	49.	33.	4520
24	26.	10.	172	88	.67	.67	1334	164	.33	.33	4640
25	.17	.17	182	90	37.	21.	1392	166	.67	.67	4763
26	.33	.33	192	92	.33	.33	1451	168	50.	34.	4887
27	26.5	10 5	202	94	.67	.67	1510	170	.33	.33	5014
28	.67	.67	212	96	38.	22.	1569	172	.67	.67	5143
29	.83	.83	223	98	.33	.33	1631	174	51.	35.	5275
30	27.	11.	234	100	.67	.67	1695	176	.33	.33	5409
31	.17	.17	245	102	39.	23.	1761	178	.67	.67	5545
32	.33	.33	256	104	.33	.33	1829	180	52.	36.	5680
33	27.5	11.5	268	106	.67	.67	1899	182	.33	.33	5820
34	.67	.67	280	108	40.	24.	1968	184	.67	.67	5962
35	.83	.83	292	110	.33	.33	2041	186	53.	37.	6106
36	28.	12.	304	112	.67	.67	2115	188	.33	.33	6252
38	.33	.33	329	114	41.	25.	2191	190	.67	.67	6400
40	.67	.67	356	116	.33	.33	2269	192	54.	38.	6552
42	29.	13.	383	118	.67	.67	2346	194	.33	.33	6704
44	.33	.33	411	120	42.	26.	2424	196	.67	.67	6859
46	.67	.67	441	122	.33	.33	2504	198	55.	39.	7016
48	30.	14.	472	124	.67	.67	2587	200	.33	.33	7178
50	.33	.33	504	126	43.	27.	2672	202	.67	.67	7339

Art. 12. Brick Arches. Since even good brick fit for large arches has far less crushing strength than good granite or limestone, and is inferior even to good sandstone, while its weight does not differ very materially from stone, it is plain that it cannot be used in arches of as great span as stone can. Some of those already built, and which have stood for many years, have a theoretical co-efficient of safety of but about 3; whereas the authorities direct us not to trust even stone with more than one-twentieth of its crushing load. This last, however, appears to the writer to be one of those hasty assumptions which, when once admitted into professional books, are difficult to be got rid of. It is his opinion that with good cement, and proper care in striking the centers, one-tenth of the ultimate strength is sufficiently secure against even the abnormal strains caused by the settling at crown, and rising at the haunches when the centers are struck. It is useless to attempt to fix limits of safety for bad materials poorly put together.

Rem. 1. The common practice of building brick arches in a series of **concentric rings,** as at *a c e e*, Fig 12, with no other bond between them than that afforded by the mortar, is censured by authorities, on the ground that the line of pressure in passing from the extrados to the intrados tends to separate the rings, and thus weaken the arch by, as it were, splitting it longitudinally. The reason for using these rings, instead of making the radial joints continuous throughout the depth *m n* of the arch, as at *b*, is to avoid the thick mortar-joints at the back of the arch, and shown in the Fig. If the center of an arch built as at *b* be struck too soon, the soft mortar in these thick joints will be so much compressed as to cause great settlement at the crown, throwing the arch out of shape, and creating such inequality of pressure as might even lead to its fall, especially if flat. As a compromise between rings and continuous joints, they are sometimes employed together, so as to get rid of **some** of the long radial joints; and at the same time to break **at intervals** the continuity of the rings. Thus in Fig 12. which is supposed to be brick-and-a-half deep, beginning at the abutment *a*, we may lay half-brick rings as far as say to *e o e;* then **cutting away** the brick *o* to the line *e e*, we may lay from *e e* to *m n* a block of bricks with continuous radial joints, the same as at *b;* and then start again with three rings; and so on alternately. A still better, but more expensive, mode would be to fill *e e*, *m n* with a regular cut-stone voussoir.

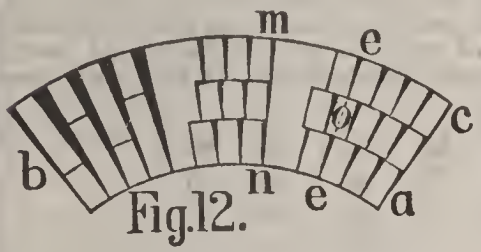


Fig.12.

The proper intervals for changing from rings to blocks will depend upon the number of the rings and the depth *c a* of the arch; reference being also had to reducing the amount of brick cutting as much as possible.

These points can be best decided on from a drawing of a portion of the arch on a scale of 3 or 4 ins to a foot. Generally the rings are made only half-brick, or about 4 to 4.5 ins thick, as at *a c;* and in Brunel's Maidenhead viaduct of two elliptic brick arches of 128 ft span, and 24.25 ft rise; the boldest brick arches yet attempted; but which have been estimated to have a co-efficient of safety of but three against crushing at the crown.

So many others of from 70 to 100 ft span have been successfully built entirely in rings of either half or whole brick thick, as to justify us in attaching but little weight to the above theoretical objection, provided first class cement be used, and time allowed it to become nearly or quite as hard as the bricks themselves, before striking the centers. Under such circumstances we should not object to a series of rings even 1.5 bricks thick, laid alternately header and stretcher, as at *b*.

If the bricks were voussoir-shaped. that is, a little thicker at one end than the other, then rings a whole-brick thick could be used without any increase in thickness of mortar-joint at the back of each ring. Still with more than one ring, the radial joints would not be continuous, as at *b*, but broken as at *ac*. Such bricks however would be more expensive to make; and moreover, in order fully to answer the intended purpose, they would have to be made of many patterns, so as to conform to the many radii used in arches; and even to the radii of the different rings, when the depth of the arch required several of them.

See foot-note, p 671.

Rem. 2. Wet the bricks before laying. See last paragraph of p 670.

Rem. 3. When the ends or faces of a brick arch are to be finished with **cut-stone voussoirs,** these had better not be inserted until some time after the completion of the brickwork, the hardening of the mortar, and a partial easing of the centers; lest they be cracked or spawled by the unequal settlements of themselves and the bricks.

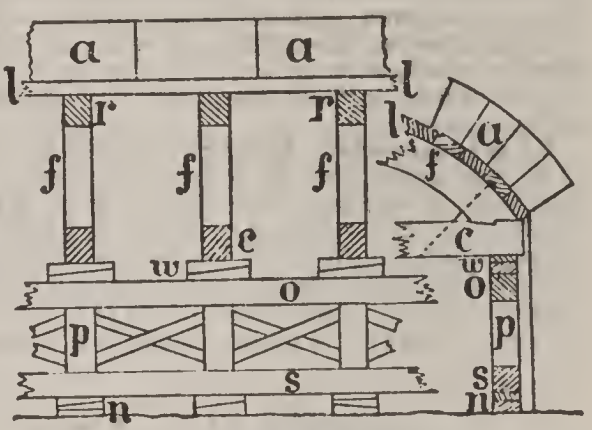
Rem. Brick arches, from their great number of joints are apt to settle much more than cut stone ones when the centers are removed, and thereby to derange the shape of the arch, and at times, without due care, even to endanger its safety, especially if it be large and flat. When the span exceeds about 30 to 35 ft, and particularly if flat, use only brick of superior quality in good **cement** mortar. With even best materials and work we advise the young engineer not to attempt brick arches for railroad bridges of greater spans than about the following. Considerably larger ones than some of them have been built, and have stood; but their coefs of safety are not in all cases satisfactory. In this table the rise is in parts of the span.

R.	S.	R.	S.	R.	S.	R.	S.	R.	S.
.5	100	1/3	88	.225	68	1/6	50	.134	35
.4	97	.29	82	1/5	60	.155	45	1/8	30
.36	93	1/4	75	.183	55	1/7	40		

On the **Filbert Street Extension of the Penna R R, in Phila,** are four brick arches of 50 ft 1 inch span, and with the very low rise of 7 ft. They are 2 ft 6 ins thick, except on their showing faces, where they are but 2 ft. The joints are in common mortar, and about 1/4 inch thick. These four arches, about 200 yards apart, with a large number of others of 26 ft span, form a viaduct. The piers between the short spans are 4 ft 3 ins thick. Those at the ends of the 50-ft spans, 18 ft 6 ins. The springing lines of all the arches are about 6 to 8 ft above the ground. One of the 50-ft arches settled 3 ins upon prematurely striking the centers; but no further settlement has been observed, although the viaduct has, since built (1880) had a very heavy freight and passenger traffic, at from 10 to 20 miles per hour. Roadbed, about 100 ft wide, giving room for 9 or 10 tracks.

CENTERS FOR ARCHES.

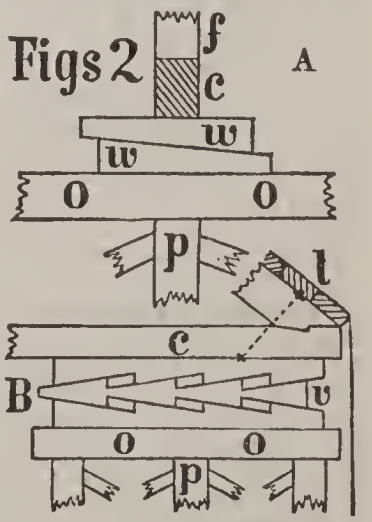
Art. 1. A center is a temporary wooden structure (built lying flat, on a full size drawing, on a fixed platform, under cover or not) for supporting an arch while it is being built. It consists of a number of trusses or **frames,** *f, f*, Fig. 1, placed from 1 to 6 ft apart from cen to cen, and covered with a flooring *l, l*, of rough boards or planks, usually laid close, and called the **sheeting** or **lagging,** immediately upon which the archstones are laid. In Fig 3, the lagging is not laid close. There is no great economy in placing the frames very far apart, on account of the greater required amount of lagging, the thickness of which increases rapidly. For the thickness of lagging see Rem 9, p 719. The frames are of many designs. Thus Figs 7 and 9, pp 554, 556, are often used for small spans (say 15 to 25 ft), their upper timbers supporting throughout their length planks on edge, with their upper edges trimmed to conform to the curve of the arch. Fig 14, p 570, covered in the same way, is sometimes used for still longer spans, say 25 to 40 feet; also Fig 28, p 594; Fig 31, p 595; and Fig 35, p 598, for still longer ones.



Figs 1.

The centers rest by the ends of their chords, *c*, upon wooden **striking wedges** *w*, Fig 1, supported by **standards** composed of posts *p*, whose tops are connected by **cap-pieces** *o;* and whose feet rest on **stringers** *s;* the whole being braced diagonally as shown.

If the ground is very firm, and the arch light, the standards may rest on it, with the interposition of **adjusting-blocks,** *n*, below the stringer, to accommodate irregularities of the surface of the ground, as in the Fig. These blocks should be somewhat double-wedge-shaped, so that by driving them the standard may be raised at any point in case it should settle a little into the ground. But for heavy arches the standards must rest on a much firmer foundation, such as short blocks of brickwork sunk a few feet into the ground, or some other device adapted to the case. Frequently projecting offsets or footings, or at times recesses, are provided in the masonry of the abutments and piers for this express purpose; and with a view to this it is well to design the center at the same time as the arch. Knowing the wt of the arch the proper dimensions of the posts may readily be found by table p 459, etc. Up to spans of 50 or 60 ft a single row of posts (one under each end of each frame) will suffice; but for much larger ones two or three rows, 2 or more feet apart may become expedient, as in the lower Fig 2.



Figs 2

The striking or lowering-wedges before alluded to are for striking or lowering the center after the completion of the arch. They consist of pairs of wedge-shaped blocks, *w w*, at A, Figs 2, of hard wood, from 1 to 2 ft long, about half as wide, and a quarter or more as thick, (sufficient to lower the center from say 2 to 6 or more inches, according to span and other circumstances,) resting on the cap *o*, of the standard, while the chord *c* of the frame rests on *them*. When the end of a frame is supported by two or more posts *p*, as at B, Fig 2, instead of upon one, the striking-wedges are sometimes made as there shown; and where B *v* is one long wedge at right angles to the abutment, and acting as four wedges which may all be lowered together by blows against the end B.

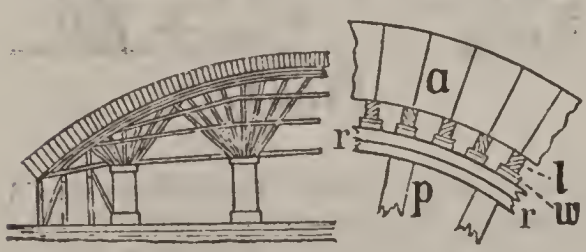
Up to spans of 60 or 80 ft, all the frames may rest on but two wedges like B *v*.

each so long as to reach **transversely** across the entire arch. Then all the frames can be lowered at one operation, as described near end of Art 9.

If we had to consider only the friction of dry wood against dry wood, the taper of these wedges might be as steep as 1 vert to 3 hor, without any danger of their sliding upon each other of their own accord; and they would then require very moderate blows to start them, or even to entirely separate them, when the center had finally to be lowered. But it is of the utmost importance, especially in large arches, that the **centers should be lowered very slowly,** otherwise the momentum acquired by so heavy a body as the arch in descending suddenly even but 2 or 3 ins, might possibly affect its shape, or even its safety.

Therefore the wedges should not have a taper steeper than about 1 in 6 or 8 for arches of less than about 50 ft span; or than 1 in 8 or 10 for larger spans. Vertical lines at equal dists apart should be drawn on the long sides of the wedges as a guide for lowering them all to the same extent at a time; and this should not exceed in all about half an inch a day in intervals of about an eighth of an inch, for 50 ft spans; or about .1 to .25 of an inch per day in all, for spans over 100 ft. Slowness is especially to be recommended in **brick arches,** not only because their greater number of joints exposes them to greater derangement of shape, but because even good brick has much less than the average crushing strength of good granite, limestone, or sandstone, and therefore is far more liable than they to crack, or even to crush (as the writer has seen) when the strains are thrown almost entirely upon their edges, as described in Art 3. For more on brick arches, see p 709.

At Gloucester Bridge, England, of first class cut stone, span 150 ft, rise 35 ft, the centers were entirely struck within the very short space of 3 hours; and the crown of the arch descended 10 ins! **At Grosvenor Bridge,** England, of first class cut stone, span 200 ft, rise 42 ft, such care was taken in easing the centers that the crown of the arch settled but 2.5 ins. This case however was marked by two or three peculiarities, all of which contributed to this favorable result. Namely, the center instead of being a series of frames supported as usual by their ends, and of course involving an appreciable, although small, degree of sagging or settlement, consisted essentially of vertical and inclined posts or struts, see Fig 3, footing on four temporary piers of masonry, 7 or 8 feet thick, built in the river, parallel to the abutments, and as long as they. These piers supported six frames (or rather six series) about 7 ft apart cen to cen, of such struts, footing on cast iron shoes. Fig 3 shows half of one series. Each frame or series consisted of four fan-like sets of posts, all in the same vertical plane. The long horizontal pieces seen extending from side to side of the arch were bolted to the struts to increase their stiffness; and other pieces for the same purpose united the six series transversely. Here each strut sustains its own share of the weight of the archstones, and transfers it directly to the unyielding foundation of the pier; whereas in the usual trussed centers, the entire load rests upon the frames, and is finally transferred to the comparatively unstable support of the posts at their ends.



Figs 3.

The tops *p* of the posts of a series varied about from 5 to 8 ft apart cen to cen; and were connected by a continuous curved rib, *r r*, of two thicknesses of 4 inch plank, bent to conform approximately to the curve of the arch. On this rib were placed pairs of striking-wedges *w* like Fig 2, about 16 ins long, 10 to 12 ins wide, and tapering 1.5 ins, so near together (varying about from 2.5 to 3.5 ft cen to cen) that there was a pair under each joint of the archstones, *a a*. On these wedges, and extending over all six of the frames, were the lagging pieces *l*, 4.5 ins thick.

This peculiar arrangement of the striking-wedges and lagging has, in large spans, great advantages over the usual one of placing them only at the ends of the frames. In the last the entire center and the entire arch are lowered together, without giving an opportunity to rectify any slight derangements of shape or inequality of bearing that may have occurred in the arch during its construction, This center, designed by Mr. Trubshaw, admits of lowering either the whole equally, or any one part a little more or less than the others. He had much experience in large arches, and stated that during the striking he found that he had an arch under better control, or could humor it better, by keeping the haunches a little down, and the crown a little up, until near the end of the operation.

Rem. 1. Instead of piers of masonry for supporting the feet of the posts, wooden cribs or piles may often be used if the arch is over water.

The principle of supporting even trussed frames by struts at points of the chord as far from the abutments as circumstances will admit of (in addition to those at the very ends) should always be applied when possible, in order to reduce their sagging to a minimum. **Steps or offsets in the masonry** of the abutments and piers may be provided for receiving the feet of such struts, when they are inclined.

Rem. 2. Screws may be used instead of wedges for lowering centers. At the Pont d'Alma, Paris, ellipse of 141.4 ft span, and 28.2 ft rise, the frames were supported by wooden pistons or plungers, the feet of which rested on **sand confined in plate-iron cylinders** 1 ft in diam and height, and having near the bottom of each a plug which could be withdrawn and replaced at pleasure, thus regulating the outflow of the sand and the descent of the center. This device succeeded perfectly, and is well worthy of adoption under arches exceeding about 60 ft span. When much larger than this the driving of the wedges on striking requires heavy blows, and becomes a somewhat awkward operation, requiring at times a battering-ram, even when the wedges are lubricated. In railroad cuttings crossed by bridges, **the earth under the arch** has been made to serve as a center, by dressing its surface to the proper curve, and then embedding in it curved timbers a few feet apart, and extending from abut to abut, for supporting the close plank lagging.

Rem. 3. All centers must yield or settle more or less under the wt of the arch, especially when supported only near their ends; and since the arch itself also settles somewhat not only when the centers are struck, but for some time after, it is advisable to make them at first a little higher than the finished arch is intended to be. This extra height, when the supports are at the ends, may be from 2 to 4 ins per 100 ft of span for cut stone arches (according to time of striking, character of masonry, workmanship, etc.), and about twice as much in brick ones.

Rem. 4. The proper time for striking centers is a disputed point among engineers, some contending that it should be done as soon as the arch is finished and sufficiently backed up; and others that the mortar should first be given time to harden. It is the writer's opinion that inasmuch as in cut-stone arches the mortar joints should be very thin; and since, in such, the mortar is at best of very little service, it is of no importance when they are struck; provided the masonry backing, and the embkt up to *y n* Fig 2, p 698, have been completed; but that in brick or rubble, the numerous joints of both of which require much mortar, (which for hardness should consist largely of cement,) 3 or 4 months, or longer, if possible, should be allowed it to harden sufficiently to prevent undue compression and consequent settlement when the centers are struck. The continuance of the centers need not interfere with traffic over the bridge.

Art. 2. The pressure of archstones against a center is very trifling until after the arch is built up so far on each side that the joints form angles of 25° or 30° with the horizontal. Theoretical discussions on this pressure make no allowance for accidental jarrings in laying the archstones, or by the accumulation of material ready for use, laborers working on it, &c. Without going into any detail, we merely advise on the score of safety not to assume it at less than about the following proportions or ratios to the weight of the entire arch, namely, in a semicircular arch .47; rise .35 span, .61; rise .25 span, .79; rise .2 span, .86; rise .167 span, or less, 1, or equal to the wt of the arch. This gives the pressure of a semicircular arch upon its centers rather less than half its wt. **The wt of the centers themselves** when supported only near the ends must be considered as part of the load borne by them.

Art. 3. We have seen that as an arch *a a a* is being gradually built upward on both sides, after passing the points *e, e*, Fig 4, where its joints form angles *a s e*, of about 30° with the horizontal *a a*, the arch begins to press more and more upon the centers; thereby tending to flatten them at the haunches, as shown at *h* in the dotted line; and consequently to raise them at the crown, as shown at *c*. But as the building goes on still higher, the added stones press much more heavily upon the centers than those below had done, and thereby tend to a final derangement of the centers just the reverse of that caused by the lower ones; namely to depress them at the crown *a*, as at *o;* and consequently to raise the haunches as at *n;* and this the more because the upper stones actually tend to lift or ease the lower ones from the lagging. In some cases where this tendency has been increased by **forcing** the keystones into place by too hard driving, the lagging under the haunches could be drawn out without any trouble before the centers were eased at all. On striking the centers this tendency to sink at crown and

rise at haunches is very apt to exhibit itself more or less dangerously in the arch-stones themselves, as in Fig 5, causing those near the crown to press very hard together at the extrados, and to separate from each other at the intrados; while near the haunches the reverse takes place. Hence the angles of the stones are frequently split and spawled off near *c* and *h* by this unequal pressure. These

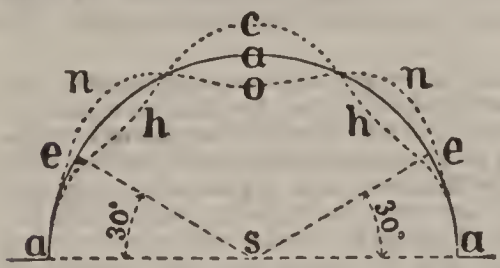


Fig. 4.

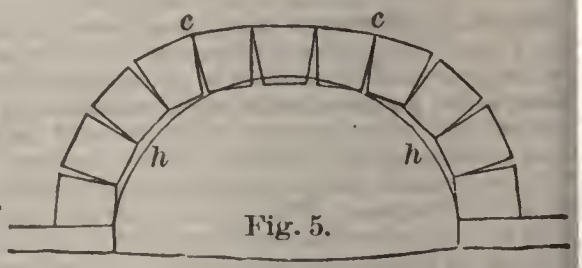


Fig. 5.

derangements are of course much more likely to be serious in high arches than in flat ones, especially if their spandrels are not sufficiently built up before lowering the centers.

In the Grosvenor bridge, before alluded to, of 200 ft span, this dangerous excess of pressure near *c* and *h* was prevented by covering the skewback joint of the springing course at each abutment with a wedge of lead 1.5 ins thick at the intrados of the arch, and running out to nothing at the extrados. Beside this a strip 9 ins wide of sheet lead was laid along the intrados edge of every joint until reaching that point at which it was judged that the line of pressure would pass from the intrados to the extrados; after which similar strips were laid along the extrados edges of the joints, up to the crown. Hence when the centers were struck, this excess of pressure merely compressed the lead, and was thus enabled to distribute itself more evenly over the entire depth of the joints. See Trans Inst Civ Eng London, vol i.

At the bridge at Neuilly, France (of 5 elliptic arches of 120 ft span, and 30 ft rise), the centers were so radically defective in design that the arches sank 13.25 ins at crown during the time of building; and 10.5 ins more during and immediately after the striking; or say 2 ft in all. Their construction made the striking very tedious and hazardous; greatly endangering the lives of the workmen and the existence of the arches. Some of the joints at the extrados at the haunches opened an inch each; and those at the intrados of the crown .25 of an inch. By the exercise of great care and humoring in lowering the centers, these openings were much reduced.

Rem. 1. Chamfering the edges of the archstones diminishes the danger of their spawling off from unequal pressure; as does also the **scraping out of the mortar of the joints** for an inch or two in depth before striking the centers.

Rem. 2. It is evident that in order to prevent, or at least to diminish the alternate derangements of the center, those of its web members which at first acted as **struts** near the haunches, Fig. 4, to prevent them from sinking as at *h*, must afterwards act as **ties** to prevent them from rising as at *n*; while those which at first acted as ties near the crown *a*, to prevent it from rising as at *c*, must afterwards act as struts to prevent it from sinking as at *o*. In other words, the principle of **counter-bracing** must be attended to as well in a frame or truss for a center, as in one for a bridge. If the web members are on the Warren or simple triangle system, as in Fig 23, p 589, this may be effected by making each member a tie-strut; or the Pratt, or the Howe system, Fig 35, p 598, may be used.

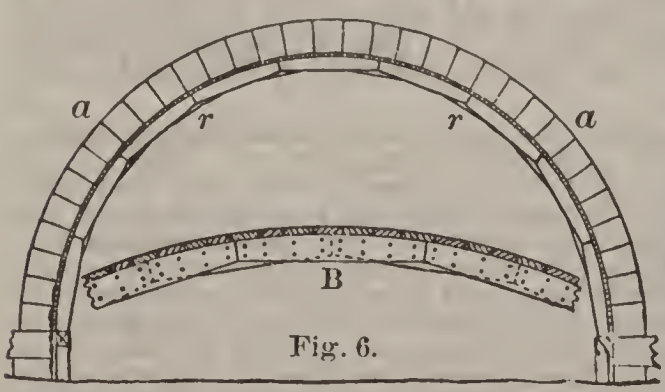


Fig. 6.

Art. 4. From the foregoing it is plain that **a simple unbraced wooden**

rch, or curved rib is, on account of its great flexibility, about as unfit a form s could be chosen for a center, except for very small spans, where a great proporonal depth of rib can be readily secured. Still the writer has seen it used for a ut-stone semicircular arch of 35 ft span, with archstones 2 ft deep. Fig 6 shows ne rib *r r*, and the arch, *a a*, drawn to a scale. Each rib consisted of two thicknesses f 2 inch plank in lengths of about 6.5 ft, trenailed together so as to break joint, s at B. Each piece of plank was 12 ins deep at middle, and 8 ins at each end; he top edge being cut to suit the curve of the arch. The treenails were 1.25 ins n diam; and 12 of them showed to each length. These ribs were placed 17 ins part from cen to cen, and steadied together by a bridging piece of inch board, 13 ns long, at each joint of the planks, or about 3.25 ft apart. Headway for traffic eing necessary under the arch, there were no chords to unite the opposite feet f the ribs. The ribs were covered with close board lagging, which also assisted n steadying them together transversely. As the arch approached about two-hirds of its height on each side, the ribs began to sink at the haunches, as at *h*, ig 4; and to rise at the crown, as at *c*. This was rectified by loading the crown ith stone to be used in completing the arch; which was then finished without urther trouble.

A still more striking example of the use of a simple unbraced ooden rib, was in the old National Turnpike bridge over Wills Creek, Virginia. his bridge, of which one arch with s center is shown in Fig 7 drawn a scale, consisted of two elliptic ut stone arches 26.5 ft wide across oadway, and of 60 ft span, and 15 rise. The archstones were 3 ft eep at crown, and 4 ft deep at kewbacks. **Each frame of he center** was a simple rib 6 ns thick, composed of three thicknesses of 2 inch oak plank in different lengths (about 7 to 15 ft) to suit the curve, nd at the same time to preserve a width of about 16 ins at the middle of each ength, and 12 ins at each of its ends. The thicknesses were well treenailed together, breaking joint and showing from 10 to 16 treenails to a length.

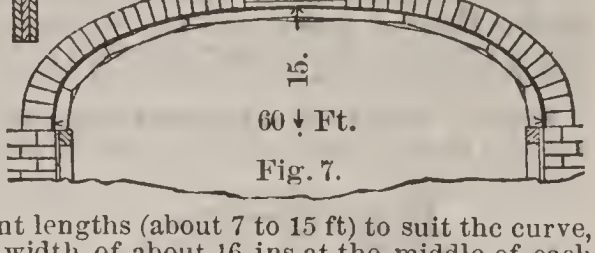


Fig. 7.

Here, as in Fig 6, there were no chords, owing to the violence of the floods in he creek. These ribs were placed 18 ins from cen to cen, and steadied against ne another by a board bridging-piece 1 ft long, at every 5 ft. These were of ourse assisted by the lagging.

When the archstones had approached to within about 12 ft of each other near he middle of the span, the sinking at the crown, and the rising at the haunches ad become so alarming that pieces of 12 × 12 oak, 00, were hastily inserted at ntervals, and well wedged against the archstones at their ends. The arch was hen finished in sections between these timbers, which were removed one by one s this was done.

Rem. 1. Such instances of partial failure are very instructive. t is indeed by such, rather than by theoretical deductions, that the proper dimenions are arrived at in a vast number of cases pertaining to engineering, machinery, &c.* Thus we might with entire confidence of no serious mishap, apply ibs of the foregoing dimensions to spans only half as great.

Rem. 2. Assuming the rib-planks to be 12 ins wide, it would, as a matter of etail, be better to make them about 10 ins wide at the ends instead of the 8 ins n Fig 6 making top curve 2 ins. To secure this, their lengths, depending on the **adius** of the rib, must not exceed those in **the following table:**

Rad of Arch.	Greatest Length.	Rad of Arch.	Greatest Length.
Feet.	Feet and Ins.	Feet.	Feet and Ins.
5	2 " 5	30	6 " 4
10	3 " 4	35	7 " 0
15	4 " 2	40	7 " 6
20	5 " 0	45	7 " 10
25	5 " 9	50	8 " 2

* The young engineer should make and preserve full notes in detail of all such as may fall within is notice; and if the professional journals would do the same thing in regard to failures which are onstantly occurring, they would greatly increase the value of their papers.

If cut 1½ times as long as this table, they will be very approximately 8 in wide at ends; or each will on top curve 4 ins.

Art. 5. In cases where all possible headway is essentia during the building of the arch, as in the two foregoing ones, the writer woul suggest the expedient rudely illustrated b Fig 8; namely to **place the center above the arch,** instead of **belov** it; and after the arch is completed in sec tions, *a a*, instead of **lowering** the cen ters, **to take them apart.** The cen ters might resemble in principle Fig 35½ p 598.

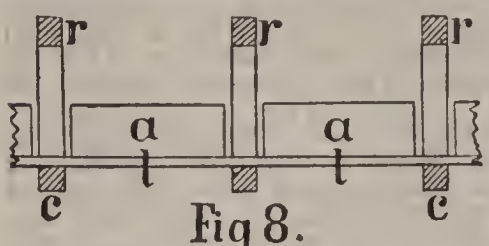


Fig 8.

Fig 8 is a transverse section through par of the center, and of the arch *a a*. Her *r c*, *r c*, *r c*, are frames of the center say 5 or ft apart; and of any depth and constructio whatever that may be necessary to insure absolute safety; and *l l* is the lagging Having built the arch from abutment to abutment in a series of sections *a*, *a*, *a*, ne cessarily separated say a foot or more by the deep frames, we may take the center apart, and then fill in the narrow intermediate sections upon a lagging suspende by iron rods from the already completed sections. Good concrete might be use for these narrow sections. In some cases it might be well to use **deep plate iron ribs** of I section, resting the lagging on the lower flange. Part of th web might be left remaining embedded in the masonry; and the upper part an both flanges removed after the arch is finished.

Art. 6. Centers with hor chords *c c* Fig 9 are objectionable (notwith standing their strength) in large spans of great rise, as on right side of the Fig, o account of the excessive lengt required for the web members and hence it will in such case usually be found expedient t adopt something analogous t what is shown on the left hand of the Fig. Here a truss *f*, shorte and shallower than that on th right hand, is substituted for th latter. At its ends provision mus be made for supporting not only itself, but the archstones below it. As the pressure of these low er archstones is comparatively small, this may usually be effected by resting the end of the frame *f* upon another and shallower frame *o a*. This may in large spans be aided by either inclined or vertical struts, either single or braced together; or as the trestles on p 755. Sometimes one shallow truss like *f* is sustained upon another truss throughout its entire length. The striking-wedges for these various supports may be placed at either their tops or their feet, as may be most convenient.

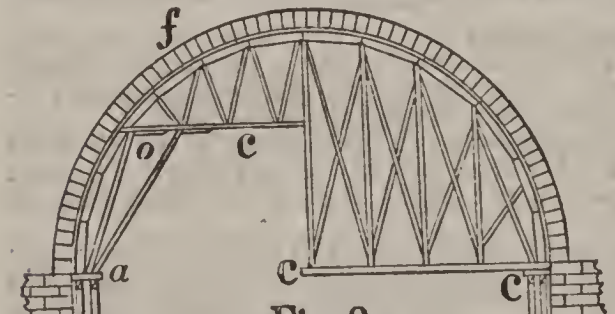


Fig 9.

Art. 7. For flat arches of 10 feet clear span, a mere board *o o* Fig 10, 12 ins deep, by 1.5 ins thick, with another piece *c* of the same thickness on top of it, trimmed to the curve, and con fined to *o o* by nailing on two cleats of nar row board, will answer every purpose, with intervals of 18 ins from cen to cen. If the upper piece also is as much as 12 ins deep at its center, the clear span may be extended to 15 ft.

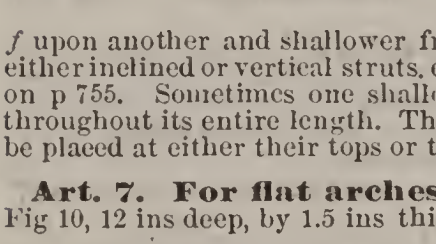


Fig 10.

For spans of 10 to 15 ft, and of any rise, two thicknesses of plank from 1 to 2 ins thick according to span; 8 to 12 ins wide at middle of each piece, in lengths as per table, Rem 2, Art 4, well **nailed** or **spiked** together, according to span, breaking joint as in Fig 6, will answer for distances of 2 to 3 ft apart cen to cen. For greater dists apart increase the thick ness of the planks proportionally.

If the centers have to be moved from place to place, to serve for other arches, then, to preserve them from injury in handling, their feet should be united by nailing on one or both sides of each frame a chord piece of about 1

neh board; and also a vertical pieee or pieces of the same size from the center f the chord to the top of the frame.

Even when they are not to be moved, the chord pieces are useful ven in so small spans, inasmuch as they render the striking easier, by not allowng the feet of the ribs to give trouble by spreading outward and pressing against ne abutments.

For spans of 15 to 30 ft, and for any rise not less than one sixth of the pan, the following dimensions, varying with the span, may be used for distanees part of 3 ft from cen to cen. ee Fig 11. **For the bow** *b*, wo thicknesses of 1 to 2 inch lank from 9 to 12 ins wide t the middle; and from 7 to 0 ins at each end, well spiked ogether breaking joint as at B, ig 6. **For the chord** *c*, two hicknesses of plank of same ize as the bow at its middle; laeed on outsides of bow, and ell spiked to its ends. **A ertical** *v*, in one pieee as ide as a bow plank, and twice s thick. Its top is placed under the bow, and is confined to it by two pieces, *o*, *o*, f bow plank twice as long as the bow plank is deep, and spiked to both *v* and the ow. The foot of *v* passes between the two thicknesses of the chord *c*, and is piked to them. **Two oblique tie-struts,** *s*, each of two pieees of bow lank, outside of the bow and vertical *v*; footing against eaeh other; and spiked o bow and *v*. These with *v* divide the bow into 4 parts.

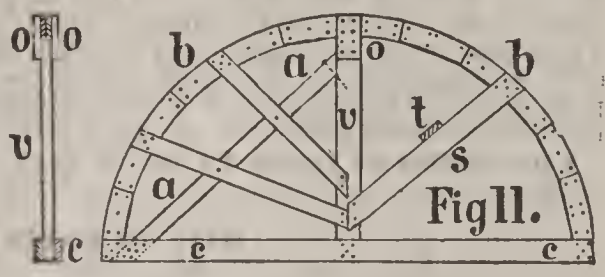


Fig 11.

Rem. 1. The above dimensions are suitable to a rise of one sixth. If the ise is one fourth, the **thickness only** of the planks may be reduced one third art; and for a rise of one third or more, we may reduce to one half.

Rem. 2. If in the larger of these spans the struts *s* should show any ineliation to bend sideways, nail on some pieces *t* from frame to frame. Also in the rger ones with rises exeeeding one third, insert four double struts *s*, instead f two; thus dividing the bow into 6 parts, as at left side of Fig. 11. For spans of 5 to 35 ft, add also two struts like *a a*, of same size as *v*.

Art. 8. For spans greater than about 30 ft, the writer believes hat as a general rule (liable to modifications according to the judgment of the ngineer in charge) the following ideas will lead to safe practiee. Namely, to dopt a bowstring truss with a simple Warren or triangular web, as at *f* on the eft side of Fig 9. The bow to rest on the chord, and each to be of a single thickness. The web members (especially in large spans) to be also of single thickness, nd placed below the bow, resting on the chords, and well strapped to both, so as o act as either ties or struts. In smaller spans the web members may each be in wo thicknesses, one bolted or trecnailed to eaeh side of the bow and chord. Other nodes will suggest themselves; but we have not space for such details.

Or a web of the Howe, or of the Pratt system, as on the right side of Fig 9 may be sed. But in refercnec to both of these it may be remarked that **the use of ong iron rods in centers** of large spans is highly objectionable, owing o the different rates of expansion between iron and wood. Therefore if these ystems are used, all the members should be of wood. **The lattice** may be used.

Even when the rise of the **arch** exeeeds .25 of the span, it is better not to let hat of the **centers exceed** that limit; but adopt the expedient shown at he left side of Fig 9, with a rise of about one sixth of the span.

Rem. 1. To fix on the number of web triangles in a Warren russ or frame for a center, find the square root of the span, and to it add one enth of the span. Divide their sum by 2, and call the quotient *n*. Divide the pan by *n*. If this quotient is a whole number use it; or if the quotient is partly lecimal, use the whole number nearest to it, as a distance in feet to be stepped off long the chord; thus dividing the chord into a number of equal parts. All the oints thus found on the chord, are the places for the **feet** of the triangles. Next, from half way between each two of these points, draw vertieal lines to the ow. The points thus found along the bow, are the places of the **tops** of the riangles. This rule will be used in connection with the following Table of Areas f Bows, as the two are dependent on each other.

In large arches **the timber of the bow should not be wasted** by rimming its upper edges to the curve of the arch, but should be left straight; and eparate pieces so trimmed, like *c* in Fig. 10, should be spiked on top of them.

The transverse area of the bow, in square inches, may be taken from the following table; and may in practice be assumed to be uniform throughout its entire length; which in fact it is quite approximately. See Rem 2.

TABLE FOR BOWSTRING CENTERS.

Table of areas in square inches at the crown of each Bow, of **properly trussed** Bowstring frames for centers of stone or brick arches. The frames to be placed 5 feet apart from cen to cen. With these areas, the combined weight of arch, center (of oak), and lagging, will in no case in the table strain the Bow at crown of the greatest spans quite 1000 lbs per square inch; diminishing gradually to 600 or 700 lbs in the smallest spans, which are more liable to casualties. The depths of the archstones may be taken **fully** equal to those in our table, 697. Although centers of moderate span are usually made of white or yellow pine, spruce, or hemlock, all of which are considerably lighter than oak, we have for safety assumed them to be of oak, in preparing our table.

For spans of from 10 to 20 feet use the same sizes as for 20 feet.

	Rise in parts of the Span.							Origina
	.5	.4	.35	.3	.25	.2	.15	.1
Span in feet.	**Areas of transverse section of Bow,** in square inches.							
20	14	17	19	21	24	29	38	59
25	18	22	25	28	33	40	53	80
30	23	28	32	37	43	51	71	103
35	28	34	40	45	54	64	87	125
40	34	41	48	55	65	77	106	150
45	40	49	57	65	76	92	126	175
50	47	57	66	76	89	107	146	203
55	53	64	75	87	102	121	166	233
60	60	73	85	99	115	135	187	263
65	68	81	95	110	129	151	209	294
70	75	90	105	122	143	168	233	325
75	83	99	115	133	157	184	256	357
80	91	108	125	145	171	201	279	390
85	99	117	136	157	185	218	302	423
90	108	127	147	169	199	235	325	457
95	115	136	158	181	214	252	348	490
100	123	146	169	194	229	270	372	524
110	133	166	191	219	260	307	420	592
120	155	187	213	246	291	345	470	660
130	172	208	237	274	323	384	520	
140	190	230	263	303	357	424	572	
150	209	252	289	333	393	466		
160	229	276	315	365	430	509		
170	250	299	343	399	469			
180	272	323	373	435	511			
190	294	347	403	472				
200	318	372	435	509				

Rem. 2. The square root of any of these areas gives in inches the **side of a square bow** of that area. The distances apart of the triangles which form the web of the frame, having first been found by Rem 1 (for said Rem and this table are dependent on each other), the above areas for bows 5 ft apart from cen to cen, suffice not only to resist the pressure along the bow, but also, as **square beams,** to sustain with a safety in no case less than about 5, the load of archstones resting upon them between the adjacent tops of two triangles; and with very trifling deflections. It is therefore unnecessary to deepen the ribs for that purpose; although it may be done (preserving the same area) in case considerations of detail should render it desirable.

As before suggested, it will generally be best, in spans exceeding 30 or 40 ft, to give the bow a rise not exceeding about one fifth or one sixth of the span; and to support the frames as at *f*, Fig 9.

The size of the chord may be the same as that of the bow; and like it uniform from end to end; care however being taken that it be not materially weakened by footing the bow upon its ends; or (when too long for single timbers) by the splicing necessary to prevent its being stretched or pulled apart by

the thrust of the bow. When, however, the chord can be placed at, or a little below the springs of the arch, all danger of this kind may be avoided by simply wedging its ends well against the faces of the abutments.

As to the size of the web members, when a bowstring truss is **fully loaded** on top of the bow, (as is approximately the case with a center and its archstones,) the strains on the web members are quite insignificant, and arise chiefly from the weight of the center itself; **but while it is being so loaded,** they are not only greater, but are constantly changing, not only in amount, but also in character—being at one period compressive, and at another tensile.

Hence it would be very tedious to calculate the dimensions of the web members. Fortunately the necessity for doing so is in a great measure obviated by the fact that a center being but a temporary structure, the timber composing it is not ultimately wasted if a greater quantity of it is used than is absolutely required. Moreover facility of workmanship is secured by not having to employ timbers of many different sizes.

Hence the writer will venture to suggest, entirely as a rule of thumb, to **give each web member half the transverse area of the bow,** taking care to make each of them a **tie-strut.**

Rem. 3. As to details of joints, we refer to the Figs on pages 611, 613; merely suggesting here the use of long and wide iron shoes where timbers are subjected to great pressure sideways.

Rem. 4. To prevent the thrust of the bow when its rise is small, from splitting off the ends of the chords, the two may be united by many more bolts than are employed in roof trusses, &c, where only one is generally placed near each end of the chord. But they may when required be inserted at intervals extending to many feet from the ends. They should have strong large washers; and may have about the same inclination as the shortest web member.

Another way of securing the same end in smaller spans, is by completely encasing the two sides of the bow and chord, to a distance of a few feet from their ends, in short pieces of board or plank spiked to both of them, and having about the same inclination as just suggested for bolts.

Rem. 5. Build up both sides of the arch at once, in order to strain the centers as little as possible.

Rem. 6. When a bridge consists of more than one arch, and they are to be built one at a time, there must be at least two centers; for a center must not be struck until the contiguous arches on both sides are finished, for fear of overturning the outer unsupported pier. Therefore if there are but two arches, they must be built at once, requiring two centers.

Rem. 7. Always use supports either vertical or inclined (and provided with striking-wedges) under the frames, and intermediate of the end supports, when possible; even if they can extend out but a few feet from the abutments, as at the left side of Fig 9.

Rem. 8. The weight of large centers and their lagging is greater for flat arches than for high ones of the same span; and also approaches nearer to that of the supported arch.

Rem. 9. Thickness of lagging. The following table gives thicknesses which will not bend more than an eighth of an inch under the weight of any probable archstones **adapted to the respective spans;** and generally not so much.

TABLE OF LAGGING.—Original.

Distance apart of frames, **in the clear.**	**Span of center in feet.**					
	10.	**20.**	**50.**	**100.**	**150.**	**200.**
	Thickness of close lagging not to bend more than $\frac{1}{8}$ inch.					
Feet.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
6	$3\frac{1}{4}$	$3\frac{5}{8}$	$4\frac{1}{4}$	$4\frac{3}{4}$	5	$5\frac{1}{4}$
5	$2\frac{1}{2}$	$2\frac{7}{8}$	$3\frac{3}{8}$	$3\frac{5}{8}$	$3\frac{7}{8}$	4
4	$1\frac{7}{8}$	$2\frac{1}{8}$	$2\frac{1}{2}$	$2\frac{3}{4}$	$2\frac{7}{8}$	3
3	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{3}{4}$	$1\frac{7}{8}$	2	2
2	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{8}$	$1\frac{1}{4}$

With thicknesses three quarters as great as these, the bending may reach a full quarter inch; which may be allowed in dists apart of 3 or more ft.

Rem. 10. Centers are framed, or put together, (like iron bridges) on a firm, level temporary floor or platform, on which a full-size drawing of a frame is

first made. As each frame is finished, it is removed to its place on the piers or abuts.

Art. 9. The Wissahickon Bridge of the Reading R R, at Philadelphia, has five arches of 65 ft span, 23 ft rise, 28 ft wide (archstones 3 ft deep, with dressed beds and joints, in cement mortar); with four cutstone piers 9.5 ft thick at top, and from 35 to 50 ft high. It contains about 15400 cub yds of masonry.* **Each center** consisted of 7 frames or trusses of hemlock timber, of the Bowstring pattern, with lattice (p 596) web-members; and as nearly as may be, of the same span and rise as the arches. They were placed 4.5 ft apart from center to center; and were supported near each end *f*, Fig 13 (a transverse section to scale) by a hemlock post *p*, 12 ins square.

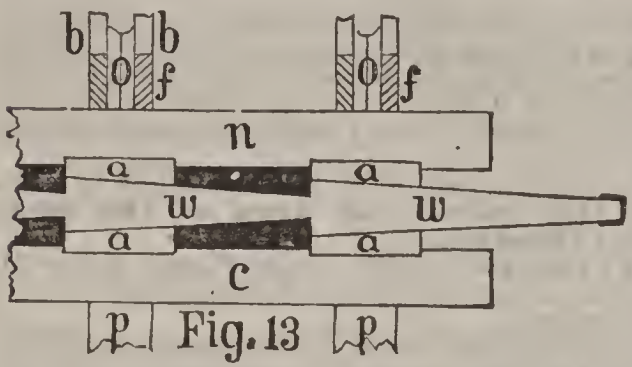


Fig. 13

The bow was of two thicknesses *b b* of hemlock plank, 6 ins apart clear, in lengths of 6 ft, with their upper edges cut to suit the curve of the arch. Each piece was 4 ins thick, by 13.5 ins deep at its middle and 12 ins at its ends. These pieces did not break joint; but at each joint were four ¾ inch bolts, with nuts and washers, uniting them with chocks or filling-in pieces. The bow, *b b*, footed on top of the ends of the chords *f f*; and the angle formed by their meeting (seen only in a side view) was (for about 2.5 ft horizontal and 5.5 ft vertical) filled up solid with vertical pieces, to afford a firmer base for resting the frame on *n*; beyond which it extends (in a side view) about 18 ins.

The chords *f* were of two thicknesses of 4 × 12 hemlock plank, 6 ins apart clear, and most of them in two or three lengths; breaking joint, and with two ¾ inch bolts, with nuts and washers, at each joint, for bolting them together, and to filling-in pieces. **The web members** of each frame were 26 lattices, *o*, of 3 × 12 inch hemlock, crossing each other about at right angles, at intervals of about 3.5 ft from center to center, and passing between the two thicknesses *b b* of the bow and *f f* of the chords. A few of the lattices were in two lengths, and the joints were not at the crossings. The lattices were connected at each crossing by two hard wood treenails 9 ins long, and 2 ins diam; and one such, 18 ins long, passed through the intersection of each end of a lattice with a bow or chord. The first lattice foot about 4 ft from the end of a chord. They do not extend above the top of the bow. All the spaces between the two thicknesses of bow or chord, where not occupied by the ends of lattices, were completely filled by chocks, well spiked.

Each frame contained about 360 cub ft of timber; and weighed about tons. They were very flexible laterally until in place, and braced together by transverse horizontal planks spiked to their chords; and by 5 others above them spiked to the lattices.

Until the keystones were placed, all the joints of the frames continued tight, under the pressure from the arch, and from the unfinished backing to the height of about 14 ft above the springing line; but after the keystones were set, all the joints of the chords alone opened from .25 to .75 of an inch; and at the same time the lagging under the *haunches* of the arches became slightly separated from the soffit of the masonry.

Each center sank but a full inch at the middle, under the pressure from the arch and 14 ft of backing.

The portion of the bridge above the piers was about two thirds completed before the centers were struck.

There was one wedge *w*, *w*, (32.5 ft long, of 12 × 12 inch *oak*) under each end of a center. It was trimmed to form 7 smaller ones *w*, *w*, each 4.5 ft long, and tapering 7 ins; one under each end of each frame *f*. They played between tapered blocks *a*, *a*, of *oak*, 2 ft long, 1 ft wide, let 1 inch into the cap *c*, or into the piece *n* on which last the frames *f*, *f*, rested. The sliding surfaces were well lubricated with tallow when put in place.

The wedges were struck with ease, at one end of a center at a time, by a oak log battering-ram 18 ft long, and nearly a ft in diam, suspended by ropes, and swung and guided by 4 men. They generally yielded and moved several inches at the second blow with a 3 or 4 ft swing. Although each wedge was loosened *entirely* within 2 or 3 minutes, thus lowering the centers *very suddenly*, yet on account of the

* This bridge, finished without accident, in 1882, reflects much credit on the late William Lorenz, Esq, Ch. Eng; on Mr. Charles W. Buchholz, Assistant in Charge; and on the skilful and energetic contractors, William & James Nolan, of Reading, Penna. These last most cordially assisted the writer in making observations during the entire progress of the work.

od character of the masonry, not the slightest crack of a mortar joint could after-
rds be detected in any part of the work. After three days the average sinking of
e keystones was only .35 of an inch; the least was $\frac{1}{4}$: and the greatest $\frac{5}{8}$ of an
ch. The heads and feet of the posts *p* compressed the hemlock caps *c*, and the
lls, about $\frac{3}{8}$ of an inch each, showing that for arches of this size the caps and sills
d better be of some harder wood, as yellow pine or oak; although probably the
mpression was facilitated by the large mortices, 3 by 12 ins, and 6 ins deep.

RAILROADS.

RAILROAD CONSTRUCTION.

TABLE OF ACRES REQUIRED per mile, and per 100 feet for different widths.

Width. Feet.	Acres per Mile.	Acres per 100 Ft.	Width. Feet.	Acres per Mile.	Acres per 100 Ft.	Width. Feet.	Acres per Mile.	Acres per 100 Ft.	Width. Feet.	Acres per Mile.	Acres per 100 Ft.
1	.121	.002	26	3.15	.060	52	6.30	.119	78	9.45	.178
2	.242	.005	27	3.27	.062	53	6.42	.122	79	9.58	.181
3	.364	.007	28	3.39	.064	54	6.55	.124	80	9.70	.184
4	.485	.009	29	3.52	.067	55	6.67	.126	81	9.82	.186
5	.606	011	30	3.64	.069	56	6 79	.129	82	9.94	.188
6	.727	.014	31	3.76	.071	57	6.91	.131	½	10.	.189
7	.848	.016	32	3.88	.073	¾	7.	.133	83	10.1	.190
8	.970	.018	33	4.00	.076	58	7.03	.133	84	10.2	.193
¼	1.	.019	34	4.12	.078	59	7.15	.135	85	10.3	.195
9	1.09	.021	35	4.24	.080	60	7.27	.138	86	10.4	.197
10	1.21	.023	36	4.36	.083	61	7.39	.140	87	10.5	.200
11	1.33	.025	37	4.48	.085	62	7.52	.142	88	10.7	.202
12	1.46	.028	38	4.61	.087	63	7.64	.145	89	10.8	.204
13	1.58	.030	39	4.73	.090	64	7.76	.147	90	10.9	.207
14	1.70	032	40	4.85	.092	65	7.88	.149	¾	11.	.209
15	1.82	.034	41	4.97	.094	66	8.	.151	91	11.0	.209
16	1.94	.037	¼	5.	.094	67	8.12	.154	92	11.2	.211
½	2.	.038	42	5.09	.096	68	8.24	.156	93	11.3	.213
17	2 06	.039	43	5.21	.099	69	8.36	.158	94	11.4	.216
18	2.18	.041	44	5.33	.101	70	8.48	.161	95	11.5	.218
19	2.30	.044	45	5.45	.103	71	8.61	.163	96	11.6	.220
20	2.42	.046	46	5.58	.106	72	8 73	.165	97	11.8	.223
21	2.55	.048	47	5.70	.108	73	8.85	.168	98	11.9	.225
22	2.67	.051	48	5.82	.110	74	8 97	.170	99	12.	.227
23	2.79	.053	49	5.94	.112	¼	9.	.170	100	12.1	.230
24	2.91	.055	½	6.	.114	75	9.09	.172			
¾	3.	.057	50	6 06	.115	76	9.21	.174			
25	3.03	.057	51	6.18	.117	77	9.33	.177			

Table of grades per mile, and per 100 feet measured horizontally, and corresponding to different angles of inclination.

Deg.	Min.	Feet per mile.	Feet per 100 ft.	Deg.	Min.	Feet per mile.	Feet per 100 ft.	Deg.	Min.	Feet per mile.	Feet per 100 ft.	Deg.	Min.	Feet per mile.	Feet per 100 ft.
0	1	1.536	.0291	0	45	69.11	1.3090	1	58	181.3	3.4341	3	26	316.8	5.9994
	2	3.072	.0582		46	70.64	1.3381	2	0	184.4	3.4924		28	319.8	6.0579
	3	4.608	.0873		47	72.18	1.3672		2	187.5	3.5506		30	322.9	6.1163
	4	6.144	.1164		48	73.72	1.3963		4	190.6	3.6087		32	326.0	6.1747
	5	7.680	.1455		49	75.26	1.4254		6	193.6	3.6669		34	329.1	6.2330
	6	9.216	.1746		50	76.80	1.4545		8	196.7	3.7250		36	332.2	6.2914
	7	10.75	.2037		51	78.33	1.4837		10	199.8	3.7833		38	335.3	6.3498
	8	12.29	.2328		52	79.87	1.5128		12	202.8	3.8416		40	338.4	6.4083
	9	13.82	.2619		53	81.40	1.5419		14	205.9	3.8999		42	341.4	6.4664
	10	15.36	.2909		54	82.94	1.5710		16	208.9	3.9581		44	344.5	6.5246
	11	16.90	.3200		55	84.47	1.6000		18	212.0	4.0163		46	347.6	6.5832
	12	18.43	.3491		56	86.01	1.6291		20	215.1	4.0746		48	350.7	6.6418
	13	19.96	.3782		57	87.54	1.6583		22	218.1	4.1329		50	353.8	6.7004
	14	21.50	.4073		58	89.08	1.6873		24	221.2	4.1911		52	356.8	6.7583
	15	23.04	.4364		59	90.62	1.7164		26	224.3	4.2494		54	359.9	6.8163
	16	24.58	.4655	1		92.16	1.7455		28	227.4	4.3076		56	363.0	6.8751
	17	26.11	.4946		2	95.23	1.8038		30	230.5	4.3659		58	366.1	6.9339
	18	27.64	.5237		4	98.30	1.8620		32	233.5	4.4242	4		369.2	6.9926
	19	29.17	.5528		6	101.4	1.9202		34	236.6	4.4826		5	376.9	7.1384
	20	30.72	.5818		8	104.5	1.9784		36	239.7	4.5409		10	384.6	7.2842
	21	32.26	.6109		10	107.5	2.0366		38	242.8	4.5993		15	392.3	7.4300
	22	33.80	.6400		12	110.6	2.0948		40	245.9	4.6576		20	400.1	7.5767
	23	35.33	.6691		14	113.6	2.1530		42	248.9	4.7159		25	407.8	7.7234
	24	36.86	.6982		16	116.7	2.2112		44	252.0	4.7742		30	415.5	7.8701
	25	38.40	.7273		18	119.8	2.2694		46	255.1	4.8325		35	423.2	8.0163
	26	39.94	.7564		20	122.9	2.3277		48	258.2	4.8908		40	431.0	8.1625
	27	41.47	.7855		22	126.0	2.3859		50	261.3	4.9492		45	438.7	8.3087
	28	43.01	.8146		24	129.1	2.4441		52	264.3	5.0075		50	446.5	8.4554
	29	44.54	.8436		26	132.1	2.5023		54	267.4	5.0658		55	454.2	8.6021
	30	46.08	.8727		28	135.2	2.5604		56	270.5	5.1241	5		461.9	8.7489
	31	47.62	.9018		30	138.3	2.6186		58	273.6	5.1824		5	469.6	8.8951
	32	49.16	.9309		32	141.3	2.6768	3		276.7	5.2407		10	477.4	9.0413
	33	50.69	.9600		34	144.4	2.7350		2	279.7	5.2990		15	485.1	9.1875
	34	52.23	.9891		36	147.4	2.7932		4	282.8	5.3573		20	492.9	9.3347
	35	53 76	1.0182		38	150.5	2.8514		6	285.9	5.4158		25	500.6	9.4819
	36	55.30	1.0472		40	153.6	2.9097		8	289.0	5.4742		30	508.4	9.6292
	37	56 83	1.0763		42	156.6	2.9679		10	292.1	5.5326		35	516.1	9.7755
	38	58.37	1.1054		44	159.7	3.0262		12	295.1	5.5909		40	523.9	9.9218
	39	59.90	1.1345		46	162.8	3.0844		14	298.2	5.6493		45	531.6	10.068
	40	61.44	1.1636		48	165.9	3.1427		16	301.3	5.7077		50	539.4	10.215
	41	62.97	1.1927		50	169.0	3.2010		18	304.4	5.7660		55	547.2	10.362
	42	64.51	1.2218		52	172.0	3.2592		20	307.5	5.8244	6		555.	10.510
	43	66.04	1.2509		54	175.1	3.3175		22	310.5	5.8827				
	44	67.57	1.2800		56	178.2	3.3758		24	313.6	5.9410				

On a turnpike road 1° 38′, or about 1 in 35, or 151 ft per mile, is the greatest slope that should be given to allow horses to trot down rapidly with safety. In crossing mountains, this is often increased to 3, or even to 5°. It should never exceed 2½°, except when absolutely necessary.

SLOPES IN FEET PER 100 FT. HORIZONTAL.

The fractions of minutes are given only to 34 ft in 100.

A clinometer graduated by the 3d column, and numbered by the first one will give at sight the slopes in feet per 100 ft horizontal. No errors. Original.

Rise in ft per 100 ft hor.	Length of slope per 100 ft hor.	Angle of slope.		Rise in ft per 100 ft hor.	Length of slope per 100 ft hor.	Angle of slope.		Rise in ft per 100 ft hor.	Length of slope per 100 ft hor.	Angle of slope.	
	Feet.	Deg.	Min.		Feet.	Deg.	Min.		Feet.	Deg.	Min.
1	100.005	0	34.4	35	105.948	19	17	69	121.495	34	36
2	100.020	1	8.7	36	106.283	19	48	70	122.066	35	0
3	100.045	1	43.1	37	106.626	20	18	71	122 642	35	23
4	100.080	2	17.5	38	106.977	20	48	72	123.223	35	45
5	100.125	2	51.8	39	107.336	21	18	73	123.810	36	8
6	100.180	3	26.0	40	107.703	21	48	74	124.403	36	30
7	100.245	4	0.3	41	108.079	22	18	75	125.000	36	52
8	100.319	4	34.4	42	108.462	22	47	76	125.603	37	14
9	100.404	5	8.6	43	108.853	23	16	77	126.210	37	36
10	100.499	5	42.6	44	109.252	23	45	78	126.823	37	57
11	100.603	6	16.6	45	109.659	24	14	79	127.440	38	19
12	100 717	6	50 6	46	110.073	24	42	80	128.062	38	40
13	100.841	7	24.4	47	110.494	25	10	81	128.690	39	1
14	100.975	7	58.2	48	110.923	25	38	82	129 321	39	21
15	101.119	8	31.9	49	111.359	26	6	83	129.958	39	42
16	101.272	9	5.4	50	111 803	26	34	84	130.599	40	2
17	101.435	9	38.9	51	112.254	27	1	85	131.244	40	22
18	101.607	10	12.2	52	112.712	27	28	86	131.894	40	42
19	101.789	10	45.5	53	113.177	27	55	87	132.548	41	1
20	101.980	11	18.6	54	113.649	28	22	88	133.207	41	21
21	102.181	11	51.6	55	114.127	28	49	89	133.869	41	40
22	102.391	12	24.5	56	114.612	29	15	90	134.536	41	59
23	102.611	12	57.2	57	115.104	29	41	91	135.207	42	18
24	102.840	13	29.8	58	115.603	30	7	92	135.882	42	37
25	103.078	14	2.2	59	116.108	30	32	93	136.561	42	55
26	103.325	14	34.5	60	116.619	30	58	94	137.244	43	14
27	103.581	15	6.6	61	117.137	31	23	95	137.931	43	32
28	103.846	15	38.5	62	117.661	31	48	96	138.622	43	50
29	104.120	16	10 3	63	118.191	32	13	97	139 316	44	8
30	104.403	16	42.0	64	118.727	32	37	98	140.014	44	25
31	104.695	17	13.4	65	119.269	33	1	99	140.716	44	43
32	104.995	17	44.7	66	119.817	33	25	100	141.421	45	00
33	105.304	18	15.8	67	120.370	33	49	101	142.130	45	17
34	105.622	18	46.7	68	120.930	34	13	102	142.843	45	34

Any hor dist is = sloping dist × cosine ang of slope.
" **sloping dist** is = hor dist ÷ cosine " " "
" **vert height** is = hor dist × tangent " " "
or = sloping dist × sine " " "

Table of grades per mile; or per 100 feet measured horizontally.

Grade in ft. per mile.	Grade in ft. per 100 ft.	Grade in ft. per mile.	Grade in ft. per 100 ft.	Grade in ft. per mile.	Grade in ft. per 100 ft.	Grade in ft. per mile	Grade in ft. per 100 ft.
1	.01894	39	.73864	77	1.45833	115	2.17803
2	.03788	40	.75758	78	1.47727	116	2.19697
3	.05682	41	.77652	79	1.49621	117	2.21591
4	.07576	42	.79545	80	1.51515	118	2.23485
5	.09470	43	.81439	81	1.53409	119	2.25379
6	-11364	44	.83333	82	1.55303	120	2.27273
7	.13258	45	.85227	83	1.57197	121	2.29167
8	.15152	46	.87121	84	1.59091	122	2.31061
9	.17045	47	.89015	85	1.60985	123	2.32955
10	.18939	48	.90909	86	1.62879	124	2.34848
11	.20833	49	.92803	87	1.64773	125	2.36742
12	.22727	50	.94697	88	1.66666	126	2.38636
13	.24621	51	.96591	89	1.68561	127	2.40530
14	.26515	52	.98485	90	1.70455	128	2.42424
15	.28409	53	1.00379	91	1.72348	129	2.44318
16	.30303	54	1.02273	92	1.74242	130	2.46212
17	.32197	55	1.04167	93	1.76136	131	2.48106
18	.34091	56	1.06061	94	1.78030	132	2.50000
19	.35985	57	1.07955	95	1.79924	133	2.51894
20	.37879	58	1.09848	96	1.81818	134	2.53788
21	.39773	59	1.11742	97	1.83712	135	2.55682
22	.41667	60	1.13636	98	1.85606	136	2.57576
23	.43561	61	1.15530	99	1.87500	137	2.59470
24	.45455	62	1.17424	100	1.89394	138	2.61364
25	.47348	63	1.19318	101	1.91288	139	2.63258
26	.49242	64	1.21212	102	1.93182	140	2.65152
27	.51136	65	1.23106	103	1.95076	141	2.67045
28	.53030	66	1.25000	104	1.96969	142	2.68939
29	.54924	67	1.26894	105	1.98864	143	2.70833
30	.56818	68	1.28788	106	2.00758	144	2.72727
31	.58712	69	1.30682	107	2.02652	145	2.74621
32	.60606	70	1.32576	108	2.04545	146	2.76515
33	.62500	71	1.34470	109	2.06439	147	2.78409
34	.64394	72	1.36364	110	2.08333	148	2.80303
35	.66288	73	1.38258	111	2.10227	149	2.82197
36	.68182	74	1.40152	112	2.12121	150	2.84091
37	.70076	75	1.42045	113	2.14015	151	2.85985
38	.71970	76	1.43939	114	2.15909	152	2.87879

If the grade per mile should consist of feet *and tenths*, add to the grade per 100 feet in the foregoing table, that corresponding to the number of tenths taken from the table below; thus, for a grade of 83.7 feet per mile, we have .81439 + .01326 = .82765 feet per 100 feet.

Ft. per Mile.	Per 100 Feet.	Ft. per Mile.	Per 100 Feet.	Ft. per Mile.	Per 100 Feet.
.05	.00094	.4	.00758	.7	.01326
.1	.00189	.45	.00852	.75	.01420
.15	.00283	.5	.00947	.8	.01515
.2	.00379	.55	.01041	.85	.01609
.25	.00473	.6	.01136	.9	.01705
.3	.00568	.65	.01230	.95	.01799
.35	.00662				

Table of Radii, Middle Ordinates, &c, of Curves. Chord 100 feet

Contains no error as great as 1 in the last figure.

Ang. of Defl.	Rad. in ft.	Defl. Dist. in ft.	Tang. Dist. in ft.	Mid. Ord.	Ang. of Defl.	Rad. in ft.	Defl. Dist. in ft.	Tang. Dist. in ft.	Mid Ord
° ′					° ′				
1	343775	.029	.014	.004	1 36	3581	2.793	1.396	.349
2	171887	.058	.029	.007	38	3508	2.851	1.425	.356
3	114592	.087	.043	.011	40	3438	2.909	1.454	.364
4	85944	.116	.058	.014	42	3370	2.967	1.483	.371
5	68755	.145	.072	.018	44	3306	3.025	1.512	.378
6	57296	.175	.087	.022	46	3243	3.084	1.542	.385
7	49111	.204	.102	.025	48	3183	3.142	1.571	.393
8	42972	.233	.116	.029	50	3125	3.200	1.600	.400
9	38197	.262	.131	.033	52	3070	3.257	1.629	.407
10	34377	.291	.145	.036	54	3016	3.316	1.658	.414
11	31252	.320	.160	.040	56	2964	3.374	1.687	.422
12	28648	.349	.174	.043	58	2913	3.433	1.716	.429
13	26444	.378	.189	.047	2	2865	3.490	1.745	.436
14	24555	.407	.203	.051	2	2818	3.549	1.774	.443
15	22918	.436	.218	.054	4	2773	3.606	1.803	.451
16	21486	.465	.232	.058	6	2729	3.664	1.832	.458
17	20222	.494	.247	.062	8	2686	3.723	1.861	.465
18	19098	.524	.262	.065	10	2645	3.781	1.890	.473
19	18094	.553	.276	.069	12	2605	3.839	1.919	.480
20	17189	.582	.291	.073	14	2566	3.897	1.948	.487
21	16370	.611	.305	.076	16	2528	3.956	1.978	.495
22	15626	.640	.320	.080	18	2491	4.014	2.007	.502
23	14947	.669	.334	.083	20	2456	4.072	2.036	.509
24	14324	.698	.349	.087	22	2421	4.131	2.065	.516
25	13751	.727	.363	.091	24	2387	4.189	2.094	.523
26	13222	.756	.378	.095	26	2355	4.246	2.123	.531
27	12732	.785	.392	.098	28	2323	4.305	2.152	.538
28	12278	.814	.407	.102	30	2292	4.363	2.182	.545
29	11854	.844	.422	.105	32	2262	4.421	2.210	.552
30	11459	.873	.436	.109	34	2232	4.480	2.240	.560
31	11090	.902	.451	.113	36	2204	4.537	2.268	.567
32	10743	.931	.465	.116	38	2176	4.596	2.298	.574
33	10417	.960	.480	.120	40	2149	4.654	2.327	.582
34	10111	.989	.494	.123	42	2122	4.713	2.356	.589
35	9822	1.018	.509	.127	44	2096	4.771	2.385	.596
36	9549	1.047	.523	.131	46	2071	4.829	2.414	.603
37	9291	1.076	.538	.134	48	2046	4.888	2.444	.611
38	9047	1.105	.552	.138	50	2022	4.946	2.473	.618
39	8815	1.134	.567	.142	52	1999	5.003	2.501	.625
40	8594	1.164	.582	.145	54	1976	5.061	2.530	.632
41	8385	1.193	.596	.149	56	1953	5.120	2.560	.640
42	8185	1.222	.611	.153	58	1932	5.176	2.588	.647
43	7995	1.251	.625	.156	3	1910	5.235	2.618	.654
44	7813	1.280	.640	.160	2	1889	5.294	2.647	.662
45	7639	1.309	.654	.164	4	1869	5.350	2.675	.669
46	7473	1.338	.669	.167	6	1848	5.411	2.705	.676
47	7314	1.367	.683	.171	8	1829	5.467	2.734	.683
48	7162	1.396	.698	.174	10	1810	5.526	2.763	.691
49	7016	1.425	.712	.178	12	1791	5.583	2.792	.698
50	6876	1.454	.727	.182	14	1772	5.643	2.821	.705
51	6741	1.483	.741	.185	16	1754	5.707	2.850	.713
52	6611	1.513	.757	.189	18	1736	5 760	2.880	.720
53	6486	1.542	.771	.193	20	1719	5.817	2.908	.727
54	6366	1.571	.786	.197	22	1702	5.875	2.937	.734
55	6251	1.600	.800	.200	24	1685	5.935	2.967	.742
56	6139	1.629	.815	.204	26	1669	5.992	2.996	.749
57	6031	1.658	.829	.207	28	1653	6.050	3.025	.756
58	5927	1.687	.844	.211	30	1637	6.108	3.054	.764
59	5827	1.716	.858	.214	32	1622	6.166	3.083	.771
1	5730	1.745	.872	.218	34	1607	6.223	3.112	.778
2	5545	1.803	.902	.225	36	1592	6.281	3.140	.785
4	5372	1.862	.931	.233	38	1577	6.341	3.170	.793
6	5209	1.920	.960	.240	40	1563	6.398	3.199	.800
8	5056	1.978	.989	.247	42	1549	6.456	3.228	.807
10	4911	2.036	1.018	.255	44	1535	6.515	3.257	.814
12	4775	2.094	1.047	.262	46	1521	6.575	3.287	.822
14	4646	2.152	1.076	.269	48	1508	6.631	3.316	.829
16	4523	2.211	1.105	.276	50	1495	6.689	3.345	.836
18	4407	2.269	1.134	.284	52	1482	6.748	3.374	.843
20	4297	2.327	1.163	.291	54	1469	6.807	3.403	.851
22	4192	2.385	1.192	.298	56	1457	6.863	3.432	.858
24	4093	2.443	1.221	.305	58	1445	6.920	3.460	.865
26	3997	2.502	1.251	.313	4	1433	6.980	3.490	.873
28	3907	2.560	1.280	.320	5	1403	7.125	3.562	.891
30	3820	2.618	1.309	.327	10	1375	7.271	3.635	.909
32	3737	2.676	1.338	.334	15	1348	7.416	3.708	.927
34	3657	2.734	1.367	.342	20	1323	7.561	3.781	.945

able of Radii, Middle Ordinates, &c, of Curves. Chord 100 feet.

(Continued.)

The Tangential Angle is always one-half of the Angle of Deflection.

ng. of Defl.	Rad. in ft.	Defl. Dist. in ft.	Tang. Dist. in ft.	Mid.* Ord.	Ang. of Defl.	Rad. in ft.	Defl. Dist. in ft.	Tang. Dist. in ft.	Mid.* Ord.
° ′					° ′				
4 25	1298	7.707	3.854	.963	10 15	559.7	17.87	8.942	2.238
30	1274	7.852	3.927	.982	30	546.4	18.30	9.160	2.292
35	1250	7.997	3.999	1.000	45	533.8	18.73	9.378	2.347
40	1228	8.143	4.072	1.018	11	521.7	19.17	9.596	2.402
45	1207	8.288	4.145	1.036	15	510.1	19.60	9.814	2.456
50	1186	8.433	4.218	1.054	30	499.1	20.04	10.03	2.511
55	1166	8.579	4.290	1.072	45	488.5	20.47	10.25	2.566
5	1146	8.724	4.363	1.091	12	478.3	20.91	10.47	2.620
5	1128	8.869	4.436	1.109	15	468.6	21.34	10.69	2.675
10	1109	9.014	4.508	1.127	30	459.3	21.77	10.90	2.730
15	1092	9.160	4.581	1.145	45	450.3	22.21	11.12	2.785
20	1075	9.305	4.654	1.164	13	441.7	22.64	11.34	2.839
25	1058	9.450	4.727	1.182	15	433.4	23.07	11.56	2.894
30	1042	9.596	4.799	1.200	30	425.4	23.51	11.77	2.949
35	1027	9.741	4.872	1.218	45	417.7	23.94	11.99	3.003
40	1012	9.886	4.945	1 236	14	410.3	24.37	12.21	3.058
45	996.9	10.03	5.017	1.255	15	403.1	24.81	12.43	3.113
50	982.6	10.18	5.090	1.273	30	396.2	25.24	12.65	3.168
55	968.8	10.32	5.163	1.291	45	389.5	25.67	12.86	3.223
6	955.4	10.47	5.235	1.309	15	383.1	26.11	13.08	3.277
5	942.3	10.61	5.308	1.327	15	376.8	26.54	13.30	3.332
10	929.6	10.76	5.381	1.346	30	370.8	26.97	13.52	3.387
15	917.2	10.90	5.453	1.364	45	364.9	27.40	13.73	3.442
20	905.1	11.05	5.526	1.382	16	359.3	27.84	13.95	3.496
25	893.4	11.19	5.599	1.400	30	348.5	28.70	14.39	3.606
30	882.0	11.34	5.672	1.418	17	338.3	29.56	14.82	3.716
35	870.8	11.48	5.744	1.437	30	328.7	30.43	15.26	3.825
40	859.9	11.63	5.817	1.455	18	319.6	31.29	15.69	3.935
45	849.3	11.77	5.890	1.473	30	311.1	32.15	16.13	4.045
50	839.0	11.92	5.962	1.491	19	302.9	33.01	16.56	4.155
55	828.9	12.07	6.035	1.510	30	295.3	33.87	17.00	4.265
7	819.0	12.21	6.108	1.528	20	287.9	34.73	17.43	4.375
5	809.4	12.36	6.180	1.546	21	274.4	36.44	18.30	4.594
10	800.0	12.50	6.253	1.564	22	262.0	38.17	19.17	4.815
15	790.8	12.65	6.326	1.582	23	250.8	39.87	20.04	5.035
20	781.8	12.79	6.398	1.600	24	240.5	41.58	20.91	5.255
25	773.1	12.94	6.471	1.619	25	231.0	43.29	21.77	5.476
30	764.5	13.08	6.544	1.637	26	222.3	44.98	22.64	5.696
35	756.1	13.23	6.616	1.655	27	214.2	46.69	23.51	5.917
40	747.9	13.37	6.689	1.673	28	206.7	48.38	24.37	6.139
45	739.9	13.52	6.762	1.691	29	199.7	50.08	25.24	6.361
50	732.0	13.66	6.835	1.710	30	193 2	51.76	26.11	6.582
55	724.3	13.81	6.907	1.728	31	187.1	53.45	26.97	6.805
8	716.8	13.95	6.980	1.746	32	181.4	55.13	27.83	7.027
15	695.1	14.39	7.198	1.801	33	176.0	56.82	28.70	7.252
30	674.7	14.82	7.416	1.855	34	171.0	58.48	29.56	7.473
45	655.5	15.26	7.634	1.910	35	166.3	60.13	30.42	7.695
9	637.3	15.69	7.852	1.965	36	161.8	61.80	31.29	7.919
15	620.1	16.13	8.070	2.019	37	157.6	63.45	32.15	8.142
30	603.8	16.56	8.288	2.074	38	153.6	65.10	33.01	8.366
45	588.4	17.00	8.506	2.128	39	149.8	66.76	33.87	8.591
10	573.7	17.43	8.724	2.183	40	146.2	68.40	34.73	8.816

For ordinates 5 ft apart, for Chords of 100 ft, see p 730.

To find tangential and deflection angles for any given rad and hord. Divide *half* the chord by the rad. The quot will be nat sine of the tangl ng. Find this tangl ang in the table of nat sines; and mult it by 2 for the def ang.

To find the def dist for chords 100 ft long. Div 10000 by the ad in feet.

To find the def dist for equal chords of any given length.)iv chord by rad. Mult quot by chord. Or div sq of chord by rad.

To find the tangl dist for equal chords of any given length. 'irst find the tangl ang as above. Divide it by 2. Find in the table of nat sines he nat sine of the quot. Mult this nat sine by the given chord. Mult prod by 2.

To find the rad to any given def ang. for equal chords of any ngth. Divide the def ang by 2. Find nat sine of the quotient. Divide half he chord by this nat sine.

* **The middle ordinate** for a rad of 600 ft or more, (chord 100 ft,) may in ractice be taken at one-fourth of the tang dist. Even in 400 ft rad it will be too short only 6 in the hird decimal.

Radii, &c, of Curves; in metres. Chord, 20 metres = 2 dekametres.

The stakes, at the ends of the 2-dekametre chords, should be numbered 2, 4, 6, &c, meaning 2, 4, 6, &c, *dekametres*. The *tangential* angle in the table will then give the amount of deflection per unit (dekametre) of measurement.

Defl angle.	Tangential angle.	Radius. Metres.	Defl dist. Metres.	Tangl dist. Metres.	Mid. ord. Metres.	Defl angle.	Tangential angle.	Radius. Metres.	Defl dist. Metres.	Tangl dist. Metres.	Mid. ord. Metres.
0° 10′	0° 5′	6875.50	.058	.029	.007	8° 0′	4° 0′	143.36	2.790	1.396	.349
20	10	3437.75	.116	.058	.015	10	5	140.44	2.848	1.425	.356
30	15	2291.84	.175	.087	.022	20	10	137.63	2.906	1.454	.364
40	20	1718.88	.233	.116	.029	30	15	134.94	2.964	1.483	.371
50	25	1375.11	.291	.145	.036	40	20	132.35	3.022	1.512	.378
1° 0′	30	1145.93	.349	.175	.044	50	25	129.85	3.080	1.541	.386
10	35	982.23	.407	.204	.051	9° 0′	30	127.45	3.138	1.570	.393
20	40	859.46	.465	.233	.058	10	35	125.14	3.196	1.599	.400
30	45	763.97	.524	.262	.065	20	40	122.91	3.254	1.629	.407
40	50	687.57	.582	.291	.073	30	45	120.76	3.312	1.658	.415
50	55	625.07	.640	.320	.080	40	50	118.68	3.370	1.687	.422
2° 0′	1° 0′	572.99	.698	.349	.087	50	55	116.68	3.428	1.716	.429
10	5	528.92	.756	.378	.095	10° 0′	5° 0′	114.74	3.486	1.745	.437
20	10	491.14	.814	.407	.102	20	10	111.05	3.602	1.803	.451
30	15	458.40	.873	.436	.109	40	20	107.58	3.718	1.861	.466
40	20	429.76	.931	.465	.116	11° 0′	30	104.33	3.834	1.919	.480
50	25	404.48	.989	.494	.124	20	40	101.28	3.950	1.977	.495
3° 0′	30	382.02	1.047	.524	.131	40	50	98.39	4.065	2.035	.509
10	35	361.91	1.105	.553	.138	12° 0′	6° 0′	95.67	4.181	2.093	.524
20	40	343.82	1.163	.582	.145	20	10	93.09	4.297	2.152	.539
30	45	327.46	1.222	.611	.153	40	20	90.65	4.413	2.210	.553
40	50	312.58	1.280	.640	.160	13° 0′	30	88.34	4.528	2.268	.568
50	55	298.99	1.338	.669	.167	20	40	86.14	4.644	2.326	.582
4° 0′	2° 0′	286.54	1.396	.698	.175	40	50	84.05	4.759	2.384	.597
10	5	275.08	1.454	.727	.182	14° 0′	7° 0′	82.06	4.875	2.442	.612
20	10	264.51	1.512	.756	.189	20	10	80.16	4.990	2.500	.626
30	15	254.71	1.570	.785	.196	40	20	78.34	5.106	2.558	.641
40	20	245.62	1.629	.814	.204	15° 0′	30	76.61	5.221	2.616	.655
50	25	237.16	1.687	.844	.211	20	40	74.96	5.336	2.674	.670
5° 0′	30	229.26	1.745	.873	.218	40	50	73.37	5.452	2.732	.685
10	35	221.87	1.803	.902	.225	16° 0′	8° 0′	71.85	5.567	2.790	.699
20	40	214.94	1.861	.931	.233	20	10	70.40	5.682	2.848	.714
30	45	208.43	1.919	.960	.240	40	20	69.00	5.797	2.906	.729
40	50	202.30	1.977	.989	.247	17° 0′	30	67.65	5.912	2.964	.743
50	55	196.53	2.035	1.018	.255	20	40	66.36	6.027	3.022	.758
6° 0′	3° 0′	191.07	2.093	1.047	.262	40	50	65.12	6.142	3.080	.772
10	5	185.91	2.152	1.076	.269	18° 0′	9° 0′	63.92	6.257	3.138	.787
20	10	181.03	2.210	1.105	.276	20	10	62.77	6.372	3.196	.802
30	15	176.39	2.268	1.134	.284	40	20	61.66	6.487	3.254	.816
40	20	171.98	2.326	1.163	.291	19° 0′	30	60.59	6.602	3.312	.831
50	25	167.79	2.384	1.192	.298	20	40	59.55	6.717	3.370	.846
7° 0′	30	163.80	2.442	1.222	.306	40	50	58.55	6.831	3.428	.860
10	35	160.00	2.500	1.251	.313	20° 0′	10° 0′	57.59	6.946	3.486	.875
20	40	156.37	2.558	1.280	.320	21° 0′	30	54.87	7.289	3.660	.919
30	45	152.90	2.616	1.309	.327	22° 0′	11° 0′	52.41	7.632	3.834	.963
40	50	149.58	2.674	1.338	.335	23° 0′	30	50.16	7.975	4.008	1.007
50	55	146.40	2.732	1.367	.342	24° 0′	12° 0′	48.10	8.316	4.181	1.051
						25° 0′	30	46.20	8.658	4.355	1.095

Radius $= \frac{\text{Half the chord}}{\text{Sine of tangential angle}} =$ Half the chord $\times$ cosecant of tangential angle.

Deflection dist $= \frac{\text{Square of chord}}{\text{Radius}} =$ Twice the chord $\times$ sine of tangential angle.

Tangential dist = Twice the chord $\times$ sine of half the tangential angle.

Middle ord = Radius $\times$ (1 — cosine of tangential angle) = Half the chord $\times$ tangent of half the tangential angle.

For curves of 60 metres, or greater, radius, **the ordinate at 5 metres** from the end of the 20-metre chord, or midway between the end of the chord and the middle ordinate, may be taken at three-fourths of the middle ordinate.

Table of Long Chords.

Lengths of Chord in ft, required to subtend from 1 to 4 stations of 100 ft each.

Ang. of Defl.	1 Sta.	2 Sta.	3 Sta.	4 Sta.	Ang. of Defl.	1 Sta.	2 Sta.	3 Sta.	4 Sta.
1°	100	200.0	300.0	400.0	¾	100	199.7	298.9	397.5
¼	100	200.0	300.0	399.9	6°	100	199.7	298.8	397.3
½	100	200.0	300.0	399.9	¼	100	199.7	298.7	397.0
¾	100	200.0	300.0	399.8	½	100	199.7	298.6	396.7
2°	100	200.0	299.9	399.7	¾	100	199.6	298.5	396.5
¼	100	200.0	299.9	399.6	7°	100	199.6	298.4	396.2
½	100	200.0	299.8	399.5	¼	100	199.6	298.3	396.0
¾	100	200.0	299.8	399.4	½	100	199.6	298.2	395.7
3°	100	200.0	299.7	399.3	¾	100	199.6	298.1	395.4
¼	100	200.0	299.7	399.2	8°	100	199.6	298.0	395.1
½	100	200.0	299.6	399.1	¼	100	199.5	297.9	394.8
¾	100	200.0	299.6	399.0	½	100	199.5	297.8	394.5
4°	100	199.9	299.6	398.9	¾	100	199.4	297.7	394.3
¼	100	199.9	299.5	398.7	9°	100	199.4	297.5	394.1
½	100	199.9	299.4	398.5	¼	100	199.4	297.4	393 7
¾	100	199 9	299.3	398.3	½	100	199.3	297.3	393.2
5°	100	199.9	299.2	398.0	¾	100	199.2	297.2	392.8
¼	100	199 8	299.1	397.8	10°	100	199.2	297.0	392.4
½	100	199.8	299.0	397.6					

Elevation of outer rail in curves theoretically is equal in ins to (square of vel in ft per sec × gauge in ins) ÷ (Rad of curve in ft × 32.2). Experience has shown that half an inch for each degree of def. angle (100 ft chords) does very well for 4 ft 8.5 ins gauge up to 40 miles per hour. At 60 miles use 1 inch per deg. In dangerous places this may be increased for safety against high winds. Approaching the curve raise the outer rail at the rate of 1 inch in about 60 or 80 ft.

If the ends of the curves are tapered off with very easy "transition" curves, part of the rise may be made on said easier curves instead of on the tangent; or all of it if they are long enough.

Table of Ordinates 5 ft apart. Chord 100 ft.

For Railroad Curves.

Ordinates for angles intermediate of those in the table can at once be found by simple proportion.

Distances of the Ordinates from the end of the 100 feet Chord.

Ang. of Defl.	Mid. 50 ft.	45 ft.	40 ft.	35 ft.	30 ft.	25 ft.	20 ft.	15 ft.	10 ft.	5 ft.
° ′										
4	.014	.014	.014	.013	.012	.010	.008	.008	.005	.003
8	.029	.029	.028	.026	.024	.022	.018	.015	.010	.005
12	.043	.043	.041	.038	.037	.033	.028	.022	.015	.008
16	.058	.058	.056	.052	.049	.044	.037	.030	.020	.011
20	.073	.072	.070	.066	.061	.055	.047	.037	.026	.014
24	.087	.086	.083	.077	.074	.066	.056	.045	.031	.017
28	.102	.101	.098	.092	.086	.077	.065	.052	.036	.019
32	.116	.115	.112	.106	.098	.088	.075	.058	.042	.022
36	.131	.130	.126	.119	.110	.099	.084	.066	.047	.024
40	.145	.144	.140	.133	.123	.110	.093	.074	.052	.027
44	.160	.158	.153	.145	.135	.121	.103	.081	.057	.030
48	.174	.172	.167	.158	.147	.132	.112	.088	.062	.033
52	.189	.187	.181	.171	.159	.143	.122	.095	.068	.035
56	.204	.202	.195	.185	.171	.154	.131	.103	.073	.038
1	.218	.216	.209	.198	.183	.164	.140	.111	.078	.041
4	.233	.231	.223	.211	.196	.175	.150	.118	.083	.043
8	.247	.245	.237	.224	.208	.186	.159	.125	.088	.046
12	.262	.260	.252	.237	.220	.196	.168	.133	.094	.049
16	.276	.274	.265	.251	.232	.207	.177	.140	.099	.052
20	.291	.288	.279	.264	.244	.218	.187	.148	.104	.055
24	.306	.303	.293	.277	.256	.229	.197	.155	.109	.057
28	.320	.317	.307	.291	.269	.240	.206	.163	.114	.060
32	.334	.331	.321	.304	.281	.251	.215	.171	.120	.063
36	.349	.345	.335	.317	.293	.262	.224	.178	.125	.066
40	.364	.360	.349	.330	.305	.273	.233	.185	.130	.069
44	.378	.374	.363	.343	.318	.284	.242	.192	.135	.072
48	.393	.389	.377	.356	.330	.295	.251	.200	.141	.075
52	.407	.403	.391	.370	.342	.305	.261	.208	.147	.077
56	.422	.418	.405	.383	.354	.316	.270	.215	.152	.080
2	.436	.432	.419	.397	.366	.327	.280	.222	.157	.083
4	.451	.446	.433	.409	.379	.338	.289	.230	.162	.086
8	.465	.461	.447	.425	.391	.349	.298	.237	.167	.088
12	.480	.475	.461	.437	.403	.360	.308	.245	.173	.090
16	.495	.490	.475	.450	.415	.371	.317	.252	.178	.093
20	.509	.504	.489	.463	.428	.382	.326	.260	.183	.096
24	.523	.518	.503	.476	.440	.393	.334	.267	.188	.099
28	.538	.533	.517	.489	.452	.404	.346	.275	.194	.102
32	.552	.547	.531	.503	.465	.415	.355	.282	.199	.104
36	.567	.562	.545	.516	.477	.425	.364	.289	.204	.107
40	.582	.576	.559	.529	.489	.436	.373	.297	.209	.110
44	.596	.590	.573	.542	.501	.447	.382	.304	.214	.113
48	.611	.605	.587	.555	.513	.458	.391	.312	.219	.116
52	.625	.619	.601	.569	.526	.469	.401	.319	.225	.118
56	.640	.634	.615	.582	.538	.480	.410	.326	.230	.121
3	.654	.648	.629	.595	.550	.491	.419	.334	.235	.124
4	.669	.662	.643	.608	.562	.502	.428	.341	.240	.127
8	.683	.677	.657	.621	.574	.512	.438	.349	.246	.130
12	.698	.691	.671	.635	.587	.523	.448	.357	.251	.132
16	.713	.705	.685	.649	.599	.534	.457	.364	.257	.135
20	.727	.720	.699	.662	.611	.545	.466	.371	.262	.138
24	.742	.734	.713	.675	.623	.556	.475	.378	.267	.141
28	.756	.749	.727	.688	.635	.567	.485	.386	.272	.144
32	.771	.763	.741	.702	.648	.578	.494	.394	.278	.146
36	.786	.777	.755	.715	.660	.589	.503	.401	.283	.149
40	.800	.792	.769	.728	.673	.600	.512	.408	.288	.152
44	.814	.806	.783	.741	.685	.611	.521	.415	.293	.155
48	.829	.821	.797	.754	.697	.621	.531	.423	.298	.158
52	.843	.835	.811	.768	.709	.632	.541	.431	.304	.160
56	.858	.850	.825	.781	.721	.643	.550	.438	.309	.163
4	.873	.864	.839	.794	.734	.655	.559	.445	.314	.166
10	.909	.900	.874	.827	.764	.682	.582	.464	.327	.173
20	.945	.936	.909	.860	.795	.709	.606	.482	.340	.179
30	.981	.972	.944	.893	.825	.736	.629	.501	.354	.186
40	1.017	1.008	.979	.926	.855	.764	.652	.519	.367	.193
50	1.054	1.044	1.014	.959	.886	.791	.676	.538	.380	.199
5	1.091	1.080	1.048	.993	.917	.818	.699	.557	.393	.207
10	1.127	1.116	1.083	1.026	.947	.845	.722	.576	.406	.214
20	1.164	1.152	1.118	1.058	.978	.872	.746	.594	.419	.220
30	1.200	1.188	1.153	1.092	1.009	.900	.769	.613	.432	.228

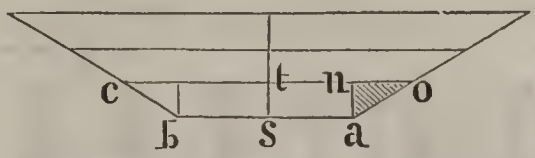
Table of Ordinates 5 ft apart. —(Continued.)

Distances of the Ordinates from the end of the 100 feet Chord.

Ang. of Defl.	Mid. 50 ft.	45 ft.	40 ft.	35 ft.	30 ft.	25 ft.	20 ft.	15 ft.	10 ft.	5 ft.
° ′										
5 40	1.236	1.224	1.188	1.124	1.039	.927	.792	.631	.445	.235
50	1.273	1.260	1.223	1.157	1.070	.954	.816	.649	.458	.241
6	1.309	1.296	1.258	1.191	1.100	.982	.839	.668	.472	.248
10	1.345	1.332	1.293	1.224	1.130	1.009	.862	.686	.485	.255
20	1.382	1.368	1.328	1.256	1.161	1.036	.886	.705	.498	.262
30	1.419	1.404	1.362	1.290	1.192	1.064	.909	.724	.511	.269
40	1.455	1·440	1.397	1.323	1.222	1.091	.932	.742	.524	.276
50	1.491	1.476	1.432	1.355	1.253	1.118	.956	.761	.537	.283
7	1.528	1.512	1.467	1.389	1.284	1.146	.979	.779	.551	.290
10	1.564	1.548	1.502	1.422	1.314	1.173	1.002	.798	.564	.297
20	1.600	1.584	1.537	1.454	1.345	1.200	1.026	.816	.576	.304
30	1.637	1.620	1.572	1.488	1.375	1.228	1.048	.835	.590	.311
40	1.673	1.656	1.607	1.521	1.405	1.255	1.071	.854	.603	.318
50	1.710	1.692	1.641	1.553	1.436	1.282	1.095	.872	.616	.324
8	1.746	1.728	1.677	1.587	1.467	1.310	1.118	.891	.629	.332
30	1.855	1.836	1.782	1.687	1.559	1.392	1.188	.946	.669	.353
9	1.965	1.944	1.886	1.787	1.651	1.474	1.258	1.002	.708	.373
30	2.074	2.052	1.991	1.887	1.742	1.556	1.328	1.057	.748	.394
10	2.183	2.161	2.096	1.987	1.834	1.637	1.398	1.114	.787	.415
30	2.292	2.269	2.201	2.087	1.926	1.719	1.468	1.170	.827	.436
11	2.401	2.377	2.306	2.186	2.018	1.802	1.538	1.226	.866	.457
30	2.511	2.486	2.411	2.286	2.110	1.884	1.609	1.282	.906	.478
12	2.620	2.594	2.516	2.386	2.203	1.967	1.680	1.339	.946	.499
30	2.730	2.703	2.621	2.485	2.295	2.049	1.750	1.395	.985	.520
13	2.839	2.811	2.726	2.585	2.387	2.132	1.820	1.451	1.025	.541
30	2.949	2.920	2.832	2.685	2.479	2.214	1.891	1.507	1.065	.562
14	3.058	3.028	2.937	2.785	2.571	2.297	1.961	1.564	1.105	.583
30	3.168	3.136	3.042	2.884	2 664	2.379	2.031	1.620	1.144	.604
15	3.277	3.245	3.147	2.984	2.756	2.462	2.102	1.676	1.184	.625
30	3.387	3.354	3.252	3.084	2.848	2.544	2.172	1.732	1.224	.646
16	3.496	3.462	3.358	3.184	2.941	2.627	2.243	1.789	1.264	.667
17	3.716	3.680	3.569	3.384	3.125	2.792	2.384	1.902	1.344	.709
18	3.935	3.897	3.779	3.584	3.310	2.958	2.525	2.014	1.424	.751
19	4.155	4.115	3.990	3.784	3.495	3.123	2.666	2.127	1.504	.793
20	4.375	4.332	4.201	3.984	3.680	3.288	2.808	2.240	1.583	.836
22	4.815	4.768	4.624	4.386	4.050	3.620	3.093	2.467	1.744	.922
24	5.255	5.204	5.048	4.789	4.423	3.952	3.379	2.695	1.905	1.008
26	5.697	5.642	5.473	5.192	4.798	4.286	3.665	2.924	2.068	1.094
28	6.139	6.079	5.898	5.595	5.171	4.622	3.952	3.154	2.232	1.181
30	6.582	6.517	6.323	5.999	5.544	4.958	4.239	3.385	2.396	1.268
32	7·027	6.957	6.751	6.406	5.922	5.297	4.530	3.619	2.565	1.356
34	7.472	7.398	7.179	6.813	6.300	5.637	4.822	3.854	2.733	1.445
36	7.918	7.841	7.609	7.222	6.679	5.978	5.115	4.090	2.901	1.535
38	8.367	8.286	8.041	7.633	7.060	6.320	5.410	4.327	3.069	1.626
40	8.816	8.731	8.474	8.044	7.442	6.663	5.705	4.565	3.238	1.718

For middle ordinates for bending rails, see p 761.

To prepare a Table, T, of Level Cuttings, for every $\frac{1}{10}$ of a foot of height, or depth. For Tables of Level Cuttings, see pp 733 to 740. For cost of Earthwork, p 742, &c.



Let the fig represent the cutting; or, if inverted, the filling; in which the horizontal lines are supposed to be $\frac{1}{10}$ foot apart. First calculate the area in square feet, of the layer *a b c o*, adjoining the roadway *a b*. Then find how many cubic yards that area gives in a distance of 100 feet. These cubic yards we will call Y; they form the first amount to be put into the Table T.

Next calculate the area in square feet of the triangle *a n o*. Multiply this area by 4. Find how many cubic yards this increased area gives in a distance of 100 feet. Or they will be found ready calculated below. We will call them *y*. This is all the preparation that is needed before commencing the table.

Exam.—Let the roadbed *a b* be 18 feet, and the side-slopes 1½ to 1. Then for the area of *a b c o*: since the side-slopes are 1½ to 1; and *s t* is .1 foot; c o must be 18.3 feet; and the mean length of *a b c o* must be 18.15 feet. Consequeutly, the area is 18.15 × .1 = 1.815 square feet; which, in a distance of 100 feet, gives 181.5 cubic feet; which is equal to $\frac{181.5}{27} = 6.7222$ cubic yards; or Y.

Next, as to the triangle *a n o*: its height *a n* being .1 foot, and its base *n o* .15 feet; its area $= \frac{.1 \times .15}{2} = \frac{.015}{2} = .0075$ square ft. This multiplied by 4, gives .03 square feet; which, in a distance of 100 feet, gives .03 × 100 = 3 cubic feet: which is equal to $\frac{3}{27} = .1111$ cubic yard; or *y*.

Having thus found Y and *y*, proceed to make out the table in the manner following, which is so plain as to require no explanation. The work should be tested about every 5 feet, by calculating the area of the full depth arrived at; multiply it by 100, and divide the product by 27 for the cubic yards The cubic yards thus found should agree with the table.

Y.....	6.7222	Y. 6.722	.1
y	.1111		
	6.8333	6.8333	
y.....	.1111		
	6.9444	13.5555	.2
		6.9444	
y.....	.1111		
	7.0555	20 5000	.3
		7.0555	
y.....	.1111		
	7.1666	27.5555	.4
		7.1666	
y.....	.1111		
	7.2777	34.7222	.5
		7.2777	
		42.0000	.6

TABLE T.

Height. Feet.	Cub. Yds.
.1	6.72 Y.
.2	13.6
.3	20 5
.4	27.6
.5	34.7
.6	42.0
&c.	

The following table contains *y*, ready calculated for different side-slopes. It plainly remains the same for all widths of roadbed.

Side-slope.	*y*	Side-slope.	*y*
¼ to 1	..... .0185	1¾ to 1	..... .1296
½ to 1	..... .0370	2 to 1	..... .1482
¾ to 1	..... .0556	2¼ to 1	..... .1667
1 to 1	..... .0741	2½ to 1	..... .1852
1¼ to 1	..... .0926	3 to 1	.... .2222
1½ to 1	..... .1111	4 to 1	.... .2963

Table 1. Level Cuttings.*

Roadway 14 feet wide, side-slopes 1½ to 1.

For single-track embankment.

Height in Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
	Cu.Yds.	Cu.Yds.	Cu.Yds.	Cu.Yds.	Cu.Yds.	Cu.Yds.	Cu.Yds.	Cu.Yds.	Cu.Yds.	Cu.Yds.
0		5.24	10.6	16.1	21.6	27.3	33.1	39 0	45.0	51.2
1	57.4	63.8	70.2	76.8	83.5	90.3	97.2	104.2	111.3	118.6
2	125 9	133.4	141.0	148.6	156.4	164.4	172.4	180.5	188.7	197.1
3	205.6	214.1	222.8	231.6	240.5	249.5	258.7	267.9	277.3	286.7
4	296.3	306.0	315.8	325.7	335.7	345.8	356.1	366.4	376.9	387.5
5	398.1	408.9	419.9	430.9	442.0	453.2	464.6	476.1	487.6	499.3
6	511.1	523.0	535.0	547.2	559.4	571.8	584.2	596.8	609.5	622.3
7	635 2	648.2	661.3	674.6	687.9	701.4	714.9	728.6	742.4	756.3
8	770.3	784.5	798.7	813.1	827.5	842.1	856.8	871.6	886.5	901.5
9	916.7	931.9	947.3	962.7	978.3	994.0	1010	1026	1042	1058
10	1074	1090	1107	1123	1140	1157	1174	1191	1208	1225
11	1243	1260	1278	1295	1313	1331	1349	1367	1385	1404
12	1422	1441	1459	1478	1497	1516	1535	1554	1574	1593
13	1613	1633	1652	1672	1692	1712	1733	1753	1773	1794
14	1815	1835	1856	1877	1898	1920	1941	1962	1984	2006
15	2028	2050	2072	2094	2116	2138	2161	2183	2206	2229
16	2252	2275	2298	2321	2344	2368	2391	2415	2439	2463
17	2487	2511	2535	2559	2584	2608	2633	2658	2683	2708
18	2733	2759	2784	2809	2835	2861	2886	2912	2938	2964
19	2991	3017	3044	3070	3097	3124	3151	3178	3205	3232
20	3259	3287	3314	3342	3370	3398	3426	3454	3482	3510
21	3539	3567	3596	3625	3654	3683	3712	3741	3771	3800
22	3830	3859	3889	3919	3949	3979	4009	4040	4070	4101
23	4132	4162	4193	4224	4255	4287	4318	4349	4381	4413
24	4444	4476	4508	4541	4573	4605	4638	4670	4703	4736
25	4769	4802	4835	4868	4901	4935	4968	5002	5036	5070
26	5104	5138	5172	5206	5241	5275	5310	5345	5380	5415
27	5450	5485	5521	5556	5592	5627	5663	5699	5735	5771
28	5807	5844	5880	5917	5953	5990	6027	6064	6101	6139
29	6176	6213	6251	6289	6326	6364	6402	6440	6479	6517
30	6556	6594	6633	6672	6711	6750	6789	6828	6867	6907
31	6946	6986	7026	7066	7106	7146	7186	7226	7267	7307
32	7348	7389	7430	7471	7512	7553	7595	7636	7678	7719
33	7761	7803	7845	7887	7929	7972	8014	8057	8099	8142
34	8185	8228	8271	8315	8358	8401	8445	8489	8532	8576
35	8620	8664	8709	8753	8798	8842	8887	8932	8976	9022
36	9067	9112	9157	9203	9248	9294	9340	9386	9432	9478
37	9524	9570	9617	9663	9710	9757	9804	9851	9898	9945
38	9993	10040	10088	10135	10183	10231	10279	10327	10375	10424
39	10472	10521	10569	10618	10667	10716	10765	10815	10864	10913
40	10963	11013	11062	11112	11162	11212	11263	11313	11364	11414
41	11465	11516	11567	11618	11669	11720	11771	11823	11874	11926
42	11978	12029	12081	12134	12186	12238	12291	12343	12396	12449
43	12502	12555	12608	12661	12715	12768	12822	12875	12929	12983
44	13037	13091	13145	13200	13254	13309	13363	13418	13473	13528
45	13583	13639	13694	13749	13805	13861	13916	13972	14028	14084
46	14141	14197	14254	14310	14367	14424	14480	14537	14595	14652
47	14709	14767	14824	14882	14940	14998	15056	15114	15172	15230
48	15289	15347	15406	15465	15524	15583	15642	15701	15761	15826
49	15880	15939	15999	16059	16119	16179	16239	16300	16360	16421
50	16481	16542	16603	16664	16725	16787	16848	16909	16971	17033
51	17094	17156	17218	17280	17343	17405	17467	17530	17593	17656
52	17719	17782	17845	17908	17971	18035	18098	18162	18226	18290
53	18354	18418	18482	18546	18611	18675	18740	18805	18870	18935
54	19000	19065	19131	19196	19262	19327	19393	19459	19525	19591
55	19657	19724	19790	19857	19923	19990	20057	20124	20191	20259
56	20326	20393	20461	20529	20596	20664	20732	20800	20869	20937
57	21005	21074	21143	21212	21280	21349	21419	21488	21557	21627
58	21696	21766	21836	21906	21976	22046	22116	22186	22257	22327
59	22398	22469	22540	22611	22682	22753	22825	22896	22968	23039
60	23111	23183	23255	23327	23399	23472	23544	23617	23689	23762

Table 2. Level Cuttings.

Roadway 24 feet wide, side-slopes 1½ to 1.

For double-track embankment.

Height in Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
	Cu.Yds.	Cu.Yds.	Cu.Yds.	Cu.Yds.	Cu.Yds.	Cu.Yds.	Cu.Yds.	Cu.Yds.	Cu.Yds.	Cu.Yds.
0		8.94	18.0	27.2	36.4	45.8	55.3	64.9	74.7	84.5
1	94.4	104.5	114.7	124.9	135.3	145.8	156.4	167.2	178.0	188.9
2	200.0	211.2	222.4	233.8	245.3	256.9	268.6	280.5	292.4	304.4
3	316.6	328.9	341.2	353.7	366.3	379.0	391.9	404 8	417.8	431.0
4	444.4	457.8	471.3	484.9	498.6	512.4	526.4	540.4	554.6	568.8
5	583.3	597.8	612.4	627.1	642.0	656.9	671.9	687.1	702.3	717.7
6	733.3	748.9	764.7	780.5	796.4	812.5	828.7	844.9	861.3	877.8
7	894.4	911.2	928.0	944.9	962.0	979.2	996.4	1014	1031	1049
8	1067	1085	1102	1121	1139	1157	1175	1194	1212	1231
9	1250	1269	1288	1307	1326	1346	1365	1385	1405	1425
10	1444	1465	1485	1505	1525	1546	1566	1587	1608	1629
11	1650	1671	1692	1714	1735	1757	1779	1800	1822	1845
12	1867	1889	1911	1934	1956	1979	2002	2025	2048	2071
13	2094	2118	2141	2165	2189	2213	2236	2261	2285	2309
14	2333	2358	2382	2407	2432	2457	2482	2507	2532	2558
15	2583	2609	2635	2661	2686	2713	2739	2765	2791	2818
16	2844	2871	2898	2925	2952	2979	3006	3034	3061	3089
17	3117	3145	3172	3201	3229	3257	3285	3314	3342	3371
18	3400	3429	3458	3487	3516	3546	3575	3605	3635	3665
19	3694	3725	3755	3785	3815	3846	3876	3907	3938	3969
20	4000	4031	4062	4094	4125	4157	4189	4221	4252	4285
21	4317	4349	4381	4414	4446	4479	4512	4545	4578	4611
22	4644	4678	4711	4745	4779	4813	4846	4881	4915	4949
23	4983	5018	5052	5087	5122	5157	5192	5227	5262	5298
24	5333	5369	5405	5441	5476	5513	5549	5585	5621	5658
25	5694	5731	5768	5805	5842	5879	5916	5954	5991	6029
26	6067	6105	6142	6181	6219	6257	6295	6334	6372	6411
27	6450	6489	6528	6567	6606	6646	6685	6725	6765	6805
28	6844	6885	6925	6965	7005	7046	7086	7127	7168	7209
29	7250	7291	7332	7374	7415	7457	7499	7541	7582	7625
30	7667	7709	7751	7794	7836	7879	7922	7965	8008	8051
31	8094	8138	8181	8225	8269	8313	8356	8401	8445	8489
32	8533	8578	8622	8667	8712	8757	8802	8847	8892	8938
33	8983	9029	9075	9121	9166	9212	9259	9305	9351	9398
34	9444	9491	9538	9585	9632	9679	9726	9774	9821	9869
35	9917	9965	10012	10061	10109	10157	10205	10254	10302	10351
36	10400	10449	10498	10547	10596	10646	10695	10745	10795	10845
37	10894	10945	10995	11045	11095	11146	11196	11247	11298	11349
38	11400	11451	11502	11554	11605	11657	11709	11761	11812	11865
39	11917	11969	12021	12074	12126	12179	12232	12285	12338	12391
40	12444	12498	12551	12605	12659	12713	12766	12821	12875	12929
41	12983	13038	13092	13147	13202	13257	13312	13367	13422	13478
42	13533	13589	13645	13701	13756	13813	13869	13925	13981	14038
43	14094	14151	14208	14265	14322	14379	14436	14494	14551	14609
44	14667	14725	14782	14840	14899	14957	15015	15074	15132	15191
45	15250	15309	15368	15427	15486	15546	15605	15665	15725	15785
46	15844	15905	15965	16025	16085	16146	16206	16267	16328	16389
47	16450	16511	16572	16634	16695	16757	16819	16881	16942	17005
48	17067	17129	17191	17254	17316	17379	17442	17505	17568	17631
49	17694	17758	17821	17885	17949	18013	18076	18141	18205	18269
50	18333	18398	18462	18527	18592	18657	18722	18787	18852	18918
51	18983	19049	19115	19181	19246	19313	19379	19445	19511	19578
52	19644	19711	19778	19845	19912	19979	20046	20114	20181	20249
53	20317	20385	20452	20521	20589	20657	20725	20794	20862	20931
54	21000	21069	21138	21207	21276	21346	21415	21485	21555	21625
55	21694	21765	21835	21905	21975	22046	22116	22187	22258	22329
56	22400	22471	22542	22614	22685	22757	22829	22901	22972	23045
57	23117	23189	23261	23334	23406	23479	23552	23625	23698	23771
58	23844	23918	23991	24065	24139	24213	24286	24361	24435	24509
59	24583	24658	24732	24807	24882	24957	25032	25107	25182	25258
60	25333	25409	25485	25561	25636	25713	25789	25865	25941	26018

For continuation to 100 feet, see TABLE 7.

Table 3. Level Cuttings.

Roadway 18 feet wide, side-slopes 1 to 1.

For single-track excavation.

Depth in Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.
0		6.70	13.5	20 3	27.3	34.3	41.3	48.5	55.7	6 .0
1	70.4	77.8	85.3	92 9	100.6	108.3	116.1	124.0	132.0	140.0
2	148.1	156.3	164.6	172.9	181.3	189.8	198.4	207.0	215.7	224.5
3	233.3	242.3	251.3	260.3	269.5	278.7	288.0	297.4	306.8	316.3
4	325.9	335.6	345.3	355.1	365.0	375.0	385.0	395.1	405.3	415.6
5	425.9	436.3	446.8	457.4	468.0	478.7	489.5	500.3	511.3	522.3
6	533.3	544.5	555.7	567.0	578.4	589.8	601.3	612.9	624.6	636.3
7	648.1	660.0	672.0	684.0	696.1	708.3	720.6	732.9	745.3	757.8
8	770.4	783.0	795.7	808.5	821.3	834.3	847.3	860.3	873.5	886.7
9	900.0	913.4	926.8	940.3	953.9	967.6	981.3	995.1	1009	1023
10	1037	1051	1065	1080	1094	1108	1123	1137	1152	1167
11	1181	1196	1211	1226	1241	1256	1272	1287	1302	1318
12	1333	1349	1365	1380	1396	1412	1428	1444	1460	1476
13	1493	1509	1525	1542	1558	1575	1592	1608	1625	1642
14	1659	1676	1693	1711	1728	1745	1763	1780	1798	1816
15	1833	1851	1869	1887	1905	1923	1941	1960	1978	1996
16	2015	2033	2052	2071	2089	2108	2127	2146	2165	2184
17	2204	2223	2242	2262	2281	2301	2321	2340	2360	2380
18	2400	2420	2440	2460	2481	2501	2521	2542	2562	2583
19	2604	2624	2645	2666	2687	2708	2729	2751	2772	2793
20	2815	2836	2858	2880	2901	2923	2945	2967	2989	3011
21	3033	3056	3078	3100	3123	3145	3168	3191	3213	3236
22	3259	3282	3305	3328	3352	3375	3398	3422	3445	3469
23	3493	3516	3540	3564	3588	3612	3636	3660	3685	3709
24	3733	3758	3782	3807	3832	3856	3881	3906	3931	3956
25	3981	4007	4032	4057	4083	4108	4134	4160	4185	4211
26	4237	4263	4289	4315	4341	4368	4394	4420	4447	4473
27	4500	4527	4553	4580	4607	4634	4661	4688	4716	4743
28	4770	4798	4825	4853	4881	4908	4936	4964	4992	5020
29	5048	5076	5105	5133	5161	5190	5218	5247	5276	5304
30	5333	5362	5391	5420	5449	5479	5508	5537	5567	5596
31	5626	5656	5685	5715	5745	5775	5805	5835	5865	5896
32	5926	5956	5987	6017	6048	6079	6109	6140	6171	6202
33	6233	6264	6296	6327	6358	6390	6421	6453	6485	6516
34	6548	6580	6612	6644	6676	6708	6741	6773	6805	6838
35	6870	6903	6936	6968	7001	7034	7067	7100	7133	7167
36	7200	7233	7267	7300	7334	7368	7401	7435	7469	7503
37	7537	7571	7605	7640	7674	7708	7743	7777	7812	7847
38	7881	7916	7951	7986	8021	8056	8092	8127	8162	8198
39	8233	8269	8305	8340	8376	8412	8448	8484	8520	8556
40	8593	8629	8665	8702	8738	8775	8812	8848	8885	8922
41	8959	8996	9033	9071	9108	9145	9183	9220	9258	9296
42	9333	9371	9409	9447	9485	9523	9561	9600	9638	9676
43	9715	9753	9792	9831	9869	9908	9947	9986	10025	10064
44	10104	10143	10182	10222	10261	10301	10341	10380	10420	10460
45	10500	10540	10580	10620	10661	10701	10741	10782	10822	10863
46	10904	10944	10985	11026	11067	11108	11149	11191	11232	11273
47	11315	11356	11398	11440	11481	11523	11565	11607	11649	11691
48	11733	11776	11818	11860	11903	11945	11988	12031	12073	12116
49	12159	12202	12245	12288	12332	12375	12418	12462	12505	12549
50	12593	12636	12680	12724	12768	12812	12856	12900	12945	12989
51	13033	13078	13122	13167	13212	13256	13301	13346	13391	13436
52	13481	13527	13572	13617	13663	13708	13754	13800	13845	13891
53	13937	13983	14029	14075	14121	14168	14214	14260	14307	14353
54	14400	14447	14493	14540	14587	14634	14681	14728	14776	14823
55	14870	14918	14965	15013	15061	15108	15156	15204	15252	15300
56	15348	15396	15445	15493	15541	15590	15638	15687	15736	15784
57	15833	15882	15931	15980	16029	16079	16128	17177	16227	16276
58	16326	16376	16425	16475	16525	16575	16625	16675	16725	16776
59	16826	16876	16927	16977	17028	17079	17129	17180	17231	17282
60	17333	17384	17436	17487	17538	17590	17641	17693	17745	17796

For continuation to 100 feet deep, see Table 7.

Table 4. Level Cuttings.

Roadway 18 feet, side-slopes 1½ to 1.

For single-track excavation.

Depth in Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.
0		6.72	13.6	20.5	27.6	34.7	42.0	49.4	56.9	64.5
1	72.2	80.1	88.0	96.1	104.2	112.5	120.9	129.4	138.0	146.7
2	155.5	164.5	173.5	182.7	191.9	201.3	210.8	220.4	230.1	240.0
3	249.9	260.0	270.1	280.4	290.8	301.3	311.9	322.6	333.4	344.5
4	355.5	366.7	378.0	389.4	400.9	412.5	424.2	436 0	448.0	460.0
5	472.2	484.5	496.9	509.4	522.0	534.7	547.6	560.5	573.6	586.7
6	600.0	613.4	626.9	640.5	654.2	668.1	682.0	696.1	710.2	724.5
7	738.9	753.4	768.0	782.7	797.6	812.5	827.6	842.7	858.0	873.4
8	888.9	904.5	920.2	936.1	952.0	968.1	984.2	1001	1017	1033
9	1050	1067	1084	1101	1118	1135	1152	1169	1187	1205
10	1222	1240	1258	1276	1294	1313	1331	1349	1368	1387
11	1406	1425	1444	1463	1482	1501	1521	1541	1560	1580
12	1600	1620	1640	1661	1681	1701	1722	1743	1764	1785
13	1806	1827	1848	1869	1891	1913	1934	1956	1978	2000
14	2022	2045	2067	2089	2112	2135	2158	2181	2204	2227
15	2250	2273	2297	2321	2344	2368	2392	2416	2440	2465
16	2489	2513	2538	2563	2588	2613	2638	2663	2688	2713
17	2739	2765	2790	2816	2842	2868	2894	2921	2947	2973
18	3000	3027	3054	3081	3108	3135	3162	3189	3217	3245
19	3272	3300	3328	3356	3384	3413	3441	3469	3498	3527
20	3556	3585	3614	3643	3672	3701	3731	3761	3790	3820
21	3850	3880	3910	3941	3971	4001	4032	4063	4094	4125
22	4156	4187	4218	4249	4281	4313	4344	4376	4408	4440
23	4472	4505	4537	4569	4602	4635	4668	4701	4734	4767
24	4800	4833	4867	4901	4934	4968	5002	5036	5070	5105
25	5139	5173	5208	5243	5278	5313	5348	5383	5418	5453
26	5489	5525	5560	5596	5632	5668	5704	5741	5777	5813
27	5850	5887	5924	5961	5998	6035	6072	6109	6147	6185
28	6222	6260	6298	6336	6374	6413	6451	6489	6528	6567
29	6606	6645	6684	6723	6762	6801	6841	6881	6920	6960
30	7000	7040	7080	7121	7161	7201	7242	7283	7324	7365
31	7406	7447	7488	7529	7571	7613	7654	7696	7738	7780
32	7822	7865	7907	7949	7992	8035	8078	8121	8164	8207
33	8250	8293	8337	8381	8424	8468	8512	8556	8600	8645
34	8689	8733	8778	8823	8868	8913	8958	9003	9048	9093
35	9139	9185	9230	9276	9322	9368	9414	9461	9507	9553
36	9600	9647	9694	9741	9788	9835	9882	9929	9977	10025
37	10072	10120	10168	10216	10264	10313	10361	10409	10458	10507
38	10556	10605	10654	10703	10752	10801	10851	10901	10950	11000
39	11050	11100	11150	11200	11251	11301	11352	11403	11454	11505
40	11556	11607	11658	11709	11761	11813	11864	11916	11968	12020
41	12072	12125	12177	12229	12282	12335	12388	12441	12494	12547
42	12600	12653	12707	12761	12814	12868	12922	12976	13030	13085
43	13139	13193	13248	13303	13358	13413	13468	13523	13578	13633
44	13689	13745	13800	13856	13912	13968	14024	14081	14137	14193
45	14250	14307	14364	14421	14478	14535	14592	14649	14707	14765
46	14822	14880	14938	14996	15054	15113	15171	15229	15288	15347
47	15406	15465	15524	15583	15642	15701	15761	15821	15880	15940
48	16000	16060	16120	16181	16241	16301	16362	16423	16484	16545
49	16606	16667	16728	16789	16851	16913	16974	17036	17098	17160
50	17222	17285	17347	17409	17472	17535	17598	17661	17724	17787
51	17850	17913	17977	18041	18104	18168	18232	18296	18360	18425
52	18489	18553	18618	18683	18748	18813	18878	18943	19008	19073
53	19139	19205	19270	19336	19402	19468	19534	19601	19667	19733
54	19800	19867	19934	20000	20068	20135	20202	20269	20337	20405
55	20472	20540	20608	20676	20744	20813	20881	20949	21018	21087
56	21156	21225	21294	21363	21432	21501	21571	21641	21710	21780
57	21850	21920	21990	22061	22131	22201	22272	22343	22414	22485
58	22556	22627	22698	22769	22841	22913	22984	23056	23128	23200
59	23272	23345	23417	23489	23562	23635	23708	23781	23854	23927
60	24000	24073	24147	24221	24294	24368	24442	24516	24590	24665

For continuation to 100 feet deep, see Table 7.

Table 5. Level Cuttings.

Roadway 28 feet wide, side-slopes 1 to 1.

For double-track excavation.

Depth in Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.
0		10.4	20.9	31.4	42.1	52.8	63.6	74.4	85.3	96.3
1	107.4	118.6	129.8	141.1	152.4	163.9	175.4	187.0	198.7	210.4
2	222.2	234.1	246.1	258.1	270.2	282.4	294.7	307.0	319.4	331.9
3	344.4	357.1	369.8	382.6	395.4	408.3	421.3	434.4	447.6	460.8
4	474.1	487.4	500.9	514.4	528.0	541.7	555.4	569.2	583.1	597.1
5	611.1	625.2	639.4	653.7	668.0	682.4	696.9	711.4	726.1	740.8
6	755.6	770.4	785.4	800.4	815.5	830.6	845.8	861.1	876.5	891.9
7	907.5	923.0	938.7	954.5	970.3	986.2	1002	1018	1034	1050
8	1067	1083	1099	1116	1132	1149	1166	1182	1199	1216
9	1233	1250	1267	1285	1302	1319	1337	1354	1372	1390
10	1407	1425	1443	1461	1479	1497	1515	1534	1552	1570
11	1589	1607	1626	1645	1664	1682	1701	1720	1739	1759
12	1778	1797	1816	1836	1855	1875	1895	1914	1934	1954
13	1974	1994	2014	2034	2055	2075	2095	2116	2136	2157
14	2178	2199	2219	2240	2261	2282	2304	2325	2346	2367
15	2389	2410	2432	2454	2475	2497	2519	2541	2563	2585
16	2607	2630	2652	2674	2697	2719	2742	2765	2788	2810
17	2833	2856	2879	2903	2926	2949	2972	2996	3019	3043
18	3067	3090	3114	3138	3162	3186	3210	3234	3259	3283
19	3307	3332	3356	3381	3406	3431	3455	3480	3505	3530
20	3556	3581	3606	3631	3657	3682	3708	3734	3759	3785
21	3811	3837	3863	3889	3915	3942	3968	3994	4021	4047
22	4074	4101	4128	4154	4181	4208	4235	4263	4290	4317
23	4344	4372	4399	4427	4455	4482	4510	4538	4566	4594
24	4622	4650	4679	4707	4735	4764	4792	4821	4850	4879
25	4907	4936	4965	4994	5024	5053	5082	5111	5141	5170
26	5200	5230	5259	5289	5319	5349	5379	5409	5439	5470
27	5500	5530	5561	5591	5622	5653	5684	5714	5745	5776
28	5807	5839	5870	5901	5932	5964	5995	6027	6059	6090
29	6122	6154	6186	6218	6250	6282	6315	6347	6379	6412
30	6444	6477	6510	6543	6575	6608	6641	6674	6708	6741
31	6774	6807	6841	6874	6908	6942	6975	7009	7043	7077
32	7111	7145	7179	7214	7248	7282	7317	7351	7386	7421
33	7456	7490	7525	7560	7595	7631	7666	7701	7736	7772
34	7807	7843	7879	7914	7950	7986	8022	8058	8094	8130
35	8167	8203	8239	8276	8312	8349	8386	8423	8459	8496
36	8533	8570	8608	8645	8682	8719	8757	8794	8832	8870
37	8907	8945	8983	9021	9059	9097	9135	9174	9212	9250
38	9289	9327	9366	9405	9444	9482	9521	9560	9599	9639
39	9678	9717	9756	9796	9835	9875	9915	9954	9994	10034
40	10074	10114	10154	10194	10235	10275	10315	10356	10396	10437
41	10478	10519	10559	10600	10641	10682	10724	10765	10806	10847
42	10889	10930	10972	11014	11055	11097	11139	11181	11223	11265
43	11307	11350	11392	11434	11477	11519	11562	11605	11648	11690
44	11733	11776	11819	11863	11906	11949	11992	12036	12079	12123
45	12167	12210	12254	12298	12342	12386	12430	12474	12519	12563
46	12607	12652	12696	12741	12786	12831	12875	12920	12965	13010
47	13056	13101	13146	13191	13237	13282	13328	13374	13419	13465
48	13511	13587	13603	13649	13695	13742	13788	13834	13881	13927
49	13974	14021	14068	14114	14161	14208	14255	14303	14350	14397
50	14444	14492	14539	14587	14635	14682	14730	14778	14826	14874
51	14922	14970	15019	15067	15115	15164	15212	15261	15310	15359
52	15407	15456	15505	15554	15604	15653	15702	15751	15801	15850
53	15900	15950	15999	16049	16099	16149	16199	16249	16299	16350
54	16400	16450	16501	16551	16602	16653	16704	16754	16805	16856
55	16907	16959	17010	17061	17112	17164	17215	17267	17319	17370
56	17422	17474	17526	17578	17630	17682	17735	17787	17839	17892
57	17944	17997	18050	18103	18155	18208	18261	18314	18368	18421
58	18474	18527	18581	18634	18688	18742	18795	18849	18903	18957
59	19011	19065	19119	19174	19228	19282	19337	19391	19446	19501
60	19556	19610	19665	19720	19775	19831	19886	19941	19996	20052

For continuation to 100 feet, see Table 7.

Table 6. Level Cuttings.

Roadway 28 ft wide, side-slopes 1½ to 1.

For double-track excavation.

Depth in Ft	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.
0		10.4	21.0	31.6	42.4	53.2	64.2	75 3	86.5	97.9
1	109.3	120.8	132.5	144.3	156.1	168.1	180.2	192.4	204.8	217.2
2	229.6	242.3	255.0	267.9	280.9	294.0	307.2	320.5	334.0	347.5
3	361.2	374.9	388.8	402.8	416.9	431.1	445.4	459.9	474.4	489.1
4	503.7	518.6	533.6	548 6	563.9	579.3	594.7	610.2	625.8	641.6
5	657.5	673.4	689.5	705.7	722.1	738.5	755 0	771 7	788.4	805.3
6	822.2	839.3	856.5	873.8	891.2	908.8	926.4	944.2	962.0	980.0
7	998.1	1016	1035	1053	1072	1090	1109	1128	1147	1166
8	1185	1204	1224	1243	1263	1283	1303	1322	1343	1363
9	1383	1403	1424	1445	1465	1486	1507	1528	1549	1571
10	1592	1614	1635	1657	1679	1701	1723	1745	1767	1790
11	1812	1835	1858	1881	1904	1927	1950	1973	1997	2020
12	2044	2068	2092	2116	2140	2164	2189	2213	2238	2262
13	2287	2312	2337	2362	2387	2413	2438	2464	2489	2515
14	2541	2567	2593	2619	2645	2672	2698	2725	2752	2779
15	2806	2833	2860	2887	2915	2942	2970	2997	3025	3053
16	3081	3109	3138	3166	3195	3223	3252	3281	3310	3339
17	3368	3397	3427	3456	3486	3516	3546	3576	3606	3636
18	3667	3697	3728	3758	3789	3820	3851	3882	3913	3944
19	3976	4007	4039	4070	4102	4134	4166	4198	4231	4263
20	4296	4328	4361	4394	4427	4460	4493	4527	4560	4594
21	4627	4661	4695	4729	4763	4797	4832	4866	4900	4935
22	4970	5005	5040	5075	5111	5146	5181	5217	5253	5288
23	5324	5360	5396	5432	5469	5505	5542	5578	5615	5652
24	5689	5726	5763	5800	5838	5875	5913	5951	5989	6027
25	6065	6103	6141	6179	6218	6257	6295	6334	6373	6412
26	6451	6491	6530	6570	6609	6649	6689	6729	6769	6809
27	6850	6890	6931	6971	7012	7053	7094	7135	7176	7217
28	7259	7300	7342	7384	7426	7468	7510	7552	7594	7637
29	7680	7722	7765	7808	7851	7894	7937	7981	8024	8067
30	8111	8155	8199	8243	8287	8331	8375	8420	8464	8509
31	8554	8598	8643	8688	8734	8779	8824	8870	8915	8961
32	9007	9053	9099	9145	9191	9238	9284	9331	9378	9425
33	9472	9519	9566	9613	9661	9708	9756	9804	9851	9900
34	9948	9997	10045	10093	10142	10190	10239	10288	10337	10386
35	10435	10484	10534	10583	10633	10683	10732	10782	10832	10882
36	10933	10983	11034	11084	11135	11186	11237	11288	11339	11391
37	11443	11494	11546	11598	11649	11701	11753	11806	11858	11910
38	11963	12016	12068	12121	12174	12227	12281	12334	12387	12441
39	12494	12548	12602	12656	12710	12764	12819	12873	12928	12982
40	13037	13092	13147	13202	13257	13312	13368	13423	13479	13535
41	13591	13647	13703	13759	13815	13872	13928	13985	14042	14099
42	14156	14213	14270	14327	14385	14442	14500	14558	14615	14673
43	14731	14790	14848	14906	14965	15024	15082	15141	15200	15259
44	15318	15378	15437	15497	15556	15616	15676	15736	15796	15856
45	15917	15977	16038	16098	16159	16220	16281	16342	16403	16465
46	16526	16587	16649	16711	16773	16835	16897	16959	17021	17084
47	17146	17209	17272	17335	17398	17461	17524	17587	17651	17714
48	17778	17842	17905	17969	18033	18098	18162	18226	18291	18356
49	18420	18485	18550	18615	18680	18746	18811	18877	18942	19008
50	19074	19140	19206	19272	19339	19405	19472	19538	19605	19672
51	19739	19806	19873	19940	20008	20075	20143	20211	20279	20347
52	20415	20483	20551	20620	20688	20757	20826	20894	20963	21032
53	21102	21171	21241	21310	21380	21450	21519	21589	21659	21730
54	21800	21870	21941	22012	22082	22153	22224	22295	22366	22438
55	22509	22581	22652	22724	22796	22868	22940	23012	23085	23157
56	23230	23302	23375	23448	23521	23594	23667	23741	23814	23888
57	23961	24035	24109	24183	24257	24331	24405	24480	24554	24629
58	24704	24779	24854	24929	25004	25079	25155	25230	25306	25381
59	25457	25533	25609	25686	25762	25838	25915	25992	26068	26145
60	26222	26299	26376	26454	26531	26609	26686	26764	26842	26920

For continuation to 100 feet, see Table 7.

Table 7. Level Cuttings.

Continuation of the six foregoing Tables of Cubic Contents, to 100 feet of height or depth.

Height or Depth in Feet.	Table 1	Table 2	Table 3	Table 4	Table 5	Table 6
	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.
61	23835	26094	17848	24739	20107	26998
.5	24201	26479	18108	25113	20386	27390
62	24570	26867	18370	25489	20667	27785
.5	24942	27257	18634	25868	20949	28183
63	25317	27650	18900	26250	21233	28583
.5	25694	28046	19168	26635	21519	28986
64	26074	28444	19437	27022	21807	29393
.5	26457	28846	19708	27413	22097	29801
65	26843	29250	19981	27806	22389	30213
.5	27231	29657	20256	28201	22682	30627
66	27622	30067	20533	28600	22978	31044
.5	28016	30479	20812	29001	23275	31464
67	28413	30894	21093	29406	23574	31887
.5	28812	31313	21375	29813	23875	32312
68	29215	31733	21659	30222	24178	32741
.5	29620	32157	21945	30635	24482	33172
69	30028	32583	22233	31050	24789	33605
.5	30438	33013	22523	31468	25097	34042
70	30852	33444	22814	31889	25407	34481
.5	31268	33879	23108	33313	25719	34924
71	31687	34317	23404	32739	26033	35369
.5	32108	34757	23701	33168	26349	35816
72	32533	35200	24000	33600	26667	36267
.5	32960	35646	24301	34035	26986	36720
73	33390	36094	24604	34472	27307	37176
.5	33823	36546	24907	34913	27631	37635
74	34259	37000	25214	35356	27956	38096
.5	34697	37457	25522	35801	28282	38561
75	35139	37917	25832	36250	28611	39028
.5	35582	38379	26144	36701	28942	39498
76	36029	38844	26458	37156	29174	39970
.5	36479	39313	26774	37613	29608	40446
77	36931	39783	27092	38072	29944	40924
.5	37386	40257	27411	38535	30282	41405
78	37844	40733	27733	39000	30622	41889
.5	38305	41213	28056	39468	30964	42375
79	38768	41694	28381	39939	31307	42865
.5	39235	42179	28708	40413	31653	43357
80	39704	42667	29037	40889	32000	43852
81	40650	43650	29700	41850	32700	44850
82	41607	44644	30370	42822	33407	45859
83	42576	45650	31048	43806	34122	46880
84	43555	46667	31733	44800	34844	47911
85	44546	47694	32426	45806	35574	48954
86	45548	48733	33126	46822	36311	50008
87	46561	49783	33833	47850	37056	51072
88	47585	50844	34548	48889	37807	52148
89	48620	51917	35270	49939	38567	53235
90	49667	53000	36000	51000	39333	54333
91	50724	54094	36737	52072	40107	55443
92	51793	55200	37481	53156	40889	56563
93	52872	56317	38233	54250	41678	57694
94	53963	57444	38993	55356	42474	58837
95	55065	58583	39759	56472	43278	59990
96	56178	59733	40533	57600	44089	61155
97	57302	60894	41315	58739	44907	62331
98	58437	62067	42104	59889	45733	63518
99	59583	63250	42900	61050	46567	64716
100	60741	64444	43704	62222	47407	65926

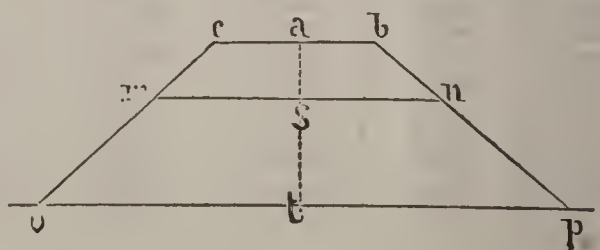
Table 8,

Of Cubic Yards in a 100-foot station of level cutting or filling, to be added to, or su tracted from, the quantities in the preceding seven tables, in case the excav tions or embankments should be increased or diminished 2 feet in width.

Cubic Yards in a length of 100 feet; breadth 2 feet; and of different depths.

Height or Depth in Feet.	Cubic Yards.	Height or Depth in Feet.	Cubic Yards.	Height or Depth in Feet.	Cubic Yards.	Height or Depth in Feet.	Cubic Yards.	Height or Depth in Feet.	Cubi Yard
.5	3.70	.5	152	.5	300	.5	448	.5	596
1	7.41	21	156	41	304	61	452	81	600
.5	11.1	.5	159	.5	307	.5	456	.5	604
2	14.8	22	163	42	311	62	459	82	607
.5	18.5	.5	167	.5	315	.5	463	.5	611
3	22.2	23	170	43	319	63	467	83	615
.5	25.9	.5	174	.5	322	.5	470	.5	619
4	29.6	24	178	44	326	64	474	84	622
.5	33.3	.5	181	.5	330	.5	478	.5	626
5	37.0	25	185	45	333	65	481	85	630
.5	40.7	.5	189	.5	337	.5	485	.5	633
6	44.4	26	193	46	341	66	489	86	637
.5	48.1	.5	196	.5	344	.5	493	.5	641
7	51.9	27	200	47	348	67	496	87	644
.5	55.6	.5	204	.5	352	.5	500	.5	648
8	59.3	28	207	48	356	68	504	88	652
.5	63.0	.5	211	.5	359	.5	507	.5	656
9	66.7	29	215	49	363	69	511	89	659
.5	70.4	.5	219	.5	367	.5	515	.5	663
10	74 1	30	222	50	370	70	519	90	667
.5	77.8	.5	226	.5	374	.5	522	.5	670
11	81.5	31	230	51	378	71	526	91	674
.5	85 2	.5	233	.5	381	.5	530	.5	678
12	88.9	32	237	52	385	72	533	92	681
.5	92.6	.5	241	.5	389	.5	537	.5	685
13	96.3	33	244	53	393	73	541	93	689
.5	100	.5	248	.5	396	.5	544	.5	693
14	104	34	252	54	400	74	548	94	696
.5	107	.5	256	.5	404	.5	552	.5	700
15	111	35	259	55	407	75	556	95	704
.5	115	.5	263	.5	411	.5	559	.5	707
16	119	36	267	56	415	76	563	96	711
.5	122	.5	270	.5	419	.5	567	.5	715
17	126	37	274	57	422	77	570	97	719
.5	130	.5	278	.5	426	.5	574	.5	722
18	133	38	281	58	430	78	578	98	726
.5	137	.5	285	.5	433	.5	581	.5	730
19	141	39	289	59	437	79	585	99	733
.5	144	.5	293	.5	441	.5	589	.5	737
20	148	40	296	60	444	80	593	100	741

Remark. **The foregoing tables of level cuttings may also b used for widths of roadway greater than those at the head of the tables.** Thus, suppose we wish to use Table 1, for a roadbed *m n*, 16 wide, instead of *c b*, which is only 14 ft, and for which the table was calculated. is only necessary first to find the vert dist *s a*, between these two roadbeds; and add it *mentally* to each height *t s*, of the given embkt, when taking out from t



able the numbers of cub yds corresponding to the heights. By this means we obtain he contents of the embkt *c b o p*, for any required dist. Next, from these contents ubtract that corresponding to the height *s a*, for the same dist. The remainder will lainly be the embkt *m n o p*.

In practice it will be sufficiently correct to take *s a* to the nearest tenth of a oot, which will save trouble in adding it mentally to the heights in the tables.

If the roadbed is narrower than the table, as, for instance, if *m n* be he width in the table, but we wish to find the contents for the width *c b*, then irst find *s a*, and calculate the cubic yards in 100 feet length of *c b m n*. Then, n taking out the cubic yards from the table, first subtract *s a* mentally from ach height; and to the cubic yards taken out for each 100 feet, opposite this *educed* height, add the cubic yards in 100 feet of *c b m n*.

To avoid trouble with contractors about the measurement of rock uts, stipulate in the contract, either that it shall conform with the theoretical ross section; or that an extra allowance of say about 2 feet of width of cut vill be made, to cover the unavoidable irregularities of the sides.

Shrinkage of Embankment. Although earth, when first dug, and oosely thrown out, *swells* about $\frac{1}{5}$ part, so that a cubic yard *in place* averages bout $1\frac{1}{5}$ or 1.2 cubic yards when dug: or 1 cubic yard dug is equal to $\frac{5}{6}$, or to 3333 of a cubic yard in place; yet when made into embankment it gradually ubsides, settles, or shrinks, into a less bulk than it occupied *before being dug*.

The following are approximate averages of the shrinkage; or, in other words, he earth measured in place in a cut, will, when made into embankment, occupy t bulk less than before by about the following proportions:

Gravel or sand	about	8 per ct;	or 1 in $12\frac{1}{2}$ less.
Clay	"	10 per ct;	or 1 in 10 less.
Loam	"	12 per ct;	or 1 in $8\frac{1}{3}$ less.
Loose vegetable surface soil	"	15 per ct;	or 1 in $6\frac{2}{3}$ less.
Puddled clay	"	25 per ct;	or 1 in 4 less.

The writer thinks, from some trials of his own, that 1 cubic yard of any hard ock in place, will make from $1\frac{2}{3}$ to $1\frac{3}{4}$ cubic yards of embankment; say on an verage 1.7 cubic yards. Or that 1 cubic yard of rock embankment requires 5882 of a cubic yard in place. He found that a solid cubic yard when broken nto fragments, made about as follows (see p 678):

	Cubic yards.	Of which there were Solid	Voids
In loose heap	1.9	52.6 per cent.	47.4 per cent.
Carelessly piled	1.75	57 "	43 "
Carefully piled	1.6	63 "	37 "
Rubble, very carelessly scabbled	1.5	67 "	33 "
Rubble, somewhat carefully scabbled	1.25	80 "	20 "

For trestles, see p 755.
For culverts and stone bridges, see pp 693, &c.

COST OF EARTHWORK.

Art. 1. It is advisable to pay for this kind of work by the cubic yard of *excavation* only; instead of allowing separate prices for excavation and embankment. By this means we get rid of the difficulty of measurements, as well as the controversies and lawsuits which often attend the determination of the allowance to be made for the settlement or subsidence of the embankments.

It is, moreover, our opinion that justice to the contractor should lead to the **English practice of paying the laborers by the cubic yard.** instead of by the day. Experience fully proves that when laborers are scarce and wages high, men can scarcely be depended upon to do three-fourths of the work which they readily accomplish when wages are low, and when fresh hands are waiting to be hired in case any are discharged. The contractor is thus placed at the mercy of his men. The writer has known the most satisfactory results to attend a system of task-work, accompanied by liberal premiums for all overwork. By this means the interests of the laborers are identified with that of the contractor; and every man takes care that the others shall do their fair share of the task.

Ellwood Morris, C E, of Philadelphia, was, we believe, the first person who properly investigated the elements of cost of earthwork, and reduced them to such a form as to enable us to calculate the total with a considerable degree of accuracy. He published his results in the Journal of the Franklin Institute in 1841. His paper forms the basis on which, with some variations, we shall consider the matter; and on which we shall extend it to wheelbarrows, as well as to carts. Throughout this paper we speak of a cubic yard considered only as solid in its place, or before it is loosened for removal. It is scarcely necessary to add that the various items can of course only be regarded as tolerably close approximations, or averages. As before stated, the men do less work when wages are high; and more when they are low. A great deal besides depends on the skill, observation, and energy of the contractor and his superintendents. It is no unusual thing to see two contractors working at the same prices, in precisely similar material, where one is making money, and the other losing it, from a want of tact in the proper distribution of his forces, keeping his roads in order, having his carts and barrows well filled, &c, &c. Uncommonly long spells of wet weather may seriously affect the cost of executing earthwork, by making it more difficult to loosen, load, or empty; besides keeping the roads in bad order for hauling.

The aggregate cost of excavating and removing earth is made up by the following items, namely:

1st. *Loosening the earth ready for the shovellers.*
2d. *Loading it by shovels into the carts or barrows.*
3d. *Hauling, or wheeling it away, including emptying and returning.*
4th. *Spreading it out into successive layers on the embankment.*
5th. *Keeping the hauling-road for carts, or the plank gangways for barrows, in good order.*
6th. *Wear, sharpening, depreciation, and interest on cost of tools.*
7th. *Superintendence, and water-carriers.*
8th. *Profit to the contractor.*

We will consider these items a little in detail, basing our calculations on the assumption that common labor costs $1 per day, of 10 working hours. The results in our tables must therefore be increased or diminished in about the same proportion as common labor costs more or less than this.

Art. 2. Loosening the earth ready for the shovellers. This is generally done either by ploughs or by picks; more cheaply by the first. A plough with two horses, and two men to manage them, at $1 per day for labor, 75 cents per day for each horse, and 37 cents per day for plough, including harness, wear, repairs, &c, or a total of $3.87, will loosen, of strong heavy soils, from 200 to 300 cubic yards a day, at from 1.93 to 1.29 cents per yard; or of ordinary loam, from 400 to 600 cubic yards a day, at from .97 to .64 of a cent per yard. Therefore, as an ordinary average, we may assume the actual cost to the contractor for loosening by the plough, as follows: strong heavy soils, 1.6 cents; common loam, .8 cent; light sandy soils, .4 cent. Very stiff pure clay, or obstinate cemented gravel, may be set down at 2.5 cents; they require three or four horses.

By the pick, a fair day's work is about 14 yards of stiff pure clay, or of cemented gravel; 25 yards of strong heavy soils; 40 yards of common loam; 60 yards of light sandy soils — all measured in place; which, at $1 per day for labor, gives, for stiff clay, 7 cents; heavy soils, 4 cents; loam, 2.5 cents; light sandy soil, 1.666 cents. Pure sand requires but very little labor for loosening; .5 of a cent will cover it.

Art. 3. Shovelling the loosened earth into carts. The amount shovelled per day depends partly upon the weight of the material, but more upon so proportioning the number of pickers and of carts to that of shovellers, as not to keep the latter waiting for either material or carts. In fairly regulated gangs, the shovellers into carts are not actually engaged in shovelling for more than six-tenths of their time, thus being unoccupied but four-tenths of it; while, under bad management, they lose considerably more than one-half of it. A shoveller can readily load into a cart one-third of a cubic yard measured in place (and which is an average working cart-load), of sandy soil, in five minutes; of loam, in six minutes; and of any of the heavy soils, in seven minutes. This would give, for a day of 10 working hours, 120 loads, or 40 cubic yards of light sandy soil; 100 loads, or 33⅓ cubic yards of loam; or 86 loads, or 28.7 yards of the heavy soils. But from these amounts we must deduct four-tenths for time necessarily lost; thus reducing the actual working quantities to 24 yards of light sandy soil, 20 yards of loam, 17.2 yards of the heavy soils. When the shovellers do less than this, there is some mismanagement.

Assuming these as fair quantities, then, at $1 per day for labor, the actual cost to the contractor for shovelling per cubic yard measured in place, will be, for sandy soils, 4.167 cents; loam, 5 cents; heavy soils, clays, &c, 5.81 cents.

In practice, the carts are not usually loaded to any less extent with the heavier soils than with the lighter ones. Nor, indeed, is there any necessity for so doing, inasmuch as the difference of weight of a cart and one third of a cubic yard of the various soils is too slight to need any attention; especially when the cart-road is kept in good order, as it will be by any contractor who understands his

own interest. Neither is it necessary to modify the load on account of any *slight inclinations* which may occur in the grading of roads. An earth-cart weighs by itself about ½ a ton.

Art. 4. Hauling away the earth; dumping, or emptying; and returning to reload. The average speed of horses in hauling is about 2½ miles per hour, or 200 feet per minute; which is equal to 100 feet of trip each way; or to 100 feet of *lead*, as the distance to which *the earth is hauled* is technically called.* Besides this, there is a loss of about four minutes in every trip, whether long or short, in waiting to load, dumping, turning, &c. Hence, every trip will occupy as many minutes as there are lengths of 100 feet each in the lead; and four minutes besides. Therefore, to find the number of trips per day over any given average lead, we divide the number of minutes in a working day by the sum of 4 added to the number of 100 feet lengths contained in the distance to which the earth has to be removed; that is,

$$\frac{\textit{The number (600) of minutes in a working day}}{4 + \textit{the number of 100-feet lengths in the lead}} = \textit{the number of trips, or loads removed per day, per cart.}$$

And since ⅓ of a cubic yard measured before being loosened, makes an average cart-load, the number of loads, divided by 3, will give the number of cubic yards removed per day by each cart; and the cubic yards divided into the total expense of a cart per day, will give the cost per cubic yard for hauling.

Remark. When removing loose *rock*, which requires more time for loading, say,

$$\frac{\textit{No. of minutes (600) in a working day}}{6 + \textit{No. of 100-feet lengths of lead}} = \textit{No. of loads removed, per day, per cart.}$$

In leads of ordinary length one driver can attend to 4 carts; which, at $1 per day, is 25 cents per cart. When labor is $1 per day, the expense of a horse is usually about 75 cents; and that of the cart, including harness, tar, repairs, &c, 25 cents, making the total daily cost per cart $1.25. The expense of the horse is the same on Sundays and on rainy days, as when at work; and this consideration is included in the 75 cents. Some contractors employ a greater number of drivers, who also help to load the carts, so that the expense is about the same in either case.

Example. How many cubic yards of loam, measured in the cut, can be hauled by a horse and cart in a day of 10 working hours, (600 minutes,) the lead, or length of haul of earth being 1000 feet, (or 10 lengths of 100 feet,) and what will be the expense to the contractor for hauling, per cubic yard, assuming the total cost of cart, horse, and driver, at $1.25?

$$\textit{Here,}\ \frac{600\ \textit{minutes}}{4 + 10\ \textit{lengths of 100 feet,}} = \frac{600}{14} = 43\ \textit{loads.} \qquad \textit{And}\ \frac{43\ \textit{loads}}{3} = 14.3\ \textit{cubic yards.}$$

$$\textit{And}\ \frac{125\ \textit{cents}}{14.3\ \textit{cub yds}} = 8.74\ \textit{cents per cubic yard.}$$

In this manner the 2d and 3d columns of the following tables have been calculated.

Art. 5. Spreading, or levelling off the earth into regular thin layers on the embankment. A bankman will spread from 50 to 100 cubic yards of either common loam, or any of the heavier soils, clays, &c, depending on their dryness. This, at $1 per day, is 1 to 2 cents per cubic yard; and we may assume 1½ cents as a fair average for such soils; while 1 cent will suffice for light sandy soils.

This expense for spreading is saved when the earth is either dumped over the end of the embankment, or is wasted; still, about ¼ cent per yard should be allowed in either case for keeping the dumping-places clear and in order.

Art. 6. Keeping the cart-road in good order for hauling. No ruts or puddles should be allowed to remain unfilled; rain should at once be led off by shallow ditches; and the road be carefully kept in good order; otherwise the labor of the horses, and the wear of carts, will be *very greatly* increased. It is usual to allow so much per cubic yard for road repairs; but we suggest so much per cubic yard, per 100 feet of lead; say $\frac{1}{10}$ of a cent.

Art. 7. Wear, sharpening, and depreciation of picks and shovels. Experience shows that about ¼ of a cent per cubic yard will cover this item.

Superintendence and water-carriers. These expenses will vary with local circumstances; but we agree with Mr. Morris, that 1½ cents per cubic yard will, under ordinary circumstances, cover both of them. An allowance of about ¼ cent may in justice be added for extra trouble in digging the side-ditches; levelling off the bottom of the cut to grade; and general trimming up. In very light cuttings this may be increased to ½ cent per cubic yard.

At ¼ cent, all the items in this article amount to 2 cents per cubic yard of cut.

Art. 8. Profit to the contractor. This may generally be set down at from 6 to 15 per cent, according to the magnitude of the work, the risks incurred, and various incidental circumstances. Out of this item the contractor generally has to pay clerks, storekeepers, and other agents, as well as the expenses of shanties, &c; although these are in most cases repaid by the profits of the stores; and by the rates of boarding and lodging paid to the contractors by the laborers.

Art. 9. A knowledge of the foregoing items enables us to calculate with tolerable accuracy the cost of removing earth. For example, let it be required to ascertain the cost per cubic yard of excavating common loam, measured in place; and of removing it into embankment, with an average haul or lead of 1000 feet; the wages of laborers being $1 per day of 10 working hours; a horse 75 cts a day; and a cart 25 cts. One driver to four carts.

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* When an entire cut is made into an embankment, the **mean haul** is the dist between centers of gravity of the cut and embkt.

		Cents.
Here we have cost of loosening, say by pick,	*Art* 2, *per cubic yard, say,*	2.50
Loading into carts,	*Art.* 3, " "	5.00
Hauling 1000 *feet, as calculated previously in example,*	*Art.* 4, "	8.74
Spreading into layers,	*Art.* 5, "	1.50
Keeping cart-road in repair,	*Art.* 6, 10 *lengths of* 100 *ft,*	1.00
Various items in	*Art.* 7,	2.00
Total cost to contractor,		20.74
Add contractor's profit, say 10 *per cent,*		2.074
Total cost per cubic yard to the company,		22.814

It is easy to construct a table like the following, of costs per cubic yard, for different lengths of lead. Columns 2 and 3 are first obtained by the Rule in Article 4; then to each amount in column 3 is added the *variable* quantity of $\frac{1}{10}$ of a cent for every 100 feet length of lead, for keeping the road in order; and the *constant* quantity (for any given kind of soil) composed of the prices per cubic yard, for loosening, loading, spreading, or wasting, &c, either taken from the preceding articles; or modified to suit particular circumstances. In this manner the tables have been prepared.

By Carts. Labor $1 per day, of 10 working hours.

Length of Lead, or distance to which the earth is hauled, in feet.	Number of cubic yards in place, hauled per day by each cart.	Cost per cubic yard in place, for hauling and emptying only.	Common Loam. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.				Strong Heavy Soils. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.			
			Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.	Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.
Feet.	Cu. Yds.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.
25	47.0	2.66	13.69	12 44	11.99	10.74	16.00	14.75	13.50	12.25
50	44.4	2.81	13.86	12.61	12.16	10.91	16.17	14.92	13.67	12.42
75	42.1	2.97	14.05	12.80	12.35	11.10	16.36	15.11	13.86	12.61
100	40.0	3.12	14.22	12.97	12.52	11.27	16.53	15.28	14.03	12.78
150	36.4	3 43	14.58	13.33	12.88	11.63	16.89	15 64	14.39	13.14
200	33.3	3.75	14.95	13.70	13.25	12.00	17.26	16.01	14.76	13.51
300	28.6	4.37	15.67	14.42	13.97	12.72	17.98	16.73	15.48	14.23
400	25 0	5.00	16.40	15.15	14.70	13.45	18.71	17.46	16.21	14.96
500	22.2	5.63	17.13	15 88	15.43	14.18	19.44	18.19	16.94	15 69
600	20.0	6.25	17.85	16.60	16.15	14.90	20.16	18.91	17.66	16.41
700	18.2	6.87	18.57	17.32	16.87	15.62	20.88	19.63	18.38	17.13
800	16.7	7.48	19.28	18.03	17.58	16.33	21.59	20.34	19.09	17.84
900	15.4	8.12	19.92	18.67	18.22	16.97	22.23	20.98	19.73	18.48
1000	14.3	8.74	20.74	19.49	19.04	17.79	23.05	21.80	20.55	19.30
1100	13.3	9.40	21.50	20.25	19.80	18.55	23.81	22.56	21.31	20.06
1200	12.5	10.0	22.20	20.95	20.50	19 25	24.51	23.26	22.01	20.76
1300	11.8	10.6	22.90	21.65	21.20	19.95	25.21	23.96	22.71	21.46
1400	11.1	11.2	23.60	22.35	21.90	20.65	25.91	24.66	23.41	22.16
1500	10.5	11.9	24.40	23.15	23.70	21.45	26.71	25.46	24.21	22.96
1600	10.0	12.5	25.10	23.85	23.40	22.15	27.41	26.16	24.91	23.66
1700	9.52	13.1	25.80	24.55	24.10	22.85	28.11	26.86	25 61	24.36
1800	9.09	13.7	26.50	25.25	24.80	23.55	28.81	27.56	26.31	25.06
1900	8.70	14.4	27.30	26 05	25.60	24.35	29.61	28.36	27.11	25.86
2000	8.33	15.0	28.00	26.75	26.30	25.05	30.31	29.06	27.81	26.56
2250	7.54	16.6	29.85	28.60	28.15	26.90	32.16	30.91	29.66	28.41
2500	6 90	18.1	31.60	30.35	29.90	28.65	33.91	32.66	31.41	30.16
½ mile	6.58	19.0	32.64	31.39	30.94	29.69	34.95	33.70	32.45	31.20
3000	5.88	21.2	35.20	33.95	33.50	32.25	37.51	36.26	35.01	33.76
3250	5.48	22.8	37.05	35.80	35.35	34.10	39.36	38.11	36.86	35.61
3500	5.13	24.3	38.80	37.55	37.10	35.85	41.11	39.86	38.61	37.36
3750	4.82	25 9	40.65	39.40	38.95	37.70	42.96	41.71	40.46	39.21
4000	4.54	27.5	42.50	41.25	40.80	39.55	44.81	43.56	42.31	41.06
4250	4.30	29.1	44.35	43.10	42.65	41.40	46.66	45.41	44.16	42.91
4500	4.08	30.6	46.10	44.85	44.40	43.15	48.41	47.16	45.91	44.66
4750	3.88	32.2	47.95	46.70	46.25	45.00	50.26	49.01	47.76	46.51
5000	3.70	33.8	49.80	48.55	48.10	46.85	52.11	50 86	49.61	48.36
1 mile	3.52	35.5	51.78	50.53	50.08	48.83	54.09	52.84	51.59	50.34
1¼ m.	2.86	43.8	61.40	60.15	59.70	58.45	63.71	62.46	61.21	59.96
1½ m.	2.40	52.1	71.02	69.77	69.32	68.07	73.33	72.08	70.83	69.58
1¾ m.	2.07	60.4	80.64	79.39	78.94	77.69	82.95	81.70	80.45	79.20
2 m.	1.82	68.7	90.26	89.01	88.56	87.31	92.57	91.32	90.07	88.82

By Carts. Labor $1 per day, of 10 working hours.

Length of Lead, or distance to which the earth is hauled, in feet.	Number of cubic yards in place, hauled per day by each cart.	Cost per cubic yard in place, for hauling and emptying only.	Pure stiff Clay, or cemented Gravel. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.				Light Sandy Soils. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.			
			Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.	Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.
Feet.	Cu. Yds.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.
25	47.0	2.66	19.00	17.75	14.50	13.25	11.52	10.77	10.25	9.50
50	44.4	2.81	19.17	17.92	14.67	13.42	11.69	10.94	10.42	9.67
75	42.1	2.97	19.36	18.11	14.86	13.61	11.88	11.13	10.61	9.86
100	40.0	3.12	19.53	18.28	15.03	13.78	12.05	11.30	10.78	10.03
150	36.4	3.43	19.89	18.64	15.39	14.14	12.41	11.66	11.14	10.39
200	33.3	3.75	20.26	19.01	15.76	14.51	12.78	12.03	11.51	10.76
300	28.6	4.37	20.98	19.73	15.48	15.23	13.50	12.75	12.23	11.48
400	25.0	5.00	21.71	20.46	17.21	15.96	14.23	13.48	12.46	12.21
500	22.2	5.63	22.44	21.19	17.94	16.69	14.96	14.21	13.69	12.94
600	20.0	6.25	23.16	21.91	18.66	17.41	15.68	14.93	14.41	13.66
700	18.2	6.87	23.88	22.63	19.38	18.13	16.40	15.65	15.13	14.38
800	16.7	7.48	24.59	23.34	20.09	18.84	17.11	16.36	15.84	15 09
900	15.4	8.12	25.23	23.98	20.73	19.48	17.75	17.00	16.48	15.73
1000	14.3	8 74	26.05	24.80	21.55	20.30	18.57	17.82	17.30	16.55
1100	13.3	9.40	26.81	25.56	22.31	21.06	19.33	18.58	18.06	17.31
1200	12.5	10.0	27.51	26.26	23.01	21.76	20.03	19.28	18.76	18.01
1300	11.8	10.6	28.21	26.96	23.71	22.46	20.73	19.98	19.46	18.71
1400	11.1	11.2	28.91	27.66	24.41	23.16	21.43	20.68	20.16	19.41
1500	10.5	11.9	29.71	28.46	25.21	23.96	22.23	21.48	20.96	20.21
1600	10.0	12.5	30.41	29.16	25.91	24.66	22.93	22.18	21.66	20.91
1700	9.52	13.1	31.11	29.86	26.61	25.36	23.63	22.88	22.36	21.61
1800	9.09	13.7	31.81	30.56	27.31	26.06	24.33	23.58	23.06	22.31
1900	8.70	14.4	32.61	31.36	28.11	26.86	25.13	24.38	23.86	23.11
2000	8.33	15.0	33.31	32.06	28.81	27.56	25.83	25.08	24.56	23.81
2250	7.54	16.6	35.16	33.91	30.66	29.41	27.68	26.93	26.41	25.66
2500	6.90	18.1	36.91	35.66	32.41	31.16	29.43	28.68	28.16	27.41
½ mile	6.58	19.0	37.95	36.70	33.45	32.20	30.47	29.72	29.20	28.45
3000	5.88	21.2	40.51	39.26	36.01	34.76	33.03	32.28	31.76	31.01
3250	5.48	22.8	42.36	41.11	37.86	36.61	34.88	34.13	33.61	32.86
3500	5.13	24.3	44.11	42.86	39.61	38.36	36.63	35.88	35.36	34.61
3750	4.82	25.9	45.96	44.71	41.46	40.21	38.48	37.73	37.21	36.46
4000	4.54	27.5	47.81	46.56	43.31	42.06	40.33	39.58	39.06	38.31
4250	4.30	29.1	49.66	48.41	45.16	43.91	42.18	41.45	40.93	40.18
4500	4.08	30.6	51.41	50.16	46.91	45.66	43.93	43.18	42.66	41.91
4750	3.88	32.2	53.26	52.01	48.76	47.51	45.78	45.03	44 51	43.76
5000	3.70	33.8	55.11	53.86	50.61	49.36	47.63	46.88	46.36	45.61
1 mile	3.52	35.5	57.09	55 84	52.59	51.34	49.61	48.86	48.34	47.59
1¼ m.	2.86	43.8	66.91	65.46	62.21	60.96	59.23	58.48	57.96	57.21
1½ m.	2.40	52.1	76.33	75.08	71.83	70.58	68.85	68.10	67.58	66.83
1¾ m.	2.07	60.4	85.95	84.70	81.45	80.20	78.47	77.72	77.20	76.45
2 m.	1.82	68.7	95.57	94.32	91.07	89.82	88.09	87.34	86.82	86.07

Art. 10. By wheelbarrows. The cost by barrows may be estimated in the same manner as by carts. See Articles 1, &c. Men in wheeling move at about the same average rate as horses do in hauling, that is, 2½ miles an hour, or 200 feet per minute, or 1 minute per every 100-feet length of lead. The time occupied in loading, emptying, &c (when, as is usual, the wheeler loads his own barrow,) is about 1.25 minutes, without regard to length of lead; besides which, the time lost in occasional short rests, in adjusting the wheeling plank, and in other incidental causes, amounts to about $\frac{1}{10}$ part of his whole time; so that we must in practice consider him as actually working but 9 hours out of his 10 working ones. Therefore

$$\frac{\textit{The number of minutes in a working day} \times .9}{1.25 + \textit{the number of 100-feet lengths of lead}} = \textit{the number of trips or of loads removed per day per barrow.}$$

See Remark, next page.

The number of loads divided by 14 will give the number of cub yards, since a cub yard, measured in place, averages about 14 loads. And the cost of a wheeler and barrow per day, (say $1 per man and 5 cents per barrow,) divided by the number of cub yards, will give the cost per yard for loading wheeling, and emptying.

Ex. How many cubic yards of common loam, measured in place, will one man load, wh and empty, per day of 10 working hours, (or 600 minutes;) the lead, or distance to which the eart removed being 1000 feet, (or 10 lengths of 100 feet;) and what will be the expense per yard, suppos the laborer and barrow to cost $1.05 per day?

$$Here, \frac{600\ minutes \times .9}{1.25 + 10\ lengths} = \frac{540}{11.25} = 48\ trips,\ or\ loads\ per\ day.$$

$$And\ \frac{48}{14} = 3.43\ cub\ yds\ per\ day.\quad And\ \frac{105\ cents}{3.43\ cub\ yds} = 30.6\ cents$$

per cub yard for loading, wheeling away, emptying, and returning. This would be increased alm inappreciably by the cost of the shovel, which, in the following tables, however, is included in cost of tools.

Rem. For *rock*, which requires more time for loading, say

$$\frac{No\ of\ minutes\ in\ a\ working\ day \times .9}{1.6 + No\ of\ 100\text{-}feet\ lengths\ of\ lead} = \frac{No\ of\ loads\ removed}{per\ day,\ per\ barrow.}$$

Art. 11. The following tables are calculated as in the case of carts, by first finding columr and 3 by means of the Rule in Art 10, and then adding to each sum in column 3, the variable quant of .1 of a cent per cubic yard per 100 feet of lead for keeping the wheeling-planks in order; and prices of loosening, spreading, superintendence, water-carrying, &c, per cubic yard, as given in preceding Articles 2 to 7.

By Wheelbarrows. Labor $1 per day, of 10 working hour

Length of Lead, or distance to which the earth is wheeled, in feet.	Number of cubic yards in place, loaded, and wheeled per day; each barrow.	Cost per cubic yard in place, for loading, wheeling, and emptying.	Common Loam. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.				Strong, Heavy Soils. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.			
			Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.	Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted
Feet.	Cu. Yds.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.
25	25.7	4.09	10.12	8.87	8.42	7.17	11.62	10.37	9.12	7.87
50	22.1	4.75	10.80	9.55	9.10	7.85	12.30	11.05	9.80	8.55
75	19.3	5.44	11.52	10.27	9.82	8.57	13.02	11.77	10.52	9.27
100	17.1	6.14	12.24	10.99	10.54	9.29	13.74	12.49	11.24	9.99
150	14.0	7.50	13.65	12.40	11.95	10.70	15.15	13.90	12.65	11.40
200	11.9	8.82	15.02	13.77	13.32	12.07	16.52	15.27	14.02	12.77
250	10.3	10.2	16.45	15.20	14.75	13.50	17.95	16.70	15 45	14.20
300	9.07	11.6	17.90	16.65	16.20	14.95	19.40	18.15	16.90	15 65
350	8.14	12.9	19.25	18.00	17.55	16.30	20.75	19.50	18 25	17.00
400	7.36	14.3	20.70	19.45	19.00	17.75	22.20	20.95	19.70	18.45
450	6.71	15.6	22.05	20.80	20.35	19.10	23.55	22.30	21.05	19.80
500	6.17	17.0	23.50	22.25	21.80	20.55	25.00	23.75	22.50	21.25
600	5.32	19.7	26.30	25.05	24.60	23.35	27.80	26.55	25.30	24.05
700	4.67	22.5	29.20	27.95	27.50	26.25	30.70	29.45	28.20	26.95
800	4.17	25.2	32.00	30.75	30.30	29.05	33.50	32 25	31.00	29.75
900	3.76	27.9	34.80	33.55	33.10	31.85	36.30	35.05	33.80	32.55
1000	3.43	30.6	37.60	36.35	35.90	34.65	39.10	37.85	36.60	35.35
1200	2.91	36.1	43.30	42.05	41.60	40.35	44.80	43.55	42.30	41.05
1400	2.53	41.5	48.90	47.65	47.20	45.95	50.40	49.15	47.90	46.65
1600	2.24	46.9	54.50	53.45	52.80	51.55	56.00	54.75	53.50	52.25
1800	2.00	52.5	60.30	59.05	58.60	57.35	61.80	60.55	59.30	58.05
2000	1.81	58.0	66.00	64.75	64.30	63.05	67.50	66.25	65.00	63.75
2200	1.66	63.3	71.50	70.25	69.80	68.55	73.00	71.75	70.50	69.25
2400	1.53	68.6	77.00	75.75	75.30	74.05	78.50	77.25	76.00	74.75
½ mile.	1.39	75.5	84.14	82.89	82.44	81.19	85.64	84.39	83.14	81.89

By Wheelbarrows. Labor $1 per day, of 10 working hours.

Length of Lead, or distance to which the earth is wheeled.	Number of cubic yards in place, loaded, and wheeled, per day; each barrow.	Cost per cubic yard in place, for loading, wheeling, and emptying.	Pure Stiff Clay, or Cemented Gravel. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.				Light Sandy Soils. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.			
			Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.	Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.
Feet.	Cu. Yds.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.
25	25.7	4.09	14.62	13.37	10.12	8.87	8.79	8.04	7.52	6.77
50	22.1	4.75	15.30	14.05	10.80	9.55	9.47	8.72	8.20	7.45
75	19.3	5 44	16.02	14.77	11.52	10.27	10.19	9.44	8.92	8.17
100	17.1	6.14	16.74	15.49	12.24	10.99	10.91	10.16	9.64	8.89
150	14 0	7.50	18.15	16.90	13.65	12.40	12.32	11.57	11.05	10.30
200	11.9	8.82	19.52	18.27	15.02	13.77	13.69	12.94	12.42	11.67
250	10.3	10.2	20.95	19.70	16.45	15.20	15.12	14.37	13.85	13.10
300	9.07	11.6	22.40	21.15	17.90	16.65	16.57	15.82	15.30	14.55
350	8.14	12.9	23.75	22.50	19.25	18.00	17.92	17.17	16.65	15.90
400	7.36	14.3	25.20	23.95	20 70	19.45	19.37	18.62	18.10	17.35
450	6.71	15.6	26.55	25.30	22.05	20.80	20.72	19.97	19.45	18.70
500	6.17	17.0	28.00	26.75	23.50	22.25	22.17	21.42	20.90	20.15
600	5.32	19.7	30.80	29.55	26.30	25.05	24.97	24.22	23.70	22.95
700	4.67	22.5	33.70	32.45	29.20	27.95	27.87	27.12	26.60	25.85
800	4.17	25.2	36.50	35.25	32.00	30.75	30.67	29.92	29.40	28.65
900	3.76	27.9	39.30	38.05	34.80	33.55	33.47	32.72	32.20	31.45
1000	3.43	30.6	42.10	40.85	37.60	36.35	36.27	35.52	35.00	34.25
1200	2.91	36.1	47.80	46.55	43.30	42.05	41.97	41.22	40.70	39.90
1400	2.53	41.5	53.40	52.15	48.90	47.65	47 57	46.82	46.30	45.55
1600	2.24	46.9	59.00	57.75	54.50	53.25	53.17	52.42	51.90	51.15
1800	2.00	52.5	64 80	63.55	60.30	59.05	58.97	58.22	57.70	56.95
2000	1.81	58.0	70.50	69.25	66.00	64.75	64.67	63.92	63.40	62.65
2200	1.66	63.3	76.00	74.75	71.50	70.25	70 17	69.42	68.90	68.15
2400	1.53	68.6	81.50	80.25	77.00	75.75	75.67	74.92	74.40	73.65
½ mile.	1.39	75.5	88.64	87.39	84.14	82.89	82.81	82.06	81.54	80.79

Art. 12. By wheeled scrapers and drag scrapers. The body of the wheeled scraper is a box of smooth sheet-steel about 3½ ft square by 15 ins deep, containing about ½ cubic yard of earth when "even full." The box is open in front (in some machines it is closed by an "end gate" when full), and can be raised and lowered, and revolved on a horizontal axis. To fill the box, it is lowered into, and held down in, the earth, while the team draws the machine forward. When full, it is raised to about a foot above ground; and, on reaching the dump, is unloaded by being overturned on its axis. All the movements of the box are made by means of levers, and without stopping the team, which thus travels constantly. The wheels have broad tires, to prevent them from cutting into the ground.

In the *drag* scraper the box, owing to the greater resistance to traction, is made much smaller. It contains about .15 to .25 cubic yard in place, and is always open in front. The operation of the drag scraper is similar to that of the wheeled scraper, except that the box, when filled, rests upon the ground and is dragged over it by the team.

Each scraper ("wheeled" or "drag") requires the constant use of a team of two horses with a driver. Besides, a number of men, depending on the shortness of the lead and the number of scrapers, are required in the pit and at the dump to load the scrapers (by holding the box down into the earth) and unload them (by tipping the box). Except in sand, or in very soft soil, it is economical to use a plow before scraping.

The severest work for the team is the filling of the box; and this occurs oftenest where the lead is shortest. Hence smaller scrapers are used on short than on long hauls. We base our calculations on the following loads:

For drag scrapers (used only on short hauls)	.2	cubic yard
For wheeled scrapers		
lead less than 100 feet	.33	"
" 100 to 300 feet	.4	"
" 400 to 500 feet	.5	"
" over 500 feet	.6	"

The daily expense per scraper, for driver's wages and the use of a 2-horse team, is about $3.50. For leads of 400 feet and over, we add 50 cts per day for use of "snatch team" to help load the larger scrapers then used. One snatch team generally serves a number of scrapers.

Owing to the fact that the teams are constantly in motion without rest, they travel somewhat more slowly than with carts. We take 150 ft per minute (or 75 ft of *lead* per minute) as an average.

In loading and unloading, the teams not only go out of their way in order to turn around, but travel more slowly than when simply hauling. To cover this we make an addition of 25 ft to each length of lead, whether long or short, for wheeled scrapers; and 15 feet for drag scrapers.

We add 1 cent per cubic yard for the cost of loading and dumping the scrapers; and estimate the approximate cost of the other items as follows:

Repairs of cart-road $\frac{1}{10}$ ct per cub yd in place for each 100 ft of lead

	Light Soils cts per cub yd in place	Heavy Soils cts per cub yd in place
Loosening		
by pick	*	5.
by shovel	*	2.
Spreading	1.	1.5
Superintendence, wear and tear etc	1.	1.

We repeat that our figures are to be regarded merely as tolerable approximations, and subject to great variations according to skill of contractor and superintendent, strength of teams, character of material moved, state of weather etc etc.

$$\text{No. of trips per day per } \textit{wheeled} \text{ scraper} = \frac{\text{No. (600) of mins in a working day}}{\text{No. of 75 ft lengths in (lead + 25 ft)}}$$

$$\text{No. of trips per day per } \textit{drag} \text{ scraper} = \frac{\text{No. (600) of mins in a working day}}{\text{No. of 75 ft lengths in (lead + 15 ft)}}$$

$$\text{No. of cub yds in place moved per day by each scraper} = \text{No. of trips per day per scraper} \times \text{No. of cub yds in place, per scraper per trip}$$

$$\text{Cost per cub yd in place, for loading, hauling, dumping and returning} = \frac{\text{Daily expense of one scraper}}{\text{No. of cub yds in place, moved per day by each scraper}} + \text{1 ct for loading and dumping}$$

$$\text{Total cost per cubic yard in place exclusive of contractor's profit} = \text{Cost per cub yd in place, for loading, hauling, dumping, and returning} + \text{.1 ct per cub yd in place for each 100 ft of lead, for repairs of road} + \text{Cost, per cub yd in place, of loosening, spreading or wasting, and superintendence &c.}$$

By Wheeled Scrapers. Labor $1 per day of 10 working hours.

Length of lead, or dist to which earth is hauled	Quantity in place, hauled per day by each scraper	Cost per cub yd in place, for loading, hauling, dumping, and returning.	Total cost, per cubic yard in place, exclusive of contractor's profit.					
			Light Soils		Heavy Soils			
					Picked and		Plowed and	
			Spread	Wasted	Spread	Wasted	Spread	Wasted
Feet	cub yds	cts	cts	cts	cts	cts	cts	cts
50	200	2.8	4.9	3.9	10	8.5	7.3	5.8
100	140	3.4	5.5	4.5	11	9.5	8.	6.5
150	105	4.3	6.5	5.5	12	11	9	7.5
200	80	5.4	7.6	6.6	13	12	10	8.5
300	56	7.3	9.6	8.6	15	14	12	11
400	50	8.5	11	10	16	15	13	12
600	43	10	13	12	18	17	15	14
800	33	13	16	15	21	20	18	17
1000	27	16	19	18	25	24	22	21

* Light soils can generally be advantageously loosened by the scrapers themselves in the act of loading.

By Drag Scrapers. Labor $1 per day of 10 working hours.

Length of lead, or dist to which earth is hauled	Quantity in place, hauled per day by each scraper	Cost, per cub yd in place, for loading, hauling, dumping and returning	Total cost, per cubic yard in place, exclusive of contractor's profit.					
			Light Soils		Heavy Soils			
					Picked and		Plowed and	
			Spread	Wasted	Spread	Wasted	Spread	Wasted
Feet	cub yds	cts	cts	cts	cts	cts	cts	cts
less than 40	220	2.6	4.6	3.6	10	8.5	7	5.5
50	140	3.5	5.5	4.5	11	9.5	8	6.5
75	100	4.5	6.6	5.6	12	11	9	8
100	80	5.4	7.5	6.5	13	12	10	9
150	54	7.5	9.6	8.6	15	14	12	11
200	42	9.3	12.	11	17	16	14	13

Both wheeled and drag scrapers are made by Western Wheel Scraper Co, Mount Pleasant, Iowa; by Kilbourne & Jacobs Mfg Co, Columbus, Ohio; by Fay Manufacturing Co, Elyria, Ohio, and others. A medium-sized wheeled scraper, weighing 450 lbs, and carrying .4 cubic yard, costs about from $50 to $70. A drag scraper weighs about 100 lbs, and costs about $14.

Art. 13. By cars and locomotive, on level track. We have based our calculations upon the following assumptions: Trains of 10 cars, each car containing 1½ cubic yards of earth measured in place. Average speed of trains, including starting and stopping, but not standing, 10 miles per hour, = 5 miles of *lead* per hour. Labor $1 per day of 10 working hours. Loosening, loading (by shovelers), spreading, wear &c of tools, superintendence, &c, the same as with carts, Arts 2, 3, 5, and 7. Loss of time in each trip for loading, unloading, &c, 9 minutes, = .15 hour. Therefore

$$\text{Number of trips per day, per train} = \frac{\text{The number (10) of hours in a working day}}{.15 + \text{the number of 5-mile lengths in the lead}}$$

$$\text{Number of cubic yards in place, per day per train} = \text{Number of trips per day per train} \times \text{Number (10) of cars in a train} \times \text{Number (1.5) of cubic yards in place in each car}$$

$$\text{Cost per cubic yard, in place, for hauling, dumping, and returning} = \frac{\text{One day's train expenses} + \text{1 day's cost of track}}{\text{Number of cubic yards in place per day per train}}$$

One day's train expenses:

Cost of 10 cars @ $100	$1000	
" locomotive	3000	
	$4000	
One day's interest at 6 per cent, on cost of train		$0.67
Wages of engine driver (who fires his own engine)		2.00
" foreman at dump		2.00
" 3 men at dump at $1		3.00
Fuel		2.00
Water		1.00
Repairs of locomotive and cars		2.33
Total daily expense of one train		$13.00

The daily expense of track, for interest and repairs, may be taken at $3 for each mile, or fraction of a mile, of lead.

Therefore,

$$\text{Cost per cubic yard in place, for hauling, dumping, and returning} = \frac{\$13 + (\$3 \text{ for each mile of lead})}{\text{Number of trips per day per train} \times \text{Number (10) of cars in a train} \times \text{Number (1.5) of cubic yards in each car}}$$

$$\text{Total cost per cubic yard in place, exclusive of contractor's profit} = \text{Cost per cub yd in place for hauling, dumping, and returning} + \text{Cost per cubic yard, in place, for loosening, loading, spreading or wasting, and superintendence, \&c. (Arts 2, 3, 5, and 7.)}$$

By Cars and Locomotive. Labor $1 per day of 10 working hours.

Length of lead, or distance to which the earth is hauled.	Number of cubic yards, in place, hauled per day by each train.	Cost per cubic yard, in place, for hauling, dumping, and returning.	Total cost per cubic yard, in place, exclusive of contractor's profit.							
			Light Sandy Soils.				Strong Heavy Soils.			
			Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.	Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.
Miles.	Cu yds.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.
1	4350	.4	9.7	8.4	8.4	7.2	13.7	12.4	11.3	10.
2	2700	.7	10.	8.8	8.8	7.5	14.	12.8	11.6	10.4
3	1950	1.1	10.4	9.2	9.2	7.9	14.5	13.3	12.1	10.9
4	1500	1.7	11.	9.7	9.7	8.5	15.	13.7	12.6	11.3
5	1200	2.3	11.6	10.4	10.4	9.1	15.6	14.4	13.2	12.
6	1050	3.	12.3	11.	11.	9.8	16.3	15.	13.9	12.6
7	900	3.8	13.1	11.8	11.8	10.6	17.1	15.8	14.7	13.4
8	750	4.9	14.2	13.	13.	11.7	18.2	17.	15.8	14.6
10	600	7.2	16.5	15.2	15.2	14.	20.5	19.2	18.	16.8

Where large amounts of work are to be done, the **steam excavator, land dredge or steam shovel** generally economizes time and money. Where the depth of cutting is less than 10 ft, so much time is lost in moving from place to place that the excavators do not work to advantage. In stiff soils, cuttings may be made about from 17 to 20 ft deep without changing the level of the machine. For greater depths in such soils the work is done in two levels, since the bucket or dipper cannot reach so high. But in sand and loose gravel, much deeper cuts may be made from a single level.

The excavator resembles a dredging machine in its appearance and operation. A large plate-steel bucket, like a dredging bucket, with a flat hinged bottom, and provided with steel cutting teeth, is forced into and dragged through the earth by steam power. It dumps its load, by means of the hinged bottom, either into cars for transportation, or upon the waste bank, as desired.

Each machine is mounted on a car of standard gauge, which can be coupled in an ordinary freight train. The car is made of wood or iron, as desired, and is provided with a locomotive attachment, by which it can be moved from point to point as the work proceeds. The machines can be used as **wrecking or derrick cars.** Each machine has a water tank, holding from 300 to 550 gallons, for the supply of its boiler.

Before beginning to excavate, the end of the car nearest the work is lifted from the track by hydraulic or screw jacks, upon which it rests while working.

In stiff soils the excavator leaves the sides of the cut nearly vertical; and the desired slope is afterwards given by pick and shovel. When the soil is hard or much frozen, it may be loosened by blasting in advance of the excavator.

Steam excavators **are made by** Osgood Dredge Co, Albany N Y; by John Souther & Co, (the "Otis" excavator) Boston Mass; by Vulcan Iron Works, Toledo O; by Industrial Works, Bay City Mich; and by Pound Manufacturing Co., Lockport N. Y.

The Osgood is made in two sizes. In No 1 the car is 34 ft × 10 ft, and its floor is 4 ft above the rails. It has a four-wheeled truck near each end. The dipper holds 2 cubic yards, struck measure. The machine weighs, complete, about 40 tons, and costs about $7500 on track at works (Albany, N Y). In the No 2 machine, the car is 28 ft × 10 ft; floor 5 ft above the rails. It has two pairs of wheels, 16 ft apart from center to center of axles. The dipper holds 1½ cubic yards, struck measure. The machine weighs, complete, about 28 tons, and costs about $6000.

The excavator has to be moved forward (as the work advances) abt 8 ft at a time. s regularly made, it can dig at a distance of 17 ft, horizontally, from the center of ne car in any direction, and can dump 12 ft above the track. In sand or gravel it kes out, while actually digging, 3 dipperfuls (= 4½ to 6 cub yards in the dipper, = 3.75 to 5 cubic yards in place) per minute; in stiff clay, 2 dipperfuls per minute = 3 to 4 cub yards in the dipper, = 2.5 to 3.33 cubic yards in place). An average ay's work (10 hours) for a "No 1" machine, including time lost in moving the maine, &c, is about 500 cubic yards in "hard-pan," and from 1200 to 1500 in sand and ravel. This allows for the usual and generally unavoidable delays in having cars eady for the excavator.

The excavators carry about 80 to 90 lbs of steam. They burn from 100 to 150 lbs f good hard or soft coal per hour; and require one engineer, one fireman, one ranesman, and 5 to 10 pitmen, including a boss. The pitmen are laborers, who tend to the jacks, lay track for the excavator and for the dump cars, assist in oving the latter, bring or pump water, &c, &c.

After reaching the site of the work, about 30 minutes are required for getting the xcavator into working condition; and an equal length of time, after completion f the work, in getting it ready for transportation.

The following figures are taken from the records of work done by a No 1 machine, om May to Nov, 1883. The material was hard clay with pockets of sand. The xpenses per day of 12 working hours, at $1.50 per such day for labor, were

Water (a very high allowance)	$ 5.00
Coal, 1½ tons bituminous	10.00
Wages of engineer	4.00
" " fireman	1.50
" " cranesman or dipper-tender	2.50
" " pit boss	3.00
" " 8 pitmen at $1.50	12.00
Oil, waste, repairs, &c (estimated)	5.00
Interest on cost ($7500) of machine	1.25
	$44.25

Reduced to our standard of $1 for labor per day of 10 working hours, this would e say $30.00 per day. Reduced to the same standard, and allowing for the greater roportional loss of time in stopping at evening and starting in the morning: the verage daily quantity excavated, measured in place, was, in shallow cutting, 530 ubic yards; in deep cutting, 1200 cubic yards; average of whole operation, 800 ubic yards. This would make the cost, per cubic yard measured in place, for oosening and loading into cars, 5.67 cts, 2.5 cts, and 3.75 cts respectively; while the ost by ploughing and shoveling, in strong heavy soils, by Arts 2 and 3, is 7.4 cts; and y picking and shoveling, say 10 cts.

Art. 14. Removing rock excavation by wheelbarrows.

cubic yard of hard rock, *in place*, or before being blasted, will weigh about 1.8 tons, if sandstone r conglomerate. (150 lbs per cubic foot:) or 2 tons if good compact granite, gneiss, limestone, or narble, (168 lbs per cubic foot.) So that, near enough for practice in the case before us, we may assume the weight of any of them to be about 1.9 tons, or 4256 lbs per cubic yard, in place; or 158 lbs er cubic foot.

Now, a solid **cubic yard, when broken up** by blasting for removal by wheelbarrows r carts, will occupy a space of about 1.8, or $1\frac{4}{5}$ cubic yards; whereas average earth, when loosened, wells to but about 1.2, or $1\frac{1}{5}$ of its original bulk in place; although, after being made into embankment, it eventually shrinks into less than its original bulk. In estimating for earth, it is assumed hat $\frac{1}{14}$ cubic yard, in place, is a fair load for a wheelbarrow. Such a cubic yard will weigh on an verage 2430 lbs, or 1.09 tons; therefore, $\frac{2430}{14} = 174$ lbs, is the weight of a barrow-load, of 2.31 cubic eet of loose earth. Assuming that a barrow of loose rock should weigh about the same as one of arth, we may take it at $\frac{1}{24}$ of a cubic yard; which gives $\frac{4256}{24} = 177$ lbs per load of loose rock, ccupying 2 cubic feet of space.

In the following table, columns 2 and 3 are prepared on the same principle as for earth, as directed n Article 10. Column 4 is made up by adding to each amount in column 3, .2 of a cent for each 100 eet length of lead, for keeping the wheeling-planks in order; and 45 cents per cubic yard, in place, s the actual cost for loosening, including tools, drilling, powder, &c; as well as moderate drainage, nd every ordinary contingency not embraced in column 3. Contractor's profits, of course, are not ere included.

Ample experience shows that when labor is at $1 per day, the foregoing 45 cents per cubic yard, in lace, is a sufficiently liberal allowance for loosening hard rock under all ordinary circumstances. n practice it will generally range between 30 and 60 cents; depending on the position of the strata, ardness, toughness, water, and other considerations. Soft shales, and other allied rocks, may frequently be loosened by pick and plough, as low as 15 to 20 cents; while, on the other hand, shallow uttings of very tough rock, with an unfavorable position of strata, especially in the bottoms of excavations, may cost $1, or even considerably more. These, however, are exceptional cases, of comparatively rare occurrence. The quarrying of average hard rock requires about ¼ to ⅓ lb of powder er cubic yard, in place; but the nature of the rock, the position of the strata, &c, may increase it

to ½ lb, or more. Soft rock frequently requires more powder than hard. A good churn-driller will drill 8 to 10 feet in depth, of holes about 2½ feet deep, and 2 inches diameter, per day, in average hard rock, at from 12 to 18 cents per foot. Drillers receive higher wages than common laborers.

Hard Rock, by Wheelbarrows.

Labor $1 per day, of 10 working hours.

Length of Lead, or distance to which the rock is wheeled.	Number of cubic yards, in place, wheeled per day by each barrow.	Cost per cubic yard, in place, for loading, wheeling, and emptying.	Total cost per cubic yard, in place, exclusive of profit to contractor.	Length of Lead, or distance to which the rock is wheeled.	Number of cubic yards, in place, wheeled per day by each barrow.	Cost per cubic yard, in place, for loading, wheeling, and emptying.	Total cost per cubic yard, in place, exclusive of profit to contractor.
Feet.	Cubic Yds.	Cents.	Cents.	Feet.	Cubic Yds.	Cents.	Cents.
25	12.2	8.64	53.7	600	2.96	35.5	81.7
50	10.7	9.81	54.9	700	2.62	40.1	86.5
75	9.58	11.0	56.2	800	2.34	44.8	91.4
100	8.66	12.1	57.3	900	2.12	49.5	96.3
150	7.26	14.5	59.8	1000	1.94	54.1	101.1
200	6.25	16.8	62.2	1200	1.65	63.6	115.0
250	5.49	19.1	64.6	1400	1.44	72.9	120.7
300	4.89	21.5	67.1	1600	1.28	82.2	130.4
350	4.41	23.8	69.5	1800	1.15	91.5	140.1
400	4.02	26.1	71.9	2000	1.04	100.8	149.8
450	3.69	28.5	74 4	2200	.953	110.2	159.6
500	3.41	30 8	76.8	2400	.879	119.5	169.3

Art. 15. Removing rock excavation by carts. A cart-load of rock may be taken at $\frac{1}{5}$ of a cubic yard, in place. This will weigh, on an average, 851 lbs; or but 41 lbs more than a cart-load of average soil. Since the cart itself will weigh about ½ a ton, the total loads are very nearly equal in both cases. Columns 2 and 3 of the following table are prepared on the same principle as for earth, as directed in Art. 4. Column 4 is made up by adding to each amount in column 3, the following items: For blasting, (and for everything except those in column 3; loading, and repairs of cart-road,) 45 cents per cubic yard, in place; for loading, 8 cents, per cubic yard, in place; and for repairs of road, .2, or $\frac{1}{5}$ of a cent for each 100-feet length of lead. Contractor's profit not included.

Hard Rock, by Carts.

Labor $1 per day, of 10 working hours.

Length of Lead, or distance to which the rock is hauled.	Number of cubic yards, in place, hauled per day, by each cart.	Cost per cubic yard, in place, for hauling, and emptying.	Total cost per cubic yard, in place, exclusive of profit to contractor.	Length of Lead, or distance to which the rock is hauled.	Number of cubic yards, in place, hauled per day, by each cart.	Cost per cubic yard, in place, for hauling, and emptying.	Total cost per cubic yard, in place, exclusive of profit to contractor.
Feet.	Cubic Yds.	Cents.	Cents.	Feet.	Cubic Yds.	Cents.	Cents.
25	19.2	6.51	59.6	1800	5.00	25 0	81.6
50	18.5	6.77	59.9	1900	4.80	26.0	82.8
75	17.8	7.03	60.2	2000	4.62	27.1	84.1
100	17.1	7.29	60.5	2250	4.21	29.7	87.2
150	16.0	7.81	61.1	2500	3.87	32.3	90.3
200	15.0	8.33	61.7	½ mile	3.70	33.7	92.0
300	13.3	9.37	63.0	3000	3.33	37.5	96.5
400	12.0	10.4	64.2	3250	3.12	40.1	99.6
500	10.9	11.5	65.5	3500	2.92	42.8	102 8
600	10 0	12.5	66.7	3750	2.76	45.3	105.8
700	9.23	13.6	68.0	4000	2.61	47.9	108.9
800	8.57	14.6	69.2	4250	2.47	50.6	112.1
900	8.00	15.6	70.4	4500	2.35	53.2	115.2
1000	7.50	16.7	71.7	4750	2.24	55.8	118.3
1100	7.06	17.7	72.9	5000	2.14	58.4	121.4
1200	6.67	18.7	74.1	1 mile	2.04	61.2	124.8
1300	6.32	19.8	75.4	1¼ "	1 67	75.0	141.2
1400	6.00	20.8	76.6	1½ "	1.41	88.8	157.6
1500	5.71	21.9	77.9	1¾ "	1.22	102.5	174.0
1600	5.45	22.9	79.1	2 "	1.08	116.3	190.4
1700	5.22	24.0	80.4	2¼ "	.962	130.0	206.8

"**Loose rock**" will cost about 30 cts per yd less; and even *solid* rock will *average* about 10 cts less than the tables.

Art. 16. Removing rock excavation by cars and locomotive, on level track. Our calculations are based upon the following assumptions: rains of 10 cars, each car containing 1 cubic yard of rock measured in place. Average speed of trains, including starting and stopping, but not standing, 10 miles per our = 5 miles of *lead* per hour. Labor $1 per day of 10 working hours. Loosening, 5 cts per cubic yard in place. Loading, 8 cts per cubic yard in place. Cost of track, r interest and repairs, $3 per day per mile of lead. The calculations are the same, in rinciple, as those in Art. 13.

Hard Rock, by Cars and Locomotive.

Labor $1 per day of 10 working hours.

ength of lead, or distance to which the rock is hauled ....miles	1	3	5	7	10
lumber of cubic yards, in place, hauled per day by each train	2900	1300	800	600	400
ost, per cubic yard in place, for hauling, dumping, and returning....cents	.6	1.7	3.5	5.7	10.8
otal cost, per cubic yard in place, exclusive of contractor's profit....cents	53.6	54.7	56.5	58.7	63.8

TUNNELS.

Tunnels for railroads should, if possible, be straight, especially when there is but a single track; inasmuch as collisions or other accidents in a tunnel would be peculiarly disastrous. A tunnel will rarely be expedient before the depth of cutting exceeds 60 feet. Firm rock of moderate hardness, and of a durable nature, is **the most favorable material** for a tunnel, especially if free from springs, and lying in horizontal strata. In soft rock, or in shales (even if hard and firm at first), or in earth, a lining of hard brick or masonry in cement, is necessary. A tunnel should have a **grade or inclination** in one direction, for ease of future drainage and ventilation. No special arrangement is essential for **ventilation** either during construction or after, if the length does not exceed about 1000 feet; but beyond that, generally during construction either shafts are resorted to, or means provided for forcing air into the tunnel through pipes from its ends. But after the work is finished, except under peculiar circumstances, nothing of the kind is necessary. Shafts often draw air downwards; and frequently, even when aided by a steep uniform grade, do not secure ventilation. The Mont Cenis tunnel under the Alps, completed in 1871, is 7½ miles long, and has no shafts, although it grades up from each end, which is the most unfavorable of all conditions for ventilation without shafts. It was made so for facilitating drainage. Its ventilation is maintained by air forced in from the ends. The Hoosac tunnel, Mass, 4½ miles long, has shafts: one of them 1030 feet deep; but they were for expediting the work. **Shafts generally cost** from 1½ to 3 times as much per cubic yard as the main tunnel, owing to the greater difficulty of excavating and removing the material, and getting rid of the water, all of which must be done by hoisting. When through earth, they must be lined as well as the tunnel; and the lining must usually be an under-pinning process. Or the lining may first be built *over* the intended shaft, and then sunk by undermining it gradually; see page 650. Their sectional area commonly varies from about 40 to 100 square feet. They have the great advantage of expediting the work by increasing the number of points at which it can be carried on; but if placed too close together, their cost more than compensates for this. The air in some tunnels, while being constructed, is much more foul than in others; so that after the work has been commenced, shafts with forced air may be expedient where they were not anticipated. In excavating the tunnel itself, a **heading** or passage-way, 5 or 8 feet high, and 3 to 12 feet wide, is driven and maintained a short distance (10 to 100 feet, or more, according to the firmness of the material) in advance of the main work. In rock, the heading is just below the top of the tunnel, so that the men can conveniently drill holes in its floor for blasting; but in earth, the heading is driven along the bottom of the tunnel, that being the most convenient for enlarging the aperture to the full tunnel size, by undermining the earth, and letting it fall. In earth, the top and sides of the heading, as well as of the tunnel, must be carefully prevented from caving in before the lining is built; and this is done by means of rows of vertical rough timber props, and horizontal caps or overhead pieces, between which and the earth rough boards are placed to form temporary supporting sides and ceiling to the excavation. The props and caps are placed first; and the boards are then driven in between them and the earthen sides of the excavation. These are gradually removed as the lining is carried forward. **The lining,** when of brick, is usually from 2 to 3 bricks thick (17 to 26 inches) at bottom, and from 1½ to 2½ bricks thick at top; and when of rough rubble in cement, about half again as thick. It is important that the bricks or stone should be of excellent hard quality, and laid in good cement. The bricks should be moulded to the shape of the arch. As the lining is finished in short lengths, and before the centers are removed, any **cavities or voids** between it and the earth should be carefully and compactly filled up. Even in rock, if much fissured, or if not of durable character, as common shale, lining is necessary. **The cross-section** of a single-track railroad tunnel, in the clear of everything, and for cars of 11 feet extreme width, should not be *less* than about 15 feet wide, by 18 feet high; nor a double-track one, less than 27 feet wide, by 24 feet high; unless in the last case the material is firm rock, in which a high arch is not necessary for lining. The roof may then be much flatter, so that a height of 20 feet may answer. With cars of 10 feet extreme width, the width of the tunnel may be reduced to 25 feet; or with 9 feet cars, to 23 feet. Many have been made 22 feet. The Mont Cenis is 26 feet wide, by 25 high. **The rate of daily progress** from *each* face of a tunnel varies from 18 inches to 9 feet of length per 24 hours, with three relays of workmen. On the Mont Cenis the ex-

remes were about 4 to 9 feet daily for a whole year, from each face. Drills worked by compressed air were employed in the headings, which were 12 feet wide by 8 feet high. Ordinarily, from 1½ to 3 feet may be taken as averages. The difference of rate of progress between a single and a double track tunnel is not so great as might be supposed; inasmuch as a larger force can be employed on the wider one. If the tunnel is in earth, the construction of the lining about makes up for the slower excavation of one in rock. In rock, with labor at $1 per day, **the cost** will usually vary with the character of the rock, from $2 to $5 per cubic yard for the main tunnel; and from $3 to $10 for the heading; while shafts will average about 50 per cent. more than heading. The cost of a single-track tunnel, when common labor is $1 per day, will generally range between $30 and $75 per foot of length. Tunnel work, however, is liable to serious contingencies which cannot be foreseen. Since the sides and roof are rough as blasted, the width and height should each be estimated to the contractor as about 18 inches or 2 feet greater than the established *clear* ones. At any rate, the mode of measurement should be clearly stated in the specifications for the work. When a tunnel is made with a uniform grade, the work generally progresses in a more satisfactory manner from the lower end, because the descent favors the drainage of the spring water that is usually met with; whereas, at the upper end, it must be removed by pumps or by bailing. The upper end has, however, the advantage of sooner getting rid of the smoke in blasting. Before commencing a tunnel, or even deciding upon one, **trial shafts** should be sunk to ascertain the nature of the material. In long ones, the greatest care and accuracy are necessary for preserving the line of direction, so that the work from both ends shall meet properly at the center.

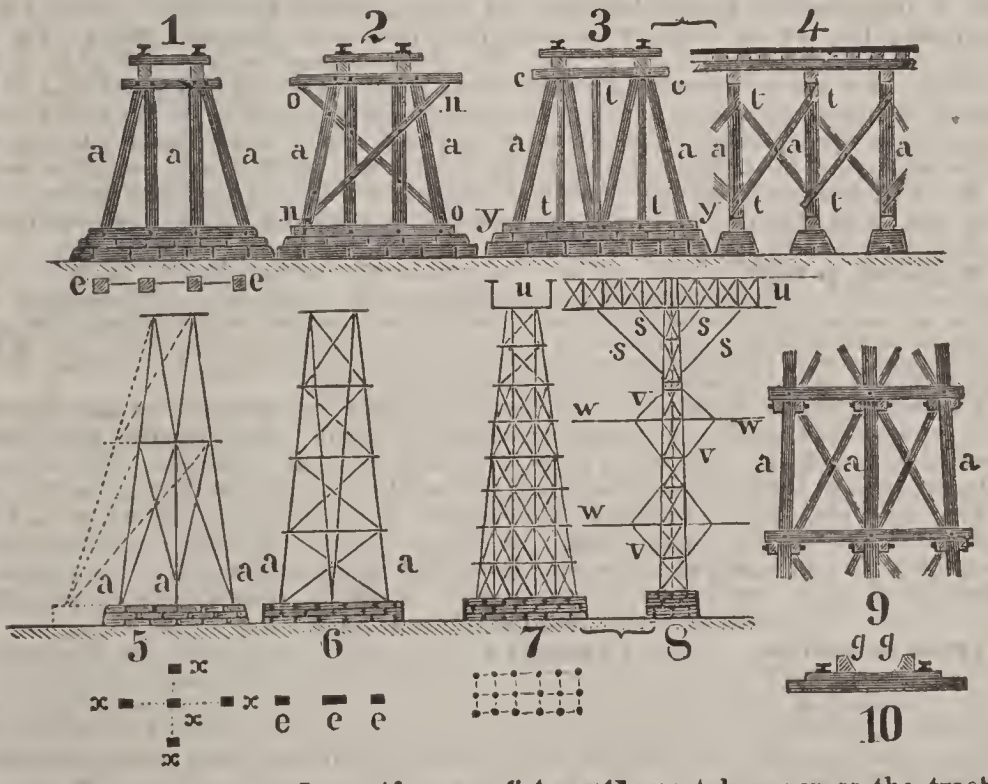
In the heading of the Vosburg (Pa) tunnel of the Lehigh Valley R R, built 1884, cross-section 7½ feet × 26 feet, the **average progress** per working day of 24 hours with two shifts of 12 hours each, was as follows; by hand drilling 2.8 feet and 2.4 feet respectively from each end; by machine drills (two rival drills in competition) 5.6 feet and 7.8 feet. The material was hard gray sandstone. For the whole tunnel the rate was about 2 feet per day.

For further information respecting tunnels, the reader is referred to Mr. H. S. Drinker's very full treatise on the subject, published by the Messrs Wiley.

For Stone bridges and culverts, see pp 693, &c.

For Trusses, see pp 547, &c.

TRESTLES.



Figs 1, 2, 3, 5, 6, 7, are **elevations of trestles;** taken across the track or roadway. We may consider Fig 1 as adapted to a height of about 10 to 20 ft; Figs 2

and 3, to heights from 20 to 30 ft; Fig 5, from 30 to 40 ft; Fig 6, from 40 to 60 ft, as rough approximations merely. A single framework, such as that shown in each of these six figures, is called a "bent." These bents of course admit of many modifications. They are usually supported by bases of masonry, as in the figures. These preserve the lower timbers from contact with the earth, which would hasten their decay. It is advisable to make these bases high enough to prevent injury from cattle, or passing vehicles, &c. Up to heights of about 40 or 50 ft, a single row of posts or uprights, *a*, *a*, *a*, Figs 1 to 9, as shown at *e e* under Figs 1 and 6, will answer. But as the height becomes greater, more posts should be introduced, as shown at *x x* under Fig 5; or two entire rows of them; or three rows, as under Fig 7; and as also in Fig 8, which is an end view of Fig 7. Figs 7 and 8 bear much resemblance to the trestles 190 ft high, with masonry bases 30 ft high (S. Seymour, C. E.), which carried the Erie Rway (now the N Y, Lake Erie & West'n R R) over the **Genesee River** at **Portage, N Y.** There each bent had 21 posts 14 ins square, at its base; and 15 posts of 12 × 12, at its top. The other timbers were 6 × 12; many of them were in pairs, embracing the posts. This single-track viaduct was begun July 1, 1851, and completed Aug. 14, 1852. It contained 1,602,000 ft (B M) of timber, and 108,862 lbs of iron. In the foundations were 9200 cub yds of masonry. The entire cost was about $140,000. It was burned down in 1875, and was replaced, in less than 3 mos, with a single-track **viaduct of wrought-iron trestles,** (described below) containing, in all, 1,340,000 lbs of iron, and 130,600 ft (B M) of timber; and costing, complete, above the masonry, about $95,000. Frequently the posts of trestles are in pairs; and the other timbers pass between; all bolted together.

In Fig 4, the posts *a*, *a*, *a*, are end views of three trestles or bents, such as Fig 3; and *t t* are diag braces extending from trestle to trestle; the two outer ones inclining in one direction; and the central one crossing them. These may be placed either intermediate of the posts, as in Fig 3; with the heads of the two outer ones confined to the cap *c c* of one trestle; and their feet to the sill *y y* of the next one; or they may all be spiked or bolted to the posts themselves, as in Fig 4. The last is the best, as it serves also directly to stiffen the posts: as do also the braces *o o*, *n n*, Fig 2. Such bracing is too frequently omitted. During the passage of trains, the backward pressure of the steam, exerted through the driving wheels against the track, produces a serious strain lengthwise of the road, and tending to upset the trestles; and the sudden application of brakes to a moving train, produces a similar strain in the opposite direction. These strains become more dangerous as the ht increases. Hence the need for such braces. Usually the outer **posts may lean** 1.5 to 2.5 ins to a ft.

The posts should not be less than about 12 ins square, except in quite low trestles; and even then not less than about 10 × 10. The diag bracing may generally be about as wide as the posts; and half as thick. The dist apart of the bents, when the roadway is supported by simple longitudinal beams, should not exceed 10 or 12 ft, for railroads. But if these beams receive support from braces beneath, like *s s*, Fig 8; or from iron truss rods, as at Fig 52, page 514, the dist may be extended to 15 or 20 or more ft. But when the trestles become very high, and contain a great deal of timber, it becomes cheaper to place them farther apart, say 30 to 60 ft; and to carry the railway upon regular framed trusses, as at *u u*, Figs 7 and 8; as in a bridge with stone piers. In the Genesee viaduct, the trestles were 50 ft apart, center to center.

When such a trestle as Fig 8 becomes very narrow in proportion to its height, we may add to its stability by introducing beams *w*, extending from trestle to trestle; and still further by inserting diag braces *v v*, as in the old Genesee viaduct.

Figs 62, p 613, "Trusses," will show how the timbers may be joined. In designing trestles, (as in wooden bridges,) it is advisable, as far as practicable, to arrange the pieces so that any one may be removed if it becomes decayed; and another put in its place. ON CURVES, additional strength should be given on the convex side; as suggested by the dotted lines in Fig 5. On very high trestles especially, (as well as on bridges,) wheel-guards, *g g*, Fig 10, either inside or outside of the rails, should never be omitted, as is commonly done.

In marshy ground, piles may be driven to support the trestles; or may be left so far above ground, as themselves to constitute the posts. Such trestles may often be used advantageously, even when to be afterward filled in by embkt. They then sustain the rails at their proper level until the embkt has reached its final settlement.

They are generally used to avoid the expense of embkt; especially when earth can only be obtained from a great dist. Even when earth and timber are equally convenient, they will rarely much exceed about half the cost of embkt; even when but about 30 ft high; but owing to their liability to decay, they should be resorted to only in case of necessity; or as a temporary expedient.

Iron trestles. At the **Crumlin** double-track iron viaduct, in England, 1500 ft long, (spans 150 ft,) they are about 180 ft high; 60 by 27 ft at base; 30 by 18 at top; each composed of 14 cast-iron posts, arranged as a long hexagon; each post being formed of 17-ft lengths of iron pipes, 1 ft outer diam, by 1 inch thick. At each 17-ft length, the pipes are firmly connected by hor iron pieces; and between these diff stages is diag bracing of 4 × 4 × ½ inch rolled T iron, arranged as in Fig 7. The viaduct contains about 3,000,000 lbs of wrought-iron and nearly as much cast-iron

The new Portage Viaduct, referred to above, consists of iron Pratt usses, resting upon six towers. Fig 11 is a longitudinal view, and Fig 12 a trans- erse view, of one of the tallest towers, 203 ft 8 is high from top of masonry to track-rail, and eighing 285,000 lbs. Each tower consists of 2 ents B B, Fig 11. Each bent has two wrought- on columns C C, Fig 12, inclining toward each ther with a batter of 1 in 8. They are 20 ft part at top, and, in the tallest tower, 69 ft 8 ins etween centers, at base. The two bents of a ower are 50 ft apart, and are connected with ach other by hor longitudinal struts S, Fig 11, nd diag tie-rods R. **The spans between owers** vary from 50 to 118 ft.

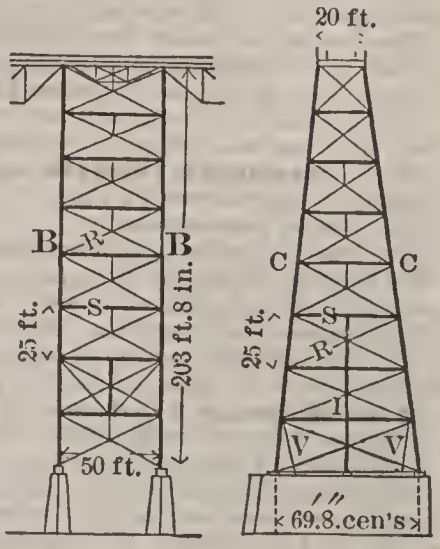
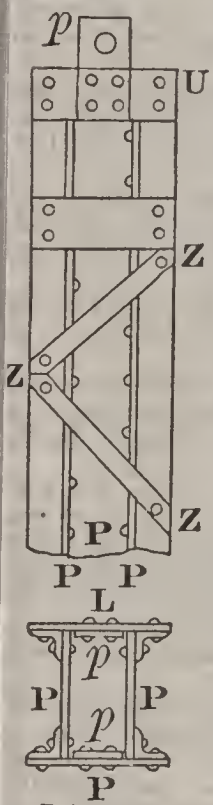


Fig.11 Fig.12

The columns are put together in lengths f 25 ft. Figs 13, 14, and 15 show the upper end f one of these lengths. Fig 13 is an elevation, een from between the feet of a bent, Fig 14 a ross section, and Fig 15 a side view. Each col s composed of three plates, P P P, and 4 angle- ars $4 \times 4 \times \frac{1}{2}$ inch, as shown. In the tallest owers, the 2 opposite *side* plates are 15 ins wide, nd from $\frac{5}{8}$ to $\frac{7}{8}$ inch thick; their thickness ncreasing with their dist from the top of the ower. The third, or *back*, plate is 17 ins $\times \frac{3}{4}$ nch throughout. The fourth side, L, has only a zig-zag lacing, Z Z Z, Fig 13, of at bars, so that the interior of the col is accessible for painting.

At the upper end U of each 25-ft length, are riveted two small iron plates *p p*, forming a tenon. The foot of the next length fits over this tenon, and is confined to it by a turned iron pin, $1\frac{7}{8}$ ins diam, passing through carefully bored holes. To this pin, the longitudinal diag rods, R, Fig 11, $1\frac{1}{4}$ ins diam, are attached. The **longitudinal hor struts,** S, Fig 11, are light latticed girders of uniform width (1 ft) and depth (2 ft). They abut against the sides of the columns, and are bolted to lugs of angle iron, riveted to them. They are connected with the corresponding *transverse* struts, S, Fig 12, by hor diag angle-bars fastened to each strut 10 ft from its end.

Fig.13



The **transverse struts** are of different designs, depending upon their lengths. At their ends they are held by pins passing through holes *o*, Fig 15, in the side plates of the columns. These pins also hold the $1\frac{1}{4}$ inch diag rods, R, Fig 12. Each of the lowest three transverse struts is in two lengths, and is supported by an **intermediate vert post,** I, Fig 12, shown in cross section by Fig 16; and each of the lowest two is connected with the corresponding strut in the other bent of the same tower by a longitudinal strut and hor diag rods.

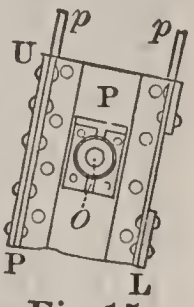


Fig.15

The cols rest upon **cast-iron pedestals.** Each pedestal is tenoned into the foot of its col. The pedestals on the north side of the bridge are doweled to cast-iron plates built into the masonry. Those on the south side are on rollers, rolling at right angles to the axis of the bridge. The two pedestals of each bent are connected with each other, and held in position, by eye-bars, which prevent them from spreading; and by struts, which prevent them from coming together when the diags are screwed up. V V, Fig 12, are anchor rods, bolted to the masonry, and attached to the columns at such a lit as not to interfere with the hor expansion of the tower.

Fig.14

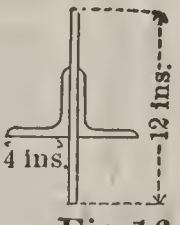


Fig.16

Cast-iron caps are tenoned and bolted into the tops of the cols. On each cap est the ends of the upper chords of two trusses, as shown in Fig 11.

One end of each long-span truss is bolted to the cap of a col. The other end rests pon rollers on the cap of the next col, but is connected with the fixed end of the djoining truss by iron loops which pass over the end pins of both trusses, and allow nly sufficient motion for contraction and expansion. The short spans *over* the owers are bolted to the caps at *both* ends, but those *between* the towers are arranged

like the long spans. Where the ends of a long, and of a short, span rest upon the same cap, they are placed respectively 3 and 6 ins from its center, so that their center of pressure coincides with the center of cross section of the col.

The towers are made strong enough for a double-track road; and the trusses are so arranged that they can be placed closer together, and additional trusses placed alongside of them; so that **the track can be doubled at any time.**

The bridge is so designed that the **greatest compressive strain** per sq inch that can come upon a col, under a double-track bridge and load, is 6600 lbs; and the **greatest tensile strain** per sq inch on a diag, 15,000 lbs. The greatest weight on the foot of any one col will be 357,500 lbs, or 155 lbs per sq inch of the 4 ft sq pedestal-stone.

The lowest section of a tower was first erected by means of a wooden framework on a flooring resting on the stone piers. A gin-pole 55 ft high was then lashed to each col, and by means of these poles the flooring and framework were raised to the tops of the columns, and used in placing the second section in position on top of the first. This process was repeated until the tower attained its full height. One of the tallest towers was erected in 11 days.

The Kinzua Viaduct, on a branch of the N Y, Lake Erie & Western R R in Penna, is single track, 2052 ft long, and its greatest height is 285 ft from the top of the masonry piers to the track-rails.

There are 21 spans of 61 ft each, placed between 20 towers; and 20 spans of 38½ ft each, over the towers. The towers are similar, in general arrangement, to those of the Portage viaduct, above described. The bents are 10 ft wide at top, and 103 ft at the feet of the tallest towers, which are 278 ft high. The 2 bents of a tower are 38½ ft apart. The legs are of Phœnix Iron Co's 4-segment columns, pattern C, p 449, $7\frac{3}{16}$ ins inner diam, and from $\frac{1}{2}$ to $\frac{13}{16}$ in thick. They are made up of lengths of about 33 ft, which are held in place, one on top of another, by plain cylindrical wrought-iron tubes about 14 ins long, and of such diam that they just fit inside of the cols. The abutting ends of two column-lengths slide over such a tube, and meet at the middle of its length. The tube is held in place by 4 bolts, which pass through the column from side to side, two of them being at right angles to the other two. These bolts hold in place the longitudinal and transverse hor braces, most of which are lattice-girders, spaced about 30 ft apart vertically.

The cols rest upon smooth iron plates, which allow a sliding movement of 1 inch transversely, and .38 inch lengthwise, of the roadway. Each plate is bolted, by two 1¼-inch bolts, from 9 to 12 ft long, to a square, pyramidal masonry pier, from 10 to 13 ft deep, 8 ft square at bottom and 4 ft square at top.

The cols have cast-iron caps, to which the ends of the lower chords of the 38½-ft spans are firmly bolted, and on which those of the 61-ft spans rest. The latter are bolted to the shorter spans through oval bolt-holes, which allow .17 inch play, longitudinally, for expansion and contraction.

Fig. 17

The greatest compression on a col is 8000 lbs per sq in. The ult load, by U S Govt experiments at Watertown Arsenal, is 35,000 lbs per sq in. The greatest tension on the diags is 15,000 lbs per sq in.

The heaviest *column* weighs about 5000 lbs. The entire structure, 3,500,000 lbs. It cost $275,000, and was built by a gang averaging 125 men, aided by 2 steam-hoisters and a traveling crane. Clarke, Reeves & Co, Phila, builders; now (1886) Phœnix Bridge Co.

Mode of erection. At each of the four corners of the site for a tower a mast 60 ft long was set up. These were guyed with ropes, and by means of them the 4 columns about 33 ft long, forming the lowest story of a tower, were erected and braced together. The masts were then raised about 30 ft, and clamped one to each of the columns, and used for raising the second story into position on top of the first, and so on, except for the top story, which was bolted together on the ground, in two pieces, and hoisted into position by the travelling crane.

Verrugas Viaduct, near Lima, Peru, 575 ft long, 252 ft high, carrying the Oroya R R (single track) over the Agua de Verrugas. Built by the late Baltimore Bridge Co, Chas. H. Latrobe, Engr, in less than four months in 1872. It contains 1,325,000 lbs of iron, and cost $165,000. There are four Fink truss spans, three of 100 ft, and one of 125 ft. They rest on 3 towers. Each tower consists of three parallel bents, 25 ft apart, and each bent has four columns, arranged as in Fig 17. The bents are 15 ft wide at top and the outer columns of each bent batter 1 in 12. The columns are all of the Phœnix segment pattern. The two outer ones of each bent have six segments each, diam 11½ ins, area of cross section, 20 sq ins. The inner ones have four segments each, diam 8 ins, area 13 sq ins. The columns are put together in lengths of 23½ ft, which are joined by cast-iron couplings. The hor braces extending from col to col are about 25 ft apart vert.

BALLAST.

Table of cubic yards of ballast per mile of road.

Side-slope of the ballast 1 to 1. Width in clear between 2 tracks 6 ft. The ties nd rails may be laid first, for carrying the ballast along the line; then raised a w ft of length at a time, and the ballast placed under them. **Deduct for ties,** below.

Depth in Ins.	Top width, Single Track.			Top width, Double Track.		
	10 Ft.	11 Ft.	12 Ft.	21 Ft.	22 Ft.	23 Ft.
	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.
12	2152	2347	2543	4303	4199	4695
18	3374	3667	3960	6600	6894	7188
24	4694	5085	5474	8996	9388	9780
30	6111	6600	7087	11490	11980	12470

A man can break 3 to 4 cub yds per day, of hard quarried stone, to a size suitable r ballast; say averaging cubes of 3 ins on an edge. Where other ballast cannot be had, hard-burnt ay is a good substitute. The slag from iron furnaces is excellent. The ties decay more rapidly hen gravel or sand is used instead of broken stone, because these do not drain off the rain, but keep e ties damp longer. For **stone crushers,** see p 680.

TIES.

In the United States **the life of a tie** is about as follows:

	Years.	Average, Years.		Years.	Average, Years.
hestnut,	5 to 12	7	White Oak,	5 to 12	7
edar,	6 to 15	9	Spruce Pine,	4 to 7	5
lemlock,	3 to 8	5			

As shown by table, Art 3, p 815, the annual **expense for renewal of ties,** n the U. S. alone, is about ten millions of dollars.

The **Penna R R,** in 1883, **used** 575,000 ties in construction on its main line nd branches (= 2552 miles of single track) and 779,000 ties in repairs.

It will often, especially in the case of the softer and more perishable woods, be rne economy to preserve ties by the injection of creosote. See p 425. Creosote reserves the *spikes.*

The writer believes that most of the fault usually ascribed to cross-ties, as well as o rail-joints, is in reality due to imperfect drainage of the roadbed. Hence, he does ot agree with those who advocate VERY LONG TIES; but considers that with good allast, on a well-drained roadbed, 8½ ft is as good as more; and that 8½ ft, by 9 ns, by 7 ins; and 2½ ft apart from center to center, is sufficient for the heaviest raffic. On many important roads they are but 8 ft; and on some only 7½ ft long; rack 4 ft 8½. On narrow-gauge roads the ties are generally from 6 to 7 ft long. The actual COST OF CUTTING DOWN THE TREES, lopping off the branches, and hewing he ties ready for hauling away to be laid, is about 6 to 9 cts per tie, at $1.75 per ay per hewer.

The narrow bases of rails resting immediately on the cross-ties, without chairs, requently produce in time such an amount of crushing in the ties as to injure them naterially even before decay begins. Burnetised ties rust the spikes away rapidly. Creosoted ones preserve them.

Cross-ties of 8½ ft, by 9 ins, by 7 ins, contain 3.719 cub ft each; and if placed 2½ ft apart rom center to center, there will be 2112 of them per mile, amounting to 291 cub yds. Therefore, if hey are completely embedded in the ballast, they will diminish its quantity by that amount. At 2 ft part there will be 2640 of them occupying 364 cub yds; and at 3 ft apart, 1760 of them; 243 cub yds.

Cubic feet contained in cross-ties of different sizes.

Dimensions.		Dimensions.		Dimensions.	
Ft. Ins. Ins.	Cub. Ft.	Ft. Ins. Ins.	Cub. Ft.	Ft. Ins. Ins.	Cub. Ft.
8 by 8 by 6	2.667	8½ by 8 by 6	2.833	9 by 8 by 6	3.000
8 9 6	3.000	8½ 9 6	3.188	9 9 6	3.375
8 9 7	3.500	8½ 9 7	3.719	9 9 7	3.938
8 10 6	3.333	8½ 10 6	3.542	9 10 6	3.750
8 10 7	3.889	8½ 10 7	4.132	9 10 7	4.375
8 10 8	4.444	8½ 10 8	4.722	9 10 8	5.000
8 12 8	5.333	8½ 12 8	5.667	9 12 8	6.000

Bessemer steel ties, furnished by International Railway Tie Co., office 210 Washington St., Boston Mass., and laid in the Boston & Maine R. R. near Boston in July 1885, under

RAILS.

Every sq inch of sectional area of rail, corresponds to 10 lbs per yard of a single rail; or to 15.7143 tons per mile of single-track road. Consequently,

$$\frac{\text{Wt in lbs per yd of rail,}}{10} \text{ or } \frac{\text{Wt in tons per mile of single-track}}{15.7143} = \text{Area of rail in sq ins.}$$

Thus, a rail of 100 tons per mile of single track, will have a section of 6.364 sq ins; and will weigh 63.64 lbs per yd of single rail. Add for turnouts, sidings, road-crossings, and a trifle for waste in cutting. When the ties are in place, and the rails distributed in piles at short intervals, a gang of 6 men can lay ½ a mile of rails per day, of single track; or after the ballast is in place, a gang of 15 men will lay about one mile of complete single-track superstructure per week.

Steel rails last from 9 to 25 years; average 15 years.

In the U. S. **steel rails weigh,** on 4 ft 8½ in and 5 ft gauges, usually from 55 to 70 lbs per yard. 56 and 60 are common. Sometimes as light as 50, and as heavy as 76 to 80. 3 ft gauge, 30 to 40, sometimes 50. 2 ft, 25. The usual **length** of rails is 30 ft. They have been made much longer, even up to 60 ft, but such lengths have not come into use to any extent. They of course have fewer joints, but the great space necessary between the rail-ends at the joints, to allow for expansion in hot weather, is a serious objection. This might be obviated by beveled joints, p 764. For sections of rails, to scale, see pp 764 and 765.

Annual production of rails in the United States, in tons of 2240 lbs.

	1872	1885
Bessemer steel	83,991	959,470
Open-hearth steel	.....	1,250
Iron	808,866	13,118
	892,857	973,838

Steel rails usually contain from .3 to .5 of one per cent of carbon.

The **Penna R R used,** in 1883, on its main line and branches (= 2552 miles of single track), 6600 tons of steel rails in construction of new lines, and 14,300 tons in repairs.

Price of Steel rails, at mill, in 1886, about $35 per ton of 2240 lbs.

Table of Middle Ordinates, to be used for the bending of rails of different lengths, so as to form portions of curves of different radii. Ordinates for lengths or radii intermediate of those in the table, may be found by simple proportion.

Def. Ang.	Radius.	LENGTHS OF RAILS.												
		30	**28**	**26**	**24**	**22**	**20**	**18**	**16**	**14**	**12**	**10**	**8**	**6**
Deg.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet
.5	11460.	.010	.008	.006	.005	.004	.004	.003	.002	.002	.001	.001	.000	.000
1.	5730.	.020	.016	.013	.011	.009	.008	.006	.005	.004	.003	.002	.001	.001
1.5	3820.	.029	.026	.021	.018	.016	.013	.010	.008	.006	.004	.003	.002	.001
2.	2865.	.038	.034	.029	.025	.021	.017	.014	.011	.008	.006	.004	.003	.001
2.5	2292.	.049	.043	.037	.031	.027	.022	.018	.014	.010	.007	.005	.003	.002
3.	1910.	.058	.051	.044	.037	.031	.026	.022	.017	.012	.009	.006	.004	.002
3.5	1637.	.070	.061	.052	.043	.037	.031	.025	.020	.015	.011	.008	.005	.003
4.	1433.	.079	.069	.060	.050	.042	.035	.029	.023	.018	.013	.009	.006	.003
4.5	1274.	.088	.077	.067	.056	.047	.039	.032	026	.020	.015	.010	.007	.004
5.	1146.	.099	.086	.074	.063	.053	.044	.035	.029	.022	.016	.011	.007	.004
5.5	1042.	.108	.094	.082	.070	.059	.048	.039	.032	.024	.018	.012	.008	.004
6.	955.4	.117	.102	.088	.076	.064	.052	.042	.034	.026	.019	.013	.008	.005
6.5	882.	.128	.112	.097	.082	.069	.057	.046	.037	.028	.021	.014	.009	.005
7.	819.	.137	.120	.104	.088	.074	.061	.049	.039	.030	.022	.015	.010	.005
7.5	764.5	.146	.127	.111	.094	.079	.065	.053	.042	.032	.024	.016	.010	.006
8.	716.8	.158	.137	.119	.100	.085	.070	.056	.045	.034	.025	.017	.011	.006
8.5	674.6	.166	.145	.126	.106	.090	.074	.060	.048	.036	.027	.018	.012	.007
9.	637.3	.175	.153	.133	.112	.095	.078	.063	.050	.038	.029	.019	.012	.007
9.5	603.8	.187	.163	.141	.119	.101	.083	.067	.054	.042	.031	.021	.013	.008
10	573.7	.196	.171	.148	.125	.106	.087	.071	.057	.045	032	.022	.014	.008
11	521.7	.216	.188	.163	.139	.117	.096	.078	.063	.049	.036	.024	.016	.009
12	478.3	.236	.206	.179	.151	.128	.105	.085	.069	.053	.039	.026	.017	.010
13	441.7	.254	.222	.192	.163	.138	.113	.092	.075	.057	.042	.028	.019	.010
14	410.3	.275	.239	.207	.175	.148	.122	.099	.080	.061	.045	.030	.020	.011
15	383.1	.295	.257	.223	.188	.159	.131	.106	.085	.065	.049	.033	.021	.012
16	359.3	.313	.273	.236	.200	.170	.139	.113	.091	.070	.052	.035	.023	.013
17	338.3	.333	.290	.252	.213	.180	.148	.120	.096	.074	.055	.037	.024	.014
18	319.6	.351	.306	.265	.225	.190	.156	.127	.102	.078	.058	.039	.025	.014
19	302.9	.371	.324	.280	.238	.201	.165	.134	.108	.082	.061	.041	.027	.015
20	287.9	.392	.341	.296	.250	.212	.174	.141	.114	.087	.066	.044	.028	.016
21	274.4	.410	.357	.309	.262	.222	.182	.148	.120	.091	.069	.046	.030	.017
22	262.	.430	.375	.325	.275	.233	.191	.155	.126	.096	.072	.048	.031	.018
23	250.8	.450	.390	.338	.287	.243	.199	.162	.131	.100	.075	.050	033	.019
24	240.5	.469	.408	.354	.299	.253	.208	.169	.137	.104	.078	.052	.034	.019
25	231.	.486	.424	.367	.311	.263	.216	.176	.142	.108	.081	.054	.035	.020
26	222.3	.506	.441	.382	.323	.274	.225	.183	.148	.112	.084	.056	.037	.021
27	214.2	.524	.457	.396	.335	.284	.233	.190	.153	.116	.087	.058	.038	022
28	206.6	.545	.475	.411	.348	.294	.242	.197	.158	.120	.090	.060	.039	.022
29	199.7	.564	.491	.424	.361	.303	.250	.203	.163	.124	.093	.062	.041	.023

For ordinates for center line of road, see pp 726 to 731.

WEIGHT OF RAILROAD SPIKES.

The hook-headed spikes *t*, commonly used for confining rails to the cross-ties, vary within the limits of the following table; the lightest ones for light rails on short local branches; and the heaviest ones for heavy rails on first-class roads. The spikes are sold in kegs usually of 150 lbs. For the weight of spikes of larger dimensions, we may near enough take that of a square bar of the same length. What is saved at the point, suffices for the addition at the head. Price, Phila, 1886, from 2 cts per lb for $\frac{9}{16}$ inch thick; to 3 for $\frac{3}{8}$.

Size in ins. Length.	Side.	No. per keg of 150 lbs.	No. per lb.	Size in ins. Length.	Side.	No. per keg of 150 lbs.	No. per lb.
$4\frac{1}{2}$ ×	$\frac{7}{16}$	526	3.5	$5\frac{1}{2}$ ×	$\frac{1}{2}$	350	2.33
$4\frac{1}{2}$ ×	$\frac{1}{2}$	400	2.66	$5\frac{1}{2}$ ×	$\frac{9}{16}$	289	1.93
5 ×	$\frac{3}{8}$	705	4.7	$5\frac{1}{2}$ ×	$\frac{5}{8}$	218	1.46
5 ×	$\frac{7}{16}$	488	3.25	6 ×	$\frac{1}{2}$	310	2.07
5 ×	$\frac{1}{2}$	390	2.6	6 ×	$\frac{9}{16}$	262	1.75
5 ×	$\frac{9}{16}$	295	1.97	6 ×	$\frac{5}{8}$	196	1.30
5 ×	$\frac{5}{8}$	257	1.71				

A size in very common use, is $5\frac{1}{2} \times \frac{9}{16}$; which weighs about $\frac{1}{2}$ lb per spike. A mile of single-track road, with 2640 cross-ties, 2 feet apart from center to center; and with rails of the ordinary length of 30 feet, or 15 ties to a rail; will have 352 rail-joints per mile; and with 4 spikes to each tie, will require 10560 spikes, weighing about 5500 lbs.

But an allowance must be made for rail guards at road-crossings, which we may assume to be 30 ft wide, or the length of a rail. A guard will usually consist of 4 extra rails for protecting the track-rails, and spiked to the 15 ties by which said track-rails are sustained. Consequently, such a crossing requires $15 \times 8 = 120$ spikes. For turnouts, sidings, loss, &c, we may roughly average 700* spikes more per mile; thus making in all (if we assume one road-crossing per mile) $10560 + 120 + 700 = 11380$ spikes per mile; or say 6000 lbs, or 40 kegs of 150 lbs.

Adhesion of spikes. Professor W. R. Johnson found that a plain spike .375, or $\frac{3}{8}$ inch square, driven $3\frac{3}{8}$ ins into seasoned Jersey yellow pine, or unseasoned chestnut, required about 2000 lbs force to extract it; from seasoned white oak, about 4000; and from well-seasoned locust, about 6000 lbs. Bevan found that a 6-penny nail, driven one inch, required the following forces to extract it: Seasoned beech 667 lbs; oak, 507; elm, 327; pine, 187.

Very careful experiments in Hanover, Germany, by Engineer Funk, give from 2465 to 3940 lbs, (mean of many experiments, about 3000 lbs,) as the force necessary to extract a plain ½ inch square iron spike, 6 ins long, wedge-pointed for one inch, (twice the thickness of the spike,) and driven 4½ ins into *white or yellow pine.* When driven 5 ins, the force reqd was about $\frac{1}{10}$ part greater. Similar spikes, $\frac{9}{16}$ inch square, 7 ins long, driven 6 ins deep, reqd from 3700 to 6745 lbs to extract them from pine; the mean of the results being 4873 lbs. In all cases *about twice as much force was reqd to extract them from oak.* The spikes were all driven *across* the grain of the wood. Experience shows that when driven *with* the grain, spikes or nails do not hold with much more than half as much force.

Jagged spikes, or twisted ones (like an auger,) or those which were either swelled or diminished near the middle of their length, all proved inferior to plain square ones. When the length of the wedge point was increased to 4 times the thickness of the spike, the resistance to drawing out was a trifle less. But see "Jag-spike" in Glossary.

When the length of the spike is fixed, there is probably no better shape than the plain square cross-section, with a wedge point twice as long as the width of the spike, as per this fig.

* This allows that turnouts and sidings amount to about 1 mile of extra track on 15 miles of road.

RAIL-JOINTS.

Art. 1. A track, being weakest at the joints between the rails, where they 're deprived of their vertical strength, has of course a greater tendency to bend at hose points; and this bending produces an irregularity in the movement of the ain, which is detrimental to both rolling-stock and track. Moreover, that end of a il upon which a loaded wheel is moving, bends more than the adjacent unloaded nd of the next rail: so that when the wheel arrives at said second rail, it imparts to s end a severe blow, which injures it. Thus, the ends of the rails are exposed to r more injury than its other portions. Nmmerons devices have been resorted to for rengthening the joints of the rails, with a view of preventing this bending entirely; r, at least, of causing the two adjacent rail-ends to bend equally, and together: so s to avoid the blows alluded to. None of these joint-fastenings, known as chairs, sh-plates, wooden blocks, &c, have proved entirely satisfactory.

Much of the deficiency ascribed to the fastenings, is, however, really due to want of stability in the oss-ties at the joints; and more attention must be directed to this latter consideration, before an ficient fastening can be obtained. Observation shows that when the joint-ties are very firmly dded, almost any of the ordinary fastenings will, (if the joint is placed *between* two ties, instead resting *upon* a tie,)* answer very well; whereas, when the cross-ties are so insecurely bedded as to ay up and down for half an inch or more under the driving-wheels of the engines, the strongest and ost effective fastenings soon become comparatively inoperative. All the parts of the best of them il! in that case become gradually loosened, warped, bent, or broken.

Experience has established the superiority of suspended joints over supported nes. Long fastenings, perhaps, possess but little superiority over short ones, where he track is not kept in good repair; for the great bearing of the former, although mparting increased firmness on a good track, becomes converted into a powerful everage, by which it accelerates its own destruction, in a bad one. An element in he injury of joints, is the omission of proper fastenings at the center of the rails. Each rail should be so firmly attached to the cross-ties at and near its center, as to ompel the contraction and expansion to take place equally from that point, toward ach end. It would probably be somewhat difficult to accomplish this perfectly. The attempts hitherto made have failed.

* In the first case the joint is called a SUSPENDED one: in the last a SUPPORTED one.

Under the extremes of temperature in the United States, **bar iron expand or contracts** about 1 part in 916; or 1 inch in 76⅓ feet; consequently, a ra 30 ft long will vary $\frac{7}{16}$ inch; and one 20 ft long fully ¼ inch.

Beside this, the rails are very liable to move or creep bodily in the direction of the heaviest trade, especially when th grade descends in the same direction; and by this process also the joint-fastening are exposed to additional strain and derangement.*

All rails appear to become elongated very slightly at their ends by use; and th renders a full allowance for contraction and expansion the more necessary.

Art. 2. "Even" joints and **"broken" joints.** If the road-bed of a yielding character, it is advisable to place the joints (on the two rails of the sam track) opposite to each other ("even joints"); because "staggered" or "broken joints are more liable to give the cars a sideways swaying motion. But if the roa bed is solid, and in good condition, the broken joints are preferable, because, wit them, the jar of passing from rail to rail is less severe than where both wheels mak that passage at the same time.

Art. 3. Beveled, or mitred joints. To lessen this jar, Mr. Sayre sug gests cutting the rails so that the vertical plane forming the rail end shall make a angle of 45° to 60° with the longitudinal vert plane of the web of the rail, instead o the usual right angle. This would permit the use of longer rails than are now lai as the great space (½ inch or more) between the ends of such long rails in col weather, would not be so serious an objection when the ends were thus cut obliquely This method of cutting the rails has been tried, with good results, but has not ye come into general use. It is claimed that a comparatively inexpensive change in th arrangement of the saws at the rolling mill, would permit the rails to be cut wit ends at any angle, as readily as with square ends, and without further increase i the cost of sawing.

Art. 4. Fish-plates, Fig 1, and **Angle-plates,** Figs 2, 3, 4, and 5, hav nearly supplanted all other forms of joint on the principal railroads of the U. S. Although the rails are almost universally of Bessemer, or similar, steel, the joint-plates are generally made of iron. They are rolled in long bars, and cut off in any desired length, generally about 2 ft; and are bolted together, and to the rails, by 4 bolts, 2 in each rail-end.

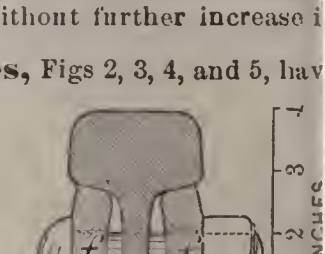


Fig.1.

Art. 5. The fish-plate joint was one of the earliest suggested. It was introduced upon the Newcastle and Frenchtown R R, in Delaware, by Robt. H. Barr, in 1843. The weight of a complete fish-plate joint, including bolts and nuts, is about 20 lbs.

Fig 1, one-fifth of real size, represents a fish-plate joint made by **Cambria Iron Co,** office 218 S 4th St, Phila, with a steel rail weighin 50 lbs per yard, by the same Co. The thickness of plate at *f* is $\frac{3}{16}$ inch. The plate weigh 4¼ lbs per lineal foot of a single plate.

Art. 6. The principal advantage of the **angle-plates** is that the spikes which are driven through slots in their flanges to confine them to the cross-ties serve also to counteract the tendency of the rails to "creep."* (See Art 1.) Wher *fish*-plates are used, the flanges of the *rails* have to be slotted for this purpose.

Fig.2.

Art. 7. Fig 2 (one-fifth of real size) shows the standard angle-plate joint (1884 of the **Phila & Reading R R.** The bars are 2 ft long, and weigh 16 lbs each

* On the St Louis bridge (steel arches) and its eastern approach (plate girders on iron columns the rails creep, in the direction of the traffic, *about a foot per day*, both up and down a grade of 8 ft per mile, and with such force that although various fastenings were used, in order to prevent th creeping, none proved effectual. The track is now adjusted daily to accommodate the creeping.

Art. 8. The wheel-tread and 76 lb steel rail, shown (one-fifth of real size) in Fig 3, are those designed by **Robt. H. Sayre, C. E.,** and used by the **Lehigh Valley R R,** under very heavy traffic. **The angle-plate joint** was designed by **Mr. John Fritz,** Supt Bethlehem Iron Co, Bethlehem, Pa, and **Mr. Sayre,** and has been in use on the **Lehigh Valley R R** for the past 12 years.

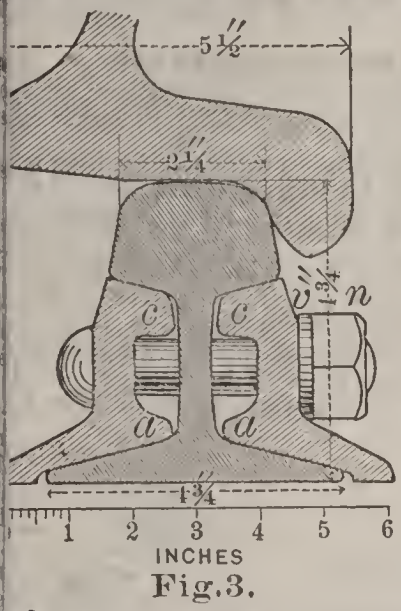


Fig.3.

These forms of wheel-tread, rail, and splice, are the result of careful study, and each detail has been modified from time to time as experience dictated, until now they are probably the most perfect in this country. Mr. Sayre places the stems of the two plates much farther apart than usual, thus giving the joint greater lateral strength; at the same time adding to its vertical strength by the support given to the lower side of the rail-head by the upper enlargement *c*; while the lower one *a* secures a full bearing on the flange of the rail. The joint, for 76 lb rail, complete, 2 ft long, with 4 bolts ⅞ inch diam, weighs 40 to 48 lbs, depending upon the thickness of the angle plate. The drilled bolt-holes in the stem of the *rail*, are 1 inch diam, to allow the rails to contract and expand.

Art. 9. Figs 4 and 5 (one-fifth of actual size) show an **angle-plate joint** made by **Cambria Iron Co,** Johnstown, Pa, office 218 S 4th St, Phila, and furnished with their **patent nut-lock,** which consists of a small piece, or "key," *p*, of Bessemer steel, semi-circular in cross-section at one end, and tapered to a horizontal edge at the other. After the nut has been screwed to its place, the key is driven close up to it, and then the pointed end of the key is bent up (as shown in Fig 5) by a special tool with a lever attached. The key is prevented from falling out sideways by the edge of the longitudinal groove, Fig 4, in the angle-plate, into which it fits. This nut-lock resembles that used with the old form

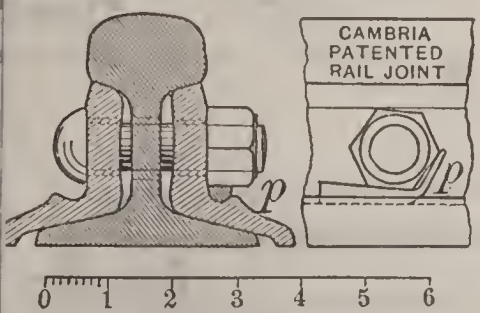


Fig.4.

Fig.5.

f Fisher-joint, see B Figs 15. See nut-lock washers, p 408.

Art. 10. Both fish- and angle-plates are apt to crack vertically about the mid-le of their length, or opposite to the joint in the rail. To obviate this, the **Samson-bar"** (which is made either of fish or of angle form) is rolled about alf inch thicker at the middle than at its ends. The thickened portion is about 8 as long, extending say 4 ins each way from the joint, but the upper edge of the bar, pon which the head of the rail bears, and in *fish*-bars the lower edge also, are made f this increased thickness throughout their length. These joints are (1886) largely sed on the Western railroads of the U. S. Their cost is about the same as that of rdinary fish- and angle-joints. They are made by Morris Sellers & Co, office No. 6 shland Block, Chicago, Ill.

Art. 11. Fish- and angle-plates, of all the patterns shown, and others, are **olled to suit different sizes and shapes of rails.** The **bolt eads** are usually round, and the **shoulders** of the bolts, immediately under he heads, are therefore made of **oval** cross-section, fitting into corresponding oval oles in the fish- or angle-plate. The bolt is thus prevented from turning when the ut is screwed on, and afterwards. Many devices have been tried, with a view to **preventing the nuts from wearing loose** (see lock-nut washers, 408). The Vulcanized Fibre Co, Wilmington, Del, furnish a **vulcanized asher,** which is intended to act as an elastic cushion, deadening shocks and ibrations. They are said to become hard, and lose their elasticity, in time.

The plates are frequently rolled, as in Figs 17, with a **longitudinal groove,** s wide as the head or nut of the bolt, and about ⅛ inch deep, running their entire ength. This groove receives either the head of the bolt, which in such cases is ade square or oblong and inserted first, and the nut afterwards screwed on; or lse the nut is first placed in the groove, and the bolt afterwards screwed into it. his is intended to prevent the unscrewing of the nut, but cannot be relied upon to lo so.

It is well to have the slots in the flanges of rails or of angle-bars so spaced that

the two spikes of a joint, driven into the same cross-tie shall not be directly opposite to each other, but "staggered," so as t diminish the danger of splitting the tie.

Joints are frequently laid with **one fish- and one angle-plate.**

In 1886, angle- and fish-plates **cost** about 2 cts per lb; bolts and nuts, 3 cts pe lb. The cost of a complete angle-joint for 70-lb rail, is from 80 cts to $1.20; of fish-joint, 60 to 80 cts.

Art. 12. It will be noticed that both fish- and angle-plates act by placing support under the *head* of the rail. **The Fisher bridge-joint,** Figs 6 to 9 made by Mr. Clark Fisher, Trenton, N J, applies the support under the *base* of th rail.

The principal feature of this joint is a flanged beam, Fig 6, about 6 ins wide an 22 ins long, which extends across, and is spiked to, the two joint-ties, as in Fig 7 The holes for the spikes are placed so that the two spikes in the same tie are no opposite to each other; and the flanges F F also are staggered, so as not to interfer with the driving of the spikes. The joint-ties T T are placed 7 inches apart in th clear. The beam has an upward camber of about one-eighth of an inch. The tw

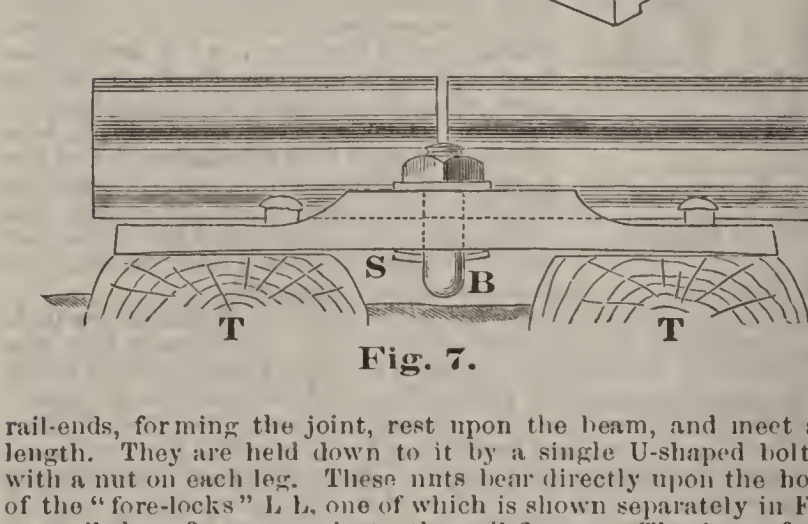


Fig.6.

Fig.9.

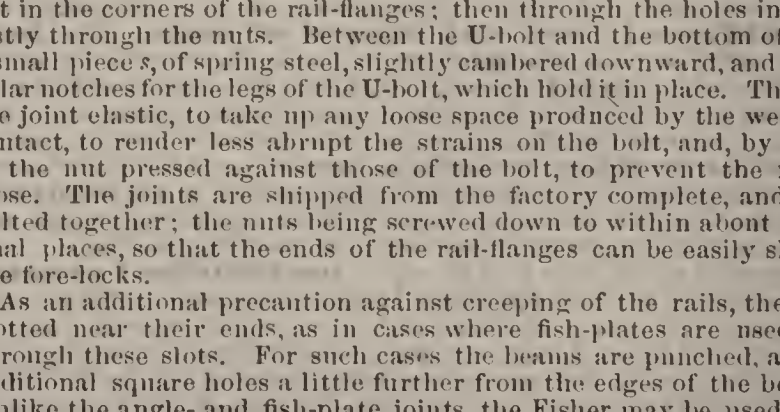


Fig. 7.

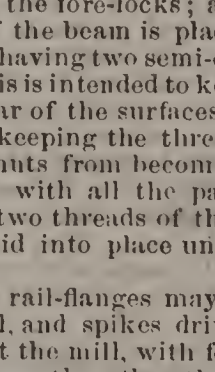


Fig. 8.

rail-ends, forming the joint, rest upon the beam, and meet at the middle of its length. They are held down to it by a single U-shaped bolt B, of 1 inch diam with a nut on each leg. These nuts bear directly upon the horizontal upper sides of the "fore-locks" L L, one of which is shown separately in Fig 9. The fore-locks are rolled to fit accurately to the rail-flanges. The legs of the U-bolt pass first through the circular holes *h h*, in the beam, Fig 6; next through rounded notches cut in the corners of the rail-flanges; then through the holes in the fore-locks; and lastly through the nuts. Between the U-bolt and the bottom of the beam is placed a small piece *s*, of spring steel, slightly cambered downward, and having two semi-circular notches for the legs of the U-bolt, which hold it in place. This is intended to keep the joint elastic, to take up any loose space produced by the wear of the surfaces in contact, to render less abrupt the strains on the bolt, and, by keeping the threads of the nut pressed against those of the bolt, to prevent the nuts from becoming loose. The joints are shipped from the factory complete, and with all the parts bolted together; the nuts being screwed down to within about two threads of their final places, so that the ends of the rail-flanges can be easily slid into place under the fore-locks.

As an additional precaution against creeping of the rails, the rail-flanges may be slotted near their ends, as in cases where fish-plates are used, and spikes driven through these slots. For such cases the beams are punched, at the mill, with four additional square holes a little further from the edges of the beam than the others. Unlike the angle- and fish-plate joints, the Fisher may be used with any section of T-rail; and the head of the rail may be made stronger by being rolled pear-shaped, which is inadmissible with fish- and angle-joints, because these require a nearly

rizontal bearing on the under side of the head. The "Fisher" requires no drill-g or punching of the stem of the rail. It **costs** about 25 per cent more than a n- or angle-joint for the same rail. Its **weight,** complete, for 65-lb rail, is about lbs.

Mr. Fisher makes also an extra strong joint with three U-bolts, heavy curves and for places liable to wash-outs. It is intended to support the nt, even if the ballast is removed from under the joint-ties. Either of the Fisher nts can be made of any desired weight. The "Fisher" is largely used on some the principal eastern roads, and with very satisfactory results. Figs 15, p 768, ow an old form of this joint.

Art. 13. The Gibbon boltless rail-joint, Figs 10, 11, and 12, invented Mr. Thos. H. Gibbon, C E, Albany, N Y, is a *supported* one. Two inches in length

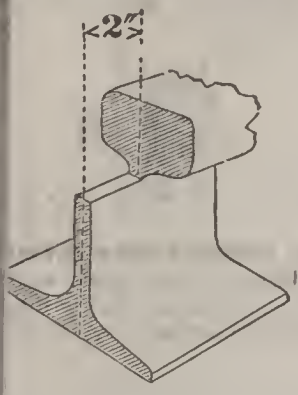


Fig. 10.

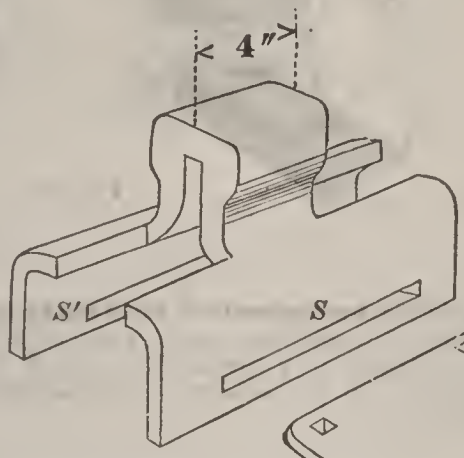


Fig. 11.

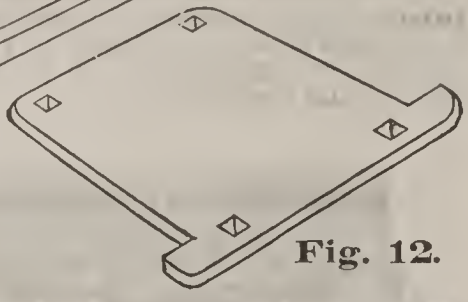


Fig. 12.

the head, at each rail-end, have to be cut off, as in Fig 10; but no further cutting, d no punching or drilling, of the rail, is required. Over the ends of two rails, thus t, and placed together in their final position on the ties, the Bessemer steel "sad-e-casting," Fig 11, is placed. The top of this casting is shaped so as to correspond th the head of the rail; and its length, 4 ins, just occupies the space cut away om the ends of the two adjoining rail-heads. The feet of the casting fit into notches t in the joint-tie at the sides of the rail. The ends of the rails are then raised ghtly, and the base-plate, Fig 12, of iron or steel, is slipped through the slot *s* in e side of the saddle-casting, then under the base of the rail, and lastly through a rresponding slot in the opposite leg of the saddle-casting. Spikes are then driven to the tie through the four holes in the base-plate, and the joint is complete. The ddle-casting and base-plate **weigh,** together, about 24 lbs, and **cost** (1886) $1.00.

Art. 14. The following are some forms of rail-joint that have fallen into disuse, have been proposed but not adopted. They illustrate early practice, and may be eful as hints.

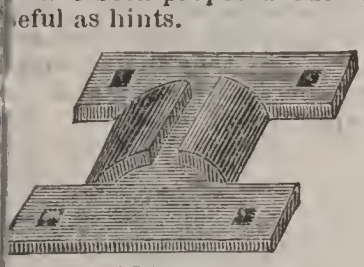


Fig. 13.

Fig 13 was an early form of supported **wrought-iron chair.** It was about 7 ins square, ¾ thick, and weighed 10 lbs. Fig 14 is a later form, still furnished, to some extent, by the Tredegar Co, of Richmond, Va.

Fig.14.

Figs 15 show **one of the earliest forms of the Fisher joint.** It as at one time largely used on the Lehigh Valley R R. (For the present form of his joint, see p 766.) It was a suspended joint; and, instead of the long supporting eam, or bridge-plate, of the present pattern, it had a plate *c*, 6 ins square. It had U-bolts, each of 1 inch diam, and the fore-locks *t t* were 6 ins long, and had two oles each. The lower side of the thread on the bolt was made horizontal as at *j*. he thread in the nut was somewhat as at *l*. In screwing on the nut, these two

threads were forced into conformity, thus employing the principle of the prese Harvey lock-nut, p 408. A nut-lock B, somewhat similar to the Cambria pate lock, p 765, was used as an additional precaution.

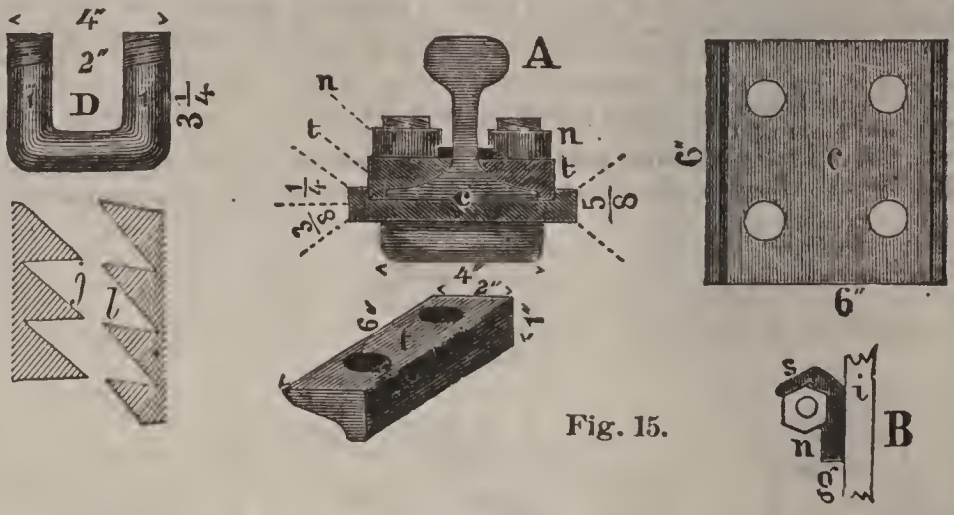


Fig. 15.

Figs 16 show a remarkable **suspended fastening, called the Ring joint;** highly approved of at one time on the Camden & Amboy road (now Penn R R, United Railroads of New Jersey Division) on which it was employed for many years, under a heavy traffic; to the almost entire exclusion of others, except for experimental comparison.

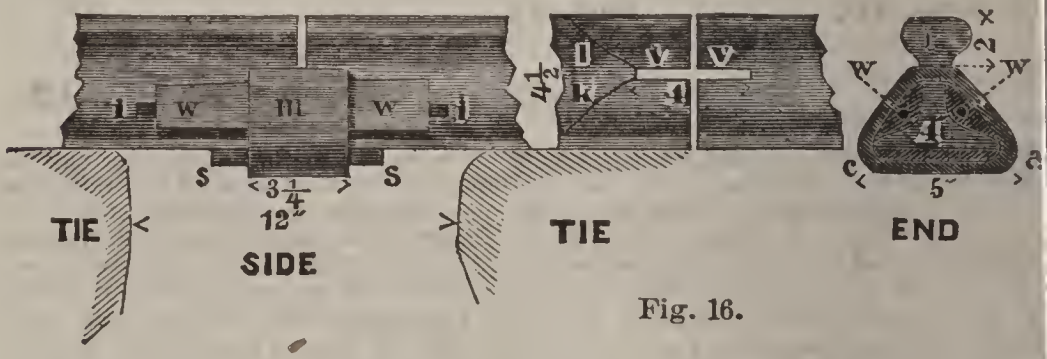


Fig. 16.

This fastening consisted of a simple welded triangular ring *t c a*, (in the end view or *m* in the side view,) ½ an inch thick, and 3¼ ins wide. This ring passed through a slot *v v*, (see middle fig,) 4 ins long, cut into the adjacent rail-ends. Two cast-iron wedges *w w*, 6 to 8 ins long, of a shape to fit the ring and the rail, were inserted between them; and a thinner one *s s*, of plate iron, below the rail. The first were cast around a cylindrical rod of rolled iron, (*w* of the end view, *i i* of the side view,) about ⅜ inch diam; and a little longer than the wedges; for increasing their strength and for preventing them from falling out from the chair in case they should break which they sometimes did. The rail-ends sometimes split, as shown at *l* and *k*.

The joint was suspended between two cross-ties, 1 ft apart in the clear.

Figs 17 represent the combined suspended joint-fastening introduced upon the hilada & Reading railroad by **J. Dutton Steele,** C E. Some of them are still n use (1884).

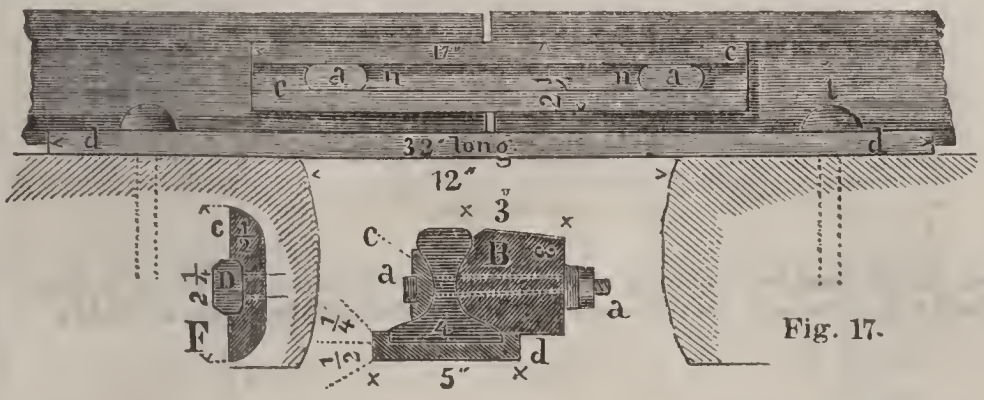


Fig. 17.

The joint is suspended between 2 cross-ties, placed 1 ft apart in the clear. B is a lock (known as Trimble's splice) of oak, 3 by 3 ins, and 3 ft long, dressed on one de to fit the outside of the rail; and c is a rolled fish, 17 ins long, (shown more in etail at F,) placed on the *inside* of the rail. This fish and the oak block are bolted ogether, through the rail, by two ¾-inch screw-bolts *a a*, 13 inches apart. Under he rail is a rolled chair *d*, 2 ft 8 ins long, 5 ins wide, and ½ inch thick; turned up ¼ inch along each of its two sides, and fastened to the 2 wooden cross-ties by 4 hook-eaded spikes, ½ inch square, by 5½ ins long. The heads of the two screw-bolts are nade somewhat oblong, (about ¾ inch by 1¼,) for fitting into the groove *n n*, seen long one side of the fish; so as to prevent the tendency of the bolts to revolve under he action of the trains, and thus unscrew the nut at the other end. The nuts, how-ver, unscrew themselves, notwithstanding this precaution. The strain on the screw-olts is great, both vert and hor; and it becomes greater as the wooden blocks in me lose (as they do) their close fit to the sides of the rails. The blocks then cease o act in perfect unison with the other parts of the fastening, in sustaining passing oads; and when the track is not kept in good order, the various parts may plainly e seen to yield and move in various directions, independently of each other. The olt-nuts then loosen; and the fish pieces, long chairs, and long bolts, become bent; nd sometimes split, or break entirely. The long wooden blocks B crush and decay onest near where their tops are in contact with the rail. They, however, have an verage life of 6 to 8 years, upon roads kept in tolerable order.

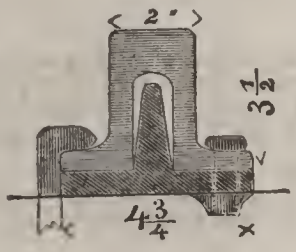


Fig. 18.

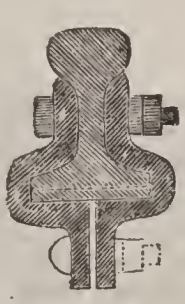


Fig. 19.

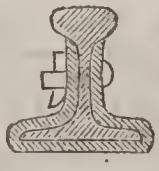


Fig. 20.

Fig 18 is a joint for U-rail. It was of rolled iron, 5 ft long, and rested on 3 ties. ; was riveted loosely to one flange of the rail, as shown on the right-hand side. It ept the rails in position very well.

Fig 19 is a joint-fastening proposed many years since by **Alex. W. Rae, C E,** Penna. It certainly possesses merit.

Fig 20 was also one of the numerous joint-fastenings suggested at an early day; ut, like the foregoing, it never came into use. In lengths of about 8 ins, it would robably make an efficient fastening; especially with the addition of a broad, thin edge between the bottom of the rail and the foot of the chair. This would dimin-h the difficulty of sliding the chairs on or off of the rail; and would thus make it sy to employ larger ones; besides insuring a firm bearing for the base of the rail pon the fastening.

TURNOUTS.

Art. 1. To enable an engine and train to pass from one track, A B, Fig 1, to another, A D, a *turnout* is introduced. **This consists essentially** of a

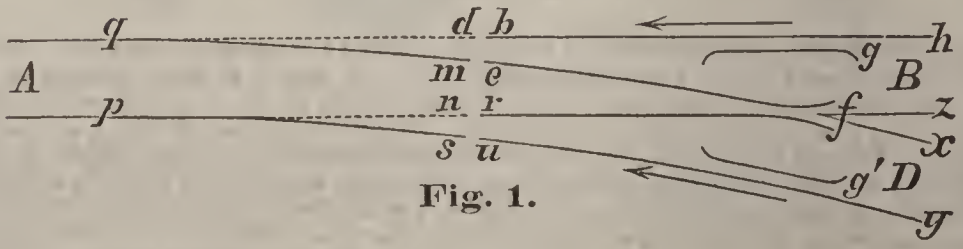


Fig. 1.

switch, *q m p s*, a **frog,** *f*, and two fixed **guard-rails,** *g* and *g'*. If a switch is made to serve for *two* turnouts, A D and A D', Fig 2, one on each side of the main track, A B, it is called a **three-throw switch.**

Fig. 2.

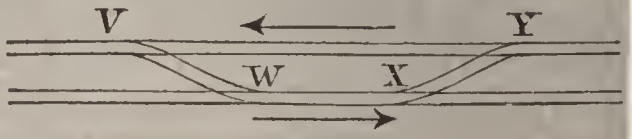


Fig. 3.

Art. 2. When a train approaches a switch in the direction of either arrow, Fig 1; or so that it passes the *frog* before reaching the *switch*, it is said to "**trail**" the switch. When it approaches in the *opposite* direction, passing the *switch* before reaching the *frog*, it is said to "**face**" the switch. Fig 3 represents a portion of a double-track road in which the trains keep to the right, as shown by the arrows. In this fig, V and W are "*trailing*" switches; and X and Y are "*facing*" switches. In order to leave the main track by a *trailing* switch, a train must move in a direction contrary to the proper one on said track.

Art. 3. Misplaced switches. A moving train, *facing* any switch, must plainly go as the switch is set, whether right or wrong. If wrong, serious accident may result. For instance, the train may run upon, and over the end of, a short trestle siding, or may collide with a train standing or moving upon the turnout. **Safety switches,** such as the Lorenz, Arts 13, &c, and Wharton, Arts 18, &c, are so arranged that trains *trailing* them can pass them safely, even if the switch is misplaced. But in the case of the plain **stub-switch,** Art 4, when misplaced, a trailing train will leave the rails at *b* and *r*, or *e* and *u*, Fig 4, and run upon the ties.

Stub-switches are frequently provided with "**safety-castings**" of iron bolted to their sides, and reaching from their toes *m* and *s*, Fig 4, several feet toward *p* and *q*. These, in case of misplacement of the switch, receive the flanges of the wheels of a trailing train, and guide the wheels safely on to the switch-rails *q m* and *p s*. **The "Tyler" switch** is arranged in this way.

Art. 4. The common blunt-ended or stub-switch consists ssentially of two rails, *q m* and *p s*, Figs 1 and 4. The ends, *q* and *p*, of these ils, where they are fixed in line with the main track, form the "**heel**" of the witch. Their other ends, *m* and *s*, form the "**toe,**" and are free to move from *d*

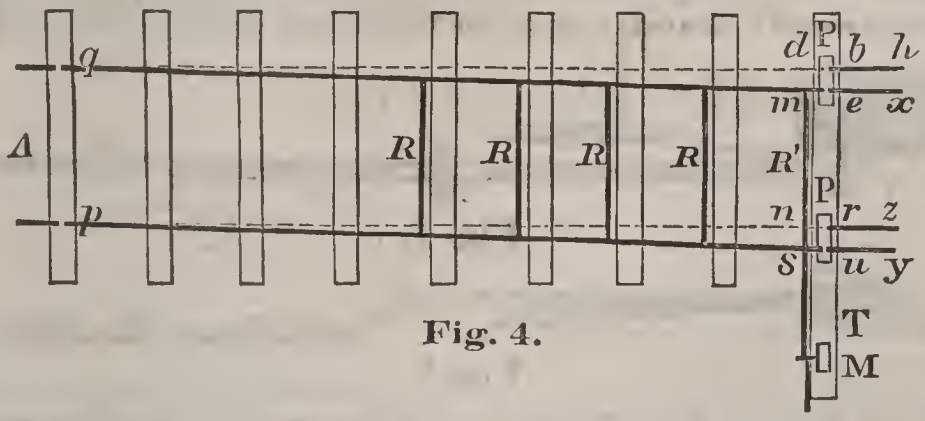


Fig. 4.

nd *n* (where they are in line with the main-track rails, *b h* and *r z*) to *m* and *s* where they are in line with the turnout-rails, *e x* and *u y*).

On main lines of road, the switch-rails are usually from 18 to 26 feet long from eel to toe. Formerly their heels, *q* and *p*, were fixed by being confined in the same airs which held the adjoining ends of the main-line rails; or by being connected ith said rails by short fish-plates. See p 764. In either case they remained prac-cally straight, even when set for the turnout. Now, however, they are generally ade long enough to extend 4 to 6 feet back from their heels, toward A; and this dditional length is spiked unyieldingly to e ties; so that, when set for the turn-ut, the switch-rails bend so as to form (at ast approximately) a part of the turn-ut curve. Their toes, *m* and *s*, in any se, rest and slide upon iron "**head-lates,**" P P, Fig 4, shown in detail Fig 5. These head plates also receive e ends, *b e* and *r u*, of the main-track nd turnout rails. In *three-throw* switches, e head-plates must of course be longer, give room for the *three* rail-ends side side.

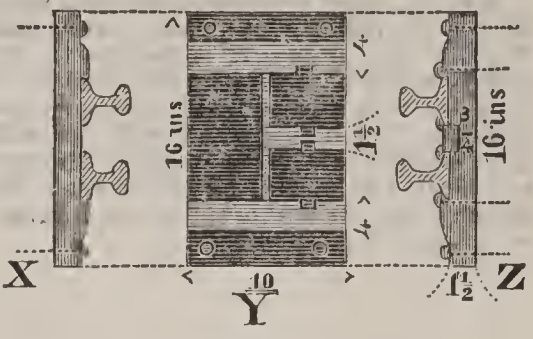


Fig. 5.

Frequently a plain strip of iron, about 3 s wide by half inch thick, is fastened to e upper surface of each tie, under the se of the rail, for the latter to slide on.

The switch-rails are connected together by from 3 to 5 transverse wrought-iron **lamp-rods,** *R R R'*, Fig 4, full 1¼ ins diam. These are fastened to the rails in rious ways; generally as shown in Fig 6. The clamp-bars should be placed, if possible, at least as low as the tops of the cross-ties, so as to avoid danger of their coming into contact with any portions of cars or engines that may become partially detached and drag on the track.

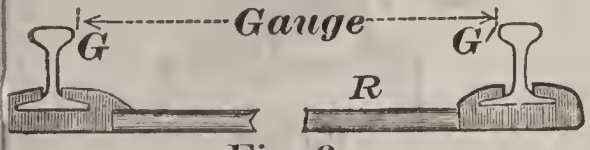


Fig. 6.

One of the clamp-bars, R', is ar the toes, *m* and *s*, Fig 4. It projects beyond the track, and is jointed, as shown Figs 7 and 8, and connected with the lever, L, by which the switch is moved. he tie, T, Fig 4, to which the head-plates are fastened, is made longer than the hers, in order to give room for the switch-stand, M, Figs 4, 7, and 8, which is bolted its upper surface. This tie should also be of larger cross-section than the others, rfectly sound, and well bedded; because upon it come severe strains due to the assage of cars and engines across the space between the rail-ends, *m* and *e*, *s* and *u*, c, Fig 4. See second paragraph of Art 10, p 773.

Art. 5. The **switch-levers,** and the **switch-stands** to which they are attached, are made in a great variety of forms. See Figs 7, 8, 9, 14, 15, and 17. That shown in Figs 7 and 8 is the "**Tumbling-lever** stand" or "**Ground-lever** stand," and, in its numerous modifications, is very largely used. It is so arranged, that, whichever way the switch is set, the crank, C, is on the dead center, so that the lateral strains of passing cars or engines can exert no tendency to turn it. **The "Greenwood" stand,** made by the Pennsylvania Steel Co, Steelton, Pa, has

Fig. 7.

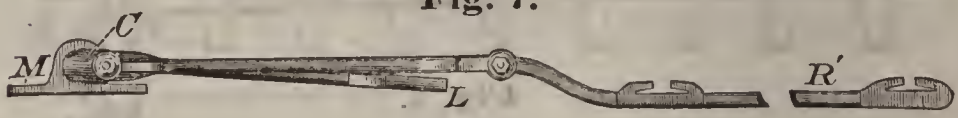


Fig. 8.

a tumbling lever, but is so arranged that (unlike the ordinary tumbling lever) it can be used for either a two-throw or a three-throw switch.

Tumbling switches are convenient because they occupy but little space. By means of a target or lantern, connected with the switch, they may be made to indicate to the engine driver the position of the switch.

When the switch is set either way, the lever is padlocked to a staple driven into the tie and passing up through the slot in the handle of the lever. The lever is frequently made with a weight of say 20 lbs on its free end, to aid in bringing it down to its proper position. **Price** of a tumbling-lever stand, 1886, $3.50 to $5.00, without target.

Art. 6. Fig 9 represents a common form of the **upright lever and stand.** The switch-rod, R' Figs 4, 7 and 8, is generally attached at the lower end, A, of the lever. The cast-iron frame, F, is fastened to the long tie, T, Fig 4, by large screws or spikes, which pass through its broad feet or flanges, B B. The top of the frame is provided with two notches, N N, and staples, to which the lever is secured by a padlock. When this stand is to be used for a *three*-throw switch, the frame has *three* notches and three staples. The upright stand may be used wherever it will not be in the way of passing trains. The target, T, at the top of the lever, by showing the position of the latter, indicates to the driver of an approaching engine which way the switch is set.

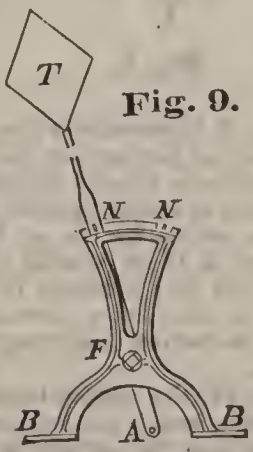
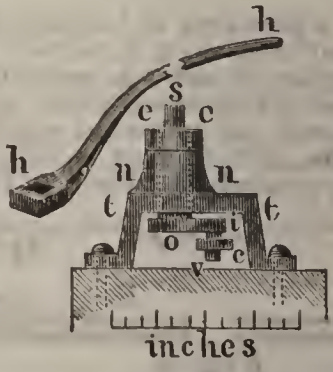


Fig. 9.



MONKEY SWITCH

Fig. 10.

Art. 7. In the "**Monkey-switch,**" Fig 10, the crank, *o i*, is moved horizontally through an arc of a circle by means of the lever, *h h*, about 3 ft long, which fits upon the square head, *s*, of the vertical spindle or pin, *s o*. The switch-rod, R' Figs 4, 7 and 8, is attached to the pin, *i v*.

Many modifications of the monkey-switch are in use. The spindle, *s o*, is frequently made long enough to bring the lever to about the level of the hand; and the lever is permanently attached to the stand, and hinged near the spindle so as to hang down, out of the way, when not in use. To the top of the spindle is frequently attached a vertical rod of any desired length, and carrying at its top a target which turns as the spindle does, and thus indicates the position of the switch.

Art. 8. All parts of the switch-stand, and the tie upon which it rests, should e perfectly rigid, because it is very important that they should hold the ends of ie switch-rails exactly in line with those of the main line and turnout. They ierefore, in view of the great strains to which they are subjected, must be strongly onstructed, and frequently looked after. See **Automatic** switch-stand, Art 12, id **De Vout's** switch-stand, Art 14.

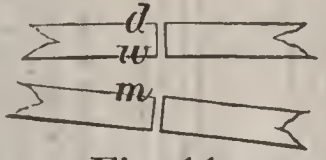


Fig. 11.

Art. 9. In Figs 1 and 4, *d q m* is called the **switch-angle.** The dist, *d m*, igs 1, 4, and 11, required for the motion of the toes, is called the **throw** of the vitch. It must be equal at least to the width, *d w*, Fig 11, of the top of the rail, addition to a width, *w m*, sufficient to allow the flanges of the wheels to pass ong readily between *b* and *e*, Fig 1, and between *r* and *u*. The tops of the rails re generally between 2 and 2½ ins wide; and about 1¾ to 2½ ins suffice for the anges. The throw, *d m*, however, is commonly about 5 ins.

The **gauge,** Fig 6, p 771, of a railroad track, is the distance between the *inner* des G G′ of the heads of its two rails. Hence these inner sides are called the **gauge ides** of the rails.

Art. 10. The stub-switch is cheaper in first cost than the improved safety vitches, Arts 13, 18, etc, but is less economical in the long run.

As it is very essential that the toes of the switch-rails should never come into ontact with the adjoining rail-ends, a space of about an inch must be allowed the toes for expansion, and for "creeping" (see p 764). This renders the blows passing trains very severe, and injurious to rolling stock, and to the rail-ends. he latter are worn away rapidly and must be frequently renewed. From the same ause the tie under the head-plate is apt to become loose in its bed.

In ordering fixtures for stub-switches, the exact section of rail, id gauge of track, should be given. **The cost of a stub-switch** with switch-and, is, 1886, from $18 to $30, according to size, finish, character of stand, &c, &c.

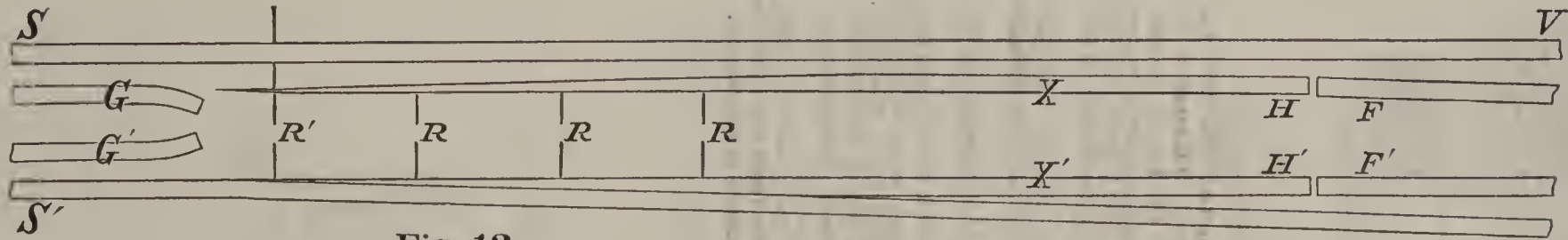


Fig. 12.

POINT SWITCH.

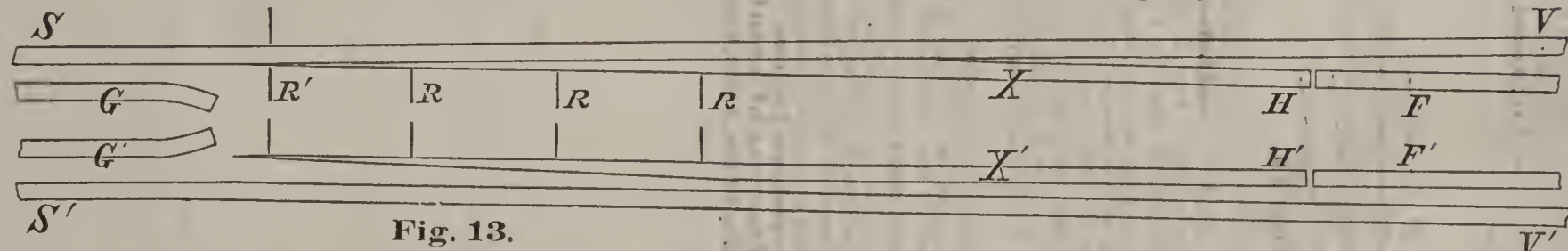


Fig. 13.

Art. 11. Split-, tongue-, or point-switches. Figs 12 and 13 represent these switches in their simplest form, and without the safety appliances described in Arts 12, 13, &c. They omit the *flanges* of the rails, and show only their *heads*. Fig 12 shows the switch set for the main line. Fig 13 shows it set for the turnout. The **"stock-rails,"** S V and S′ V′, are continuous, and are spiked to the ties throughout. Sometimes the **switch-rails,** X and X′, are made long enough to be spiked to the ties for some feet back from their heels, H H′, as is now generally done with stub-switches, see Art 4; but usually their heels are fastened by fish-plates to the ends of the rails, F and F′, which lead to the frog, and they are then not otherwise fastened. They are therefore free to move about their heels as fulcrums, without changing their shape. Iron plates are fastened to the ties under the feet of the switch-rails, to facilitate their sliding. It will be noticed that in the split-switch the **heels** are in the places occupied by the **toes** in the stub-switch, and *vice versa*. The switch-rails, X and X′, are straight. When the switch is set for the main line, as in Fig 12, the gauge side of X′ coincides with that of S′ and F′. **The toe of each switch-rail is pointed,** and so shaped as to fit close up to the head of the rail, S V or S′ V′, with which it is in contact. Hence the name "*point*"-switch. In Europe, where most of the switches are of this type, the name "points" is applied to *any* switch. For some distance back from the toes, where the switch-rails are thinnest, their tops are made a little *lower* than in the rest of their length, so that the weight of the train does not come upon said thinnest portion. The heels, H H′, are placed so that the heads of the switch-rails, X and X′, at those points, are about 2 ins, in the clear, from those of the stock-rails, S V and S′ V′, in order to allow room for the passage of the wheel-flanges; and, theoretically, the same clear distance from the stock-rail would be sufficient for the *points;* but *they* are given a throw of about 3½ or 4 ins, in order to avoid all possibility of a wheel-flange striking the point. As an additional precaution, short **guard-rails** are sometimes spiked to the ties about 2 ins from the stock-rails. Their ends are shown at G and G′. The switch-rails are connected with each other by means of **connecting-bars,** R R′, as in the stub-switch. As improved by the Penna Steel Co, and shown in Fig 16, these connecting-bars are jointed, so that, while *rigid* [illegible] Point-switches may be moved by any of the ordinary

Art. 12. The "Automatic" switch-stand, made by the Penna Steel Co, is shown in hor section by Fig 14, and in vert section by Fig 15; and is designed

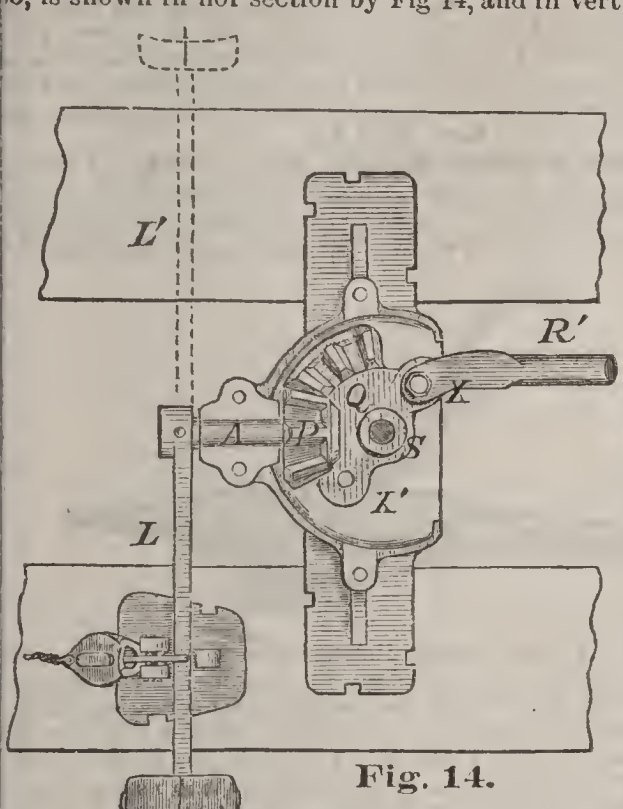


Fig. 14.

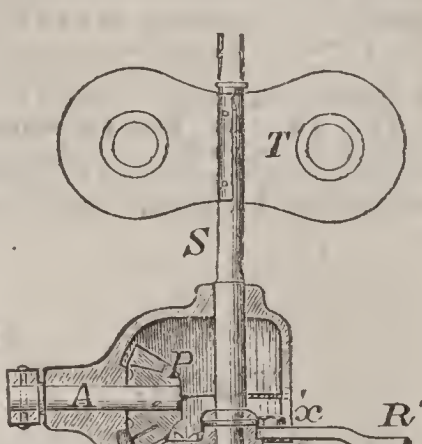


Fig. 15.

especially for split-switches like those shown in Figs 12 and 13. It is operated by a tumbling-lever, L, Fig 14. To the hor axis, A, of the lever, is fastened the beveled pinion, P. This engages in the teeth of the quadrant, Q, and moves it horizontally through a quarter of a circle, when the lever L is thrown from its position L to that of L', shown by the dotted lines. The rod, R', from the switch, is attached at X, or at X', to this quadrant; and the movement of the latter thus sets the switch, and at the same time turns the target, T, Fig 15, or lantern, fixed to the vert spindle, S, thus indicating the position of the switch. The gearing is enclosed in a cast-iron case, as shown in Fig 5. If a train on the *turnout*, moving in the direction of the arrow, Fig 1 (or "trailing"), approaches a split-switch provided with the automatic stand, and set (through oversight or otherwise) for the *main* track, as in Fig 12, the flange of the first wheel, pressing between rails X' and S' V', will push the switch-rails into the proper position, Fig 13, for the *turnout;* at the same time necessarily throwing the weighted lever over into the reverse position and turning the target so as to indicate that the switch is set for the turnout. A similar movement of the switch, in the opposite direction, takes place if a trailing train on the *main* track approaches the switch when set for the *turnout*, as in Fig 13. Hence the term "automatic," as applied to this stand. The switch is thus made a *safety* switch.

The cost of the automatic switch-stand (1886) is about $15.

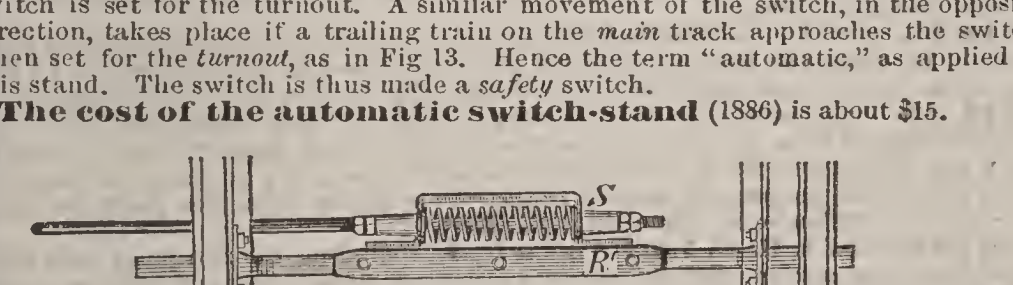


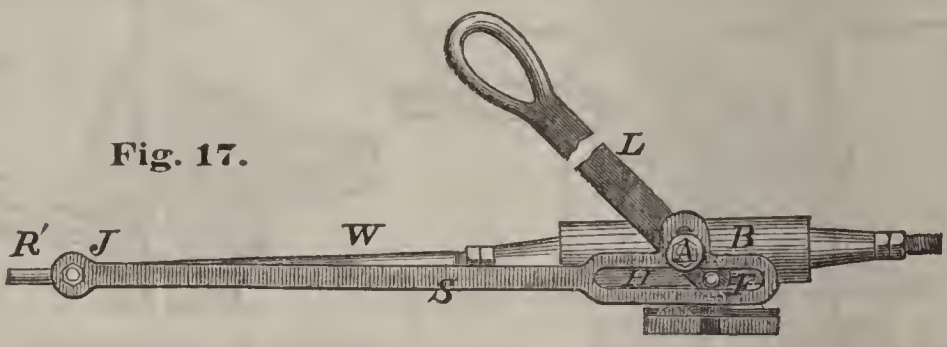
Fig. 16.

Art. 13. The Lorenz safety-switch, designed by the late Wm. Lorenz, Esq, Ch Eng of the Phila & Reading R R, is a split-switch, in which the connecting-bar, R', nearest to the toes, is provided with a **spring,** S, Fig 16, placed sometimes between the rails, as there shown; sometimes outside of the track. This spring permits the moving of the switch-rails by the wheels of a trailing train, as does the automatic switch-stand, Figs 14 and 15; but, after the passage of each wheel, the spring returns the switch-rails to their original position. The blow of the

switch-rails against the stock-rails, thus occasioned, is injurious to both, and liable to break the former. It is also urged as an objection to these switches that the spring may permit the switchman to force the switch-lever to its place, even although some obstacle, as a small stone, lodged between the switch-rail and stock-rail, prevents the two from coming into contact, so that both points remain an inch or so away from the stock-rails and in danger of being struck by the wheel-flanges of approaching trains "facing" the switch. **See Art 14.**

Art. 14. De Vout's safety switch-stand, Fig 17, made by Penna Steel Co, is designed to remedy this. In this stand, the spring is placed in, and secured to, a semi-cylindrical iron spring-case or box, B; to the opposite sides of which are fixed two hor axles. One of these is shown at A. This axle passes through the

Fig. 17.



switch-lever, L, near its fulcrum, F. It also passes through the inverted T-shaped slot, H, in the rigid bar, S, which, together with the bar, W, attached to the spring-case, is jointed, at J, to the switch-rod, R′. When the switch is properly set, either for the main line or for the turnout, the axle, A, is in the hor part of the slot, H, and immediately under the vert part, so that there is no obstruction to the movement of the switch, and a trailing train will open a misplaced switch as explained in Arts 12 and 13. But when the lever is raised, for the purpose of setting the switch in the other position, the axle, A, rises into the vert part of the slot, as in the fig, lifting the spring-case with it. If now any obstruction prevents the switch-rail from being pressed home, the rigid bar, S, by means of the axle, A, prevents the lever, L, from moving farther. **Cost of De Vout's stand,** 1886, is about $12, without target.

Art. 15. Theory would require that **the lengths of the switch-rails,** in split-switches, should vary with the radius of the turnout curve, and formerly they were so made. Where this radius is such that a No 10 frog (see Art 26) is required, the switch-rails should, theoretically, be 28 ft long. But in practice a uniform length of 15 ft (just half the usual length of the steel rail from which the switch-rails are cut) for all turnouts, gives the best results, combining economy of manufacture with greater strength, and greater ease of handling, than are possible with much longer rails.

Art. 16. It will be noticed that in point-switches (as also in the Wharton switch, Arts 18, &c) there can be no such jar as that occasioned in the stub-switch by the long space between the toes of the switch-rails and the ends of the adjoining rails.

Art. 17. It is important that the thin portions of each switch-rail should be carefully shaped so as to receive throughout a firm lateral support from the stock-rail when in contact with it. Otherwise the switch-rails are in danger of bending under the lateral pressure of passing trains. This might throw the point out from the stock-rail, endangering the train.

The **price of a 15-ft Lorenz switch,** including the two point-rails with connecting-bars, spring, spring-fixtures, switch-rod, slide-plates, and rail-braces, but exclusive of stock-rails, lever, and stand, is, 1886, about $30 to $40, according to weight of rail, gauge, &c.

Lorenz switches about 7½ ft long are made **for yard use.** Price, including the same items as in the full-sized switches, 1886, about $25 to $33.

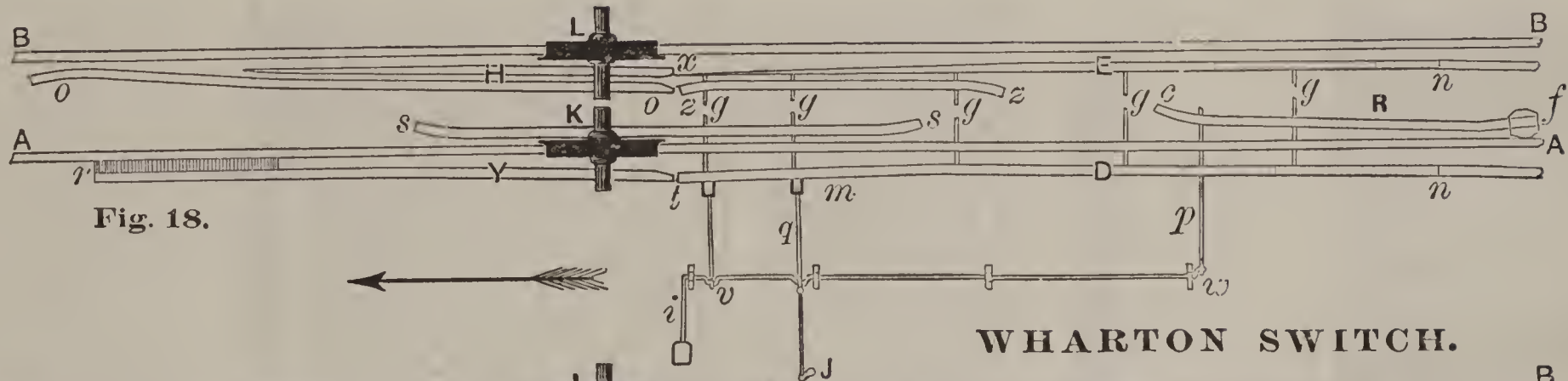


Fig. 18.

WHARTON SWITCH.

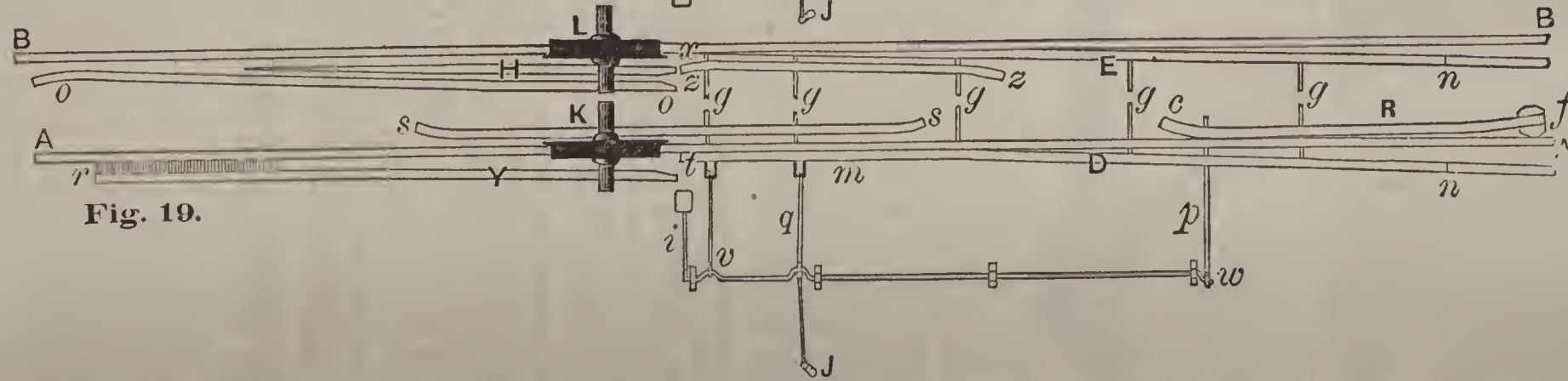


Fig. 19.

Art. 18. The Wharton patent safety-switch. Our Figs 18 and 19 show the main features of this switch in its present form (adopted 1884). We omit the *flanges* of the rails, showing only their *heads*.

Fig 18 shows the switch set for the main line; Fig 19 shows it set for the turnout. A A and B B are the main-track rails. They are continuous, except, of course, at the frog, and are spiked to the cross-ties throughout, and are not moved or broken in the working of the switch. E and D are the switch-rails, 18 ft long, or longer if desired. Their *heels* are at *n n* (the places of the *toes* in the *stub*-switch, Art 4), where they are fastened to the ends of the turnout rails by fish-plate joints.

Switch-rail D is blunt ended. At its toe *t*, its top is level with that of A A; but from *t* it rises, until, at *m*, 3 ft 6 ins from the toe, its

top is 1¾ inches higher than that of A A. This enables the flange of wheel, K, in passing to or from the turnout, to pass over the main rail, A A, without touching it. D retains this elevation from *m* to its heel, *n*. Beyond *n*, the turnout rail, continuous with D, slopes gently downward, until its top is again level with those of the main-track rails. The unbroken main rails, and this carrying of the flanges over the main rail, are special features of the Wharton switch. **The pointed switch-rail, E,** is elevated, like D, to prevent the lateral rocking of cars and engine which would otherwise result from the elevation of D.

The guard-rail, z z, is firmly bolted to the pointed switch-rail, E, and of course moves with it. Sufficient space is left between their heads to allow the passage of the wheel-flanges. Said guard-rail prevents the flange of wheel, K, from touching the head of rail, A A, in coming down the incline, *m t*, in Fig 19. As in other switches, the switch-rails, D and E, are connected together by **clamp-bars, g.** These pass underneath the main rail, A A. The switch-rails rest and slide upon iron plates fastened to the tops of the cross-ties.

Art. 19. The switch is operated by means of the **tumbling-lever, i,** and the cranks, *v w*. The lever, when the switch is set either way, lies so as to bring the crank *v* a little *below* its center, so that any lateral pressure on the switch-rails, instead of misplacing the switch, only presses the lever closer to the cross-tie to which the switch-stand is bolted. The weight on the end of the lever aids in bringing it down to its proper position. When the lever is moved from its position in Fig 18 to that in Fig 19, the switch-rails are pushed close up against the main rails. The point, *x*, of E, then fits close up against the stem, and under the head, of the main rail, B B; so that the wheel, L, moving in either direction, can pass the point, *x*, without striking it. **The fixed guard-rail, s s,** tends to keep L away from B B, and thus acts as an additional precaution against its striking *x*. At J is shown, in cross-section, a vertical iron rod, which carries, at its top, the **signal target** or lantern. This signal apparatus is operated automatically by the moving of the switch; the two being connected by the rod *q*.

Art. 20. Provision against accident in case of misplacement of the switch. Case 1. As explained in Art 3, p 770, trains moving in a direction *opposite* to the arrow, or "*facing*" the switch, must go as the switch is set, whether right or wrong. On this account it is usual, on **double-track** roads, to use, as far as possible, only *trailing* switches. See Art 2. For **single-track,** the Wharton may be so arranged that it will not remain set for the turnout unless actually held so by the switchman.

Case 2. If a *trailing* train comes from the *turnout* while the switch has, through oversight or otherwise, been left set for the *main line*, as in Fig 18; the wheels run upon the **safety-rails,** H and Y; and the **guide-rail,** *o o*, guides them safely on to the main-track rails, A A and B B. The guide-rail, *o o*, is firmly bolted to H, with room between for the passage of the flanges. In order that wheel, K, in this operation, may pass safely from the safety-rail, Y, to the main rail, A A, the former inclines gently upward from *t* toward *r*, and thus lifts wheel, K, so that its flange passes over rail A A without touching it. **The steel-plated casting, r,** keeps Y at its proper distance from A A, and prevents a wheel, K, with narrow tread, from dropping between the two when passing from one to the other in the operation just described.

Case 3. A *trailing* train moving along the *main track* while the switch is set for the *turnout*, as in Fig 19. **The cranks, v w,** are so arranged (see figs), that when the switch-rails are pushed up against the main-track rails, as in Fig 19, the end, *c*, of **the movable guard-rail, R** (moving in the opposite direction about its fulcrum in the chair, *f*), is also brought close up against the main rail, A A. In this case, the flange of the first wheel, K, of the train, pushes aside *c*, which, pulling the rod, *p*, attached to it, throws the crank, *w* (and through it the entire switch), into the proper position, Fig 18, for the main track. The crank, *w*, is placed on its shaft at such an angle as to be readily operated in this way.

Art. 21. Until recently, safety-*castings* were used in place of the safety-*rails*, H and Y. But when these castings were made long enough to avoid a too abrupt movement of the train they were found to be in danger of breaking. The switch-rail, E, and its guard-rail, *z z*, were heretofore rolled in one piece, called a **grooved rail.** The present arrangement was adopted in order to give these rails less lateral rigidity, and thus adapt them better to the passage of the heavy rolling stock now in use. In the earlier forms of this switch, the rail D rose from *t* 2½ inches in 2 feet; but this was found to be too steep an incline, and the present one, of 1¾ ins in 3½ ft, was therefore substituted.

The Wharton R R Switch Co, office 125 S 4th St, Phila, make these, as well as point and common stub, switches. **The cost of a Wharton switch,** 1886, is from $70 to $100, exclusive of safety-rails, which cost about $40 extra. Target, $10 extra.

Art. 22. Frogs. The frog is a contrivance for allowing the flange of the wheel on the rail $e\,x$, Fig 1, to cross the rail $r\,z$; and that of the wheel on $r\,z$, to cross $e\,x$. **The first contrivance for this purpose was a bar,** approximately of the shape of the rail, pivoted at the point where the center lines of the rails $e\,x$ and $r\,z$ cross each other, and free to move horizontally about this pivot, so that it could form a portion of $e\,x$ when the train was passing to or from the *turnout*, or a portion of $r\,z$ when the train was using the *main track*. Sometimes the pivot

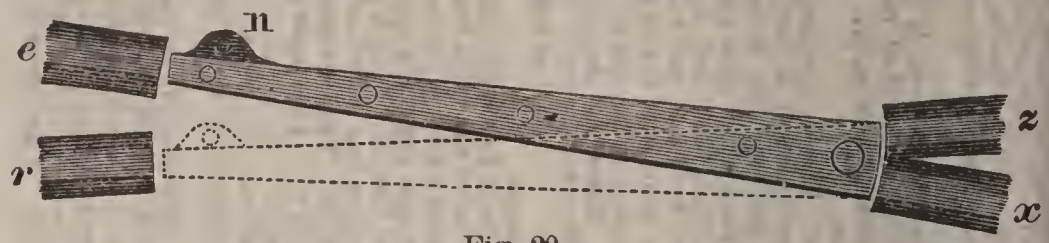


Fig. 20.

passed through one *end* of the bar, as in Fig 20, and sometimes through its *center*, as in Fig 21. Such bars were generally moved by a rod (attached at n) and lever, similar to those used for switches; and they then, of course, required an attendant; but many attempts have been made to use such frogs by connecting them with the switch by means of rods, &c, so that the bar should move automatically when the switch was turned. Owing to the considerable distance (80 ft, more or less) between the frog and switch, it has been found difficult to secure simultaneous movements of the switch and frog, and the contrivances referred to have not come into extensive use. Such bars, while they avoid the jar produced by wheels passing across the throat of the frog (Art 35), labor under the same disadvantage as the stub-switch, Art 10, in requiring a liberal allowance of space between their ends and those of the adjoining rails, to avoid any possibility of their coming into contact.

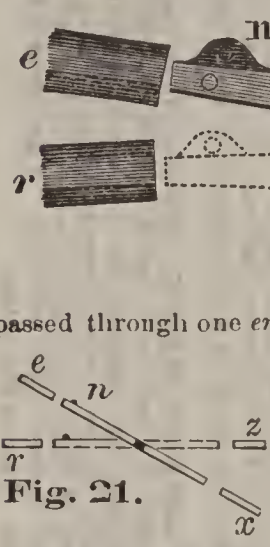


Fig. 21.

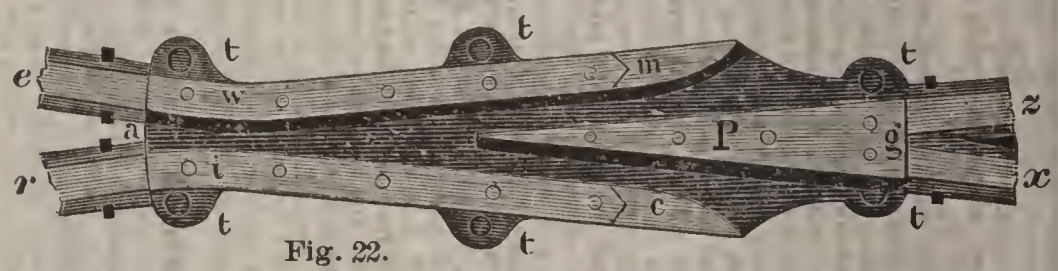


Fig. 22.

Art. 23. These bars were soon superseded by rigid **cast-iron frogs,** Figs 22 and 23. These were hardened by chilling, so as better to resist the action of

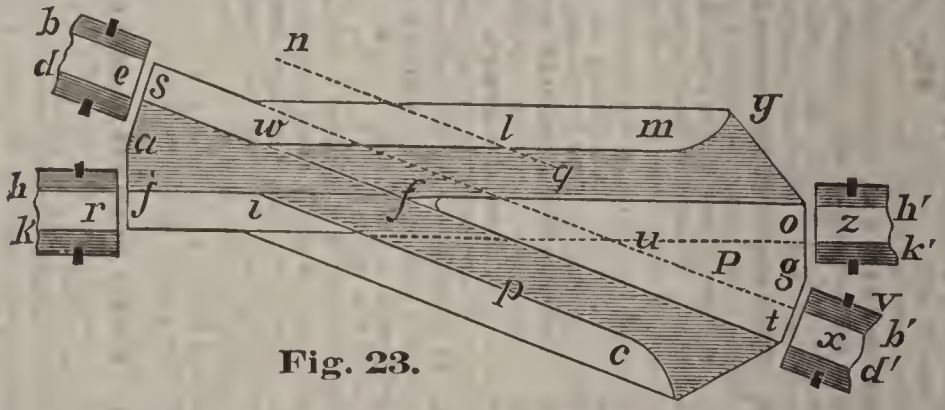


Fig. 23.

passing wheels; but even with this precaution they wore out so much more rapidly than the rails, that the **wings,** $w\,m$ and $i\,c$, and the **tongue,** P, were capped with steel from ½ inch to 1 inch thick, bolted or riveted to their upper surfaces.

The triangle, P, called the tongue of the frog, is the meeting-point of the two rails, $f z$ and $f x$, Fig 1; while the wings, $w m$ and $i c$, are continuations of the rails e and r. The wings give support to the treads of the wheels in passing over the spaces between the point and w and i, which spaces are left for the passage of the flanges.

The channel is called the **mouth** of the frog at a, Figs 22 and 23; and its **throat** at the narrowest part, $w i$. That part of the tongue back of u, Fig 23, or between u and g, is called its **heel.**

The channel is made about 2 ins deep to prevent the flanges from touching its bottom.

The projections, $t t$, Fig 22, are for bolting the frog to the wooden cross ties.

Although one side of the frog forms a part of the turnout curve, its shortness warrants us in making both sides, $j o$, $s t$, Fig 23, straight.

Art. 24. Guide-rails, or guard-rails, $g g'$, Fig 1. Suppose wheels to be rolling from A toward B, Fig 1, on the main track; the switch-rails being in the dotted positions. On arriving opposite the frog, some irregularity of motion might cause the flanges of the wheels running along the rail, $r z$, to press laterally against said rail. Consequently, after passing the throat, $w i$, Fig 22, they would press against the wing, $i c$; and passing between c and P, they would leave the track: or strike the sharp end of P, breaking it, and endangering the train. To prevent this, the guard-rail, g, Fig 1, is placed so near the rail, $b h$ (say 1¾ to 2 ins from it), that the flanges at $b h$, while passing between it and g, prevent those at the opposite rail from pressing against the wing, $i c$, Fig 22, and from striking the point; and guide them safely along their proper channel, $i m$. Similarly, if wheels be rolling from A toward D, Fig 1 (the switch-rails being in the positions, $q m$, $p s$), the centrifugal force due to the curve would cause the flanges to press against the rail, $e x$, and against the wing, $w m$, Fig 22, thus rendering the train liable to the same kind of accident as in the preceding case. This is prevented, in the same manner as before, by the guard-rail, g', Fig 1, which keeps the flanges in their proper channel, $w c$, Fig 22.

The narrow flange-way between the guard-rail, g, Fig 1, and the rail, $b h$, **should extend** at least a foot each way from a point directly opposite the point, f, Fig 23, of the frog. In a distance of at least about 2 ft more at each of its ends the guard-rail should flare out to about 3 ins from the rail, $b h$, so as to guide the flanges into the narrow channel. The same with g'.

Guard-rails have to resist a strong side pressure, and should be very firmly secured to the wooden cross-ties. This is usually done by bolting against them two or more stout blocks of wood, or of cast-iron, which, in turn, are bolted to the ties.

Art. 25. The cast-iron frog, as first made, had **no provision for fastening it to the rails;** but was simply bolted to the cross-ties. It was afterwards provided with a recess at each end, of the exact shape and size of the end of the rail. The rail ends were inserted into these recesses, and the frog was thus kept in line with the rail. In frogs made of rails, the same purpose is served by fish- or angle-plates, p 764, by which the ends of the frog are secured to those of the rails.

Art. 26. The length, a g, Fig 23, of a cast-iron frog, usually varies from 4 to 8 ft; and depends upon the angle, $o f t$, at which the rails, $e x$ and $r z$, Figs 1 and 23, cross each other. This is called the **frog-angle.** This angle may be expressed either in degs and mins, or in the number of times the width of the tongue on any line, as $o t$, Fig 23, is contained in the distance, $g f$, from the point, f, to the center, g, of that line. This number is called the **frog number.** Thus, if the angle, $o f t$, Fig 23, is such that the length, $g f$, is 3, 4, or 10, &c, times the width, $o t$, the frog is called a No 3, 4, or 10, &c, frog. Fig 23 is a No 3; Fig 22, No 5. Frogs are usually made of Nos 4 to 12; sometimes with half numbers, as 7½, 8½, &c.

Art. 27. Draw two parallel lines, $b b'$, $d d'$, for the top of rail, $e x$, Fig 23, and $h h'$, $k k'$, for that of rail, $r z$; crossing each other at the required angle. Then the intersection, f, of lines $d d'$ and $h h'$ is the theoretical **point of the frog.** As this point would be too narrow and weak for service, it is in practice rounded off where the tongue is about ½ inch wide, as shown. If the frog is to be simply abutted to the rail-ends, $z x$, Fig 23, as in some cast-iron frogs, the length, $f g$, need be only great enough to give a width, $t o$, sufficient to accommodate the rail-ends, z and x, and the heads of the two spikes at v which confine them to the ties. If desired, a portion of the flange of each rail-end may be cut away so that the rail-heads come together; thus diminishing the width necessary for $t o$, and, of course, the distance, $f g$. In the case of cast-iron frogs provided with recesses for holding the rail-ends, as in Art 25, the width, $o t$, and length, $f g$, must of course be greater.

In frogs made of rails, the length must be such that the rail-ends, z and x, Fig 23, are far enough apart to give room for fitting to their inner sides the splice-plates by which they are connected with the frog. Where *angle*-plates are used, this distance must be greater than in the case of *fish*-plates.

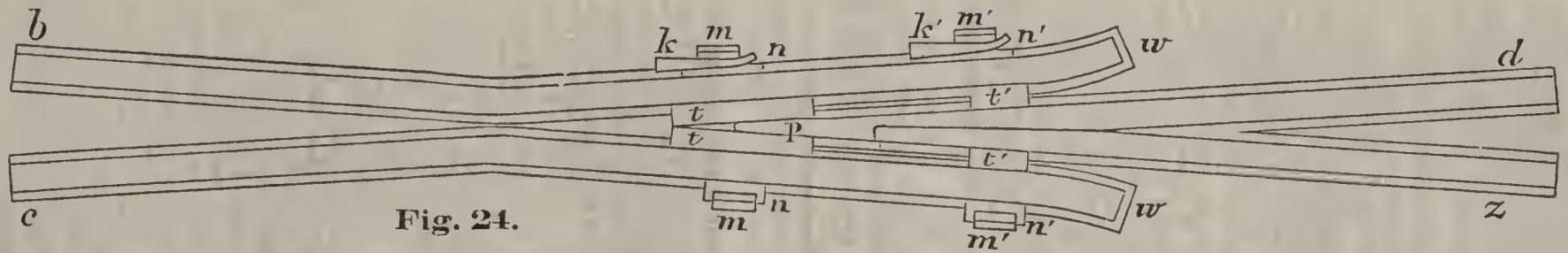


Fig. 24.

Art. 28. Draw $w\,l$, Fig 23, parallel to $f\,o$, and distant from it about 1¾ ins, or barely sufficient to allow the ready passage of the flanges of the wheels. Also, in like manner, draw $i\,p$, parallel to $f\,t$. Make $i\,j$ and $w\,s$ long enough to give a width, $s\,j$, sufficient to accommodate the width of the rail-ends (with or without provision for admitting them to recesses, or for splicing them to the ends of the frog, as the case may be); and to allow the corners at w and i to be rounded off.

Art. 29. Draw $n\,q$ parallel to $i\,p$ and distant from it about an inch more than the width of the widest tires (probably 6¼ ins) used on the road. Then the wing should be made straight at least as far as from w to l. From l to y, the inner edge of the wing should be flared outward, as in the fig, so as to guide wheels entering that end of the frog. In cast-iron frogs this distance, $l\,y$, should be at least 6 ins (more if the length of the frog permits). In frogs made of rails, $l\,y$ is made somewhat greater.

Art. 30. Cast-iron frogs are being superseded by **frogs made of T rails.** While the rails were made of *iron*, such frogs, although used to some extent, were found to wear away too rapidly. With *steel* rails greater hardness is secured.

Art. 31. Our Figs 24 and 27 show **frogs made from steel rails** by the Penna Steel Co, Steelton, Pa. The two rails, $d\,z$, composing the point are shaped by machinery and without being heated, and are fitted and dovetailed together, and then fastened to each other by several rivets passing through their webs. The webs of the four rail-ends, b, c, d, and z, are drilled for fish- or angle-plate splices, p 764, which confine them to the adjoining ends of the track-rails. They thus form, as it were, part of the track, and are therefore less liable than cast-iron frogs to wear loose under the passage of trains. Figs 24 and 27 show the full lengths of the frogs.

Art. 32. Figs 24, 25, and 26 show a **"*keyed* stiff frog,"** in which all the parts are immovable, as in the cast-iron frog, and as distinguished from the spring-rail frog, Fig 27, in which one rail, s, is movable. In Figs 24, 25, and 26 the wing-rails, $w\,w$, and the point, p, are shown bound together by heavy wrought-iron **clamps,** $m\,m$, $m'\,m'$, which enclose their flanges, and in which they are secured, and held at the proper distance apart, by solid iron **throat-pieces,** $t\,t$, $t'\,t'$, **blocks,** $n\,n$, $n'\,n'$, and tapering split **keys** or wedges, k, k'. After the keys are driven firmly home, their split ends are spread open to prevent the keys from falling out. The throat-pieces, t, extend beyond the point, p, thus bracing it against lateral strains. They are, of course, made low enough to clear the flanges of the wheels. The throat-pieces, t, and the brace-blocks, n, are **prevented from sliding longitudinally** from their proper positions by the heads of the rivets shown in Figs 25 and 26. These rivets are driven through holes in the webs of the several rails; and their heads fit

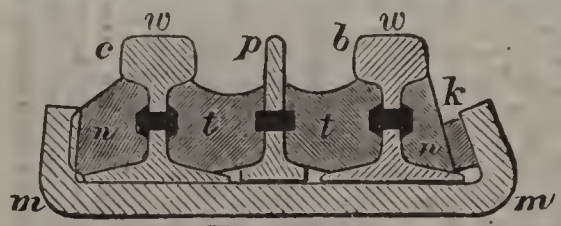


Fig. 25.

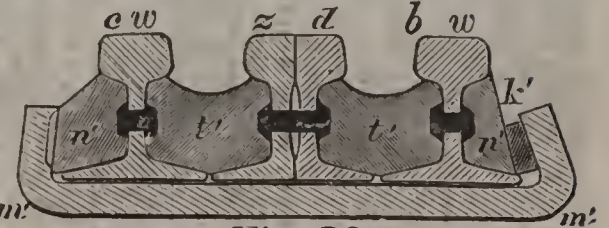


Fig. 26.

nto recesses let into the sides of the throat-pieces and blocks. **The clamps re prevented from sliding** by clips riveted on the flanges of the rails. reat care is taken to have all the **adjoining surfaces in full contact vith each other,** throughout, so as to diminish the liability to wear.

Art. 33. The Co also make stiff frogs in which the parts are held together by **olts** passing through the rails and the throat-pieces; and others in which there re no throat-pieces, and in which the four rails forming the frog are riveted to a rought-iron **plate.** (A spring-rail frog, of the plate pattern, is shown in Fig 27, . 784.) All of these frogs can be made of any desired length and to any desired ngle. **The standard length** of stiff frogs from *b c* to *d z*, for Nos 4 to 8, nclusive, is 8 ft; Nos 9 and 10, 9 ft; Nos 11 and 12, 10 ft; No 15, 12 ft. **Prices,** 886, $18 to $25, according to weight of rails and the length of frog.

Art. 34. Figs 26 A and 26 B show the **Mansfield patent elastic frog,** nade by the Mansfield Elastic Frog Co, New Haven, Conn. These frogs have been

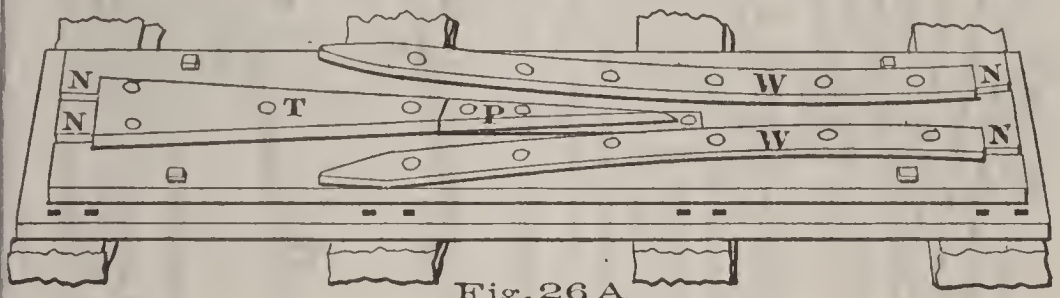


Fig. 26 A

Fig. 26 B

argely used, with good results. Fig 26 B is a vert longitudinal section through the enter of the frog. Elasticity is given by two **creosoted white-oak planks,**) O. Formerly india-rubber was used in place of these. The lower plank is 1½ ns thick; the upper one about 2 ins, depending upon the ht of the rail used. I I I re three rolled-iron plates, from ⅜ to ½ inch thick. The point P, tongue T, and vings W W, are of hammered cast-steel; and their tops are 1¾ ins above the sur- ace of the upper iron plate, to allow the wheel flanges to clear the latter. These arious parts are fastened together by countersunk bolts of Ulster iron, as shown. t will be noticed that the point is secured laterally by being let into the upper iron late and upper oak plank, so as to rest on the second iron plate. The ends of the rog are notched, as shown at N N, Fig 26 A, to receive the webs and flanges of the rack-rail. **Price,** 1886, $40 for medium size. The Mansfield Co make also an lastic steel-rail frog, spring-rail frogs, crossing frogs, safety switches, etc.

Art. 35. The object in reducing the width of the channels or some distance each way from the point, *f*, Fig 23, so as barely to admit the langes freely, is to allow the treads of the wheels to have as much bearing as pos- ible upon the wings and tongue while moving over the broadest part of the chan- nel near *f*. In frogs shorter than about No 4, it is difficult to secure sufficient bearing for the treads, even with the utmost allowable contraction of the channel, when the width of the tires is, as usual, about 6 ins. In the earliest frogs this diffi- culty was partially overcome by gradually **raising the bottom of the channel** between the point and wings, so that the wheels, in traversing that part, ran upon their *flanges* instead of upon their *treads*. The jar occasioned by the tread, in striking against wing and point, was thus avoided. This arrangement is still used in *crossings*, where the tracks cross at a very obtuse angle, and where, consequently, the wings can give little or no support to the treads. The flanges, however, soon cut gutters in the bottoms of the channels, and thus increase their depths, so that the treads strike the wings and point, as in the ordinary frog.

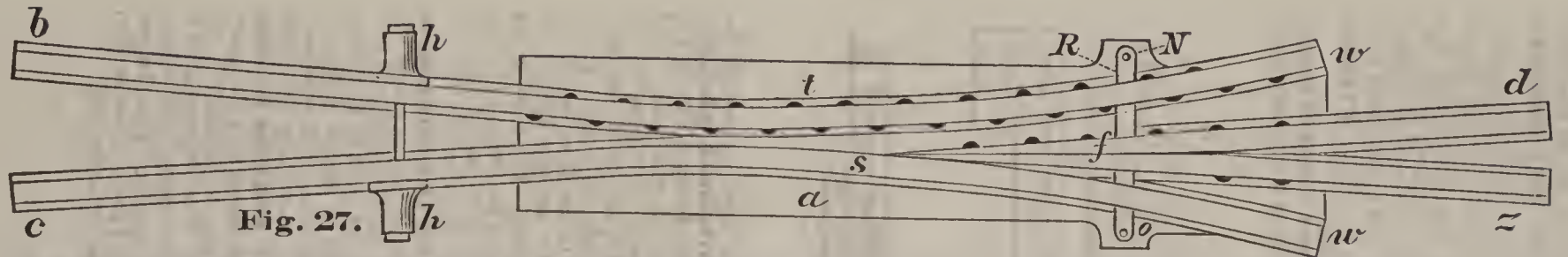


Fig. 27.

Art. 36. Fig 27 represents the Penna Steel Co's **spring-rail frog;** and Fig 28 illustrates its operation. (For convenience of illustration we show the *plate* pattern, although the *keyed* pattern has decided advantages, and is generally preferred.) The rail, *t*, of the turnout is riveted to the wrought-iron plate, *a*, Fig 27, as is also the point; the two being fixed at the proper distance (about 1¾ ins) apart, to allow the passage of the flange of wheel G. The **rail,** *s*, of the main track, called the **spring-rail,** is held in place only by the splice-plate at *c* (which confines its end to that of the adjoining main-track rail, *r c*, Fig 28); and by the **springs,** *h h*, which keep its head pressed against that of the point-rail while no trains are passing to or from the *turnout*. The flange of the spring-rail, on the side next to the point-rail, is removed in order to allow the two heads to come together. The treads of wheels, as G, moving in *either* direction on the *main* track, have thus at all times a full bearing throughout the frog, and no throat to cross; but those, as J and L, running on the rail, *e x*, of the *turnout*, and passing the frog in either direction, must cross the throat between rails *f* and *t*, Fig 28, as in a stiff frog.

In order that such wheels may pass the frog, rail *s* must evidently be moved from its position in contact with rail *f*, in order that the wheel-flanges may pass between the two. This is done as follows:

The flange of a wheel, L, moving from *x* to *e*, presses in between the point and the spring-rail, and thus pushes the latter aside.

When a wheel, J, moving from *e* to *x*, approaches the frog, the guard-rail, *g'*, pressing against wheel K, causes the opposite wheel, J, to press against the spring-rail, *s*, and push it away from the point.

In either case the springs, *h h*, after the passage of each wheel, restore the spring-rail, *s*, to its position, close to the point *f*, as in the figs. The bar, R, Fig 27, passes through narrow slots cut in the webs of all the rails, and is held in place by the bolts N and O. This bar prevents its end of the spring-rail from rising when the weight on said rail is concentrated near its other end, *c*; and the bolt, O, prevents it from moving out too far.

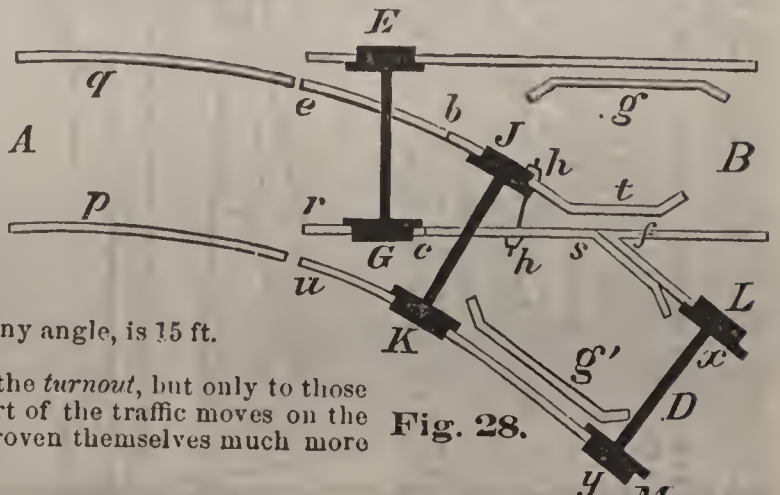


Fig. 28.

The standard length for either style of **spring-rail frog,** and for any angle, is 15 ft.

Prices, 1886, **Plate** pattern, Fig 27, $27 to $34. **Keyed** pattern, $26 to $32.

Since spring-rail frogs do not present a full bearing to wheels entering or leaving the *turnout*, but only to those passing to and fro on the *main track*, they are most useful where the greater part of the traffic moves on the main line, and where but few trains use the turnout. In such positions they have proven themselves much more durable than the stiff-rail frogs.

Art. 37. In Wood's self-acting frog, which was largely and successfully used for many years on the Camden & Amboy R R, under heavy trains moving at high speeds, and which, like those just described, was made entirely of rails; the point, *f*, Fig 28, was firmly fixed to the cross-ties; and *both* of the wing-rails, *t* and *s*, while rigidly fixed at a proper distance *from each other*, were free to move, as a whole, about their heels, *b* and *c*, so that either *t* or *s* could be brought into contact with the point, *f*. It thus provided an unbroken bearing to trains on the *turnout*, as well as to those on the main track. There was no spring; and the frog remained as it was set by the first pair of wheels of a train, until another train, using the other track, set it over the other way.

If this frog is set as in Fig 28, and an engine moves from *e* toward *x* (so that the wings are moved by means of the guard-rail, *g'*, and the wheel at the opposite track, *e x*), the motion of the wings has to take place while the weight of the engine itself is resting partly on the wing-rail, *e t*; and, consequently, their sliding transversely of the track is attended with great friction. The same effect is produced if *t* is in contact with *f*, and an engine moves from A to B. This, however, did not, in practice, seem to interfere with the efficiency of the frog.

Art. 38. In ordering frogs, give the frog angle or number, and the exact cross-section of the rail used on the road.

For spring-rail frogs, specify also whether the turnout is to the right or left hand. In the case of *stiff* frogs, this is not necessary.

Art. 39. The laying-out of Turnouts.

The words *heel* and *toe* are used in this article *with reference to the common or stub switch*, Fig 4, in which the *heels* are at *q* and *p*, and the *toes* at *m* and *s*. In the Wharton, Lorenz, and some other switches, the positions of heel and toe will be seen to be the reverse of this.

The formulas given in our early editions for finding frog dist, rad of turnout, &c, were based upon the old practice of regarding the straight switch-rails, *q m*, *p s*, Fig 4, as forming a tangent to the turnout curve, which last was considered as beginning at the *toes*, *m* and *s*, of the switch-rails. **The modern practice** is to curve the switch-rails so as to form a part of the turnout curve; the latter being supposed to begin at the *heels*, *q* and *p*, of the switch. This view of the case admits of simpler formulas.

In each of the Figs 29, 30, and 31, *w p z* represents the main track. The **frog distance,** *p f*, is a straight line drawn from the theoretical *point of frog*, *f*, to the heel, *p*, of that switch-rail which, when opened, forms the *inner* rail of the *turnout*. Formerly, when the turnout curve was taken as starting at the *toe* of the switch, the frog dist was a straight line from the theoretical point of frog to the toe, *m*, Fig 4, of the outer switch-rail, *q m*, when opened.

As already remarked, frogs are usually made of Nos 4 to 12; sometimes with half numbers; and the turnout radii, &c, are made to conform to them.

Scrupulous accuracy is not necessary in these matters. Thus, a deviation, either way, of say 3 per cent in the length of the turnout radius from that given by the table or the formulas, will be almost inappreciable. So too, if a frog number should be used, intermediate of those in the first column of the table, the other dimensions may be found, approximately enough, by using quantities similarly intermediate. A rail almost always has to be cut in two in order to fill up the frog dist; and the exact length of the piece can be found by actual measurement at the time of cutting it.

Rem. When the turnout leaves a straight track, as in Fig 29, the **frog angle** is equal to the central angle, *f c o*. When the main track is curved, and the turnout curves in the **opposite** direction (Fig 30), it is equal to the **sum** (*v f u*) of the central angles, *f c o*, *f n o*; and when the two curve in **the same** direction (Fig 31), it is equal to the **diff** (*n f c*) of the central angles, *f c o*, *f n o*.

Art. 40. To lay out a turnout, *p x*, Fig 29, from a straight track, *p z*. From the column of radii in the table below, select one, *c o*, suitable for the turnout; together with the corresponding frog number, frog dist, *p f*, and switch length. Place the frog so that the main-line side of its tongue shall be at *f z*, *precisely* in line with the inner edge of the rail, *w z*, and its theoretical point, *f*, at the tabular frog dist, *p f*, from the starting-point, *p*. Stretch a string from *q* (opposite *p*) to *f*; and from it lay off the three ordinates from the table; thus finding three points (in addition to *q* and *f*) in the outer curve. Do not, however, drive stakes at these points; but as each of them is found, measure off from it, inward, half the gauge of the track; and *there* drive stakes. Do the same from *q* and *f*. The five stakes will all then be in the dotted center line of the turnout, Fig 29; and will serve as guides to the work, without being liable to be displaced. The dimensions in the table below are found by **the following formulas,** the main track being straight:

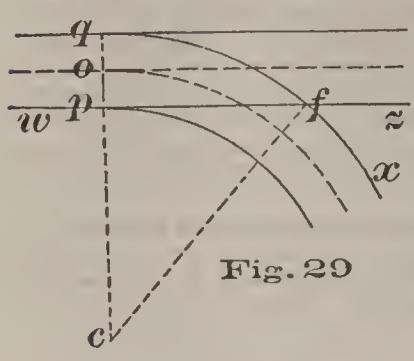


Fig. 29

Tangent of **half frog angle** = Gauge ÷ Frog dist.

Frog No..... = $\sqrt{}$ Radius *c o* ÷ Twice the gauge.
Or, **Frog No**..... = Half the cotangent of half the frog angle.
Radius c o..... = Twice the gauge × Square of frog number.
Or, **Radius c o**..... = (Frog dist *p f* ÷ Sine of frog angle) — half the gauge.
Or, **Radius c o**..... = (Gauge ÷ Versed sine of frog angle) — half the gauge.
Frog dist p f..... = Frog number × Twice the gauge.
Or, **Frog dist p f**..... = Gauge *p q* ÷ Tangent of half the frog angle.
Or, **Frog dist p f**..... = (Rad *c o* + half the gauge) × Sine of frog angle.
Middle ord..... = ¼ gauge, approx enough.
Each side ord... = ¾ mid ord = $\frac{3}{16}$ (or .188 of the) gauge, approx enough.

Switch Length = approx enough $\sqrt{\dfrac{\text{Throw in ft} \times 10000}{\text{Tangential dist for chords of 100 ft, for rad } c\,o \text{ of turnout curve. See table, p 726 to 728.}}}$

TABLE OF TURNOUTS FROM A STRAIGHT TRACK. Fig 29.

Gauge 4 ft 8½ ins. Throw of switch 5 ins.

For any other gauge, the *frog angle* for any given frog number remains the same as in the table. The other items may be taken, approx enough, to vary directly as the gauge.

Frog Number	Frog Angle	Turnout Radius c o	Deflang of Turnout Curve	Frog Dist p f	Middle Ordinate	Side Ords	Stub Switch Length
	° ′	Feet.	° ′	Feet.	Feet.	Feet.	Feet.
12	4 46	1356	4 14	113.0	1.177	.883	34
11½	4 58	1245	4 36	108.3	1.177	.883	32
11	5 12	1139	5 2	103.6	1.177	.883	31
10½	5 28	1038	5 31	98.9	1.177	.883	29
10	5 44	942	6 5	94.2	1.177	.883	28
9½	6 2	850	6 45	89.5	1.177	.883	27
9	6 22	763	7 31	84.7	1.177	.883	25
8½	6 44	680	8 26	80.0	1.177	.883	24
8	7 10	603	9 31	75.3	1.177	.883	22
7½	7 38	530	10 50	70.6	1.177	.883	21
7	8 10	461	12 27	65.9	1.177	.883	20
6½	8 48	398	14 26	61.2	1.177	.883	18
6	9 32	339	16 58	56.5	1.177	.883	17
5½	10 24	285	20 13	51.8	1.177	.883	15
5	11 26	235	24 32	47.1	1.177	.883	14
4½	12 40	191	30 24	42.4	1.177	.883	13
4	14 14	151	38 46	37.7	1.177	.883	11

Remark. The switch lengths in the Table merely denote the **shortest** length of **Stub** switch that will at the same time form part of the turnout curve, and give 5 ins throw. **Pointed,** or **split-rail** switches, like the Lorenz, &c, require only half this throw. In practice all kinds are frequently made much shorter than the table requires, thereby sharpening the beginning of the curve.

Art. 41. To lay out a turnout from a curved main track.

There are two Cases:

Case 1, Fig 30; when the two curves deflect in **opposite** directions.

Case 2. Fig 31; when the two curves deflect in **the same** direction.

Having determined approx upon a radius for the turnout curve, take from the table, p 726, its corresponding *deflection* angle, and that for the main curve. *In Case* 1, find the *sum* of these two angles. *In Case* 2, find their *difference.* In the table, p 786, find the *deflection* angle (not the frog angle) nearest to the sum or diff just found. The **frog number, switch length** and **frog distance** *p f*, in the table, opposite the deflection angle thus selected, are the proper ones for the turnout. *Theoretically* we should, in Case 1, *add* to the tabular frog distance *p f* about half an inch per 100 feet for each degree of deflection angle of the easier of the two curves; and, in Case 2, *deduct* it; but this refinement is unnecessary.

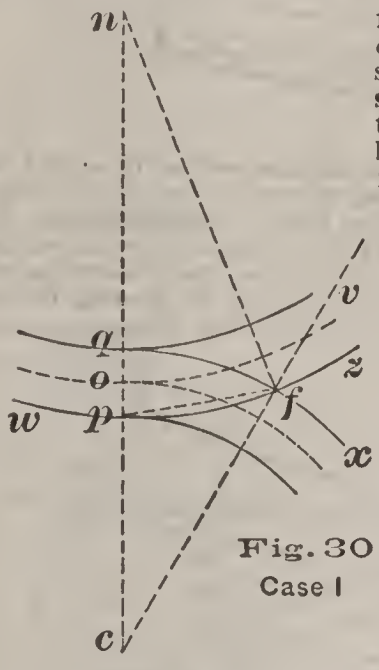


Fig. 30
Case I

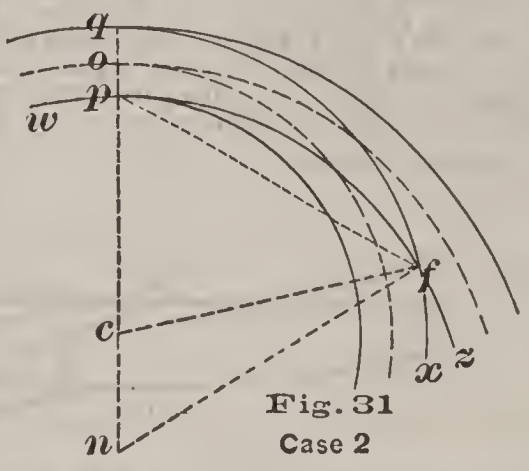


Fig. 31
Case 2

Deflection angle of turnout curve

(in Case 1) = Tabular deflection angle (p 786) — deflection angle of main curve,

(in Case 2) = Tabular deflection angle (p 786) + deflection angle of main curve.

For **radius** *c o* **of turnout curve,** having its deflection angle, see table, p 726.

Ex. Rad of main curve, 2865 ft. Rad selected approx for turnout, 716.8 ft. Here the defl angles (table, p 726) are, respectively, 2° and 8°.

In Case 1; 8° + 2° = 10°. Nearest defl angles in table, p 786, 10° 50′ and 9° 31′.

If we select 10° 50′, we have Frog No, 7½; Switch L'gth, 21 ft; Frog Dist = 70.6 ft + *twice* ½ in = say 70.7 ft; Defl Angle for turnout = 10° 50′ — 2° = 8° 50′; Rad *c o* (table, p 726), 649 ft. If we select 9° 31′, we have Frog No, 8; Switch L'gth, 22 ft; Frog Dist = 75.3 ft + twice ½ in = say 75.4 ft; Defl Angle = 9° 31′ — 2° = 7° 31′; Rad *c o*, say 763 ft.

In Case 2; 8° — 2° = 6°. Tabular defl angles, p 786, 6° 5′ and 5° 31′. 6° 5′ gives Frog No, 10; Switch L'gth, 28 ft; Frog Dist = 94.2 ft — twice ½ in = say 94.1 ft; Defl Angle = 6° 5′ + 2° = 8° 5′; Rad, say 709 ft. 5° 31′ gives Frog No, 10½; Switch L'gth, 29 ft; Frog Dist = 98.9 ft — twice ½ in = say 98.8 ft; Defl Angle = 5° 31′ + 2° = 7° 31′; Rad, say 763 ft.

The frog dist *p f* may also be found thus:

$$\text{Tangent of half } f\,n\,o = \frac{\text{Gauge} \times \text{Frog No}}{\text{Radius } n\,o.}$$

Frog Dist *p f* = *n p* × twice the sine of half *f n o.*

Place the frog with the main-line side of its tongue at *f z*, in line with the inner edge of the frog-rail, *p z*, of the main line, and with its theoretical point, *f*, at the dist, *p f* (found as above), from the heel, *p*, of the *inner* switch-rail. Stretch a string from *f* to the heel, *q*, of the *outer* switch-rail. Measure the dist, *q f*, divide it into four equal parts, and lay off three ordinates, found thus:

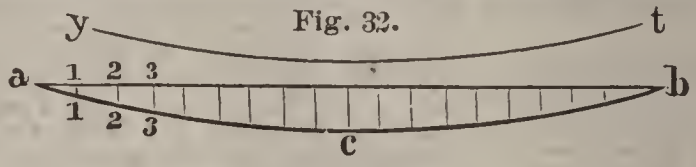
Middle ord = (Square of half *q f*) ÷ twice the rad of turnout curve. Each side ord = three-fourths of middle ord.

These three ords. and the points *q* and *f*, give us 5 points of the outer rail of the turnout curve; and from these we measure, inward, half the gauge, and drive 5 center guide-stakes, as in Art 40.

Art. 42. To find frog dists, &c, by means of a drawing to scale. The frog dist can generally be found near enough for practice, from a drawing on a scale of about ¼ or ⅛ inch to a foot. And so in the many cases where turnouts cross tracks in various directions, in and about stations, depots, &c.

Figs 32 and 33 are intended merely to furnish a few general hints in regard to

Fig. 32.



such drawings. For instance, the curves of a main track, as well as those of a turnout, generally have radii too large to admit of being drawn on a scale of ¼ inch to a foot, by a pair of dividers or compasses. But they may be managed thus: Draw any straight line, *a b*, Fig 32, to represent by scale a 100-ft chord of the curve, divide it into twenty 5-ft parts, *a* 1, 1 2, 2 3, &c, and lay off by scale the 19 corresponding ordinates, 1 1, 2 2, 3 3, &c, taken from the table on page 730. By joining the ends of these, we obtain the reqd curve, *a c b*, of the main track; and of course can draw

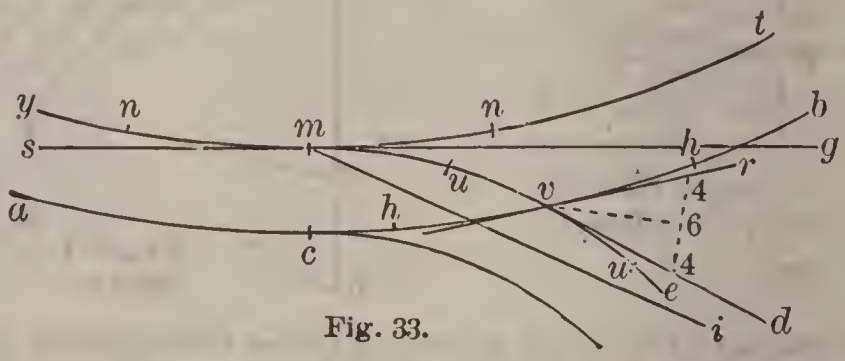


Fig. 33.

the inner line, *y t*, distant from it by scale the width of track, say 4 ft 8½ ins. Now let *a c b* and *y t*, Fig 33, be a curved main track so drawn; and let any point *m* be taken as the starting-point of the turnout, *m v*, &c. On each side of *m* measure off any two equidistant points, *n* and *n*, in the same curve; and through *m* draw *s g*, parallel to *n n*. Then is *m g* a tang to the curve, *y m t*, at *m*. Having determined on the rad of the turnout curve, *m v e*, draw that curve by the same process as before; first laying off the angle, *g m i*, equal to the tangential angle of the curve, taken from the table, p 726. Then, beginning at *m*, lay off 5-feet dists along *m i*; and from them, as in Fig 32, draw the ords corresponding to the turnout curve. Through the ends of these ords draw the curve, *m v e*. itself. Then the frog dist will be the straight dist from c to *v*, and can be measured by the scale, within a few inches; or near enough for practice. The middle ord of the arc, *m v*, cannot be found correctly by so small a scale as ¼ inch to a foot, but should be calculated thus: From the square of the rad take the square of half the chord, *m v*. Take the sq rt of the rem. Subtract this sq rt from the rad. If two other ords should be desired, half way between *m* and *v* and the center one, they may each be taken as ¾ of the center one. Make the switch-rail long enough to leave 2½ ins at its toe between *m n* and *m u*.

The frog angle at *v* will be equal to the angle, *r v d*, formed between the tang, *v r*, to the curve, *a c b*; and the tang, *v d*, to the curve, *m v e*. These tangs are found in the same way as *m g*; namely, for the tang, *v r*, lay off from *v* two equidistant points, *h* and *h*, on the curve, *a c b*; and through *v* draw *v r* parallel to *h h*. Also, for *v d*, lay off from *v* any equidistant points, *u* and *u*, on the curve, *m v e*, and through *v* draw *v d* parallel to them. This angle may be measured by a protractor. Or, if on the two tangs we make *v* 4 and *v* 4 equal to each other, and draw the dotted line 4 4; and from its center at 6 draw 6 *v*; then 6 *v* divided by 4 4 will give the No of the frog. With care, and a little ingenuity, the young student will be able, by similar processes, to solve graphically any turnout case that may present itself. The method by a drawing has great advantages over the tedious and complicated calculations which otherwise become necessary in cases where curved and straight tracks intersect each other in various directions. The drawing serves as a check against serious errors, which would be detected at once by eye. None of the graphical measurements will be strictly accurate; but with care, none of the errors need be of practical importance. The ordinates for bending rails so as to suit turnout curves can be found from the table, p 761. All of Art 42 may be done on the ground.

Art. 43. An experienced track-layer, with a good eye, can place his own guide-stakes by trial on the ground; and by them lay his turnouts with an accuracy as practically useful as the most scrupulous calculations of the engineer can secure.

The following example, Fig 34, of a turnout from a straight track, Y Z, exhibits a common case, in which all the work may be performed on the ground, without pre-

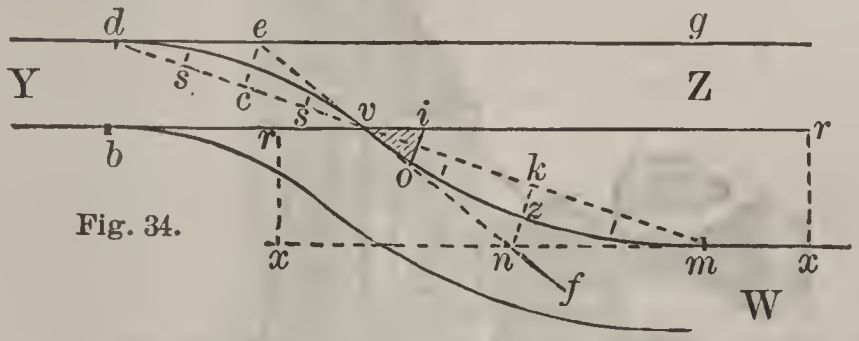


Fig. 34.

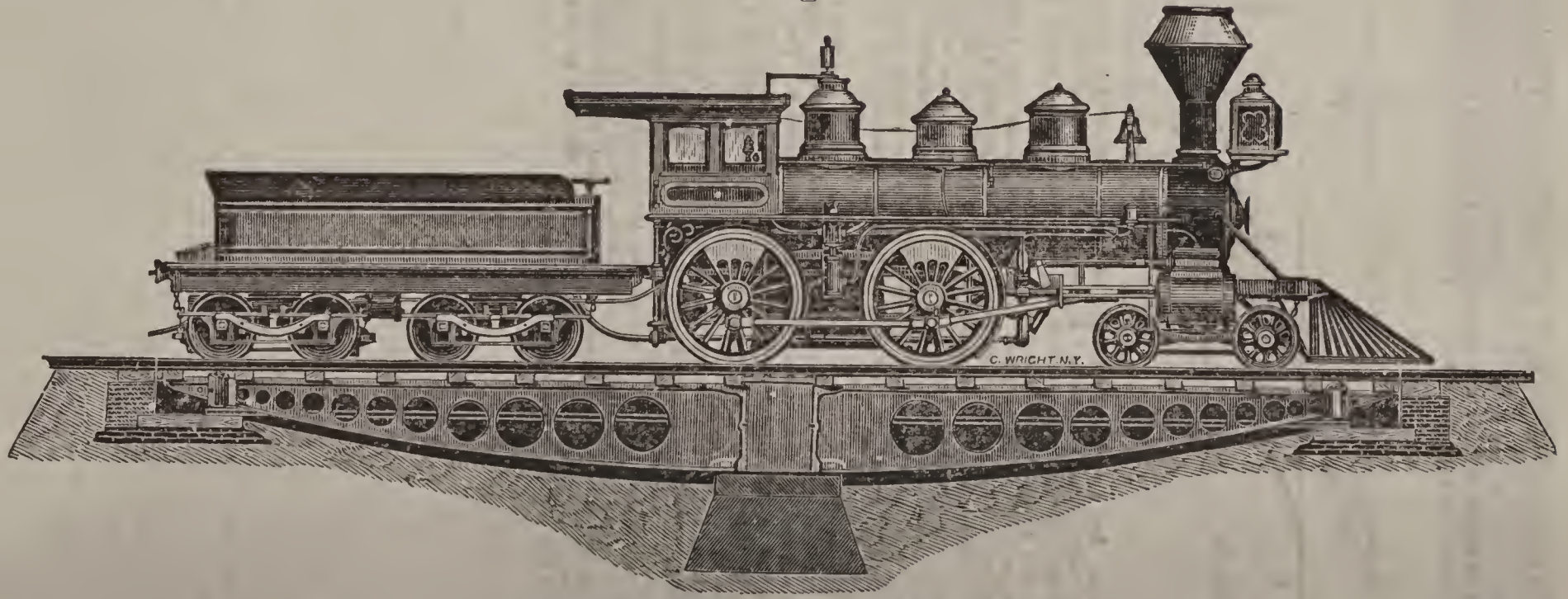
vious calculation. Let *i v o* be the tongue of a frog, with which the assistant has been directed to make a turnout from Y Z; and that he has received no instructions more than that the turnout must start at *d*, and terminate in a track, W, to be laid parallel to Y Z, and distant from it *r x* or *r x*, equal to 6 ft.

Place the tongue of the frog by guess near where it must come, having its edge, *v i*, *precisely* in line with the inner or flange edge of the rail, *b r*. Then stretch a piece of twine along the edge, *o v*, of the frog, and extending to *d g*. Try by measure whether *v e* is then equal to *e d*; and if it is not, move the frog along the line, *b r*, until those two dists become equal. Then is *v* the proper place for the point of the frog; *b v* is the frog dist; one-half of *c e* is the length of the middle ord of the turnout curve, *d v*; and if two intermediate ords are needed at *s* and *s*, each of them will be ¾ of said middle one.

The frog being now placed, proceed thus: Place two stakes and tacks, *x* and *x*, at the reqd inter-track dist, *r x* and *r x*, of 6 ft from the rails, *b r*. Then range by pieces of twine *x x* and *v f*, to find the point, *n*, of intersection. Then measure *n v*, and make *n m* equal to it. Then is *m* the end of the reverse curve, *v m*, of the turnout. The ords of this curve may be found as before; one-half of *n k* being the middle one, &c.

Rem. It may frequently be of use to remember that in any arc, as *v m*, of a circle; *v n* and *m n* being tangs from the ends of the arc; one-half of the dist, *k n*, is the middle ord, *k z*, of the curve; near enough for most practical purposes, *whenever the length of the chord*, v m, *of the arc is not greater than one-half the rad of the circle of which the arc is a part.* Or, within the same limit, vice versa, if we make *k n* equal to twice *k z*, then will *n* be very approximately the point at which two tangs from the ends of the arc will meet. Also, the middle ord of the half arc, *v z* or *z m*, may be taken as ¼ of the middle ord, *k z*, of the whole arc.

Fig. 1.



SELLERS' CAST-IRON TURNTABLE.

BY WILLIAM SELLERS & CO., PHILADA.

TURNTABLES.

Art. 1. A turntable is a platform, usually from 40 to 60 ft long and about 6 to 10 ft wide (see Fig 1,) upon which a locomotive and its tender may be run, and then be turned around hor through any portion of a circle; and thus be transferred from one track to another forming any angle with it. The table is supported by a pivot under its center; and by wheels or rollers under its two ends. Frequently other rollers are added between the center and ends. Beneath the platform is excavated a circular pit about 4 or 5 ft deep, having its circumf lined with a wall of masonry or brick about 2 ft thick, capped with either cut stone or wood. The diam of the pit in clear of this lining is about 2 ins greater than the length of the turntable. The lining is generally built with a step, as seen in Fig 1, for supporting the circular rail on which the end rollers travel; or, instead of this step, a detached support may be used for this circular rail, as at *u*, Fig 11. At the center of the pit is a solid well-founded mass of masonry or timber, for the pivot to rest on, as seen in Fig 1. This, as well as the step for the end rollers, should be very firm, and perfectly level; otherwise the platform will be hard to work. The platform is frequently floored across for a width of 6 to 10 ft to furnish a pathway across the pit, without stepping down into it; especially when under cover of a building. At first they were floored over so as to cover the entire circular pit; but this increased not only their cost but their wt, so as to make them difficult to turn; besides causing much expense for repairs; with greater trouble in making them. It is therefore rarely done at present, except where want of space sometimes renders it necessary in indoor turntables.

For the minimum length of a turntable, add from 1½ to 2 ft to the total wheel base (p 805) of the longest locomotive and tender for which it is to be used; but a turntable should be several feet longer than is necessary for merely allowing the engine and tender to stand on it; for the increased length enables the engine-men to move them a little backward or forward, so as to balance them chiefly upon the central support; and thus relieve the end rollers. By this means the friction while turning is confined as much as possible to the center of motion; and is there-

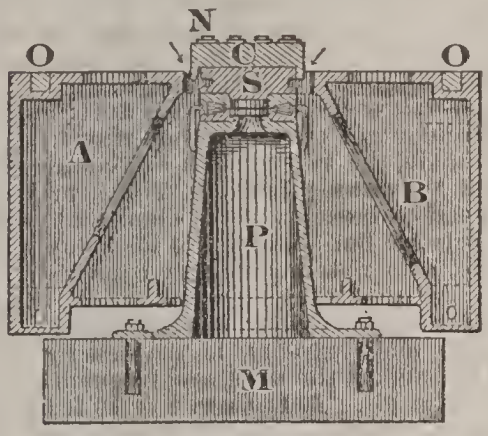


Fig. 2

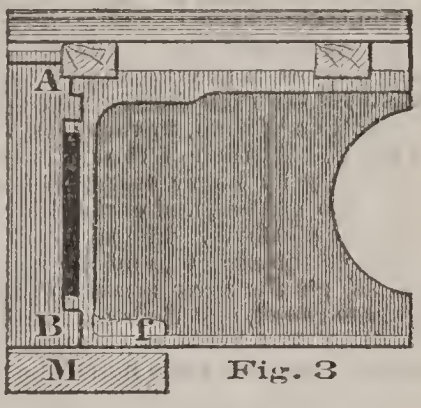


Fig. 3

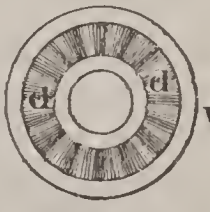


Fig. 5

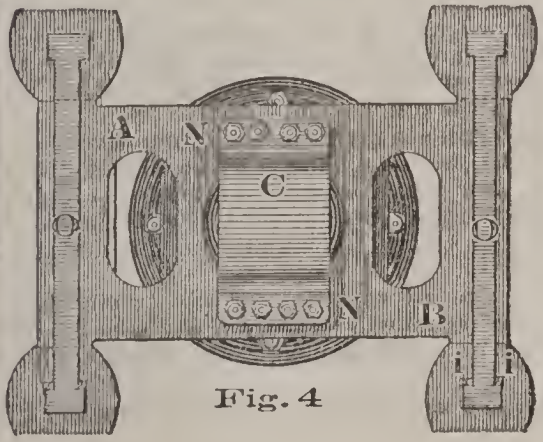


Fig. 4

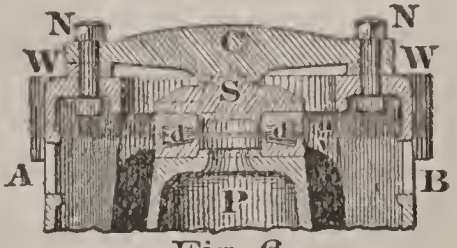


Fig. 6

fore more readily overcome than if it were allowed to act at the circumf. The engine-men soon learn, by feeling, the proper spot for stopping the engine so as thus to balance the platform.

Art. 2. Figs 1 to 6 represent the **Sellers cast-iron turntable** of Wm Sellers & Co., Phila. It consists of two cast-iron girders of about 1¾ ins average thickness, perforated by circular openings to save metal. One of these girders is shown in Fig 1; and parts of one in Figs 3 and 4. Each girder is in two separate pieces, which are fastened, as shown in Figs 1, 3, and 4, to a hollow cast-iron "center-box," A B, Figs 2, 3, 4, and 6, by means of 2¼ inch screw-bolts, at *f*, Fig 3; and by hor bars, *o o*, of rolled iron about 3¾ ins square, fitting into sunk recesses on top of the boxing, and tightened in place by wedges, *i i*, screw-bolted beneath.

Art. 3. The sides of the **center-box** are about 1¾ ins thick. It is suspended from the **steel cap,** C, by 8 screw-bolts 2 ins diam. On its lower side this cap has a semi-cylindrical groove extending across it, transversely of the track, as shown in Fig 6. This groove fits over a corresponding semi-cylindrical ridge on the top of the cast-iron **"socket,"** *s* (so called), on which the cap thus rests. The socket, in turn, rests upon the upper one, *u*, of two annular steel plates, *u* and *v*, which form a **circular box** containing 15 **steel conical anti-friction rollers,** *d d*, Figs 2, 5, and 6. These are about 3 ins in length, and in greatest diam. They have no axles, but merely lie loosely in the lower part, *v*, of the circular box; filling its circumf with the exception of about ½ inch left for play. In the direction of their axes they have ⅛ inch play. The lid, *u*, of the circular box, rests upon the tops of the rollers, which separate it from *v* by about ½ inch. *v* rests upon the top of the hollow **cast-iron post,** P, which, by means of its flanges, is bolted to the cap-stone, M, of the **foundation pier.**

In order to insure a perfect bearing of the revolving surfaces upon each other, and thus diminish the liability to abrasion, the rollers, *d d*, and the insides of the box in contact with them, are accurately finished, as are also the top and bottom of the roller-box, and the surfaces of the socket, *s*, and post, P, in contact with them. The rollers are oiled by means of the spaces shown by the arrows in Fig 2.

Art. 4. Adjustment of the height of the table. By turning the nuts, N N, of the 8 screw-bolts which support the table, the latter may be raised or lowered 1 or 2 ins; the cap, C, socket, *s*, and roller-box, *u v*, remaining stationary on top of the post, P. All turntables should have the means of making such adjustment. Before the nuts, N N, are finally tightened up, the **blocks, w w, of hard wood,** cut to the proper thickness for the desired ht of the table, are inserted between the cap, C, and the top of the center-box, A B, as shown.

The ht of the table should be such that each of the wheels at its outer ends shall be ¼ inch, in the clear, from the circular rail on which they travel.

Art. 5. At each of the outer ends of the table, the two girders are connected transversely by heavy cast-iron beams, called **"cross-girts."** These project beyond the girders, and carry the cast-iron **end-wheels,** 20 ins diam, 2 at each end of the platform. The treads of these wheels are but about 3 or 4 ins below the bottoms of the girders, and the wheels therefore do not require any considerable depth of pit for their accommodation. In order that they may roll freely, their treads are coned, and their axles are made radial to the circular turntable pit. Intermediate transverse connection between the main girders, is secured by the **wooden cross-ties** notched upon them to support the rails, and frequently 10 or 12 ft long, for giving a wide footway across the pit. **A lever,** 8 or 10 ft long, fitting into a staple, is used for turning the table, not on account of friction, but as a handle for the workmen.

Art. 6. The semi-cylindrical shape of the joint between the cap, C, and the socket, *s*, permits the slight longitudinal rocking motion of the turntable which takes place when a locomotive comes upon it or leaves it; but prevents it from tipping sideways, as it was apt to do, when, as formerly, the cap rested directly upon the lid *u* of the roller-box, and the top of the post P was hemispherical, forming a ball-and-socket joint with a casting upon which the roller-box rested.

Art. 7. Ten **sizes of these turntables** are furnished for locomotive use. The **diameters** are as follows: 70 feet, 65 feet, 60 feet, 56 feet, 54 feet, 50 feet (two patterns), 45 feet, 40 feet, 30 feet and 12 feet. For prices, weights, dimensions of pit and foundations, timber required, etc; address William Sellers & Co, 1600 Hamilton St., Philadelphia. Machinery for turning, being considered unnecessary, is not attached, unless specially ordered. Its cost is extra.

The entire cost of excavating and lining the pit; foundation for pivot; circular rail for end rollers, &c, complete for a 56-ft turntable will vary from $1200 to $2500 in addition, depending on the class of materials and workmanship; and whether the bottom of the pit is paved or not.

Art. 8. The girders of turntables are now very generally made of

olled-iron plates, each girder being in one piece; and the two main girders e then connected with each other, near their centers, by two **cross-girders** I or channel beams, or of plate-iron, one on each side of the central pivot. These oss-girders, and the sides of the main girders between them, form a sort of rectan-lar box, corresponding to the cast-iron center-box of the Sellers table, Arts 2 and The arrangement of the center bearing apparatus, and the manner in which it connected with the cross-girders, differ in the tables of the several makers.

Art. 9. In the plate-iron turntables made by the **Edge Moor** Iron Co, Wil-ngton, Del, the cross-girders, G G, Fig 8, are of plate-iron; and, at their ends, have nges. F F, of angle-iron, by which they are riveted to the main-girders, E. In the -ft tables the cross-girders are 26 ins apart in the clear.

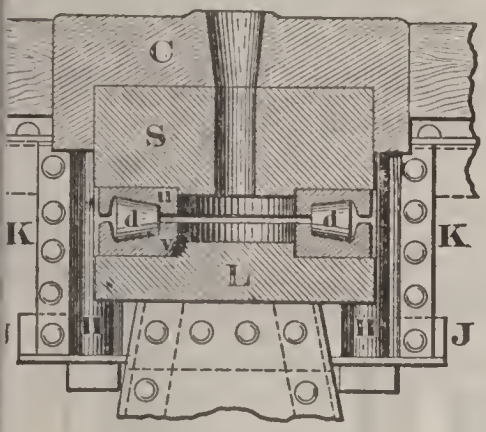


Fig. 7

Enlarged transverse section of roller-box &c, on center line of Fig 8.

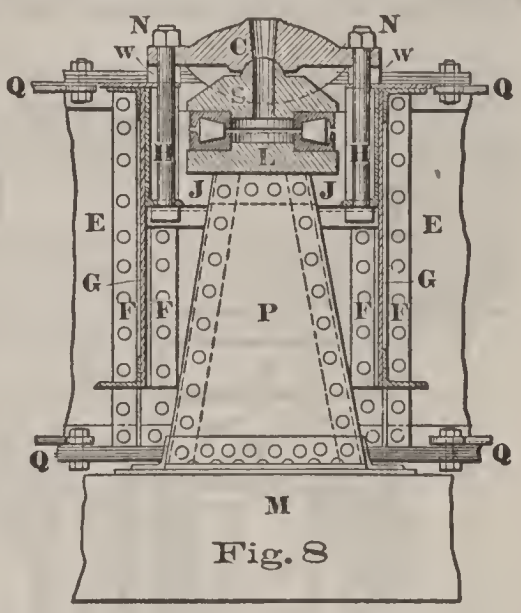


Fig. 8

Art. 10. The bolts, H, by which the table is suspended from the cap, C, are in number, and are arranged in two rows of three bolts each, one row being on ch side of the post, P, and between it and one end of the turntable. To each cross-der is riveted a short hor bar of angle-iron, J J. Through the hor flange of each these two bars pass the 3 bolts, H, of one row. The hor flange, bearing all the of its girder and load, rests upon the heads of these 3 bolts. To prevent the nge from yielding under its load, vert struts of angle-iron are riveted to the web the cross-girder between the bolts. Two of these struts are shown at K K, Fig 7. eir ends abut against the upper flange of the cross-girder, and against J J. The 6 bolts, H H, pass up through the flanges of the cap, C (three bolts through ch flange), and their nuts rest upon its top.

Art. 11. The cap, C, **is held in place** on the socket, *s*, by means of nges which extend down from it on both sides, as shown in Fig 7.

The **rollers,** *d d*, and the roller-box, *u v*, are those made by Wm. Sellers & Co, hila, Art 3.

The ht of the table may be adjusted, within a range of 2 or 3 ins, by eans of the nuts, N N, as in the Sellers table, Art 4.

The lower part, *v*, of the roller-box, instead of resting directly upon the post, P, in the Sellers table, Art 3, rests upon an iron casting, L, which, in turn, rests upon e post, P, and is held in place upon it by a lug which projects down into it.

The post is built up of plate- and angle-irons riveted together, and may be a hol-w truncated square pyramid, as shown, or of other shapes. Those presenting the ape of a cross in hor section have the advantage of being accessible for painting d inspection.

Art. 12. The main girders are braced by hor diag rods, whose ends e shown at Q, Fig 8. These rods are provided with sleeve-swivel turn-buckles, by hich they may be tightened or loosened.

The remarks in Art 5 on the **end wheels** of the Sellers table apply also to ose of the Edge Moor, except that in the latter the wheels are 25 ins in diam, and e cross-girts which carry them are of angle-iron. The wheels are fastened to their les, which turn in brass bearings enclosed in cast-iron journal-boxes. Each box provided with means for raising or lowering it; and the brasses are rounded on p, so as to insure a uniform bearing, no matter what inclination may be given to e axle by the vert adjustment of the journal-boxes. The brasses may be readily

replaced when worn out. The lower part of the journal-box is filled with oil an waste for lubrication. The several parts are made large, in order to lessen the fri tion per square inch, and to withstand the shocks to which they are subjected.

The Edge Moor tables are made 50, 54, 56, and 60 ft long, and are designed to turn the heaviest "Consolidation" locomotive and tender without subjecting any part of the iron work of the table to a greater tensile or compressive strain than 10000 lbs per square inch.

Art. 13. The approximate **weights and prices of Edge Moor turntables** are given on the accompanying table.

Length.	Finished wt.	Price, 1886, on cars at Edge Moor.
50 ft	18900 lbs	$1100
54 ft	22600 lbs	$1450
56 ft	24300 lbs	$1500
60 ft	26400 lbs	$1600

Average cost, 1886, say 6 cts per lb.

Art. 14. Fig 9 shows the center bearing arrangement of the turntable patente and furnished by **Mr. C. O. H. Fritzsche,** 71 Broadway, New York. The lef side of the fig is in section; the right side is in elevation.

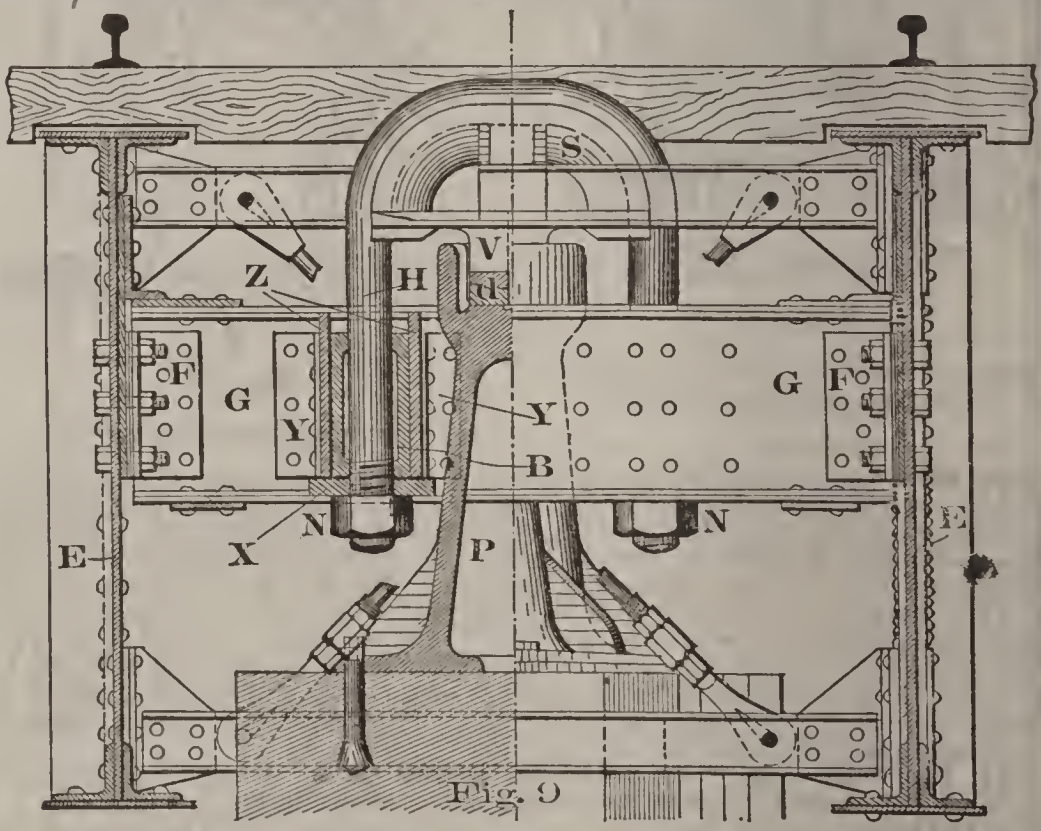


Fig. 9

Here each **cross-girder,** G, is a heavy rolled-iron I beam, to each end of which are riveted flanges, F, of angle-iron. These flanges are *bolted* to the webs of the main girders, E, in order that iron "**adjusting-plates**" of any required thickness may be inserted between either angle-flange and the web of the main girder which it supports. This permits a transverse adjustment of the table, so that the center of gravity of its transverse cross-section may be brought immediately over the vert axis of the post, P.

Art. 15. Four rolled-iron plates, of which two (or a *pair*) are shown at Z Z, extend from one cross-girder to the other, and are fastened to the webs of the latter by angle-pieces, Y Y. The lower edges of these plates rest upon rolled-iron washers, X. Each washer supports two of the four plates, Z Z, and rests, in turn, upon one of the nuts, N N, of the single inverted U-bolt, H. The upper part of this U-bolt rests in a cast-iron saddle, S; the neck, V, of which enters the top of the cast-iron post, P, and rests upon the upper one of two steel discs, *d*, which, finally, rest on the bottom of the cylindrical cavity in the top of the post. **The object in the use of the single U-bolt** is to reduce, as far as possible, the number of points of support, and thus to reduce, also, the uncertainty as to the proportion of the total load sustained by each. Its shape, and the manner in which it rests in its saddle, S, render the table a rigid mass revolving on the center post.

Art. 16. Oil is applied through openings in the side of the saddle-casting, S. These openings communicate with a vert hole through its center, and thus with a similar hole through each disc. The two faces of the discs in contact with each other, form a segment of a sphere; and each face has three radial gutters, extending from its cen to its circumf. Channels are cut in the sides, and around the lower edge, of the cylindrical cavity in the top of the post. Thus all the parts which revolve upon one another can be kept bathed in oil.

Art. 17. Each leg of the U-bolt passes through, and is held in place by, a **cast-iron box,** B. These boxes are held in position by the edges of the four *inner* angles (corresponding to Y Y, but not shown in the fig) *between* the plates Z Z.

Sometimes the two washers, X, four plates, Z Z, and the two boxes, B, are omitted, and two **cast-iron boxes** of another shape are substituted in their places, one for each leg of the U-bolt. These boxes are not fastened to the cross-girders, but the upper flanges of the latter rest upon the upper corners of the boxes, which are fitted to them. The cross-girders, in such cases, are prevented from spreading apart, by bolts passing through both of them, close to the cast-iron boxes.

Art. 18. The following gives the **weights of iron** in several of these turntables now in use; with the **wt of locomotive and tender** which they turn; the **max load** on the end carriage; and that on the pivot pier. This last, of course, includes the wt of track and cross-ties in addition to the other wts named.

Diam of turntable. ft.	Wt of iron in turntable. lbs.	Wt of loco and tender. lbs.	Max load on end wheels. lbs.	Max load on centre pivot. lbs.
50	17500	124000	57000	149000
50	20500	150000	64000	177000
55	22500	124000	61000	154000
55	25500	161000	68000	194000
60	27500	169000	74000	206000

Prices, accompanied by strain sheets based upon the wheel loads of loco and tender, furnished upon application.

Art. 19. The Greenleaf turntable, Clements A. Greenleaf, M E, Indianapolis, Ind, patentee and manufacturer; is made in a variety of forms. **Its distinguishing feature** consists in a series of 27 **cylindrical steel rollers,** R R, Fig 10, $2\frac{7}{16}$ ins diam, with their axes vert. These rollers are arranged in a circle around the post, P, P, *near its base.* They have no axles, but are held in a circular cast-steel box attached to the cross-girder, as in the fig, or in a circular groove in the center-box when this last is of cast-iron. The post is made cylindrical externally, near its foot, as shown, in order to give a full bearing to the vert sides of the rollers. **Their object** is to prevent the tipping of the turntable, while turning, even although the locomotive is not exactly balanced on the table (Art. 1) and thus to prevent the end wheels from bearing on their circular rail.

Art. 20. Conical rollers, *d d*, in a **roller-box,** *u v*, are used, as in the Sellers (Art 3) and other tables. **The cap,** C, rests and fits upon a **cast-iron hemisphere,** S, a cylindrical dowel on the bottom of which enters and fits the space in the center of the roller-box. **Oil is supplied** to the roller-box through the vert passage, Q, and its branches.

Art. 21. The post here shown is made of eight 60-lb steel T rails, P P P, &c, with their flanges outward. To these flanges is riveted an iron-plate, O O, $\frac{3}{8}$ inch thick, forming a conical shell or covering for the post. The steel casting, L (at the top of which is the path for the vert rollers, R), is held to the wrought-iron base-plate, K, by long rivets which pass between the feet of the rails, P, of the post. A wrought-iron "tire," T, is shrunk around the ends of the rails. When this form of post is used, the **arrangement for hanging the table to the cap** is similar to that in the Edge Moor table, Art 10. **Hollow cast-iron posts,** however, are more largely used, and are recommended as being better and cheaper.

With cast-iron posts a cast-iron center-box is used, closely surrounding the post, and furnished with flanges, by which it is riveted to plate-iron cross-girders; but the arrangement of the cap, C, hemisphere, S, and rollers, is the same as in Fig 10.

Art. 22. In all cases **the table is suspended from the cap** by 8 bolts, arranged in a circle. The holes in the cap are made a little larger in diam than the bolts, to allow the slight hor movement of the center-box, caused by the tipping of the table when an engine comes upon it, or leaves it, because the

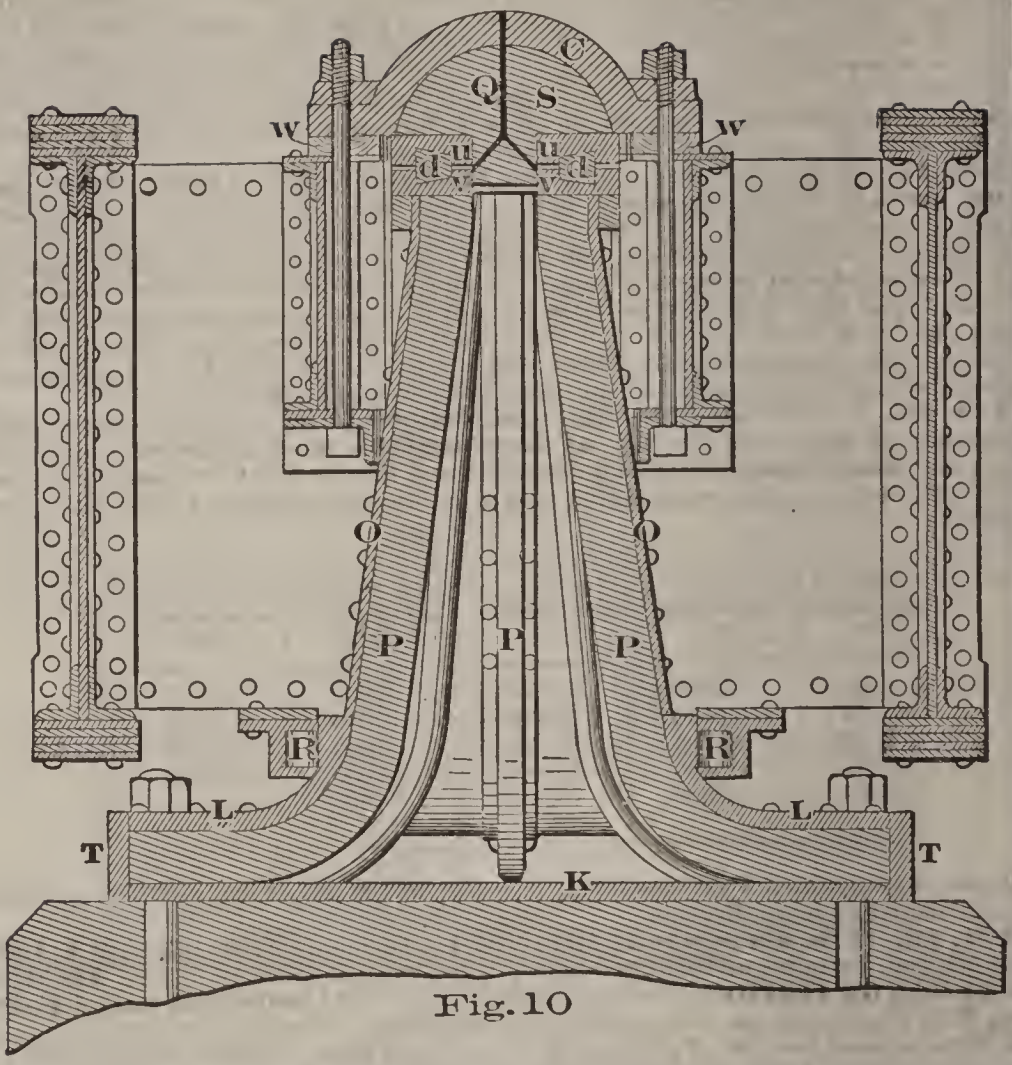


Fig. 10

vert rollers, R R, prevent this motion from taking place near the *foot* of the post, as it does in other tables.

Art. 23. These tables are made both of wrought- and of cast-iron, and are largely used (especially on Western roads) with very satisfactory results.

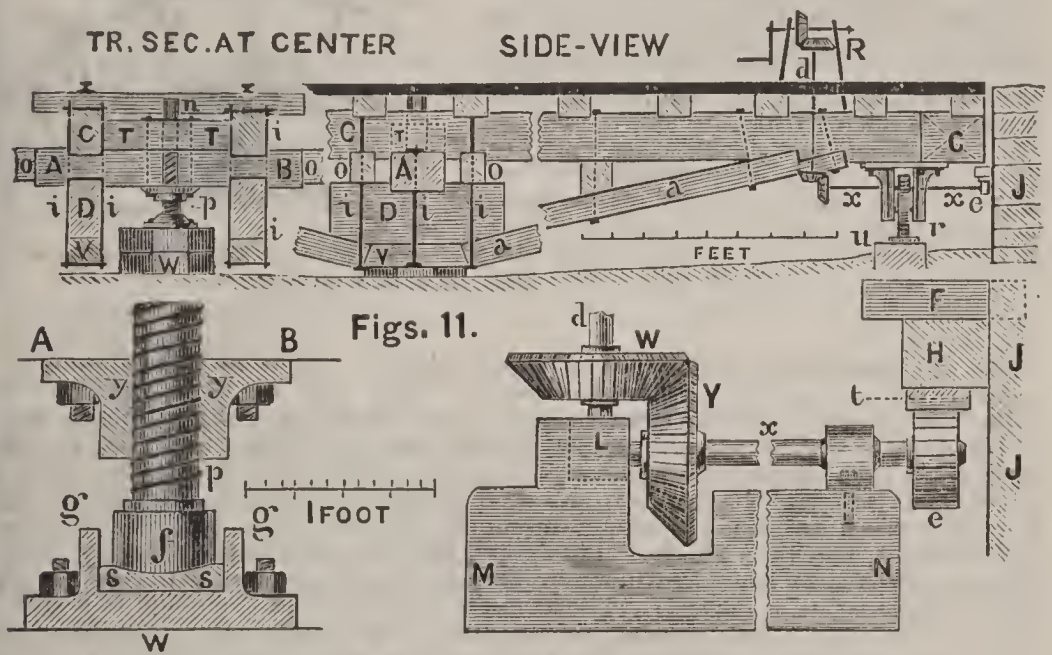
A Greenleaf **wrought-iron** turntable, 60 ft long, **weighs** 28000 lbs, and **costs,** 1886, $1680, say 6 cts per lb; **cast-iron,** same length, 32000 lbs, $1300, say 4 cts per lb. **A center,** complete, with cast-iron center-post and center-box, costs about $600.

Art. 24. The wrought-iron turntable made by the **Union Bridge Co** (late **Kellogg & Maurice**), of **Athens, Pa,** also **has vert rollers** travelling around the post near its base; but the rollers are only two in number. They are held in place by vert axles, and are fixed one on each *side* of the post, or so that a hor line, joining their centers, is at right angles to the longitudinal axis of the turntable. They thus aid in preventing sideways tipping, but not in balancing the locomotive longitudinally. They therefore bring no great strains upon the post or its foundation.

Art. 25. Mr. H. T. Stock, Chicago, Ill, furnishes a turntable in which the main girders are heavy rolled I-beams, trussed on their upper sides with a central vert strut and inclined tension-rods like Fig 52, p 514, inverted. Price of a 40-ft table, 1886, $800; 60 ft, $1500.

Wooden turntables, with none but two common wheel rollers at each end of the platform, are sometimes resorted to from motives of original cost. They are, however, much harder to turn, generally requiring two men, aided by wheelwork; and are more liable to get out of order; and more expensive to repair. They are made of a great variety of patterns, both as regards the girders, and the central pivots, end rollers, &c. Frequently an addition is made of 8 to 12 small rollers travelling on a circular rail of 6 to 12 feet diameter, around the pivot as a center. These are intended to sustain the whole weight; the end rollers being so adjusted as to touch their rail only when the platform rocks or tilts as the engine enters or leaves it. Therefore, there is less resistance from friction than when, as in Figs 11, there are only the end rollers *r*. In this last case, the engine and tender cannot be balanced so precisely upon the slender central pivot, as to prevent a great part of the weight from being thrown upon the end rollers; thus materially increasing the frictional resistance.

In plan, these wooden platforms are sometimes in shape of a cross; that is, in addition to the main platform for the engine, there is another transverse or at right angles to it, also extending across the pit; and having end rollers travelling on the circular rail. Thus, in Figs 11, (which show one of the many modes of framing a table which has only a central pivot *f*, and end rollers *r*,) the main platform rests on the girders *c*, which are strengthened below by braces *a*; while the transverse one rests on the timbers *o*, *o*, which must be imagined to extend across the pit. One-half, or one arm, of this transverse platform is intended to carry the wheelwork R *x x*, for turning the platform; and the other arm serves merely as a balance to it: therefore, neither of them requires to be very strong. It is important to connect the four ends of the two platforms by four beams, as the whole structure is thereby materially stiffened. In the figures the wheelwork R *x x* is for convenience improperly shown as if it stood upon the main platform.



Figs. 11.

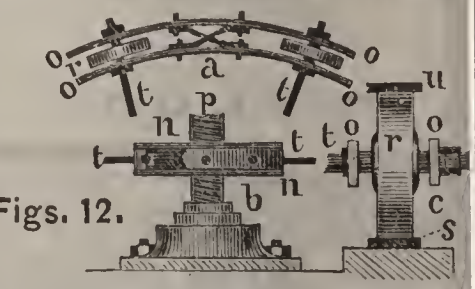
The figures need but little explanation. They represent an actual 45-foot platform, which has been in use for some years. The convex foot *f* of the central pivot, about 6 inches diameter, should be faced with steel: and should rest on a steel *step s s*. This should be kept well oiled; and protected from dust by a leather collar around *p*, and resting on *g g*. Its upper part, about 4 inches diameter, is cut into a screw with square threads about ¼ inch thick, for a distance of about 15 inches. It works in a female screw in the strong cast-iron nut *y y*; and serves for raising the whole platform when necessary. When not in use for this purpose, it is keyed tight to the platform, (by a key at

its head *n*,) so as to revolve with it. Strong screw-bolts *i i* connect the several timbers at the center of the platform.

R is a light cast-iron stand supporting two bevel wheels about 1 foot diameter which give motion by means of an axle *d*, 1¾ inches diameter to two similar ones below, shown more plainly at W and Y. These last give motion by the axle *x* to the pinion *e*, (6 inches diameter, and 2¼ inches face,) which turns the platform by working into a circular rack *t*, (teeth horizontal, 1 inch pitch; 3½ inches face,) which surrounds the entire pit. This rack is spiked to the under side of a continuous wooden curb H, which is upheld by pieces F, a few feet apart, which are let into the wall J J, which lines the pit. The short beam M N, (about 6 feet,) which carries the lower wheelwork, is suspended strongly from the beams of the transverse platform above it. Instead of the two lower bevel wheels W Y, and the horizontal axle *x*, a more simple arrangement is to place the pinion *e* at the lower end of the vertical axle *d*; and let it work into a rack with *vertical* teeth at *u*, on the inner face of the stone foundation of the circular rail. For this purpose the stand R should be directly over *u*. There are two cast-iron rollers *r*, 2 feet diameter, 3 inch face, under each end of the main platform; and one under each end of the secondary one.

Although this kind of platform necessarily has much friction, yet one man can generally turn a 45-foot one by means of the wheelwork, when loaded with a heavy engine and tender. Indeed, he may do it with some difficulty by hand only, while all is new and in perfect order; but when old, and the circular railway uneven and dirty, it requires two men at the winches to do it with entire ease.

As before remarked, the resistance to turning is diminished by employing a set of from 8 to 12 rollers or wheels *r*, Figs 12, about a foot to 15 inches in diameter, so arranged as to form a circle 8 to 12 feet diameter around the pivot. When this is done, the main girders of the platform are placed 8 to 12 feet apart; and long cross-ties are used for supporting the railway track. Also, the main girders are sometimes trussed by iron rods, as in the swing bridge on page 593; but instead of one post *a c*, it is best to have two, 6 or 8 feet apart at foot, and meeting at top. The width of platform must then be sufficient to allow the engine to pass the posts on either side of it. Ten feet will suffice.



Figs. 12.

Fig 12 shows the arrangement of these rollers *r*, which revolve upon a circular track *s*: while the platform rests on their tops by the track *u*. The rollers *r* are held between two wrought-iron rings *o*, *o*, about 3 inches deep, ½ inch thick, which also are carried by the rollers. From each roller a radial tie-rod *t*, 1 inch diameter, extends to a ring *n n*, which surrounds the pivot *p* closely, but not tightly, so as to revolve independently of it. These tie-rods keep the rings *o o* at their proper distance from the pivot, so that the rollers cannot leave the rails *s* and *u*. Between each two rollers, the rings *o o* should be strengthened by some arrangement like *a*, to prevent change of shape. The pivot *p* may be as in Figs 11. There must, of course, be the usual two rollers under each end of the platform, for sustaining the engine as it goes on or off; but during the act of turning the platform, the whole weight should rest on the central rollers. Such a platform of 50 feet length can, if carefully made, be turned, together with an engine and tender, by one man, by means of a wooden lever 12 to 15 feet long, inserted in a staple for that purpose; and therefore may dispense with the transverse platform for sustaining wheelwork.

Such rollers as have just been described, in connection with friction rollers, Fig 5, form perhaps **the best arrangement for a large turning bridge.** At least one end of a platform must be provided with a **catch** or **stop** for arresting its motion at the moment it has reached the proper spot.

common mode is shown at Fig 13. It consists of a wrought-iron bar *m n*, 4 et long, 3 inches wide, and ¾ thick; hinged at its end *m*, which is confined to the top of the platform. Its outer end *n* is formed into a ring *V* for lifting it. A strong casting *e e* (or in longitudinal section at *t t*,) about 15 inches long, 3 inches wide, and 1 inch thick, is also firmly bolted to the top of the platform; and the stop-bar *m n* rests in its recess *r*, while the platform is being turned. A similar casting *a a* is well bolted to the wooden or stone coping *c c*, which surrounds the top of the lining wall of the pit. When the stop-bar reaches this last casting, as the platform revolves, it rises up one of *its* little inclined planes *t t*, and falls into the recess of *a a*, bringing the platform to a stand. When the platform is to be started again, the bar is fted out of its recess by the ring *V*, until it passes the casting; when it is again id upon the coping *c c*, and moves with the platform; or, if required, the inge at *m* allows it to be turned entirely over on its back. When there is a ansverse platform, the proper place for the stop is at that end which carries e turning gear; as it is there handy to the men who do the turning. If there only a main platform, the stop may be placed midway of the rails. Some- mes a **spring catch** is placed at each end of the platform; and each catch loosened from its hold at the same instant by a long double-acting lever. All e details of a platform admit of much variety.

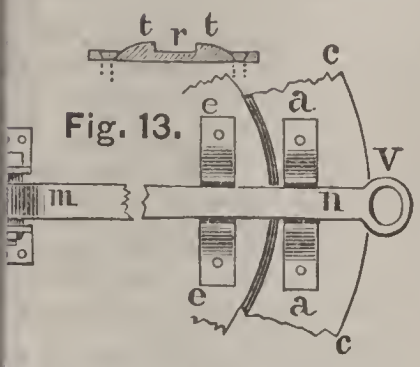


Fig. 13.

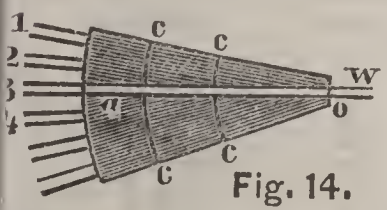


Fig. 14.

Instead of the friction *rollers*, Fig 5, friction *balls* 5 or 6 inches diameter, of polished steel, are sometimes used. The pivots also are made in many shapes.

Platforms like *o a*, Fig 14, revolving around one end *o* as a center of motion, are sometimes useful. The shaded space is he pit. If an engine approaching along the track W, is intended to pass on to ny one of the tracks 1, 2, 3, 4, the platform is first put into the required posi- ion, and the engine passes at once without detention. If the platform is long, t will be necessary to have roller-wheels not only under the moving end *a*, but t one or two other points, as indicated by the roller rails *c c*.

Engine houses, of brick, cost from $1000 to $1200 per engine stall, exclu- ive of the foundations.

The cost of a complete set of shops of brick, for the thorough re- pair of about 20 locomotives, and of the corresponding number of passenger nd other cars; together with suitable smith shop, foundry, car shop, boiler hop, copper and brass shop, paint shop, store rooms, lumber shed, offices, &c; ompletely furnished with steam power, lathes, planing machines, scales, and ll other necessary tools and appliances, will be about from $75000 to $100000 ex- clusive of ground. A large yard, of at least an acre, should adjoin the buildings. A moderate establishment, for the repairs of a few engines only, may be built nd furnished for $25000.

WATER STATIONS.

Water stations are points along a railroad, at which the engines stop t take in water. **Their distance apart** varies (like that of the fuel sta tions, which accompany them,) from about 6 miles, on roads doing a very larg business; to 15 or 20 miles on those which run but few trains. Much depends however, upon where water can be had. It has at times to be conducted i pipes for 2 or 3 miles or more. The object in having them near together is t prevent delay from many engines being obliged to use the same station. T prevent interruption to travel, they are frequently placed upon a side track A supply of water is kept on hand at the station, usually in large wooden tub or tanks, enclosed in frame tank-houses. The tank-house stands near the track leaving only about 2 to 4 feet clearance for the cars. It is two stories high; th tank being in the upper one; and having its bottom about 10 or 12 feet above th rails. In the lower story is usually the pump for pumping up the water into th tank; and a stove for preventing the water from freezing in winter.*

The tanks are usually circular; and a few inches greater in diameter at th bottom than at the top, so that the iron hoops may drive tight. **Thei capacity** generally varies from 6000 to 40000 gallons, (rarely 80000 or more, depending on the number of engines to be supplied. **A tender-tank holds** from 1000 to 3000 gallons; and **an engine evaporates** from 20 to 150 gal lons per mile, depending on the class of engine; weight of train; steepness of grade, &c. Perhaps 40 gallons will be a tolerably full average for passenger, and 80 for freight engines. **The following are the contents of tanks** of different inner diameters, and depths of water. U. S. gallons of 231 cubic inches; or 7.4805 gallons to a cubic foot.

Diam.	Depth.	Contents.		Diam.	Depth.	Contents.	
Ft.	Ft.	Gallons.	Cub. Ft.	Ft.	Ft.	Gallons.	Cub. Ft.
12	8	6767	905	24	12	40607	5429
14	9	10363	1385	26	13	51628	6902
16	9	13535	1810	28	14	64481	8621
18	10	19034	2545	30	15	79310	10603
20	10	23499	3142	32	16	96253	12868
22	11	31277	4181	34	17	115451	15435

Cypress or any of the pines answer very well for tanks. The staves may be about 2½ inches thick for the smaller ones; to 4 or 5 inches for the largest. The bottoms may be the same. The staves should be planed by machinery to suit the curve precisely. Nothing is then needed between the staves to produce tightness. A single wooden dowel is inserted between each two near the top, merely to hold them in place while being put together. The bottom is dowelled together; and simply inserted into a groove very accurately cut, about an inch deep, around the inner circumference of the tub, at a few inches above the bottoms of the staves.

One of 20 feet diameter, and 12 feet deep, may have 9 hoops of good iron; placed several inches nearer together at the bottom of the tank than at the top. Their width 3 inches; the thickness of the lower two, ¼ inch; thence gradually diminishing until the top one is but half as thick. The lower two are driven close together. These dimensions will allow for the rivet-holes for riveting together the overlapping ends; and for a moderate strain in driving the hoops firmly into place.† Three rivets of ½ inch diameter, and 3 inches apart, in line, are sufficient for a joint of a lower hoop. One of 34 feet diameter, 17 deep, may have 12 hoops; the lower ones 4 inches by ½; with three ¾-inch rivets to a lower hoop-joint.

The bottom planks of the tank must bear firmly upon their supporting joists, or bearers.

A tank must have an **inlet-pipe** by which the water may enter it; a **waste-pipe** for preventing overflow; and a **discharge or feed-pipe** 7 or 8 inches diameter, in or near the bottom; through which the water flows out to the tender. The inner end of the discharge-pipe is covered by a valve, to be opened at will by the engine man, by means of an outside cord and lever. To

*A frame tank-house, 18 feet square, with stone foundations for both house and tank, will by itself **cost** $400 to $600. A brick or stone one somewhat more.

†Such a tank, put up in its place, will **cost** about $400. Geo. J. Burkhardt's Sons, North Broad St below Cambria, Philadelphia, make tanks their specialty; and are provided with machinery which secures perfect accuracy of joint in every part. Their work is sought from great distances.

its outer end is generally attached a flexible canvas and gum-elastic hose about 7 or 8 inches diameter, and 8 or 10 feet long, through which the water enters the tender-tank. Or, instead of a hose, the feed-pipe may be prolonged by a metallic pipe, or nozzle, sufficiently long to reach the tender; and so jointed as, when not in use, to swing to one side, or to be raised to a vertical position, (in the last case it is called *a drop,*) so as not to be in the way of passing trains.

The same tank may supply two engines on different tracks, at once. The tanks are very durable.

The patent frost-proof tank of John Burnham, Batavia, Illinois, is simply an ordinary tank, in which the water is prevented from freezing by means, 1st, of a circular roof which protects a ceiling of joists, between which is a layer of mortar; 2d, by an air-space obtained by a similar ceiling beneath the timbers on which the tank rests. Although the sides are entirely unprotected, no house is necessary; but merely strong posts and beams on a stone foundation, for the support of the tank.* The supply pipes are in boxes made of boards and tar-paper.

Tanks are frequently made rectangular, with vertical sides of posts lined with plank, and braced across in both directions by iron rods. They are more apt to leak than circular ones. They have been made of iron; but wood seems to be preferred.

The water for supplying the tanks, may be pumped by hand, steam, horse, wind, hydraulic ram, or otherwise, from a running stream; from a pond made by damming the stream if very small or irregular; from a cistern below the tank; or from a common well. Many roads doing a business of 10 or 12 engines daily in each direction, depend entirely upon wells; and pump by hand; generally two men to a pump. Those doing a very large business, when the supply cannot be obtained by gravity, mostly use steam. **The windmill** is the most economical power; and when well made, is very little liable to get out of order. Of course it will not work during a calm; but this objection may be obviated in most cases by having the tanks large enough to hold a supply for several days.† Steam, however, is most reliable.

The following table will give some idea of the **power required in a steam engine for the pumping.** In ordering an engine, specify not its number of horse-powers, but the number of gallons it must raise in a given number of hours, to a given height; with a given steam pressure, (say about 60 to 80 lbs per square inch.) The pump should be sufficiently powerful not to have to work at night; and should be capable of performing at least 25 per cent. more than its required duty.

A fair average horse should pump in 8 hours the quantities contained in the first 3 columns; to the height in the 4th column; or sufficient to supply the number of locomotives in the 5th column, with about 2000 gallons each. Two men should do about one-third as much.‡

Cub. Ft.	Lbs.	Gals.	Ht. Ft.	No. of Locos.	Cub. Ft.	Lbs.	Gals.	Ht. Ft.	No. of Locos.
1600	100000	11968	100	6	4571	285714	34194	35	17
2000	125000	14960	80	7½	5333	333333	39893	30	20
2667	166666	19946	60	10	6400	400000	47872	25	24
3200	200000	23936	50	12	8000	500000	59840	20	30
3555	222222	26596	45	13¼	10667	666667	79787	15	40
4000	250000	29920	40	15	16000	1000000	119680	10	60

A reservoir, with a stand-pipe, or water column, is preferable to the ordinary tank, when the locality admits of it; being less liable than the pump to get out of order; and being cheaper in the end. The reservoir is supposed to be filled by water flowing into it by gravity; and to have its bottom at

*The U. S. Wind-Engine and Pump Co, of Batavia, Ill, make a specialty of the construction and erection of these tanks, complete in every detail, ready for use. They also make windmills.

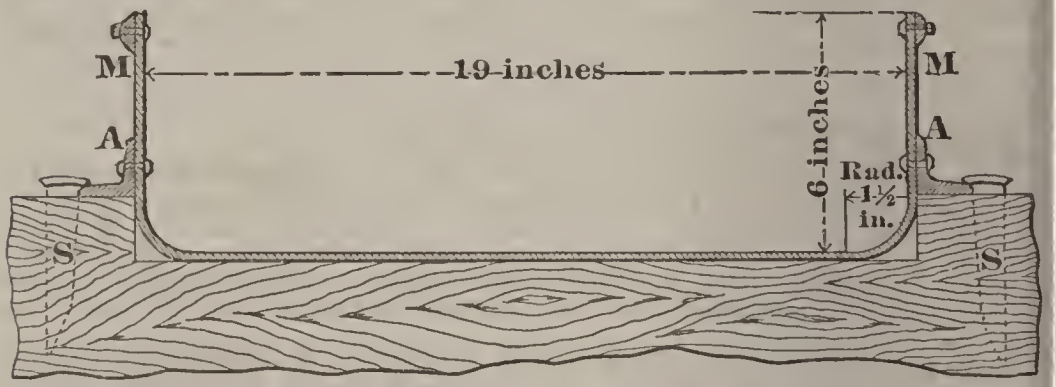
†Andrew J. Corcoran, No. 76 John St, New York, furnishes excellent machines. He also, when desired, provides pumps, &c, complete. The cost of windmill alone, for railway stations, varies from about $450 for 18 feet diameter; to $1500 for 36 feet diameter, at the factory.

‡**The cost of a direct acting steam pump,** with its boilers, &c, fixed in place, ready for work, and capable of the duty of the above table, may be roughly set down at about $450; twice the duty, $600; 4 times, $750; 6 times, $900; 10 times, $1300; 20 times, $2000. Add cost of engine-house. Made by Henry R. Worthington, 86 Liberty St, New York; Geo. F. Blake Manufacturing Co, 44 Washington St, Boston; and by many establishments in most of our large cities.

least about 8 feet above the rails; or at any greater height whatever that the ground and the height of the water may require. It may be excavated in the ground; lined with brick or masonry in cement; with a bottom of concrete; or it may be built above ground, according to the locality. It may be roofed and covered in, or not; and it may be near the tracks, or at a considerable distance from them, according to circumstances.* From its bottom, an iron pipe from 8 to 12 inches diameter, is carried (generally underground,) to within a few feet of the track. At that point it turns vertically upward to about 8 or 10 feet above the track, forming a **stand-pipe, or water-column;** from the upper end of which the water flows (through either a hose or a jointed nozzle,) as in the case of a tank. Several such pipes, or one larger one, may be laid, for the supply of two or more engines at once, through as many stand-pipes. Where the pipe makes its bend, and becomes vertical, is a valve for opening and closing it; and which may be worked by a hand-wheel placed at such a height as to be easily reached by the engine man.† A valve on the principle of those for street pipes, page 301, is best.

On some of the more important lines, the **tenders of fast trains scoop up water, while running,** from a long **trough,** or **"track tank"** laid between the rails. The tanks are about ¼ mile long. They must of course be level, and they therefore require a level track.

As originally introduced in England, by Ramsbottom, the trough was of cast iron, in lengths of about 6 ft. These were bolted together by means of flanges at their ends. The ends were not in contact with each other, but were separated by a strip of vulcanized rubber.



Our figure is a vertical cross-section of the standard track tank of the Pennsylvania Railroad, 1884. It is of $\frac{3}{16}$ inch rolled plate-iron, the sheets of which are 6 ins long. The lengths overlap each other 2 ins; leaving 5 ft as their *showing* length. The sheets are cut slightly tapering, so that at one end of each length the trough is $\frac{3}{16}$ in. deeper than at the other, and the tops are thus kept flush with each other throughout. The joints are double riveted with ⅜ inch rivets, about 1½ ins from center to center, and staggered. At each end of the trough, the bottom slopes upward, and in a length of 6 ft, comes to the level of the tops of the sides. The cross-ties are notched, as shown, to receive the trough, which is loosely held to them by two spikes, S and S, in each tie. The heads of the spikes fit over the horizontal flanges of the $1\frac{1}{2} \times 1\frac{1}{2}$ inch angle bars, A and A. M and M are mouldings of $1\frac{1}{8} \times 1$ inch bar-iron. The angles and the mouldings are in lengths of 15 ft, and are riveted to the sides of the trough continuously throughout its length.

The **scoop** on the tender is **lowered** into the trough, **and raised** from it by means of a lever on the fireman's platform, and is not permitted to touch the bottom of the trough.

The **trough is supplied** with water by means of pipes leading from an adjacent tank. The supply is regulated by a man in charge.

To prevent the water from **freezing in winter,** steam is led to the trough from the boiler of the pumping engine, through iron pipes laid under ground alongside of the track. These pipes are provided with branches which introduce the steam to the trough at every 40 ft of its length. The steam-pipes are protected by wooden boxes, and are furnished with valves for regulating the supply of steam.

* An uncovered reservoir 50 ft diam by 12 ft deep, lined with brick or masonry, will usually cost from $2500 to $3500, according to circumstances.

† The price of a cast-iron water-column, of 6-inch bore; with bed-plate; holding-down bolts and washers; connecting-pipes; swing-joint with copper arm 9 ft long; valve; hand-wheel; &c complete, ready to set up, (by Isaac S. Cassin & Co, Philada, in 1886,) is $475 at the shop less 20 per cent discount.

Evaporation from Locomotives. In addition to what is said on page 800, in the passage preceding the table, we may state that the evaporation is usually from 6 to 7 lbs of water to 1 lb of fair coal. Hence if we take the average at 6½ lbs, or say .8 of a gallon of water to 1 lb of coal, and assume, as on page 800, that a passenger engine evaporates an average of 40 gallons per mile, and a freight engine 80 gallons, we shall have very nearly 2¼ tons of coal consumed per 100 miles by the former; and 4½ tons by the latter. The evaporation from a heavily tasked powerful engine may amount to 150 gallons or more per mile; but such is an exceptional case.

Theoretical thickness near bottom of sheet-iron water tanks, single riveted; safety 4; ultimate strength of the iron 40000 lbs per square inch, but reduced say one-half by punching the rivet holes. Although safe against the *pressure of the water*, some are plainly far too thin for *handling*.

Depth in Feet.	INNER DIAMETER IN FEET.							
	5	10	15	20	25	30	35	40
	THICKNESS IN INCHES.							
1	.0026	.0052	.0078	.0104	.0130	.0156	.0182	.0208
5	.0130	.0260	.0391	.0520	.0651	.0781	.0911	.1042
10	.0260	.0521	.0781	.1042	.1302	.1562	.1823	.2083
15	.0391	.0781	.1172	.1562	.1953	.2344	.2734	.3125
20	.0521	.1042	.1562	.2084	.2604	.3125	.3645	.4166
25	.0651	.1302	.1953	.2604	.3255	.3906	.4557	.5208
30	.0781	.1562	.2344	.3124	.3906	.4687	.5470	.6250

Railroad track scales are made by Riehlé Bros, office, 4th and Chestnut Sts, Phila, and by Fairbanks & Co, St Johnsbury, Vt. **Price-list** of Riehlé track scales. Discount, 1886, about 45 per cent. The **capacities** are in tons of 2000 lbs or 2240 lbs, as may be desired.

Capacity.	Length. ft.	Price. $	Capacity.	Length. ft.	Price. $
10	12	350	65	40 to 65	1850
15	12 to 15	400	75	40 to 85	2200
20	12 to 16	600	100	50 to 112	2800
30	20 to 32	850	150	60 to 123	3200
40	30 to 40	975	150	100 to 150	3700
50	40 to 50	1100			

Post-and-rail fences, in panels 8¼ ft long; 5 rails; usually cost between 10 to 100 cents per panel, including the putting up; or from $512 to $1280 per mile of road fenced on both sides, with 1280 panels.

Fence-posts are usually of chestnut, cedar, or white oak. They last about 10 years on an average. The usual size is 2 to 3 ins thick × 6 to 7 ins wide, 8 ft long, 5 ft above ground. Their cost varies greatly; say from 5½ to 25 cts each; average, 10 to 15.

Worm fences seven rails high, with two rails on end at each angle, cost about ¼th less. Labor $1.75 per day. The scarcity or abundance of timber chiefly influences the price; as is also the case with ties.

The Glidden Barbed Steel Wire fence, made by I. L. Ellwood & Co., at De Kalb, Illinois, has (1882) come into extensive use; many thousands of miles of it having already been put up. Its cost per mile of single row of fence, put up, including the wooden posts and all labor, will usually range from $150 to $250, depending on the height of fence, the varying market price of wire, labor, &c.

A way-station house, 30 × 60 feet, surrounded by a platform 12 feet wide, protected by projecting roof; for passengers and freight; will cost from $6000 to $10000, according to finish and completeness, at eastern city prices.

Approximate average estimate for a mile of single-track railway. Labor $1.75 per day.

Grubbing and clearing, (average of entire road,) 3 *acres at* $50	$ 150
Grading; 20000 *cub yds of earth excavation, at* 35 *cts*	7000
" 2000 *cub yds of rock excavation, at* $1.00	2000
Masonry of culverts, drains, abutments of small bridges, retaining-walls, &c; 400 *cub yds, at* $8, *average*	3200
Ballast; 3000 *cub yds broken stone, at* $1.00	3000
Cross-ties; 2640, *at* 60 *cts, delivered*	1584
Rails; (60 *lbs to a yard;*) 96 *tons, at* $30, *delivered*	2880
Spikes	150
Rail-joints	300
Sub-delivery of materials along the line	300
Laying track	600
Fencing: (average of entire road,) supposing only ½ *of its length to be fenced*	450
Small wooden bridges, trestles, sidings, road-crossings, cattle guards, &c, &c	1000
Land damages	1000
Engineering, superintendence, officers of Co, stationery, instruments, rents, printing, law expenses, and other incidentals	2380
Total	$26000

Add for depots, shops, Engine-houses, Passenger and Freight Stations, Platforms, Wood Sheds, Water Stations with their tanks and pumps, Telegraph, Engines, Cars, Weigh Scales, Tools, &c, &c. Also for large bridges, tunnels, Turnouts, &c.

ROLLING STOCK.

LOCOMOTIVES.

Dimensions, Weights, &c, of Locomotives.

The following lists of the dimensions, weights, &c, of some of the principal sizes of locomotives made by the Baldwin Locomotive Works, Phila; Burnham, Parry, Williams & Co, proprietors; will give an idea of the present usual **proportions of locomotives and tenders as made in the United States.**

In the designation of the **class,** the first number (8, 10, &c) is the total number of wheels of the locomotive. The second (20, 26, &c) is an *arbitrary* number indicating the diameter of the cylinders. The *letter* (C, D, or E) indicates the number (4, 6, or 8, respectively) of *driving*-wheels.

The wheel-base is the distance from center to center of the front and back wheels. For minimum **length of turntable,** add 1½ to 2 ft to the *total* wheel-base of locomotive and tender, which is = wheel-base of locomotive + wheel-base of tender + distance between centers of front tender wheel and hind engine wheel.

Under "**Service,**" **P** means *passenger;* **F,** *freight;* **M,** *mixed;* **S,** *switching.*

For gauge of 4 ft 8 1-2 ins.

Dimensions.

Class.	Service.	Type.	Cylinders. Diam.	Cylinders. Stroke.	Driving Wheels. No.	Driving Wheels. Diam.	Wheel-base. Locomotive. Driv'rs	Wheel-base. Locomotive. Total.	Wheel-base. Tender.	Wheel-base. Loco and tender.	Extreme lgth loco and tender.	Extreme width.	Height of top of chimney above top of rails.
			Ins.	Ins.		Ins.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.
8—14—C	P	"American."	10	20	4	45 to 51	5 6	16 4	5 1	27 6	41 3	7 9	12 4
8—20—C	P F		13	22	4	49 to 57	7 0	20 6	14 2	42 0	47 8	8 4	13 0
8—30—C	"		18	22	4	61 to 66	8 6	22 5	14 0	44 4	54 6	8 6	13 6
10—26—D	F M	"10-wheel."	16	24	6	51 to 56	12 6	22 8	13 5	42 2	54 9	9 4	13 0
10—28—D	"		17	24	6	51 to 56	12 10	23 0	13 1	42 11	55 1	8 11	13 0
10—32—D	"		19	24	6	54 to 60	13 6	23 8	13 11	44 10	55 9	9 0	13 6
8—18—D	F	"Mogul."	12	18	6	37 to 41	10 0	16 0	10 11	33 0	41 8	8 4	12 6
8—26—D	"		16	24	6	45 to 51	14 2	21 6	13 5	42 2	54 6	9 0	13 0
8—32—D	"		19	24	6	54 to 60	15 2	22 6	13 11	43 10	56 2	9 2	13 8
10—34—E	"	"Consolidation."	20	24	8	48 to 50	14 9	22 10	14 4	46 2	58 6	9 6	14 0
10—36—E	"		21	24	8	48 to 50	14 9	22 10	14 4	46 2	57 3	9 8	14 0
4—26—C	S		16	24	4	48 to 54	7 6	7 6	13 5	34 6	47 2	9 0	13 0
6—28—D	S		17	24	6	45 to 49	11 0	11 0	13 1	35 5	48 4	9 0	13 4

Weights, &c.

Class.	Service.	Type.	Weights in working order. Locomotive. Greatest on 1 pair of drivers. lbs	tons	On all drivers. lbs	tons	Total. lbs	tons	Tender loaded. lbs	tons	Total loco and tender. lbs	tons	Capacity of tender. Coal or Wood. lbs	cords	Water. Gals of 231 cu in	lbs
8.14.C	P	"American."	10500	4.6	21000	9.4	36000	16.1	26000	11 6	62000	27.7	8000	1.25	1000	8300
8.20.C	P F		17500	7.8	35000	15.6	56000	25.0	34200	15.3	90200	40.3	9000	1.60	1400	11700
8.30.C	"		27000	12.0	54000	24.1	82000	36.6	55500	24.8	137500	61.4	12300	2.90	2400	19900
10.26.D	FM	"10-wheel."	18000	8.0	58000	25.9	78000	34.8	47800	21.3	125800	56.2	11800	1.70	2000	16700
10.28.D	"		19000	8.5	63000	28.1	84000	37.5	51600	23.0	135600	60.5	12000	1.80	2200	18400
10.32.D	"		22000	9.8	72000	32.1	96000	42.9	59300	26.5	155300	69.3	12500	2.00	2600	21700
8.18.D	F	"Mogul."	11000	4.9	33000	14.7	40000	17.9	21600	9.6	61600	27.5	8500	1.50	1200	10000
8.26.D	"		21000	9.4	63000	28.1	75000	33.5	47800	21.3	122800	54.8	11800	1.70	2000	16700
8.32.D	"		25000	11.2	80000	35.7	94000	42.0	59300	26.5	153300	68.4	12500	2.00	2600	21700
10.34.E	"	"Consolidation."	23000	10.3	88000	39.3	104000	46.4	59300	26.5	163300	72.9	12500	2.00	2600	21700
10.36.E	"		23000	10.3	96000	42.9	112000	50.0	63400	28.3	175400	78.3	13000	2.20	2800	23300
4.26.C	S		28000	12 5	56000	25.0	56000	25.0	47800	21.3	103800	46.3	11800	1.70	2000	16700
6.28.D	S		24000	10.7	69000	30.8	69000	30.8	51600	23.0	120600	53.8	12000	1.80	2200	18400

For gauge of 3 ft.

Dimensions.

Class.	Service.	Cylinders. Diam.	Cylinders. Stroke.	Driving Wheels. No.	Driving Wheels. Diam.	Wheel-base. Locomotive. Drivers	Wheel-base. Locomotive. Total.	Wheel-base. Tender.	Wheel-base. Loco and tender.	Extreme l'gth loco and tender.	Extreme width.	Height of top of chimney above top of rails.
		Ins.	Ins.		Ins.	Ft. Ins.	Ft. Ins.	Ft. Ins.	Ft. Ins.	Ft. In.	Ft. Ins.	Ft. Ins.
8—18—C	P	12	16	4	43	7 2	18 3	11 4	35 8	44 7	7 4	12 0
8—22—C	"	14	18	4	45	8 2	20 1	13 2	40 5	48 6	7 10	12 4
8—20—D	F	13	18	6	37	12 0	17 10	11 4	35 3	47 1	7 4	12 2
8—22—D	"	14	18	6	39	12 0	18 4	13 2	38 8	45 5	7 8	13 0
10—24—E	"	15	18	8	37	11 4	17 10	13 5	39 11	49 5	8 1	12 5
10—26—E	"	16	20	8	37	11 9	18 1	13 0	40 9	50 0	8 4	13 7
4—18—C	S	12	16	4	37	6 0	6 0	.....	6 0	37 0	7 10	13 0
6—22—D	"	14	18	6	37	9 6	9 6	.....	9 6	43 6	7 4	11 1

Weights, &c.

Class.	Service.	Weights in working order. Locomotive. Greatest on 1 pair of drivers. lbs	tons	On all drivers. lbs	tons	Total. lbs	tons	Tender loaded. lbs	tons	Total loco and tender. lbs	tons	Capacity of tender or tank. Coal or Wood. lbs	cords	Water. Gals of 231 cu in	lbs
8—18—C	P	13000	5.8	24000	10.7	37000	16.5	31000	13.8	68000	30.4	9000	1.3	1200	1000
8—22—C	"	16000	7.1	33000	14.7	48000	21.3	35000	15.6	83000	37.0	9775	1.4	1400	1170
8—20—D	F	13000	5.8	39000	17.4	46000	20.5	31000	13.8	77000	34.4	9000	1.3	1200	1000
8—22—D	"	14000	6.2	42130	18.8	50330	22.5	35000	15.6	85330	38.1	9775	1.4	1400	1170
10—24—E	"	12000	5.4	48000	21.3	56000	25.0	37000	16.5	93000	41.5	10300	1.5	1500	1245
10—26—E	"	15000	6.7	58000	25.9	67000	29.9	38500	17.2	105500	47.1	9775	1.4	1600	1328
4—18—C	S	15000	6.7	28000	12.5	28000	12.5	.....	....	28000	12.5	1800	.5	400	332
6—22—D	"	15000	6.7	48000	21.3	48000	21.3	.....	....	48000	21.3	2000	.5	500	415

Standard passenger (Class N) and freight (Class I) locomotives of Pennsylvania Railroad, 1885. Gauge, 4 ft 9 ins.

Dimensions.

Class.	Cylinders. Diam.	Cylinders. Stroke.	Driving Wheels. No.	Driving Wheels. Diam.	Wheel-base. Locomotive. Drivers	Wheel-base. Locomotive. Total.	Wheel-base. Tender.	Wheel-base. Loco and tender.	Extreme l'gth loco and tender.	Extreme width.	Height of top of chimney above top of rails.
	Ins.	Ins.		Ins.	Ft. Ins.	Ft. Ins.	Ft. Ins.	Ft. Ins.	Ft. Ins.	Ft. Ins.	Ft. Ins.
N	17	24	4	62	8 6	23 2	14 10	45 2	54 7	9 0	15 0
I	20	24	8	50	13 8	21 6	15 4	47 7	56 0	9 3	15 11

Weights, &c.

Class.	Weights in working order. Locomotive. Greatest on 1 pair of drivers. lbs	tons	On all drivers. lbs	tons	Total. lbs	tons	Tender. lbs	tons	Locomotive and tender. lbs	tons	Capacity of tender. Coal. lbs	tons	Water. Gals of 231 cu ins.	lbs
N	30800	13.8	57700	25.8	91300	40.8	50500	22.5	141800	63.3	8000	3.6	2400	20000
I	22200	9.9	80500	35.9	92700	41.4	56650	25.3	149350	66.7	8000	3.6	3000	25000

The following are the principal **dimensions, &c,** of five recent and **exceptionally large** types of locomotives:

	"Decapod."* Largest loco built by the Baldwin Wks up to 1887.	"El Gobernador," Centl Pacific R R, 1884, cyls 21 × 36.	"Class K," Engine "No. 10, &c, Penna R R	"Class P," Penna R R.	Phila & Reading R R.
Service	Freight.	Freight.	Fast Pass'r.	Passenger.	Fast Pass'r.
Gauge	4 ft 8½ ins	4 ft 8½ ins	4 ft 9 ins	4 ft 9 ins	4 ft 8½ ins
Driving wheels:					
Number	10	10	4	4	4
Diam	45 ins	57 ins	78 ins	68 ins	68 ins
Truck wheels:					
Number	2	4	4	4	4
Wheel-base:					
Locomotives:					
Drivers	17 ft 0 ins	19 ft 7 ins	7 ft 9 ins	7 ft 9 ins	7 ft 0 ins
Total	24 ft 4 ins	28 ft 11 ins	22 ft 7½ ins	22 ft 7½ ins	21 ft 1 in
Tender	14 ft 5 ins	15 ft 2 ins	15 ft 4 ins	15 ft 4 ins	16 ft 10 ins
Loco and tender	49 ft 2 ins	52 ft 7 ins	47 ft 8 ins	47 ft 8 ins	50 ft 8½ ins
Extreme length:					
Loco and tender	63 ft 3½ ins	63 ft 8 ins	58 ft 6 ins	58 ft 7 ins	58 ft 8⅝ ins
Height:					
Top of chimney above top of rail	14 ft 6 ins	16 ft 2 ins	15 ft 0 ins	15 ft 0 ins	14 ft 4 ins
Weight in working order, lbs					
Locomotive:					
Greatest on 1 pair of drivers	27000	25800	33600	34700	36000
On all drivers	134000	121600	59000	59450	71950
Total	148000	152000	92700	100600	103850
Tender	82000	85650	56300	56300	65000
Loco and tender	230000	237650	149000	156900	168850
Capacity of tender:					
Coal: lbs	16000	10000	12000	12000	12000
Water:					
Gals of 231 cubic ins	3600	3000	2400	2400	3570

From the above lists it will be seen that **the weight of road locomotives per foot of their total wheel-base** varies from .9 ton in light passenger engines for 3 ft gauge, to 2.5 tons in very heavy freight engines for standard gauge.

The cost of locomotives and tenders is about from 9 to 12 cts per pound of locomotive alone, varying with the details of the specifications, &c.

Steel driving tires are usually of open hearth steel, and are from 2½ to 3½ inches thick; occasionally 4 inches. Flanges average 1⅛ to 1¼ inches deep, 1¼ inches thick. **Prices,** delivered in New York: rough, 5 cts per lb; bored, 5½ cts. Standard Steel Works, makers, Lewistown, Pa; office, 220 S 4th St, Phila. The inner diameter of the tire is usually made $\frac{1}{1200}$ (or $\frac{1}{100}$ inch per foot) less than diameter of the wheel center upon which it is to go. Heating the tire increases its inner diameter so that it can be placed upon the wheel center. Its contraction, or tendency to contract, in cooling, binds it fast.

The treads are re-turned in a lathe as often as from $\frac{3}{16}$ to ¼ inch wears off from their thickness. (= ⅜ to ½ inch in *diameter*) and are abandoned when worn down to about 1¼ inches thick. On passenger engines, they run about 60000 miles between turnings; on freight engines, 35000.

*Built 1886 for Northern Pacific R R, Nos 500, 501; **Extreme width** 10 ft. The original "Decapod," built by same works in 1884 for Dom Pedro II R R, Brazil (5 feet 3 inches gauge), and described in our last edition, was a little lighter.

Driving axles are of iron, or of a softer steel than the tires. They are usually from 5 to 7½ ins diam.

Passenger engines usually carry **fuel and water sufficient for 40 o 50 miles;** some, 50 to 60. Freight trains, enough for 20 to 25 miles. Roads, divisions, with steep grades require the fuel and water stations to be nearer togethe than where the grades are easy.

Performance of Locomotives.

The following gives the **loads** (exclusive of locomotive and tender) which th above described **Baldwin engines** will haul, at their usual speed, on a straigl track and **on different grades** varying from a level to 3 ft per 100 ft, or 158 ft per mile. The loads are based upon the assumption that the so-called "**adhe sion**" of the locomotive is one-fourth of the weight on all the drivers, and tha the condition of road and cars is such that the **frictional resistance** of th cars does not exceed 7 lbs per ton of 2240 lbs of their weight. These are ordinaril favorable conditions. The adhesion is seldom less than one-fifth, or more than on third, of the weight on the drivers.

The **resistance of cars to motion,** on a level track, and with cars an track in fair order, is usually taken at about from 6 to 8 lbs per ton of 2240 lb With everything in perfect order, it may fall as low as 5, or even 4 lbs per ton. O the other hand, if the wheels are not truly round, and if the journals are not we lubricated, it may greatly exceed 10 or 12 lbs. See p 374 *e*.

Gauge 4 ft 8 1-2 ins.

Loads in tons of 2240 lbs (exclusive of locomotive and tender).

Class. *	Service. *	Type.	On a grade of: 0 per cent = 0 ft per mile.	½ per cent = 26.4 ft per mile.	1 per cent = 52.8 ft per mile.	1½ per cent = 79.2 ft per mile.	2 per cent = 105.6 ft per mile.	2½ per cent = 132 ft per mile.	3 per cent = 158.4 ft per mile.
8—14—C	P	"American"	500	220	130	85	60	45	35
8—20—C	P F	"American"	1000	435	255	170	125	95	75
8—30—C	"	"American"	1550	675	395	265	200	150	115
10—26—D	F M	"10-wheel"	1685	740	435	305	220	175	135
10—28—D	"	"10-wheel"	1830	780	460	320	235	185	150
10—32—D	"	"10-wheel"	2090	915	540	375	270	215	175
8—18—D	F	"Mogul"	930	405	240	160	120	95	75
8—26—D	"	"Mogul"	1830	800	475	330	245	195	155
8—32—D	"	"Mogul"	2330	1026	610	412	315	245	195
10—34—E	"	"Consolidation"	2560	1130	670	465	350	275	220
10—36—E	"	"Consolidation"	2740	1205	720	495	370	290	235
4—26—C	S	"Consolidation"	1635	720	430	300	225	175	140
6—28—D	"	"Consolidation"	1865	820	490	340	255	200	160

Gauge 3 ft.

Loads in tons of 2240 lbs (exclusive of locomotive and tender).

Class. *	Service. *	Type.	On a grade of: 0 per cent = 0 ft per mile.	½ per cent = 26.4 ft per mile.	1 per cent = 52.8 ft per mile.	1½ per cent = 79.2 ft per mile.	2 per cent = 105.6 ft per mile.	2½ per cent = 132 ft per mile.	3 per cent = 158.4 ft per mile.
8—18—C	P	"American"	650	252	148	103	73	55	43
8—22—C	"	"American"	790	308	182	123	90	68	53
8—20—D	F	"10-wheel"	900	353	215	145	112	83	66
8—22—D	"	"10-wheel"	990	392	235	162	120	92	74
10—24—E	"	"Mogul"	1160	460	275	191	142	116	89
10—26—E	"	"Mogul"	1450	580	350	243	181	142	114
4—18—C	S	"Consolidation"	825	335	205	145	110	90	75
6—22—D	"	"Consolidation"	1320	540	330	235	180	145	120

* See foot note, p. 809.

The tractive power of a locomotive, in lbs $= \dfrac{\text{Square of diam of one piston in ins.} \times \text{Single length of stroke in ins.} \times \text{Average steam pressure in the cylinders in lbs per sq inch}}{\text{Diameter of driving-wheel in inches}}$

rovided the result does not exceed the adhesion. If it does, the tractive power equal to the adhesion.

The *initial* **steam pressure in the cylinders** is always less than the oiler pressure; and the disproportion increases with the speed. Thus, at 8 or 10 iles an hour, the boiler pressure may be about 110 lbs per square inch; and the linder pressure from 90 to 100 lbs, while at a speed of 30 or 40 miles, the proporon may be as 110 to 60 or 70 lbs. The *average* cylinder pressure is ascertained by eans of an indicator applied to the cylinder; and its proportion to the *initial* ressure depends upon how early in the stroke the supply of steam from boiler to linder is cut off; or, in other words, upon the extent to which the steam is used *pansively.*

The power and speed of locomotives, and their consumption of fuel and water, vary greatly with circumstances, such as grades and rvature; condition of track and rolling stock; number of cars in train: diamers, number and distance apart, of car wheels; manner of coupling the cars; skill locomotive runner and fireman, &c, &c. The following records of actual performce will serve as indications:

Baldwin engines. Anthracite passenger engine, class 8—28—C,* American" type, hauls 9 loaded passenger cars, 216 tons besides weight of engine id tender, 52 tons, up a grade 1771 ft long, averaging 107 ft per mile, at 10 miles r hour. At the foot of the grade is a curve of 225 ft radius, on which the grade 100 ft per mile. A similar engine hauls 4 passenger cars, 1 parlor car and 1 bagge car, 145 tons, engine and tender, 59 tons, 59 miles over a nearly level road, with w and easy curves, in 1½ hours. Boiler pressure about 125 lbs per square inch. ur such runs consumed 10000 lbs of coal. **Bituminous passenger engine,** class 8—30—C, "American" type, hauls 4 passenger cars, 1 sleeper, and 3 ggage and mail cars, 150 tons, engine and tender, 61.4 tons, 11.55 miles in 28 min-es, up continuous grades, mostly of about 70 ft per mile, and over nearly continuus reversed curves of from 1° to 7°. **Freight engines,** class 10—30—D, "10 heel" type, haul the following trains: with bituminous coal, 18 cars, 337 tons, gine and tender, 65 tons, up a grade of 79.2 ft per mile, with a curve of 819 ft dius, 885 ft long; 48 cars, 386 tons, engine and tender, 65 tons, up a grade of 62 ft r mile, with 4° curves; 40 cars, 785 tons, engine and tender, 65 tons, up a grade 21 ft per mile, on 5° curves. **Freight engines,** class 8—26—D, "Mogul" pe, haul 45 cars, 300 tons, engine and tender, 55 tons, up grades of 83 ft per mile, ith a 2° curve; starting on the grade; boiler pressure about 130 lbs; also, 37 cars, 5 tons, engine and tender, 55 tons, up a grade of 85 ft per mile, with curves of 9° d 10°. **Freight engines,** class 10—34—E, "Consolidation" type, haul 90 rs, about 2000 tons, engine and tender, 68 tons, 45.5 miles in 4 hours 21 minutes, er a nearly level road with easy curves, consuming 1.8 to 2.7 lbs of bituminous al per loaded car per mile; also, with anthracite coal, 33 cars, 264 tons, engine and nder, 68 tons, up a grade of 96 ft per mile, 12 miles long, with many curves of 573 radius, at from 10 to 20 miles per hour; also, with anthracite, 25 loaded 4-wheel al cars, 235 tons, engine and tender, 68 tons, up a grade of 126 ft per mile.

Central Pacific R R, 1883. Engines with 4 drivers, 56 ins diam, cylinders × 24 ins. Weight on drivers, 47550 lbs = 21.25 tons; weight of engine and tender, in working order, 133000 lbs = 59.4 tons. Average train, 51 freight cars, 800 tons. aximum grades, 52 ft per mile. Sharpest curve, 5°. Runs of 84 and 145 miles. otal distance run, 3000 miles. Average speed about 10 miles per hour. Maximum oiler pressure 125 lbs per square inch. Bituminous coal consumed 73 lbs per train ile; or 30.8 train miles per ton of coal; or 11 ton miles per lb of coal. Water aporated 41 gallons per mile; 4.71 lbs per lb of coal.

On the **Boston & Albany R R, passenger engines** with 4 drivers 3/4 ins diam, cylinders 18 × 22 ins, with 6 passenger cars, run between Boston and ringfield, 97 miles, in about 4¼ hours, consuming 8000 lbs coal, 400 lbs wood for ghting, and evaporating 47800 lbs of water; or say 6 lbs per lb of coal. Boiler essure 130 to 150 lbs.

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* In the Baldwin Works classification ("8—30—C" etc) the first number (8, 10 etc) the total number of wheels of the loco. The second indicates *arbitrarily* the diam the cyls; thus, 12 means 9 ins; 14, 10; 16, 11; 18, 12; 20, 13; 22, 14; 24, 15; 26, 5; 28, 17; 30, 18; 32, 19; 34, 20; and 36, 21 ins diam. The *letter* (C, D, or E) indiates the number (4, 6, or 8, respectively) of *driving* wheels. In the "Service" column of opposite table, P means passenger; F, freight; M, mixed; and S, switching.

Average of main line of Phila & Reading R R, 1884.

Trains.	Average speed.	No. of cars.	Weight of train.			Coal consumed.		
			Cars.	L. & T.	Total.	Kind.	per train-mile	Per ton-mile
	miles per hour.		tons.	tons.	tons.		lbs.	lbs.
Passenger.....	30	8	157	66	223	Anth	68.4	.31
Freight	20	40	835	72	907	Waste	103.8	.11
Coal:								
Up (empty)	15	165	578	72	660	" }	138	
Down (full)	12	145	1320	72	1392	" }		

"Coal consumed" includes amount used in firing up. "Ton" = 2240 lbs. Th "waste" is finely broken anthracite, or **"culm,"** the refuse of collieries, & burned on the special grates designed for it by Mr. John E. Wootten.

On the Phila & Reading R R, in 1883, the **cost of fuel per freight trai mile** was, for anthracite, 12.4 cts; for "culm," 2.3 cts. The fast passenger engine ran 55 miles in 76 minutes, with 15 full passenger cars, consuming 53 lbs culm pe minute = 73 lbs per mile, and evaporating 55 gallons of water per minute = 76 ga lons per mile.

The average **consumption of bituminous coal by passenge engines,** on 9 roads, in 1882, was 52.44 lbs per mile run; greatest, 67; least, 34.

Wood fuel. A ton (2240 lbs) of good anthracite or bituminous coal is abou equal to 1½ cords of good dry, hard, mixed woods (chiefly white oak); or to 2 cord of such soft ones as hemlock, white, and common yellow pine. Much of the inferic bituminous coal of Illinois is hardly equal (per ton) to a cord of average wood.

A cord is 4 × 4 × 8 ft, or 128 cub ft. A cord of good dry, white oak, (next to hick ory, the best wood for fuel,) weighs 3500 lbs or 1.563 tons. Dry hemlock, white, common yellow pine, (all of them inferior for fuel,) about .9 ton. Perfectly gree woods generally weigh about ⅛ to ¼ more than when partially dried for locomotiv use; in other words, a cord of wood, in its partial drying, loses from ¼ to ⅓ ton o water, and still contains a large quantity of it. Since this water causes a grea waste of heat, green wood should never be used as fuel. The values of woods as fu are in nearly the same proportion as their weights per cord when perfectly dry.

The run of freight and passenger trains throughout the U States is 20 to 40 miles per cord, and 30 to 60 miles per ton; or say 40 to 80 pound of coal per mile; but very heavy freight trains will burn from 100 to 200 pound or more of coal per mile; = 22.4 to 11.2 miles per ton. Much depends upo the adaptation of the engine to the kind of fuel used. A good coal-burner may be ba for wood, and vice versa; so that trials with the same engine may give very erroneou results as to the comparative merits of the two kinds of fuel. When wood is used, abou .2 cord; or when coal, about ⅓ cord of wood, must be used for kindling, and gettin up steam ready for running; and this item is the same for a long run as for a shor one; so that long roads have in this respect an advantage over short ones, in ecor omy of fuel. Wood has the disadvantage of emitting more sparks; and is, moreove nearly twice as heavy as coal, for the performance of equal duty; and is, therefor more expensive to handle. It also occupies 4 or 5 times as much space as coal.

Up grades greatly increase the consumption of fuel. Thus, on a road 95 mile long, with grades mostly of less than 6 ft per mile, and with very few exceeding 1 ft per mile, with coal trains of 734 tons descending, and 291 tons (empty) ascendin at about 10 miles per hour each way, the coal consumption per 100 miles for eac ton of total train (including engine and tender) was 14.5 lbs descending, and 36.6 lb ascending.

On first-class roads **a passenger engine will average about 3500 miles per year,** or say 100 miles per day; a freight engine 25000 miles per yea or say 70 miles per day.

Locomotive expenses per 100 miles run, will average about as follows:

	Passenger.	Freight.
Fuel	3.00	6.00
Water.....	1.00	2.00
Oil, waste, &c.....	.70	.90
Repairs	4.00	8.00
Engineer and fireman.....	5.00	6 00
Putting away, cleaning, and getting out	1 50	2.00
Locomotive superintendence.....	.30	.50
	$15.50	$25.40

This is all that is usually stated in annual reports of expenditures; but inasmuch an engine in active service, even under a judicious system of repairs, generally comes worthless, (except as old iron,) in say 16 years on an average, an additional lowance of about 6 per ct on the first cost, or about $500 to $800 = say $2 per 100 iles, should be made annually for **depreciation of each engine.**

CARS.

sual dimensions, weights, and capacities. Approx prices in 1886. For 4 ft 8½ in gauge.

	Length of body. ft.	Width. ft.	Height above rail. ft.	Weight, empty. lbs.	Nominal capacity, in passengers, or lbs.	Approx cost, 1886. $
ssenger.....	44 to 52*	9⅓ to 10	14	40000 to 50000	50 to 60	4000 to 6000
rlor	50 to 66*	9½ to 11	"	50000 to 60000	30 to 40	8000 to 12000
eper.....	" *	"	"	60000 to 72000	25 to 50	10000 to 15000
ggage, mail, and xpress.....	45 to 55*	9⅔	"	40000 to 50000	.....	2000 to 4000
x and cattle.....	28 to 36½	8 to 9½	11 to 13†	20000 to 28000	40000 to 50000	400 to 550
ndola.....	"	8 to 9¼	6 to 7½‡	16000 to 22000	"	300 to 450
tform.....	"	8½ to 9	3½ to 4½‡	14000 to 19000	30000 to 40000	"
l, 8 wheels.....	22 to 35	8 to 9	7 to 8½‡	18000 to 22000	35000 to 50000	400 to 450
" 4 wheels.....	12 to 16	6 to 8	6½ to 7 ‡	5600 to 9600	11000 to 20000	200 to 250
mp	14	8	6½	9000	18000	100

Add 6 ft in all for the two platforms. These are usually from 3 ft 6 ins to 4 ft above the rails
Add from 1 to 2 ft for projecting brake rod and handle.
Add about 1 ft for projecting brake rod and handle.

On narrow gauge (3 ft and 3½ ft) roads, there is but little uniformity in building. The dimensions and capacities of the cars are now not much less than ose of corresponding cars for standard (4 ft 8½ ins) gauge; while the weights and ts are usually from 15 to 20 per cent less. The following are about the averages: ssenger, 45 ft long, 8⅓ ft wide, $3000. Parlor, 45 ft long, 8⅓ ft wide, $9000, ggage, &c, 35 ft long, 8⅓ ft wide, $1800. Freight and coal, 24 to 30 ft long, 7 to ½ ft wide, $200 to $400.

Dimensions, &c, of iron-frame cars, built by the United States Tube lling Stock Co; offlce, No 3 Broad St, New York.

	Length. ft.	Width. ft.	Depth of body. ft.	Weight, empty. lbs.	Nominal capacity. lbs.	Cost.
For 4 ft 8½ ins gauge.						
and cattle.....	33½	8½	6¼	21500	60000	About 10 per cent greater than that of corresponding first-class wooden cars.
ndola.....	34 to 34½	7½ to 8½	1¼	16000 to 19000	50000 to 60000	
tform.....	34	8	.....	14000	60000	
l, 8 wheels.....	34½	8	2½	18900	60000	
For 3 ft gauge.						
and cattle	28	7	6	18000	40000	
ndola.....	"	"	2	10500	"	
tform.....	"	"	.....	9500	"	
l, 8 wheels.....	"	"	2	10400	"	

or data respecting cars, &c, we are indebted to Jackson & Sharp Co and Dure & Co, Wilmington. ; Allison Mfg Co, makers of cars and dealers in wheels and axles, Phila; Harrisburg (Pa) Car ; Co; Erie (Pa) Car Works, Lim; Youngstown (O) Car Mfg Co; Michigan Car Co, Detroit; Mid-town (Pa) Car Works; J G Brill & Co, Phila.; Gilbert Car Mfg Co, Troy, N Y.

The **average life** of a passenger car is about 16 years. Average **annual pairs,** including painting, $300 to $700; for mail and express cars, $150 to $300; ight cars, $75 to $150; 4-wheel coal cars, $20 to $30. On the Phila & Reading, in 83, repairs were as follows: passenger cars, $0.156 per 100 passenger-miles; freight rs, $0.133 per 100 ton-miles; coal cars, $0.078 per 100 ton-miles.

Allowing 125 lbs per passenger, a *full* car-load of passengers (50 to 60 in number) uld weigh but from 6250 to 7500 lbs, or say 3 tons; while the cars themselves igh say 20 tons, or nearly **7 tons of dead load to 1 paying ton of assengers.** But, as a general rule, passenger trains are not *more* than half-ed; thus making the proportion about 13 to 1. In England, the cars are heavier

in proportion to the number of passengers carried; and the dead load on some lin is from 20 to 30 times the paying load. The above table shows that when **freigh cars** are loaded to their nominal capacity, there is but about **1-2 ton of dea load per ton of paying load**; or, with cars half loaded, 1 to 1.

From the table, p 814, it will be seen that the average cost, in the United State of moving a passenger one mile is 2¼ times that of moving a ton of freight on mile, while the receipts per passenger-mile are not quite double those per freigh ton-mile.

The **resistance of cars to motion,** on a level track, and with cars an track in fair order, is usually taken at about from 6 to 8 lbs per ton of 2240 lb With everything in perfect order, it may fall as low as 5, or even 4, lbs per ton On the other hand, if the wheels are not truly round, and if the journals are nc well lubricated, it may greatly exceed 10 or 12 lbs. See p 374 *e*.

The wheels for passenger and freight cars are usually about 3 to 33 ins diam; and those of coal cars 26 to 28. The cast-iron wheels made by Messi A. Whitney & Sons, of Philada, weigh as follows, per single wheel. Cone, 1 in 32:

Diameter of Wheels.	Usual Gauge.				Narrow Gauge.			
	Single Plate.		Double Plate.		Single Plate.		Double Plate.	
	4 in Tread.	4¾ in Tread.	4 in Tread.	4¾ in Tread.	3½ in Tread.	3¾ in Tread.	3½ in Tread.	3¾ in Tread.
	lbs	lbs	lbs	lbs	lbs	lbs	lbs	lbs
20 in.....	260	270	.....	.....	200 250	205 255	.....	.....
22 in.....	.....	.....	.....	.....	245	255	.....	.....
24 in.....	345 355	360 370	365	380	265 320 330	275 330 340	340	350
26 in.....	360 375	380 395	380	400	285 335 350	300 350 360	345	360
28 in.....	405 440	425 460	440	460	320 365 400	335 380 415	380	395
30 in.....	445 485	470 510	480	505	405 445	425 465	460	470
33 in.....	510 545	540 575	525 545 560	555 575 590	.....	.....	.....	.....
36 in.....	585	640	.....	.....	.....	.....	.....	.....
38 in.....	685	.....	.....	.....	.....	.....	.....	.....

The weights by other makers do not differ from these materially.

The diam of car or engine wheels does not include the flanges; but is th least diam from tread to tread.

Price of cast chilled wheels, in Phila, in 1886, about 2 cts per lb.

Good chilled cast wheels. 33 ins diam, will run about 50,00 miles; usually about 40,000; rarely 60,000 or much higher. 42-inch wheels (rarel used) average about one-third higher. On first-class roads about 1 to 1¼ per cen of all the car-wheels are cracked or broken annually.

In the steel-tired **"paper car-wheels"** of the Allen Paper Car-Wheel Co office 240 Broadway, New York, the hub is of cast-iron or steel; the "center," o main body of the wheel, of compressed straw-board confined between two circula plates of rolled iron; and the tire is of rolled steel. The bolts, which confine the iron plates to the paper center, pass also through flanges on the outside of the hub and the inside of the tire. To secure elasticity, the bolt-holes in the flange of the tire are slightly elongated, and the circular iron plates are made of a little less diam than the inside of the tire, so that the plates and tire do not come into contact, bu the weight of car and load is transferred from the hubs to the tires through the paper centers only. These wheels are now largely used on passenger, parlor, and sleeping cars, and on the trucks of locomotives. The principal sizes for 4 ft 8½ in

gauge are 33 inch and 42 inch; treads, for either diam, 3⅞ and 4¼ ins. A 42-inch **paper wheel costs,** 1886, from $85 to $95, according to width of tread; and veighs 1085 lbs; of which 165 lbs are paper; 550 lbs tire; 160 lbs side-plates; 180 bs hub; and 30 lbs bolts. The steel tires on 42-inch wheels run about 100,000 miles etween turnings, and about 400,000 miles before having to be abandoned. A new ire is then placed upon the old center, at a cost of about $55. Allowance must be ade for the fact that these wheels are generally under sleeping or parlor cars or rst-class passenger cars, on through trains which make few stops; and that they re therefore subjected to less of the destructive action of the brakes than are com- on wheels. Besides, the great majority of the latter are used under freight cars, here they have rough usage, due to the inferior character of the *springs* on such ars, &c. The average cost of turning steel-tired wheels is about $1 to $1.25 per nnum per pair. Axles are said to run several times longer with paper wheels than ith cast-iron ones.

Wheels of cast- and of wrought-iron with steel-tires are being largely used ex- erimentally under high-class cars. They run longer than chilled cast-iron wheels, ut are more costly.

Axles. Standard dimensions adopted by the Master Car Builders' and Master Mechanics' Associations in 1879: **Length,** total, 6 ft 11¼ ins; between ubs, 4 ft 0½ in; each wheel-seat, 7 ins; each journal, 7 ins. **Diam,** at middle, ⅞ ins; at hubs, 4⅞ ins; at journals, 3¾ ins. **Weight,** finished, 347 lbs per xle. The **diam** at middle was increased to 4¼ ins by the Master Car Builders' Association in 1884. This change of course increased the **weight** slightly.

Cost of wrought-iron axles, hammered or rolled, 1886, 2½ to 3 cts per lb.

For Standard Railway Time, see p 396.

RAILROAD STATISTICS.

Art. 1. In the following, most of the figures for 1880 are based upon the **U. S Census** for that year; those for 1884, upon **Poor's Manual.**

IN THE UNITED STATES.

	1880.	1884.
Plant.		
Miles built in one year	7174	3977
(In 1881, 9789; in 1882, 11596.)		
Miles in operation	87801	125152
Gauge, Percentage of all, 1880. Single-track miles.		
3 ft 5	5191	
4 ft 8½ ins 66	71403	
4 ft 9 ins 11	12335	
5 ft (Southern gauge) 11	12282	
Cost of road, exclusive of rolling stock,		
per mile, in dollars	46800	
total, in millions of dollars	4112	
Rolling stock in operation.		
Number of locomotives	17412	24587
" passenger cars	12330	17993
" baggage, mail, and express cars	4475	5911
" freight cars	375312	798399
" other "	80138	
Cost of rolling stock,		
per mile of road, in dollars	4761	
total, in millions of dollars	418	
Cost of road and equipment,		
per mile, in dollars	51561	55330
total, in millions of dollars	4530	6925
Operation.		
For one year.		
Passengers carried one mile, per mile of road	65392	70143
Tons of freight carried one mile, per mile of road	368514	357367
Gross earnings,		
per mile of road, from passengers, dollars	1641	1653
" " " freight "	4740	4018
" " " mails, &c "	230	429
" " total "	6611	6100
per passenger-mile, from passengers, "	.0251	.0236
" ton-mile, from freight "	.0129	.0112
passenger earnings ÷ total earnings	.2483	.2709
freight " ÷ "	.7169	.6588
mail, &c, " ÷ "	.0348	.0703
gross earnings ÷ total investment	.1136	.1102
Expenses. (For details, see Art 3.)		
per mile of road dollars	4019	3970
cost of moving freight, per ton-mile "	.0076	
(Penna R R, 1883, $.0056.)		
cost of moving passengers, per passenger-mile "	.0171	
(Penna R R, 1883, $.0163.)		
expenses ÷ gross earnings	.6078	.6508
Net earnings.		
Net earnings ÷ total investment	.0504	.0385

Art. 2. UNITED STATES BY DIVISIONS, 1884.

	Eastern States.	Middle States.	Southern States.	Western States.	Pacific States.	Total, U. S.
Plant.						
iles in operation	6405	18256	19826	72704	7961	125152
ost of road and equipment per mile, dollars	52166	92306	42338	48418	68549	55330
Operation. For one year.						
ross earnings per mile, $..	9142	12177	3524	5199	4348	6100
xpenses per mile, $	6564	7951	2322	3339	2615	3970
xpenses÷Gross earnings.	.718	.653	.659	.642	.601	.651

Art. 3. Items of total annual expenses for maintenance and operation of all the railroads of the United States in 1880.

	$ per mile of road.	per cent of total.	per cent of earnings.
pairs of road-bed and track	451	11.23	6.82
enewals of rails (total $17243950)	197	4.89	2.97
" " ties (total $10741577)	122	3.04	1.85
pairs of bridges	102	2.55	1.55
" " buildings	87	2.17	1.32
" " fences, crossings, &c	17	.42	.25
legraph expenses	41	1.01	.62
xes	152	3.77	2.29
intenance of road and real estate	1169	29.08	17.67
pairs, &c, of locomotives	249	6.19	3.76
" " passenger, baggage, and mail cars	120	2.99	1.82
" " freight cars	257	6.40	3.89
pairs, &c, of rolling stock (Including renewals and additions.)	.626	15.58	9.47
ssenger train expenses	137	3.41	2.07
eight " "	330	8.21	4.99
el for locomotives	374	9.31	5.66
ter supply, oil, and waste	70	1.74	1.06
ges of locomotive runners and firemen	310	7.72	4.69
ents and station service and supplies	451	11.23	6.82
laries of officers and clerks	139	3.46	2.10
vertising, insurance, legal expenses, stationery, and printing	123	3.06	1.87
mages to persons and property	40	.98	.60
ndries	250	6.22	3.78
nning and general expenses	2224	55.34	33.64
gregate annual expenses	4019	100.00	60.78

Each of these items is, however, subject to great variation, not only on diff roads, t on the same road, from year to year. A road with many bridges, deep cuts, high

embkts, &c, to keep in repair, will have heavier maintenance of way than one whi has but few; and this item may be but small one year, and twice as great the ne Fuel may be cheap on one road, and dear on another; thus materially affecting t item of motive power. And so with the other items. Sometimes maintenance way exceeds motive power and cars together; at others, conducting transportati is fully half the total expense.

The total annual expenses on railroads in the United Stat nsually range between 65 and 130 cents per train mile; that is, per mile actua run by trains. Also, between 1 and 2 cents per ton of freight, and per passeng carried one mile. When a road does a very large business, and of such a charac that the trains may be heavy, and the cars full, (as in coal-carrying roads,) the e pense per train mile becomes large; but that per ton or passenger small; and vi versa, although on coal roads half the train miles are with empty cars.

Art. 4. Gross annual earnings per mile, per passeng mile, and per ton mile, of some of the principal U S rai roads in 1880.

	Length miles.	From passrs per mile of road.	From passrs per passr mile.	From frt per mile of road.	From 1 per to mile.
Pennsylvania R R	1806	$4700	$.0242	$15615	$.008
New York Central & Hudson River	994	6651	.0200	21794	.008
Baltimore & Ohio	1487	1812	.0206	10310	.008
Central Pacific	2447	2237	.0303	4577	.024
Chicago, Burlington, & Quincy	1805	1532	.0240	7202	.011
Philadelphia & Reading	780	3429	.0201	17200	.016
Union Pacific	1215	2624	.0320	7154	.019
Wabash, St Louis, & Pacific	1730	1220	.0271	4382	.008
Atchison, Topeka, & Santa Fé	1398	1144	.0606	3974	.020
Average of United States	87801	1641	.0251	4740	.012

Art. 5. Annual earnings and expenses of some of the pri cipal railroads of the United States in 1880.

	Length miles.	Gross earnings per mile of road.	Expenses per mile of road.	Expen ÷ gros earnin
Pennsylvania R R	1806	$20,315	$12,267	.585
New York Central & Hudson River	994	28,445	17,969	.609
Baltimore & Ohio	1487	12,122	7,035	.571
Central Pacific	2447	6,814	3,340	.470
Chicago, Burlington, & Quincy	1805	8,734	4,454	.497
Philadelphia & Reading	780	20,629	11,754	.568
Union Pacific	1215	9,778	4,507	.426
Wabash, St Louis, & Pacific	1730	5,602	3,942	.678
Atchison, Topeka, & Santa Fé	1398	5,118	2,408	.458
Total, United States	87801	6,611	4,019	.608

The following table of expenses in past years will serve for comparison with t above.

Table of Annual Expenses of some U S Railroads.*

Names of Companies.	Per Mile of Road. $	Per Train Mile. cts.	Per ct. of Recpts.
igh Valley, 1860..	.....	103	50
" " 1862	.....	178	79½
" " 1868 and 1869 .....about...	.....	.....	57
" " 1872	.....	.....	65½
timore & Ohio, main stem, 1859	4434	54	46½
" " " " 1860	4254	49	41
" " " " 1865	.....	.....	56
" " " " 1866	.....	.....	58
" " " " 1872	.....	.....	57½
t Tennessee & Georgia, 1872	.....	.....	69½
nphis & Charleston, 1860	2617	89½	46½
rgia Central, 1872.	4180	105	50
na Central, main line from Phila to Pittsburg, 358 miles, 1859, exclusive of State tonnage tax	7848	87	53½
" " 1860, " " " " "	.....	.....	56
" " 1861, " " " " "	.....	.....	50
" " 1868, tonnage tax repealed	.....	.....	69
" " 1869, " " " .....about...	32000	.....	70¾
" " 1872, " " "	.....	.....	63¼
la & Reading, 1859	.....	.....	54¼
" 1860	.....	91	51
" 1868, 365 miles of main road and branches...	17200	144	71
" 1869	.....	.....	61⅓
" 1872	.....	.....	66½
th Pennsylvania, 1860, 54 miles long	3213	76	45
" 1862.	3240	68	45
" 1867.	9534	.....	58
" 1868.	.....	.....	57
" 1872.	.....	.....	55
necticut; average of all the railroads, 1861	3781	95	57
ssachusetts; " " " " 1861	3785	85	60
" averages of 19 years previous	2700 to 4300	70 to 110	
" 1867 .....average...	.....	.....	71
ena & Chicago, 1859	.....	93	60
" 1860	3102	73	55½
la, Wilmington & Baltimore, main stem, 1859	4586	116	41
" " " " 1860.	7100	167	58
" " " " 1861.	7785	.....	51
" " " " 1867.	17380	187	68½
v York; all the R R in the State, average,† 1859	4964	106	68½
" " " " " " 1861	5100	.....	65
" " " " " " 1867	13856	.....	76
v Jersey R R and Transportation, 1861	12213	84½	41½
isville & Nashville, 1861.	.....	85	38
la & West Chester, 1861, 27 miles.	2274	59	47
" 1862.	2282	.....	44
" 1872.	7030	104	62
la, Germantown & Norristown, 1861, 20 miles.	6405	56	50
" " 1862.	6405	53¾	48½
" " 1867.	18208	128	60¾
v York & Erie, 1861	6461	.....	64½
" " 1867, with its branches, 784 miles in all	14545	177	79½
v York Central, 1861	8360	.....	63½
" " 1867, with its branches, 696 miles in all....	15620	170	77¾
glish R R, averages for 1856–7–8	.....	66	50
tch " " " " " "	.....	56	44
h " " " " " "	.....	52	40

Annual reports often omit the lengths of the roads and branches; and as these frequently vary
year to year, it is possible that the table may contain some errors in the first column.
528 miles in operation. Total exps equalled 1.56 cts per passenger or ton carried 1 mile. Dead
ht of cars, equal to 1.19 tons per passenger; and to 1.74 tons per ton of freight.

Art. 6. Statistics of several U. S. **narrow-gauge railroads** for 188 from Poor's Manual.

	Gauge, Feet.	Length, Miles.	Rolling Stock.				Cost of road and equipment per mile.	Gross Annual Earnings per mile of road.	Annual Expenses per mile of road.	Expenses ÷ gross
			Locomotives.	Passr Cars.	Mail, &c, Cars.	Freight Cars.				
Bridgton & Saco River, Maine.....	2	16	2	2	1	16	$12167	$1112	$ 834	.7
Profile & Franconia Notch, N H.....	3	14	3	7	....	6	15430	1346	640	.4
Camden, Gloucester & Mt Ephraim, N J.	3	6	....	....	....	....	13645	2868	2642	.9
Bradford, Bordell & Kinzua, Pa.....	3	39	5	5	2	69	14922	1793	1717	.9
Denver & Rio Grande, Col.....	3	1685	239	115	71	5676	35000	3519	2573	.7

The weights of the steel rails used on narrow gauge roads vary from 30 to 40 per yard.

Art. 7. Miles of railroad in the world at the close of 1883. Amer (U S, 125152), 143335. Europe, 114313. Total, 279856.

In **Great Britain,** in 1883, there were 16000 miles of railroad. Gross ea ings for *half* year, $10130 per mile. Expenses for *half* year, $5364 per mile. penses ÷ Gross earnings = .53.

GLOSSARY OF TERMS.

bacus; the flat square member on top of a column.
bsciss or *abscissa;* any portion of the axis of a curve, from the vertex to any point from which he leaves the axis at right angles, and extends to meet the curve itself; said line being called an *inate.* An abscise and ordinate together are called *co-ordinates.*
cclivity; an upward slope, or ascent of ground, &c.
dit; a horizontal passage into a mine, &c.
dze; a well-known curved cutting instrument, for dressing or chipping horizontal surfaces.
lternating motion; up and down, or backward and forward, instead of revolving, &c.
ngle-bead, or plaster bead: a bead nailed to projecting angles in rooms, to protect the plaster on r edges from injury.
ngle-block; a triangular block against which the ends of the braces and counters abut in a Howe ge.
ngular velocity. See p 365.
nneal; to toughen some of the metals, glass, &c, by first heating them, and then causing them to very slowly. This process however lessens the tensile strength.
nticlinal axis; in geology; a line from which the strata of rocks slope away downward in oppo- directions, like the slates on the roof of a house; the ridge of the roof representing the axis.
pex; a point in either chord of a truss, where two web members meet.
pron; a covering of timber, stone, or metal, to protect a surface against the action of water flow- over it. Has many other meanings.
rbor. See Journal.
rchitrave; that part of an entablature which is next above the columns. Applies also when there no columns. Also, the mouldings around the sides and tops of doors and windows, attached to er the inner or outer face of the wall.
rris; a sharp edge formed by any two surfaces which meet at an angle. The edges of a brick are ses.
shler; a facing of cut stone, applied to a backing of rubble or rough masonry, or brickwork.
strayal; a small moulding, about semi-circular or semi-elliptic, and either plain or ornamented by ving.
xis; an imaginary line passing through a body, which may be supposed to revolve around it; as diam of a sphere. Any piece that passes through and supports a body which revolves; in which it is called an axle, or shaft.
xle-box. See Journal-box.
xletree; an axle which remains fixed while the wheel revolves around it, as in wagons, &c.
zimuth. The azimuth of a body is that arc of the horizon that is included between the meridian le at the given place, and another great circle passing through the body.
acking; the rough masonry of a wall faced with finer work. Earth deposited behind a retaining- l, &c.
alance-beams; the long top beams of lock-gates, by which they are pushed open or shut.
alk; a large beam of timber.
allast; broken stone, sand or gravel, &c, on which railroad cross-ties are laid.
all-cock; a cistern valve at one end of a lever, at the other end of which is a floating ball. The rises and falls with the water in the cistern; and thus opens or shuts the valve.
all-valve. See Valve.
argeboards; boards nailed against the outer face of a wall, along the slopes of a gable end of a se, to hide the rafters, &c; and to make a neat finish.
ascule bridge; a hinged lift-bridge furnished with a counterpoise.
atter, (sometimes affectedly *batir,*) or *talus;* the sloping backward of a face of masonry.
ay; on bridges, &c, sometimes a panel; sometimes a span.
ead; an ornament either composed of a straight cylindrical rod; or carved or cast in that shape ny surface.
earing; the course by a compass. The span or length in the clear between the points of support beam, &c. The points of support themselves of a beam, shaft, axle, pivot, &c.
ed-mouldings; ornamental mouldings on the lower face of a projecting cornice, &c.
ed-plate; a large plate of iron laid as a foundation for something to rest on.
eetle; a heavy wooden rammer, such as pavers use.
ell-crank. See Crank.
ench-mark; a level mark cut at the foot of a tree for future reference, as being more permanent n a stake.
erm, or *berme;* a horizontal surface, as if for a pathway, and forming a kind of step along the face sloping ground. In canals, the level top of the embankment opposite and corresponding to the path is called the berm.
Bessemer steel is formed by forcing air into a mass of melted cast iron; by which means the excess carbon in the iron is separated from it, until only enough remains to constitute cast steel. The rbon is *chemically* united with the steel, but *mechanically* with the iron.
Beton; concrete of hydraulic cement, with broken stone and bricks, gravel, &c.
Bevel; the slope formed by trimming away a sharp edge, as of a board, &c. Edges of common awing rulers and scales are usually bevelled. See 13. p 613.
Bevel gear; cog-wheels with teeth so formed that the wheels can work into each other at an angle.
Bilge; the nearly flat part of the bottom of a ship on each side of the keel. Also, the swelled part a barrel, &c. To bilge is to spring a leak in the bilge, or to be broken there.
Bitts; the small boring points used with a brace.
Blast-pipes; in a locomotive; those through which the waste steam passes from the cylinder into e smoke-pipe, and thus creates an artificial draft in the chimney, or smoke-pipe.
Boasting; dressing stone with a broad chisel called a boaster, and mallet. The boaster gives a noother surface after the use of the point, or the narrow chisel called a tool.
Body; the thickness of a lubricant or other liquid. Also, the *measure* of that thickness, expressed the number of seconds in which a given quantity of the oil, at a given temperature, flows through given aperture.

Bolster; a timber, or a thick iron plate, placed between the end of a bridge and its seat on abutment.

Bond; the disposing of the blocks of stone or brickwork so as to form the whole into a firm st ture, by a judicious overlapping of each other, so as to break joint. Applies also to timber, &c various ways.

Bonnet; a cap over the end of a pipe, &c. A cast-iron plate bolted down as a covering over aperture.

Bore; inner diameter of a hollow cylinder.

Borrow-pit; a pit dug in order to obtain material for an embankment.

Boss; an increase of the diameter at any part of a shaft for any purpose. A projection in sh of a segment of a sphere, or somewhat so, whether for use or for ornament; often carved, or cast

Box-drain; a square or rectangular drain of masonry or timber, under a railroad, &c.

Brace; a kind of curved handle used for boring holes with bitts. The head of the brace rema stationary, being pressed against by the body of the person using it, while the other part with bitt is turned round by his hand. Also, an inclined beam, bar, or strut, for sustaining compressi

Bracket; a projecting piece of board, &c, frequently triangular, the vertical leg attached to face of a wall, and the horizontal one supporting a shelf, &c. Often made in ornamental shapes, supporting busts, clocks, &c. Also, the supports for shafting: as pendent, wall, and pedestal brack

Brake; an arrangement for preventing or diminishing motion by means of friction. The fric is usually applied at the circumference of a revolving wheel, by means of levers. On railroads, car-brakes should be worked by steam, as those of Loughridge, Westinghouse, and Creamer. A such a handle as that of a common pump.

Brass is composed of copper and zinc.

Brasses; fittings of brass in many plummer-blocks, and in other positions, for diminishing friction of revolving journals which rest upon them.

Braze; to unite pieces of iron, copper, or brass, by means of a hard solder, called spelter sold and composed, like brass, of copper and zinc, but in other proportions.

Break joint; to so overlap pieces that the joints shall not occur at the same place, and thus p duce a bad bond.

Breast-summer; a beam of wood, iron, or stone, supporting a wall over a door or other openi a kind of lintel.

Breast-wall; one built to prevent the falling of a vertical face cut into the *natural* soil; in tinction to a retaining-wall or revetment, which is built to sustain earth *deposited* behind it.

Breech; the hind part of a cannon, &c.

Bridge, or *bridge-piece*, or *bridge bar;* a narrow strip placed across an opening, for support something without closing too much of the opening.

Bronze is composed of copper and tin.

Bulkhead; on ships, &c, the timber partitions across them. Also, a long face of wharf para to the stream.

Buoy; a floating body, fastened by a chain or rope to some sunk body, as a guide for finding latter. Sometimes also used to indicate channels, shoals, rocks, &c.

Burnish; to polish by rubbing; chiefly applies to metals.

Bush; to line a circular hole by a ring of metal, to prevent the hole from wearing larger. A when a piece is cut out, and another piece neatly inserted into the cavity, the last piece is sometim said to be bushed in; sometimes it is called a plug.

Butt-joint; one in which the ends of the two pieces abut together without overlapping, and joined by one or more separate pieces called covers or welts, which reach across the joint and fastened to both pieces.

Buttress; a vertical projecting piece of brickwork or masonry, built in front of a wall strengthen it.

Caisson; a large wooden box with sides that may be detached and floated away.

Caliber; the inner diameter, or bore.

Calipers; compasses or dividers with curved legs, for measuring outside and inside diameters.

Calk..or *caulk:* to fill seams or joints with something to prevent leaking.

Calking iron, a tool for forcing calking into a joint.

Cam, or *cam*, or *wiper;* a piece fixed upon a revolving shaft in such a manner as to produce alternating or reciprocating motion in something in contact with the cam. An eccentric.

Camber; a slight upward curve given to a beam or truss, to allow for settling.

Camel; a kind of barges or hollow floating vessels, which, when filled with water, are fastened the sides of a ship; and the water being then pumped out, they rise by their buoyancy; and lift ship so that she can float in shallower water.

Cantilevers; projecting pieces for supporting an upper balcony, &c.

Cants, rims, or *shroudings;* the pieces forming the ends of the buckets of water-wheels, to prev the water from spilling endwise.

Capstan; a long hollow rope-drum surrounding a strong vertical pivot, upon the head of whic rests, and around which it turns. Its top is a thick projecting circular piece, having holes around outer edge or circumference, for the insertion of the ends of levers; or capstan-bars. It is a kind vertical windlass.

Case-harden; to convert the outer surface of wrought iron into steel, by heating it while in cont with charcoal.

Casemate; in fortification: the small apartment in which a cannon stands.

Castors; rollers usually combined with swivels; as those used under heavy furniture, &c.

Causeway; a raised footway or roadway.

Cavetto; a moulding consisting of a receding quadrant of a circle.

Cementation; the process of converting wrought iron into steel, by heating it in contact with ch coal. This process produces blisters on the steel bars; hence *blister* steel. These are removed, the steel compacted, by reheating it, and then subjecting it to a tilt-hammer. It is then *tilted* st or *shear* steel. Or if the blister steel is broken up; remelted; and then run into ingots or blocks is called *cast*, or *ingot steel;* which is harder and closer-grained than tilted steel. It may be softe and thus become less brittle, by annealing. The ingots may be converted into bars by either roll or hammering, the same as shear and blister.

Center; the supports of an arch while being built.

Center of gravity. See p. 347, &c.

Center of gyration. See Radius of Gyration p 440.

ıter of oscillation, or *of vibration*. See Rem 2, of Pendulums, p 365.
ıter of percussion, in a moving body, is that point which would strike an opposing body with er force than any other point would. If the opposiug body is immovable, it will receive *all* the of a rigid moving body which strikes with its center of percussion. Sec Pendulum, page 365.
ispool; a shallow well for receiving waste water, filth, &c.
amfer; means much the same as bevel; but applies more especially when two edges are cut away to form either a chamfer-groove, (see 14, p 613, of Trusses.) or a projecting sharp edge.
eeks; two flat parallel pieces confiniug something between tnem. See *w*, at 15, of Figs 21½, of ses, p 583.
illing, chill-hardening, or *chill-casting;* giving great hardness to the outside of cast-iron, by ng it into a mould made of iron instead of wood. The iron mould causes the outside or skin of sting to cool very rapidly; and this for some unknown reason increases its hardness. This pro s frequently confounded with case-hardening.
ock; any piece used for filling up a chance hole, or vacancy.
uck; the arrangement attached to the revolving shaft, arbor, or mandril of a lathe, for holding hing to be turned.
urn-drill; a long iron bar, with a cutting end of steel; much used in quarrying, and worked by ng it and letting it fall. When worked by blows of a hammer or sledge it is called a jumper.
ma, or *cyma;* a moulding nearly in shape of an S. When the upper part is concave, it is called a recta; when convex, a cima reversa. See page 151.
ack valve. See Valves.
amp; a piece fastened by tongue and groove, transversely along the end of others, to keep them warping. A kind of open collar, which, being closed by a clamp-screw, holds tight what it surds. See Cramp.
ap boards; short thin boards, shingle-shaped, and used instead of shingles.
aw, a split provided at the end of an iron bar, or of a hammer, &c, to take hold of the heads of or spikes for drawing them out; as in a common claw-hammer.
eat; a piece merely bolted to another to serve as a support for something else; as at 7, 8, 10, . 613, of Trusses. Often used on shipboard for fastening ropes to, as at 11. Also a piece of 1 nailed across two or more other boards, for holding them together, as is often done in tempodoors, &c.
evis. See Shackle.
ck. See Ratchet.
p; a fastening like that on the tops of the Y's of a spirit level; being a kind of half collar opening hiuge.
tch; applied to various arrangements at the ends of separate shafts, and which by clutching or ing into each other cause both shafts to revolve together. A kind of coupling.
ck; a kind of valve for the discharge of liquids, air, steam, &c.
efficient; or a *Constant* of friction, safety, or strength, &c, may usually be taken to be a numwhich shows the proportion (or rather the ratio) which friction, safety, tensile strength, &c, bear certain something else which is not generally expressed at the time, but is well understood. Thus, n we say that the coeff of friction of one body upon another is $\frac{1}{10}$, &c. it is understood that the ion is in the proportion of $\frac{1}{10}$th of the *pressure* which produces it. A coeff of safety of 3, means the safety has a proportion or ratio of 3 to 1 to the *theoretical breaking load*. A coeff of 500 lbs, 20 tons, &c, of tensile strength of any material, denotes that said strength is in the proportion 0 lbs, or of 20 tons, &c, to each *square inch of transverse section*. &c. Same as *Modulus*.
ffer-dam; an enclosure built in the water, and then pumped dry, so as to permit masonry or work to be carried on inside of it.
g; the tooth of a cog-wheel.
llar; a flat ring surrounding anything closely.
llar-beam; a horizontal timber stretching from one to another of two rafters which meet at top; above the main tie-beam. See 21, p 613.
ncrete; artificial stone formed by mixing broken stone, gravel, &c, with common lime. When raulic cement is used instead of lime, the mixture is called beton. The terms "lime concrete" "cement concrete" would be convenient.
nnecting-rod; a piece which connects a crank with something which moves it, or to which it s motion.
nsole; a kind of ornamental bracket, somewhat in shape of an S; much used in cornices, &c, supporting ornamental mouldings above it.
ping; flat plates of stone, iron, &c, placed on the tops of walls exposed to the weather.
rbel; a horizontal projecting piece which assists in supporting one resting upon it which projects farther.
re; anything serving as a mould for anything else to be formed around. A term much used in ndries.
rnice; the ornamental projection at the eaves of a building, or at the top of a pier, or of any other ecture.
tter-bolt, or *key-bolt;* a bolt which, instead of a screw and nut at one end, has a slot cut through ear that end, for the insertion of a wedge-shaped key or cotter, for keeping it in its place. Sometes the ends of these keys are split, so as to spread open after being inserted, so as not to be jolted of place.
unterfort; vertical projections of masonry or brickwork built at intervals along the *back* of a wall trengthen it; and generally of very little use.
unter-shaft; a secondary shaft or axle which receives motion from the principal one.
untersunk. See Ream.
unter-weight; or counter-balance; any weight used to balance another.
uplings; a term of very general application to arrangements for connecting two shafts so that y shall revolve together.
ver; see "butt-joint."
ver; In re-rolling iron and steel from piles of small pieces, a large bar or slab, called a cover, of same width and length as the pile, is employed to form the bottom of the pile, and a similar slab the top. The covers serve to hold the pile together; and, after rolling, they form unbroken top bottom surfaces of the finished plate, bar, rail, I beam, &c.

Crab; a short shaft or axle, which serves as a rope-drum in raising weights; and is revolved ei by cog-wheels, a winch, or by levers or handspikes, inserted in holes around its circumference li windlass, or capstan, of which it is a variety. It may be either vertical or horizontal. It is o set in a frame, to be carried from place to place. Also the *whole machine* is called a crab.

Cradle; applied to various kinds of timber supports, which partly enclose the mass sustained.

Cramp; a short bar of metal, having its two ends bent downward at right angles for insertion two adjoining pieces of stone, wood, &c, to hold them together. Much used at the ends of coping-sto Also a similar bent piece, with a set-screw passing through one of the bent ends, for holding th tight between it and the other end. This last is also called a clamp.

Crane; a hoisting machine consisting of a revolving vertical *post* or *stalk;* a projecting *jib;* a *stay* for sustaining the outer end of the jib. The stay may be either a strut or a tie. There also cog-wheels, a rope drum or barrel, with a winch, ropes, pulleys, &c. In a crane the post, and stay do not change their relative positions, as they do in a derrick.

Crank; a double bend at right angles, somewhat like a Z, at the end of a shaft or axle, and form a kind of handle by which the axle may be made to revolve. Sometimes, as in common grindsto this crank is formed of a separate piece removable at pleasure. That part of this piece which has square opening in it for fitting it to the square end of the axle, is called the *crank-arm;* and the o part the *crank-handle. A bell-crank* consists of 4 bends at right angles at the center of an axle, fo ing in it a kind of U. A *double crank* consists of two bell cranks arranged thus, $_{U}^{\Omega}$. The ben the U forms the *crank-wrist.* The term bell-crank is applied also to those used in fixing common dw ing house bells: and to larger ones on the same principle. A *crank-pin* is a pin projecting from volving *wheel,* disk, or other body, and serving as a crank-handle. A *crank-shaft* is a shaft wi has a crank in it, or at its end. A *cranked* shaft has it *in* it only. A ship or other vessel is sai be crank when its breadth is so small in proportion to its depth as to make it liable to upset easily when the same liability is caused by want of sufficient ballast.

Crest; that top part of a dam over which the water pours.

Cross-cut saw; a large horizontal saw worked by two men, one at each end.

Cross-head; a piece attached across the end (or near it) of another piece, and at right angles t so as to form a kind of T or cross. Often seen on piston rods, which they serve to keep in plac resting on the slides, or guides.

Crowbar; a bar of iron used as a lever for various purposes; often pointed at one end.

Crown, or *contrate wheel;* a cog-wheel in which the teeth stand not upon its outer circumferenc usual, but upon the plane of its circle.

Curb; a broad flat circular ring of wood, iron, or stone, placed under the bottoms of circular w as in a well, or shaft, to prevent unequal settlement; or built into the walls at intervals, for the s purpose. Has many other meanings.

Cut-off; an arrangement for cutting off the steam from a cylinder before the piston has made full stroke. Also a channel cut through a narrow neck of land, to straighten the course of a rive

Cutwater, or starling; the projecting ends of a bridge pier, &c, usually so shaped as to allow wa ice, &c, to strike them with but little injury.

Damper; a door or valve to regulate the admission of air to a furnace, stove, &c.

Dead load; the cars, engine, &c, in a train; non-paying load.

Dead-load; in a bridge, the weight of the bridge itself, with flooring, roof, &c; as distinguis from the *live* load of passing trains, vehicles, pedestrians, &c.

Dead points: those two points in the revolution of a crank, where the crank arm is parallel w the rod which connects it with the moving power; and at which said rod exerts no tendency to t the crank.

Declination, of the sun, or of a star, is its angle north or south of the earth's equator at the t of observation.

Declivity; a downward slope or descent of ground, &c.

Dentils; blocks constituting ornaments in a cornice; placed at short intervals apart, they resem teeth. When, instead of mere blocks, they are handsomely carved in various shapes, they are ca modillions.

Derrick; a kind of crane, differing from common ones, chiefly in the fact that the rope or ch which forms the stay may be let out or hauled in at pleasure, thus raising or lowering the inclina of a jib; thereby enabling the raised load to be placed vertically at the required spot. This can be done with a crane, which, therefore, is not as well adapted for laying heavy masonry, especi at great heights.

Diaphram; a thin plate or partition placed across a tube or other hollow body.

Die; that part of a stamp that gives the impression. Dies are also two flat plates of hardened s on an edge of each of which is hollowed out a semicircular half of a short female screw. When t plates are put in contact they form a complete female screw, like that in a nut; and being stro held together by an iron boxing called the die-stocks, which have long handles for revolving them, t constitute a mould or cutter for forming threads on a male screw. Also the main body of a pedes

Dip; in geology, either the angle which the slope of a stratum forms with a horizontal; or direction by compass, toward which it slopes. In surveying, the inclination at which an unbalan compass-needle rests on its pivot after being magnetized.

Disk; a flat circular piece.

Dock; an artificial enclosure, either partial or total, in which ships and other vessels are pla for being loaded or unloaded, or repaired. The first is a *wet* dock; the last a *dry* one.

Dog-iron; a short bar of iron, forming a kind of cramp, with its ends bent down at right an and pointed, so as to hold together two pieces into which they are driven. Often used for tempor purposes. It is also called a dog-iron when only one end is bent down and pointed for driving,

her end being formed into an eye or a handle by which the piece into which the other end is driven ay be hauled or towed away.

Donkey-engine; a small steam engine attached to a large one, and fed from the same boiler. It is ied for pumping water into the boiler.

Double crank. See Crank.

Double keys. See K. of Trusses; page 613.

Dovetail; a joint like 20. page 613; it is a poor one for timber when there is much strain, ing then apt to draw out more or less.

Dowel; a straight pin of wood or metal, inserted part way into each of two faces which it unites.

Draft; the depth to which a floating vessel sinks in the water; in other words the water it draws.

Draught; a drawing. A narrow level stripe which a stonecutter first cuts around the edges of a ough stone, to guide him in dressing off the face thus enclosed by the draught.

Draw-plate; a plate of very hard steel, pierced with small circular holes of different diameters, rough which in succession rods of iron are drawn, and thus lengthened out into wire. Sometimes ie holes are drilled through diamond or ruby, &c, instead of steel.

Drift; a horizontal or inclined passage-way, or small tunnel, in mines, &c. To float away with a irrent. Trees, &c, carried along by freshets.

Drip; a small channel cut under the lower projecting edge of coping, &c, so that rain when it eaches that point will drip or fall off, instead of finding its way horizontally beneath to the wall, hich it would make damp.

Drop; short pieces of nearly complete cylinders, placed at small distances apart, in a row like eeth, as an ornament to cornices, &c.

Drum; a revolving cylinder around which ropes or belts either travel or are wound. When nar- ow and used with belts they are called pulleys.

Dry-rot; decay in such portions of the timber of houses, bridges, &c, as are exposed to dampness, specially in confined warm situations. The timber in cellars and basement stories is more liable to than in other parts, owing to the greater dampness absorbed by the brickwork from the ground. Contact with lime or mortar hastens dry rot. The ends of girders, joists, &c, resting on damp walls, ay be partially protected by placing pieces of slate or sheet iron under them. The painting or tar- ing of *unseasoned* timber expedites internal dry rot. A thorough soaking of timber in a solution of 8 grains of quicklime to 1 gallon of water is said to be a preventive of dry-rot; but the best process or that purpose is saturation with creosote or carbolic acid.

Dyke; mounds of earth, &c, built to prevent overflow from rivers or the sea. A kind of geological regularity or disturbance, consisting of a stratum of rock injected as it were by volcanic action, be- ween or across strata of rocks of another kind. A levee.

Eccentric; a circular plate or pulley, surrounded by a loose ring, and attached to a revolving haft, and moving around with it, but not having the same center; for producing an alternate motion. Often used instead of a crank, as they do not weaken the axle by requiring it to be bent. There are many modifications.

Escarpment; a nearly vertical natural face of rock or soil.

Escutcheon; the little outside movable plate that protects the keyhole of a lock from dust.

Eye; a circular hole in a flat bar, &c, for receiving a pin, or for other purposes.

Eye and strap; a hinge common for outside shutters, &c, one part consisting of an iron strap one nd of which is forged into a pin at right angles to it; and the other part, of a spike with an eye, hrough which the pin passes. When the eye is on the strap, and the pin on the spike, it is called a ook and strap. Such hinges are sometimes called "backflaps."

Eye-bolt; a bolt which has an eye at one end.

Face-wall; one built to sustain a face cut into natural earth, in distinction to a retaining-wall, which supports earth deposited behind it.

Fall; the rope used with pulleys in hoisting.

False-works; the scaffold, center, or other temporary supports for a structure while it is being uilt. In very swift streams it is sometimes necessary to sink cribs filled with stone, as a base for alse-works to foot upon.

Fascines; bundles of twigs and small branches, for forming foundations on soft ground.

Fatigue; of materials; the increase of weakness produced by frequent bending; or by sustaining heavy loads for a long time.

Faucet; a short tube for emptying liquids from a cask, &c; the flow is stopped by a spigot. The wider end of a common cast-iron water or gas pipe.

Feather; a slightly projecting narrow rib lengthwise of a shaft, and which, catching into a corre- sponding groove in anything that surrounds and slides along the shaft, will hold it fast at any required part of the length of the feather. Has other applications.

Feather-edge; when one edge of a board, &c, is thinner than the other.

Felloe, or *felly;* the circular rim of a wheel, into which the outer ends of the spokes fit; and which is often surrounded by a tire.

Felt; a kind of coarse fabric or cloth made of fibres of hair, wool, coarse paper, &c, by pressure, and not by weaving.

Fender; a piece for protecting one thing from being broken or injured by blows from another; frequently vertical timbers along the outer faces of wharves, to prevent injury from the rubbing of vessels.

Fender-piles; piles driven to ward off accidental floating bodies.

Ferrule; a broad metallic ring or thimble put around anything to keep it from splitting or breaking. A small sleeve.

Fillet; a plain narrow flat moulding in a cornice, &c. See Flatband.

Fish; to join two beams, &c, by fastening other long pieces to their sides.

Flags; broad flat stones for paving.

Flange; a projecting ledge or rim.

Flashings; broad strips of sheet lead, copper, tin, &c, with one edge inserted into the joints of brickwork or masonry an inch or two above a roof, &c; and projecting out several inches, so as to be flattened down close to the roof, to prevent rain from leaking through the joint between the roof and the brick chimney, &c, which projects above it.

Flasks; upper and lower; the two parts of the box which contains the mould into which melted iron is poured for castings.

Flatting; causing painting to have a dead or dull, instead of a glossy finish, by using turpentine instead of oil in the last coat.

Fliers; a straight flight of steps in a stairway.

Floodgate; a gate to let off excess of water in floods, or at other times.

Flume; a ditch, trough, or other channel of moderate size for conducting water. The ditches c culverts through which surplus water passes from an upper to a lower reach of a canal.

Flush; forming an even continuous line or surface. To clean out a line of pipes, sewers, gutter &c, by letting on a sudden rush of water. The splitting of the edges of stones under pressure.

Fluxes; various substances used to prevent the instantaneous formation of rust when welding tw pieces of hot metal together. Such rust would cause a weak weld. Borax is used for wrought iron a mixture of borax and sal ammoniac for steel; chloride of zinc for zinc; sal ammoniac for coppe or brass; tallow or resin for lead.

Fly-wheel; a heavy revolving wheel for equalizing the motion of machinery.

Foaming; an undue amount of boiling, caused by grease or dirt in a boiler.

Follower; any cog-wheel that is driven by another; that other is the *leader.* See also p 615.

Forceps; any tools for holding things, as by pincers, or pliers.

Forebay, or penstock; the reservoir from which the water passes immediately to a water-wheel.

Forge; to work wrought iron into shape by first softening it by heat, and then hammering it int the required form.

Forge-hammer; a heavy hammer for forging large pieces; and worked by machinery.

Foxtail; a thin wedge inserted into a slit at the lower end of a pin, so that as the pin is drive down, the wedge enters it and causes it to swell, and hold more firmly.

Frame; to put together pieces of timber or metal so as to form a truss, door, or other structure The thing so framed.

Friction-rollers; hard cylinders placed under a body, that it may be moved more readily than by sliding.

Friction-wheels; wheels so placed that the journals of a shaft may rest upon their rims, and th be enabled to revolve with diminished friction. See page 374 a.

Frieze; in architecture, the portion between the architrave and cornice. The term is often applie when there is no architrave.

Fulcrum; the point about which a lever turns.

Furrings; pieces placed upon others which are too low, merely to bring their upper surfaces up t a required level; as is often done with joists, when one or more are too low; a kind of check.

Fuze, or fuse; to melt. A slow match, which, by burning for some time before the fire reaches th powder, gives the men engaged in blasting, time to get out of the way of flying fragments of stone.

Gasket; rope-yarn or hemp, used for stuffing at the joints of water-pipes, &c.

Gearing; a train of cog-wheels. *Now much supplanted by belts.*

Gib; the piece of metal somewhat of this shape, ⊣, often used in the same hole with a wedge shaped key for confining pieces together. In common use for fastening the strap to the stub-end of the connecting-rod of an engine.

Gin; a revolving vertical axis, usually furnished with a rope-drum, and having one or more long arms or levers, by means of which it is worked by horses walking in a circle around it. Used for hoisting. Cotton-gin, a machine for separating cotton from its seeds.

Girder; a beam larger than a common joist, and used for a similar purpose.

Glacis; in fortification, an easy slope of earth.

Gland. See Stuffing-box. Also, a kind of coupling for shafts.

Glue; a cement for wood, prepared chiefly from the gelatine furnished by boiling the parings of hides. Good glue will hold two pieces of wood together with a force of from 400 to 750 lbs per sq in.

Governor; two balls so attached to an upright revolving axis as to fly outward by their centrifugal force, and thus regulate a valve.

Grapnel; a kind of compound hook with several curved points, for finding things in deep water.

Grillage; a kind of network of timbers laid crossing each other at right angles; frequently placed on the heads of piles, for supporting piers of bridges, and other masonry. See p 641.

Groin; an arch formed by two segmental arches or vaults intersecting each other at right angles. Also, a kind of pier built from the shore outward, to intercept shingle or gravel).

Groove; a small channel. A triangular one is called a chamfered groove.

Ground-swell; waves which continue after a storm has ceased; or caused by storms at a distance.

Grout; thin mortar, to be poured into the interstices between stones or bricks.

Gudgeons; the metal journals of a horizontal shaft, such as that of a water-wheel. For moderate speeds

$$\left.\begin{matrix}\text{Diam, ins}\\ \text{if of cast-iron}\end{matrix}\right\} = \sqrt{\frac{\text{Weight in lbs on one gudgeon}}{20}}$$

For wrought-iron, add one-twentieth.

Gun-metal, or bronze; a compound of copper and tin, sometimes used for cannon. Also, a quality of cast iron fit for the same purpose.

Gussets; plain triangular pieces of plate iron, riveted by their vertical and horizontal legs to the sides, tops, and bottoms of box-girders, tubular bridges, &c, inside, for strengthening their angles.

Guys; ropes or chains used to prevent anything from swinging or moving about.

Gyrate; to revolve around a central axis, or point.

Halving; to notch together two timbers which cross each other, so deeply that the joint thickness shall equal only that of one whole timber.

Hammer dress; to dress the face of a stone by slight blows of a hammer with a cutting edge. The *patent* hammer for such purposes has several such edges placed parallel to each other, each of which may be removed and replaced at pleasure.

Hand-lever; in an engine, a lever to be worked by hand instead of by steam.

Handspike; a wooden lever for working a capstan or windlass; or other purposes.

Hand-wheel; a wheel used instead of a spanner, wrench, winch, or lever of any kind, for screwing nuts, or for raising weights, or for steering with a rudder, &c.

Hangers, or pendent brackets; fixtures projecting below a ceiling, to support the journals of long lines of shafting; and for other purpose. Should be "self-adjusting."

Hasp; a piece of metal with an opening for folding it over a staple.

Hatchway; a horizontal opening or doorway in a floor, or in the deck of a vessel.

Haunches; the parts of an arch from the keystone to the skewback.

Head-block; a block on which a pillow-block rests.

Header; a stone or brick laid lengthwise at right angles to the face of the masonry.

Heading; in tunnelling, a small driftway or passage excavated in advance of the main body of the tunnel, but forming part of it; for facilitating the work.

Headway; the clear height overhead. Progress.

Heel-post; that on which a lock gate turns on its pivot.

Helve; the handle of an axe.

Hinge; those commonly used on the doors of dwellings are called butts, or butt hinges. (Eye and strap, .) *Rising hinges* are such as cause the door to rise a little as it is opened, and thus cause the door to shut itself.

Hip roof, or *hipped roof;* one that slopes four ways; thus forming angles called hips.

Hoarding; a temporary close fence of boards, placed around a work in progress, to exclude ragglers.

Holding-plates, or *anchors;* strong broad plates of iron sunk into the ground, and generally surrounded by masonry; for resisting the pull of the cables of suspension bridges; and for other similar purposes.

Hook and strap. See Eye and strap.

Horses; the sloping timbers which carry the steps in a staircase.

Housings; in rolling mills, &c, the vertical supports for the boxes in which the journals revolve.

Hub, or *nave;* the central part of a wheel, through which the axletree passes, and from which the spokes radiate.

Impost; the upper part of a pier from which an arch springs.

Ingot; a lump of cast metal, generally somewhat wedge shaped. A pig of cast iron is an ingot.

Invert; an inverted arch frequently built under openings, in order to distribute the pressure more evenly over the foundation.

Jack; a raising instrument, consisting of an iron rack, in connection with a short stout timber which supports it, and worked by cog-wheels and a winch. A *screw-jack* is a large screw working in a strong frame, the base of which serves for it to stand on; and which is caused to revolve and rise, carrying the load on top of it, by turning a nut, or otherwise.

Jack-rafters, or common rafters; small rafters laid on the purlins of a roof, for supporting the shingling laths, &c.

Jag-spike; a spike whose sides are jagged or notched, with the mistaken idea that its holding power is thereby much increased. If a spike or bolt is first put into its place loosely, and then has melted lead run around it, the jagging does assist; but not when it is driven into wood.

Jambs; the sides of an opening through a wall, &c; as door, window, and fireplace jambs.

Jamb-linings; the facing of woodwork with which jambs are covered and hidden.

Jaw; an opening, often V-shaped, the inner edges of which are for holding something in place.

Jettie, or *jetty;* a pier, mound, or mole projecting into the water; as a wharf-pier, &c.

Jib; the upper projecting member or arm of a crane, supported by the stay.

Jig-saw; a very narrow thin saw worked vertically by machinery, and used for sawing curved ornaments in boards.

Joggle; a joint like that at 3 or 4, &c, p 613, of Trusses, for receiving the pressure of a strut at right angles or nearly so. Also applied to squared blocks of stone sometimes inserted between courses of masonry to prevent sliding, &c.

Joist; binding joists are girders for sustaining common joists. The common ones are then called *ridging* joists. *Ceiling* joists are small ones under roof trusses, or under girders, and for sustaining merely the plastered ceiling.

Journal-box; a fixture upon which a journal rests and revolves, instead of a plummer-block.

Journals; the cylindrical supporting ends of a horizontal revolving shaft. Their length is usually about 1 to 1¼ times their diam. In lines of shafting 4 diams. To find the diam, see Gudgeon.

Jumper; a drill used for boring holes in stone by aid of blows of a sledge-hammer.

Kedge; a small anchor.

Keepers; the pieces of metal or wood which keep a sliding bolt in its place, and guide it in sliding.

Kerf; the opening or narrow slit made in sawing.

Key-bolt. See Cotter-bolt.

Keystone; the center stone of an arch.

Kibble; the bucket used for raising earth, stone, &c, from shafts or mines.

King-post, king-rod; the center post, vertical piece, or rod, in a truss; all those on each side of it are queen-posts, or queen-rods. Frequently called simply kings and queens.

Knee; a piece of metal or wood bent at an angle; to serve as a bracket, or as a means of uniting two surfaces which form with each other a similar angle.

Lagging, or *sheeting;* a covering of loose plank; as that placed upon centers, and supporting the archstones. Also, an outer wooden casing to locomotive boilers and others.

Landing; the resting-place at the end of a flight of stairs.

Lantern wheel. See Trundle.

Lap; to place one piece upon another, with the edge of one reaching beyond that of the other.

Lap-welding; welding together pieces that have first been lapped; in distinction to butt-welding.

Lead, (pronounced *leed;*) in steam-engines, a certain amount of opening of the port-valve before each stroke of the piston begins. The distance to which earth is hauled or wheeled.

Leader; a cog-wheel that gives motion to the next one or follower.

Leading-beam; leading-pile; one placed as a guide for placing others.

Leading-wheels; in a locomotive, those frequently placed in front of the driving-wheels.

Leaves; the cogs of pinions.

Ledge; a part projecting over like a shelf; a rock so projecting. A narrow strip of board nailed across other boards, to hold them together, as in temporary ledge-doors.

Lewis; an arrangement composed of 2 or 3 pieces of metal let into a wedge-shaped hole in a block of stone, by which to raise the block.

Lighter; a scow, raft, or other vessel, used for unloading vessels out from the shore.

Linch-pin; a pin near the end of an axle, to hold the wheel on.

Link; one of the divisions of a chain; or a piece shaped like one.

Link-motion; a device for regulating the movement of the main or port valve in a steam-engine.

Lintel; a horizontal beam across an opening in a wall, as seen in windows, doors, &c. When of wide span, and supporting heavy brickwork or masonry, it is called a breast-summer, or bressummer.

Lock; those common door locks which are entirely concealed within the thickness of the door, are called mortice locks; those which are screwed against the face of a door, rim locks. It must be remembered that locks are "right and left."

Louvre; a kind of vertical window, frequently at the tops of roofs of depots, &c, provided with horizontal slats, which permit ventilation, and exclude rain.

Lozenge; the shape of a rhomb; often called diamond-shaped.

Lug; in casting, small projections from the general surface, and for various purposes, such as for lifting the body; or for a flange for joining it to another; or for a support for something else.

Mallet; the wooden hammer used by stonecutters.

Mandrel; an iron rod used as a core around which a flat piece may be bent into a cylindrical shape. Also the shaft that carries the chuck of a lathe.

Manhole; an opening by which a man can enter a boiler, culvert, &c. to clean or repair it.

Mattock; a kind of pick with broad edges for digging.

Maul; a heavy wooden hammer.

Mean, arithmetical: half the sum of two numbers.

" , *geometrical;* the sq rt of the product of two numbers.

Mean-proportional; the same as the geometrical mean.

Meridian; a north and south line. Noon.

Mitre-joint; a joint formed along the diagonal line where the ends of two pieces are united at an angle with each other.

Mitre-sill; the sill against which the lock gates of a canal shut.

Modulus; a datum serving as a means of comparison. Same as *constant* or *coefficient.*

Modulus of elasticity; see p 434. *Modulus of Rupture,* p 485.

Moment; tendency of force acting with leverage. See p 335.

Moment of inertia. See p 486.

Moment of rupture, or *of bending;* the tendency which any load or force exerts to break or bend a body by the aid of leverage. Its amount is found in foot-pounds by multiplying the force in lbs, by the length of leverage in feet between it and that part of the body upon which the tendency is exerted.

Moment of stability. See Art 69, of Force in Rigid Bodies.

Momentum; moving force. See page 310, Force in Rigid Bodies.

Monkey; the hammer or ram of a pile-driver.

Monkey-wrench, or *screw-wrench;* a spanner, the gripping end of which can be adjusted by means of a screw to fit objects of different sizes.

Moorings; fixtures to which ships, &c, can make fast.

Mortise; a hole cut in one piece, for receiving the tenon which projects from another piece.

Muck; soft surface soil containing much vegetable matter.

Muntins, or *mullions:* the vertical pieces which separate the panes in a window-sash.

Nailing-blocks; blocks of wood inserted in walls of stone or brick, for nailing washboards, &c, to.

Nave; the main body of a building, having connecting wings or aisles on each side of it. The hub of a wheel.

Newel; the open space surrounded by a stairway.

Newel-post; a vertical post sometimes used for sustaining the outer ends of steps. Also the large baluster often placed at the foot of a stairway.

Nippers; pincers. An arrangement of two curved arms for catching hold of anything.

Normal; perpendicular to. According to rule, or to correct principles.

Nosing; the slight projection often given to the front edge of the tread of a step; usually rounded.

Nut, or *burr;* the short piece with a central female screw, used on the end of a screw-bolt, &c, for keeping it in place.

Ogee; a moulding in shape of an S. the same as a cima.

Ordinate; a line drawn at right angles from the axis of a curve, and extending to the curve.

Oscillate; to swing backward and forward like a pendulum.

Out of wind, pronounced *wynd;* perfectly straight or flat.

Ovolo; a projecting convex moulding of quarter of a circle; when it is concave it is a cavetto, or hollow.

Packing; the material placed in a stuffing-box, &c, to prevent leaks.

Packing-pieces; short pieces inserted between two others which are to be riveted or bolted together, to prevent their coming in contact with each other.

Pall, or *pawl.* See Ratchet.

Parapet; a wall or any kind of fence or railing to prevent persons from falling off.

Parcel; to wrap canvas or rags round a rope.

Parge; to make the inside of a flue smooth by plastering it.

Patent hammer; a hammer with several parallel sharp edges for dressing stone.

Pay. To cover a surface with tar, pitch, &c. A ship word.

Pay out. To slacken, or let out rope.

Pediment; the triangular space in the face of a wall that is included between the two sloping sides of the roof and a line joining the eaves.

Penstock. See Forebay.

Pier; the support of two adjacent arches. The wall space between windows, &c. A structure built out into the water.

Pierre-perdue; lost stone; *random stone,* or rough stones thrown into the water, and let find their own slope.

Pilaster; a thin flat projection from the face of a wall, as a kind of ornamental substitute for a column.

Pile-planks; planks driven like piles.

Pillow-block, or *plummer-block;* a kind of metal chair or support, upon which the journals of horizontal shafts are generally made to rest, and on which they revolve.

Pinion; a small cog-wheel which gives motion to a larger one.

Pintle; a vertical projecting pin like that often placed at the tops of crane-posts, and over which the holding rings at the tops of the wooden guys fit. Also, such as is used for the hinges of rudders, or of window-shutters to turn around.

Pitch; the slope of a roof, &c. The distance from center to center of the teeth of a cog-wheel, or the threads of a screw. Boiled tar. Also the dist apart of rivets, &c.

Pitman; a connecting-rod for transmitting motion from a prime mover to machinery at a distance, moved by it.

Pit-saw; a large saw worked vertically by two men, one of whom (the pitman) stands in a pit.

Pivot; the lower end of a vertical revolving shaft, whether a part of the shaft itself, or attached to it. It should be flat; and both it and the step or socket upon which it rests should be of hard steel. If a steel pivot has to revolve rapidly and continuously, it is well to proportion its diam, so as not to have to sustain *more* than 250 lbs per sq inch; otherwise it will wear quickly. Dust and grit should for the same reason be carefully guarded against. Pivots which revolve but seldom, and slowly, as those of a railroad turntable, may be trusted with half a ton, or even a whole ton per sq inch. As a rude rule, cast-iron pivots should not be loaded with more than half as much as steel ones. A steel one may be welded to the foot of its cast-iron shaft; or may be inserted part way into it; and the whole strengthened by iron bands shrunk on.

Planish; to polish metals by rubbing with a hard smooth tool.

Plant; the outfit of machinery, &c, necessary for carrying on any kind of work.

Plaster-bead; a small vertical strip of iron or wood nailed along projecting angles in rooms, to protect the plaster at those parts.

Platband; a plain, flat, wide, slightly projecting strip, generally for ornament. When narrow, it is called a fillet.

Pliers; a kind of pincers.

Plinth; the square lowest member of the base of a column or pillar.

Plug; a piece inserted to stop a hole. *Screw-plug*, a plug that is screwed into a hole.

Plumb; vertical.

Plummet, or *plumb-bob;* a weight at the lower end of a string, for testing verticality.

Plunger; a kind of solid piston, or one without a valve.

Point; a kind of pointed chisel for dressing stone. To put a finish to masonry by touching up the outer mortar joints. To dress stone with a point and mallet.

Pole-plate; a longitudinal timber resting on the ends of tie-beams of roofs; and for supporting the feet of the common or jack rafters, when such are used.

Port; the opening or passage controlled by a valve.

Prime; to put on the first coat of paint. Priming also is when water passes into a steam cylinder along with the steam.

Projection. If parallel straight lines be imagined to be drawn in any one given direction, from every point in any surface *s*, whether flat, curved, or irregular, then if all these lines be supposed to be intersected or cut by a plane, either at right angles to their direction, or obliquely, the figure which their cross-section thus made would form upon said plane, is called the projection of the surface *s*. If such lines be supposed to be drawn from a person's face, in a direction in front of him, and to be cut by a plane at right angles to their direction, their projection on the plane would be the person's full-face portrait. If the lines be drawn *sideways* from his face, the projection will be his profile. The projection of a globe upon a flat plane, will evidently be a circle if the plane cuts the lines at right angles; and an ellipse if it cuts them obliquely. Shadows cast by the sun are projections.

Puddle; earth well rammed into a trench, &c, to prevent leaking. A process for converting cast iron into wrought by a puddling furnace.

Pug-mill; a mill for tempering clay for bricks or pottery, &c.

Pulley; a circular hoop which carries a belt in machinery.

Puppet; in machinery, a small short pedestal or stand. *Puppet-valve.* See Valves.

Purlins; the horizontal pieces placed on rafters, for supporting the roof covering.

Put-logs, or *put-locks;* horizontal pieces supporting the floor of a scaffold; one end being inserted into *put-log holes* left for that purpose in the masonry.

Quay; a wharf.

Quoin: the hollow into which a quoin-post of a canal lock-gate fits. Stones, usually dressed, placed along the vertical angles of buildings, chiefly for ornament.

Quoin-post; the vertical post on which a lock-gate turns. The heel-post.

Rabbet, or *rebate;* a half groove along the edge of a board, &c. See 16, p 613, of Trusses, where two rabbets are shown overlapping each other.

Race; the channel which conducts water either to or from a water-wheel: the first is a head race; the last a tail race. The waves produced by the meeting of strong opposing currents; also, a rapid tideway, or roost.

Rack and pinion; the rack is a straight row of cogs on a bar, and called a rack-bar; and the pinion is a small cog-wheel working into it.

Radius of gyration. See Center of gyration. Rad of gyr is $= \sqrt{\frac{\text{moment of inertia}}{\text{weight of body}}}$

Rag-bolt. See Jag-spike.

Rag-wheel, or *Sprocket-wheel;* one with teeth or pins which catch into the links of a chain.

Ram; the hammer of a pile-driver.

Random stone; rip rap, or rough stones thrown promiscuously into the water, to form a foundation, &c.

Rasp; a coarse file.

Ratchet and pall; the former is sometimes a straight bar, at others a wheel: in either case it is furnished with teeth between which the pall drops and prevents backward motion. Used for safety in hoisting machinery, &c. The pall is sometimes called a click.

Ream. A hole, wider at top than at bottom, (see 19, p 613, of Trusses,) through which a screw, bolt, &c, is to be inserted, so that its head shall not project above the general surface, is said to be reamed, or reamed out; or to be countersunk.

Rebate. See Rabbit, above.

Reciprocal of a number is the quotient found by dividing 1 by that number.

Reciprocating motion. See Alternating motion.

Re-entering angle; an angle or corner projecting inward. See Salient, below.

Revetment; steep facing of stone to the sides of a ditch or parapet in fortification. A retaining-wall.

Rib; the curved pieces which form the arches of iron or wooden bridges, &c. Also, those to which the outer planking of a sailing vessel, &c, are fastened.

Ridge of a roof; its peak, or the sharp edge along its very top. Has various similar applications.

Ridge-pole, ridge-piece, or *ridge-plate;* the highest horizontal timber in a roof, extending from top to top of the several pairs of rafters of the trusses; for supporting the heads of the jack-rafters.

Right and left; a lock which in its proper position suits one flap of a pair of folding doors, will not suit if fastened to the other flap; nor even to the same flap if required to open to the right instead of to the left, or vice versa, according to whether it is a right or a left-hand lock. And so with many other things, as, for instance, certain arrangements for working railway switches, &c. Right and left boots and shoes are a familiar illustration; also, right and left screws. Therefore, in ordering several of anything, it is necessary to consider whether they may all be of the same pattern, or whether some must be right-hand, and others left-hand ones.

Right shore of a river; that which is on the right hand when descending the river.

Right-solid body; one which has its axis at right angles to its base; when not so, it is *oblique.*

Ring-bolt; a bolt with an eye and a ring at one end.

Rip-rap. See Random stone.

Roadstead; anchorage at some distance from shore.

Rock-shaft; a shaft which only rocks or makes part of a revolution each way, instead of revolving entirely around.

Rockwork; squared masonry in which the face is left rough to give a rustic appearance.

Rubble; masonry of rough, undressed stones. Scabbled rubble has only the roughest irregularities knocked off by a hammer. Ranged rubble has the stones in each course rudely dressed to nearly a uniform height.

Rundle, or *round;* the step of a ladder.

Rustic; much the same as rockwork.

Saddle; the rollers and fixtures on top of the piers of a suspension bridge, to accommodate expansion and contraction of the cables. The top piece of a stone cornice of a pediment. Has many other applications.

Sag; to bend downward.

Salient; projecting outward. See Re-entering, above.

Sandbag; a bag filled with sand for stopping leaks.

Scabble; to dress off the rougher projections of stones for rubble masonry, with a stone-axe, or scabbling hammer.

Scantling; the depth and breadth of pieces of timber; thus we say, a scantling of 8 by 10 ins, &c.

Scarf; the uniting of two pieces by a long joint, aided by bolts, &c.

Scarp; a steep slope. In fortification the inner slope of a ditch.

Scotia; a receding moulding consisting of a semi-circle or semi-ellipse, or similar figure.

Screeds; long narrow strips of plaster put on horizontally along a wall, and carefully faced out of wind, to serve as guides for afterward plastering the wide intervals between them.

Screw-bolt, a bolt with a screw cut on one end of it.

Screw-jack. See Jack.

Screw-wrench. See wrench.

Scribe; to trim off the edge of a board, &c, so as to make it fit closely at all points, to an irregular surface. The lower edges of an open caisson are scribed to fit the irregularities of a rocky river bottom.

Scroll; an ornamental form consisting of volutes or spirals arranged somewhat in the shape of S.

Scupper nails; nails with broad heads for nailing down canvas, &c.

Scuppers; on shipboard, holes for allowing water to flow off from the deck into the sea.

Scuttle; a small hatchway. To make holes in a vessel to cause sinking.

Sea-wall; a wall built to prevent encroachment of the sea.

Secret nailing; so nailing down a floor by nails along the edges of the boards, that the nail-heads do not show.

Serve; to wrap twine or yarn, &c, closely round a rope to keep it from rubbing.

Set-screw, or *tightening-screw;* a screw for merely pressing one thing tightly against another at will; such as that which confines the movable leg of a pair of dividers in its socket.

Shackle, or *clevis:* a link in a chain shaped like a U, and so arranged that by drawing out a bolt or pin, which fits into two holes at the ends of the U, the chain can be separated at that point.

Shaft; a vertical pit like a well. The body of a column. A large axle.

Shank; the body of a bolt exclusive of its head. The long straight part of many things, as of an anchor, a key, &c.

Shears, or *sheers;* two tall timbers or poles, with their feet some distance apart, and their tops fastened together; and supporting hoisting tackle.

Sheave; a wheel or round block with a groove around its circumference for guiding a rope.

Sheeting, or *sheathing;* covering a surface with boards, sheet iron, felt, &c.

Shingle; the pebbles on a seashore.

Shoes; certain fittings at the ends of pieces; as the pointed iron shoes for piles. The wall shoes into which the lower ends of iron rafters generally fit, &c.

Shore; a prop.

Shot; the edge of a board is said to be shot when it is planed perfectly straight.

Shrink. When an iron hoop or band is first heated, and then at once placed upon the body which it is intended to surround, it shrinks or contracts as it cools, and therefore clasps the body more firmly. This is called *shrinking on* the hoop.

Shuttle; a small gate for admitting water to a water-wheel, or out of a canal lock, &c.

Siding; a short piece of railroad track, parallel to the main one, to serve as a passing-place.

Silt; soft fine mud deposited by rivers, &c.

Siphon culvert; a culvert built in shape of a U, for carrying a stream under an obstacle, and allowing it afterward to rise again to its natural level. The term is improper, inasmuch as the principle of the siphon is not involved.

Skewback; the inclined stone from which an arch springs.

Skids; vertical fenders, on a ship's sides. Two parallel timbers for rolling things upon.

Skirting; narrow boards nailed along a wall, as the washboards in dwellings.

Sledge; a heavy hammer.

Sleeper; any lower or foundation piece in contact with the ground.

Sleeve; a hollow cylinder slid over two pieces to hold them together.

Slide-bars, or *slides;* bars for anything to slide along; as those for the cross-heads of piston-rods, &c. Often called guides.

Slings; pieces of rope or chain to be put around stones, &c, for raising them by.

Slip; the sliding down of the sides of earth-cuts or banks. A long narrow water space or dock between two wharf-piers.

Slope-wall; a wall, generally thin and of rubble stone, used to preserve slopes from the action of water in the banks of canals, rivers, reservoirs, &c; or from the action of rain.

Slot; a long narrow hole cut through anything.

Sluice; a water-channel of wood, masonry, &c; or a mere trench. The flow is usually regulated by a sluice-gate.

Smoke-box; in locomotives, that space in front of the boiler, through which the smoke passes to the chimney.

Snag; a lug with a hole through it, for a bolt.

Socket; a cavity made in one piece for receiving a projection from, or the end of, another piece; as that into which the movable leg of a pair of dividers fits.

Soffit; the lower or underneath surface of an arch, cornice, window, or door-opening, &c.

Solder; a compound of different metals, which when melted is used for uniting pieces of metal also heated. Soft solder is a compound of lead and tin, and is used for uniting lead or tin. There are various hard solders, such as spelter solder, composed of copper and zinc, for uniting iron, copper, or brass.

Sole; that lining around a water-wheel which forms the bottoms of the buckets.

Spandrel; the space, or the masonry, &c, between the back or extrados of an arch and the roadway.

Spanner; a kind of wrench, consisting of a handle or lever with a square eye at one end of it; much used for tightening up the nuts upon screw-bolts, &c. The eye fits over or surrounds the nut.

Spar; a beam; but generally applied to round pieces like masts, &c.

Spelter; zinc.

Spigot; the pin or stopper of a faucet. The smaller end of a common cast-iron water or gas pipe.

Spindle; a thin delicate shaft or axle.

Splay; to widen or flare, like the jambs of a common fireplace, or those of many windows; or like the wing-walls of most culverts.

Splice; to unite two pieces firmly together.

Springer; the lowest stone of an arch.

Sprocket-wheel, or *rag-wheel;* one with teeth or pins which catch in the links of a chain.

Spur-wheel; a common cog-wheel, in which the teeth radiate from a common cen, like those of a spur.

Square; in roofing; 100 square feet.

Square-head; a square termination like that upon which a watch-key fits for winding; or that upon which the eye of the handle of a common grindstone fits for turning it, &c.

Staging; the temporary flooring of a scaffold, platform, &c.

Stanchion; a vertical prop or strut.

Standing-bolt, or *stud-bolt;* a bolt with a screw cut upon each end; one end to be screwed permanently into something, and the other end to hold by means of a nut something else that may be required to be removed at times.

Stand-pipe. See p. 298.

Staple; a kind of double pin in shape of a U; its two sharp points are driven into timber, and curved part is left projecting, to receive a hoop, pin, or hasp, &c.

Starlings; the projecting up and down-stream ends or cutwaters of a bridge pier.

Stay; variously applied to props, struts, and ties, for staying anything or keeping it in place.

Stay-bolts; long bolts placed across the inside of a boiler, &c, to give it greater strength.

Steam-chest; the iron box in locomotive engines and others, through which the steam is admitted to the cylinders.

Steam-pipe; the one which leads steam from a boiler to the steam-chest.

Step; a cavity in a piece for receiving the pivot of an upright shaft; or the end of any upright piece.

Stiles; the flat vertical pieces between and at the sides of the panels in doors, &c.

Stock; the eye with handles for turning it, in which the dies for the cutting of screws are held.

Stove-up, or *stoved*, or *upset;* when a rod of iron is heated at one end, and then hammered endwise so that that part becomes of greater diameter or stouter than the remainder. The heads of bolts are frequently made in one piece with the shank in this way; and the screw ends of long screw-rods are often upset, so that the cutting of the threads of the screw may not reduce the strength of the bar.

Strap; a long thin narrow piece of metal bolted to two bodies to hold them together. A strap-hinge is a strap fastened to a shutter, &c, and having an eye or a pin at one end for fitting it to the other part of the hinge which is attached to the wall.

Stratum; a layer, or bed; as the natural ones in rocks, &c.

Stretcher; a brick, or a block of masonry laid *lengthwise* of a wall. A frame for stretching anything upon.

Stretcher-course; a course of masonry all of stretchers, without any headers.

Strike; an imaginary horizontal line drawn upon the inclined face of a stratum of rocks. Thus, if the slates or shingles on a roof represent inclined strata of rocks, then either the ridge or the eaves of the roof, or any horizontal line between them, will represent their strike. The inclination is called the *dip* of the strata; and the strike is always at right angles to it by compass.

String; variously applied to longitudinal pieces.

String-board; the boarding (often ornamented) at the outer ends of steps in staircases. It hides the *horses*, as the inclined timbers which carry the steps are called.

String-course; a long horizontal course of brick or masonry projecting a little beyond the others; and often introduced for ornament.

Stringer; any longitudinal timber or beam, &c.

Strut; a prop. A piece that sustains compression, whether vertical or inclined.

Strut-tie, or *tie-strut;* a piece adapted to sustain both tension and compression.

Stub-end; a blunt end.

Stud; a short stout projecting pin. A prop. The vertical pieces in a stud partition.

Stud-bolt. See Standing-bolt.

Stuffing-box; a small boxing on the end of a steam cylinder, and surrounding the piston-rod like a collar; or in other positions where a rod is required to move backward and forward, or to revolve, in an opening through any kind of partition, without allowing the escape of steam, air, or water, &c, as the case may be. The box is filled with greased hemp or other packing, which is kept pressed close around the moving rod by means of a top-piece or kind of cover called the *gland*, which may be screwed down more or less tightly upon it at pleasure. The rod passes through the gland also.

Sumpt, or *sump;* a draining well into which rain or other water may be led by little ditches from different parts of a work to which it would do injury.

Surbase; the inside horizontal mouldings just under a window-sill. Also those around the top of a pedestal, or of wainscoting, &c.

Swage, or *swedge;* a kind of hammer, on the face of which is a semi-cylindrical, or other shaped groove or indentation; and which, being held upon a piece of hot iron and struck by a heavy hammer, leaves the shape of the indentation upon the iron.

Switch; the movable tongue or rail by which a train is directed from one track to another.

Swivels; devices for permitting one piece to turn readily in various directions upon another, without danger of entanglement or separation. At 13, p 583, of Trusses, is a tightening swivel; the castors under the legs of heavy furniture are swivelled rollers.

Synclinal axis; in geology, a valley axis, or one toward which the strata of rocks slope downward from opposite directions. The line of the gutter in a valley roof may represent such an axis.

T's; pieces of metal in that shape, whether to serve as straps, or for other purposes. So also with L's, S's, W's, +'s, &c. See figs to Welded Iron Tubes, p 405.

Tackle; a combination of ropes and pulleys.

Talus; the same as batter.

Tamp; to fill up with sand or earth, &c, the remainder of the hole in which the powder has been poured for blasting rock. To compact earth generally, as under cross-ties, &c.

Tap; a kind of screw made of hard steel, and having a square head which may be grasped by a wrench for turning it around, and thus forcing it through a hole around the inside of which it cuts an interior screw. To strike with moderate force. To make an opening in the side of any vessel.

Tappet; a pin or short arm projecting from a revolving shaft; or from an alternating bar, and intended to come into contact with, or tap, something at each revolution or stroke.

Teeth; or cogs of wheels.

Temper; to change the hardness of metals by first heating, and then plunging them into water, oil, &c. To mix mortar, or to prepare clay for bricks, &c.

Templet; the outline of a moulding or other article, cut out of sheet metal or thin wood, to serve as a pattern for stonecutters, carpenters, &c.

Tenon; a projecting tongue fitting into a corresponding cavity called a mortise.

Terra cotta; baked clay. Brick is a coarse kind.

Thimble; an iron ring with its outer face curved into a continuous groove. A rope being doubled around this and tied, the thimble acts as an eye for it, and prevents that part of the rope from wearing. Also, a short piece of tube slid over another piece, or over a rod, &c, to strengthen a joint, &c.

Thread; the continuous spiral projection or worm of a screw.

Through-stone; a stone that extends entirely through a wall.

Throw; the radius, or distance to which a crank "throws out" its arm. Applies in the same way to lathes. Some use it to express the diameter instead of the radius. To avoid mistakes, the terms "single" and "double" throw might be used.

Tie; any piece that sustains tension or pull.

Tie-strut; a piece adapted to sustain either tension or compression.

Tightning-ring. See 14, of Figs 21½, of Trusses.

Tightning-screw. See Set-screw.

Tire; the iron ring placed around the outer circumference of the felloe of a wheel.

Toggle joint. In Fig 16, of Force in page 323, suppose *a m* and *a n* to be two stiff bars, hinged together at *a*. It is plain that if we press downward at *a*, the result will be a great pushing force against any bodies placed at the ends *m* and *n*. Such an arrangement of two bars for producing such pressure is a toggle-joint.

Tongue; a long slightly projecting strip to be inserted into a corresponding groove, as in tongued and grooved floors.

Tooling; dressing stone by means of a *tool* and mallet; the tool being a chisel with a cutting edge of 1 to 2 inches wide. Tooling is generally done in parallel stripes across the stone.

Torus; a projecting semi-circular, or semi-elliptic moulding; often used in the bases of columns. It is the reverse of a scotia.

Trailing-wheels; in a locomotive, those sometimes placed behind the driving-wheels.

Train; a number of cog-wheels working into each other.

Tramway; any two smooth parallel tracks upon which wheels without flanges may run. In rail

tramways the rails themselves have flanges; but in wide stone ones for common vehicles, none are required.

Transom; a beam across the opening for a door, &c. Also, a horizontal piece dividing a high window into two stories, &c, &c. Also, an opening above a door, for ventilation or light.

Tread; the horizontal part of a step.

Treadle; a kind of foot-lever, for turning a lathe, grindstone, &c, by the foot.

Treenail; a long wooden pin.

Trimmer; a short cross-timber framed into two joists so as to sustain the ends of intermediate joists, to prevent the latter from entering a chimney-flue, or interfering with a window, &c.

Trip-hammer, or *tilt-hammer;* a large hammer worked by camb machinery, and used for heavy iron work, especially for hammering irregular masses into the shape of bars, &c.

Truck; a kind of small wagon consisting of a platform on two or more low wheels. Also, those frames and wheels usually placed under railroad cars and engines, and which, by means of a pintle connecting the two, allow them to vibrate or move laterally to some extent independently of each other.

Trundle, lantern-wheel, or *wallower;* used instead of a cog-wheel, and consisting of two parallel circular pieces some distance apart, and united by a central axis, and by cylindrical rods placed around and parallel to the axis, to serve instead of cogs or teeth.

Trunk; a long wooden boxing forming a water channel.

Trunnions; cylindrical projections, as at the sides of a cannon, forming as it were an interrupted axle or shaft for supporting the cannon on its carriage; and allowing it to revolve vertically through some distance.

Tumbler; a kind of spring catch, which at the proper moment falls or tumbles into a notch or hole prepared for it in a piece; thus holding the piece in position until the tumbler is lifted out of the notch.

Tumbling-bay; see "waste-weir."

Tumbling-shaft; in locomotives, a shaft used in the "link motion."

Turnbuckle; variously applied, as to the ordinary fastenings at the outer face of a wall, for holding window-shutters back when opened; also, to the tightening-swivel at 13, page 583, of Trusses, &c.

Turntable; the well-known arrangement for turning locomotives at rest. See page 791.

Undermine; to excavate beneath anything.

Underpin; to add to the height of a wall already constructed, by excavating and building beneath it. Also, to introduce additional support of any kind beneath anything already completed.

Upset. See Stove-up.

Valves; various devices for permitting or stopping at pleasure the flow of water, steam, gas, &c. A SAFETY VALVE is one so balanced as to open of itself when the pressure becomes too great for safety. A SLIDE VALVE is one that slides backward and forward over the opening through which the flow takes place. A BALL VALVE, or spherical valve, is a sphere, which in any position fits the opening. When the pressure below it raises it off from its seat, it is prevented from rolling away by means of a kind of open caging which surrounds it. A CONICAL or PUPPET VALVE is a horizontal slice of a cone, which fits into a corresponding conical seat made in the opening. In rising and falling it is kept in position by a vertical valve-stem or spindle, which passes through its center, and which plays through guide-holes in bridge-pieces placed above and below the valve. A TRAP, CLACK, FLAP, or DOOR VALVE, is a plate with hinges like a door. When two such valves are used, with their hinged edges adjacent to each other, so that in opening and shutting they flap like the wings of a butterfly, they constitute a butterfly valve. A THROTTLE VALVE is one which when closed forms a partition across a pipe; and opens by partially revolving upon an axis placed along its diameter. A ROTARY VALVE works like a common stopcock. A SNIFTING VALVE is one which lets out steam under water; and is so called from the sniffing noise thereby produced. The PORT VALVE is the sliding one which admits steam from the steam-chest into the cylinders. A DOUBLE SEAT, or DOUBLE-BEAT VALVE is a peculiar one with two seats, one above the other: and so arranged that the pressure of steam or water against it when shut, does not oppose its being opened. A CUP VALVE is in shape of an inverted cylindrical cup, with a length somewhat greater than its diameter. Its lower or open edge is ground to fit the seat over which it rests. As this cup rises and falls, it is kept in place by a cylindrical caging closed at top, and having for its sides four or more vertical pieces, against the inner sides of which the sides of the cup play. A CHECK VALVE is any kind so placed as to check or prevent the return of the fluid after its passage through the valve into the pipe or vessel beyond it.

Vault; an arch long in comparison with its span. The space covered by such an arch.

Veneer; a very thin sheet of ornamental wood glued over a more common variety.

Wainscot; a wooden facing to walls in rooms, instead of plaster, or over a facing of plaster; usually not more than 3 or 4 feet high above the floor.

Wales; long longitudinal timbers in the sides of a ship, coffer-dam, caisson, &c.

Wallow; a water-wheel, &c, is said to wallow when it does not revolve evenly on its journals.

Wallower. See Trundle.

Wall-plate, or *raising-plate;* a timber laid along the tops of walls for the roof trusses or rafters to rest on, so as to distribute their weight more equally upon the wall.

Warped; twisted, as a board, or the face of a stone, &c, which is not perfectly flat. *To warp;* to haul a vessel ahead by means of an anchor dropped some distance ahead. To flood an extent of ground with water for a short time to increase its fertility.

Washboards; boards nailed around the walls of rooms at the floor, so as to prevent injury to the plaster when washing the floors.

Washers; broad pieces of metal surrounding a bolt, and placed between the faces of the timber through which the bolt passes, and the head and nut of the bolt, so as to distribute the pressure over a larger surface, and prevent the timber from being crushed when the bolt is tightly screwed up.

Waste-weir; an overfall provided along a canal, &c, at which the water may discharge itself in case of becoming too high by rain, &c. Sometimes called a tumbling-bay.

Watch-tackle; ropes running in different directions from a boat, and used in bringing it into a desired position.

Water-shed; the sloping ground from which rain-water descends into a stream.

Water-table; a slight projection of the lower masonry or brickwork on the outside of a wall, an reaching to a few feet above the ground surface, as a partial protection against rain, or as ornamen

Ways; the inclined timbers along which a vessel glides when being launched.

Weather-boards; boards used instead of bricks or masonry for the outsides of a building, or bridge &c. They are nailed to vertical and inclined indoor timbers; and may be either vertical or hor When hor, they are so placed that the lower edge of one overlaps the upper edge of the one below When vert, their edges should be tongued and grooved; and narrow slips be nailed over the vert joints to keep out rain, &c.

Weir, or *wier;* a dam, or an overfall.

Weld; to join two pieces of metal together by first softening them by heat, and then hammering them in contact with each other. In this operation fluxes are used.

Welt; see "butt-joint."

Wharf; a level space upon which vessels lying along its sides can discharge their cargoes; or from which they can receive them.

Wheel-base; the distance from center to center from the extreme front wheels, to the extreme hin ones in a locomotive, car, &c.

Wicket; a small door or gate made in a larger one; as the shuttle or valve in a lock-gate, for letting out the water.

Winch; a handle bent at right angles, and used for turning an axis; that of a common grindstone

Wind. See Out of wind.

Winders; those steps (often triangular) in a staircase by which we wind, or turn angles.

Windlass; the wheel and axle, or winch and drum, as often used in common wells. Also, a horizontal shaft on shipboard, by which the anchor is raised; the windlass being revolved by means of wooden levers called *handspikes.*

Wing-dam; a projection carried out part way across a shallow stream, so as to force all the water to flow deeper through the channel thus contracted.

Wings; applied in many ways to projections. The flanges which radiate out from a gudgeon; and by which it is fastened to the shaft. Small buildings projecting from a main one. The wings or flaring wing-walls of a culvert or bridge.

Wing-walls; the retaining-walls which flare out from the ends of bridges, culverts, &c.

Wiper. See Camb.

Working-beam, or *walking-beam;* a beam vibrating vertically on a rock-shaft at its center, as seen in some steam-engines; one end of it having a connection with the piston-rod; and the other end with a crank, or with a pump-rod, &c.

Worm; the so-called endless screw, which by revolving without advancing gives motion to a cog-wheel (worm-wheel), the teeth of which catch in the thread of the screw.

Wrench; a long handle having at one end an eye or jaw which may catch hold of anything to be twisted or turned around, as a screw-nut, &c. When it has a jaw which by means of a screw is adaptable to nuts, &c, of different sizes, it is a monkey-wrench, or screw-wrench.

INDEX.

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